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# Decentralized event-triggered $H_\infty$ control for neural networks subject to cyber-attacks

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## ABSTRACT

This paper addresses the problem of decentralized event-triggered  $H_\infty$  control for neural networks subject to limited network-bandwidth and cyber-attacks. In order to alleviate the network transmission burden, a decentralized event-triggered scheme is employed to determine whether the sensor measurements should be sent out or not. Each sensor can decide the transmitted sensor measurements locally according to the corresponding event-triggered condition. It is assumed that the network transmissions may be modified by the occurrence of the random cyber-attacks. A Bernoulli distributed variable is employed to reflect the success ration of the launched cyber-attacks. The Lyapunov method is employed to derive a sufficient condition such that the closed-loop system is asymptotically stable and achieves the prescribed  $H_\infty$  level. Moreover, the desired  $H_\infty$  controller gains are derived provided that the sufficient condition is satisfied. Finally, illustrative examples are utilized to show the usefulness of the obtained results.

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## 1. Introduction

Due to the successful applications in signal processing, combinatorial pattern recognition and so on, neural networks have been an hot topic and received much attention in the last few decades [29,34,36]. An important issue on an unstable neural network is to design a suitable controller to stabilize it. Up to date, a number of notable results on this issue are available in the literature. To mention a few, network-based  $H_\infty$  quantized control is considered in [14] for a class of neural networks; The finite stabilization problem for delayed neural networks is investigated in [30]; and the guaranteed cost control is concerned with [31] for Markov jump neural networks with distributed delay, fading channel and event-triggered scheme.

In many network-based industrial systems, transmission resources such as communication network-bandwidth and sensor energy are limited [15,20,23,32,35,37]. It is significant to execute only necessary transmission tasks so that limited communication resources can be reasonably used. Recently, increasing attention has been paid to designing proper transmission strategies in order to save the precious network resources [7,8,11,19,28,38]. There are a number of methods available in the literature [9,10,17,39,40]. A useful one is called a time-triggered mechanism, under which the signal transmission

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is executed in a periodic way. This mechanism definitely produces unnecessary or redundant signals to be transmitted, leading to a congested network traffic. As a result, system performance is degraded. To overcome the drawback of the time-triggered mechanism, several event-triggered schemes are proposed to stop those unnecessary signals transmitted, and thus the quality of service of a communication network is improved. For example, in [33], an event-triggered scheme is proposed, which is dependent on the sampled-data error between the current sampled state and the latest transmitted one. In [41], an event-triggered communication scheme is introduced to design output tracking controllers for networked T-S fuzzy system. Asynchronous event-triggered control is considered in [12] for decentralized networked systems.

The security problems in networked control systems have attracted much attention in the control community [6,16,22], due to the fact that large amount of data need to be transferred through networked communication channel which is exposed to cyber-attacks. The cyber-attacks aim to exploit the vulnerabilities in communication links and send incorrect control actions to the operators in control centers. Recently, some efficient methods have been proposed to defend against cyber-attacks and make perfect attacks impossible. For instance, in [2], the deception attacks are considered in dealing with consensus via an event-triggered control scheme; The coordinated cyber-attacks in hybrid state estimation are discussed in [1]. In [24], a resilient event-triggered communication scheme is introduced for a multi-area power system with bandwidth-limited communication channel; and  $H_\infty$  filter design for neural networks with a hybrid triggered scheme and deception attacks is investigated in [18]. However, for the stabilization problem of a neural network, few results have been reported to investigate the cyber-attacks when a decentralized event-triggered scheme is employed. This motivates the current study.

Based on the above discussions, this paper aims to investigate the problem of event-triggered  $H_\infty$  control for neural networks subject to cyber-attacks. The main contributions of this paper include: (1) A closed-loop system model is proposed by considering the limited network resources and randomly occurring cyber-attacks; (2) A decentralized event-triggered-scheme is employed to reduce the utilization of the network resources. With this scheme, all transmitted nodes needn't to be synchronous and each sensor can determine its transmitted signals locally; (3) The randomly occurring cyber-attacks, governed by a Bernoulli distributed variable, are modeled as a nonlinear function satisfying a restraining condition. (4) A new effective control method is presented to stabilize the studied system.

Notation:  $\mathbb{R}^n$  and  $\mathbb{R}^{n \times m}$  denote the  $n$ -dimensional Euclidean space and the set of  $n \times m$  real matrices;  $\text{Prob}\{X\}$  stands for the occurrence probability of event  $X$ ;  $\mathcal{E}$  denotes the expectation operator; the superscript  $T$  stands for matrix transposition; the notation  $X > 0$ , for  $X \in \mathbb{R}^{n \times n}$  means that the matrix  $X$  is real symmetric positive definite;  $I$  is the identity matrix of appropriate dimension; for a matrix  $B$  and two symmetric matrices  $A$  and  $C$ ,  $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$  denotes a symmetric matrix, where  $*$  denotes the entries implied by symmetry.

## 2. Preliminaries

In this paper, the neural network model addressed is described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Eg(x(t - \eta(t))) + D\omega(t) \\ z(t) = Cx(t) \end{cases} \quad (1)$$

where  $x(t) = [x_1(t) \ x_2(t) \ \dots \ x_n(t)]^T$  is the state variable with  $n$  neurons;  $u(t) \in \mathbb{R}^m$  is the control input; and  $z(t)$  is the controlled output variable;  $g(x(t)) = [g_1(x_1(t)) \ g_2(x_2(t)) \ \dots \ g_n(x_n(t))]^T$  denotes the neuron activation function;  $\eta(t)$  represents the time-varying delay satisfying  $0 \leq \eta(t) \leq \eta_M$ , where  $\eta_M$  is a constant;  $A = \text{diag}\{a_1, a_2, \dots, a_n\}$  is a diagonal matrix;  $B, E, D$  and  $C$  are known real matrices with appropriate dimensions.

Throughout this paper, the neuron active function  $g(x)$  is assumed to satisfy the following assumption:

**Assumption 1.** For  $i \in \{1, 2, \dots, n\}$ , the activation function  $g_i(x)$  satisfies  $g_i(0) = 0$  and  $\forall s_1 \neq s_2$

$$\phi_{gi}^- \leq \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \leq \phi_{gi}^+ \quad (2)$$

where  $\phi_{gi}^-$  and  $\phi_{gi}^+$  are known constants.

**Assumption 2.** The communication network delay at sampled instant  $t_k^i$  in the  $i$ th sensor is denoted by  $\tau_{t_k^i}$ . It is assumed that  $0 < \tau_{t_k^i} < \bar{\tau}^i$ ,  $\bar{\tau} = \max_{i \in \{1, 2, \dots, n\}} \{\bar{\tau}^i\}$ .

The diagram of the controlled neural network is shown in Fig. 1. The sensors and the corresponding controllers exchange information via a communication network subject to time-delay and cyber-attacks. Particularly, with consideration of the limited network-bandwidth, the event-generators are employed at each sensor side to reduce the unnecessary network transmissions. The newly sampled signals are carried to the corresponding event generators directly. Only the sampled signals, which violate the event-triggered condition, can gain access to the network transmission. It is assumed that the cyber-attacks occur randomly which aim to make the communication signals dishonest.

The objective of this paper is to stabilize the neuron states based on the neural network (1). Considering the limited network resources and the randomly occurring cyber-attacks, the closed-loop system model will be established step by step in the following.

