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# Event-triggered output feedback *H*∞ control for networked Markovian jump systems with quantizations



Liju[a](#page-0-0)n Zha $^{\rm a}$ , Jian-an Fang $^{\rm a, *}$  $^{\rm a, *}$  $^{\rm a, *}$ , Xiaofan Li $^{\rm a}$ , Jinliang Liu $^{\rm b}$  $^{\rm b}$  $^{\rm b}$ 

<span id="page-0-0"></span><sup>a</sup> *College of Information Science and Technology, Donghua University, Shanghai 201620, PR China*

<span id="page-0-2"></span><sup>b</sup> *College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, PR China*

## a r t i c l e i n f o

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## a b s t r a c t

This paper is concerned with  $H_{\infty}$  output feedback control of event-triggered Markovian jump systems with measured output quantizations. An event-triggered mechanism and a quantization scheme are designed to reduce the occupancy of the limited network resources. The measured output is directly transmitted to the event generator, whether the released signal can be sent out to the controller through the quantizer depends on the triggering condition. A new Markovian jump time-delay system model with external disturbances is then presented and sufficient conditions are derived to guarantee the stability of the resulting closed-loop system and prescribed performance. Criteria for codesigning both the output feedback gain and the trigger parameters are established. Finally, a numerical example confirms the effectiveness of the proposed method.

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#### **1. Introduction**

As a special class of stochastic hybrid systems, Markovian jump systems have been extensively studied in the past years, in which the jumps are modeled by the transitions of a Markov chain [\[1–9\]](#page-12-0). Such systems have the advantage of better describing physical systems with abrupt variations. Many industrial plants can be modeled by Markovian jump systems, such as manufacturing systems, networked control systems, economics systems. Much effort has been devoted to the stability analysis and controller synthesis for Markovian jump systems.

With the rapid development of computer and networking technologies, networked control systems have gained considerable research attention due to the advantage of cost effectiveness, simplicity in installation and maintenance, and high reliability [10-15]. In most practical systems, control system components(reference input, plant output, control input, etc.) may located in different places, signals are required to be transmitted via communication networks. However, insertion of network bring some interesting and challenging problems such as random network-induced delays, data packet dropout and so on. Therefore, it is necessary to develop methods that deal with these issues more effectively. To overcome these problems, signal quantization has recently become an active research topic. It is indispensable to quantize signals before it being transmitted through a communication channel with limited bandwidth. Until now, quantization problems have been investigated for linear or nonlinear systems with quantized measurement or quantized input [\[16–18\]](#page-12-2). Specifically, the authors in [\[16\]](#page-12-2) investigate the control design problem of event-triggered networked systems with both state and control input quantizations. In [\[17\]](#page-12-3), the authors address the problem of *H*∞ filtering for a class of discrete time T-S model based fuzzy systems with quantized measurements.

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<span id="page-0-1"></span><sup>∗</sup> Corresponding author. Fax: +86 25 85481197. *E-mail address:* [jafang@dhu.edu.cn](mailto:jafang@dhu.edu.cn) (J.-a. Fang).

More recently, different event-triggering schemes have been proposed in the literature as an alternative approach to minimize the use of the communication resources. Compared with the conventional time-triggered communication, the event-triggering schemes can largely reduce the sampling rates in the single processor systems while preserving the control performance. It is because the transmission rates can be adjusted in a certain way depending on the current sampled data within the system. In the last few years, event triggered schemes have motivated a lot of interesting research [\[19–24\]](#page-12-4). For example, in [\[20\]](#page-12-5), the authors investigate event-triggered  $H_{\infty}$  controller design for networked control systems considering the effect of network transmission delay. The authors in [\[21\]](#page-12-6) investigate the problem of event-triggered output-feedback *H*<sub>∞</sub> control for networked control systems with non-uniform sampling. The authors in [\[22\]](#page-12-7) design a dynamic discrete state dependent event-triggered scheme, based on the event triggered scheme, the problem of *H*∞ control for networked Markovian jump system is studied. In [\[23\]](#page-12-8), the authors investigate the problem of fault detection for nonlinear discrete-time networked systems under an event-triggered scheme. The authors in [\[24\]](#page-12-9) focus on the reliable control for event-triggered networked control systems with probabilistic sensor and actuator faults. To the best of the authors' knowledge, up to now, little work has been done to deal with the event-triggered *H*∞ output feedback control for Markovian jump systems with measured output quantizations. This motivates the research of this work.

Inspired by the results mentioned above, we will focus on event-triggered *H*∞ output feedback control for Markovian jump systems with output quantizations. The main contributions are summarized as follows: (i) the dynamic discrete eventtriggered scheme is presented and different Markovian modes have different triggered thresholds. (ii) the effects of networkinduced delays and control input quantizations, event-triggering schemes are involved in a unified framework. (iii) the  $H_{\infty}$ performance criterion and *H*∞ output feedback controller are derived for Markovian jump systems considering measured output quantizations.

The paper is organized as follows. In Section [2,](#page-1-0) a new Markovian jump time-delay system model with external disturbances is formulated considering the effect of the event triggered scheme and measured output quantizations. In Section [3,](#page-3-0) the stability and  $H_{\infty}$  performance of the closed-loop system are analyzed. Moreover, criteria for co-designing both controller and the trigger parameters are derived. A simulation example is given to illustrate the effectiveness of the established theoretical results in Section [4.](#page-7-0)

Notation:  $\mathbb{R}^n$  and  $\mathbb{R}^{n\times m}$  denote the *n*-dimensional Euclidean space, and the set of  $n\times m$  real matrices; the superscript "*T*" stands for matrix transposition; *I* is the identity matrix of appropriate dimension; the notation  $X > 0$  (respectively,  $X \geq 0$ ), for  $X \in \mathbb{R}^{n \times n}$  means that the matrix *X* is real symmetric positive definite (respectively, positive semi-definite). For a matrix *B* and two symmetric matrices *A* and C,  $\begin{bmatrix}A&*\\B&C\end{bmatrix}$  denotes a symmetric matrix, where  $*$  denotes the entries implied by symmetry.

