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Research Article

Event-based finite-time state estimation for Markovian jump systems with quantizations and randomly occurring nonlinear perturbations

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This paper is concerned with finite-time state estimation for Markovian jump systems with quantizations and randomly occurring nonlinearities under event-triggered scheme. The event triggered scheme and the quantization effects are used to reduce the data transmission and ease the network bandwidth burden. The randomly occurring nonlinearities are taken into account, which are governed by a Bernoulli distributed stochastic sequence. Based on stochastic analysis and linear matrix inequality techniques, sufficient conditions of stochastic finite-time boundedness and stochastic H_∞ finite-time boundedness are firstly derived for the existence of the desired estimator. Then, the explicit expression of the gain of the desired estimator are developed in terms of a set of linear matrix inequalities. Finally, a numerical example is employed to demonstrate the usefulness of the theoretical results.

1. Introduction

As an important class of stochastic hybrid systems, Markovian jump systems (MJSs) have received an extensive attention [1–5], in which the modes can switch from one to another at different time in structure and parameters. MJSs are used to describe the phenomena of random abrupt variation, possibly caused by the component failures, sudden environmental disturbances, changes in the interconnections of subsystems, and so forth. Some important results in systems and control theory have been reported [6–9]. Particularly, delay-dependent H_∞ filtering is investigated in [6] for singular Markovian jump time-delay systems. State estimation problem for discrete Markovian jumping neural networks with time delay is discussed in [7]. The authors in [8] address the problem of H_∞ control for networked MJSs under event-triggered scheme. The authors in [9] investigate the problem of robust H_∞ control for MJSs with partially known transition probabilities and nonlinearities.

As we know, the bandwidth of the transport network is limited, it is essential to develop appropriate transmission strategies to reduce the bandwidth utilization of the transport network. Recently, much attention has been paid to deal with the issue [10–17]. The following two methods are usually applied to reduce the communication burden: (i) the first is quantization strategy, which aims to reduce the size of the data. For example, in [10], the authors investigate the H_∞ filtering of continuous Markov jump linear systems with general transition probabilities and output quantization. The authors in [11] are concerned with the stability analysis of networked control systems with dynamic quantization, variable sampling intervals and communication delays. (ii) the second one is event-triggered scheme. The recently proposed event-triggered scheme has been proved to be an effective method to reduce the data transmission frequency in the network. The key idea is that whether or not the current sampled data will be sent out is judged by an event generator with a pre-specified threshold, and those unselected data is discarded without any further processing. Many results with regard to event-triggered control are reported, to be specific, in [12], the authors are concerned with event-triggered H_∞ controller design for networked control systems. The reliable control design is investigated in [13] for networked control system under event-triggered scheme. The authors in [14] investigate decentralized control for a class of interconnected system under decentralized state-dependent triggering scheme. The authors in [15] propose a discrete event-triggered communication scheme for a class of networked Takagi-Sugeno fuzzy systems. However, when considering the two methods at the same time, up to mow, no results about state estimation for MJSs can be found, which is the first motivation of this study.

It is well known that most of the researches focuse on the asymptotical or exponential stability of closed loop systems over an infinite time. However, in practical applications, it is usually expected that the system state does not exceed certain specified

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bounds in a given finite time interval, for example, large values of the system states are not acceptable in the presence of saturations. It is necessary to deal with transient behavior [18,19]. Finite-time stability is introduced in [20] to ensure the system trajectories within given bounds in a given finite time interval. Based on the concept of finite-time stability, the authors in [21] investigate the non-fragile and robust finite-time H_∞ control problem for a class of uncertain Markovian jump nonlinear systems with bounded parametric uncertainties and norm-bounded disturbance. The problem of finite-time H_∞ static output feedback control of Markov jump systems is considered in [22]. In [23], the authors are concerned with the problem of reliable finite-time H_∞ filtering for discrete time-varying delay systems with Markovian jump and randomly occurring nonlinearities. The problem of finite time stabilization for a class of delayed neural networks is investigated in [24]. However, there are no results available coping with the finite-time event-triggered state estimation for MJSs with quantizations and randomly occurring nonlinearities, which motivates the current work.

Motivated by the above discussions, in this paper, the problem of event-based finite-time state estimation is investigated for Markovian jump systems with quantizations and randomly occurring nonlinearities. A state estimator is designed to guarantee the state estimation error dynamic system to be stochastic finite boundedness and satisfy a prescribed H_∞ performance level in a finite-time interval. The main contributions of this paper can be summarized as follows.

(1) The event-triggered scheme and the quantization strategy are introduced into the model of MJSs. The measured outputs are directly transmitted to the event generator. Only when the measured output satisfies a specified triggering condition, can it be transmitted though the quantizer to the state estimator.
(2) An estimation error system is constructed, which includes the influence of the event-triggered scheme, the quantization strategy and randomly occurring nonlinearities.
(3) Sufficient conditions are derived which can ensure the estimation error dynamic stochastically H_∞ finite boundedness. Moreover, a new co-design algorithm is developed to design the desired estimator gains and event-triggered parameters. Finally, a numerical simulation example is employed to demonstrate the usefulness of the obtained results.

The remainder of this paper is organized as follows. Section 2 describes the modeling process when considering event-triggered scheme and the quantization. The main results concerning the state estimator design conditions are presented in Section 3. A simulation is given in Section 4 to demonstrate the usefulness of the proposed method. Finally, Section 5 concludes the paper.

Notation: R^n and $R^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n\times m$ real matrices; the superscript T stands for matrix transposition; I is the identity matrix of appropriate dimension; the symbol $\otimes$ denotes the Kronecker product; the notation $X>0$ (respectively, $X\ge 0$), for $X\in\mathbb{R}^{n\times n}$, means that the matrix X is real symmetric positive definite (respectively, positive semi-definite); $Prob\{X\}$ denotes probability of event X to occur; $Sym\{X\}$ denotes the expression X^T+X; $\mathcal{E}$ denotes the expectation operator; for a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C\end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System description

Consider the following continuous-time Markovian jump system (MJS)

$$\begin{cases}\dot{x}(t)=A_{r_t}x(t)+A_{\omega r_t}\omega(t)+\alpha(t)h_{r_t}(x)+(1-\alpha(t))g_{r_t}(x)\\ y(t)=C_{r_t}x(t)\\ z(t)=L_{r_t}x(t)\end{cases} \tag{1}$$

where $x(t)\in R^n$ is the state variable, $y(t)\in R^m$ is the measured output, $z(t)\in R^p$ is the signal to be estimated, $\omega(t)\in R^q$ is the external disturbance with $\omega(t)\in\mathcal{L}_2[0,\infty)$, respectively; A_{r_t}, $A_{\omega r_t}$, C_{r_t}, and L_{r_t} are known real constant matrices with appropriate dimensions. $\{r_t, t\ge 0\}$ is continuous-time Markov jump process taking values in a finite space $S=\{1,2,\cdots,r\}$. The transition probability matrix $\Pi=(\pi_{ij})_{r\times r}$ are given by

$$Prob\{r_{t+h}=j|r_t=i\}=\begin{cases}\pi_{ij}h+o(h), & i\ne j\\ 1+\pi_{ij}h+o(h), & i=j\end{cases}$$

where $h>0$, $\lim_{h\to 0}\frac{o(h)}{h}=0$, $\pi_{ij}\ge 0$, for $j\ne i$. $\pi_{ii}=-\sum_{j=1,j\ne i}^{N}\pi_{ij}$.

The stochastic variable $\alpha(t)$ is Bernoulli-distributed white sequences taking values on 0 or 1 with $Prob\{\alpha(t)=1\}=\bar{\alpha}$, $Prob\{\alpha(t)=0\}=1-\bar{\alpha}$. The nonlinear functions $h_{r_t}(x)$ and $g_{r_t}(x)$ are assumed to satisfy $h_{r_0}(0)=0$, $g_{r_0}(0)=0$ and the following second-bounded conditions:

$$\begin{aligned}&[h_{r_t}(x)-h_{r_t}(y)-\phi_{1r_t}^h(x-y)]^T[h_{r_t}(x)-h_{r_t}(y)-\phi_{2r_t}^h(x-y)]\le 0\\ &[g_{r_t}(x)-g_{r_t}(y)-\phi_{1r_t}^g(x-y)]^T[g_{r_t}(x)-g_{r_t}(y)-\phi_{2r_t}^g(x-y)]\le 0\end{aligned} \tag{2}$$

where $x,y\in R^n$, $\phi_{1r_t}^h$, $\phi_{2r_t}^h$, $\phi_{1r_t}^g$ and $\phi_{2r_t}^g$ are real matrices with compatible dimensions.

