



Reliable control for hybrid-driven T–S fuzzy systems with actuator faults and probabilistic nonlinear perturbations

Lijuan Zha^a, Jian-an Fang^{a,*}, Jinliang Liu^b, Engang Tian^c

^a College of Information Science and Technology, Donghua University, Shanghai, PR China

^b College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, Jiangsu, PR China

^c School of Electrical and Automation Engineering, Nanjing Normal University, Nanjing, Jiangsu, PR China

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Abstract

This paper is concerned with the reliable controller design for hybrid-driven nonlinear systems via T–S fuzzy model with probabilistic actuator faults and probabilistic nonlinear perturbations. To reduce unnecessary transmissions in the network, a hybrid-driven scheme is introduced, in which the communication transmission scheme can be selected between time-driven scheme and event-driven scheme. The switch law of the communication schemes is modeled as Bernoulli distributed stochastic variable. Considering the network-induced delay, nonlinear perturbations and the probabilistic actuator faults, a new model is constructed under the hybrid driven scheme. By using Lyapunov–Krasovskii functionals and stochastic analysis techniques, sufficient conditions are established which ensure the asymptotical stability of the augmented system under hybrid driven scheme. Furthermore, a unified co-design algorithm for the desired controller and the hybrid-driven scheme are developed. Finally, a typical simulation example is used to demonstrate the validity of obtained results.

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* Corresponding author.

E-mail address: jafang@dhu.edu.cn (J.-a. Fang).

1. Introduction

During the past few years, fuzzy systems based on Takagi–Sugeno (T–S) model have been widely recognized as an efficient approach to approximate nonlinear systems with arbitrary precision. Research in modeling and control of nonlinear systems by T–S fuzzy model has received a great deal of attention, because they can be analyzed by the linear system theories and described by a family of IF-THEN rules. There are some outstanding results reported [1–9]. In [1], the authors concentrate on the fault detection filtering problem for nonlinear switched stochastic system in the T–S fuzzy framework. In [3], the authors investigate network-based output tracking control for a T–S fuzzy system that cannot be stabilized by a non-delayed fuzzy static output feedback controller. The authors study the distributed event-triggered H_∞ filtering for a class of nonlinear system in [5]. The authors investigate the reliable filter design with strict dissipativity for a class of T–S fuzzy time-delay systems in [9]. It should be observed that the premises in the controlled fuzzy plant and the one in the fuzzy controller are often asynchronous due to the presence of network-induced delays. How to deal with the asynchronous premises in networked control systems is still an open problem. The existing results are not enough, more effort should be paid to deal with this problem, which motivates the present investigation.

Due to the increasing number of components in practical systems, the presence of actuator faults is unavoidable, which may result in deterioration of system performance and even instability of the closed-loop system. Therefore, it is essential and significant to increase system reliability and security. The studies of reliable control scheme have received considerable attention of control community and lots of outstanding results have been obtained [10–12]. In [10], the problem of adaptive fault tolerant tracking controller for a class of uncertain nonlinear systems with input quantization and actuator faults is investigated. In [11], the fault-tolerant stabilization problem has been studied for a class of nonlinear systems with uncertain parameters. The authors in [12] deal with the problem of reliable and robust H_∞ static output feedback controller synthesis for continuous-time nonlinear stochastic systems with actuator faults. Despite these efforts, the reliable control for T–S fuzzy systems with probabilistic nonlinear perturbation under hybrid-driven scheme has not drawn any attention in the present literature.

Networked control systems have become an active research area in recent years due to the advantage of simple installation, low cost, easier maintenance, etc. However, the introduction of communication networks can bring some problems. Especially, due to the limited network bandwidth, the phenomena of time delay, packet dropout and noise interference are often encountered, which may lead to performance degradation and system instability. Fortunately, much effort has been made on how to reduce the amount of network transmission and improve the efficient of network utilization. Different strategies have been proposed in the existing publications [13–20]. Generally speaking, the researches tackling with this problem can be classified into two categories:(i) The first one is the time-driven scheme. With this method, network transmission is assigned periodically. The fixed sampling interval is chosen to guarantee a desired performance under the worst conditions such as external disturbances, uncertainties, time-delays and so on. However, when a control system runs in a steady state, there is no need to update the measurement and control signal, such a method will lead to unnecessary waste of network resources. (ii) The second one is the event-driven scheme. In order to overcome the drawback of the time-driven scheme, the event-driven scheme has been proposed in [19], in which the task is executed only when the pre-defined condition

is satisfied. The event-driven scheme has been proved to be an effective way in saving the network resource while maintains satisfactory system performance. The main advantages of the event-driven scheme can be summarized as low transmission frequency, reduction in the release times of the sensor and reduction in calculation cost of controller. Recently, on the basis of the work in [19], different problems have been investigated under different event-driven schemes [13–18,20]. For example, the authors in [13] investigate decentralized control for a class of interconnected system. In [17], the authors are concerned with H_∞ filter design for a class of neural network systems with event-triggered communication scheme and quantization.

Different from above works, taking the random actuator faults and probabilistic nonlinear perturbations into consideration, the reliable control for hybrid-driven T–S fuzzy systems is investigated in this paper. Compared with some past works, the adopted hybrid-driven scheme pays attention to the variation of the network loads, which is more flexible in saving the network resources. In our hybrid-driven scheme, a Bernoulli distributed stochastic variable is used to describe the access status of time-driven scheme and the event-driven scheme. Such a method is suitable to guarantee the desired performance of networked control systems under the worst conditions of external disturbances, uncertainties, time-delays and so on. The hybrid-driven scheme is also capable of improving the network resources utilization while maintaining satisfactory system performance especially when there is no negligible measurement variation.

However, to the best of our knowledge, no investigations have been involved about the problem of reliable control for hybrid-driven T–S fuzzy systems with probabilistic nonlinear perturbations under hybrid-driven scheme. Compared with the existing researches, the reliable control is firstly addressed for hybrid-driven T–S fuzzy systems with random actuator faults and probabilistic nonlinear perturbations. The main contributions of this paper are as follows: (1) A Bernoulli distributed stochastic variable is used to describe the switch between different communication schemes in the hybrid-driven scheme, which is efficient to make full use of the network capacity. (2) Considering the actuator faults, probabilistic nonlinear perturbations and network-induced delays, a new model is established under the proposed hybrid-driven scheme. (3) Based on the model, sufficient conditions are obtained to ensure the desired system performance and the explicit design method of controller gains are derived. (4) The premises in the controlled plant and the one in the fuzzy controller operate in an asynchronous way in this paper.

The rest of this paper is organized as follows. In Section 2, the issue on the implementation of the hybrid-driven scheme is presented. Section 3 gives the sufficient conditions for the stabilization of the augmented system. Furthermore, the explicit controller design method is presented. An illustrative example is provided in Section 4 to show the effectiveness of the obtained results. Finally, Section 5 concludes the paper.

Notation: \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote the n -dimensional Eculidean space, and the set of $n \times m$ real matrices; the superscript T stands for matrix transposition; I is the identity matrix of appropriate dimension; the notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n \times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite); $Prob\{X\}$ denotes probability of event X to occur; $Sym\{X\}$ denotes the expression $X^T + X$; \mathcal{E} denotes the expectation operator; for a matrix B and two symmetric matrices A and C , $\begin{bmatrix} A & \\ & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

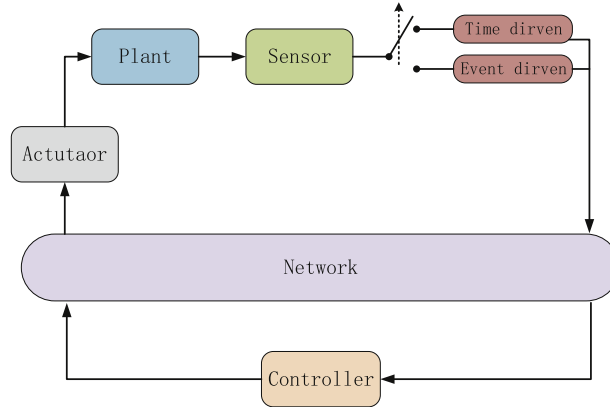


Fig. 1. The structure of T–S fuzzy systems under hybrid-driven scheme.

2. System description

The control problem with unreliable communication links and probabilistic nonlinear perturbations is shown in Fig. 1, in which the sensor is clock driven, the controller and the actuator are event driven. The nonlinear control system can be described by the following T–S fuzzy system:

Plant Rule i : IF $\theta_1(t)$ is M_1^i , and $\theta_2(t)$ is M_2^i , ..., and $\theta_p(t)$ is M_p^i , then

$$\dot{x}(t) = A_i x(t) + B_i u(t) + \beta(t) E_i h(x) + (1 - \beta(t)) F_i g(x) \tag{1}$$

where $i = 1, \dots, r$, $x(t) \in R^n$ denotes the system state vector, $u(t) \in R^m$ is the control input, $\theta_i(t)$ denotes the vector of fuzzy premise variables, $\theta(t) = [\theta_1(t) \ \theta_2(t) \ \dots \ \theta_p(t)]^T$, $M_j^i (j = 1, \dots, p)$ are the fuzzy sets, A_i, B_i, E_i and $F_i (i = 1, 2, \dots, r)$ are constant matrices of appropriate dimensions. The initial condition of the system (1) is given by $x(t_0) = x_0$; $h(x)$ and $g(x)$ are nonlinear functions satisfying

$$\begin{cases} \|h(x)\|_2 \leq \|\Phi_1 x(t)\|_2 \\ \|g(x)\|_2 \leq \|\Phi_2 x(t)\|_2 \end{cases} \tag{2}$$

where Φ_1 and Φ_2 are real matrices with compatible dimensions, $h(x)$ and $g(x)$ are the abbreviation of $h(t, x(t))$ and $g(t, x(t))$, respectively. $\beta(t)$ is a Bernoulli distributed random variable, which is assumed to be known prior through statistical tests

$$\mathcal{E}\{\beta(t)\} = Prob\{\beta(t) = 1\} = \bar{\beta}$$

Remark 1. Nonlinear perturbations are ubiquitous in practical systems. In this paper, a Bernoulli distributed random variable is used to distinguish the different probabilities of $h(x)$ and $g(x)$ occurring. For example, when $h(x)$ and $g(x)$ have the same lower and upper bounds, the random variable can describe the variation of the nonlinear perturbations. Similar nonlinearities can be found in [13,21].