#### <span id="page-1-0"></span>**2. System description**

Consider the following Markovian jump system (MJS)

<span id="page-1-1"></span>
$$
\begin{cases}\n\dot{x}(t) = A(r_t)x(t) + B(r_t)u(t) + B_{\omega}(r_t)\omega(t) \\
y(t) = C_1(r_t)x(t) \\
z(t) = C_2(r_t)x(t) + D(r_t)u(t)\n\end{cases} (1)
$$

where  $x(t) \in R^n$  is the state variable,  $y(t) \in R^m$  is the measured output,  $u(t) \in R^w$  is the controlled input,  $z(t) \in R^p$ is the controlled output,  $\omega(t) \in R^q$  is the external disturbance with  $\omega(t) \in \mathcal{L}_2[0,\infty)$ , respectively;  $A(r_t)$ ,  $B(r_t)$ ,  $B_{\omega}(r_t)$ ,  $C_1(r_t)$ ,  $C_2(r_t)$  and  $D(r_t)$  are known real constant matrices with appropriate dimensions.  $C_1(r_t)$  is assumed to be full row rank matrix.  $\{r_t, t \geq 0\}$  is a homogeneous finite-state Markov jump process with right continuous trajectory values in a finite set  $S = \{1, 2, \ldots, r\}$ . The transition probability matrix  $\Pi = (\pi_{ii})(i, j \in S)$  given by

$$
P\{r_{t+h} = j | r_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ij}h + o(h), & i = j \end{cases}
$$
 (2)

where  $h > 0$ ,  $\lim_{h \to 0} \frac{o(h)}{h} = 0$ ,  $\pi_{ij} \geq 0$ , for  $j \neq i$  is the transition rate from mode *i* at time *t* to mode *j* at time  $t + h$ ,  $\pi_{ii} = -\sum_{j=1,j\neq i}^{N} \pi_{ij}.$  For the networked Markovian jump system shown in [Fig. 1,](#page-2-0) we assume the following:

- 1. Sensors are clock driven. The measured outputs are sampled at a constant period *h*. The set of sampled instants is represented by  $S_1 = \{j h | j = 1, 2, ...\}$
- 2. The controller and the actuator are event driven. The logic ZOH is used to hold the control input when there is no latest control signal arrived at the actuator. The set of transmission instants is represented by  $S_2 = \{t_k h | t_k \in \mathbb{N}\} \subseteq S_1$ . All transmitted signals are time-stamped.
- 3.  $\tau_{t_k}$  denote the network-induced delay calculated from the time instant  $t_k$  when the event generator releases the sampled signal to the time instant when it arrives at the controller side,  $\bar{\tau}$  is the upper bound of  $\tau_{t_k}.$

As is well known, in output feedback control for networked control systems, the value of the plant's output is sent to the network controller through the communication network, based on the received information of the plant, the controller sends the control output back to the actuator (located at the plant side) through the communication network. To reduce the

<span id="page-2-0"></span>

<span id="page-2-1"></span>**Fig. 1.** The structure of an event-triggered NCS with a quantizer.

communication burden of communication network in [Fig. 1,](#page-2-0) an event generator and the quantizers *f*(·) are employed. The event generator is constructed between the sensor and the quantizer  $f(\cdot)$ , the sampled data  $y(kh)$  is directly transmitted to the event generator, whether the new output information of the plant needs to be sent to the network controller through the quantizer depends on the following condition [\[20\]](#page-12-5):

$$
(y(t_k h) - y(t_k h + lh))^T \Omega (r_{t_k h + lh}) (y(t_k h) - y(t_k h + lh)) \leq \sigma (r_{t_k h + lh}) y^T (t_k h + lh) \Omega (r_{t_k h + lh}) y(t_k h + lh)
$$
\n(3)

where  $\sigma(r_{t_k h+lh}) \in [0, 1)$ ,  $\Omega(r_{t_k h+lh})$  is a symmetric positive definite matrix,  $l = 1, 2, ..., y(t_k h+lh)$  is the current sampled output,  $y(t_k^h h)$  is the latest transmitted data. If the current sampled data violate the event-triggered threshold condition [\(3\),](#page-2-1) the data will be sent out to the controller side.

For the convenience of analysis later, the holding interval time of the logic ZOH can be divided into some subintervals:

$$
[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_k+l+1}) = \bigcup_{l=1}^d \Upsilon_{l,k}
$$

where  $\Upsilon_{l,k} = [t_k h + lh + \tau_{t_k+l}, t_k h + lh + \tau_{t_k+l+1}), l = 1, 2, ..., d.$ 

Define the network allowable equivalent delay  $\eta(t) = t - t_k h - lh$ , it follows that  $0 \leq \tau_{t_k} \leq \eta(t) \leq h + \bar{\tau} \equiv \eta_M$ .

In order to include the effect of the event triggered mechanism [\(3\)](#page-2-1) in deriving system stability and stabilization criterion, let state error  $e_k(t) = x(t_k h) - x(t_k h + lh)$ , it can be seen that for  $t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_k+l+1})$ , the event-triggered threshold condition  $(3)$  can be rewritten as

$$
e_k^T(t)C_1^T(r_t)\Omega(r_{t_kh+lh})C_1(r_t)e_k(t) \leq \sigma(r_{t_kh+lh})x^T(t-\eta(t))C_1^T(r_t)\Omega(r_{t_kh+lh})C_1(r_t)x(t-\eta(t)).
$$
\n(4)

**Remark 1.** The parameters  $\sigma(r_{t_k h+lh})$  is used to determine how frequently the signal should be released.  $\sigma(r_{t_k h+lh})=0$ means that all sampled signals are released at each sampling instant, in such a case, the event-triggered mechanism [\(3\)](#page-2-1) is degraded into the periodic sampling scheme.

**Remark 2.** Under the event triggered scheme, assume that the release times are  $t_1h$ ,  $t_2h$ , ... and the time-varying delay in the network communication is  $\tau_k$ , the quantized measurement will arrive at the actuator at  $t_1h + \tau_{t_1}, t_2h + \tau_{t_2}, \ldots$ 

**Remark 3.** The role of the logic ZOH at the actuator is to store the latest control output, which implies that the outdated information will be discarded actively by the logic ZOH but the most recent valid one. The actuator keeps the control signal unchanged until the output of the logic ZOH being updated to a new data.

