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Resilient event-triggered consensus control for nonlinear muti-agent systems with DoS attacks[☆]

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Abstract

The distributed event-triggered secure consensus control is discussed for multi-agent systems (MASs) subject to DoS attacks and controller gain variation. In order to reduce unnecessary network traffic in communication channel, a resilient distributed event-triggered scheme is adopted at each agent to decide whether the sampled signal should be transmitted or not. The event-triggered scheme in this paper can be applicable to MASs under denial-of-service (DoS) attacks. We assume the information of DoS attacks, such as the attack period and the consecutive attack duration, can be detected. Under the introduced communication scheme and the occurrence of DoS attacks, a new sufficient condition is achieved which can guarantee the security consensus performance of the established system model. Moreover, the explicit expressions of the triggering matrices and the controller gain are presented. Finally, simulation results are provided to verify the effectiveness of the obtained theoretical results.

1. Introduction

Consensus problem has been one of the most important and typical behaviors in multi-agent systems (MASs) [1–3]. Increasing attention has been attracted on the consensus control

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problem of MASs due to its widespread applications in various areas such as smart robots, power systems, formation control of vehicles, sensor networks and so on. A rich body of publications has been available [4–8]. For example, in [4], the authors addressed the event-triggered consensus tracking control for switched stochastic nonlinear MASs. A novel neural network control approach was been studied for nonlinear MASs in [5]. However, most of the literature assume that the network communication resources are unlimited and the communication channel between sensor and controller are safe. The observer-based consensus control was been addressed in [6] for MASs with lossy sensors and cyber-attacks. In fact, many factors can affect the desired system performance and result in secure crisis. Specially, the data transmission in network communication channel may be interrupted by malicious agent, which prompts us to investigate secure consensus control of MASs.

System secure is an important problem and stirring an ever-increasing research interests, due to the fact that network information transmission is susceptible to be corrupted by adversaries [9–11]. Recently, considerable effort has been devoted to investigate secure control problem against malicious attacks [12–14] including deception attacks [15–17], denial of service (DoS) attacks [18,19] and replay attacks [20] and so on. Various results dealing with different type of cyber attacks can be available [15,17,20,21]. For instance, in [15], secure control problem was studied for neural networks under deception attacks and decentralized event-triggered scheme. Security controller design of event-triggered networked control systems was dealt with subject to periodic DoS attacks. The distributed event-triggered control strategy was designed in [17] for networked control systems with stochastic cyber-attacks. The authors in [20] studied the distributed Kalman fusion estimation for cyber-physical systems under bandwidth constraints and replay attacks. A dynamic output feedback controller was designed in [21] for a class of stochastic nonlinear systems subject to deception attacks. Up to now, the influences of DoS attacks on consensus of nonlinear MASs have not been fully explored yet, which motivates our present work.

In recent years, networked control systems have attracted much attention due to the rapid development of network technology [22,23]. The issue of how to design appropriate data transmission strategies for networked control systems has been one of the critical problems, which aims to save the limited communication bandwidth and energy consumption while ensure the desired system performance. There has been a growing number of publications in the literature [24–27]. The existing transmission method can be categorized as follows: time-triggered method [28], continuous event-triggered method [29], discrete event-triggered method [30,31] and hybrid-triggered method [32]. Since the event-triggered method can effectively reduce the communication cost and alleviate network congestion, it has been widely studied. Specifically, the authors in [29] investigated leader-following consensus of heterogeneous MASs under aperiodic sampling and DoS attack. In [29], the authors studied the consensus problem for nonlinear MASs under continuous event-triggered scheme and cyber attacks. A discrete event-triggered controller was designed in [30] for networked control systems in which the sensor data did not need to be transmitted continuously. In [33], event-triggered H_∞ load frequency control was discussed for multiarea power systems subject to DoS attacks and deception attacks. However, the majority of the available event-triggered control strategies are not suitable to the addressed MASs in the presence of DoS attacks. Further developments are needed to deal with the control problem under discrete event-triggered scheme and DoS attacks.

It is noticed from the above mentioned references that the information transmission channel is likely to be interrupted by the occurrence of DoS attacks in practical systems, leading to system performance decreased and even unstable. Moreover, It is necessary to utilize the scarce resources effectively while guaranteeing the desired system performance. With these in mind, in this paper, event-triggered consensus control problem for MASs under periodic attacks will be addressed, which is still an interesting and challenging task. The main contributions can be described as follows.

(1) The majority of the previous event-triggered control method of MASs did not consider the impact of the DoS attacks, which can not be applied directly in the occurrence of DoS attacks. In this paper, a distributed event-triggered scheme is developed to improve the network communication efficiency, which can be adapt to the consensus control of MASs under circumstance of DoS attacks. Each agent can decide its own triggering instants based on the error between the current state and the latest released one.
(2) The parameters of controller and event generators are co-designed based on the constructed model of the discussed MASs. Compared with the published results, the obtained distributed event-triggered controller is resilient to DoS attacks and the controller gain variation, which is capable of guaranteeing the desired security performance of the addressed MASs.

This paper is outlined as follows. The basic graph theory and problem formulation are given in Section 2. In Section 3, the main proposed theoretical results are proposed. A simulation example is presented in Section 4 to show the usefulness of our proposed method. Finally, conclusions are drawn in Section 5.

Notation: $\mathbb{N}$ stands for the set of positive integer; I_N represents N-dimensional identity matrix; The notation $Q>0$ stands for Q is a real symmetric and positive definite; I is the identity matrix of appropriate dimension; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices; For matrices A and B, $A\otimes B$ describes their Kronecker product; $\lambda_{max}A$ ($\lambda_{min}A$) denotes the maximum (minimum) eigenvalue of matrix A; If $\varsigma(t)$ is a continuous function, $\| \varsigma(t) \|_h \triangleq \sup_{-h\leq s\leq 0}\{\| \dot{\varsigma}(t+s), \| \varsigma(t+s) \|\}$; The superscript T denotes matrix transposition; For $x(t)\in\mathbb{R}^n$, $\sqrt{x^T(t)x(t)}$ is its vector norm. For matrix Y and symmetric matrices X and Z, $\begin{bmatrix} X & * \\ Y & Z \end{bmatrix}$ is a symmetric matrix, in which * is used to describe the entries implied by symmetry.

2. Problem formulation

Graph theory: An undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ is used to describe the communication topology of a multiagent system, where $\mathcal{V}=\{1,2,\ldots,N\}$ represents the node set, $\mathcal{E}=\{(i,j)|i,j\in\mathcal{V}, i\neq j\}$ denotes the edge set and $\mathcal{A}=(a_{ij})_{N\times N}$ is the weighted adjacency matrix. Edge $(i,j)\in\mathcal{E}$ indicates that agent i can get information from agent j. $\mathcal{N}_i=\{j|(i,j)\in\mathcal{E}\}$ is the set of neighbors of agent i. $a_{ij}>0$ if $(i,j)\in\mathcal{E}$, otherwise, $a_{ij}=0$. Define $\mathcal{D}=diag\{deg_1, deg_2,\ldots,deg_N\}$ as the in-degree matrix with $deg_i=\sum_{j\in\mathcal{N}_i}a_{ij}$ for agent i. The Laplacian matrix $\mathcal{L}=(l_{ij})_{N\times N}$ of the graph $\mathcal{G}$ is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$.

Consider the following nonlinear MAS consisting of N agents, in which the ith agent is modeled by

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t)+E_1f(x_i(t))+E_2h(x_i(t-d(t))) \tag{1}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector of the ith agent, $u_i(t)$ represents the control input vector of the ith agent, $f(x_i(t))$ and $h(x_i(t))$ are nonlinear vector-valued functions satisfying certain assumptions, which will be given later. $d(t)$ is a time-varying delay and $0 < d(t) < d_M$. A, B, E_1 and E_2 are known matrices with appropriate dimensions.

Similar to [34], we assume $f(x_i(t))$ and $h(x_i(t-d(t)))$ satisfy the following conditions:

$$\| f(x_i(t)) \| \leq \| Mx_i(t) \| \tag{2}$$

$$\| h(x_i(t-d(t))) \| \leq \| Hx_i(t-d(t)) \| \tag{3}$$

In this paper, the secure consensus problem will be considered for system (1) when considering distributed event-triggered scheme, DoS attacks and controller gain variation. The system modeling method will be presented step by step as follows:

Consider the constriction of communication energy among agents, it is necessary to reduce the data transmission rate. For each agent i, an event generator is implemented to decide whether its state should be transmitted to its neighbors or not. The event-triggered condition is designed as follows:

$$(e_k^i(t))^T \Omega_i e_k^i(t) \leq \sigma x_i^T(t_k^i h + r^i h)\Omega_i x_i(t_k^i h + r^i h) \tag{4}$$

where $e_k^i(t)$ is the error between the latest transmission instant $x_i(t_k^i h)$ and the current sampling instant $x_i(t_k^i h + r^i h)$ for the ith agent, that is, $e_k^i(t) = x_i(t_k^i h) - x_i(t_k^i h + r^i h)$, $i \in \{1, 2, \ldots, N\}$, $\sigma \in [0, 1)$ is constant, Ω_i is the triggering matrix. $t_k^i h$ is the latest released instant, h is the sampling period. $r^i \in \mathbb{N}$. Once condition (4) holds, the state $x_i(t_k^i h + r^i h)$ of agent i will be discarded, otherwise, it will be transmitted to its neighbors, and $x_i(t_{k+1}^i h) = x_i(t_k^i h + r^i h)$

Assumption 1. The discussed MAS (1) is under a fixed network topology with a undirected spanning tree. DoS attacks can only destroy the control input of each agent. The topology graph cannot be changed.