By using a center-average defuzzifier, product inference and a singleton fuzzifier, the global dynamic of the system (1) is inferred as

$$\dot{x}(t) = \sum_{i=1}^r h_i(\theta(t)) [A_i x(t) + B_i u(t) + \beta(t) E_i h(x) + (1 - \beta(t)) F_i g(x)] \tag{3}$$

where $h_i(\theta(t)) = \frac{\mu_i(\theta(t))}{\sum_{j=1}^r \mu_j(\theta(t))}$, $\mu_i(\theta(t)) = \prod_{j=1}^p M_j^i(\theta_j(t))$, $M_j^i(\theta_j(t))$ is the membership value of $\theta_j(t)$ in M_j^i . For notational simplicity, we use h_i to represent $h_i(\theta(t))$.

As shown in Fig. 1, the system (1) is controlled over a communication network, hybrid-driven scheme is adopted, that is, time-driven scheme and event-driven scheme can work in different frequencies [22]. In this paper, we use a Bernoulli distributed random variable $\alpha(t)$ to describe the switch law between the two communication schemes. The information of $\alpha(t)$ can be obtained through statistical method. Define

$$\begin{cases} \alpha(t) = 1, & \text{if time-driven scheme is active} \\ \alpha(t) = 0, & \text{if event-driven scheme is active} \end{cases} \tag{4}$$

where $\mathcal{E}\{\alpha(t)\} = Prob\{\alpha(t) = 1\} = \bar{\alpha}$. At each sampling instant, only one of the two communication strategies is active.

Remark 2. The hybrid-driven scheme adopted in this paper is aimed to reduce the limited network resource and deal with the unavoidable occurrence of worst conditions (such as external disturbances and time-delays).

In networked control systems, due to the existence of the network-induced delays in sensor to controller path, the available premise variables for the fuzzy rules in the controller side and the premises in the system may be asynchronous. Specially, for $t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}]$, in (3), the premise variables is $\theta_i(t)$, only $\theta_i(t_k h)$ is available at the controller. In view of this, the controller $u(t)$ can be designed in a form of

Control rule i : IF $\theta_1(t_k h)$ is M_1^i , and $\theta_2(t_k h)$ is M_2^i , ..., and $\theta_p(t_k h)$ is M_p^i , then

$$u(t) = K_i x(t_k h), t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}] \tag{5}$$

where $K_i (i = 1, \dots, r)$ are controller gains to be determined, τ_{t_k} is the network-induced delay, and $t_k h$ is the time stamp of latest control signal received by the actuator.

Remark 3. Due to the presence of network-induced delays, it is unreasonable to employ the identical premise variables in the fuzzy plant and the fuzzy controller. The asynchronous characteristic between the plant and the controller leads to mismatch membership functions, which makes the analysis and the design of the system more complex. Fortunately, taking the asynchronous operation and network-induced delays into consideration, different new methods have been proposed [23–26]. Based on these works, the premises in the controlled plant and the one in the fuzzy controller operate in an asynchronous way in this paper.

In the following, based on (5), the controller design method of the hybrid-driven system will be given step by step considering the hybrid-driven scheme, random actuator faults and communication delay.

Assumption 1. In this technical note, the probability of the time-driven scheme (or the event-driven scheme) activated is assumed to be known a priori.

Assumption 2. Throughout this paper, τ_{t_k} is network-induced delays, which contains the delay from the sensor to the controller and from the controller to the actuator, and the computational and waiting delays. $x(t_k h)$ denotes the set of sensor measurements which are successfully arrive at the actuator. $x(t_0 h)$ is the initial condition.

Remark 4. The sojourn probability $\bar{\alpha}$ can be derived through the following statistical method [27]:

$$\bar{\alpha} = \lim_{n \rightarrow \infty} \frac{k_i}{n}$$

where $k_i, n \in \mathbb{Z}^+, k_i$ is the times of $\alpha(t) = 1$ in the interval $[1, n]$.

If the time-driven scheme is employed, the corresponding sensor measurement will be sent out to the controller through communication network periodically. Considering the network-induced time delay, similar to [28], let $d(t) = t - t_k h$, this leads to $0 \leq \tau_{t_k} \leq d(t) \leq t_{k+1} h - t_k h + \tau_{t_{k+1}} \triangleq d_M$. Then, the overall controller can be represented in the following form

$$u(t) = \sum_{j=1}^r h_j(\theta(t_k h)) K_j x(t - d(t)), t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}] \tag{6}$$

In order to reduce the unnecessary data transmission, an alternative event-driven scheme is introduced in the other channel, whether or not the current sampled data can be send out is dependent on the following condition [19]:

$$f(e_k(t), \sigma) = e_k^T(t) \Omega e_k(t) - \sigma^2 x^T(t_k h) \Omega x(t_k h) \leq 0 \tag{7}$$

where $e_k(t) = x(t_k h) - x(t_k h + jh)$, $x(t_k h)$ is the latest transmitted measurement, $\sigma \in [0, 1)$, Ω is the matrix with appropriate dimension. The current sampled data $x(t_k h + jh)$ can be transmitted as long as the state error $e_k(t)$ violates condition (7).

Similar to [20], the holding interval of the ZOH $t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}]$ can be divided into $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}] = \bigcup_{l=0}^d \Upsilon(l, k)$, $\Upsilon(l, k) = [t_k h + lh + \tau_{t_{k+l}}, t_k h + lh + h + \tau_{t_{k+l+1}}]$, $l = 1, \dots, d, d = t_{k+1} - t_k - 1$. Set $\tau(t) = t - t_k h - lh, 0 \leq \tau_{t_k} \leq \tau(t) \leq h + \tau_{t_{k+l+1}} \triangleq \tau_M$, the transmitted state can be written as $x(t_k h) = x(t - \tau(t)) + e_k(t)$, then, the defuzzified output of the controller is

$$u(t) = \sum_{j=1}^r h_j(\theta(t_k h)) K_j [x(t - \tau(t)) + e_k(t)], t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}] \tag{8}$$

In order to cope with the effect of hybrid-driven scheme, the following controller is utilized:

$$u(t) = \alpha(t) \sum_{j=1}^r h_j(\theta(t_k h)) K_j x(t - d(t)) + (1 - \alpha(t)) \sum_{j=1}^r h_j(\theta(t_k h)) K_j [x(t - \tau(t)) + e_k(t)], t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}] \tag{9}$$

Remark 5. Because the hybrid-driven scheme is utilized, the controller in (9) is more general. When $\alpha(t) = 1$, the signal $x(t_k h)$ received by the controller can be expressed as $x(t - d(t))$ under the time-driven scheme; when $\alpha(t) = 0$, the signal $x(t_k h)$ received by the controller can be rewritten as $x(t - \tau(t)) + e_k(t)$ under the event-driven scheme.

Remark 6. Time-driven scheme and the event-driven scheme can be selected according to the actual network loads and the system performance. When network induced delay and network congestion are unacceptable, the event-driven scheme can be chosen to save the network bandwidth. Especially, when the control system is in a steady state, the energy utilization of the related components can be reduced under event-driven scheme. Otherwise, when the fluctuation of the state is very obvious, the time-driven scheme can be selected.

Similar to the existing method in [28] and [29], the random failure of multi-actuators in the system is modeled as

$$u^F(t) = Wu(t) \tag{10}$$

where $W = \text{diag}\{w_1(t), w_2(t), \dots, w_m(t)\}$ with $w_s(t) (s = 1, \dots, m)$ being m uncorrelated random variables taking values on the interval $[0, \vartheta]$, in which ϑ is a constant and $\vartheta \geq 1$. The expectations and variances of $w_s(t)$ are \bar{w}_s and $\delta_{w_s}^2$. From the definitions of W , we can get $\mathcal{E}\{W\} = \text{diag}\{\bar{w}_1, \bar{w}_2, \dots, \bar{w}_m\} = \sum_{s=1}^{s=m} \bar{w}_s D_s$, $D_s = \text{diag}\{\underbrace{0, \dots, 0}_{s-1}, 1, \underbrace{0, \dots, 0}_{m-s}\}$

Remark 7. Actuator failure is often encountered in the system, which may result in unbearable system performance and even instability of the system. In this paper, a stochastic modeling method is used in (10). Particularly, $w_i(t) = 1$ means there is no fault in the i th actuator at time t ; $w_i(t) = 0$ means complete failure of the i th actuator or packet loss during the transmission from the controller to the actuator; $w_i(t) \in (0, 1)$ or $w_i(t) \in (1, \vartheta)$ means partial fault or data distortion occurring, that is, the received signal at the i th actuator may be either smaller or larger than the real one.