Remark 1. It is well known that the network-induced nonlinear disturbances are ubiquitous. In this paper, we introduce nonlinear functions $h_{r_t}(x)$ and $g_{r_t}(x)$ to represent the changeable type/intensity of the nonlinearities. The random variable $\alpha(t)$ is employed to describe the probabilistic switches between $h_{r_t}(x)$ and $g_{r_t}(x)$ according to Bernoulli distribution. Note that random variables of similar kind can be found in [25,26].

Throughout this paper, we make the following assumptions:

(1) The sensor is time driven and the set of sampling instants of the sensor is $S_1=\{h,2h,\cdots\}$, where h is the sampling period.
(2) As is shown in Fig. 1, the sampled signals are directly transmitted to the event generator, the released signals are then quantized. The set of release instants of the event generator are denoted by $S_2=\{t_1h,t_2h,\cdots\}$.
(3) The quantized signals are transmitted to the remote state estimator through wireless network. During the transmitting procedure, the network-induced delays are described by τ_k, $\tau_k\in[0,\bar{\tau})$, $\bar{\tau}$ is non-negative integer, which represent the maximum network-induced delay.
(4) The disturbance $\omega(t)$ satisfies

$$\mathcal{E}\left\{\int_0^{+\infty}\omega^T(t)\omega(t)dt\right\}\le d^2 \tag{3}$$

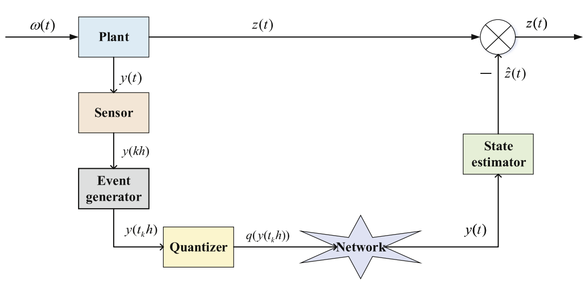

Fig. 1. Block diagram of an event-triggered MJSs with a quantizer.

In this paper, we take the effects of the event triggered scheme and the quantization into account, the following H_∞ state estimator for the network (1) will be adopted:

$$\begin{cases} \dot{\hat{x}}(t) = A_{r_t}\hat{x}(t) + \bar{\alpha}h_{r_t}(x) + (1-\bar{\alpha})g_{r_t}(x) + K_{r_t}[\hat{y}(t) - \tilde{y}(t)] \\ \hat{y}(t) = C_{r_t}\hat{x}(t) \\ \hat{z}(t) = L_{r_t}\hat{x}(t) \end{cases} \tag{4}$$

where K_{r_t} is the state estimator gain to be estimated.

In the following, when the system transits to the *ith* mode, the corresponding system matrices and the nonlinear functions are denoted by A_i, $A_{\omega i}$, C_i, L_i, h_i and g_i for simplicity.

Remark 2. The presence of the disturbances $\omega(t)$ are undesired input signals which affect the system output, the sensors and the plants causing accidents or unnecessary costs. There are some interesting research about the disturbance rejection [27–32]. For example, in [27], a hybrid controller with observer is designed for the estimation and rejection of a disturbance; The authors in [29] introduce the structure control of the disturbance rejection in two electromechanical process. In this paper, the objective is to design the finite-time state estimator for system (1) and obtain an estimate $\hat{z}(t)$ of the signal $z(t)$ such that the desired performance criteria are minimized in the estimation error sense.

Remark 3. It should be noted that $\tilde{y}(t)$ is the real input of the state estimator. In the following, we will explain that the signal $\tilde{y}(t)$ can be influenced by the event generator scheme, the quantization and network-induced delay.

As is well known, the network resources is limited, unnecessary communication can lead to a waste of communication resources. It is necessary to design a strategy to reduce the transmission frequency and communication cost. In order to remove the requirement for continuous communication, on the basis of reference [12], we introduce an event-based protocol

$$e_k^T(t)\Omega_{r_{t_kh+jh}}e_k(t) \le \sigma_{r_{t_kh+jh}}y^T(t_kh+jh)\Omega_{r_{t_kh+jh}}y(t_kh+jh) \tag{5}$$

where $e_k(t) = y(t_kh) - y(t_kh+jh)$, $\Omega_{r_{t_kh+jh}}$ is a symmetric positive definite matrix, $y(t_kh+jh)$ is the current measured output, $y(t_kh)$ is the latest transmitted data, $j = 1, 2, \cdots$. $\sigma_{r_{t_kh+jh}} \in [0, 1)$.

It should be noted that only the signals exceed the condition (5), can they be transmitted to the corresponding quantizer. The one satisfying the condition (5) will not be sent to the quantizer. Similar to [33], the holding interval $\Lambda = [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$ can be divided into interval like subset $\Lambda = \bigcup \Lambda_j$, $\Lambda_j = [t_kh + jh + \tau_{t_k}, t_kh + jh + h + \tau_{t_{k+1}})$, $j = 1, 2, \cdots, t_{k+1} - t_k - 1$.

Remark 4. Under the event triggered condition (5), the release times are assumed to be $t_0h, t_1h, t_2h, \cdots$, where $t_0 = 0$ is the initial time. $s_ih = t_{k+1}h - t_kh$ represents the release period between the latest transmitted data and the next transmitted one.

Remark 5. The event-triggered scheme has been widely used to reduce the utilization of the network resources. It should be pointed out that the triggering parameters depend on the system mode in this paper. If the newly sampled data violates the event-based protocol (5), it can be transmitted to the quantizer.

Define $\tau(t) = t - t_kh - jh$, it follows that $0 \le \tau(t) \le \bar{\tau}$. Substituting that definition of $\tau(t)$ into (5), the event-based protocol (5) can be written as

$$e_k^T(t)\Omega_ie_k(t) \le \sigma_iy^T(t-\tau(t))\Omega_iy(t-\tau(t)) \tag{6}$$

To further reduce the communication burden, quantizers are employed. The quantizer $q(\cdot)$ is defined as $q(y) = [q_1(y_1)\ q_2(y_2)\ \cdots\ q_m(y_m)]$, where $q_s(y_s)(s = 1, 2, \cdots, m)$ can be defined as

$$q_s(x_s) = \begin{cases} u_l^{(s)}, & if \frac{1}{1+\delta_{q_s}}u_l^{(s)} < y_s \le \frac{1}{1-\delta_{q_s}}u_l^{(s)}, y_s > 0 \\ 0, & if y_s = 0 \\ -q_s(-y_s), & if y_s < 0 \end{cases} \tag{7}$$

where $\delta_{q_s} = (1-\rho_{q_s})/(1+\rho_{q_s})(0 < \rho_{q_s} < 1)$, ρ_{q_s} is the quantization density and it is a given constant. The set of quantization levels is presented by [34,35]: $\mathcal{U}_s = \{\pm u_l^{(s)}, u_l^{(s)} = \rho_{q_s}^l u_0^{(s)}, l = \pm 1, \pm, 2, \cdots\} \bigcup \{\pm u_0^{(s)}\} \bigcup \{0\}$ with $u_0^{(s)} > 0$. Based on the above definition, using the sector bound approach, the measurements with quantization effects $q(y)$ can be expressed as

$$q(y) = (I + \Delta_q)y \tag{8}$$

in which $\Delta_q = diag\{\Delta_{q_1}, \Delta_{q_2}, \cdots, \Delta_{q_m}\}, \Delta_{q_s} \in [-\delta_{q_s}, \delta_{q_s}], s = 1, 2, \cdots, m$

Based on the above analysis, considering the effect of event generator and the quantizer, the actually received signal $\tilde{y}(t)$ of the designed estimator can be described as

$$\tilde{y}(t) = (I + \Delta_q)y(t_kh) \tag{9}$$

Recall the definition of $e_k(t)$ and $\tau(t)$, (9) can be rewritten as

$$\tilde{y}(t) = (I + \Delta_q)(e_k(t) + C_ix(t-\tau(t))) \tag{10}$$

Define $e(t) = x(t) - \hat{x}(t)$, $\tilde{z}(t) = z(t) - \hat{z}(t)$, combining (1), (4) and (10), the following estimation error system can be obtained

$$\begin{cases} \dot{e}(t) = A_ie(t) + A_{\omega i}\omega(t) + \alpha(t)h_i(x) - \bar{\alpha}h_i(x) + (1-\alpha(t))g_i(x) \\ -(1-\bar{\alpha})g_i(x) - K_i[C_i\hat{x}(t) - (I+\Delta_q)e_k(t) - (I+\Delta_q)C_ix(t-\tau(t))] \\ \tilde{z}(t) = L_ie(t) \end{cases} \tag{11}$$