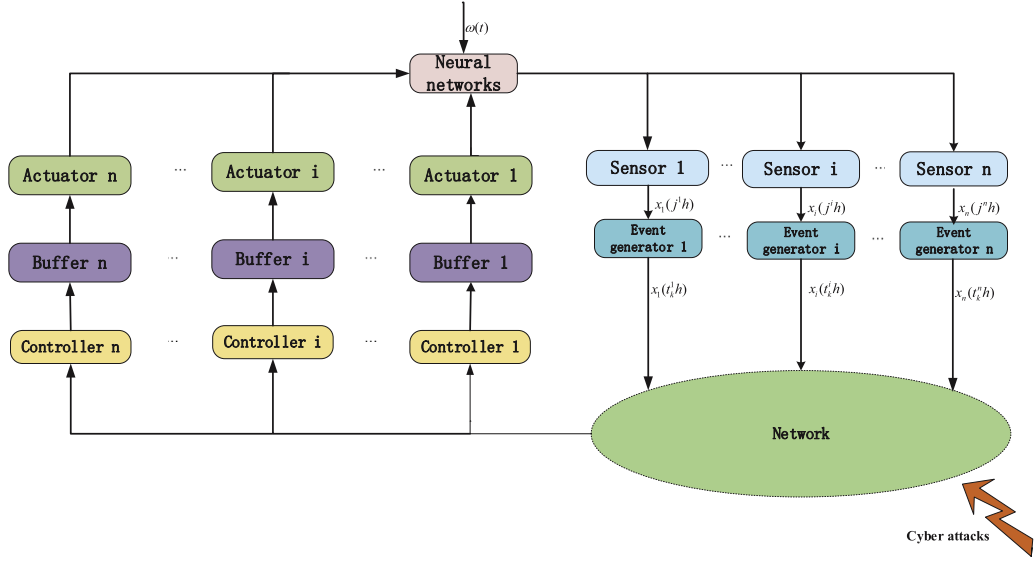


Fig. 1. The structure of decentralized event-triggered neural networks with cyber-attacks.

In order to avoid redundant network transmissions, a decentralized event-triggered scheme is introduced to determine whether local sampled measurements in each sensor should be sent out to the communication network or not. The event-triggered condition in the  $i$ th sensor is predefined as follows:

$$e_i^T(t)\Omega_i e_i(t) < \sigma_i x_i^T(t_k^i h + j^i h)\Omega_i x_i(t_k^i h + j^i h) \quad (3)$$

where  $\sigma_i \in [0, 1)$ ,  $\Omega_i > 0$  is a weighting matrix,  $i \in \{1, 2, \dots, n\}$ ,  $h$  is the sampling interval,  $j^i h$  is the sampling instant in the  $i$ th sensor,  $e_i(t) = x_i(t_k^i h) - x_i(t_k^i h + j^i h)$ ,  $x_i(t_k^i h)$  and  $x_i(t_k^i h + j^i h)$  represent the latest transmitted signal and current sampled signal, respectively.

**Remark 1.** In the  $i$ th event generator, the latest released instant is  $t_k^i h$ , the current sampled instant is  $t_k^i h + j^i h$ . It should be noted that  $\{t_1^i, t_2^i, \dots\} \subseteq \{h, 2h, \dots, j^i h, \dots\}$ ,  $i \in \{1, 2, \dots, n\}$ .

The holding interval  $[t_k^i h, t_{k+1}^i h)$  of ZOH can be partitioned into several subsets  $\bigcup_{j^i=0}^{j_M^i} \Upsilon_{j^i}$ ,  $\Upsilon_{j^i} = [t_k^i h + j^i h + \tau_{t_k^i + j^i}, t_k^i h + j^i h + \tau_{t_k^i + j^i + 1})$ ,  $j^i = 0, 1, \dots, j_M^i$ ,  $j_M^i = t_{k+1}^i - t_k^i - 1$ .

Similar to [26], at the actuator side, we set a series of buffers which aim to store the controller outputs with time-stamps. The actuators can update the controlled inputs by selecting the controller outputs with the same time-stamp from the buffers. Thus, the set of update time  $t_k h$  for the input of the actuators is as follows:  $t_{k+1} h = t_k h + j h$ ,  $j h = \operatorname{argmin}_{i \in \{1, 2, \dots, n\}} \{j^i h\}$  which can be obtained from (3).

Define  $\tau(t) = t - t_k h - j h$ , it is easy to know that  $0 \leq \tau_{t_k} \leq \tau(t) \leq \tau_M$ ,  $\tau_M = h + \tau_{t_k + j + 1}$ , based on (3), the following inequalities can be derived for  $n$  channels:

$$e^T(t)\Omega e(t) < \sigma x^T(t - \tau(t))\Omega x(t - \tau(t)) \quad (4)$$

where  $e(t) = x(t_k h) - x(t_{k+1} h + j h)$ ,  $\sigma = \operatorname{diag}\{\sigma_1, \sigma_2, \dots, \sigma_n\}$ ,  $\Omega = \operatorname{diag}\{\Omega_1, \Omega_2, \dots, \Omega_n\}$ ,

In this paper, we assume the data is transmitted via a vulnerable communication network, which can be an objective of a hacker to alter the transmitted information. Considering the limited access to the network resources for a hacker, the controller input can be destroyed or modified by the malicious cyber-attacks randomly. A new model of controller design method reflecting the decentralized event-triggered scheme and cyber-attacks is proposed as follows:

$$u(t) = \alpha(t_k) K x(t_k h) + (1 - \alpha(t_k)) K f(x(t - d(t))), \quad (5)$$

where  $\alpha(t_k) \in \{0, 1\}$  with the following statistical properties:  $\operatorname{Prob}\{\alpha(t_k) = 1\} = \bar{\alpha}$ ,  $\operatorname{Prob}\{\alpha(t_k) = 0\} = 1 - \bar{\alpha}$ ,  $K$  is the controller gain to be designed later,  $f(x(t - d(t)))$  is the function of cyber-attacks,  $f(x(t)) = [f_1(x_1(t)) \quad f_2(x_2(t)) \quad \dots \quad f_n(x_n(t))]^T$ ,  $d(t) \in (0, d_M)$ ,  $d_M$  is a positive constant.

**Remark 2.** Notice that the communication link between event generators and controllers is vulnerable to cyber-attacks. The real triggered measurements may be attacked by the randomly occurring cyber-attacks when transmitted through the communication network. Thus, the operator in the control center maybe send out wrong control actions.

Combining (1) and (5), we obtain

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \alpha(t_k)BKx(t_k h) + (1 - \alpha(t_k))BKf(x(t - d(t))) + Eg(x(t - \eta(t))) + D\omega(t), \\ t &\in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) \end{aligned} \tag{6}$$

Recalling the definition of  $\tau(t)$  and the characteristics of  $\alpha(t_k)$ , (6) can be written as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \tilde{\alpha}BK[x(t - \tau(t)) + e(t)] + (1 - \tilde{\alpha})BKf(x(t - d(t))) + Eg(x(t - \eta(t))) \\ &\quad + D\omega(t) + (\alpha(t_k) - \tilde{\alpha})BK[x(t - \tau(t)) + e(t) - f(x(t - d(t)))], \\ t &\in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) \end{aligned} \tag{7}$$

**Remark 3.** In this paper, the success of the launched cyber-attack is assumed to obey Bernoulli distribution.  $\alpha(t_k) = 1$  means the real sensor measurements are received by the controllers,  $\alpha(t_k) = 0$  means the sensor measurements which have access to the communication network are attacked.

The following assumption and lemmas are presented which is useful in deriving our main results.

**Assumption 3.** The cyber-attack function  $f_i(x)$  satisfies  $f_i(0) = 0$  and  $\forall i \in \{1, 2, \dots, n\}, s_1 \neq s_2$

$$\phi_{fi}^- \leq \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \leq \phi_{fi}^+ \tag{8}$$

where  $\phi_{fi}^-$  and  $\phi_{fi}^+$  are known constants.