Combing (3), (9) and (10), the closed-loop nonlinear networked control system becomes

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h)) \{A_i x(t) + \alpha(t)B_i W K_j x(t - d(t)) \\ & + (1 - \alpha(t))B_i W K_j [x(t - \tau(t)) + e_k(t)] \\ & + \beta(t)E_i h(x) + (1 - \beta(t))F_i g(x)\}, \quad t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}] \end{aligned} \tag{11}$$

From the definitions of W , $\alpha(t)$ and $\beta(t)$, (11) can be rewritten as

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h)) \{A_{ij} + (\alpha(t) - \bar{\alpha})B_i \bar{W} A_{ij2} \\ & + B_i (W - \bar{W}) [A_{ij1} + (\alpha(t) - \bar{\alpha})A_{ij2}] + A_{ij3}\}, \quad t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}] \end{aligned} \tag{12}$$

where

$$\begin{aligned} A_{ij} &= A_i x(t) + B_i \bar{W} A_{ij1} + \bar{\beta}(t)E_i h(x) + (1 - \bar{\beta}(t))F_i g(x) \\ A_{ij1} &= \bar{\alpha}(t)K_j x(t - d(t)) + (1 - \bar{\alpha}(t))K_j [x(t - \tau(t)) + e_k(t)] \\ A_{ij2} &= K_j x(t - d(t)) - K_j x(t - \tau(t)) - K_j e_k(t) \\ A_{ij3} &= \beta(t)E_i h(x) + (1 - \beta(t))F_i g(x) \end{aligned}$$

Under the hybrid-driven scheme, the purpose of this paper is to design the reliable controller and the triggering parameters to stabilize the T–S fuzzy systems with random nonlinear perturbations, communication delay and probabilistic actuator faults. In order to deal with this

problem, similar to [25,26], we assume the asynchronous errors of the grades of memberships satisfy the following inequality

$$|h_j(\theta(t)) - h_j(\theta(t_kh))| \leq \mu_p, p = 1, 2, \dots, r \tag{13}$$

Lemma 1. [30] For any vectors $x, y \in R^n$, and positive definite matrix $Q \in R^n \times n$, the following inequality holds:

$$2x^T y \leq x^T Qx + y^T Q^{-1}y$$

Lemma 2. [31] Suppose $\tau(t) \in [\tau_m, \tau_M]$, $d(t) \in [0, d_M]$, $\Xi_1, \Xi_2, \Xi_3, \Xi_4$ and Ω are matrices with appropriate dimensions, then

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + d(t)\Xi_3 + (d_M - d(t))\Xi_4 + \Omega < 0 \tag{14}$$

if and only if

$$(\tau_M - \tau_m)\Xi_1 + d_M\Xi_3 + \Omega < 0$$

$$(\tau_M - \tau_m)\Xi_2 + d_M\Xi_3 + \Omega < 0$$

$$(\tau_M - \tau_m)\Xi_1 + d_M\Xi_4 + \Omega < 0$$

$$(\tau_M - \tau_m)\Xi_2 + d_M\Xi_4 + \Omega < 0$$

3. Main results

In this section, we will develop an approach for deriving stability conditions and controller design for the T–S fuzzy closed-loop system (12). We now state and establish the following result.

Theorem 1. For given scalars $\bar{\alpha}, \bar{\beta}, \tau_m, d_M, w_s, \delta_a, \delta_b, \delta_{ws}(s = 1, \dots, m), \sigma, \mu_p$ ($p = 1, 2, \dots, r$) and matrix K_j , under the hybrid-driven scheme (4), the augmented system (12) is asymptotically stable if there exist positive matrix $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, \Omega > 0, U, M_{ij}, N_{ij}, T_{ij}$ and S_{ij} with appropriate dimensions such that for $l = 1, 2, 3, 4$

$$\Phi^{ii}(l) < 0, i = 1, 2, \dots, r \tag{15}$$

$$\Phi^{ij}(l) + \Phi^{ji}(l) < 0, i, j = 1, 2, \dots, r \tag{16}$$

$$U + \Xi^{ij}(l) > 0, i, j = 1, 2, \dots, r \tag{17}$$

where

$$\Xi^{ij}(l) = \begin{bmatrix} \Xi_{11} + \Gamma + \Gamma^T & * & * & * & * & * \\ \Xi_{21} & \Xi_{22} & * & * & * & * \\ \Xi_{31} & 0 & \Xi_{33} & * & * & * \\ \Xi_{41} & 0 & 0 & \Xi_{44} & * & * \\ \Xi_{51} & 0 & 0 & 0 & \Xi_{55} & * \\ \Xi_{61}(l) & 0 & 0 & 0 & 0 & \Xi_{66} \end{bmatrix} < 0$$

$$\Xi_{11} = \begin{bmatrix} PA_i + A_i^T P + Q_1 + Q_2 & * & * & * & * & * & * & * \\ \hat{\alpha} K_j^T \bar{W}^T B_i^T P & 0 & * & * & * & * & * & * \\ 0 & 0 & -Q_1 & * & * & * & * & * \\ \hat{\alpha} K_j^T \bar{W}^T B_i^T P & 0 & 0 & \sigma^2 \Omega & * & * & * & * \\ 0 & 0 & 0 & 0 & -Q_2 & * & * & * \\ \hat{\alpha} K_j^T \bar{W}^T B_i^T P & 0 & 0 & 0 & 0 & -\Omega & * & * \\ \hat{\beta} E_i^T P & 0 & 0 & 0 & 0 & 0 & -P & * \\ \hat{\beta} F_i^T P & 0 & 0 & 0 & 0 & 0 & 0 & -P \end{bmatrix}$$

$$\Gamma = [M_{ij} + T_{ij} \quad -M_{ij} + N_{ij} \quad -N_{ij} \quad -T_{ij} + S_{ij} \quad -S_{ij} \quad 0 \quad 0 \quad 0], \quad \mathcal{B}_{ij} = PB_i \bar{W} K_j$$

$$\hat{\alpha} = 1 - \bar{\alpha}, \hat{\beta} = 1 - \bar{\beta}, \delta_a = \sqrt{\bar{\alpha}\hat{\alpha}}, \delta_b = \sqrt{\bar{\beta}\hat{\beta}}, \Phi^{ij}(l) = \Xi^{ij}(l) + \sum_{p=1}^r \mu_p (U + \Xi^{ip}(l))$$

$$\Xi_{21} = \begin{bmatrix} \sqrt{d_M} PA_i & \sqrt{d_M} \bar{\alpha} \mathcal{B}_{ij} & 0 & \sqrt{d_M} \hat{\alpha} \mathcal{B}_{ij} & 0 & \sqrt{d_M} \hat{\alpha} \mathcal{B}_{ij} & \sqrt{d_M} \hat{\beta} PE_i & \sqrt{d_M} \hat{\beta} PF_i \\ \sqrt{\tau_M} PA_i & \sqrt{\tau_M} \bar{\alpha} \mathcal{B}_{ij} & 0 & \sqrt{\tau_M} \hat{\alpha} \mathcal{B}_{ij} & 0 & \sqrt{\tau_M} \hat{\alpha} \mathcal{B}_{ij} & \sqrt{\tau_M} \hat{\beta} PE_i & \sqrt{\tau_M} \hat{\beta} PF_i \end{bmatrix}$$

$$\Xi_{31} = \begin{bmatrix} 0 & \sqrt{d_M} \delta_a \mathcal{B}_{ij} & 0 & -\sqrt{d_M} \delta_a \mathcal{B}_{ij} & 0 & -\sqrt{d_M} \delta_a \mathcal{B}_{ij} & 0 & 0 \\ 0 & \sqrt{\tau_M} \delta_a \mathcal{B}_{ij} & 0 & -\sqrt{\tau_M} \delta_a \mathcal{B}_{ij} & 0 & -\sqrt{\tau_M} \delta_a \mathcal{B}_{ij} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_b \sqrt{d_M} PE_i & -\delta_b \sqrt{d_M} PF_i \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_b \sqrt{\tau_M} PE_i & -\delta_b \sqrt{\tau_M} PF_i \end{bmatrix}$$

$$\Xi_{22} = \text{diag}\{-PR_1^{-1}P, -PR_2^{-1}P\}, \quad \Xi_{33} = \text{diag}\{-PR_1^{-1}P, -PR_2^{-1}P, -PR_1^{-1}P, -PR_2^{-1}P\}$$

$$\Xi_{41} = \begin{bmatrix} 0 & \bar{\alpha} \sqrt{d_M} \bar{D} & 0 & \hat{\alpha} \sqrt{d_M} \bar{D} & 0 & \hat{\alpha} \sqrt{d_M} \bar{D} & 0 & 0 \\ 0 & \bar{\alpha} \sqrt{\tau_M} \bar{D} & 0 & \hat{\alpha} \sqrt{\tau_M} \bar{D} & 0 & \hat{\alpha} \sqrt{\tau_M} \bar{D} & 0 & 0 \\ 0 & \delta_a \sqrt{d_M} \bar{D} & 0 & -\delta_a \sqrt{d_M} \bar{D} & 0 & -\delta_a \sqrt{d_M} \bar{D} & 0 & 0 \\ 0 & \delta_a \sqrt{\tau_M} \bar{D} & 0 & -\delta_a \sqrt{\tau_M} \bar{D} & 0 & -\delta_a \sqrt{\tau_M} \bar{D} & 0 & 0 \end{bmatrix}, \quad \bar{D} = \begin{bmatrix} \delta_{w1} PB_i D_1 K_j \\ \vdots \\ \delta_{w1} PB_i D_m K_j \end{bmatrix}$$

$$\Xi_{44} = \text{diag}\{\mathcal{R}_1, \mathcal{R}_2, \mathcal{R}_1, \mathcal{R}_2\}, \quad \mathcal{R}_k = \text{diag}\{\underbrace{-PR_k^{-1}P, \dots, -PR_k^{-1}P}_m\}, k = 1, 2$$

$$\Xi_{51} = \begin{bmatrix} P\Phi_1 & 0_{1 \times 7} \\ P\Phi_2 & 0_{1 \times 7} \end{bmatrix}, \quad \Xi_{55} = \text{diag}\{-P, -P\}, \quad \Xi_{66} = \text{diag}\{-R_1, -R_2\}$$

$$\Xi_{61}(1) = \begin{bmatrix} \sqrt{d_M} M_{ij}^T \\ \sqrt{\tau_M} S_{ij}^T \end{bmatrix}, \quad \Xi_{61}(2) = \begin{bmatrix} \sqrt{d_M} M_{ij}^T \\ \sqrt{\tau_M} T_{ij}^T \end{bmatrix}, \quad \Xi_{61}(3) = \begin{bmatrix} \sqrt{d_M} N_{ij}^T \\ \sqrt{\tau_M} S_{ij}^T \end{bmatrix}, \quad \Xi_{61}(4) = \begin{bmatrix} \sqrt{d_M} N_{ij}^T \\ \sqrt{\tau_M} T_{ij}^T \end{bmatrix}$$