Let $\eta(t) = [x^T(t)\ e^T(t)]^T$, an augmented system can be obtained as follows:

$$\begin{cases} \dot{\eta}(t) = \bar{A}_i\eta(t) + \bar{A}_{qi}\eta(t-\tau(t)) + \bar{A}_{ei}e_k(t) + \bar{A}_{\omega i}\omega(t) + \bar{\alpha}\bar{h}_i(t) \\ +(1-\bar{\alpha})\bar{g}_i(t) + (\alpha(t)-\bar{\alpha})S\bar{h}_i(t) - (\alpha(t)-\bar{\alpha})S\bar{g}_i(t) \\ \tilde{z}(t) = \bar{L}_i\eta(t) \end{cases} \tag{12}$$

where

$$\bar{A}_i = \begin{bmatrix} A_i & 0 \\ -K_iC_i & A_i + K_iC_i \end{bmatrix}, \quad \bar{A}_{qi} = \begin{bmatrix} 0 & 0 \\ K_i(I+\Delta_q)C_i & 0 \end{bmatrix},$$

$$\bar{A}_{ei} = \begin{bmatrix} 0 \\ K_i((I+\Delta_q)) \end{bmatrix}, \quad \bar{A}_{\omega i} = \begin{bmatrix} A_{\omega i} \\ A_{\omega i} \end{bmatrix}$$

$$S = \begin{bmatrix} I & 0 \\ I & 0 \end{bmatrix}, \quad \bar{L}_i = \begin{bmatrix} 0 & L_i \end{bmatrix}, \quad \bar{h}_i(t) = \begin{bmatrix} h_i^T(x) & h_i^T(x) - h_i^T(\hat{x}) \end{bmatrix}^T,$$

$$\bar{g}_i(t) = \begin{bmatrix} g_i^T(x) & g_i^T(x) - g_i^T(\hat{x}) \end{bmatrix}^T$$

Remark 6. From (5), we can deduce that

$$e_k^T(t)\Omega_ie_k(t) \le \sigma_i\eta^T(t-\tau(t))H^TC_i^T\Omega_iC_iH\eta(t-\tau(t)), \quad H = \begin{bmatrix} I & 0 \end{bmatrix} \tag{13}$$

The following definitions and lemmas are necessary, which will be used in the proof of our main results.

Definition 1 (*Stochastically finite-time stability(SFTS)[36]*). For given time constant T, the augmented system (12) with $\omega(t) = 0$ is said to be SFTS with respect to (c_1, c_2, T, R) with $0 < c_1 < c_2$ and $R > 0$, if

$$\sup_{-\tau\le t_0\le 0} \mathcal{E}\{\eta^T(t_0)R\eta(t_0),\ \dot\eta^T(t_0)R\dot\eta(t_0)\}$$
$$\le c_1^2 \Rightarrow \mathcal{E}\{\eta^T(t)R\eta(t)\} < c_2^2,\ t\in[0,T] \tag{14}$$

Definition 2 (*Stochastically finite-time bounded (SFTB)[21]*). The augmented system (12) is said to be SFTB with respect to (c_1, c_2, T, R, d) with $0 < c_1 < c_2$ and $R > 0$, if the constrained relation (14) holds.

Definition 3 (*[21] (Stochastically H_∞ finite-time bounded (S H_∞ FTB)))*. The augmented system (12) is said to be S H_∞ FTB with respect to $(c_1, c_2, T, R, \gamma, d)$, if the augmented system (12) is SFTB with respect to (c_1, c_2, T, R, d) and under zero initial condition, it holds that

$$\mathcal{E}\left\{\int_0^T z^T(t)z(t)dt\right\} < \gamma^2\mathcal{E}\left\{\int_0^T \omega^T(t)\omega(t)dt\right\} \tag{15}$$

Lemma 1. *([37])Consider the Markov jump system with $\tau(t)$ that satisfies $0<\tau(t)\le\bar\tau$. For any matrices $X\in R^{n\times n}$ and $U\in R^{n\times n}$ that satisfy $\begin{bmatrix} X & U\\ U^T & X\end{bmatrix}\ge 0$, the following inequality holds:*

$$-\bar\tau\int_{t-\tau}^{t}\dot x^T(s)X\dot x(s)\le\begin{bmatrix}x(t)\\ x(t-\tau(t))\\ x(t-\bar\tau)\end{bmatrix}^T\begin{bmatrix}-X & * & *\\ X^T-U^T & -2X+U+U^T & *\\ U^T & X^T-U^T & -X\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-\tau(t))\\ x(t-\bar\tau)\end{bmatrix} \tag{16}$$

Lemma 2. *([38])Given matrices $F_1=F_1^T$, F_2 and F_3 of appropriate dimensions, we have $F_1+F_3\Delta(k)F_2+F_2^T\Delta^T(k)F_3^T<0$ for all $\Delta(k)$ satisfying $\Delta^T(k)\Delta(k)\le I$, if and only if there exists a positive scalar $\varepsilon<0$, such that $F_1+\varepsilon^{-1}F_3F_3^T+\varepsilon F_2^TF_2<0$*

3. Main results

In this section, we will develop an approach for both the finite-time bounded and the state estimator design. Firstly, we give the finite time bounded for the augmented system (12).

Theorem 1. *For given scalars $\bar\alpha$, τ_M, σ_i, β, $T>0$, $d>0$ and $c_1>0$, under the event-triggered scheme (5), the augmented system (12) is stochastically finite-time stability with respect to (c_1, c_2, T, R, d), if there exist positive scalars $\lambda_l(l=1,2,3,4,5)$, upper bound c_2, $0<c_1<c_2$, μ_1, μ_2, $P_i>0$, $Q_i>0$, $R>0$, $\Omega_i>0(i\in S)$, H_i and M_i, with appropriate dimensions such that*

$$\Xi=\begin{bmatrix}\Xi_{11} & * & *\\ \Xi_{21} & -P_iQ_2^{-1}P_i & *\\ \Xi_{31} & 0 & \Xi_{33}\end{bmatrix}<0 \tag{17}$$

$$\lambda_1R<P_i<\lambda_2R,\quad Q_1<\lambda_3R,\quad Q_2<\lambda_4R,\quad \Delta=\lambda_2+\tau_M\lambda_3+\tau_M^2\lambda_4 \tag{18}$$

$$\begin{bmatrix}Q_2 & M_i\\ M_i^T & Q_2\end{bmatrix}\ge 0,\quad \Delta c_1^2+\lambda_5d^2<e^{-\beta T}\lambda_1c_2^2,\quad 0<H_i<\lambda_5I \tag{19}$$

where

$$\Xi_{11}=\begin{bmatrix}\Gamma_{11} & * & * & * & * & * & *\\ \Gamma_{21} & \Gamma_{22} & * & * & * & * & *\\ \frac{1}{\tau_M}M_i^T & \frac{1}{\tau_M}Q_2-\frac{1}{\tau_M}M_i^T & -e^{\beta\tau_M}Q_1-\frac{1}{\tau_M}Q_2 & * & * & * & *\\ \bar A_{ei}^TP_i & 0 & 0 & -\Omega_i & * & * & *\\ \bar A_{\omega i}^TP_i & 0 & 0 & 0 & -H_i & * & *\\ \bar P_i+\mu_1\Phi_{2i}^h & 0 & 0 & 0 & 0 & \hat Q_2-\mu_1I & *\\ (1-\bar\alpha)P_i+\mu_2\Phi_{2i}^g & 0 & 0 & 0 & 0 & -\hat Q_2 & \hat Q_2-\mu_2I\end{bmatrix}$$

$$\Gamma_{11}=P_i\bar A_i+\bar A_i^TP_i-\beta P_i+Q_1-\frac{1}{\tau_M}Q_2-\mu_1\Phi_{1i}^h-\mu_2\Phi_{1i}^g+\pi_{ii}P_i$$

$$\Gamma_{21}=\bar A_{qi}^TP_i+\frac{1}{\tau_M}Q_2-\frac{1}{\tau_M}M_i^T,\quad \Gamma_{22}=-\frac{2}{\tau_M}Q_2+\frac{1}{\tau_M}M_i+\frac{1}{\tau_M}M_i^T+\sigma_iH^TC_i^T\Omega_iC_iH,$$