**Lemma 1.** [25] Assume  $\tau(t) \in [0, \bar{\tau}]$ , for any matrices  $X \in \mathbb{R}^{n \times n}$  and  $U \in \mathbb{R}^{n \times n}$  that satisfy  $\begin{bmatrix} X & U \\ U^T & X \end{bmatrix} \geq 0$ , the following inequality holds:

$$-\bar{\tau} \int_{t-\bar{\tau}}^t \dot{x}^T(s)X\dot{x}(s) \leq \begin{bmatrix} x(t) \\ x(t - \tau(t)) \\ x(t - \bar{\tau}) \end{bmatrix}^T \begin{bmatrix} -X & * & * \\ X^T - U^T & -2X + U + U^T & * \\ U^T & X^T - U^T & -X \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \tau(t)) \\ x(t - \bar{\tau}) \end{bmatrix} \tag{9}$$

**Lemma 2.** [14] For a full column rank matrix  $M \in \mathbb{R}^{n \times m}$ , the singular decomposition is  $M = U\Sigma V^T$ , in which  $U$  and  $V$  are orthogonal matrices,  $\Sigma \in \mathbb{R}^{m \times n}$  is a rectangular diagonal matrix with positive real numbers. Let  $S$  is matrix of the form  $S = U\text{diag}\{R_1, R_2\}U^T$ , then, there exists  $X \in \mathbb{R}^{m \times m}$  such that  $SM = MX$ .

**Lemma 3.** [21] For  $x(t)$  and  $g(x(t))$  in Assumption 1, and  $x(t)$  and  $f(x(t))$  in Assumption 3, one has the following inequalities for positive semi-definite diagonal matrices  $U$  and  $V$

$$\begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix}^T \begin{bmatrix} -U\Phi_g^- & U\Phi_g^+ \\ \Phi_g^+U & -U \end{bmatrix} \begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix} \geq 0 \tag{10}$$

$$\begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix}^T \begin{bmatrix} -V\Phi_f^- & V\Phi_f^+ \\ \Phi_f^+V & -V \end{bmatrix} \begin{bmatrix} x(t) \\ f(x(t)) \end{bmatrix} \geq 0 \tag{11}$$

where

$$\begin{aligned} \Phi_g^- &= \text{diag}\{\phi_{g1}^-\phi_{g1}^+, \phi_{g2}^-\phi_{g2}^+, \dots, \phi_{gn}^-\phi_{gn}^+\}, \Phi_g^+ = \text{diag}\left\{\frac{\phi_{g1}^- + \phi_{g1}^+}{2}, \frac{\phi_{g2}^- + \phi_{g2}^+}{2}, \dots, \frac{\phi_{gn}^- + \phi_{gn}^+}{2}\right\} \\ \Phi_h^- &= \text{diag}\{\phi_{h1}^-\phi_{h1}^+, \phi_{h2}^-\phi_{h2}^+, \dots, \phi_{hn}^-\phi_{hn}^+\}, \Phi_h^+ = \text{diag}\left\{\frac{\phi_{h1}^- + \phi_{h1}^+}{2}, \frac{\phi_{h2}^- + \phi_{h2}^+}{2}, \dots, \frac{\phi_{hn}^- + \phi_{hn}^+}{2}\right\} \end{aligned}$$

### 3. Main results

The following Theorem 1 presents a sufficient condition to ensure the asymptotical stability of the closed-loop system (7).

**Theorem 1.** Let the following parameters  $\eta_M, \tau_M, d_M, \tilde{\alpha}, \gamma, \varepsilon_r, r = 1, 2, 3, \sigma$  and matrix  $K$  be given, the closed-loop system (7) is asymptotically stable if there exist positive matrices  $P, Q_1, Q_2, Q_3, R_1, R_2, R_3$ , and appropriate dimensioned matrices  $M_l (l = 1, 2, 3)$ , diagonal matrices  $\Omega, U$  and  $V$  such that

$$\begin{bmatrix} \Pi_{11} & * & * \\ \Pi_{21} & \mathcal{R} & * \\ \Pi_{31} & 0 & \mathcal{R} \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} R_l & * \\ M_l & R_l \end{bmatrix} > 0, l = 1, 2, 3 \tag{13}$$

where

$$\begin{aligned} \Pi_{11} &= \begin{bmatrix} \Sigma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}, \Sigma_{11} = \begin{bmatrix} \Gamma_1 & * & * \\ R_1 - M_1 & -2R_1 + M_1 + M_1^T - U\Phi_g^- & * \\ M_1 & R_1 - M_1 & -Q_1 - R_1 \end{bmatrix}, \\ \Gamma_1 &= PA + A^T P + Q_1 + Q_2 + Q_3 - R_1 - R_2 - R_3 + C^T C, \\ \Sigma_{21} &= \begin{bmatrix} \tilde{\alpha}K^T B^T P + R_2 - M_2 & 0 & 0 \\ M_2 & 0 & 0 \\ R_3 - M_3 & 0 & 0 \\ M_3 & 0 & 0 \end{bmatrix}, \Sigma_{22} = \begin{bmatrix} \Gamma_2 & * & * & * \\ R_2 - M_2 & -Q_2 - R_2 & * & * \\ 0 & 0 & \Gamma_3 & * \\ 0 & 0 & R_3 - M_3 & -Q_3 - R_3 \end{bmatrix}, \\ \Gamma_2 &= -2R_2 + M_2 + M_2^T + \sigma\Omega, \Gamma_3 = -2R_3 + M_3 + M_3^T - V\Phi_f^-, \\ \Sigma_{31} &= \begin{bmatrix} E^T P & \Phi_g^+ U & 0 \\ (1 - \tilde{\alpha})K^T B^T P & 0 & 0 \\ \alpha K^T B^T P & 0 & 0 \\ D^T P & 0 & 0 \end{bmatrix}, \Sigma_{32} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \Phi_f^+ V & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \Sigma_{33} &= \text{diag}\{-U, -V, -\Omega, \gamma^2 I\}, \Pi_{21} = [\eta_M \Gamma_4^T \quad \tau_M \Gamma_4^T \quad d_M \Gamma_4^T]^T, \\ \Pi_{31} &= [\eta_M \Gamma_5^T \quad \tau_M \Gamma_5^T \quad d_M \Gamma_5^T]^T, \Gamma_4 = [\Gamma_{41} \quad \Gamma_{42}] \\ \Gamma_5 &= [0_{1 \times 3} \quad \delta PBK \quad 0_{1 \times 4} \quad -\delta PBK \quad \delta PBK \quad 0], \delta = \sqrt{\tilde{\alpha}(1 - \tilde{\alpha})} \\ \Gamma_{41} &= [PA \quad 0 \quad 0 \quad \tilde{\alpha}PBK \quad 0 \quad 0 \quad 0], \Gamma_{42} = [PE \quad (1 - \tilde{\alpha})PBK \quad \tilde{\alpha}PBK \quad PD] \\ \mathcal{R} &= \text{diag}\{-PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\} \end{aligned}$$

**Proof.** Choose the following Lyapunov–Krasovskii functional for the closed-loop system (7):

$$\begin{aligned} V(x(t)) &= x^T(t)Px(t) + \int_{t-\eta_M}^t x^T(s)R_1x(s)ds + \int_{t-\tau_M}^t x^T(s)R_2x(s)ds + \int_{t-d_M}^t x^T(s)R_3x(s)ds \\ &\quad + \eta_M \int_{t-\eta_M}^t \int_s^t \dot{x}^T(v)R_1\dot{x}(v)dvds + \tau_M \int_{t-\tau_M}^t \int_s^t \dot{x}^T(v)R_2\dot{x}(v)dvds + d_M \int_{t-d_M}^t \int_s^t \dot{x}^T(v)R_3\dot{x}(v)dvds \end{aligned} \tag{14}$$

Taking mathematical expectation of the derivative of  $V(x(t))$ , we have

$$\begin{aligned} \mathbb{E}\{\dot{V}(x(t))\} &= 2x^T(t)Px(t) + x^T(t)(R_1 + R_2 + R_3)x(t) - x^T(t - \eta_M)R_1x(t - \eta_M) \\ &\quad - x^T(t - \tau_M)R_2x(t - \tau_M) - x^T(t - d_M)R_3x(t - d_M) + \mathbb{E}\{\dot{x}^T(t)\mathfrak{R}\dot{x}(t)\} \\ &\quad - \eta_M \int_{t-\eta_M}^t \dot{x}^T(s)R_1\dot{x}(s)ds - \tau_M \int_{t-\tau_M}^t \dot{x}^T(s)R_2\dot{x}(s)ds - d_M \int_{t-d_M}^t \dot{x}^T(s)R_3\dot{x}(s)ds \end{aligned} \tag{15}$$

where  $\mathfrak{R} = \eta_M^2 R_1 + \tau_M^2 R_2 + d_M^2 R_3$ .