 $[f_1(y_1(t_kh))$   $f_2(y_2(t_kh))$   $\cdots$   $f_m(y_m(t_kh))$ , where  $f_s(y_s(t_kh))$   $(s = 1, 2, \ldots, m)$  can be described by To further reduce the use of the network bandwidth, a quantizer  $f(\cdot)$  is employed, which is defined as  $f(y(t_k h)) =$ 

<span id="page-2-2"></span>
$$
f_{s}(y_{s}(t_{k}h)) = \begin{cases} u_{l}^{(s)}, & \text{if } \frac{1}{1+\delta_{f_{s}}} u_{l}^{(s)} < y_{s}(t_{k}h) \leq \frac{1}{1-\delta_{f_{s}}} u_{l}^{(s)}, x_{s} > 0\\ 0, & \text{if } y_{s}(t_{k}h) = 0\\ -f_{s}(-x_{s}), & \text{if } y_{s}(t_{k}h) < 0 \end{cases}
$$
(5)

with  $\delta_{f_s} = (1 - \rho_{f_s})/(1 + \rho_{f_s}) (0 < \rho_{f_s} < 1)$ ,  $\rho_{f_s}$  is a given constant, which is called the quantization density. Moreover, the set of quantization levels is illustrated [\[16\]](#page-12-2):  $u_s= \{\pm u_l^{(s)}, u_l^{(s)}=\rho_{fs}^lu_0^{(s)}, l=\pm 1,\pm,2,\ldots\}\bigcup \{\pm u_0^{(s)}\}\bigcup \{0\}$  with  $u_0^{(s)}>0$ . Based on the above analysis, considering the event-triggered mechanism, the quantization scheme and network-induced delays in communication networks, using the sector bound approach [\[25\]](#page-12-10), the measured output received by the controller can be expressed as

$$
\bar{y}(t) = f(y(t_k h)) = (I + \Delta_f)C_1(r_t)[x(t - \eta(t)) + e_k(t)]
$$
\n(6)

in which  $\Delta_f$  = diag $\{\Delta_{f_1}, \Delta_{f_2}, \ldots, \Delta_{f_m}\}, \Delta_{f_s}$  ∈ [− $\delta_f, \delta_f,$ ], s = 1, 2, ..., *m*. Define an output feedback controller

<span id="page-3-1"></span>
$$
u(t) = K(r_t)(I + \Delta_f)C_1(r_t)[x(t - \eta(t)) + e_k(t)]
$$
\n(7)

where  $K(r_t)$  is a gain matrix to be determined later.

**Remark 4.** Noting that the quantization influence can be transformed into norm bounded uncertainties, ∆*<sup>f</sup>* accounts for the gain variations in a realistic way. By applying the sector-bound approach, ∆*<sup>f</sup>* can be eliminated, which results in δindependent conditions.

Substituting  $(7)$  into  $(1)$  leads to the following closed loop system

<span id="page-3-2"></span>
$$
\begin{cases}\n\dot{x}(t) = A(r_t)x(t) + B(r_t)K(r_t)(I + \Delta_f)C_1(r_t)[x(t - \eta(t)) + e_k(t)] + B_\omega(r_t)\omega(t) \\
z(t) = C_2(r_t)x(t) + D(r_t)K(r_t)(I + \Delta_f)C_1(r_t)[x(t - \eta(t)) + e_k(t)].\n\end{cases}
$$
\n(8)

For simplicity, in this paper, when  $r(t) = i$ ,  $i \in S$ , a matrix  $M(r(t))$  will be denoted by  $M_i$ ; for example,  $A(r(t))$  is denoted by  $A_i$ ,  $B(r(t))$  by  $B_i$ ,  $C_1(r_t)$  by  $C_{1i}$ ,  $C_2(r_t)$  by  $C_{2i}$ ,  $K(r_t)$  by  $K_i$ ,  $B_{\omega}(r_t)$  by  $B_{\omega i}$  and so on.

**Remark 5.** Note that not all of the sampler data will be transmitted to the controller through the quantizer. Due to the introduction of event generator, only the one violates the event triggered condition, can it be sent out to the quantizer.

At the end of this section, let us introduce some important lemmas, which will play an important role in obtaining our main results.

**Definition 1** ([\[26\]](#page-12-11)). For a given function  $V: C^b_{F_0}([- \tau_M,0],\mathbb{R}^n)\times S$ , its infinitesimal operator  $\mathcal L$  is defined as

$$
\mathcal{L}(V\eta(t)) = \lim_{\Delta \to 0^+} \frac{1}{\Delta} [\mathbb{E}(V(\eta_t + \Delta)|\eta_t) - V(\eta_t)]. \tag{9}
$$

**Lemma 1** ([\[27\]](#page-12-12)). For any vectors  $x, y \in R^n$ , and positive definite matrix  $Q \in R^{n \times n}$ , the following inequality holds:

<span id="page-3-4"></span> $2x^T y \le x^T Q x + y^T Q^{-1} y.$ 

**Lemma 2** ([\[28\]](#page-12-13)). Suppose  $\eta(t) \in [\eta_0, \eta_1]$ ,  $\mathcal{Z}_1$ ,  $\mathcal{Z}_2$  and  $\Omega$  are matrices with appropriate dimensions, then

$$
(\eta_1(t) - \eta_0)E_1 + (\eta_1 - \eta(t))E_2 + \Omega < 0 \tag{10}
$$

*if and only if*

<span id="page-3-7"></span> $(\eta_1 - \eta_0)E_1 + \Omega < 0$  $(\eta_1 - \eta_0) \mathcal{Z}_2 + \Omega < 0.$ 

**Lemma 3** ([\[29\]](#page-12-14)). Given matrices  $F_1 = F_1^T$ ,  $F_2$  and  $F_3$  of appropriate dimensions, we have  $F_1 + F_3 \Delta(k) F_2 + F_2^T \Delta^T(k) F_3^T < 0$  for all  $\Delta$ (k) satisfying  $\Delta^T$ (k) $\Delta$ (k)  $\leq$  I, if and only if there exists a positive scalar  $\varepsilon$   $<$  0, such that  $F_1 + \varepsilon^{-1} F_3 F_3^T + \varepsilon F_2^T F_2 < 0$ .

### <span id="page-3-0"></span>**3. Main results**

<span id="page-3-6"></span>In this section, we will develop an approach for stability analysis and controller synthesis of system [\(8\).](#page-3-2)

**Theorem 1.** For given scalars  $\gamma$ ,  $\eta$ <sub>M</sub>,  $\sigma_i$  and matrix K<sub>i</sub>, under the communication scheme, the augmented system [\(8\)](#page-3-2) is *asymptotically stable with an H<sub>∞</sub> performance index*  $\gamma$ *, if there exist matrices*  $P_i > 0$ *,*  $Q_i > 0$ *, Q, R,*  $\Omega_i > 0$  *(i ∈ S), and*  $M_i$ ,  $N_i$  with appropriate dimensions such that for  $l = 1, 2$ 

<span id="page-3-5"></span>
$$
E(l) = \begin{bmatrix} S_{11} + \Gamma + \Gamma^T & * & * & * \\ S_{21} & -I & * & * \\ S_{31} & 0 & -P_i R^{-1} P_i & * \\ S_{41}(l) & 0 & 0 & -R \end{bmatrix} < 0
$$
\n(11)