Proof. Choose the following Lyapunov functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{18}$$

where

$$V_1(t) = x^T(t)Px(t)$$

$$V_2(t) = \int_{t-d_M}^t x^T(s)Q_1x(s)ds + \int_{t-\tau_M}^t x^T(s)Q_2x(s)ds$$

$$V_3(t) = \int_{t-d_M}^t \int_s^t \dot{x}^T(v)R_1\dot{x}(v)dvd s + \int_{t-\tau_M}^t \int_s^t \dot{x}^T(v)R_2\dot{x}(v)dvd s$$

Taking derivation on $V_i(t)$ and evaluating their expectation

$$\mathcal{E}\{\dot{V}_1(t)\} = 2x^T(t)PA_{ij} \tag{19}$$

$$\mathcal{E}\{\dot{V}_2(t)\} = x^T(t)(Q_1 + Q_2)x(t) - x^T(t - d_M)Q_1x(t - d_M) - x^T(t - \tau_M)Q_2x(t - \tau_M) \tag{20}$$

$$\mathcal{E}\{\dot{V}_3(t)\} = \dot{x}^T(t)(d_M R_1 + \tau_M R_2)\dot{x}(t) - \int_{t-d_M}^t \dot{x}^T(s)R_1\dot{x}(s)ds - \int_{t-\tau_M}^t \dot{x}^T(s)R_1\dot{x}(s)ds \tag{21}$$

Notice that

$$\begin{aligned} \dot{x}^T(t)(d_M R_1 + \tau_M R_2)\dot{x}(t) &= \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h)) \left\{ \mathcal{A}_{ij}^T \bar{R} \mathcal{A}_{ij} + \delta_a^2 \mathcal{A}_{ij2}^T \bar{W}^T B_i^T \bar{R} B_i \bar{W} \mathcal{A}_{ij2} \right. \\ &+ \delta_b^2 [E_i h(t) - F_i g(t)]^T \bar{R} [E_i h(t) - F_i g(t)] + \sum_{s=1}^m \delta_{ws}^2 (B_i D_s \mathcal{A}_{ij1})^T \bar{R} (B_i D_s \mathcal{A}_{ij1}) \\ &\left. + \sum_{s=1}^m \delta_{ws}^2 \delta_a^2 (B_i D_s \mathcal{A}_{ij2})^T \bar{R} (B_i D_s \mathcal{A}_{ij2}) \right\} \end{aligned} \tag{22}$$

in which $\bar{R} = d_M R_1 + \tau_M R_2$, \mathcal{A}_{ij} , \mathcal{A}_{ij1} and \mathcal{A}_{ij2} have been denoted in (12).

Employing the free-weighting matrices method [32,33], for matrices M_{ij} , N_{ij} , T_{ij} and S_{ij} of appropriate dimensions, we have

$$2 \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h))\zeta^T(t)M_{ij} \left[x(t) - x(t - d(t)) - \int_{t-d(t)}^t \dot{x}(s)ds \right] = 0 \tag{23}$$

$$2 \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h))\zeta^T(t)N_{ij} \left[x(t - d(t)) - x(t - d_M) - \int_{t-d_M}^{t-d(t)} \dot{x}(s)ds \right] = 0 \tag{24}$$

$$2 \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h))\zeta^T(t)T_{ij} \left[x(t) - x(t - \tau(t)) - \int_{t-\tau(t)}^t \dot{x}(s)ds \right] = 0 \tag{25}$$

$$2 \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h))\zeta^T(t)S_{ij} \left[x(t - \tau(t)) - x(t - \tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds \right] = 0 \tag{26}$$

where $\zeta^T(t) = [x^T(t) \ x^T(t - d(t)) \ x^T(t - d_M) \ x^T(t - \tau(t)) \ x^T(t - \tau_M) \ e_k^T(t) \ h^T(x) \ g^T(x)]$

By Lemma 1, we can get there exist real matrices R_1 and R_2 such that

$$-2\zeta^T(t)M_{ij} \int_{t-d(t)}^t \dot{x}(s)ds \leq d(t)\zeta^T(t)M_{ij}R_1^{-1}M_{ij}^T\zeta(t) + \int_{t-d(t)}^t \dot{x}^T(s)R_1\dot{x}(s)ds \tag{27}$$

$$-2\zeta^T(t)N_{ij} \int_{t-d_M}^{t-d(t)} \dot{x}(s)ds \leq (d_M - d(t))\zeta^T(t)N_{ij}R_1^{-1}N_{ij}^T\zeta(t) + \int_{t-d_M}^{t-d(t)} \dot{x}^T(s)R_1\dot{x}(s)ds \tag{28}$$

$$-2\zeta^T(t)T_{ij} \int_{t-\tau(t)}^t \dot{x}(s)ds \leq \tau(t)\zeta^T(t)T_{ij}R_2^{-1}T_{ij}^T\zeta(t) + \int_{t-\tau(t)}^t \dot{x}^T(s)R_2\dot{x}(s)ds \tag{29}$$

$$-2\zeta^T(t)S_{ij} \int_{t-\tau_M}^{t-\tau(t)} \dot{x}(s)ds \leq (\tau_M - \tau(t))\zeta^T(t)S_{ij}R_2^{-1}S_{ij}^T\zeta(t) + \int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s)R_2\dot{x}(s)ds \tag{30}$$

From the definition in (2), we can get

$$x^T(t)\Phi_1^T P \Phi_1 x(t) - h^T(x)Ph(x) \geq 0 \tag{31}$$

$$x^T(t)\Phi_2^T P\Phi_2 x(t) - g^T(x)Pg(x) \geq 0 \tag{32}$$

Recalling the definition of $\tau(t)$, (7) becomes

$$\sigma^2 x^T(t - \tau(t))\Omega x(t - \tau(t)) - e_k^T(t)\Omega e_k(t) \geq 0 \tag{33}$$

Combining (19)–(33), we derive that

$$\begin{aligned} \mathcal{E}\{\dot{V}(t)\} &\leq \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h)) \{ 2x^T(t)P\mathcal{A}_{ij} + x^T(t)(Q_1 + Q_2)x(t) \\ &\quad - x^T(t - d_M)Q_1x(t - d_M) - x^T(t - \tau_M)Q_2x(t - \tau_M) + \zeta^T(t)M_{ij}[x(t) - x(t - d(t))] \\ &\quad + \mathcal{A}_{ij}^T \bar{R}\mathcal{A}_{ij} + \delta_a^2 \mathcal{A}_{ij2}^T \bar{W}^T B_i^T \bar{R}B_i \bar{W} \mathcal{A}_{ij2} + \delta_b^2 [E_i h(t) - F_i g(t)]^T \bar{R}[E_i h(t) - F_i g(t)] \\ &\quad + \sum_{s=1}^m \delta_{ws}^2 (B_i D_s \mathcal{A}_{ij1})^T \bar{R}(B_i D_s \mathcal{A}_{ij1}) + \sum_{s=1}^m \delta_{ws}^2 \delta_a^2 (B_i D_s \mathcal{A}_{ij2})^T \bar{R}(B_i D_s \mathcal{A}_{ij2}) \\ &\quad + \zeta^T(t)N_{ij}[x(t - d(t)) - x(t - d_M)] + \zeta^T(t)T_{ij}[x(t) - x(t - \tau(t))] \\ &\quad + \zeta^T(t)S_{ij}[x(t - \tau(t)) - x(t - \tau_M)] + d(t)\zeta^T(t)M_{ij}R_1^{-1}M_{ij}^T \zeta(t) \\ &\quad + (d_M - d(t))\zeta^T(t)N_{ij}R_1^{-1}N_{ij}^T \zeta(t) \\ &\quad + \tau(t)\zeta^T(t)T_{ij}R_2^{-1}T_{ij}^T \zeta(t) + (\tau_M - \tau(t))\zeta^T(t)S_{ij}R_2^{-1}S_{ij}^T \zeta(t) + \sigma^2 x^T(t - \tau(t))\Omega x(t - \tau(t)) \\ &\quad - e_k^T(t)\Omega e_k(t) + x^T(t)\Phi_1^T P\Phi_1 x(t) - h^T(x)Ph(x) + x^T(t)\Phi_2^T P\Phi_2 x(t) - g^T(x)Pg(x) \} \\ &\leq \sum_{i=1}^r \sum_{j=1, i \leq j}^r h_i(\theta(t))h_j(\theta(t_k h)) \{ \zeta^T(t)(\Xi_{11} + \Gamma + \Gamma^T)\zeta(t) + \mathcal{A}_{ij}^T \bar{R}\mathcal{A}_{ij} \\ &\quad + \delta_a^2 \mathcal{A}_{ij2}^T \bar{W}^T B_i^T \bar{R}B_i \bar{W} \mathcal{A}_{ij2} \\ &\quad + \sum_{s=1}^m \delta_{ws}^2 \delta_a^2 (B_i D_s \mathcal{A}_{ij2})^T \bar{R}(B_i D_s \mathcal{A}_{ij2}) + \delta_b^2 [E_i h(t) - F_i g(t)]^T \bar{R}[E_i h(t) - F_i g(t)] \\ &\quad + \sum_{s=1}^m \delta_{ws}^2 (B_i D_s \mathcal{A}_{ij1})^T \bar{R}(B_i D_s \mathcal{A}_{ij1}) - h^T(x)Ph(x) + d(t)\zeta^T(t)M_{ij}R_1^{-1}M_{ij}^T \zeta(t) \\ &\quad + (d_M - d(t))\zeta^T(t)N_{ij}R_1^{-1}N_{ij}^T \zeta(t) + \tau(t)\zeta^T(t)T_{ij}R_2^{-1}T_{ij}^T \zeta(t) \\ &\quad + (\tau_M - \tau(t))\zeta^T(t)S_{ij}R_2^{-1}S_{ij}^T \zeta(t) \\ &\quad + x^T(t)\Phi_1^T P\Phi_1 x(t) + x^T(t)\Phi_2^T P\Phi_2 x(t) - g^T(x)Pg(x) \} \end{aligned} \tag{34}$$