Based on the distributed event-triggered scheme, the event-triggered controller is designed as

$$u_i(t) = (K + \Delta K) \sum_{j \in \mathcal{N}_i} a_{ij}(x_j(t_k^j h) - x_i(t_k^i h)), t \in [t_k^i h, t_{k+1}^i h) \tag{5}$$

in which K is the controller gain to be determined later, ΔK denotes the structure $C\Delta(t)D$, C and D are the given constant matrices, $\Delta(t)$ is an unknown time-varying matrix and satisfies $\Delta^T(t)\Delta(t) \leq I$.

Remark 1. Considering implementation errors or unknown noise, controllers inevitably have some gain variation which could degenerate the desired control performance or make system unstable. Hence, it is necessary to investigate the non-fragile controller design problem. In this paper, ΔK is the corresponding uncertain matrix on implementation.

Note that the communication channel between the sensors and the corresponding controllers in MASs are vulnerable to DoS attacks which have significant impact on the consensus performance. In this paper, the attacker is assumed to be power-constraint which needs to supply energy for the next attack. There exists an infinite sequence of time intervals $[nT, nT + T), n = 0, 1, 2, \ldots,$ $\mathcal{I}_{1,n} = [nT, nT + T_{off})$ denotes the interval during which the network works in a good condition, $\mathcal{I}_{2,n} = [nT + T_{off}, nT + T)$ is the interval during which the DoS attacks happen and the network communication is paralyzed. $T_{off} \in [T_{off}^{min}, +\infty)$ denotes the length of the time interval during which the network transmission is possible, T_{off}^{min} is a given positive constant.

Remark 2. With consideration of the fact that DoS attacks blocking the network communication intermittently, the event-triggered instants in this paper are

$$t_{k,n}^{i}h = \{t_{k_m,n}^{i}h | t_{k_m,n}^{i}h \ satisfying (4), t_{k_m,n}^{i}h \in \mathcal{I}_{1,n}\} \bigcup \{nT\} \tag{6}$$

where $k_m, n, m \in \mathbb{N}$. $t_{k,n}^{i}h$ is the instant at which the k^{th} sampled state of agent i is being sent to the corresponding controller in n^{th} DoS attack occurring. Specifically, the event-triggered instant is nT if there is no event being triggered.

Remark 3. It should be pointed out that the presence of DoSs attacks is not taken into consideration in most of the published results about the event-triggered consensus control of MASs, which results that the existing event-triggered control strategies can not guarantee the desired system performance. The modified event-triggered scheme (6) in this paper can not only reduce the network transmissions, but also it can deal with the impact of the DoSs attacks. Therefore, the developed event-triggered control method can be applied in the situation of DoSs occurring.

Based on the above description, the following control protocol is designed:

$$u_i(t) = \begin{cases} (K + \Delta K) \sum_{j \in \mathcal{N}_i} a_{ij} (x_j(t_{k,n}^{j}h) - x_i(t_{k,n}^{i}h)), & t \in [t_{k,n}^{i}h, t_{k+1,n}^{i}h) \bigcap \mathcal{I}_{1,n} \\ 0, \quad t \in \mathcal{I}_{2,n} \end{cases} \tag{7}$$

where $\{t_{k,n}^{i}h\}$ is the set of released instants at which the sampled sensor measurements are sent to the controller successfully. $k \in \{0, 1, 2, \ldots, k(n)\}$, $k(n) = \sup\{k \in \mathbb{N} | nT + T_{off} \geq t_{k,n}^{i}h, n \in \mathbb{N}\} \triangleq \mathcal{K}(n)$ $t_{0,n}^{i}h \triangleq (n-1)T, h < T$.

Similar to [18], the intervals $[t_{k,n}^{i}h, t_{k+1,n}^{i}h), k \in \mathcal{K}(n)$ can be divided as $[t_{k,n}^{i}h, t_{k+1,n}^{i}h) = \bigcup_{m=0}^{d_i} L_{i,k,n}^{m}$, where $L_{i,k,n}^{m} = [t_{k,n}^{i}h + mh, t_{k,n}^{i}h + mh + h)$, $m = 0, 1, 2, \ldots, d_i$, $L_{i,k,n}^{d_i} = [t_{k,n}^{i}h + d_i h, t_{k+1,n}^{i}h)$.

Note that $\mathcal{I}_{1,n} = \bigcup_{k=0}^{k(n)} \{[t_{k,n}^{i}h, t_{k+1,n}^{i}h) \bigcap \mathcal{I}_{1,n}\} = \bigcup_{k=0}^{k(n)} \bigcup_{m=0}^{d_i} \{L_{i,k,n}^{m} \bigcap \mathcal{I}_{1,n}\}$. Set $\Phi_{i,k,n}^{m} = L_{i,k,n}^{m} \bigcap \mathcal{I}_{1,n}$, Then $\mathcal{I}_{1,n} = \bigcup_{k=0}^{k(n)} \bigcup_{m=0}^{d_i} \Phi_{i,k,n}^{m}$.

Define

$$\eta_{k,n}^{i}(t) = \begin{cases} t - t_{k,n}^{i}h, & t \in \Phi_{i,k,n}^{0} \\ t - t_{k,n}^{i}h - h, & t \in \Phi_{i,k,n}^{1} \\ \vdots \\ t - t_{k,n}^{i}h - d_i h, & t \in \Phi_{i,k,n}^{d_i} \end{cases} \tag{8}$$

and

$$e_{k,n}^{i}(t) = \begin{cases} 0, & t \in \Phi_{i,k,n}^{0} \\ x_i(t_{k,n}^{i}h) - x_i(t_{k,n}^{i}h + h), & t \in \Phi_{i,k,n}^{1} \\ \vdots \\ x_i(t_{k,n}^{i}h) - x_i(t_{k,n}^{i}h + d_i h), & t \in \Phi_{i,k,n}^{d_i} \end{cases} \tag{9}$$

Obviously, based on the definitions of $\eta_{k,n}^{i}(t)$ and $e_{k,n}^{i}(t)$, we get $0 \leq \eta_{k,n}^{i}(t) < h$ and $x(t_{k,n}^{i}h) = e_{k,n}^{i}(t) + x(t - \eta_{k,n}^{i}(t))$, $t \in [t_{k,n}^{i}h, t_{k+1,n}^{i}h) \bigcap \mathcal{I}_{1,n}$.

Then, considering the event-triggered scheme and the DoS attacks, the controller (7) can be rewritten as

$$u_i(t) = \begin{cases} (K+\Delta K)\sum_{j\in\mathcal{N}_i} a_{ij}\tilde{x}_i(t), & t \in [t^i_{k,n}h, t^i_{k+1,n}h)\bigcap \mathcal{I}_{1,n} \\ 0, \quad t \in \mathcal{I}_{2,n} \end{cases} \tag{10}$$

in which $\tilde{x}_i(t) = e^j_{k,n}(t) + x_j(t-\eta^j_{k,n}(t)) - e^i_{k,n}(t) - x_i(t-\eta^i_{k,n}(t))$.

Remark 4. In the presence of DoS attacks, the released signal cannot arrive at the controller successfully. In order to deal with this situation, a resilient event-triggered condition (6) is developed in this paper with consideration of the DoS attacks. Based on the designed event-triggered scheme, the controller (10) can be implemented to guarantee the secure consensus performance of the discussed MASs if the DoSs occur.

For convenience, denote

$$\begin{aligned} x(t) &= \begin{bmatrix} x_1^T(t) & x_2^T(t) & \cdots & x_N^T(t) \end{bmatrix}^T \\ f(x(t)) &= \begin{bmatrix} f^T(x_1(t)) & f^T(x_2(t)) & \cdots & f^T(x_N(t)) \end{bmatrix}^T \\ h(x(t)) &= \begin{bmatrix} h^T(x_1(t-d(t))) & h^T(x_2(t-d(t))) & \cdots & h^T(x_N(t-d(t))) \end{bmatrix}^T \\ x(t-\eta_{k,n}(t)) &= \begin{bmatrix} x_1^T(t-\eta^1_{k,n}(t)) & x_2^T(t-\eta^2_{k,n}(t)) & \cdots & x_N^T(t-\eta^N_{k,n}(t)) \end{bmatrix}^T \\ e_{k,n}(t) &= \begin{bmatrix} (e^1_{k,n}(t))^T & (e^2_{k,n}(t))^T & \cdots & (e^N_{k,n}(t))^T \end{bmatrix}^T \end{aligned}$$