<span id="page-3-3"></span>
$$
\sum_{j=1} \pi_{ij} Q_j \leq Q \tag{12}
$$

*where*

$$
E_{11} = \begin{bmatrix} P_i A_i + A_i^T P_i + Q_i + \eta_M Q + \sum_{j=1}^r \pi_{ij} P_j & * & * & * & * \\ C_{1i}^T (I + \Delta_f)^T K_i^T B_i^T P_i & \sigma_i C_{1i}^T \Omega_i C_{1i} & * & * & * \\ 0 & 0 & -Q_i & * & * \\ C_{1i}^T (I + \Delta_f)^T K_i^T B_i^T P_i & 0 & 0 & -C_{1i}^T \Omega_i C_{1i} & * \\ B_{\omega i}^T P_i & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}
$$
  
\n
$$
E_{21} = \begin{bmatrix} C_{2i} & D_i K_i (I + \Delta_f) C_{1i} & 0 & D_i K_i (I + \Delta_f) C_{1i} & 0 \end{bmatrix}
$$
  
\n
$$
E_{31} = \begin{bmatrix} \sqrt{\eta_M} P_i A_i & \sqrt{\eta_M} P_i B_i K_i (I + \Delta_f) C_{1i} & 0 & \sqrt{\eta_M} P_i B_i K_i (I + \Delta_f) C_{1i} & P_i B_{\omega i} \end{bmatrix}
$$
  
\n
$$
\Omega_{41}(1) = \begin{bmatrix} \sqrt{\eta_M} M_i^T \end{bmatrix}, \qquad \Omega_{41}(2) = \begin{bmatrix} \sqrt{\eta_M} N_i^T \end{bmatrix}, \qquad \Gamma = \begin{bmatrix} M_i & -M_i + N_i & -N_i & 0 & 0 \end{bmatrix}
$$
  
\n
$$
M_i^T = \begin{bmatrix} M_i^T & M_{i2}^T & M_{i3}^T & M_{i4}^T & 0 \end{bmatrix}^T, \qquad N_i^T = \begin{bmatrix} N_i^T & N_i^T & N_i^T & N_i^T & 0 \end{bmatrix}^T.
$$

**Proof.** Construct a Lyapunov–Krasovskii functional for system [\(8\)](#page-3-2)

$$
V(x(t), r_t, t) = V_1(x(t), r_t, t) + V_2(x(t), r_t, t) + V_3(x(t), r_t, t) + V_4(x(t), r_t, t)
$$
\n(13)

where

$$
V_1(x(t), r_t, t) = x^T(t)P(r_t)x(t)
$$
  
\n
$$
V_2(x(t), r_t, t) = \int_{t-\eta_M}^t x^T(s)Q(r_t)x(s)ds
$$
  
\n
$$
V_3(x(t), r_t, t) = \int_{t-\eta_M}^t \int_s^t x^T(v)Qx(v)dvds
$$
  
\n
$$
V_4(x(t), r_t, t) = \int_{t-\eta_M}^t \int_s^t \dot{x}^T(v)R\dot{x}(v)dvds.
$$

Taking the derivative of  $V(x(t), r_t, t)$  along the trajectory of the system [\(8\)](#page-3-2) for *i*,  $i \in S$ , applying free weighing matrix method [\[30](#page-12-15)[,31\]](#page-12-16), we have

$$
\mathcal{L}V(t) = 2\dot{x}^{T}(t)P_{i}\mathcal{A}_{i} + x^{T}(t)\sum_{j=1}^{r}\pi_{ij}P_{j}x(t) + x^{T}(t)Q_{i}x(t) - x^{T}(t - \eta_{M})Q_{i}x(t - \eta_{M}) + \eta_{M}x^{T}(t)Qx(t)
$$
  
+ 
$$
\int_{t-\eta_{M}}^{t} x^{T}(s)\sum_{j=1}^{r}\pi_{ij}Q_{j}x(s)ds - \int_{t-\eta_{M}}^{t} x^{T}(s)Qx(s)ds + \eta_{M}\dot{x}^{T}(t)R\dot{x}(t) - \int_{t-\eta_{M}}^{t}\dot{x}^{T}(s)R\dot{x}(s)ds
$$
  
+  $\Gamma_{1} + \Gamma_{2}$  (14)

where

<span id="page-4-0"></span>
$$
\mathcal{A}_{i} = A_{i}x(t) + B_{i}K_{i}(I + \Delta_{f})C_{1i}x(t - \eta(t)) + B_{i}K_{i}(I + \Delta_{f})C_{1i}e_{k}(t) + B_{\omega i}\omega(t)
$$
\n
$$
\Gamma_{1} = 2\xi^{T}(t)M_{i} \left[ x(t) - x(t - \eta(t)) - \int_{t - \eta(t)}^{t} \dot{x}(s)ds \right]
$$
\n
$$
\Gamma_{2} = 2\xi^{T}(t)N_{i} \left[ x(t - \eta(t)) - x(t - \eta_{M}) - \int_{t - \eta_{M}}^{t - \eta(t)} \dot{x}(s)ds \right]
$$

in which  $\xi^T(t) = \left[ x^T(t) \quad x^T(t - \eta(t)) \quad x^T(t - \eta_M) \quad e_k^T(t) \quad \omega(t) \right]^T$ . From [\(12\),](#page-3-3) we have

$$
\int_{t-\eta_M}^t x^T(t) \sum_{j=1}^r \pi_{ij} Q_j x(t) ds \le \int_{t-\eta_M}^t x^T(t) \sum_{j=1}^r Q x(t) ds.
$$
\n(15)

It is also seen that for any  $R > 0$ 

$$
-2\xi^{T}(t)M_{i}\int_{t-\eta(t)}^{t}\dot{x}(s)ds \leq \eta(t)\xi^{T}(t)M_{i}R^{-1}M_{i}^{T}\xi(t) + \int_{t-\eta(t)}^{t}\dot{x}^{T}(s)R\dot{x}(s)ds
$$
\n(16)

$$
-2\xi^{T}(t)N_{i}\int_{t-\eta_{M}}^{t-\eta(t)}\dot{x}(s)ds \leq (\eta_{M}-\eta(t))\xi^{T}(t)N_{i}R^{-1}N_{i}^{T}\xi(t)+\int_{t-\eta_{M}}^{t-\eta(t)}\dot{x}^{T}(s)R\dot{x}(s)ds.
$$
\n(17)