By using Schur complements, one can see that $\mathcal{E}\{\dot{V}(t)\} < 0$ can be ensured by

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h)) \Xi^{ij}(l) < 0 \tag{35}$$

The following slack matrix U is introduced to relax the design results

$$\sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))(h_j(\theta(t_k h)) - h_j(\theta(t)))U = 0 \tag{36}$$

If $U + \Xi^{ij}(l) > 0$, substituting (36) into (35), we can get

$$\begin{aligned} & \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) [h_j(\theta(t)) \Xi^{ij}(l) + (h_j(\theta(t_k)) - h_j(\theta(t)))(U + \Xi^{ij}(l))] \\ & \leq \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t)) h_j(\theta(t)) \left[\Xi^{ij}(l) + \sum_{p=1}^r \mu_p (U + \Xi^{ip}(l)) \right] \\ & = \sum_{i=1}^r \sum_{j=1}^r h_i^2(\theta(t)) \Phi^{ii}(l) + \sum_{i=1}^r \sum_{i < j \leq r} h_i(\theta(t)) h_j(\theta(t)) (\Phi^{ij}(l) + \Phi^{ji}(l)) \end{aligned} \tag{37}$$

where $\Phi^{ij}(l) = \Xi^{ij}(l) + \sum_{p=1}^r \mu_p (U + \Xi^{ip}(l))$

According to (15) and (16), it follows from (37) that (35) is satisfied. Then, we can have $\mathcal{E}\{\dot{V}(t)\} < 0$.

This completes the proof. \square

Sufficient conditions are given in Theorem 1, which guarantee the mean-square stability of the augmented system (12). Based on the results in Theorem 1, we are in a position to design the controller in the form of (9) in the following theorem.

Theorem 2. For given scalars $\bar{\alpha}, \bar{\beta}, \tau_M, d_M, w_s, \delta_a, \delta_b, \delta_{ws}(s = 1, \dots, m), \sigma, \mu_p$ ($p = 1, 2, \dots, r$), ε_1 and ε_2 , under the hybrid-driven scheme (4), the augmented system (12) is asymptotically stable with controller feedback gains $K_j = Y_j X^{-1}$, if there exist positive matrix $X > 0, \tilde{Q}_1 > 0, \tilde{Q}_2 > 0, \tilde{R}_1 > 0, \tilde{R}_2 > 0, \tilde{\Omega} > 0, \tilde{U}, \tilde{M}_{ij}, \tilde{N}_{ij}, \tilde{T}_{ij}$ and \tilde{S}_{ij} with appropriate dimensions such that for $l = 1, 2, 3, 4$

$$\tilde{\Phi}^{ii}(l) < 0, i = 1, 2, \dots, r \tag{38}$$

$$\tilde{\Phi}^{ij}(l) + \tilde{\Phi}^{ji}(l) < 0, i, j = 1, 2, \dots, r \tag{39}$$

$$\tilde{U} + \tilde{\Xi}^{ij}(l) > 0, i, j = 1, 2, \dots, r \tag{40}$$

where

$$\begin{aligned} \tilde{\Phi}^{ij}(l) &= \tilde{\Xi}^{ij}(l) + \sum_{p=1}^r \mu_p (\tilde{U} + \tilde{\Xi}^{ip}(l)) \\ \tilde{\Xi}^{ij}(l) &= \begin{bmatrix} \tilde{\Xi}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * & * & * \\ \tilde{\Xi}_{21} & \tilde{\Xi}_{22} & * & * & * & * \\ \tilde{\Xi}_{31} & 0 & \tilde{\Xi}_{33} & * & * & * \\ \tilde{\Xi}_{41} & 0 & 0 & \tilde{\Xi}_{44} & * & * \\ \tilde{\Xi}_{51} & 0 & 0 & 0 & \tilde{\Xi}_{55} & * \\ \tilde{\Xi}_{61}(l) & 0 & 0 & 0 & 0 & \tilde{\Xi}_{66} \end{bmatrix} < 0, i, j = 1, 2, \dots, r, i \leq j. \end{aligned}$$

$$\tilde{\Xi}_{11} = \begin{bmatrix} A_i X + X A_i^T + \tilde{Q}_1 + \tilde{Q}_2 & * & * & * & * & * & * & * \\ \tilde{\alpha} Y_j^T \tilde{W}^T B_i^T & 0 & * & * & * & * & * & * \\ 0 & 0 & -\tilde{Q}_1 & * & * & * & * & * \\ \hat{\alpha} Y_j^T \tilde{W}^T B_i^T & 0 & 0 & \sigma^2 \tilde{\Omega} & * & * & * & * \\ 0 & 0 & 0 & 0 & -\tilde{Q}_2 & * & * & * \\ \hat{\alpha} Y_j^T \tilde{W}^T B_i^T & 0 & 0 & 0 & 0 & -\tilde{\Omega} & * & * \\ \hat{\beta} X E_i^T & 0 & 0 & 0 & 0 & 0 & -X & * \\ \hat{\beta} X F_i^T & 0 & 0 & 0 & 0 & 0 & 0 & -X \end{bmatrix}$$

$$\tilde{\Gamma} = [\tilde{M}_{ij} + \tilde{T}_{ij} \quad -\tilde{M}_{ij} + \tilde{N}_{ij} \quad -\tilde{N}_{ij} \quad -\tilde{T}_{ij} + \tilde{S}_{ij} \quad -\tilde{S}_{ij} \quad 0 \quad 0 \quad 0], \quad \tilde{B}_{ij} = B_i \tilde{W} Y_j$$

$$\tilde{\Xi}_{21} = \begin{bmatrix} \sqrt{d_M} A_i X & \sqrt{d_M} \tilde{\alpha} \tilde{B}_{ij} & 0 & \sqrt{d_M} \hat{\alpha} \tilde{B}_{ij} & 0 & \sqrt{d_M} \hat{\alpha} \tilde{B}_{ij} & \sqrt{d_M} \hat{\beta} E_i X & \sqrt{d_M} \hat{\beta} F_i X \\ \sqrt{\tau_M} A_i X & \sqrt{\tau_M} \tilde{\alpha} \tilde{B}_{ij} & 0 & \sqrt{\tau_M} \hat{\alpha} \tilde{B}_{ij} & 0 & \sqrt{\tau_M} \hat{\alpha} \tilde{B}_{ij} & \sqrt{\tau_M} \hat{\beta} E_i X & \sqrt{\tau_M} \hat{\beta} F_i X \end{bmatrix}$$

$$\tilde{\Xi}_{31} = \begin{bmatrix} 0 & \sqrt{d_M} \delta_a \tilde{B}_{ij} & 0 & -\sqrt{d_M} \delta_a \tilde{B}_{ij} & 0 & -\sqrt{d_M} \delta_a \tilde{B}_{ij} & 0 & 0 \\ 0 & \sqrt{\tau_M} \delta_a \tilde{B}_{ij} & 0 & -\sqrt{\tau_M} \delta_a \tilde{B}_{ij} & 0 & -\sqrt{\tau_M} \delta_a \tilde{B}_{ij} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_b \sqrt{d_M} E_i X & -\delta_b \sqrt{d_M} F_i X \\ 0 & 0 & 0 & 0 & 0 & 0 & \delta_b \sqrt{\tau_M} E_i X & -\delta_b \sqrt{\tau_M} F_i X \end{bmatrix}$$

$$\tilde{\Xi}_{22} = \text{diag}\{-2\varepsilon_1 X + \varepsilon_1^2 \tilde{R}_1, -2\varepsilon_2 X + \varepsilon_2^2 \tilde{R}_2\}, \quad \tilde{\Xi}_{33} = \text{diag}\{\tilde{\Xi}_{22}, \tilde{\Xi}_{22}\}$$

$$\tilde{\Xi}_{41} = \begin{bmatrix} 0 & \tilde{\alpha} \sqrt{d_M} \tilde{D} & 0 & \hat{\alpha} \sqrt{d_M} \tilde{D} & 0 & \hat{\alpha} \sqrt{d_M} \tilde{D} & 0 & 0 \\ 0 & \tilde{\alpha} \sqrt{\tau_M} \tilde{D} & 0 & \hat{\alpha} \sqrt{\tau_M} \tilde{D} & 0 & \hat{\alpha} \sqrt{\tau_M} \tilde{D} & 0 & 0 \\ 0 & \delta_a \sqrt{d_M} \tilde{D} & 0 & -\delta_a \sqrt{d_M} \tilde{D} & 0 & -\delta_a \sqrt{d_M} \tilde{D} & 0 & 0 \\ 0 & \delta_a \sqrt{\tau_M} \tilde{D} & 0 & -\delta_a \sqrt{\tau_M} \tilde{D} & 0 & -\delta_a \sqrt{\tau_M} \tilde{D} & 0 & 0 \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} \delta_{w1} B_i D_1 Y_j \\ \vdots \\ \delta_{w1} B_i D_m Y_j \end{bmatrix}$$

$$\tilde{\Xi}_{44} = \text{diag}\{\tilde{R}_1, \tilde{R}_2, \tilde{R}_1, \tilde{R}_2\}, \quad \tilde{R}_k = \text{diag}\{\underbrace{-2\varepsilon_k X + \varepsilon_k^2 \tilde{R}_k, \dots, -2\varepsilon_k X + \varepsilon_k^2 \tilde{R}_k}_m\}, \quad k = 1, 2$$