Based on the Kronecker product, the system (1) with (10) can be further written as

$$\dot{x}(t) = \begin{cases} (I_N\otimes A)x(t) + (\mathcal{L}\otimes(B(K+\Delta K))x(t-\eta_{k,n}(t)) + (\mathcal{L}\otimes(B(K+\Delta K))e_{k,n}(t) \\ \quad + (I_N\otimes E_1)f(x(t)) + (I_N\otimes E_2)h(x(t-d(t))), t\in[t^i_{k,n}h, t^i_{k+1,n}h)\bigcap\mathcal{I}_{1,n} \\ (I_N\otimes A)x(t) + (I_N\otimes E_1)f(x(t)) + (I_N\otimes E_2)h(x(t-d(t))), t\in\mathcal{I}_{2,n} \end{cases} \tag{11}$$

and we can obtain $e_{k,n}(t)$ satisfy the following inequality:

$$e^T_{k,n}(t)\Omega e_{k,n}(t) \le \sigma x^T(t-\eta_{k,n}(t))\Omega x(t-\eta_{k,n}(t)) \tag{12}$$

where $\Omega = diag\{\Omega_1, \Omega_2, \ldots, \Omega_N\}$

Inspired by [18], we define $\theta(t) = \{1, 2\}$ as the state of the MAS (1). $\theta(t) = 1$ denotes $t \in \bigcup_{n\in\mathbb{N}}\mathcal{I}_{1,n}$, $\theta(t) = 2$ indicates $t \in \bigcup_{n\in\mathbb{N}}\mathcal{I}_{2,n}$. Set $\delta_{l,n} = \begin{cases} nT, & l=1 \\ nT+T_{off}, & l=2 \end{cases}$, obviously, $\mathcal{I}_{l,n} = [\delta_{l,n}, \delta_{3-l,n+l-1})$, $\theta(\delta_{l,n}) = l, \theta(\delta^-_{l,n}) = 3-l$

According to the definition of $\theta(t)$, Eq. (11) can be expressed as the following switched system:

$$\begin{aligned} \dot{x}(t) = {} & (I_N\otimes A)x(t) + \mathcal{B}_{\theta(t)}x(t-\eta_{k,n}(t)) + \mathcal{B}_{\theta(t)}e_{k,n}(t) + (I_N\otimes E_1)f(x(t)) \\ & + (I_N\otimes E_2)h(x(t-d(t))), t\in[\delta_{l,n}, \delta_{3-l,n+l-1}) \end{aligned} \tag{13}$$

where $\mathcal{B}_1 = \mathcal{L}\otimes(B(K+\Delta K))$, $\mathcal{B}_2 = 0$, the initial condition of the state $x(t)$ is $x(t) = \varsigma(t), t\in[-h, 0]$, $\varsigma(t)$ is a continuous function on $[-h, 0]$.

Remark 5. For $t \in \mathcal{I}_{2,n}$, DoS attacks happen, there are no control inputs available for agent i. For $t \in \mathcal{I}_{1,n}$, the released signal at the event generator is delivered and reach the controller. Therefore, the discussed MAS can be modeled as a switching system (13) between two modes.

The control objective of this paper is to tackle the resilient event-triggered consensus control of nonlinear MASs (13) subject to DoS attacks. The following lemmas are provided which are necessary for the development of the main results:

Lemma 1. *[35] For any matrices $\Xi \in \mathbb{R}^{n\times n}$ and $\Lambda \in \mathbb{R}^{n\times n}$ that satisfy $\begin{bmatrix} \Xi & \Lambda \\ \Lambda^T & \Xi \end{bmatrix} \geq 0$, the following inequality holds for $\vartheta(t) \in [0, \bar{\vartheta}]$:*

$$-\bar{\vartheta}\int_{t-\bar{\vartheta}}^{t} \dot{x}^T(s)\Xi\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-\vartheta(t)) \\ x(t-\bar{\vartheta}) \end{bmatrix}^T \begin{bmatrix} -\Xi & * & * \\ \Xi^T - \Lambda^T & -2\Xi + \Lambda + \Lambda^T & * \\ \Lambda^T & \Xi^T - \Lambda^T & -\Xi \end{bmatrix} \times \begin{bmatrix} x(t) \\ x(t-\vartheta(t)) \\ x(t-\bar{\vartheta}) \end{bmatrix} \tag{14}$$

Lemma 2. *Suppose nT, T and T_{off} are known, for given h, α_1, α_2, σ and matrices K, M, H, the function defined in Eq. (50) satisfies*

$$V_l(t) \leq e^{2(-1)^l\alpha_l(t-\delta_{l,n})}V_l(\delta_{l,n}), t \in [\delta_{l,n}, \delta_{3-l,n+l-1}), l = 1, 2, n \in \mathbb{N} \tag{15}$$

provided that there exist matrices $P_l > 0$, $Q_{1l} > 0$, $Q_{2l} > 0$, $R_{1l} > 0$, $R_{2l} > 0$, matrices $\Omega > 0$, U_{1l} and U_{2l} with compatible dimensions satisfying the following inequalities:

$$\Xi^l = \begin{bmatrix} \Xi_{11}^l & * & * & * & * \\ h\Gamma^l & -(I_N \otimes P_l)R_{1l}^{-1}(I_N \otimes P_l) & * & * & * \\ d_M\Gamma^l & 0 & -(I_N \otimes P_l)R_{2l}^{-1}(I_N \otimes P_l) & * & * \\ \Xi_{41} & 0 & 0 & -I & * \\ \Xi_{51} & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} R_{11} & U_{11} \\ U_{11}^T & R_{11} \end{bmatrix} > 0, \quad \begin{bmatrix} R_{21} & U_{21} \\ U_{21}^T & R_{21} \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} R_{12} & U_{12} \\ U_{12}^T & R_{12} \end{bmatrix} > 0, \quad \begin{bmatrix} R_{22} & U_{22} \\ U_{22}^T & R_{22} \end{bmatrix} > 0 \tag{18}$$

where

$$\Xi_{11}^1 = \begin{bmatrix} \Pi_{11}^1 & * & * & * & * & * & * & * \\ \Pi_{32}^1 & \Pi_{22}^1 & * & * & * & * & * & * \\ e^{-2\alpha_1 h}U_{21}^T & \Pi_{32}^1 & \Pi_{33}^1 & * & * & * & * & * \\ \Pi_{41}^1 & 0 & 0 & \Pi_{44}^1 & * & * & * & * \\ e^{-2\alpha_1 h}U_{11}^T & 0 & 0 & \Pi_{54}^1 & \Pi_{55}^1 & * & * & * \\ \Pi_{61}^1 & 0 & 0 & 0 & 0 & -\Omega & * & * \\ I_N \otimes E_1^T P_1 & 0 & 0 & 0 & 0 & 0 & -I & * \\ I_N \otimes E_2^T P_1 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\Pi_{11}^1 = I_N \otimes P_1 A + I_N \otimes A^T P_1 + Q_{11} + Q_{21} - e^{-2\alpha_1 h} R_{21} - e^{-2\alpha_1 h} R_{11} + 2\alpha_1 I_N \otimes P_1$$
$$\Pi_{22}^1 = e^{-2\alpha_1 h}(-2R_{21} + U_{21} + U_{21}^T),\ \Pi_{32}^1 = e^{-2\alpha_1 h}(R_{21} - U_{21}^T)$$
$$\Pi_{33}^1 = -e^{-2\alpha_1 h}(Q_{21} + R_{21}),\ (\Pi_{41}^1)^T = e^{-2\alpha_1 h}(R_{11} - U_{11}) + \mathcal{L} \otimes P_1 B(K + \Delta K)$$
$$\Pi_{44}^1 = \sigma\Omega + e^{-2\alpha_1 h}(U_{11} + U_{11}^T - 2R_{11}),\ \Pi_{54}^1 = e^{-2\alpha_1 h}(R_{11} - U_{11}^T)$$
$$\Pi_{55}^1 = e^{-2\alpha_1 h}(R_{11} - Q_{11}),\ (\Pi_{61}^1)^T = \mathcal{L} \otimes P_1 B(K + \Delta K)$$
$$\Gamma^1 = \begin{bmatrix} I_N \otimes P_1 A & 0 & 0 & (\Pi_{61}^1)^T & 0 & (\Pi_{61}^1)^T & I_N \otimes P_1 E_1 & I_N \otimes P_1 E_2 \end{bmatrix}$$
$$\Xi_{41} = \begin{bmatrix} I_N \otimes M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Xi_{51} = \begin{bmatrix} 0 & I_N \otimes H & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Xi_{11}^2 = \begin{bmatrix} \Pi_{11}^2 & * & * & * & * & * & * \\ e^{2\alpha_2 h}(R_{22} - U_{22}^T) & \Pi_{22}^2 & * & * & * & * & * \\ e^{2\alpha_2 h} U_{22}^T & \Pi_{32}^2 & \Pi_{33}^2 & * & * & * & * \\ \Pi_{54}^2 & 0 & 0 & \Pi_{44}^2 & * & * & * \\ e^{2\alpha_2 h} U_{12}^T & 0 & 0 & \Pi_{54}^2 & \Pi_{55}^2 & * & * \\ I_N \otimes E_1^T P_2 & 0 & 0 & 0 & 0 & -I & * \\ I_N \otimes E_2^T P_2 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$
$$\Pi_{11}^2 = I_N \otimes P_2 A + I_N \otimes A^T P_2 + Q_{12} + Q_{22} - e^{2\alpha_2 h}(R_{22} + R_{12}) - 2\alpha_2 I_N \otimes P_2$$
$$\Pi_{22}^2 = e^{2\alpha_2 h}(-2R_{22} + U_{22} + U_{22}^T),\ \Pi_{32}^2 = e^{2\alpha_2 h}(R_{22} - U_{22}^T)$$
$$\Pi_{44}^2 = e^{2\alpha_2 h}(U_{12} + U_{12}^T - 2R_{12}),\ \Pi_{33}^2 = -e^{2\alpha_2 h}(Q_{22} + R_{22})$$
$$\Pi_{55}^2 = e^{2\alpha_2 h}(Q_{12} + R_{12}),\ \Pi_{54}^2 = e^{2\alpha_2 h}(R_{12} - U_{12}^T)$$
$$\Gamma^2 = \begin{bmatrix} I_N \otimes P_2 A & 0 & 0 & 0 & 0 & I_N \otimes P_2 E_1 & I_N \otimes P_2 E_2 \end{bmatrix}$$

Proof. See Appendix A. □

3. Main results

Based on the result in Lemma 2, we are now ready to present the exponential stability criteria of the discussed system (13).