Combining  $(14)$ – $(17)$  and the event-triggered scheme  $(3)$ , we have

<span id="page-5-0"></span> $\blacksquare$ 

$$
\mathcal{L}V(t) - \gamma^2 \omega^T(t)\omega(t) + z^T(t)z(t) \leq \xi^T(t)(\mathcal{E}_{11} + \Gamma + \Gamma^T)\xi(t) + z^T(t)z(t) + \eta_M \dot{x}^T(t)R\dot{x}(t) + (\eta_M - \eta(t))\xi^T(t)N_iR^{-1}N_i^T\xi(t) + \eta(t)\xi^T(t)M_iR^{-1}M_i^T\xi(t).
$$
\n(18)

Applying Schur complements and [Lemma 2,](#page-3-4) it can be concluded that [\(11\)](#page-3-5) guarantees  $\mathcal{L}V(t) \leq \gamma^2 \omega^T(t)\omega(t) - z^T(t)z(t)$  in [\(18\).](#page-5-0) The remaining part of the proof is similar to those in [\[32](#page-12-17)[,33\]](#page-12-18) and so omitted here for simplicity. Hence the closed-loop system [\(8\)](#page-3-2) is asymptotically stable with an  $H_{\infty}$  norm bound  $\gamma$  if [\(11\)](#page-3-5) and [\(12\)](#page-3-3) holds. This completes the proof.

<span id="page-5-4"></span>Based on [Theorem 1,](#page-3-6) we are in a position to design the output feedback controller for system [\(8\).](#page-3-2)

**Theorem 2.** For given constants  $\gamma$  ,  $\sigma_i$  ,  $\eta_M$  ,  $\varepsilon_1$  ,  $\varepsilon_2$  ,  $s_i$  is a small enough positive scalar, the augmented system [\(8\)](#page-3-2) with the feedback gain  $K_i=Z_iY_i^{-1}$  under the communication scheme [\(3\)](#page-2-1) is asymptotically stable with an  $H_\infty$  performance index  $\gamma$  if there exist real matrices  $X_i > 0$ ,  $\bar{Q}_i > 0$ ,  $\bar{Q}$ ,  $\bar{R} > 0$ ( $i = 1, 2, 3$ ),  $\bar{\Omega}_i > 0$ , and  $\bar{M}_i$ ,  $\bar{N}_i$  with appropriate dimensions such that for  $l = 1, 2$ 

<span id="page-5-1"></span>
$$
\begin{bmatrix} \bar{\Xi}(l)^T & * & * & * \\ \bar{\Sigma}_{21} & -\varepsilon_2 l & * & * \\ \bar{\Sigma}_{31} & 0 & -\varepsilon_2 l & * \\ \bar{\Sigma}_{41} & 0 & 0 & \Lambda \end{bmatrix} < 0
$$
\n(19)

<span id="page-5-3"></span>
$$
\begin{bmatrix} \pi_{ii}\bar{Q}_i & * \\ \gamma & \Delta \end{bmatrix} < \bar{Q}
$$
 (20)

<span id="page-5-2"></span>
$$
s_i \rightarrow 0, \begin{bmatrix} s_i I & * \\ Y_i C_{1i} - C_{1i} X_i & -I \end{bmatrix} < 0 \tag{21}
$$

*where*

$$
\begin{aligned}\n\tilde{z}_{(l)} &= \begin{bmatrix}\n\tilde{z}_{11} + \bar{I} + \bar{I}^T & * & * & * \\
\tilde{z}_{21} & -I & * & * \\
\tilde{z}_{31} & 0 & -2\epsilon_1 X_i + \epsilon^2 \bar{R} & * \\
\tilde{z}_{41}(l) & 0 & 0 & -\bar{R}\n\end{bmatrix} \\
\tilde{z}_{11} &= \begin{bmatrix}\nA_i X_i + X_i A_i^T + \bar{Q}_i + \eta_M \bar{Q} + \pi_{ii} X_i & * & * & * \\
C_{1i}^T Z_i^T B_i^T & \sigma_i \bar{Q}_i & * & * & * \\
0 & 0 & -\bar{Q}_i & * & * \\
C_{1i}^T Z_i^T B_i^T & 0 & 0 & -\bar{Q}_i & * \\
B_{\omega i}^T & 0 & 0 & 0 & -\gamma^2 I\n\end{bmatrix} \\
\tilde{z}_{21} &= \begin{bmatrix} C_{2i} X_i & D_i Z_i C_{1i} & 0 & D_i Z_i C_{1i} & 0 \\
\sqrt{\eta_M} A_i X_i & \sqrt{\eta_M} B_i Z_i C_{1i} & 0 & \sqrt{\eta_M} B_i Z_i C_{1i} & B_{\omega i}\n\end{bmatrix} \\
\Omega_{41}(1) &= \begin{bmatrix} \sqrt{\eta_M} \bar{M}_i^T \end{bmatrix}, \qquad \Omega_{41}(2) = \begin{bmatrix} \sqrt{\eta_M} \bar{N}_i^T \end{bmatrix} \\
\tilde{z}_{31} &= \begin{bmatrix} 0 & \delta_f Z_i C_{1i} & 0 & \delta_f Z_i C_{1i} & 0 & 0 & 0 & 0 \end{bmatrix} \\
A &= \text{diag}(-X_1, \ldots, -X_{i-1}, -X_{i+1}, \ldots, -X_r), \qquad \tilde{z}_{41} = \begin{bmatrix} \Theta & 0 & 0 & 0 & 0 & 0 & 0 & 0 &
$$

**Proof.** Due to

 $(R - \varepsilon_1^{-1} P_i) R^{-1} (R - \varepsilon_1^{-1} P_i) \geq 0,$ 

we can get

$$
-P_iR^{-1}P_i \le -2\varepsilon_1P_i + \varepsilon_1^2R.
$$

Substituting  $-P_iR^{-1}P_i$  with  $-2\varepsilon_1P_i + \varepsilon_1^2R$  (*i* = 1, 2, 3) into [\(11\),](#page-3-5) one can have

$$
E(l) = \begin{bmatrix} \Sigma_{11} + \Gamma + \Gamma^T & * & * & * \\ \Sigma_{21} & -I & * & * \\ \Sigma_{31} & 0 & -2\varepsilon_1 P_i + \varepsilon_1^2 R & * \\ \Sigma_{41}(l) & 0 & 0 & -R \end{bmatrix} < 0.
$$
 (22)