$$\hat Q_2=\bar\alpha(1-\bar\alpha)S^TQ_2S$$

$$\Phi_{1i}^h=I\otimes sym\frac{1}{2}\phi_{1i}^{hT}\phi_{2i}^h,\quad \Phi_{1i}^g=I\otimes sym\frac{1}{2}\phi_{1i}^{gT}\phi_{2i}^g$$

$$\Phi_{2i}^h=I\otimes sym\frac{\phi_{1i}^h+\phi_{2i}^h}{2},\quad \Phi_{2i}^g=I\otimes sym\frac{\phi_{1i}^g+\phi_{2i}^g}{2}$$

$$\Xi_{21}=\begin{bmatrix}\sqrt{\tau_M}P_i\bar A_i & \sqrt{\tau_M}P_i\bar A_{qi} & \sqrt{\tau_M}P_i\bar A_{ei} & \sqrt{\tau_M}P_i\bar A_{\omega i} & \bar\alpha\sqrt{\tau_M}P_i & (1-\bar\alpha)\sqrt{\tau_M}P_i\end{bmatrix}$$

$$\Xi_{31}=\begin{bmatrix}\Pi & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Pi=\begin{bmatrix}\sqrt{\pi_{i1}}I & \cdots & \sqrt{\pi_{i,i-1}}I & \sqrt{\pi_{i,i+1}}I & \cdots & \sqrt{\pi_{ir}}I\end{bmatrix}^T$$

$$\Xi_{33}=diag\{-P_1^{-1},\cdots,-P_{i-1}^{-1},-P_{i+1}^{-1},\cdots,-P_r^{-1}\}$$

$$P_i=diag\{P_{1i},P_{2i}\},\quad Q_1=diag\{Q_{11},Q_{12}\},\quad Q_2=diag\{R_1,R_2\}$$

Proof. Select a Lyapunov-Krasovskii functional for system (12)

$$V(\eta(t),i,t)=V_1(\eta(t),i,t)+V_2(\eta(t),i,t)+V_3(\eta(t),i,t) \tag{20}$$

where

$$V_1(\eta(t),i,t)=\eta^T(t)P_i\eta(t)$$
$$V_2(\eta(t),i,t)=\int_{t-\tau_M}^{t}e^{\beta(t-s)}\eta^T(s)Q_1\eta(s)ds$$
$$V_3(\eta(t),i,t)=\int_{t-\tau_M}^{t}\int_s^t e^{\beta(t-s)}\eta^T(v)Q_2\eta(v)dvds$$

Calculating the derivative of $V(\eta(t),i,t)$ along the trajectory of the system (12) for i, $i\in S$, we have

$$\mathcal{E}\{\dot V_1(\eta(t),i,t)\}=2\dot\eta^T(t)P_i\mathcal{A}+\eta^T(t)\sum_{j=1}^{r}\pi_{ij}P_j\eta(t) \tag{21}$$

$$\mathcal{E}\{\dot V_2(\eta(t),i,t)\}=\beta V_2(\eta(t),i,t)+\eta^T(t)Q_1\eta(t)-e^{\beta\tau_M}\eta^T(t-\tau_M)Q_1x(t-\tau_M) \tag{22}$$

$$\mathcal{E}\{\dot V_3(\eta(t),i,t)\}=\beta V_3(\eta(t),i,t)+\tau_M\dot\eta^T(t)Q_2\dot\eta(t)-\int_{t-\tau_M}^{t}e^{\beta(t-s)}\dot\eta^T(s)Q_2\dot\eta(s)ds \tag{23}$$

in which $\mathcal{A}=\bar A_i\eta(t)+\bar A_{qi}\eta(t-\tau(t))+\bar A_{ei}e_k(t)+\bar A_{\omega i}\omega(t)+\bar\alpha\bar h_i(t)+(1-\bar\alpha)\bar g_i(t)$

By Lemma 1, for Q_2 and M_i satisfy $\begin{bmatrix}Q_2 & M_i\\ M_i^T & Q_2\end{bmatrix}\ge 0$, the following inequality holds

$$-\int_{t-\tau_M}^{t}e^{\beta(t-s)}\dot\eta^T(s)Q_2\dot\eta(s)ds\le\frac{1}{\tau_M}\begin{bmatrix}\eta(t)\\ \eta(t-\tau(t))\\ \eta(t-\tau_M)\end{bmatrix}^T\begin{bmatrix}-Q_2 & * & *\\ Q_2-M_i^T & -2Q_2+M_i+M_i^T & *\\ M_i^T & Q_2-M_i^T & -Q_2\end{bmatrix}\begin{bmatrix}\eta(t)\\ \eta(t-\tau(t))\\ \eta(t-\tau_M)\end{bmatrix} \tag{24}$$

Notice that (2) implies that

$$[\bar{h}_i(t) - (I \otimes \phi_{1i}^h)\eta(t)]^T [\bar{h}_i(t) - (I \otimes \phi_{2i}^h)\eta(t)] \le 0 \tag{25}$$

$$[\bar{g}_i(t) - (I \otimes \phi_{1i}^g)\eta(t)]^T [\bar{g}_i(t) - (I \otimes \phi_{2i}^g)\eta(t)] \le 0 \tag{26}$$

Then, combining (13) and (21)–(26), we have

$$\begin{aligned}
&\mathcal{E}\{\dot{V}(\eta(t), i, t)\} \\
&\le \beta V(\eta(t), i, t) + 2\dot{\eta}^T(t)P_i\mathcal{A} + \eta^T(t)\sum_{j=1}^{r}\pi_{ij}P_j\eta(t) \\
&\quad + \eta^T(t)Q_1\eta(t) - e^{\beta\tau_M}\eta^T(t-\tau_M)Q_1\eta(t-\tau_M) + \tau_M\dot{\eta}^T(t)Q_2\dot{\eta}(t) \\
&\quad + \frac{1}{\tau_M}\begin{bmatrix} \eta(t) \\ \eta(t-\tau(t)) \\ \eta(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -Q_2 & * & * \\ Q_2 - M_i^T & -2Q_2 + M_i + M_i^T & * \\ M_i^T & Q_2 - M_i^T & -Q_2 \end{bmatrix} \\
&\quad \begin{bmatrix} \eta(t) \\ \eta(t-\tau(t)) \\ \eta(t-\tau_M) \end{bmatrix} - e_k^T(t)\Omega_i e_k(t) + \sigma_{r_{t_kh+jh}}\eta^T(t-\tau(t))H^TC_i^T\Omega_iC_iH\eta(t-\tau(t)) \\
&\quad - \mu_1[\bar{h}_i(t) - (I \otimes \phi_{1i}^f)\eta(t)]^T[\bar{h}_i(t) - (I \otimes \phi_{2i}^f)\eta(t)] \\
&\quad - \mu_2[\bar{g}_i(t) - (I \otimes \phi_{1i}^g)\eta(t)]^T[\bar{g}_i(t) - (I \otimes \phi_{2i}^g)\eta(t)] \\
&\le \beta V(\eta(t), i, t) + \zeta^T(t)\Xi_{11}\zeta(t) \\
&\quad + \omega^T(t)H_i\omega(t) + \eta^T(t)\sum_{j=1, j\neq i}^{r}\pi_{ij}P_j\eta(t) + \tau_M\dot{\eta}^T(t)Q_2\dot{\eta}(t)
\end{aligned} \tag{27}$$

where $\zeta^T(t) = \left[\eta(t)\ \eta(t-\tau(t))\ \eta(t-\tau_M)\ e_k(t)\ \omega(t)\ \bar{h}(s)\ \bar{g}(x)\right]^T$

From (17) and (27), recalling $0 < H_i < \lambda_5 I$, we can get

$$\begin{aligned}
\mathcal{E}\{\dot{V}(\eta(t), i, t)\} &\le \mathcal{E}\{\beta V(\eta(t), i, t) + \omega^T(t)H_i\omega(t)\} \\
&\le \mathcal{E}\{\beta V(\eta(t), i, t) + \lambda_5\omega^T(t)\omega(t)\}
\end{aligned} \tag{28}$$

It can be obtained from (28) that

$$\mathcal{E}\left[\frac{d}{dt}\left(e^{-\beta t}V(\eta(t), i, t)\right)\right] < \mathcal{E}\left\{\lambda_5 e^{-\beta t}\omega^T(t)\omega(t)\right\} \tag{29}$$

Integrating (29) from 0 to t with $t \in [0, T]$, we have

$$\int_0^t \mathcal{E}\left[e^{-\beta t}V(\eta(s), i, s)\right]ds < \lambda_5\int_0^t \mathcal{E}\left\{e^{-\beta s}\omega^T(s)\omega(s)\right\}ds \tag{30}$$