Theorem 1. *Assume nT, T and T_{off} are known, let the following parameters h, $\alpha_l \in (0, +\infty)$, $\lambda_l \in (1, +\infty)$ $(l = 1, 2)$, σ and matrices K, M, H are given, if there exist matrices $P_l > 0$, $Q_{1l} > 0$, $Q_{2l} > 0$, $R_{1l} > 0$, $R_{2l} > 0$, matrices $\Omega > 0$, U_{1l} and U_{2l} with compatible dimensions satisfying the following conditions:*

$$0 < \phi \triangleq 2\alpha_1 T_{off} - 2\alpha_2(T - T_{off}) - 2(\alpha_1 + \alpha_2)h - ln(\lambda_1\lambda_2) \tag{19}$$

$$I_N \otimes P_1 \le \lambda_2(I_N \otimes P_2) \tag{20}$$

$$I_N \otimes P_2 \le \lambda_1 e^{2(\alpha_1+\alpha_2)h}(\alpha_1 + \alpha_2)h \tag{21}$$

$$Q_{1l} \le \lambda_{3-l} Q_{1\ 3-l} \tag{22}$$

$$Q_{2l} \le \lambda_{3-l} Q_{2\ 3-l} \tag{23}$$

$$R_{1l} \le \lambda_{3-l} R_{1\ 3-l} \tag{24}$$

$$R_{2l} \le \lambda_{3-l} R_{2\ 3-l} \tag{25}$$

Then, the switched system (13) is exponential stable with a decay rate $\gamma = \frac{\phi}{2T}$

Proof. According to Lemma 2, it is easy to find

$$V(t) \le \begin{cases} e^{-2\alpha_l (t-\delta_{1,n})} V_1(\delta_{1,n}), t \in [\delta_{1,n}, \delta_{2,n}) \\ e^{2\alpha_2 (t-\delta_{2,n})} V_2(\delta_{2,n}), t \in [\delta_{2,n}, \delta_{1,n+1}) \end{cases} \tag{26}$$

From Eqs. (20)–(25), we can deduce

$$\begin{cases} \lambda_2 V_2(\delta_{1,n}^-) - V_1(\delta_{1,n}) \ge 0 \\ e^{2(\alpha_1+\alpha_2)h} \lambda_1 V_1(\delta_{2,n}^-) - V_2(\delta_{2,n}) \ge 0 \end{cases} \tag{27}$$

Noticed that $\forall t \ge 0$, there exists $n \in \mathbb{N}$ satisfying $t \in [\delta_{1,n}, \delta_{2,n})$ or $t \in [\delta_{2,n}, \delta_{1,n+1})$. We will consider two cases below.

For $t \in [\delta_{1,n}, \delta_{2,n})$, similar to the method in [18,19], one may conclude from Eqs. (26) and (27) that

$$\begin{aligned} V(t) &\le \lambda_2 e^{-2\alpha_l (t-\delta_{1,n})} V_1(\delta_{1,n}^-) \\ &\le \vdots \\ &\le e^{-\phi n} V_1(\delta_{1,0}) = e^{-\phi n} V_1(0) \end{aligned} \tag{28}$$

Since $t < \delta_{2,n} = nT + T_{off}$, that is $n > \frac{t - T_{off}}{T}$. Then, we can get

$$V(t) \le V_1(\delta_{1,0}) e^{-\frac{\phi}{T} t} e^{\frac{\phi T_{off}}{T}} \tag{29}$$

For $t \in [\delta_{2,n}, \delta_{1,n+1})$, similar to above, we can obtain

$$V(t) \le \frac{V_1(\delta_{1,0})}{\lambda_2} e^{-\frac{\phi}{T} t} \tag{30}$$

Define $a_0 = \max\{e^{\frac{\phi T_{off}}{T}}, \frac{1}{\lambda_2}\}$, $a_1 = \min\{\lambda_{min}(P_1), \lambda_{min}(P_2)\}$, $a_2 = \max\{\lambda_{max}(P_1), \lambda_{max}(P_2)\}$, $a_3 = a_2 + h\lambda_{max}(Q_{11} + Q_{21}) + \frac{h^2}{2}\lambda_{max}(R_{11} + R_{21})$. Recalling the definition of $V(t)$ in Eq. (50), it follows from Eqs. (29) and (30) that:

$$V(t) \ge a_1 \parallel x(t) \parallel^2, \qquad V_1(0) \le a_3 \parallel \varsigma(0) \parallel_h^2 \tag{31}$$

Combining Eqs. (30) and (31), we can obtain

$$\parallel x(t) \parallel \le \sqrt{\frac{a_0 a_3}{a_1}} e^{\gamma t} \parallel \varsigma(0) \parallel^2, \qquad \forall t \ge 0 \tag{32}$$

which proves that the system (13) is exponential stable with a decay rate $\gamma = \frac{\phi}{2T}$.

On the basis of Theorem 1, we will provide the controller design method in the following theorem: □

Theorem 2. *Assume nT, T and T_{off} are known. For given parameters h, $\alpha_l \in (0, +\infty)$, $\lambda_l \in (1, +\infty)$, $(l = 1, 2)$, ε_{11}, ε_{21}, ε_3, σ, ε_l, μ_l, ν_l, χ_l and matrix M, H, system (13) with controller gain $K = Y_1 X_1^{-1}$ can be exponential stable with a decay rate $\gamma = \frac{\phi}{2T}$, if there exist matrices Y_1, $\hat{Q}_{1l} > 0$, $\hat{Q}_{2l} > 0$, $\hat{R}_{1l} > 0$, $\hat{R}_{2l} > 0$, $\hat{\Omega}$, $\hat{U}_{1l}$ and $\hat{U}_{2l}$ with compatible dimensions such that Eq. (19) and the following linear matrix inequalities hold:*

$$\hat{\Xi}^1 = \begin{bmatrix} \hat{\Xi}^1_{11} & * & * & * & * & * & * \\ h\hat{\Gamma}^1 & \hat{\Xi}^1_{22} & * & * & * & * & * \\ d_M\hat{\Gamma}^1 & 0 & \hat{\Xi}^1_{33} & * & * & * & * \\ \hat{\Xi}^1_{41} & 0 & 0 & -I & * & * & * \\ \hat{\Xi}^1_{51} & 0 & 0 & 0 & -I & * & * \\ \hat{\Xi}^1_{61} & \varepsilon_3 h\hat{\Gamma}^2 & \varepsilon_3 d_M\hat{\Gamma}^2 & 0 & 0 & -\varepsilon_3 I & * \\ \hat{\Xi}^1_{71} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{33}$$

$$\hat{\Xi}^2 = \begin{bmatrix} \hat{\Xi}^2_{11} & * & * & * & * \\ h\hat{\Gamma}^2 & \hat{\Xi}^2_{22} & * & * & * \\ d_M\hat{\Gamma}^2 & 0 & \hat{\Xi}^2_{33} & * & * \\ \hat{\Xi}^2_{41} & 0 & 0 & -I & * \\ \hat{\Xi}^2_{51} & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{34}$$

$$\begin{bmatrix} \hat{R}_{11} & \hat{U}_{11} \\ \hat{U}^T_{11} & \hat{R}_{11} \end{bmatrix} > 0, \quad \begin{bmatrix} \hat{R}_{21} & \hat{U}_{21} \\ \hat{U}^T_{21} & \hat{R}_{21} \end{bmatrix} > 0 \tag{35}$$

$$\begin{bmatrix} \hat{R}_{12} & \hat{U}_{12} \\ \hat{U}^T_{12} & \hat{R}_{12} \end{bmatrix} > 0, \quad \begin{bmatrix} \hat{R}_{22} & \hat{U}_{22} \\ \hat{U}^T_{22} & \hat{R}_{22} \end{bmatrix} > 0 \tag{36}$$

$$\begin{bmatrix} \lambda_2 I_N \otimes X_2 & * \\ I_N \otimes X_2 & -I_N \otimes X_1 \end{bmatrix} < 0 \tag{37}$$

$$\begin{bmatrix} e^{2(\alpha_1+\alpha_2)h}\lambda_1 I_N \otimes X_1 & * \\ I_N \otimes X_1 & -I_N \otimes X_2 \end{bmatrix} < 0 \tag{38}$$

$$\begin{bmatrix} -\lambda_{3-l}\hat{Q}_{1\ 3-l} & I_N \otimes X_{3-l} \\ I_N \otimes X_{3-l} & -2\varepsilon_l(I_N \otimes X_l) + \varepsilon_l^2\hat{Q}_{1\ 3-l} \end{bmatrix} < 0 \tag{39}$$