By Lemma 1, the following inequalities hold if there exist  $M_l (l = 1, 2, 3)$  satisfying (13):

$$-\eta_M \int_{t-\eta_M}^t \dot{x}^T(s)R_1\dot{x}(s)ds \leq \xi_1^T(t)\Psi_1\xi_1(t) \tag{16}$$

$$-\tau_M \int_{t-\tau_M}^t \dot{x}^T(s)R_2\dot{x}(s)ds \leq \xi_2^T(t)\Psi_2\xi_2(t) \tag{17}$$

$$-d_M \int_{t-d_M}^t \dot{x}^T(s)R_3\dot{x}(s)ds \leq \xi_3^T(t)\Psi_3\xi_3(t) \tag{18}$$

where

$$\xi_1^T(t) = [x^T(t) \quad x^T(t - \eta(t)) \quad x^T(t - \eta_M)]$$

$$\xi_2^T(t) = [x^T(t) \quad x^T(t - \tau(t)) \quad x^T(t - \tau_M)]$$

$$\xi_3^T(t) = [x^T(t) \quad x^T(t - d(t)) \quad x^T(t - d_M)]$$

$$\Psi_l = \begin{bmatrix} -R_l & * & * \\ R_l - M_l & -2R_l + M_l + M_l^T & * \\ M_l & R_l - M_l & -R_l \end{bmatrix}, \quad l = 1, 2, 3$$

Noting that

$$\mathbb{E}\{\dot{x}^T(t)\mathfrak{R}\dot{x}(t)\} = \mathcal{A}^T \mathfrak{R} \mathcal{A} + \tilde{\alpha}(1 - \tilde{\alpha})B^T \mathfrak{R} B \tag{19}$$

in which  $\mathcal{A} = Ax(t) + \tilde{\alpha}BK[x(t - \tau(t)) + e(t)] + (1 - \tilde{\alpha})BKf(x(t - d(t))) + Eg(x(t - \eta(t))) + D\omega(t)$ ,  $\mathcal{B} = BK[x(t - \tau(t)) + e(t) - f(x(t - d(t)))]$

From Lemma 3, it is not difficult to know that there exist  $U$  and  $V$  such that

$$\begin{bmatrix} x(t - \eta(t)) \\ g(x(t - \eta(t))) \end{bmatrix}^T \begin{bmatrix} -U\Phi_g^- & U\Phi_g^+ \\ \Phi_g^+U & -U \end{bmatrix} \begin{bmatrix} x(t - \eta(t)) \\ g(x(t - \eta(t))) \end{bmatrix} \geq 0 \tag{20}$$

$$\begin{bmatrix} x(t - d(t)) \\ f(x(t - d(t))) \end{bmatrix}^T \begin{bmatrix} -V\Phi_f^- & V\Phi_f^+ \\ \Phi_f^+V & -V \end{bmatrix} \begin{bmatrix} x(t - d(t)) \\ f(x(t - d(t))) \end{bmatrix} \geq 0 \tag{21}$$

Combining (15)–(21) and (4), then it implies that

$$\begin{aligned} \mathbb{E}\{\dot{V}(x(t))\} &\leq 2x^T(t)Px(t) + x^T(t)(Q_1 + Q_2 + Q_3)x(t) - x^T(t - \eta_M)Q_1x(t - \eta_M) \\ &\quad - x^T(t - \tau_M)Q_2x(t - \tau_M) - x^T(t - d_M)Q_3x(t - d_M) + \xi_1^T(t)\Psi_1\xi_1(t) \\ &\quad + \xi_2^T(t)\Psi_2\xi_2(t) + \xi_3^T(t)\Psi_3\xi_3(t) + \mathcal{A}^T\mathfrak{R}\mathcal{A} + \tilde{\alpha}(1 - \tilde{\alpha})\mathcal{B}^T\mathfrak{R}\mathcal{B} \\ &\quad + \begin{bmatrix} x(t - \eta(t)) \\ g(x(t - \eta(t))) \end{bmatrix}^T \begin{bmatrix} -U\Phi_g^- & U\Phi_g^+ \\ \Phi_g^+U & -U \end{bmatrix} \begin{bmatrix} x(t - \eta(t)) \\ g(x(t - \eta(t))) \end{bmatrix} \\ &\quad + \begin{bmatrix} x(t - d(t)) \\ f(x(t - d(t))) \end{bmatrix}^T \begin{bmatrix} -V\Phi_f^- & V\Phi_f^+ \\ \Phi_f^+V & -V \end{bmatrix} \begin{bmatrix} x(t - d(t)) \\ f(x(t - d(t))) \end{bmatrix} \\ &\quad - e^T(t)\Omega e(t) + \sigma x^T(t - \tau(t))\Omega x(t - \tau(t)) \\ &= \xi^T(t)\Pi_{11}\xi(t) + \mathcal{A}^T\mathfrak{R}\mathcal{A} + \tilde{\alpha}(1 - \tilde{\alpha})\mathcal{B}^T\mathfrak{R}\mathcal{B} \end{aligned} \tag{22}$$

where

$$\begin{aligned} \xi^T(t) &= [\xi_1^T(t) \quad x^T(t - \tau(t)) \quad x^T(t - \tau_M) \quad x^T(t - d(t)) \quad x^T(t - d_M) \quad \xi_4(t)], \\ \xi_4(t) &= [g^T(x(t - \eta(t))) \quad f^T(x(t - d(t))) \quad e^T(t) \quad \omega^T(t)] \end{aligned}$$

By the Schur complement, we can derive that (12) is equivalent to  $\mathbb{E}\{\dot{V}(x(t))\} < \gamma^2\omega^T(t)\omega(t) - z^T(t)z(t)$ . This completes the proof.  $\square$

Based on Theorem 1, we are now ready to present the controller design approach for the closed-loop system (7).

**Assumption 4.** In order to tackle with the nonlinear terms in Theorem 1, similar to [14],  $B$  is assumed to be full column rank, the singular decomposition for  $B$  can be denoted by  $B = M \begin{bmatrix} B_0 \\ 0 \end{bmatrix} N$ .

**Theorem 2.** Let the following parameters  $\eta_M, \tau_M, d_M, \tilde{\alpha}, \gamma, \varepsilon_r, r = 1, 2, 3$  and  $\sigma$  be given, the closed-loop system (7) is asymptotically stable if there exist positive matrices  $P, Q_1, Q_2, Q_3, R_1, R_2, R_3$ , and appropriate dimensioned matrices  $M_l (l = 1, 2, 3)$ , diagonal matrices  $\Omega, U, V$  and a matrix  $Y$  such that

$$\begin{bmatrix} \tilde{\Pi}_{11} & * & * \\ \tilde{\Pi}_{21} & \tilde{\mathcal{R}} & * \\ \tilde{\Pi}_{31} & 0 & \tilde{\mathcal{R}} \end{bmatrix} < 0 \tag{23}$$

$$\begin{bmatrix} R_l & * \\ M_l & R_l \end{bmatrix} > 0, l = 1, 2, 3 \tag{24}$$

where

$$\tilde{\Pi}_{11} = \begin{bmatrix} \Sigma_{11} & * & * \\ \tilde{\Sigma}_{21} & \Sigma_{22} & * \\ \tilde{\Sigma}_{31} & \Sigma_{32} & \Sigma_{33} \end{bmatrix}, \tilde{\Sigma}_{21} = \begin{bmatrix} \tilde{\alpha}Y^TB^T + R_2 - M_2 & 0 & 0 \\ M_2 & 0 & 0 \\ R_3 - M_3 & 0 & 0 \\ M_3 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Sigma}_{31} = \begin{bmatrix} E^TP & \Phi_g^+U & 0 \\ (1 - \tilde{\alpha})Y^TB^T & 0 & 0 \\ \tilde{\alpha}Y^TB^T & 0 & 0 \\ D^TP & 0 & 0 \end{bmatrix}, \tilde{\Pi}_{21} = [\eta_M\tilde{\Gamma}_4^T \quad \tau_M\tilde{\Gamma}_4^T \quad d_M\tilde{\Gamma}_4^T]^T,$$