From (30), we can find that

$$\mathcal{E}\left[e^{-\beta t}V(\eta(t), i, t)\right] < V(\eta(0), r_0, 0) + \lambda_5\mathcal{E}\left\{\int_0^t \omega^T(s)\omega(s)ds\right\} \tag{31}$$

Notice that

$$V(\eta(0), r_0, 0) \le \sup_{-\tau \le t_0 \le 0}\mathcal{E}\left\{\eta^T(t_0)R\eta(t_0), \dot{\eta}^T(t_0)R\dot{\eta}(t_0)\right\}\Delta \le \Delta c_1^2 \tag{32}$$

where $\Delta = \lambda_2 + \tau_M\lambda_3 + \tau_M^2\lambda_4$. In view of $\lambda_1 R < P_i$, for all $t \in [0, T]$, we have

$$\mathcal{E}\left[V(\eta(t), i, t)\right] > \lambda_1\eta^T(t)R\eta(t) \tag{33}$$

Thus, we can derive from (31)–(33) and (3) that

$$\eta^T(t)R\eta(t) < \frac{e^{\beta t}(\Delta c_1^2 + \lambda_5 d^2)}{\lambda_1} \tag{34}$$

Therefore, the system (12) is finite-time bounded, $\eta^T(t)R\eta(t) < c_2^2$. This completes the proof. □

Remark 7. It is a challenging problem on how to estimate the integral term with time delay information $-\int_{t-\tau_M}^{t} e^{\beta(t-s)}\dot{\eta}^T(s)Q_2\dot{\eta}(s)ds$. Very recently, several effective methods are developed to find the upper bound of the integral term, for example, the free-weighting-matrix approach in [39,40] and the reciprocally convex lemma proposed in [37], Besides, the authors in [41] develops two relaxed integral inequalities to estimate the integral term. In this paper, the widely used method of reciprocally convex lemma is employed.

Theorem 1 presents sufficient conditions which ensure the augmented system (12) to be stochastically finite-time stability. By employing the same method in Theorem 1, we can derive the following Theorem 2, in which sufficient conditions are given for the stochastically H_∞ finite-time bounded of the augmented system (12).

Theorem 2. *For given scalars $\bar{\alpha}$, τ_M, σ_i, β, $T > 0$, $d > 0$ and $c_1 > 0$, under the event-triggered scheme (5), the augmented system (12) is stochastically H_∞ finite-time bounded with respect to $(c_1, c_2, T, R, \gamma, d)$, if there exist positive scalars $\lambda_l (l = 1, 2, 3, 4, 5)$, μ_1, μ_2, $0 < c_1 < c_2$, $P_i > 0$, $Q_i > 0$, $R > 0$, $\Omega_i > 0$ $(i \in S)$, and M_i, with appropriate dimensions such that*

$$\bar{\Xi} = \begin{bmatrix} \bar{\Xi}_{11} & * & * & * \\ \bar{\Xi}_{21} & -P_iQ_2^{-1}P_i & * & * \\ \bar{L}_i & 0 & -I & * \\ \Xi_{41} & 0 & 0 & \Xi_{44} \end{bmatrix} < 0 \tag{35}$$

$$\lambda_1 R < P_i < \lambda_2 R,\ Q_1 < \lambda_3 R,\ Q_2 < \lambda_4 R \tag{36}$$

$$\begin{bmatrix} Q_2 & M_i \\ M_i^T & Q_2 \end{bmatrix} \ge 0, \quad \Delta c_1^2 + \gamma^2 d^2 < e^{-\beta T}\lambda_1 c_2^2, \quad \Xi_{41} = \Xi_{31},\ \Xi_{44} = \Xi_{33} \tag{37}$$

where

$$\bar{\Xi}_{11} = \begin{bmatrix} \Gamma_{11} & * & * & * & * & * & * \\ \Gamma_{21} & \Gamma_{22} & * & * & * & * & * \\ M_i^T & Q_2 - M_i^T & -e^{\beta\tau_M}Q_1 - Q_2 & * & * & * & * \\ \bar{A}_{ei}^TP_i & 0 & 0 & -\Omega_i & * & * & * \\ \bar{A}_{\omega i}^TP_i & 0 & 0 & 0 & -\gamma^2 I & * & * \\ \bar{P}_i + \mu_1\Phi_{2i}^h & 0 & 0 & 0 & 0 & \hat{Q}_2 - \mu_1 I & * \\ (1-\bar{\alpha})P_i + \mu_2\Phi_{2i}^g & 0 & 0 & 0 & 0 & -\hat{Q}_2 & \hat{Q}_2 - \mu_2 I \end{bmatrix}$$

Other variables are the same as Theorem 1

Proof. Construct the same Lyapunov-Krasovskii functional as in Theorem 1. Similar to the proof in Theorem 1, we have

$$\begin{aligned}
&\mathcal{E}\left\{\dot{V}(\eta(t), i, t)\right\} - \beta\mathcal{E}\left\{V(\eta(t), i, t)\right\} - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \\
&< 0
\end{aligned} \tag{38}$$

Further, from (38), we get

$$\mathcal{E}\left\{\frac{d}{dt}\left(e^{-\beta t}V(\eta(t), i, t)\right)\right\} < e^{-\beta t}\left[\gamma^2\omega^T(t)\omega(t) - \tilde{z}^T(t)\tilde{z}(t)\right] \tag{39}$$

Under zero initial condition, integrating (40) from 0 to T, we have

$$\begin{aligned}
&0 < \mathcal{E}\left\{\left(e^{-\beta t}V(\eta(t), i, t)\right)\right\} < \int_0^T \mathcal{E} \\
&\quad \left\{e^{-\beta t}[\gamma^2\omega^T(t)\omega(t) - \tilde{z}^T(t)\tilde{z}(t)]\right\}dt
\end{aligned} \tag{40}$$

Further, we can get

$$\begin{aligned}
\mathcal{E}\left\{e^{-\beta T}\int_0^T \tilde{z}^T(t)\tilde{z}(t)dt\right\} &< \mathcal{E}\left\{\int_0^T e^{-\beta t}\tilde{z}^T(t)\tilde{z}(t)dt\right\} \\
&< \mathcal{E}\left\{\gamma^2\int_0^T e^{-\beta t}\omega^T(t)\omega(t)dt\right\} \\
&< \mathcal{E}\left\{\gamma^2\int_0^T \omega^T(t)\omega(t)dt\right\}
\end{aligned} \tag{41}$$

Therefore, the proof is completed. □

In the following, we will deal with the estimator design problem by considering the effect of event-triggered scheme and quantization. Based on Theorem 2, sufficient conditions are established for the existence of the desired estimator and the explicit expression of the desired parameter is provided in the following Theorem.

Theorem 3. *For given scalars $\bar{\alpha}$, τ_M, σ_i, β, $T>0$, $d>0$, ε_1, ε_2, θ_ν, $\nu=1,\cdots,i-1,i+1,\cdots,1$ and $c_1>0$, under the event-triggered scheme (5), the augmented system (12) is stochastically H_∞ finite-time bounded with respect to $(c_1, c_2, T, R, \gamma, d)$, if there exist positive scalars $\lambda_l (l=1,2,3,4)$, μ_1, μ_2, $0<c_1<c_2$, $Y_i>0$, $\bar{P}_i>0$, $Q_i>0$, $R>0$, $\Omega_i>0$ $(i\in S)$, and M_i, with appropriate dimensions such that*

$$\tilde{\Xi}=\begin{bmatrix}\tilde{\Xi}_{11} & * & * & * & * \\ \tilde{\Xi}_{21} & -2\varepsilon_1 P_i+\varepsilon^2 Q_2 & * & * & * \\ \bar{L}_i & 0 & -I & * & * \\ \tilde{\Xi}_{41} & 0 & 0 & \tilde{\Xi}_{44} & * \\ \tilde{\Xi}_{51} & 0 & 0 & 0 & \tilde{\Xi}_{55}\end{bmatrix}<0 \tag{42}$$

$$\lambda_1 R<P_i<\lambda_2 R,\ Q_1<\lambda_3 R,\ Q_2<\lambda_4 R \tag{43}$$

$$\begin{bmatrix}Q_2 & M_i \\ M_i^T & Q_2\end{bmatrix}\geq 0,\quad \Delta c_1^2+\gamma^2 d^2<e^{-\beta T}\lambda_1 c_2^2 \tag{44}$$

where

$$\tilde{\Xi}_{11}=\begin{bmatrix}\tilde{\Gamma}_{11} & * & * & * & * & * & * \\ \tilde{\Gamma}_{21} & \Gamma_{22} & * & * & * & * & * \\ M_i^T & Q_2-M_i^T & -e^{\beta\tau_M}Q_1-Q_2 & * & * & * & * \\ \tilde{A}_{ei} & 0 & 0 & -\Omega_i & * & * & * \\ \bar{A}_{\omega i}^T P_i & 0 & 0 & 0 & -\gamma^2 I & * & * \\ \bar{P}_i+\mu_1\Phi_{2i}^h & 0 & 0 & 0 & 0 & \hat{Q}_2-\mu_1 I & * \\ (1-\bar{\alpha})P_i+\mu_2\Phi_{2i}^g & 0 & 0 & 0 & 0 & -\hat{Q}_2 & \hat{Q}_2-\mu_2 I\end{bmatrix}$$