$$\begin{bmatrix} -\lambda_{3-l}\hat{Q}_{2\ 3-l} & I_N \otimes X_{3-l} \\ I_N \otimes X_{3-l} & -2\mu_l(I_N \otimes X_l) + \mu_l^2\hat{Q}_{2\ 3-l} \end{bmatrix} < 0 \tag{40}$$

$$\begin{bmatrix} -\lambda_{3-l}\hat{R}_{1\ 3-l} & I_N \otimes X_{3-l} \\ I_N \otimes X_{3-l} & -2\nu_l(I_N \otimes X_l) + \nu_l^2\hat{R}_{1\ 3-l} \end{bmatrix} < 0 \tag{41}$$

$$\begin{bmatrix} -\lambda_{3-l}\hat{R}_{2\ 3-l} & I_N \otimes X_{3-l} \\ I_N \otimes X_{3-l} & -2\chi_l(I_N \otimes X_l) + \chi_l^2\hat{R}_{2\ 3-l} \end{bmatrix} < 0(l=1,2) \tag{42}$$

where

$$\hat{\Pi}_{11}^1 = I_N \otimes AX_1 + I_N \otimes A^TX_1 + \hat{Q}_{11} + \hat{Q}_{21} - e^{-2\alpha_1 h}\hat{R}_{21} - e^{-2\alpha_1 h}\hat{R}_{11} + 2\alpha_1 I_N \otimes X_1$$
$$\hat{\Pi}_{22}^1 = e^{-2\alpha_1 h}(-2\hat{R}_{21} + \hat{U}_{21} + \hat{U}_{21}^T),\ \hat{\Pi}_{32}^1 = e^{-2\alpha_1 h}(\hat{R}_{21} - \hat{U}_{21}^T)$$
$$\Pi_{33}^1 = -e^{-2\alpha_1 h}(\hat{Q}_{21} + \hat{R}_{21}),\ (\hat{\Pi}_{41}^1)^T = e^{-2\alpha_1 h}(\hat{R}_{11} - \hat{U}_{11}) + \mathcal{L} \otimes BY_1$$
$$\hat{\Pi}_{44}^1 = \sigma\hat{\Omega} + e^{-2\alpha_1 h}(\hat{U}_{11} + \hat{U}_{11}^T - 2\hat{R}_{11}),\ \Pi_{54}^1 = e^{-2\alpha_1 h}(\hat{R}_{11} - \hat{U}_{11}^T)$$
$$\hat{\Pi}_{55}^1 = e^{-2\alpha_1 h}(\hat{R}_{11} - \hat{Q}_{11}),\ (\hat{\Pi}_{61}^1)^T = \mathcal{L} \otimes BY_1$$
$$\hat{\Xi}_{61}^1 = \begin{bmatrix} \varepsilon_3\hat{\Gamma}^2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (\hat{\Gamma}^2)^T = \mathcal{L} \otimes BC$$
$$\hat{\Xi}_{11}^1 = \begin{bmatrix} \hat{\Pi}_{11}^1 & * & * & * & * & * & * & * \\ \hat{\Pi}_{32}^1 & \hat{\Pi}_{22}^1 & * & * & * & * & * & * \\ e^{-2\alpha_1 h}\hat{U}_{21}^T & \hat{\Pi}_{32}^1 & \hat{\Pi}_{33}^1 & * & * & * & * & * \\ \hat{\Pi}_{41}^1 & 0 & 0 & \hat{\Pi}_{44}^1 & * & * & * & * \\ e^{-2\alpha_1 h}\hat{U}_{11}^T & 0 & 0 & \hat{\Pi}_{54}^1 & \hat{\Pi}_{55}^1 & * & * & * \\ \hat{\Pi}_{61}^1 & 0 & 0 & 0 & 0 & -\hat{\Omega} & * & * \\ I_N \otimes E_1^T & 0 & 0 & 0 & 0 & 0 & -I & * \\ I_N \otimes E_2^T & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$
$$\hat{\Xi}_{71}^1 = \begin{bmatrix} 0 & 0 & 0 & I_N \otimes DX_1 & 0 & I_N \otimes DX_1 & 0 & 0 \end{bmatrix}$$
$$\hat{\Xi}_{11}^2 = \begin{bmatrix} \hat{\Pi}_{11}^2 & * & * & * & * & * & * \\ e^{2\alpha_2 h}(\hat{R}_{22} - \hat{U}_{22}^T) & \hat{\Pi}_{22}^2 & * & * & * & * & * \\ e^{2\alpha_2 h}\hat{U}_{22}^T & \hat{\Pi}_{32}^2 & \hat{\Pi}_{33}^2 & * & * & * & * \\ \hat{\Pi}_{54}^2 & 0 & 0 & \hat{\Pi}_{44}^2 & * & * & * \\ e^{2\alpha_2 h}\hat{U}_{12}^T & 0 & 0 & \hat{\Pi}_{54}^2 & \hat{\Pi}_{55}^2 & * & * \\ I_N \otimes E_1^T & 0 & 0 & 0 & 0 & -I & * \\ I_N \otimes E_2^T & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$
$$\Pi_{11}^2 = I_N \otimes AX_2 + I_N \otimes X_2A^T + \hat{Q}_{12} + \hat{Q}_{22} - e^{2\alpha_2 h}(\hat{R}_{22} + \hat{R}_{12}) - 2\alpha_2 I_N \otimes X_2$$
$$\Pi_{22}^2 = e^{2\alpha_2 h}(-2\hat{R}_{22} + \hat{U}_{22} + \hat{U}_{22}^T),\ \hat{\Pi}_{32}^2 = e^{2\alpha_2 h}(\hat{R}_{22} - \hat{U}_{22}^T)$$
$$\Pi_{44}^2 = e^{2\alpha_2 h}(\hat{U}_{12} + \hat{U}_{12}^T - 2\hat{R}_{12}),\ \hat{\Pi}_{33}^2 = -e^{2\alpha_2 h}(\hat{Q}_{22} + \hat{R}_{22})$$
$$\hat{\Pi}_{55}^2 = e^{2\alpha_2 h}(\hat{Q}_{12} + \hat{R}_{12}),\ \hat{\Pi}_{54}^2 = e^{2\alpha_2 h}(\hat{R}_{12} - \hat{U}_{12}^T)$$
$$\hat{\Gamma}^1 = \begin{bmatrix} I_N \otimes AX_1 & 0 & 0 & \mathcal{L} \otimes BY_1 & 0 & \mathcal{L} \otimes BY_1 & I_N \otimes E_1 & I_N \otimes E_2 \end{bmatrix}$$
$$\hat{\Gamma}^2 = \begin{bmatrix} I_N \otimes AX_2 & 0 & 0 & 0 & 0 & I_N \otimes E_1 & I_N \otimes E_2 \end{bmatrix}$$
$$\hat{\Xi}_{22}^l = -2\varepsilon_{1l}(I_N \otimes X_l) + \varepsilon_{1l}^2 R_{1l}, \quad \hat{\Xi}_{33}^l = -2\varepsilon_{2l}(I_N \otimes X_l) + \varepsilon_{2l}^2 R_{2l}$$
$$\hat{\Xi}_{41}^l = \begin{bmatrix} I_N \otimes MX_l & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\hat{\Xi}_{51}^l = \begin{bmatrix} 0 & I_N \otimes HX_l & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, l = 1, 2$$

Proof. Note that the inequality (16) with $l = 1$ can be equivalently expressed as

$$\tilde{\Xi}^1 + \Upsilon_1 (I_N \otimes F(t))\Upsilon_2 + \Upsilon_2^T (I_N \otimes F(t))\Upsilon_1^T < 0 \tag{43}$$

where

$$\Upsilon_1^T = \begin{bmatrix} (L \otimes P_1 BC)^T & 0_{7\times 1} & h(L \otimes P_1 BC)^T & (L \otimes P_1 BC)^T & 0 & 0 \end{bmatrix}$$

$$\Upsilon_2 = \begin{bmatrix} 0 & 0 & 0 & I_N \otimes D & 0 & I_N \otimes D & 0_{1\times 6} \end{bmatrix}$$

$$\tilde{\Xi}^1 = \begin{bmatrix} \tilde{\Xi}_{11}^1 & * & * & * & * \\ h\tilde{\Gamma}^1 & -(I_N \otimes P_1)R_{11}^{-1}(I_N \otimes P_1) & * & * & * \\ d_M \tilde{\Gamma}^1 & 0 & -(I_N \otimes P_1)R_{21}^{-1}(I_N \otimes P_1) & * & * \\ \Xi_{41} & 0 & 0 & -I & * \\ \Xi_{51} & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\tilde{\Xi}_{11}^1 = \begin{bmatrix} \Pi_{11}^1 & * & * & * & * & * & * & * \\ \Pi_{32}^1 & \Pi_{22}^1 & * & * & * & * & * & * \\ e^{-2\alpha_1 h} U_{21}^T & \Pi_{32}^1 & \Pi_{33}^1 & * & * & * & * & * \\ \tilde{\Pi}_{41}^1 & 0 & 0 & \Pi_{44}^1 & * & * & * & * \\ e^{-2\alpha_1 h} U_{11}^T & 0 & 0 & \Pi_{54}^1 & \Pi_{55}^1 & * & * & * \\ \tilde{\Pi}_{61}^1 & 0 & 0 & 0 & 0 & -\Omega & * & * \\ I_N \otimes E_1^T P_1 & 0 & 0 & 0 & 0 & 0 & -I & * \\ I_N \otimes E_2^T P_1 & 0 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

$$(\Pi_{41}^1)^T = e^{-2\alpha_1 h}(R_{11} - U_{11}) + \mathcal{L} \otimes P_1 BK, \quad (\Pi_{61}^1)^T = \mathcal{L} \otimes P_1 BK$$

$$\tilde{\Gamma}^1 = \begin{bmatrix} I_N \otimes P_1 A & 0 & 0 & (\tilde{\Pi}_{61}^1)^T & 0 & (\tilde{\Pi}_{61}^1)^T & I_N \otimes P_1 E_1 & I_N \otimes P_1 E_2 \end{bmatrix}$$