$$\tilde{\Pi}_{31} = [\eta_M\tilde{\Gamma}_5^T \quad \tau_M\tilde{\Gamma}_5^T \quad d_M\tilde{\Gamma}_5^T]^T, \tilde{\Gamma}_4 = [\tilde{\Gamma}_{41} \quad \tilde{\Gamma}_{42}]$$

$$\tilde{\Gamma}_5 = [0_{1 \times 3} \quad \delta BY \quad 0_{1 \times 4} \quad -\delta BY \quad \delta BY \quad 0]$$

$$\tilde{\Gamma}_{41} = [PA \quad 0 \quad 0 \quad \tilde{\alpha}BY \quad 0 \quad 0 \quad 0], \tilde{\Gamma}_{42} = [PE \quad (1 - \tilde{\alpha})BY \quad \tilde{\alpha}BY \quad PD]$$

$$\bar{\mathcal{R}} = \text{diag}\{-2\varepsilon_1 P + \varepsilon_1^2 R_1, -2\varepsilon_2 P + \varepsilon_2^2 R_2, -2\varepsilon_3 P + \varepsilon_3^2 R_3\}$$

where  $P = M \text{diag}\{P_1, P_2\} M^T$ ,  $P_1 \in \mathbb{R}^{m \times m}$ ,  $P_2 \in \mathbb{R}^{(n-m) \times (n-m)}$ . Other symbols have been defined in [Theorem 1](#). The controller gain can be designed as

$$K = P_0^{-1} Y, P_0 = (B_0 N)^{-1} P_1 B_0 N \quad (25)$$

in which  $B_0$  and  $N$  can be seen in [Assumption 4](#).

**Proof.** Since  $P = M \text{diag}\{P_1, P_2\} M^T$  and the singular decomposition  $B = M \begin{bmatrix} B_0 \\ 0 \end{bmatrix} N$  in [Assumption 4](#), according to [Lemma 2](#), there exists a new variable  $P_0$  satisfying  $PB = BP_0$ , from

$$M \text{diag}\{P_1, P_2\} M^T M \begin{bmatrix} B_0 \\ 0 \end{bmatrix} N = M \begin{bmatrix} B_0 \\ 0 \end{bmatrix} N P_0 \quad (26)$$

we get

$$P_0 = (B_0 N)^{-1} P_1 B_0 N \quad (27)$$

Substitute  $PBK$  with  $BP_0K$ , and define  $Y = P_0K$ , it follows from [\(12\)](#) that

$$\begin{bmatrix} \bar{\Pi}_{11} & * & * \\ \bar{\Pi}_{21} & \mathcal{R} & * \\ \bar{\Pi}_{31} & 0 & \mathcal{R} \end{bmatrix} < 0 \quad (28)$$

In terms of the following inequality:

$$(R_k - \varepsilon_k^{-1} P) R_k^{-1} (R_k - \varepsilon_k^{-1} P) \geq 0, (k = 1, 2, 3) \quad (29)$$

One has that

$$-PR_k^{-1}P \leq -2\varepsilon_k P + \varepsilon_k^2 R_k \quad (30)$$

Replace  $-PR_k^{-1}P$  with  $-2\varepsilon_k P + \varepsilon_k^2 R_k$  in [\(28\)](#), [\(23\)](#) can be obtained. This completes the proof.  $\square$

**Remark 4.** It should be pointed out that the event-triggered condition in this paper is obviously different from the ones in [\[3–5,13\]](#). The event-triggered control schemes in [\[3–5,13\]](#) make a decision by the receiving information including the measurement of agent  $i$  and its neighbors. However, the event-triggered condition in this paper only requires the local sensor own measurements.

**Remark 5.** It should be pointed out that the term  $PBK$  is nonlinear in [Theorem 1](#) when designing the controller for the discussed closed-loop system. To overcome the difficulty, motivated by the work in [\[14\]](#), the structure of  $P$  is  $P = M \text{diag}\{P_1, P_2\} M^T$ , the singular value decomposition of  $B$  is denoted by  $B = M \begin{bmatrix} B_0 \\ 0 \end{bmatrix} N$ . From [Lemma 2](#),  $PB = BP_0$ ,  $P_0$  is a new variable. By this method,  $P_0K$  can be defined as a new matrix variable, then the controller design problem is turned into the feasibility of inequalities [\(23\)](#) and [\(24\)](#). Moreover, the controller gain can be derived from [\(25\)](#).

**Remark 6.** [Theorem 2](#) is derived based on a normal Lyapunov–Krasovskii functional (LKF) in [\(14\)](#) and Jensen integral inequality. If we choose some proper augmented LKF [\[42,43\]](#) and employ the Bessel–Legendre inequality [\[27\]](#), it is expected to derive some less conservative results.

When the neural network is under the decentralized event-triggered scheme, and the network is without cyber-attacks, the closed-loop system [\(7\)](#) reduces to the following forms:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + BK[x(t - \tau(t)) + e(t)] + Eg(x(t - \eta(t))) + D\omega(t), \\ t &\in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1}) \end{aligned} \quad (31)$$

Similar to the proof in [Theorem 2](#), we can obtain the following Corollary.

**Corollary 1.** For given  $\eta_M, \tau_M, \gamma, \varepsilon_r, r = 1, 2$  and  $\sigma$ , the closed-loop system [\(31\)](#) is asymptotically stable if there exist positive matrices  $P, Q_1, Q_2, R_1, R_2$ , and appropriate dimensioned matrices  $M_l (l = 1, 2)$ , diagonal matrices  $\Omega, U$ , and a matrix  $Y$  such that

$$\begin{bmatrix} \hat{\Pi}_{11} & * & * \\ \hat{\Pi}_{21} & \hat{\Pi}_{22} & * \\ \hat{\Pi}_{31} & 0 & \hat{\mathcal{R}} \end{bmatrix} < 0 \quad (32)$$

$$\begin{bmatrix} R_l & * \\ M_l & R_l \end{bmatrix} > 0, l = 1, 2 \quad (33)$$

where

$$\hat{\Pi}_{11} = \begin{bmatrix} \hat{\Gamma}_1 & * & * \\ R_1 - M_1 & -2R_1 + M_1 + M_1^T - U\Phi_g^- & * \\ M_1 & R_1 - M_1 & * \\ Y^T B^T + R_2 - M_2 & 0 & -Q_1 - R_1 \\ & & 0 & \Gamma_2 \end{bmatrix},$$

$$\hat{\Gamma}_1 = PA + A^T P + Q_1 + Q_2 - R_1 - R_2 + C^T C,$$

$$\hat{\Pi}_{21} = \begin{bmatrix} M_2 & 0 & 0 & R_2 - M_2 \\ E^T P & \Phi_g^+ U & 0 & 0 \\ Y^T B^T & 0 & 0 & 0 \\ D^T P & 0 & 0 & 0 \end{bmatrix},$$

$$\hat{\Pi}_{22} = \text{diag}\{-Q_2 - R_2, -U, -\Omega, -\gamma^2 I\},$$

$$\hat{\Pi}_{31} = \begin{bmatrix} \eta_M PA & 0 & 0 & \eta_M BY \\ \tau_M PA & 0 & 0 & \tau_M BY \end{bmatrix}^T, \hat{\Pi}_{32} = \begin{bmatrix} 0 & \eta_M PE & \eta_M BY & \eta_M PD \\ 0 & \tau_M PE & \tau_M BY & \tau_M PD \end{bmatrix}$$

$$\hat{R} = \text{diag}\{-2\varepsilon_1 P + \varepsilon_1^2 R_1, -2\varepsilon_2 P + \varepsilon_2^2 R_2\}$$

where  $P = M \text{diag}\{P_1, P_2\} M^T$ ,  $P_1 \in \mathbb{R}^{m \times m}$ ,  $P_2 \in \mathbb{R}^{(n-m) \times (n-m)}$ ,  $\Gamma_2$  has been defined in Theorem 1. The controller gain can be designed as

$$K = P_0^{-1} Y, P_0 = (B_0 N)^{-1} P_1 B_0 N \tag{34}$$

in which  $B_0$  and  $N$  can be seen in Assumption 4.