Noticed that the inequalities  $(11)$  can be equivalently expressed as

$$
\hat{\Xi}(l) + sym\{H_B^T H_f\} + H_I^T \sum_{j=1, j \neq i}^r \pi_{ij} P_j H_l < 0
$$
\n(23)

where

$$
\hat{E}(l) = \begin{bmatrix}\n\hat{E}_{11} + \Gamma + \Gamma^T & * & * & * & * \\
\hat{E}_{21} & -I & * & * & * \\
\hat{E}_{31} & 0 & -2\varepsilon_1 P_i + \varepsilon_1^2 R & * \\
& \Sigma_{41}(l) & 0 & 0 & -R\n\end{bmatrix} < 0
$$
\n
$$
\hat{E}_{11} = \begin{bmatrix}\nP_i A_i + A_i^T P_i + Q_i + \eta_M Q + \pi_{ii} P_i & * & * & * \\
C_{1i}^T K_i^T B_i^T P_i & \sigma_i C_{1i}^T \Omega_i C_{1i} & * & * & * \\
0 & 0 & -Q_i & * & * \\
& & C_{1i}^T K_i^T B_i^T P_i & 0 & 0 & -C_{1i}^T \Omega_i C_{1i} & * \\
& & & B_{\omega i}^T P_i & 0 & 0 & 0 & -\gamma^2 I\n\end{bmatrix}
$$
\n
$$
\hat{E}_{21} = \begin{bmatrix} C_{2i} & D_i K_i C_{1i} & 0 & D_i K_i C_{1i} & B_{\omega i} \end{bmatrix}
$$
\n
$$
\hat{E}_{31} = \begin{bmatrix} \sqrt{\eta_M} P_i A_i & \sqrt{\eta_M} P_i B_i K_i C_{1i} & 0 & \sqrt{\eta_M} P_i B_i K_i C_{1i} & P_i B_{\omega i} \end{bmatrix}
$$
\n
$$
H_B = \begin{bmatrix} B_i^T P_i & 0 & 0 & 0 & 0 & D_i^T & \sqrt{\eta_M} B_i^T P_i & 0 \end{bmatrix}
$$
\n
$$
H_I = \begin{bmatrix} 0 & K_i \Delta_f C_{1i} & 0 & K_i \Delta_f C_{1i} & 0 & 0 & 0 & 0 \end{bmatrix}
$$

Using [Lemma 3,](#page-3-7) it follows from [\(24\)](#page-6-0) that there exist scalars  $\varepsilon_2 > 0$  such that

$$
\hat{E}(l) + \varepsilon_2 H_B^T H_B + \varepsilon_2^{-1} \delta_f \bar{H}_f^T \bar{H}_f + H_l^T \sum_{j=1}^r \pi_{ij} P_j H_l < 0 \tag{24}
$$

where

 $\bar{H}_f = \begin{bmatrix} 0 & K_i C_{1i} & 0 & K_i C_{1i} & 0 & 0 & 0 & 0 \end{bmatrix}.$ 

By Schur complement, [\(24\)](#page-6-0) is equivalent to

$$
\begin{bmatrix} \hat{\Xi}(l)^{T} & * & * & * \\ H_{B} & -\varepsilon_{2}l & * & * \\ H_{f} & 0 & -\varepsilon_{2}l & * \\ H_{I} & 0 & 0 & \hat{\Lambda} \end{bmatrix} < 0
$$
\n(25)

where

$$
\hat{\Lambda} = \text{diag}\{-P_1^{-1}, \ldots, -P_{i-1}^{-1}, -P_{i+1}^{-1}, \ldots, -P_r^{-1}\}.
$$

Define  $X_i = P_i^{-1}$ ,  $X_i R X_i = \overline{R}$ ,  $X_i Q X_i = \overline{Q}$ ,  $X_i Q_i X_i = \overline{Q}_i$ ,  $X_i M_i X_i = \overline{M}_i$ ,  $X_i N_i X_i = \overline{N}_i$ ,  $X_i C_{1i}^T \Omega_i C_{1i} X_i = \overline{\Omega}_i$ , pre- and post-multiplying both sides of [\(25\)](#page-6-1) with diag{ $X_i$ ,  $X_i$ ,  $X_i$ ,  $X_i$ ,  $I$ ,  $I$ ,  $X_i$ ,  $X_i$ ,  $I$ ,  $I$ ,  $I$ ,  $\ldots$ ,  $I$ }, we obtain



<span id="page-6-2"></span><span id="page-6-1"></span><span id="page-6-0"></span>*r*−1

in which

$$
\hat{E}(l) = \begin{bmatrix}\n\hat{E}_{11} + \bar{I} + \bar{I}^T & * & * & * & * \\
\hat{E}_{21} & -l & * & * & * \\
\hat{E}_{31} & 0 & -2\varepsilon_1 X_i + \varepsilon^2 \bar{R} & * \\
\bar{E}_{41}(l) & 0 & 0 & -\bar{R}\n\end{bmatrix}
$$
\n
$$
\hat{E}_{11} = \begin{bmatrix}\nA_i X_i + X_i A_i^T + \bar{Q}_i + \eta_M \bar{Q} + \pi_{ii} X_i & * & * & * \\
X_i C_{1i}^T K_i^T B_i^T & \sigma_i \bar{Q}_i & * & * & * \\
0 & 0 & -\bar{Q}_i & * & * \\
0 & 0 & -\bar{Q}_i & * & * \\
X_i C_{1i}^T K_i^T B_i^T & 0 & 0 & -\bar{Q}_i & * \\
B_{\omega i}^T & 0 & 0 & 0 & -\gamma^2 I\n\end{bmatrix}
$$
\n
$$
\hat{E}_{21} = \begin{bmatrix}\nC_{2i} X_i & D_i K_i C_{1i} X_i & 0 & D_i K_i C_{1i} X_i & 0\n\end{bmatrix}
$$
\n
$$
\hat{E}_{31} = \begin{bmatrix}\n\sqrt{\eta_M} A_i X_i & \sqrt{\eta_M} B_i K_i C_{1i} X_i & 0 & \sqrt{\eta_M} B_i K_i C_{1i} X_i & B_{\omega i}\n\end{bmatrix}
$$
\n
$$
\hat{\Sigma}_{31} = \begin{bmatrix}\n0 & \delta_f K_i C_{1i} X_i & 0 & \delta_f K_i C_{1i} X_i & 0 & 0 & 0 & 0\n\end{bmatrix}.
$$

Noticed that there are many coupling nonlinear terms  $B_iK_iC_{1i}X_i$ ,  $D_iK_iC_{1i}X_i$  and  $K_iC_{1i}X_i$  in [\(26\).](#page-6-2) In order to put the solution in the LMI setting, inspired by the work in [\[34\]](#page-12-19), we define  $K_i = Z_i Y_i^{-1}$ ,  $Y_i C_{1i} = C_{1i} X_i$ . Then, one can see that [\(26\)](#page-6-2) is equivalent to [\(19\).](#page-5-1)