Notice that there exists a positive scalar ε_3 such that the following inequality can be obtained from Eq. (43):

$$\tilde{\Xi}^1 + \varepsilon_3 \Upsilon_1 \Upsilon_1^T + \varepsilon_3^{-1} \Upsilon_2^T \Upsilon_2 < 0 \tag{44}$$

By using Schur complement, Eq. (44) is equivalent to

$$\begin{bmatrix} \tilde{\Xi}^1 & * & * \\ \varepsilon_3 \Upsilon_1^T & -\varepsilon_3 I & * \\ \Upsilon_2 & 0 & -\varepsilon_3 I < 0 \end{bmatrix} < 0 \tag{45}$$

Considering

$$[R_{l1} - \varepsilon_{l1}^{-1}(I_N \otimes P_1)]R_{l1}^{-1}[R_{l1} - \varepsilon_{l1}^{-1}(I_N \otimes P_1)] \geq 0 (l = 1, 2) \tag{46}$$

we can get

$$-(I_N \otimes P_1)R_{11}^{-1}(I_N \otimes P_1) \leq -2\varepsilon_{11}(I_N \otimes P_1) + \varepsilon_{11}^2 R_{11} \tag{47}$$

$$-(I_N \otimes P_1)R_{21}^{-1}(I_N \otimes P_1) \leq -2\varepsilon_{21}(I_N \otimes P_1) + \varepsilon_{21}^2 R_{21} \tag{48}$$

Replace $-(I_N \otimes P_1)R_{11}^{-1}(I_N \otimes P_1)$ and $-(I_N \otimes P_1)R_{21}^{-1}(I_N \otimes P_1)$ by $-2\varepsilon_{11}(I_N \otimes P_1) + \varepsilon_{11}^2 R_{11}$ and $-2\varepsilon_{21}(I_N \otimes P_1) + \varepsilon_{21}^2 R_{21}$ in Eq. (45), respectively, one can get

$$\begin{bmatrix} \breve{\Xi}^1 & * & * \\ \varepsilon_3 \Upsilon_1^T & -\varepsilon_3 I & * \\ \Upsilon_2 & 0 & -\varepsilon_3 I < 0 \end{bmatrix} < 0 \tag{49}$$

in which

$$\breve{\Xi}^1 = \begin{bmatrix} \tilde{\Xi}_{11}^1 & * & * & * & * \\ h\tilde{\Gamma}^1 & \breve{\Xi}_{22}^1 & * & * & * \\ d_M\tilde{\Gamma}^1 & 0 & \breve{\Xi}_{33}^1 & * & * \\ \Xi_{41} & 0 & 0 & -I & * \\ \Xi_{51} & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\breve{\Xi}_{22}^1 = -2\varepsilon_{11}(I_N \otimes P_1) + \varepsilon_{11}^2 R_{11}$$

$$\breve{\Xi}_{33}^1 = -2\varepsilon_{21}(I_N \otimes P_1) + \varepsilon_{11}^2 R_{21}$$

Denote $X_l = P_l^{-1}$, $\tilde{X}_l = I_N \otimes X_l$, $(l = 1, 2)$, $J = diag\{\underbrace{\tilde{X}_1, \ldots, \tilde{X}_1}_{6}, I_N, I_N, \tilde{X}_1, \tilde{X}_1, I_N, I_N, I_N, I_N\}$,

and introduce new variables $Y_1 = KX_1$, $\hat{Q}_{1l} = \tilde{X}_1 Q_{1l}\tilde{X}_1$, $\hat{Q}_{2l} = \tilde{X}_1 Q_{2l}\tilde{X}_1$, $\hat{R}_{1l} = \tilde{X}_1 R_{1l}\tilde{X}_1$, $\hat{R}_{2l} = \tilde{X}_1 R_{2l}\tilde{X}_1$, $\hat{\Omega} = \tilde{X}_1\Omega\tilde{X}_1$, $\hat{U}_{1l} = \tilde{X}_l U_{1l}\tilde{X}_l$, $\hat{U}_{2l} = \tilde{X}_l U_{2l}\tilde{X}_l$. Pre- and post-multiplying both sides of Eq. (49) with J, and pre- and post-multiplying both sides of Ξ^1 in Eq. (16), we find that Eqs. (33) and (34) hold. Correspondingly, Eqs. (35) and (36) can be derived from Eqs. (17) and (18), respectively. Moreover, inspired by [18], taking congruent transformation to Eqs. (20) and (21) by $\tilde{X}_2$ and $\tilde{X}_1$, respectively, and applying the Schur complement, we can conclude that Eqs. (37) and (38) are equivalent to Eqs. (20) and (21), respectively. By using similar derivations, we can find Eqs. (39)–(42) can be ensured by Eqs. (22)–(25), respectively. This completes the proof. □

Remark 6. Many results on event-triggered consensus problem can be available in the literature. However, most of the existing results on event-triggered consensus problem do not consider the occurrence of DoS attacks and controller variation, which may degrade control performance of the addressed system. Different from the available results, in this paper, we firstly address the event-triggered consensus control problem for MASs with consideration of the influences of the controller gain variations, the DoS attacks and the limited bandwidth. Thus, the obtained sufficient conditions in Theorem 2 and the designed controllers are able to deal with these situations and ensure a stable and satisfactory consensus control for the addressed MASs, while the existing ones can not be applied directly here.

4. Illustrative example

In this section, to verify the main results, a simulation example will be provided. We consider an nonlinear MAS with six agents in which the control input are vulnerable to DoS attacks.

The dynamics of the ith agent is modeled by Eq. (1) with [29]

$$A = \begin{bmatrix} -1.175 & 0.9871 \\ -8.458 & -0.8776 \end{bmatrix}, B = \begin{bmatrix} -0.194 \\ -10.29 \end{bmatrix}, E_1 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.1 \end{bmatrix},$$

$$E_2 = \begin{bmatrix} 0.2 & 0.1 \\ 0 & 0.2 \end{bmatrix}, C = \begin{bmatrix} 0.03 & 0.01 \end{bmatrix}, D = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}$$

and

$$f(x_i) = \begin{bmatrix} \tanh(0.4x_{i1}(t)) \\ \sin(0.3x_{i2}(t)) \end{bmatrix}, h(x_i(t - d(t))) = \begin{bmatrix} \tanh(0.5x_{i1}(t - d(t))) \\ \sin(0.2x_{i2}(t - d(t))) \end{bmatrix}$$

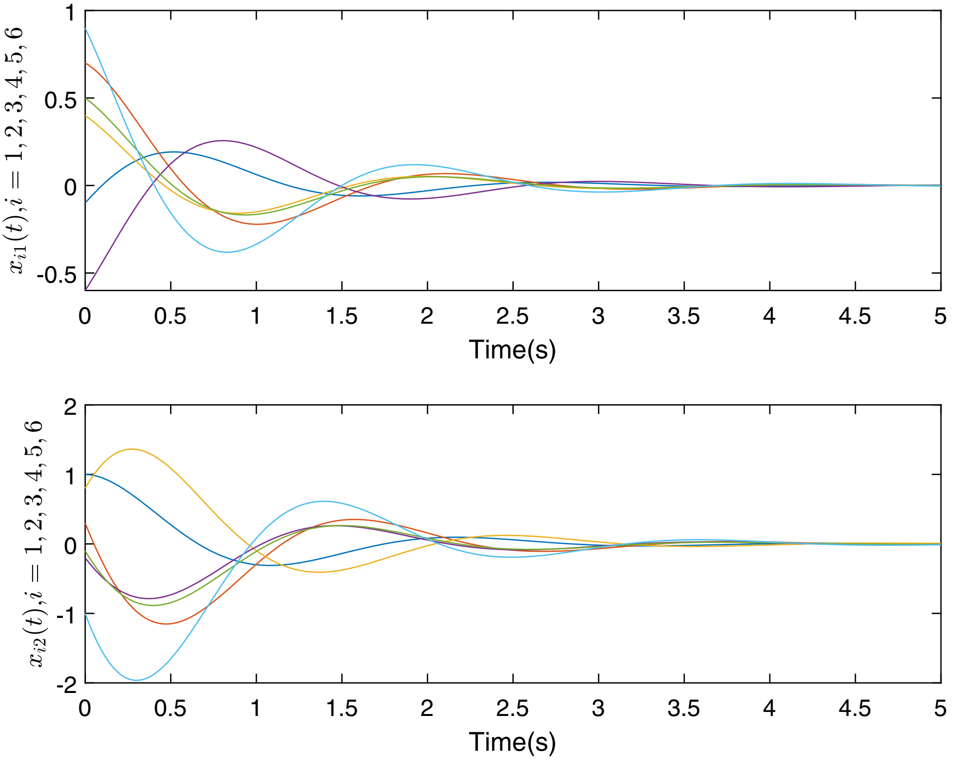

Fig. 1. State response of the nonlinear MAS with $T = 1$ s and $T_{off} = 0.75$ s.