### 4. Numerical examples

In this section, two simulation examples are provided to illustrate the effectiveness of the developed approach in the previous section.

**Example 1.** The corresponding parameters of system (1) are given as follows:

$$A = \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & -0.1 \end{bmatrix}, B = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.1 \end{bmatrix}, E = \begin{bmatrix} 0.1 & 0.2 & 0 \\ -0.1 & 0.2 & 0.1 \\ -0.2 & 0 & -0.3 \end{bmatrix}, C = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}, g(x) = \begin{bmatrix} \tanh(0.03x_1(t)) \\ \tanh(0.06x_2(t)) \\ \tanh(0.03x_3(t)) \end{bmatrix}, \omega(t) = \begin{cases} 1, & 5 \leq t \leq 10 \\ 0, & \text{else} \\ -1, & 15 \leq t \leq 20 \end{cases}$$

The cyber-attack function is assumed to be

$$f(x) = \begin{bmatrix} \tanh(0.04x_1(t)) \\ \tanh(0.04x_2(t)) \\ \tanh(0.04x_3(t)) \end{bmatrix}$$

It is easy to obtain that  $\Phi_g^- = \text{diag}\{0, 0, 0\}$ ,  $\Phi_g^+ = \text{diag}\{0.015, 0.03, 0.015\}$ ,  $\Phi_f^- = \text{diag}\{0, 0, 0\}$ ,  $\Phi_f^+ = \text{diag}\{0.02, 0.02, 0.02\}$ .

In the following, we will consider two cases for neural network. In Case 1, the event-triggered scheme and the cyber-attacks are both considered in neural network. In Case 2, the event-triggered scheme is employed in neural network without cyber-attacks.

**Case 1.** Suppose that  $\bar{\alpha} = 0.8$ ,  $\eta_M = 0.5$ ,  $d_M = 0.5$ ,  $\tau_M = 0.5$ ,  $\gamma = 1$ ,  $\sigma = 0.1$ ,  $h = 0.1$ ,  $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$ , by employing the MATLAB LMI Toolbox, from the LMI constraints (23) and (24) in Theorem 2, we can obtain the following feasible solutions:

$$P = \begin{bmatrix} 13.4444 & -2.1657 & -2.3122 \\ -2.1657 & 8.4437 & -1.1200 \\ -2.3122 & -1.1200 & 11.3531 \end{bmatrix}, Y = [-4.5193 \quad -20.8923 \quad -3.6645],$$

$$G = \begin{bmatrix} 9.9045 & 0 & 0 \\ 0 & 8.1964 & 0 \\ 0 & 0 & 1.8943 \end{bmatrix}, U = \begin{bmatrix} 18.1332 & 0 & 0 \\ 0 & 14.7018 & 0 \\ 0 & 0 & 8.9410 \end{bmatrix},$$

$$V = \begin{bmatrix} 12.3354 & 0 & 0 \\ 0 & 14.3981 & 0 \\ 0 & 0 & 13.3825 \end{bmatrix}$$

Applying (25) in Theorem 2, we derive the corresponding control gain

$$K = [-0.6645 \quad -3.0720 \quad -0.5388]$$



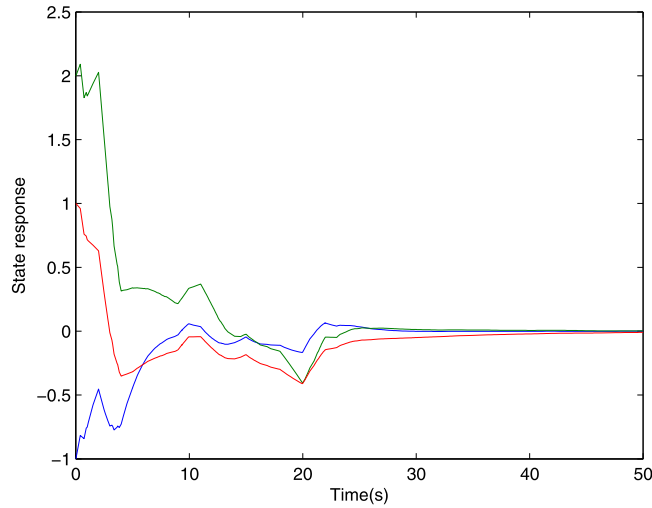


Fig. 2. State response (Case 1).

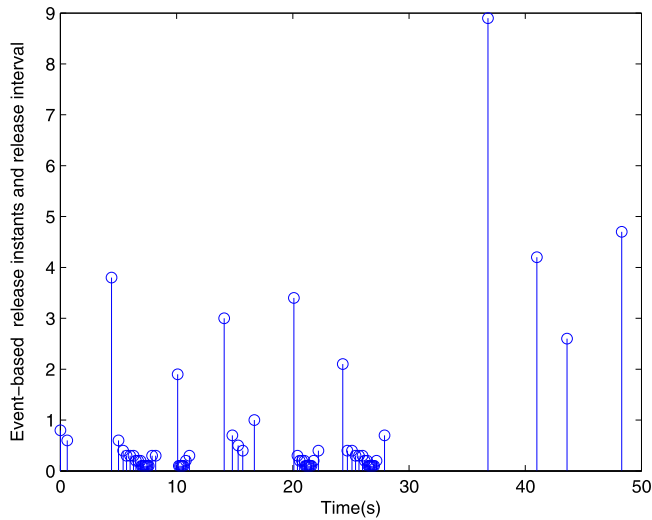


Fig. 3. Release instants and intervals on sensor node 1(Case 1).

The state trajectories of the controlled neural network is shown in Fig. 2, from which we can see that the closed-loop system converges to zero asymptotically. Figs. 3–5 present the release instants and the corresponding release intervals on sensor node 1, sensor node 2 and sensor node 3, respectively. The transmitted sensor measurements are 66, 51 and 49 on sensor node 1, sensor node 2 and sensor node 3, respectively, which means the average transmission rate is 11.07%.

**Case 2.** When the neural network is under event-triggered scheme but without cyber-attacks, set  $\eta_M = 0.6$ ,  $\tau_M = 0.6$ ,  $\gamma = 1$ ,  $\sigma = 0.2$ ,  $h = 0.1$ ,  $\varepsilon_1 = \varepsilon_2 = 1$ . Based on the MATLAB LMI Toolbox, from Corollary 1, we can get the controller gain and the triggering matrix as follows:

$$K = \begin{bmatrix} -0.3708 & -2.1362 & -0.0501 \end{bmatrix}, G = \begin{bmatrix} 5.9709 & 0 & 0 \\ 0 & 3.9676 & 0 \\ 0 & 0 & 0.2388 \end{bmatrix},$$

The state response is shown in Fig. 6. Figs. 7–9 present release instants and intervals on sensor nodes 1, 2, and 3, respectively, where the average transmission rate is 11.4% of the total 1500 sampled measurements.

From the simulation results in the above two cases, it can be found that the stability of the controlled system can be guaranteed under the designed control approach, and the decentralized event-triggered scheme can reduce data transmission frequency compared with time-triggered scheme, numerous unnecessary sampled measurements are discarded, which alleviates the transmission load greatly.