Since  $Y_iC_{1i}=C_{1i}X_i$  is not a strict inequality, from  $Y_iC_{1i}=C_{1i}X_i$ , we have  $(Y_iC_{1i}-C_{1i}X_i)^T(Y_iC_{1i}-C_{1i}X_i)=0$ , which can be transformed into the following

$$
s_i \rightarrow 0, \begin{bmatrix} s_i I & * \\ Y_i C_{1i} - C_{1i} X_i & -I \end{bmatrix} < 0 \tag{27}
$$

which is just [\(21\).](#page-5-2)

Meanwhile pre- and post-multiplying [\(20\)](#page-5-3) with *X<sup>i</sup>* , we have

<span id="page-7-1"></span>
$$
\sum_{j=1}^r \pi_{ij} X_i Q_j X_i \le \bar{Q}.
$$
\n(28)

By Schur complement, [\(28\)](#page-7-1) is equivalent to [\(20\).](#page-5-3)

That is, if [\(19\)–\(21\)](#page-5-1) hold, the closed loop system [\(8\)](#page-3-2) is asymptotically stable with an  $H_{\infty}$  performance index  $\gamma$ . This completes the proof.

**Remark 6.** In the process of deriving the main result, the main difficulty is how to deal with the nonlinear terms *BiKiC*1*iX<sup>i</sup>* ,  $D_i K_i C_{1i} X_i$  and  $K_i C_{1i} X_i$  in [\(26\).](#page-6-2) Inspired by the work in [\[34,](#page-12-19)[35\]](#page-12-20), we overcome the difficulty successfully and obtain the new static output feedback controller design method  $K_i = Z_i Y_i^{-1}$  for Markovian jump systems.

**Remark 7.** Although some feasible results have been developed in the existing literature to deal with the static output feedback problem [\[36,](#page-12-21)[37](#page-12-22)[,34](#page-12-19)[,35\]](#page-12-20), they are difficult to be applied directly to deal with the case that there exist quantization and event-triggered scheme in networked Markovian jump systems. Compared with the existing static output feedback results, there is no constraint that system output matrices C<sub>1i</sub> is a nonsingular matrix. The new method obtained in [Theorem 2](#page-5-4) is undoubtedly effective to design event-triggered output feedback *H*∞ controller for networked Markovian jump systems with quantizations.

**Remark 8.** It should be pointed out that the triggered parameters  $\sigma_i$  will vary depending on the modes of the Markovian jump system. From [Theorem 2,](#page-5-4) we can observe that the *H*∞ control performance and the network resource usage are related to the event-triggered parameters  $\sigma_i$  and  $\Omega_i$ .

#### <span id="page-7-0"></span>**4. Simulation examples**

In order to demonstrate the effectiveness of the proposed method, a special system of  $(8)$  is operated with two operating modes that capture the abrupt changes. The system parameters are given as follows:

$$
A_1 = \begin{bmatrix} -0.2 & 0.8 \\ 0 & -1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.1 \\ -0.1 \end{bmatrix}, \qquad B_{w1} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \qquad C_{11} = \begin{bmatrix} 1 & 0.5 \end{bmatrix},
$$
  

$$
C_{21} = \begin{bmatrix} 0.2 & 0.15 \\ 0.1 & 0.3 \end{bmatrix}, \qquad D_1 = \begin{bmatrix} 0.5 \\ 0.2 \end{bmatrix}
$$

$$
A_2 = \begin{bmatrix} -1 & -0.6 \\ 0.8 & -1.5 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -0.5 \\ 0 \end{bmatrix}, \qquad B_{w2} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, \qquad C_{12} = \begin{bmatrix} 1 & -4 \end{bmatrix},
$$
  
\n
$$
C_{22} = \begin{bmatrix} 0.2 & 0.1 \\ -0.2 & 0.4 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}.
$$

The corresponding transition probability matrices of the Markov process are supposed to be

$$
\Pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}.
$$

The initial state and the external disturbance are given as  $x_0 = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$ . and

$$
\omega(t) = \begin{cases} 1, & 5 \le t \le 10 \\ -1, & 10 \le t \le 15 \\ 0, & \text{else.} \end{cases}
$$
(29)

In the following, we will consider three cases for this Markovian jump system. In case 1, it employs the different triggered parameters for different jump modes and the effect of quantization is taken into account. In case 2, we employ quantization and the same triggered parameters for different jump modes. In case 3, without quantization, the same triggered parameters is applied for different jump modes.

**Case 1**: Consider a specific simulation set with  $\sigma_1 = 0.2$ ,  $\sigma_2 = 0.1$ ,  $\gamma = 2$ ,  $s_1 = 0.001$ ,  $s_2 = 0.001$  and  $\varepsilon_1 = \varepsilon_2 = 1$ ,  $\eta_M = 0.5$ , the quantization densities in [\(5\)](#page-2-2) are chosen as  $\delta_{fs} = 0.5$ , based on Matlab/LMIs toolbox and applying [Theorem 2,](#page-5-4) we can get the controller feedback gain  $K_1 = -0.1759$  and  $K_2 = -0.0043$ , the corresponding trigger matrix  $\Omega_1 = 0.1214$ and  $\Omega_2 = 1.3953$ , respectively. The maximum allowable bound of  $\eta_M$  is 0.8302 s. Suppose that  $\tau_{th} = 0$ , letting  $\bar{\tau} = 0$ , from  $\eta_M = h + \bar{\tau}$ , it can be known that the maximum allowable sampling period is 0.8302*s*. The probabilities of switching between modes can be seen from [Fig. 2.](#page-9-0) The state trajectories of [\(8\)](#page-3-2) are shown in [Fig. 3.](#page-9-1) The event-triggering release instants and intervals are shown in [Fig. 4.](#page-9-2)

**Case 2**: For given  $\sigma_1 = 0.2$ ,  $\sigma_2 = 0.2$ ,  $\gamma = 2$  and  $\varepsilon_1 = \varepsilon_2 = 1$ ,  $\eta_M = 0.5$ ,  $\delta_k = 0.5$ ,  $s_1 = 0.001$ ,  $s_2 = 0.001$ , we can obtain  $K_1 = -0.1759$  and  $K_2 = -0.0035$ ; the corresponding trigger matrix  $\Omega_1 = 0.1216$  and  $\Omega_2 = 0.6982$ , respectively. The probabilities of switching between modes can be seen from [Fig. 6.](#page-10-0) The simulation result for the responses of  $x(t)$  is shown in [Fig. 7.](#page-10-1) The event-triggering release instants and intervals are shown in [Fig. 5.](#page-10-2)

**Case 3:** Setting  $\sigma_1 = 0.2$ ,  $\sigma_2 = 0.2$ ,  $\gamma = 2$ ,  $s_1 = 0.001$ ,  $s_2 = 0.001$  and  $\varepsilon_1 = \varepsilon_2 = 1$ ,  $\eta_M = 0.5$ , the controller gains are  $K_1 = -0.3389$ ,  $K_2 = 0.0222$  and the corresponding trigger matrix are  $\Omega_1 = 0.3606$ ,  $\Omega_2 = 0.7700$ , respectively. The probabilities of switching between modes can be seen from [Fig. 8.](#page-11-0) The state response of *x*(*t*) and the event-triggering release instants and intervals are shown in [Figs. 9](#page-11-1) and [10,](#page-11-2) respectively.