The Laplacian matrix of the communication graph in the nonlinear MAS can be derived as

$$\mathcal{L} = \begin{bmatrix} 2 & -1 & 0 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Our objective is to joint-design the distributed triggering matrices and the controller gains. Suppose that the DoS attacks occur periodically to paralyze the controlled MAS with $T = 1$ s and $T_{off} = 0.75$ s. Let the sampled period $h = 0.01$, triggering parameter $\sigma = 0.1$, $\alpha_1 = 0.0015$, $\alpha_2 = 0.5$, $d_M = 0.05$, $\varepsilon_3 = 1$, $\varepsilon_{11} = 1$, $\varepsilon_{21} = 1$, $\varepsilon_1 = 1$, $\varepsilon_2 = 1$, $\lambda_l = 1.05$, $\lambda_2 = 1.05$, $\varepsilon_l = 1$, $\varepsilon_2 = 1$, $\nu_1 = 8$, $\nu_2 = 8$, $\chi_1 = 1$, $\chi_2 = 1$. By applying Theorem 2, a feasible solution is derived as follows:

$$K = \begin{bmatrix} -0.0019 \\ 0.0046 \end{bmatrix}^T, G_1 = G_2 = \begin{bmatrix} 5.0034 & 0.0970 \\ 0.0970 & 9.6711 \end{bmatrix}, G_3 = \begin{bmatrix} 5.0035 & 0.0968 \\ 0.0968 & 9.6690 \end{bmatrix}$$

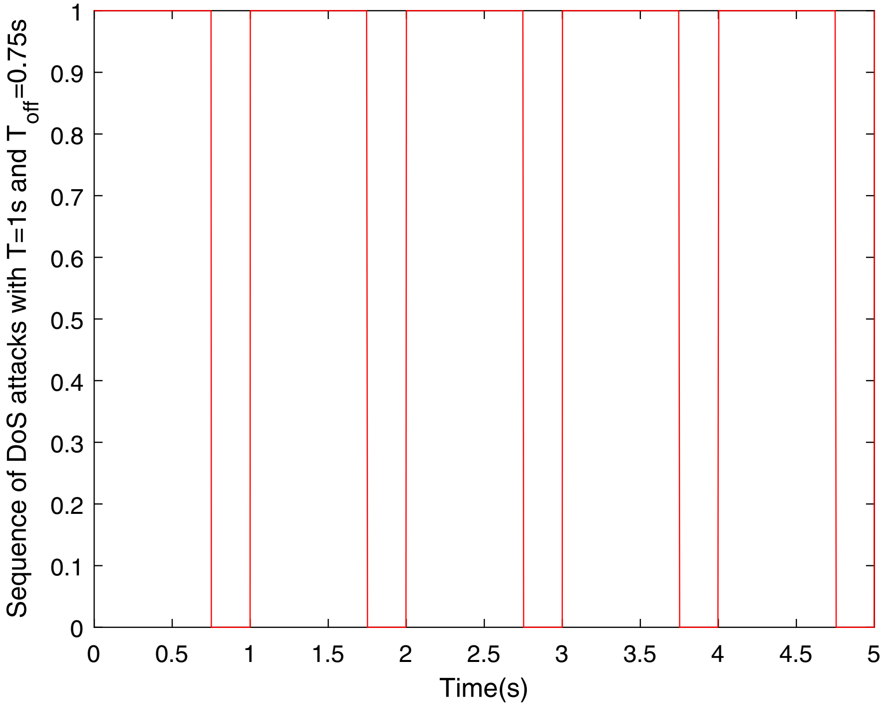

Fig. 2. Attack sequence.

$$G_4 = G_5 = G_6 = \begin{bmatrix} 5.0024 & 0.1013 \\ 0.1013 & 9.6435 \end{bmatrix}$$

In the following, choosing the initial conditions

$$x_1(0) = \begin{bmatrix} 0.1 & 1 \end{bmatrix}^T, \; x_2(0) = \begin{bmatrix} 0.7 & 0.3 \end{bmatrix}^T, \; x_3(0) = \begin{bmatrix} 0.4 & -0.2 \end{bmatrix}^T$$
$$x_4(0) = \begin{bmatrix} -0.6 & 0.8 \end{bmatrix}^T, \; x_5(0) = \begin{bmatrix} 0.5 & -0.1 \end{bmatrix}^T, \; x_6(0) = \begin{bmatrix} 0.9 & -1 \end{bmatrix}^T$$

Then, the state responses of the MAS (1) are shown in Fig. 1. Fig. 2 denotes the sequence of the DoS attack occurrence. The release instants and the intervals of agent 1 is depicted in Fig. 3. From the above simulation results, we can find that the developed approach can eliminate the impact of the DoS attack and alleviate the communication burden. Moreover, the MAS can achieve consensus. The theoretical control methods developed in this paper have been confirmed by the above simulation results.

As shown in Fig. 1, the addressed event-triggered nonlinear MASs can achieve consensus control even under DoS attacks and controller gain variation. Some results about consensus control for MASs can be available (see [14,28,29] and the references therein). However, the authors in [14] and [28] did not take the limited network resources into condition. The drawback of the event-triggered scheme in [29] is that it need to supervise the agent state continuously and check whether it satisfies the triggering conditions or not, while the event-triggered scheme in this paper can not only improve the efficiency of the network utilization, but also it is more easier to be realized than the one in [29]. Moreover, the results obtained

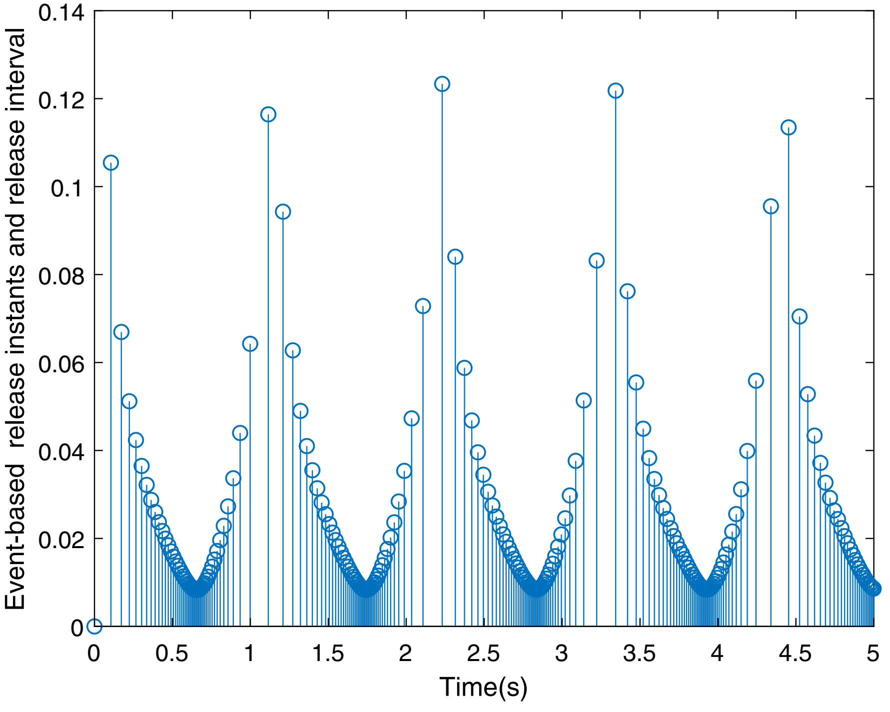

Fig. 3. Instants and intervals of agent 1 with $T = 1$ s and $T_{off} = 0.75$ s.

in this paper can be applied to the consensus problem of MASs subject to limited network resources, DoS attacks and controller gain variation.

5. Conclusion

In this paper, a new event-triggered secure consensus control of nonlinear MASs under DoS attacks has been studied. An event-triggered controller is proposed with consideration of the periodic DoS attacks. Considering the inaccurate implementation and DoS attacks, the discussed system is modeled as a switched system with two modes. Based on the model of MASs, by using the method of piecewise Lyapunov functional, exponential stability conditions are achieved in the form of linear matrix inequalities. Furthermore, the desired controller and triggering parameters are obtained. An illustrative example is used to demonstrate the effectiveness of the obtained control strategy in this paper. In our future work, we will extend the designed method in this paper to the studies of observer-based consensus of heterogeneous MASs and event-triggered predictive control for nonlinear MASs.