$\tilde{\Gamma}_{11}=\tilde{A}_i+\tilde{A}_i^T-\beta P_i+Q_1-Q_2-\mu_1\Phi_{1i}^h-\mu_2\Phi_{1i}^g+\pi_{ii}P_i,\quad \tilde{A}_{ei}=\begin{bmatrix}0 & Y_i^T\end{bmatrix}$

$\tilde{\Gamma}_{21}=\tilde{A}_{qi}+Q_2-M_i^T,\quad \tilde{A}_i=\begin{bmatrix}P_{1i}A_i & 0 \\ -Y_iC_i & P_{2i}A_i+P_{2i}Y_i\end{bmatrix},\quad \tilde{A}_{qi}=\begin{bmatrix}0 & C_i^TY_i^T \\ 0 & 0\end{bmatrix}$

$\tilde{\Xi}_{21}=\begin{bmatrix}\sqrt{\tau_M}\tilde{A}_i & \sqrt{\tau_M}\tilde{A}_{qi} & \sqrt{\tau_M}\tilde{A}_{ei} & \sqrt{\tau_M}P_i\bar{A}_{\omega i} & \bar{\alpha}\sqrt{\tau_M}P_i & (1-\bar{\alpha})\sqrt{\tau_M}P_i\end{bmatrix}$

$\tilde{\Xi}_{41}=\begin{bmatrix}\varepsilon_2 Y_i^T & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_q\tilde{C}_i & 0 & \delta_q I & 0 & 0 & 0\end{bmatrix}^T,\quad \tilde{C}_i=\begin{bmatrix}C_i & 0\end{bmatrix}$

$\tilde{\Xi}_{44}=\begin{bmatrix}-\varepsilon_2 I & 0 \\ 0 & -\varepsilon_2 I\end{bmatrix},\quad \tilde{\Xi}_{51}=\begin{bmatrix}\bar{\Pi} & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$

$\bar{\Pi}=\begin{bmatrix}\sqrt{\pi_{i1}}\bar{P}_i & \cdots & \sqrt{\pi_{i,i-1}}\bar{P}_i & \sqrt{\pi_{i,i+1}}\bar{P}_i & \cdots & \sqrt{\pi_i}\bar{P}_i\end{bmatrix}^T,\quad \bar{P}_i=diag\{P_{2i}, P_{2i}\}$

$\tilde{\Xi}_{55}=diag\{-\theta_1\bar{P}_i+\theta_1^2P_1,\cdots,-2\theta_{i-1}\bar{P}_i+\theta_{i-1}^2P_{i-1},-2\theta_{i+1}\bar{P}_i+\theta_{i+1}^2P_{i+1},\cdots,-2\theta_r\bar{P}_i+\theta_r^2P_r\}$

Other parameters are defined in Theorem 1, Moreover, if the above inequality is solvable, the desired estimator gain can be determined by

$$K_i=P_{2i}^{-1}Y_i \tag{45}$$

Proof. First, we denote $\bar{A}_{qi}=\check{A}_i+\check{A}_{qi}$, $\bar{A}_{ei}=\check{A}_{ei}+\check{A}_{eqi}$, where

$$\check{A}_i=\begin{bmatrix}0 & 0 \\ K_iC_i & 0\end{bmatrix},\quad \check{A}_{qi}=\begin{bmatrix}0 & 0 \\ K_i\Delta_qC_i & 0\end{bmatrix}=\bar{K}_i\bar{C}_{q1},\ \check{A}_{ei}=\bar{K}_i$$

$$\check{A}_{eqi}=\begin{bmatrix}0 \\ K_i\Delta_q\end{bmatrix}=\bar{K}_i\bar{C}_{q2},\ \bar{K}_i=\begin{bmatrix}0 \\ K_i\end{bmatrix},\quad \bar{C}_{q1}=\begin{bmatrix}\Delta_qC_i & 0\end{bmatrix},\ \bar{C}_{q2}=\Delta_q$$

Due to

$$(Q_2-\varepsilon_1^{-1}P_i)Q_2^{-1}(Q_2-\varepsilon_1^{-1}P_i)\geq 0,$$

we can get

$$-P_iQ_2^{-1}P_i\leq -2\varepsilon_1P_i+\varepsilon_1^2Q_2 \tag{46}$$

By utilizing Lemma 2, it follows from (35) and (51) that there exist scalars $\varepsilon_2>0$ such that

$$\Psi+\varepsilon_2H_k^TH_k+\varepsilon_2^{-1}H_q^TH_q<0 \tag{47}$$

where

$$\Psi=\begin{bmatrix}\Psi_{11} & * & * & * \\ \Psi_{21} & -2\varepsilon_1P_i+\varepsilon_1^2Q_2 & * & * \\ \bar{L}_i & 0 & -I & * \\ \Xi_{41} & 0 & 0 & \Xi_{44}\end{bmatrix},\quad \check{\Gamma}_{21}=\check{A}_i^TP_i+Q_2-M_i^T$$

$$\Psi_{11}=\begin{bmatrix}\Gamma_{11} & * & * & * & * & * & * \\ \check{\Gamma}_{21} & \Gamma_{22} & * & * & * & * & * \\ M_i^T & Q_2-M_i^T & -e^{\beta\tau_M}Q_1-Q_2 & * & * & * & * \\ \check{A}_{ei}^TP_i & 0 & 0 & -\Omega_i & * & * & * \\ \bar{A}_{\omega i}^TP_i & 0 & 0 & 0 & -\gamma^2I & * & * \\ \bar{P}_i+\mu_1\Phi_{2i}^h & 0 & 0 & 0 & 0 & \hat{Q}_2-\mu_1I & * \\ (1-\bar{\alpha})P_i+\mu_2\Phi_{2i}^g & 0 & 0 & 0 & 0 & -\hat{Q}_2 & \hat{Q}_2-\mu_2I\end{bmatrix}$$

$H_k=\begin{bmatrix}\bar{K}_i^TP_i & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\tau_M}\bar{K}_i^TP_i\end{bmatrix}$

$H_q=\begin{bmatrix}0 & \bar{C}_{q1} & 0 & \bar{C}_{q2} & 0 & 0 & 0 & 0\end{bmatrix}$

$\Psi_{21}=\begin{bmatrix}\sqrt{\tau_M}P_i\bar{A}_i & \sqrt{\tau_M}P_i\check{A}_i & 0 & \sqrt{\tau_M}P_i\check{A}_{ei} & \sqrt{\tau_M}P_i\bar{A}_{\omega i} & \bar{\alpha}\sqrt{\tau_M}P_i & (1-\bar{\alpha})\sqrt{\tau_M}P_i\end{bmatrix}$

By Schur complement, (47) is equivalent to

$$\Psi=\begin{bmatrix}\Psi_{11} & * & * & * & * \\ \Pi_{21} & -2\varepsilon_1P_i+\varepsilon_1^2Q_2 & * & * & * \\ \bar{L}_i & 0 & -I & * & * \\ \Psi_{41} & \Psi_{42} & 0 & \Xi_{44} & * \\ \Psi_{51} & 0 & 0 & 0 & \Psi_{55}\end{bmatrix}<0 \tag{48}$$

in which

$$\Psi_{41}=\begin{bmatrix}\varepsilon_2\bar{K}_i^TP_i & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \delta_q\tilde{C}_i & 0 & \delta_qI & 0 & 0 & 0\end{bmatrix}^T,$$

$$\Psi_{42}=\begin{bmatrix}\varepsilon_2\sqrt{\tau_M}\bar{K}_i^TP_i \\ 0\end{bmatrix},\ \Psi_{51}=\Xi_{31},\quad \Psi_{55}=\Xi_{33}$$