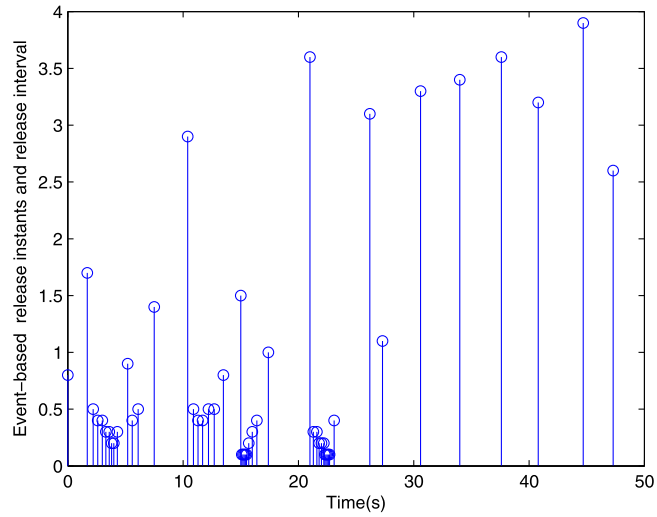


Fig. 4. Release instants and intervals on sensor node 2 (Case 1).

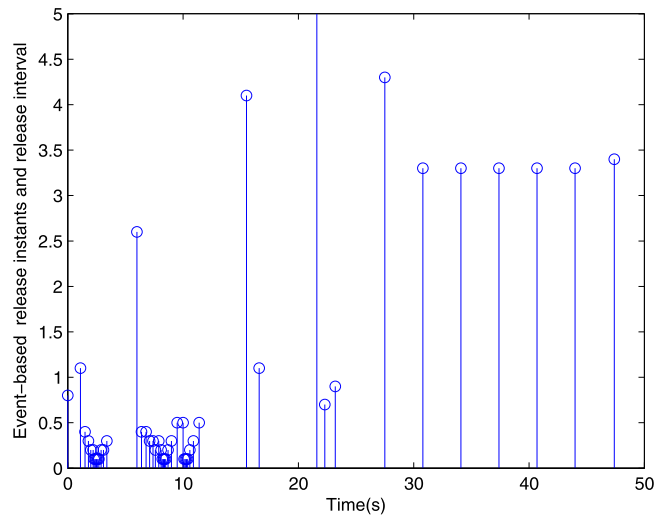


Fig. 5. Release instants and intervals on sensor node 3 (Case 1).

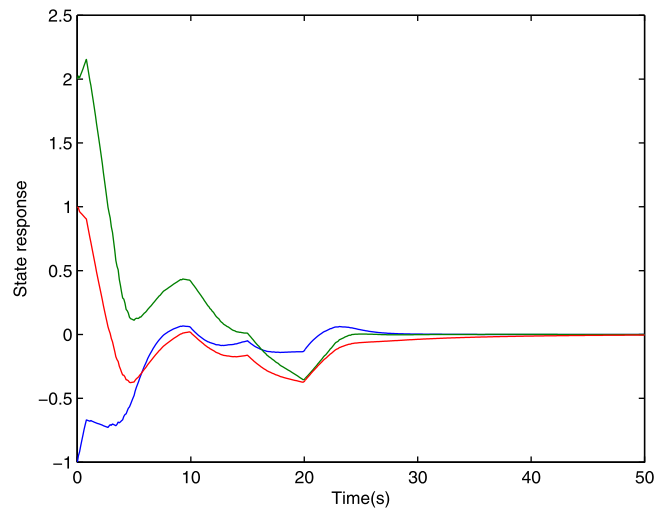


Fig. 6. State response (Case 2).

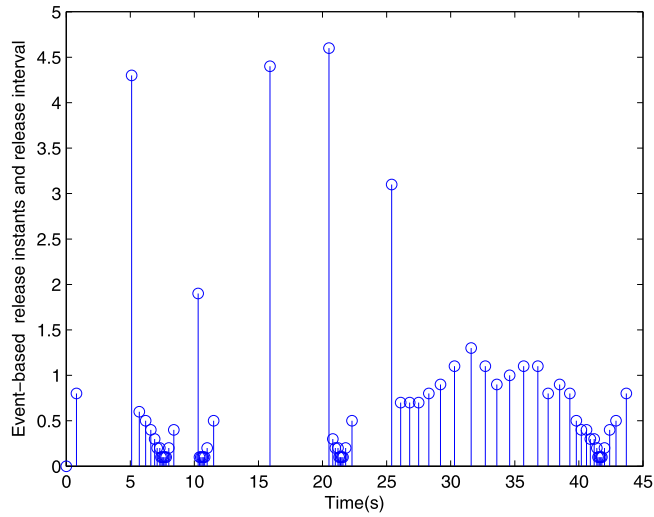


Fig. 7. Release instants and intervals on sensor node 1 (Case 2).

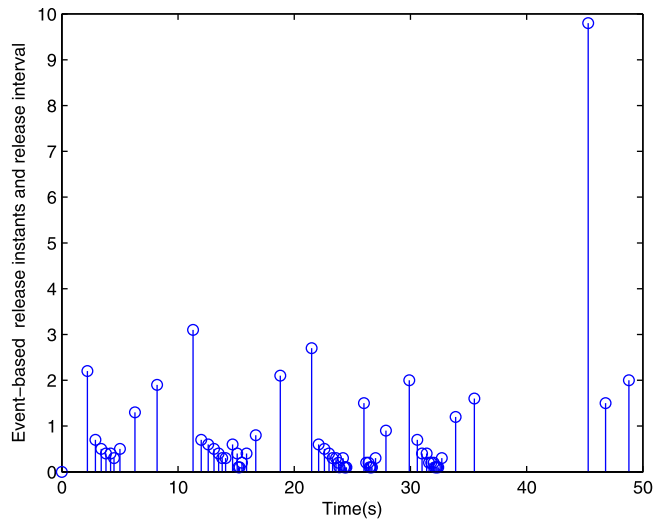


Fig. 8. Release instants and intervals on sensor node 2 (Case 2).

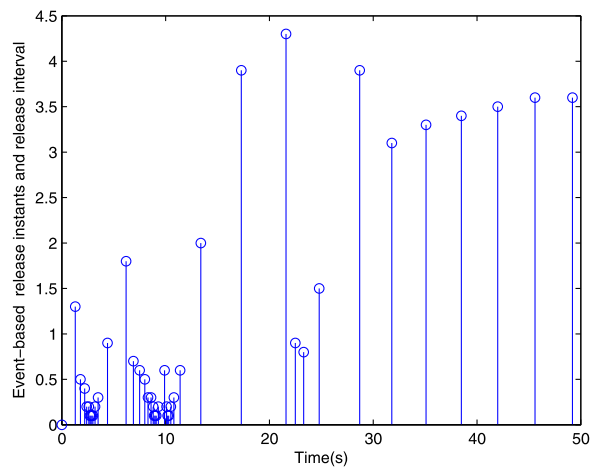


Fig. 9. Release instants and intervals on sensor node 3 (Case 2).

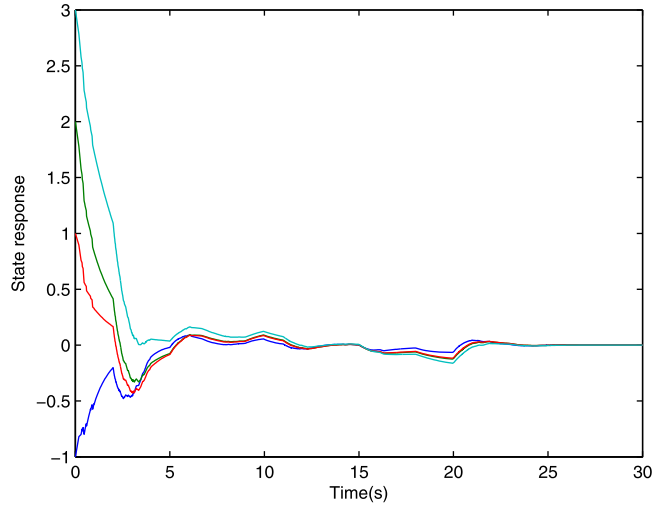


Fig. 10. State response (Example 2).