$$\tilde{\Xi}_{51} = \begin{bmatrix} \Phi_1 X & 0_{1 \times 7} \\ \Phi_2 X & 0_{1 \times 7} \end{bmatrix}, \quad \tilde{\Xi}_{55} = \text{diag}\{-X, -X\}, \quad \tilde{\Xi}_{66} = \text{diag}\{-\tilde{R}_1, -\tilde{R}_2\}$$

$$\tilde{\Xi}_{61}(1) = \begin{bmatrix} \sqrt{d_M} \tilde{M}_{ij}^T \\ \sqrt{\tau_M} \tilde{S}_{ij}^T \end{bmatrix}, \quad \tilde{\Xi}_{61}(2) = \begin{bmatrix} \sqrt{d_M} \tilde{M}_{ij}^T \\ \sqrt{\tau_M} \tilde{T}_{ij}^T \end{bmatrix}, \quad \tilde{\Xi}_{61}(3) = \begin{bmatrix} \sqrt{d_M} \tilde{N}_{ij}^T \\ \sqrt{\tau_M} \tilde{S}_{ij}^T \end{bmatrix}, \quad \tilde{\Xi}_{61}(4) = \begin{bmatrix} \sqrt{d_M} \tilde{N}_{ij}^T \\ \sqrt{\tau_M} \tilde{T}_{ij}^T \end{bmatrix}$$

Proof. From

$$(R_k - \varepsilon_k P) R_k^{-1} (R_k - \varepsilon_k P) \geq 0, \quad (k = 1, 2) \tag{41}$$

we can get

$$-P R_k^{-1} P \leq -2\varepsilon_k P + \varepsilon_k^2 R_k, \quad (k = 1, 2) \tag{42}$$

By using (42), from(15), we can obtain

$$\bar{\Phi}^{ii}(l) < 0, \quad i = 1, 2, \dots, r \tag{43}$$

$$\bar{\Phi}^{ij}(l) + \bar{\Phi}^{ji}(l) < 0, \quad i, j = 1, 2, \dots, r \tag{44}$$

$$U + \bar{\Xi}^{ij}(l) > 0, \quad i, j = 1, 2, \dots, r \tag{45}$$

where

$$\bar{\Phi}^{ij}(l) = \bar{\Xi}^{ij}(l) + \sum_{p=1}^r \mu_p(U + \bar{\Xi}^{ip}(l))$$

$$\bar{\Xi}^{ij}(l) = \begin{bmatrix} \bar{\Xi}_{11} + \Gamma + \Gamma^T & * & * & * & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * & * & * & * \\ \bar{\Xi}_{31} & 0 & \bar{\Xi}_{33} & * & * & * \\ \bar{\Xi}_{41} & 0 & 0 & \bar{\Xi}_{44} & * & * \\ \bar{\Xi}_{51}(l) & 0 & 0 & 0 & \bar{\Xi}_{55} & * \\ \bar{\Xi}_{61} & 0 & 0 & 0 & 0 & \bar{\Xi}_{66} \end{bmatrix} < 0, \quad j = 1, 2, \dots, r, \quad i \leq j.$$

$$\bar{\Xi}_{22} = \text{diag}\{-2\varepsilon_1 P + \varepsilon_1^2 R_1, -2\varepsilon_2 P + \varepsilon_2^2 R_2\}, \quad \bar{\Xi}_{33} = \text{diag}\{\bar{\Xi}_{22}, \bar{\Xi}_{22}\}$$

$$\bar{\Xi}_{44} = \text{diag}\{\bar{R}_1, \bar{R}_2, \bar{R}_1, \bar{R}_2\}, \quad \bar{R}_k = \text{diag}\{\underbrace{-2\varepsilon_k P + \varepsilon_k^2 R_k, \dots, -2\varepsilon_k P + \varepsilon_k^2 R_k}_m\}, \quad k = 1, 2$$

Define $X = P^{-1}$, $Y_j = K_j X$, $XQ_1 X = \tilde{Q}_1$, $XQ_2 X = \tilde{Q}_2$, $XR_1 X = \tilde{R}_1$, $XR_2 X = \tilde{R}_2$, $X\Omega X = \tilde{\Omega}$, $XM_{ij} X = \tilde{M}_{ij}$, $XN_{ij} X = \tilde{N}_{ij}$, $XT_{ij} X = \tilde{T}_{ij}$, $XS_{ij} X = \tilde{S}_{ij}$, $JUJ^T = \tilde{U}$, pre-multiply and pos-multiply (43) with $J = \text{diag}\{X, X, \dots, X\}$ and its transpose, (38) can be derived from (43). Similar operation to (54) and (55), we have (39) and (40).

This completes the proof. \square

Remark 8. When the system (1) is under the time-driven scheme, substituting (6) and (10) into (3) we can obtain

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h))\{A_i x(t) + B_i W K_j x(t - d(t)) + \beta(t)E_i h(t) + (1 - \beta(t)F_i g(t))\}t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}]$$
(46)

Remark 9. When the system (1) is under event-driven scheme (7), substituting (8) into (3), we get

$$\dot{x}(t) = \sum_{i=1}^r \sum_{j=1}^r h_i(\theta(t))h_j(\theta(t_k h))\{A_i x(t) + B_i W K_j(x(t - \tau(t)) + e_k(t)) + \beta(t)E_i h(t) + (1 - \beta(t)F_i g(t))\}t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}]$$
(47)

Similar to the proof in Theorem 2, the following corollaries can be easily obtained.

Corollary 1. For given scalars $\bar{\beta}$, d_M , w_s , δ_b , $\delta_{ws}(s = 1, \dots, m)$, $\mu_p(p = 1, 2, \dots, r)$, ε_1 , under the time-driven scheme, the system (46) is asymptotically stable with controller feedback gains $K_j = Y_j X^{-1}$, if there exist positive matrix $X > 0$, $\tilde{Q}_1 > 0$, $\tilde{R}_1 > 0$, \tilde{M}_{ij} , \tilde{N}_{ij} , \tilde{U}_1 with appropriate dimensions such that for $l = 1, 2$

$$\Phi_1^{ii}(l) < 0, \quad i = 1, 2, \dots, r$$
(48)

$$\Phi_1^{ij}(l) + \Phi_1^{ji}(l) < 0, \quad i, j = 1, 2, \dots, r$$
(49)

$$\tilde{U}_1 + \Pi_1^{ij}(l) > 0, \quad i, j = 1, 2, \dots, r$$
(50)

where

$$\Phi_1^{ij}(l) = \Pi_1^{ij}(l) + \sum_{p=1}^r \mu_p(\tilde{U}_1 + \Pi_1^{ip}(l))$$

$$\Pi_1^{ij}(l) = \begin{bmatrix} \Pi_{11} + \Delta + \Delta^T & * & * & * & * & * \\ \Pi_{21} & -2\varepsilon_1 X + \varepsilon_1^2 \tilde{R}_1 & * & * & * & * \\ \Pi_{31} & 0 & \Pi_{33} & * & * & * \\ \Phi_1 X & 0 & 0 & -X & * & * \\ \Phi_1 X & 0 & 0 & 0 & -X & * \\ \Pi_{61}(l) & 0 & 0 & 0 & 0 & -\tilde{R}_1 \end{bmatrix} < 0, i, j = 1, 2, \dots, r, i \leq j.$$

$$\Pi_{11} = \begin{bmatrix} A_i X + X A_i^T + \tilde{Q}_1 & * & * & * & * \\ Y_j^T \tilde{W}^T B_i^T & 0 & * & * & * \\ 0 & 0 & -\tilde{Q}_1 & * & * \\ \tilde{\beta} X E_i^T & 0 & 0 & -X & * \\ \tilde{\beta} X F_i^T & 0 & 0 & 0 & -X \end{bmatrix}$$

$$\Pi_{21} = [\sqrt{d_M} A_i X \quad \sqrt{d_M} B_i \tilde{W} Y_j \quad 0 \quad \sqrt{d_M} \tilde{\beta} E_i X \quad \sqrt{d_M} \tilde{\beta} F_i X]$$

$$\Pi_{31} = \begin{bmatrix} 0 & \sqrt{d_M} \tilde{D} & 0 & 0 & 0 \\ 0 & 0 & 0 & \delta_b \sqrt{d_M} E_i X & -\delta_b \sqrt{d_M} F_i X \end{bmatrix}, \tilde{D} = \begin{bmatrix} \delta_{w1} B_i D_1 Y_j \\ \vdots \\ \delta_{w1} B_i D_m Y_j \end{bmatrix}$$

$$\Pi_{33} = \text{diag}\{\tilde{R}_1, -2\varepsilon_1 X + \varepsilon_1^2 \tilde{R}_1\}, \Delta = [\tilde{M}_{ij} \quad -\tilde{M}_{ij} + \tilde{N}_{ij} \quad -\tilde{N}_{ij} \quad 0 \quad 0]$$

$$\tilde{\Xi}_{61}(1) = [\sqrt{d_M} \tilde{M}_{ij}^T], \quad \tilde{\Xi}_{61}(2) = [\sqrt{d_M} \tilde{N}_{ij}^T]$$

Other symbols have been defined in Theorem 2.