Denoting $Y=P_{2i}K_i$, pre- and post-multiplying both sides of (48) with $diag\{I, I, \cdots, I, \underbrace{\bar{P}_i, \cdots, \bar{P}_i}_{r-1}\}$, we can easily obtain

$$\tilde{\Psi}=\begin{bmatrix}\tilde{\Xi}_{11} & * & * & * & * \\ \tilde{\Xi}_{21} & -2\varepsilon_1P_i+\varepsilon_1^2Q_2 & * & * & * \\ \bar{L}_i & 0 & -I & * & * \\ \tilde{\Xi}_{41} & 0 & 0 & \tilde{\Xi}_{44} & * \\ \tilde{\Xi}_{51} & 0 & 0 & 0 & \tilde{\Psi}_{55}\end{bmatrix}<0 \tag{49}$$

where

$$\tilde{\Psi}_{55}=diag\{-P_{2i}P_1^{-1}P_{2i},\cdots,-P_{2i}P_{i-1}^{-1}P_{2i},-P_{2i}P_{i+1}^{-1}P_{2i},\cdots,-P_{2i}P_r^{-1}P_{2i}\} \tag{50}$$

Notice that

$$-P_{2i}P_\nu^{-1}P_{2i}\leq -2\theta_\nu P_{2i}+\theta_\nu^2P_\nu,\ \nu=1,\cdots,i-1,i+1,\cdots,r \tag{51}$$

Replace $-P_{2i}P_\nu^{-1}P_{2i}$ by $-2\theta_\nu P_{2i}+\theta_\nu^2P_\nu$, we can get (3) the proof is then complete. □

Remark 8. There are three main aspects leading to the design of state estimator more complicated, i.e. event-triggered scheme, quantizations and randomly occurring nonlinear perturbation. In

Theorem 3, sufficient conditions are derived which guarantee the augmented system (12) stochastically H_∞ finite-time bounded with respect to $(c_1, c_2, T, R, \gamma, d)$ and the explicit expression of desired state estimator gains are given in terms of the feasibility of a linear matrix inequality. From Theorem 3, we can observe that the H_∞ performance requirement and the network resource usage are related to the event-triggered parameters and the quantization method. The triggered parameters σ_i can be adjusted according to performance of the Markovian jump systems.

Remark 9. The gain can be determined by equality $K_i = P_{2i}^{-1}Y_i$. It should be pointed out that the designed state estimator is constrained by inequalities (42), (43) and (44). Noticed that when inequalities (42), (43) and (44) are feasible, the matrix Ω_i, P_{2i} and Y_i are obtained. Then, the gain matrix of the state estimator can be derived from $K_i = P_{2i}^{-1}Y_i$. The gain of the state estimator is dependent on the feasible solution of the inequalities (42), (43) and (44).

Remark 10. In many practical applications, the minimum value of $\gamma^2 + c_2^2$ is expected. The feasible conditions in Theorem 3 can be described as the following optimization problem

$$\min \varrho_1\gamma^2 + \varrho_1 c_2^2$$
$$s.\,t.\ (42-44) \tag{52}$$

where ϱ_1 and ϱ_2 are relative weighting coefficients.

4. Simulation examples

To validate the effectiveness and feasibility of the proposed method, we operate the following example. Consider the following networked Markov jump system involving two modes with the following parameters:

$$A_1 = \begin{bmatrix} -0.8 & 0 \\ 0.8 & -1 \end{bmatrix},\quad A_{\omega 1} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix},\quad C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad L_1 = \begin{bmatrix} 0.2 & 0.15 \\ 0.1 & 0.3 \end{bmatrix}$$
$$A_2 = \begin{bmatrix} -0.5 & 1 \\ 0 & -1 \end{bmatrix},\quad A_{\omega 2} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix},\quad C_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.6 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 0.2 & 0.1 \\ -0.2 & 0.4 \end{bmatrix}$$

The nonlinear functions are chosen as

$$h_1 = \begin{bmatrix} 0.04x_1 - \tan(0.3x_1) + 0.15x_2 \\ 0.09x_2\tan(0.7x_2) \end{bmatrix},\quad g_1 = \begin{bmatrix} -0.3x_1\tan(0.2x_1) \\ 0.8x_2 + \tan(0.6x_2) \end{bmatrix}$$
$$h_2 = \begin{bmatrix} 0.4x_1 - \tan(0.3x_1) \\ 0.9x_2 - \tan(0.7x_2) \end{bmatrix},\quad g_2 = \begin{bmatrix} 0.3x_1 - \tan(0.2x_1) \\ 0.8x_2 - \tan(0.6x_2) \end{bmatrix}$$

It is easy to see that the constraint (2) can be met with

$$\phi_{11}^h = \begin{bmatrix} -0.06 & 0 \\ 0 & 0 \end{bmatrix},\quad \phi_{21}^h = \begin{bmatrix} 0 & 0 \\ 0 & 0.02 \end{bmatrix},\quad \phi_{11}^g = \begin{bmatrix} 0 & 0 \\ 0 & 0.04 \end{bmatrix},$$
$$\phi_{21}^g = \begin{bmatrix} -0.02 & 0 \\ 0 & 0 \end{bmatrix}$$
$$\phi_{12}^h = \begin{bmatrix} 0 & 0 \\ 0 & 0.01 \end{bmatrix},\quad \phi_{22}^h = \begin{bmatrix} 0.02 & 0 \\ 0 & 0 \end{bmatrix},\quad \phi_{12}^g = \begin{bmatrix} -0.03 & 0 \\ 0 & 0 \end{bmatrix},$$
$$\phi_{22}^g = \begin{bmatrix} 0 & 0 \\ 0 & 0.01 \end{bmatrix}$$

The exogenous disturbance inputs are selected as

$$\omega(t) = \begin{cases} 0.5, & 5 \le t \le 10 \\ -0.5, & 15 \le t \le 20 \\ 0, & else \end{cases}$$

The switching between the two modes is described by the transition probability matrix $\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}$. The initial state is given as $x_0 = \begin{bmatrix} 0.3 & -0.3 \end{bmatrix}^T$.

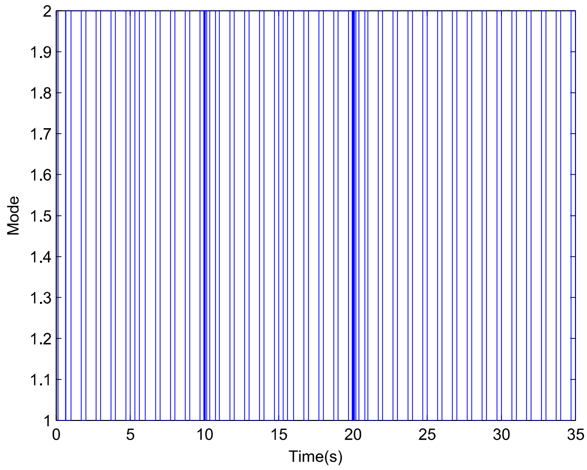

Fig. 2. The probabilities of switching between modes.

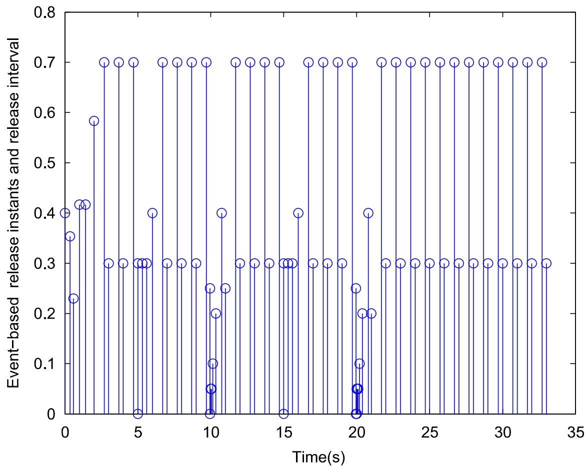

Fig. 3. Release instants and intervals in case 1.

With the parameters given above, it is aimed to co-design an event triggered scheme (5) and a state estimator (4) for the Markovian jump systems (1). In the following, we will consider two possible cases which are used to illustrate the impact of the triggered parameters on the system performance. Case 1 employs the same triggering parameters. Case 2 employs the dynamically adjusted parameters. For the convenience of analysis, the values of $\bar{\alpha}$, δ_{q_s}, τ_M, d, ε_1, ε_2, θ_1, θ_2, β, R and $T$are chosen the same for the following two cases except the triggering parameters σ_1 and σ_2.