Appendix A

Proof of Lemma 2. Construct the following Lyapunov–Krasovskii functional candidate as described in [18] for system (13):

$$V_{\theta(t)}(x(t)) = V_{1,\theta(t)}(x(t)) + V_{2,\theta(t)}(x(t)) + V_{3,\theta(t)}(x(t)) \tag{50}$$

where

$$V_{1,\theta(t)}(x(t)) = x^T(t)(I_N \otimes P_{\theta(t)})x(t)$$
$$V_{2,\theta(t)}(x(t)) = \int_{t-h}^{t} \varphi(\bullet)x^T(s)Q_{1\theta(t)}x(s)ds + \int_{t-d_M}^{t} \varphi(\bullet)x^T(s)Q_{2\theta(t)}x(s)ds$$
$$V_{3,\theta(t)}(x(t)) = h\int_{t-h}^{t}\int_{s}^{t} \varphi(\bullet)\dot{x}^T(v)R_{1\theta(t)}\dot{x}(v)dvds + d_M\int_{t-d_M}^{t}\int_{s}^{t} \varphi(\bullet)\dot{x}^T(v)R_{2\theta(t)}\dot{x}(v)dvds$$
$$\varphi(\bullet) = e^{2(-1)^{\theta(t)}\alpha_{\theta(t)}(t-s)}$$

with $P_{\theta(t)} > 0$, $Q_{1\theta(t)} > 0$, $Q_{2\theta(t)} > 0$, $R_{1\theta(t)} > 0$ and $R_{2\theta(t)} > 0$.

$\forall k \in \mathcal{K}(n)$, $n \in \mathbb{N}$, taking the derivative of $V_1(t)$ for $t \in [t^i_{k,n}h, t^i_{k+1,n}h) \bigcap \mathcal{I}_{1,n}$ along the trajectory of the system (13), we have

$$\begin{aligned}\dot{V}_1(t) \le& -2\alpha_1 V_1(t) + 2\alpha_1 x^T(t)(I_N \otimes P_1)x(t) + 2\dot{x}^T(t)(I_N \otimes P_1)[(I_N \otimes A)x(t)\\ &+ (\mathcal{L} \otimes (B(K+\Delta K)))x(t-\eta_{k,n}(t)) + (\mathcal{L} \otimes (B(K+\Delta K)))e_{k,n}(t) + (I_N \otimes E_1)f(x(t))\\ &+ (I_N \otimes E_2)h(x(t-d(t)))] + x^T(t)Q_{11}x(t) - e^{-2\alpha_1 h}x^T(t-h)Q_{11}x(t-h)\\ &+ x^T(t)Q_{21}x(t) - e^{-2\alpha_1 h}x^T(t-d_M)Q_{21}x(t-d_M) + \dot{x}^T(t)(h^2R_{11} + d_M^2R_{21})\dot{x}(t)\\ &- he^{-2\alpha_1 h}\int_{t-h}^{t}\dot{x}^T(s)R_{11}\dot{x}(s)ds - d_M e^{-2\alpha_1 h}\int_{t-d_M}^{t}\dot{x}^T(s)R_{21}\dot{x}(s)ds \end{aligned} \quad (51)$$

Applying Lemma 1, for R_{11}, R_{21}, U_{11} and U_{21} satisfy Eq. (17), one can get

$$-h\int_{t-h}^{t}\dot{x}^T(s)R_{11}\dot{x}(s)ds \le \zeta_1^T(t)\Sigma_1\zeta_1(t) \quad (52)$$

$$-d_M\int_{t-d_M}^{t}\dot{x}^T(s)R_{21}\dot{x}(s)ds \le \zeta_2^T(t)\Sigma_2\zeta_2(t) \quad (53)$$

in which $\zeta_1(t) = [x^T(t) \quad x^T(t-\eta_{k,n}(t)) \quad x^T(t-h)]^T$, $\zeta_2(t) = [x^T(t) \quad x^T(t-d(t)) \quad x^T(t-d_M)]^T$, $\Sigma_1 = \begin{bmatrix} -R_{11} & * & * \\ R_{11}-U_{11}^T & -2R_{11}+U_{11}+U_{11}^T & * \\ U_{11}^T & R_{11}-U_{11}^T & -2R_{11}\end{bmatrix}$, $\Sigma_2 = \begin{bmatrix} -R_{21} & * & * \\ R_{21}-U_{21}^T & -2R_{21}+U_{21}+U_{21}^T & * \\ U_{21}^T & R_{21}-U_{21}^T & -2R_{21}\end{bmatrix}$

Notice that Eqs. (2) and (3) imply

$$f^T(x(t))f(x(t)) \le x^T(t)(I_N \otimes M)^T(I_N \otimes M)x(t) \quad (54)$$

$$h^T(x(t-d(t)))h(x(t-d(t))) \le x^T(t-d(t))(I_N \otimes H)^T(I_N \otimes H)x(t-d(t)) \quad (55)$$

Combine Eqs. (12) and (51)–(55), we obtain

$$\begin{aligned}\dot{V}_1(t) \le& -2\alpha_1 V_1(t) + \Xi^1_{11} + \dot{x}^T(t)(h^2R_{11} + d_M^2R_{21})\dot{x}(t) + x^T(t)(I_N \otimes M)^T(I_N \otimes M)x(t)\\ &+ x^T(t-d(t))(I_N \otimes H)^T(I_N \otimes H)x(t-d(t))\end{aligned} \quad (56)$$

Using Schur complement, $\Xi^1 < 0$ in Eq. (16) can ensure

$$\begin{aligned}&\Xi^1_{11} + \dot{x}^T(t)(h^2R_{11} + d_M^2R_{21})\dot{x}(t) + x^T(t)(I_N \otimes M)^T(I_N \otimes M)x(t)\\ &+ x^T(t-d(t))(I_N \otimes H)^T(I_N \otimes H)x(t-d(t)) < 0\end{aligned} \quad (57)$$

which implies

$$\dot{V}_1(t) \le -2\alpha_1 V_1(t), \qquad t \in [t^i_{k,n}h, t^i_{k+1,n}h) \bigcap \mathcal{I}_{1,n} \tag{58}$$

Due to the arbitrary of k, it yields that

$$V_1(t) \le e^{-2\alpha_1(t-\delta_{1,n})} V_1(\delta_{1,n}), t \in [\delta_{1,n}, \delta_{2,n}), n \in \mathbb{N} \tag{59}$$

When $t \in [\delta_{2,n}, \delta_{1,n+1}), n \in \mathbb{N}$, the time derivative of $V_2(t)$ along Eq. (13) becomes

$$\begin{aligned}\dot{V}_2(t) \le\ & -2\alpha_2 V_2(t) - 2\alpha_2 x^T(t)(I_N \otimes P_2)x(t) + 2\dot{x}^T(t)(I_N \otimes P_2)[(I_N \otimes A)x(t) \\ & + (I_N \otimes E_1)f(x(t)) + (I_N \otimes E_2)h(x(t-d(t)))] + x^T(t)(Q_{12}+Q_{22})x(t) \\ & - e^{2\alpha_2 h}x^T(t-h)Q_{12}x(t-h) - e^{2\alpha_2 h}x^T(t-d_M)Q_{22}x(t-d_M) \\ & + \dot{x}^T(t)(h^2R_{12} + d_M^2 R_{22})\dot{x}(t) - he^{2\alpha_2 h}\int_{t-h}^{t} \dot{x}^T(s)R_{12}\dot{x}(s)ds \\ & - d_M e^{2\alpha_2 h}\int_{t-d_M}^{t} \dot{x}^T(s)R_{22}\dot{x}(s)ds \end{aligned} \tag{60}$$

By Lemma 1, from Eq. (18), we have

$$-h\int_{t-h}^{t} \dot{x}^T(s)R_{12}\dot{x}(s)ds \le \zeta_1^T(t)\Sigma_3\zeta_1(t) \tag{61}$$

$$-d_M\int_{t-d_M}^{t} \dot{x}^T(s)R_{22}\dot{x}(s)ds \le \zeta_2^T(t)\Sigma_4\zeta_2(t) \tag{62}$$

where

$$\Sigma_3 = \begin{bmatrix} -R_{12} & * & * \\ R_{12} - U_{12}^T & -2R_{12} + U_{12} + U_{12}^T & * \\ U_{12}^T & R_{12} - U_{12}^T & -2R_{12} \end{bmatrix}$$

$$\Sigma_4 = \begin{bmatrix} -R_{22} & * & * \\ R_{22} - U_{22}^T & -2R_{22} + U_{22} + U_{22}^T & * \\ U_{22}^T & R_{22} - U_{22}^T & -2R_{22} \end{bmatrix}$$

From Eqs. (54)–(55) and Eqs. (60)–(62), it follows:

$$\begin{aligned}\dot{V}_2(t) \le\ & 2\alpha_2 V_2(t) + \Xi^2_{11} + \dot{x}^T(t)(h^2R_{12} + d_M^2R_{22})\dot{x}(t) + x^T(t)(I_N \otimes M)^T(I_N \otimes M)x(t) \\ & + x^T(t-d(t))(I_N \otimes H)^T(I_N \otimes H)x(t-d(t)) \end{aligned} \tag{63}$$

Using Schur complement, it is clear that $\Xi^2 < 0$ in Eq. (16) can guarantee

$$\begin{aligned}& \Xi^2_{11} + \dot{x}^T(t)(h^2R_{12} + d_M^2R_{22})\dot{x}(t) + x^T(t)(I_N \otimes M)^T(I_N \otimes M)x(t) \\ & + x^T(t-d(t))(I_N \otimes H)^T(I_N \otimes H)x(t-d(t)) < 0 \end{aligned} \tag{64}$$

which implies

$$\dot{V}_2(t) \le 2\alpha_2 V_2(t) \tag{65}$$

It yields that

$$V_2(t) \le e^{2\alpha_2(t-\delta_{2,n})} V_2(\delta_{2,n}), t \in [\delta_{2,n}, \delta_{1,n+1}), n \in \mathbb{N} \tag{66}$$

This complete the proof. □

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