Corollary 2. For given scalars $\tilde{\beta}$, τ_M , w_s , δ_b , $\delta_{ws}(s = 1, \dots, m)$, $\mu_p(p = 1, 2, \dots, r)$, ε_2 , σ and Ω , under the event-driven scheme (7), the system (47) is asymptotically stable with controller feedback gains $K_j = Y_j X^{-1}$, if there exist positive matrix \tilde{U}_2 , $X > 0$, $\tilde{Q}_2 > 0$, $\tilde{R}_2 > 0$, \tilde{T}_{ij} , \tilde{S}_{ij} , with appropriate dimensions such that for $l = 1, 2$

$$\Phi_2^{ii}(l) < 0, i = 1, 2, \dots, r \tag{51}$$

$$\Phi_2^{ij}(l) + \Phi_2^{ji}(l) < 0, i, j = 1, 2, \dots, r \tag{52}$$

$$\tilde{U}_2 + \Pi_2^{ij}(l) > 0, i, j = 1, 2, \dots, r \tag{53}$$

where

$$\Phi_2^{ij}(l) = \Pi_2^{ij}(l) + \sum_{p=1}^r \mu_p(\tilde{U}_2 + \Pi_2^{ip}(l))$$

$$\Pi_2^{ij}(l) = \begin{bmatrix} \tilde{\Pi}_{11} + \Lambda + \Lambda^T & * & * & * & * & * \\ \tilde{\Pi}_{21} & -2\varepsilon_1 X + \varepsilon_1^2 \tilde{R}_1 & * & * & * & * \\ \tilde{\Pi}_{31} & 0 & \tilde{\Pi}_{33} & * & * & * \\ \tilde{\Phi}_1 X & 0 & 0 & -X & * & * \\ \tilde{\Phi}_1 X & 0 & 0 & 0 & -X & * \\ \tilde{\Pi}_{61}(l) & 0 & 0 & 0 & 0 & -\tilde{R}_1 \end{bmatrix} < 0$$

$$\bar{\Pi}_{11} = \begin{bmatrix} A_i X + X A_i^T + \tilde{Q}_2 & * & * & * & * & * \\ Y_j^T \bar{W}^T B_i^T & \sigma^2 \tilde{\Omega} & * & * & * & * \\ 0 & 0 & -\tilde{Q}_2 & * & * & * \\ Y_j^T \bar{W}^T B_i^T & 0 & 0 & -\tilde{\Omega} & * & * \\ \hat{\beta} X E_i^T & 0 & 0 & 0 & -X & * \\ \hat{\beta} X F_i^T & 0 & 0 & 0 & 0 & -X \end{bmatrix}$$

$$\bar{\Pi}_{21} = [\sqrt{\tau_M} A_i X \quad \sqrt{\tau_M} B_i \bar{W} Y_j \quad 0 \quad \sqrt{\tau_M} B_i \bar{W} Y_j \quad \sqrt{\tau_M} \hat{\beta} E_i X \quad \sqrt{\tau_M} \hat{\beta} F_i X]$$

$$\Pi_{31} = \begin{bmatrix} 0 & \sqrt{\tau_M} \tilde{D} & 0 & \sqrt{\tau_M} \tilde{D} & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_b \sqrt{\tau_M} E_i X & -\delta_b \sqrt{\tau_M} F_i X \end{bmatrix}$$

$$\Pi_{33} = \text{diag}\{\tilde{R}_2, -2\varepsilon_2 X + \varepsilon_2^2 \tilde{R}_2\}, \quad \Lambda = [\tilde{T}_{ij} \quad -\tilde{T}_{ij} + \tilde{S}_{ij} \quad -\tilde{S}_{ij} \quad 0 \quad 0 \quad 0]$$

$$\tilde{\Xi}_{61}(1) = [\sqrt{\tau_M} \tilde{T}_{ij}^T], \quad \tilde{\Xi}_{61}(2) = [\sqrt{\tau_M} \tilde{S}_{ij}^T]$$

Other symbols have been defined in Theorem 2.

Remark 10. One can see that, in the designed hybrid-driven scheme, the frequency of the data transmission is determined by $\alpha(t)$ and σ . When $\alpha(t) = 1$, more signals will be transmitted and the feasible solutions for Corollary 1 is easily to find out. When $\alpha(t) = 0$, the system is under the event-driven scheme. A larger σ brings a larger release interval, fewer numbers of packets are released and the smaller τ_M is obtained. However, too large σ may result in difficulty for Corollary 2 to find the feasible solutions. Therefore, we can make adjustment for the release interval and the system control performance by choosing $\bar{\alpha}$ and σ .

Remark 11. It should be noted that Corollary 1 and Corollary 2 can be seen as special cases of Theorem 2. Theorem 2 gives the co-design method of the controller and the hybrid-driven scheme (4) for the augmented system (12). Corollary 1 presents the controller design method of the system (46). Corollary 2 provides the co-design method of the desired controller gain and the event-driven scheme (7) for system (47).

4. Simulation examples

Consider the following nonlinear mass-spring system [20]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.01x_1 - 0.67x_1^3 + u + \omega \end{cases} \tag{54}$$

where $x_1 \in [-1, 1]$, $x_2 \in [-0.8, 0.8]$, $\omega = \beta(t)E_i h(x) + (1 - \beta(t))g(x)$ with nonlinearities bounds $\Phi_1 = \text{diag}\{-0.06, 0\}$ and $\Phi_2 = \text{diag}\{0, 0.02\}$, ω is nonlinear perturbation. The following conditions hold obviously for $x_1 \in [-1, 1]$: $0.67x_1 \geq 0.67x_1^3 \geq 0$ when $x_1 \geq 0$; $0.67x_1 \leq 0.67x_1^3 \leq 0$ when $x_1 \leq 0$. Choose the membership functions $h_1(x_1) = 1 - x_1^2$ and $h_2(x_1) = 1 - h_1(x_1)$, the nonlinear system (54) can be described by the following T-S fuzzy model:

Rule i : If x_1 is h_i , then

$$\dot{x}(t) = \sum_{i=1}^r h_i(\theta(t)) [A_i x(t) + B_i u(t) + \beta(t) E_i h(t) + (1 - \beta(t)) F_i g(t)] \tag{55}$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.68 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad F_1 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0 & 1 \\ -0.01 & 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad F_2 = \begin{bmatrix} 0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$$

The initial state is given as $x_0 = [-0.5 \quad -0.1]^T$. The nonlinear functions are chosen as

$$h(x) = \begin{bmatrix} 0.04x_1 - \tan(0.3x_1) + 0.15x_2 \\ 0.09x_2 \tan(0.7x_2) \end{bmatrix}, \quad g(x) = \begin{bmatrix} -0.3x_1 \tan(0.2x_1) \\ 0.8x_2 + \tan(0.6x_2) \end{bmatrix}$$

In the following, three possible cases are discussed, which can illustrate the effectiveness of the proposed results.

Case 1: When $\alpha(t) = 1$, the system is under time-driven scheme, for given $\bar{\beta} = 0.2$, $d_M = 0.7$, $w_1 = 0.95$, $\delta_b = 0.02$, $\delta_{w_1} = 0.05$, and $\varepsilon_1 = 1$, $\mu_p = 1 (p = 1, 2)$, based on Matlab/LMIs toolbox and applying [Corollary 1](#), we can get the controller feedback gains as follows

$$K_1 = [0.1427 \quad -0.8578], \quad K_2 = [-0.5360 \quad -1.0240] \tag{56}$$

The state response and the probabilistic actuator faults are shown in [Fig. 2](#) and [Fig. 3](#), respectively.

Case 2: When $\alpha(t) = 0$, the system is under single event-driven scheme, for given $\bar{\beta} = 0.2$, $\tau_M = 0.8$, $\sigma = 0.1$, $w_1 = 0.95$, $\delta_b = 0.02$, $\delta_{w_1} = 0.05$, $\mu_p = 1 (p = 1, 2)$ and $\varepsilon_2 = 1$, by

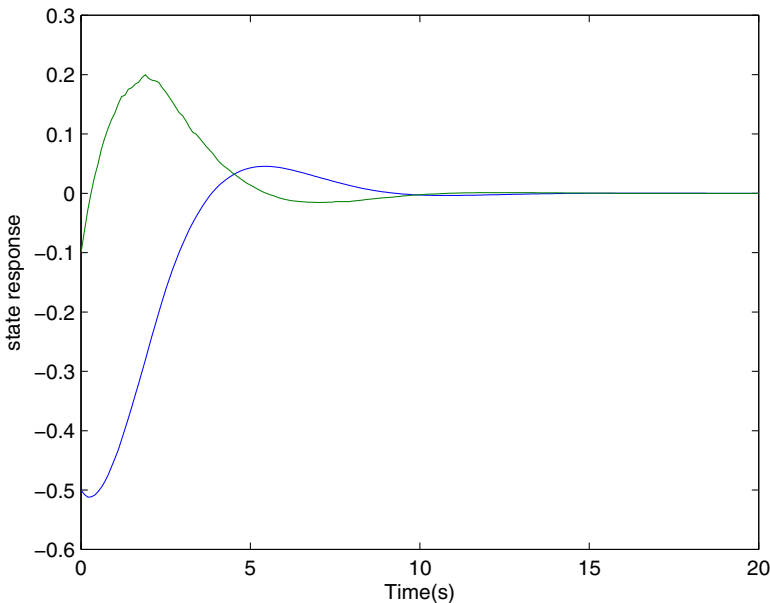


Fig. 2. The state response under feedback gain in case 1.

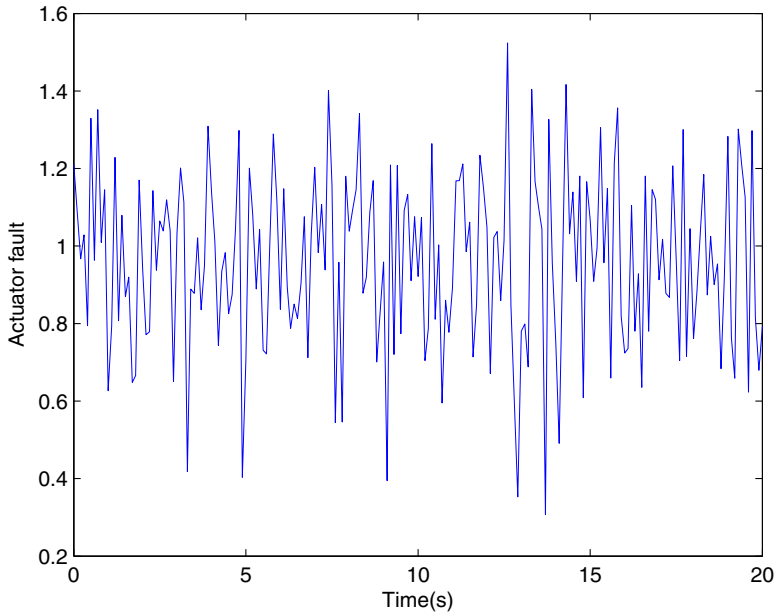


Fig. 3. The probabilistic actuator faults in case 1.