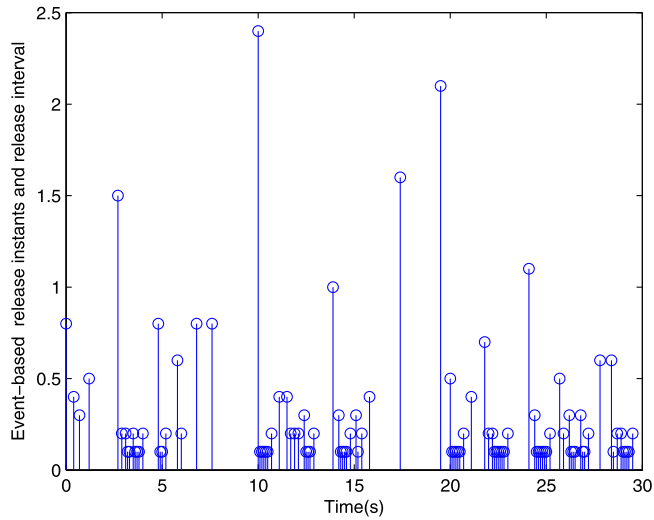


Fig. 11. Release instants and intervals on sensor node 1 (Example 2).

**Example 2.** Consider system (1) with

$$A = \text{diag}\{-1.2769, -0.6231, -0.9230, -0.4480\}, \quad B = [1 \quad 1 \quad 1 \quad 1]^T,$$

$$E = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix}, \quad C = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.1 \\ 0.1 \\ 0.1 \\ 0.1 \end{bmatrix}$$

$$g(x) = \begin{bmatrix} \tanh(0.03x_1(t)) \\ \tanh(0.06x_2(t)) \\ \tanh(0.03x_3(t)) \\ \tanh(0.03x_3(t)) \end{bmatrix}, \quad \omega(t) = \begin{cases} 1, & 5 \leq t \leq 10 \\ 0, & \text{else} \\ -1, & 15 \leq t \leq 20 \end{cases}$$

The cyber-attack function is

$$f(x) = \begin{bmatrix} \tanh(0.04x_1(t)) \\ \tanh(0.04x_2(t)) \\ \tanh(0.04x_3(t)) \\ \tanh(0.04x_3(t)) \end{bmatrix}$$

Clearly,  $\Phi_g^- = \text{diag}\{0, 0, 0, 0\}$ ,  $\Phi_g^+ = \text{diag}\{0.015, 0.03, 0.015, 0.015\}$ ,  $\Phi_f^- = \text{diag}\{0, 0, 0, 0\}$ ,  $\Phi_f^+ = \text{diag}\{0.02, 0.02, 0.02, 0.02\}$ .

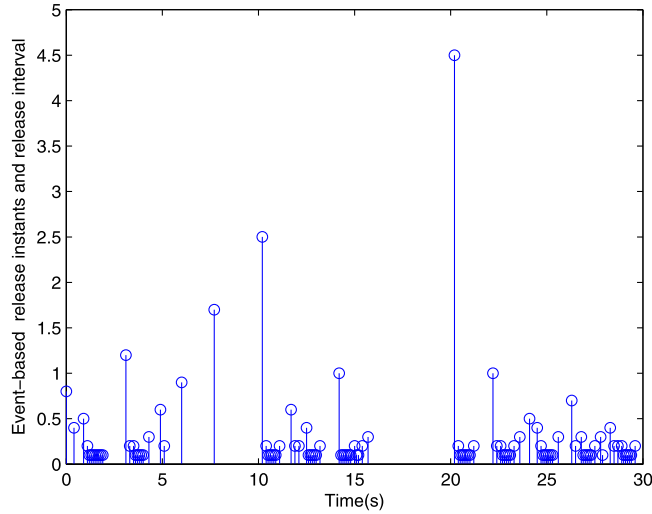


Fig. 12. Release instants and intervals on sensor node 2 (Example 2).

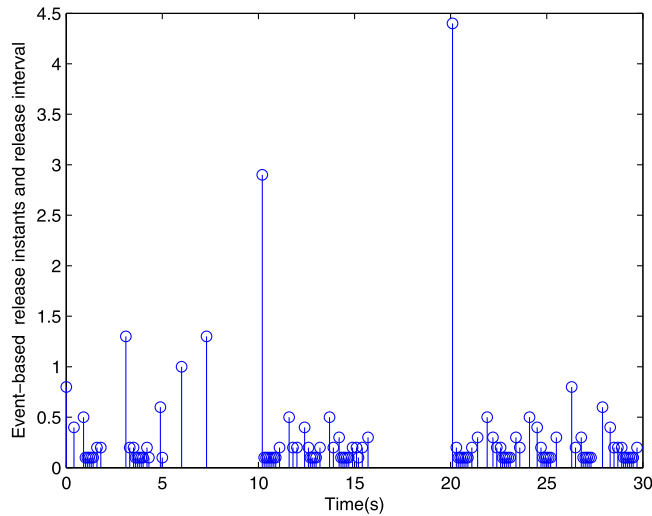


Fig. 13. Release instants and intervals on sensor node 3 (Example 2).

Choose  $\bar{\alpha} = 0.8$ ,  $\eta_M = 0.4$ ,  $d_M = 0.4$ ,  $\tau_M = 0.3$ ,  $\gamma = 1$ ,  $\sigma = 0.1$ ,  $h = 0.1$ ,  $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$ . Applying Theorem 2, the corresponding controller gain matrix and the trigger matrix are

$$K = \begin{bmatrix} -0.0625 & -0.5065 & -0.2758 & -0.3406 \end{bmatrix}, G = \begin{bmatrix} 9.8267 & 0 & 0 & 0 \\ 0 & 22.6337 & 0 & 0 \\ 0 & 0 & 11.8197 & 0 \\ 0 & 0 & 0 & 7.4354 \end{bmatrix}$$

and the transmitted sensor measurements are 99, 105, 106 and 99, respectively. That is the average transmission rate is 34.08% of the total 1200 sampled measurements. The release instants and the corresponding release intervals of sensor nodes 1, 2, 3, 4 are shown in Figs. 11–14. Fig. 10 depicts the response of the state. From the simulation results, it can be seen that the average release intervals decrease and the derived control method and triggering matrix can stabilize the neural networks with limited network-bandwidth and cyber-attacks.

### 5. Conclusions

In this paper, the event-triggered  $H_\infty$  control problem for neural network with limited network-bandwidth and cyber-attacks is investigated. A decentralized event-triggered scheme is employed to avoid unnecessary transmissions, which can reduce the controller updates. The randomly occurring cyber-attacks aiming to reduce the network reliability is modeled as

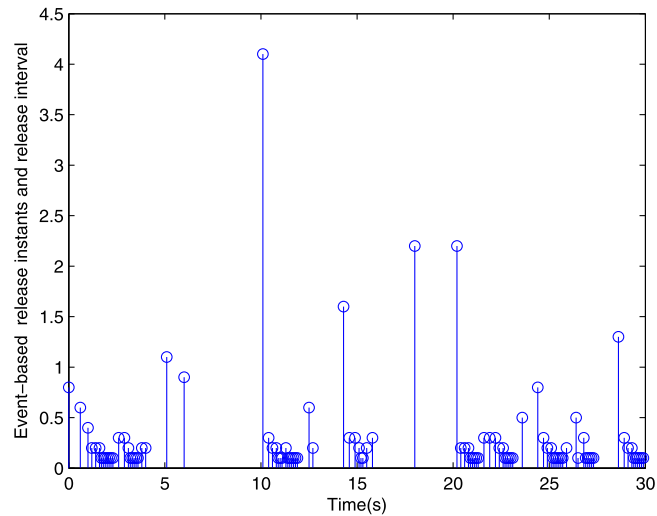


Fig. 14. Release instants and intervals on sensor node 4 (Example 2).

a nonlinear function. Desired controllers have been designed to guarantee the closed-loop system asymptotically stable. The usefulness of the obtained results has been illustrated by simulation results.

## Acknowledgments

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