Case 1: Setting the dynamically adjusted triggering parameters $\sigma_1 = 0.25$, $\sigma_2 = 0.15$, for given $\bar{\alpha} = 0.8$, $\delta_{q_s} = 0.9$, $\tau_M = 0.1$, d=2.5, $\varepsilon_1 = 1$, $\varepsilon_2 = 1$, $\theta_1 = 1$, $\theta_2 = 1$, $\beta = 0.5$, $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, T=3, based on Matlab/LMIs toolbox and applying Theorem 3, combine (42)–(44) and $K_i = P_{2i}^{-1}Y_i$, we can get $\gamma = 3.3391$, c_2=7.0328 ($\varrho_1 = 1$, $\varrho_2 = 1$), and the desired estimator parameters and event-triggered matrix as follows

$$K_1 = \begin{bmatrix} -0.0516 & -0.0506 \\ -0.0370 & 0.0244 \end{bmatrix},\quad K_2 = \begin{bmatrix} -0.0387 & -0.0592 \\ -0.0379 & 0.0205 \end{bmatrix}$$
$$\Omega_1 = \begin{bmatrix} 5.4640 & -0.4325 \\ -0.4325 & 5.9486 \end{bmatrix},\quad \Omega_2 = \begin{bmatrix} 5.8080 & -0.2328 \\ -0.2328 & 5.9777 \end{bmatrix}$$

The probabilities switching between modes can be seen from Fig. 2. The release instants and intervals are shown in Fig. 3. In the

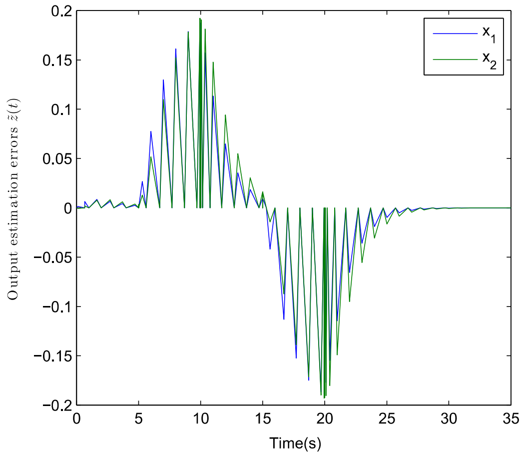

Fig. 4. Output estimation errors $\tilde{z}(t)$ in case 1.

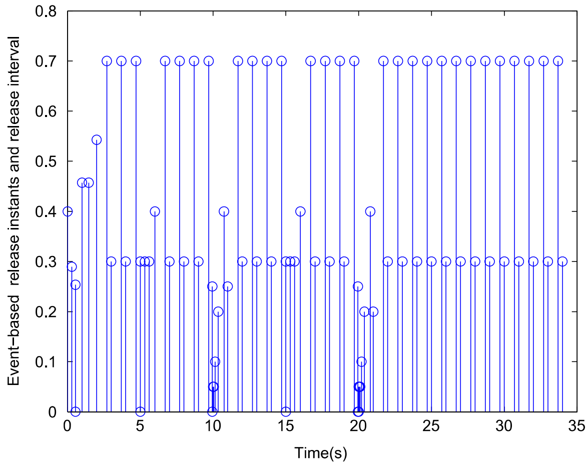

Fig. 6. Release instants and intervals in case 2.

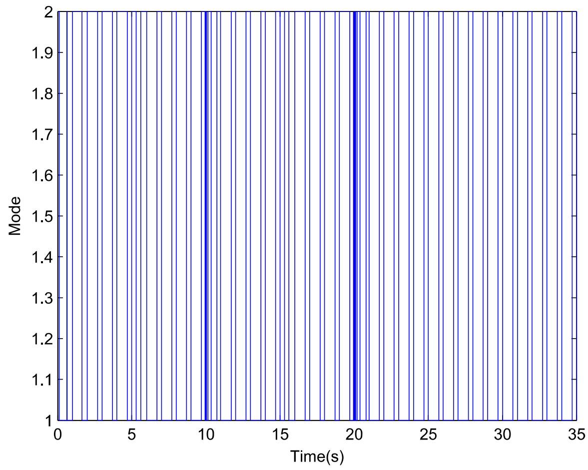

Fig. 5. The probabilities of switching between modes in case 2.

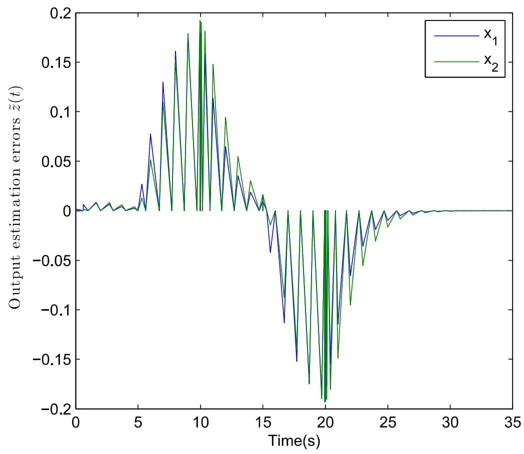

Fig. 7. Output estimation errors $\tilde{z}(t)$ in case 2.

simulation of 35 s, only 88 sampled data are released, which takes 25.1% of the sampled signals.

The estimation output error $\tilde{z}(t)$ is shown in Fig.4, where the estimation output error converges to zero.

Case 2: Setting the same triggering parameters are chosen as $\sigma_1 = \sigma_2 = 0.25$, for given $\bar{\alpha} = 0.95$, $\delta_{q_s} = 0.9$, $\tau_M = 0.1$, d=2.5, $\varepsilon_1 = 1$, $\varepsilon_2 = 1$, $\theta_1 = 1$, $\theta_2 = 1$, $\beta = 0.5$, $R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, T=3, we can obtain $\gamma = 3.3364$, $c_2 = 7.0283(\varrho_1 = 1, \varrho_2 = 1)$, the desired estimator parameters and event-triggered matrix are derived as follows

$$K_1 = \begin{bmatrix} -0.0515 & -0.0508 \\ -0.0370 & 0.0248 \end{bmatrix}, \quad K_2 = \begin{bmatrix} -0.0389 & -0.0593 \\ -0.0380 & 0.0206 \end{bmatrix}$$

$$\Omega_1 = \begin{bmatrix} 5.4582 & -0.4328 \\ -0.4328 & 5.9416 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} 5.4239 & -0.3698 \\ -0.3698 & 5.6851 \end{bmatrix}$$

Fig.5 depicts a possible system mode evaluation. The release instants and intervals are given in Fig.6. The simulation results for $t \in [0, 35]$ show that 92 sampled data are transmitted, which takes 26.3% of the sampled signals. Fig.7 shows the estimation output error, which indicates that the estimation output error converges to zero.

From Figs. 4 and 7, it can be seen that the estimation error system is stochastically H_∞ finite-time bounded and the designed estimator is effective and feasible. Compared with the time-triggered scheme, 25.1% and 26.3% sampled data are transmitted for case 1 and case 2, respectively. The simulation results in case 1 and case 2 show that the event-triggered scheme can reduce the communication load in the network-based MJSs, the maximum release interval of the event generator is 0.7 s. Moreover, Figs. 3 and 6 illustrate that the number of the sampled-data by dynamic triggered scheme is less than those by making use the same event-triggered scheme. The obtained results in this article not only reduce the network transmission frequency while preserve the desired performance but also guarantee the estimation error dynamic system stochastically H_∞ finite-time bounded.

Remark 11. It should be observed that the novelty of the results in this paper pays more attention to finite-time state estimation and the limited network resources in Markovian jump systems with randomly occurring nonlinearities. An modified event-triggered method is employed to save the network resources. The similar event-triggered scheme was firstly proposed in [12], however, the authors only considered the controller design problem. They didn't consider the finite-time state estimation for Markovian jump systems with randomly occurring nonlinearities. From the simulation results, we can see that the state estimator design method obtained in this paper is effective.

5. Conclusion

In this paper, the problem of event-based finite-time state estimation is investigated for Markovian jump systems with quantizations and randomly occurring nonlinearities. An event triggered communication scheme and a quantizer are introduced into the framework to reduce the network bandwidth utilization. Sufficient conditions of stochastically finite-time boundedness and stochastically H_∞ finite-time boundedness are established for the estimation error system. Furthermore, the explicit expressions of the desired estimator gains are derived. A simulation example has highlighted the usefulness of the proposed method. Future research directions will include the problems of event-based finite-time H_∞ filtering and event-based non-fragile finite-time output feedback H_∞ control for Markovian jump systems with network-induced time delays.

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