Corollary 2, the controller feedback gains and event-triggered matrix are derived as follows

$$K_1 = [0.2703 \quad -0.7947], \quad K_2 = [-0.3706 \quad -0.9810], \quad \Omega = \begin{bmatrix} 0.8269 & -0.3998 \\ -0.3998 & 0.8311 \end{bmatrix} \quad (57)$$

Fig. 4 depicts the state response. The release instants and intervals are given in **Fig. 5**.

Case 3: When the system is under hybrid-driven scheme, for given $\bar{\alpha} = 0.25$, $\bar{\beta} = 0.12$, $d_M = 0.5$, $\tau_M = 0.7$, $\sigma = 0.1$, $w_1 = 0.95$, $\delta_b = 0.02$, $\delta_{w_1} = 0.15$, $\varepsilon_1 = 1$, $\mu_p = 1$ ($p = 1, 2$) and $\varepsilon_2 = 1$, applying **Theorem 2**, the controller feedback gains and event-triggered matrix are as follows

$$K_1 = [0.2765 \quad -0.9109], \quad K_2 = [-0.3652 \quad -1.0611],$$

$$\Omega = \begin{bmatrix} 0.0503 & -0.0340 \\ -0.0340 & 0.0491 \end{bmatrix} \quad (58)$$

The state response is shown in **Fig. 6**. The stochastic variables $\alpha(t)$ in this simulation is plotted in **Fig. 7**.

The simulation results in **Figs. 2, 4** and **6** have confirmed that the designed reliable controllers can stabilize the T–S fuzzy systems with probabilistic actuator faults and stochastic nonlinear perturbations. **Fig. 5** illustrates that the amount of transmission is reduced by using event-driven scheme. The hybrid driven scheme in this paper is more general, which can reduce the number of transmissions and guarantee the discussed system achieve a satisfactory performance.

Remark 12. From the simulation results, we can see that the obtained results are effective in saving the scarce network resources and stabilize the discussed system when the random

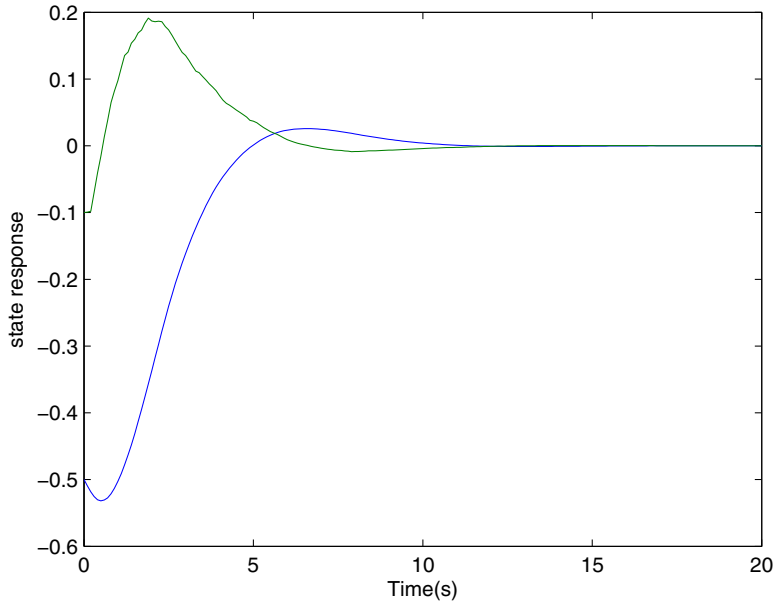


Fig. 4. The state response under feedback gain in case 2.

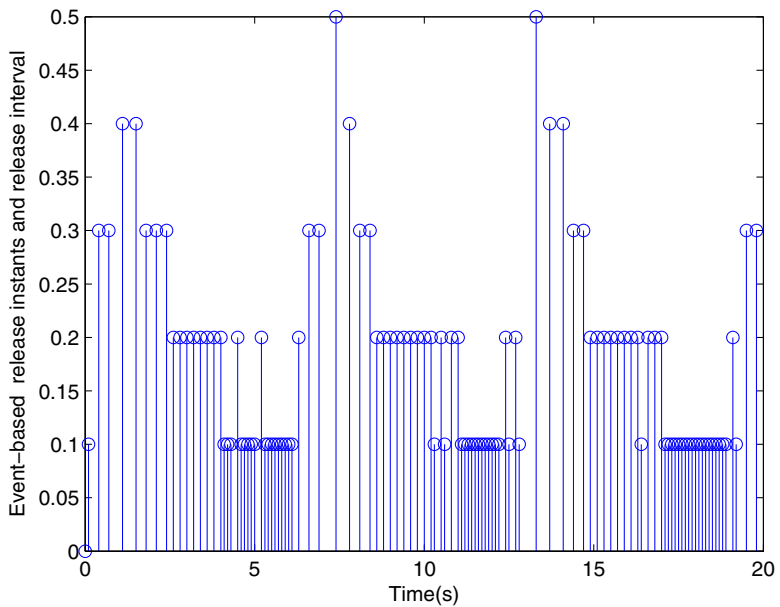


Fig. 5. Release instants and intervals in case 2.

actuator faults and probabilistic nonlinear perturbations happen. Moreover, the hybrid driven scheme covers the features in the time-driven scheme and event-driven scheme.

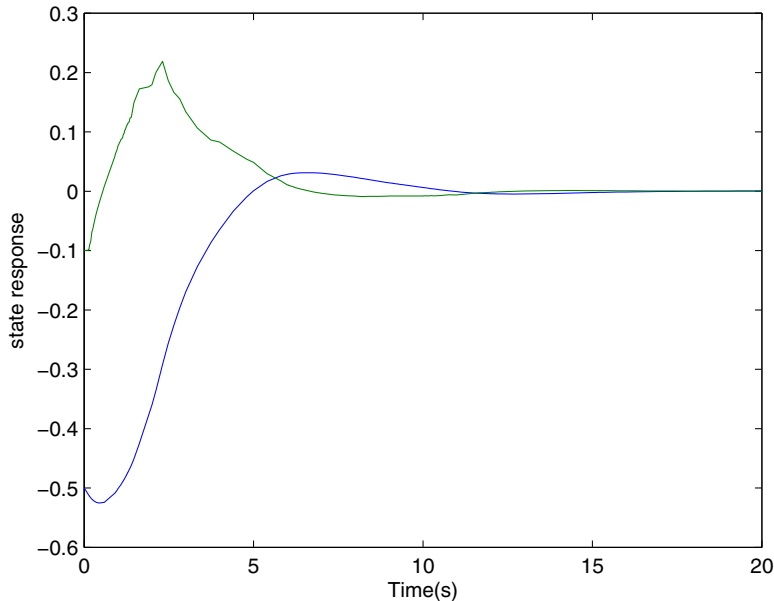


Fig. 6. The state response under feedback gain in case 3.

5. Conclusion

This paper discusses the reliable control for a class of hybrid-driven nonlinear networked control systems via T–S fuzzy model with random actuator faults and nonlinear perturbations. The randomly occurring actuator faults are governed by a set of unrelated random variables satisfying certain probabilistic distribution. Considering the effects of random actuator faults and the random nonlinear perturbations, a new model is developed under the hybrid-driven scheme in this paper. By using Lyapunov functional method and linear matrix inequality technique, sufficient conditions for the asymptotical stability of the augmented system are obtained. Furthermore, the explicit expressions of the desired controller gains and the parameters of the hybrid-driven scheme are derived. A simulation example is used to highlight the application and effectiveness of the proposed method.

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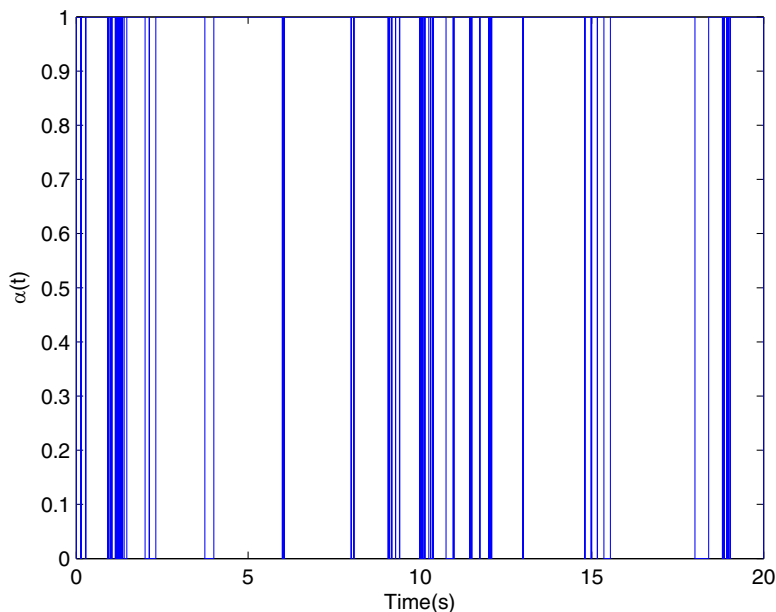


Fig. 7. The Bernoulli stochastic variable $\alpha(t)$ with $\bar{\alpha} = 0.25$ in case 3.

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