From the simulation results in [Figs. 3,](#page-9-1) [7](#page-10-1) and [9,](#page-11-1) we can see that although there exist slight difference of the trajectories, the studied system can be stabilized while ensuring the desired control performance. The event triggering method can improve the utilization of the network bandwidth, which is shown in [Figs. 4,](#page-9-2) [5](#page-10-2) and [10.](#page-11-2) As to Markovian jump systems, the dynamic event-triggering scheme with quantization is better than constant event-triggering scheme, which has been illustrated in Case 1 and Case 2. [Figs. 5](#page-10-2) and [10](#page-11-2) illustrate that the quantization method can further save the network resources. In a word, the simulation results can confirm that the proposed method is effective.

**Remark 9.** It should be pointed out that  $\varepsilon_1$  and  $\varepsilon_2$  are used to adjust the feasible solutions, which may affect the conservative of the results. How to choose appropriate values of  $\varepsilon_1$  and  $\varepsilon_2$  is still an open problem. Traditionally, the values of  $\varepsilon_1$  and  $\varepsilon_2$ are selected in a finite number of different values by means of simulation. The optimal values of  $\varepsilon_1$  and  $\varepsilon_2$  can show less design conservatism.

**Remark 10.** In [\[22\]](#page-12-7), the authors investigate the state feedback control for networked Markovian jump system under event-triggered scheme. In our paper, we investigate output feedback control for networked Markovian jump system with quantization and event-triggered scheme. It should be pointed out that state feedback control and output feedback control are two different feedback strategies. The state feedback, which requires all state variables to be accessible, may simplify the considered problem, however, the obtained results are restrictive in engineering when the full state estimation is difficult to obtain. In practical control applications and realizations, the full state information is difficult to obtain while the system output signals are easy to access. This promotes the research into output feedback control in this paper.

## **5. Conclusion**

The problem of *H*∞ output feedback control for event triggered Markovian jump system with input quantized is studied in this paper. Event triggered communication schemes and a quantizer are introduced into the framework to reduce the network bandwidth utilization and improve the desired system performance. A new Markovian jump system model has been utilized to describe the prosperities of the event trigger and the effect of quantizations. Sufficient conditions are established to ensure the stability of the discussed system and to achieve a prescribed performance. Criteria for stability with

<span id="page-9-0"></span>

**Fig. 2.** The probabilities of switching between modes in case 1.

<span id="page-9-1"></span>

**Fig. 3.** The state response of closed-loop system in case 1.

<span id="page-9-2"></span>

**Fig. 4.** The release instants and release interval in mode 1.

an *H*∞ norm bound and *H*∞ output feedback controller design have been derived. A simulation example has highlighted the effectiveness of the proposed method.

<span id="page-10-2"></span>

**Fig. 5.** The release instants and release interval in mode 2.

<span id="page-10-0"></span>

**Fig. 6.** The probabilities of switching between modes in case 2.

<span id="page-10-1"></span>

**Fig. 7.** The state response of closed-loop system in case 2.

<span id="page-11-0"></span>

**Fig. 8.** The probabilities of switching between modes in case 3.

<span id="page-11-1"></span>



<span id="page-11-2"></span>

**Fig. 10.** The release instants and release interval in mode 3.

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#### **References**

- <span id="page-12-0"></span>[1] E. T[ian, D. Yue, G. Wei, Robust control for Markovian jump systems with partially known transition probabilities and nonlinearities, J. Franklin Inst. B](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref1) 350 (8) (2013) 2069–2083.
- [2] Y. Kao, J. Xie, C. Wang, H. Karimi, A sliding mode approach to *H*∞ [non-fragile observer-based control design for uncertain Markovian neutral-type](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref2) stochastic systems, Automatica 52 (2015) 218–226.
- [3] F. Li, H. Shen, Finite-time *H*∞ [synchronization control for semi-Markov jump delayed neural networks with randomly occurring uncertainties,](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref3) Neurocomputing 166 (2015) 447–454.
- [4] J. Cheng, H. Park Ju, Y. Liu, Z. Liu, L. Tang, Finite-time *H*∞ fuzzy control of nonlinear Markovian jump delayed systems with partly uncertain transition descriptions, Fuzzy Sets and Systems (2016) [http://dx.doi.org/10.1016/j.fss.2016.06.007.](http://dx.doi.org/10.1016/j.fss.2016.06.007)
- [5] S. L[i, Z. Xiang, Stochastic stability analysis and image-gain controller design for positive Markov jump systems with time-varying delays, Nonlinear](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref5) Anal. Hybrid Syst. 22 (2016) 31–42.
- [6] M. [Liu, D.W.C. Ho, Y. Niu, Stabilization of Markovian jump linear system over networks with random communication delay, Automatica 45 \(2\) \(2009\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref6) 416–421.
- [7] J. Wang, H. Wang, A. Xue, et al., Delay-dependent *H*∞ [control for singular Markovian jump systems with time delay, Nonlinear Anal. Hybrid Syst. 8](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref7) (8) (2013) 1–12.
- [8] J. Li[u, E. Tian, Z. Gu, et al., State estimation for Markovian jumping genetic regulatory networks with random delays, Commun. Nonlinear Sci. Numer.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref8) Simul. 19 (19) (2014) 2479–2492.
- [9] L. Xiong, H. Zhang, Y. Li, et al., Improved stability and *H*∞ [performance for neutral systems with uncertain Markovian jump, Nonlinear Anal. Hybrid](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref9) Syst. 19 (2016) 13–25.
- <span id="page-12-1"></span>[10] H. Li, J. Wang, H.K. Lam, Q. Zhou, H. Du, Adaptive sliding mode control for interval type-2 fuzzy systems, IEEE Trans. Syst. Man Cybern. Syst. (2016) [http://dx.doi.org/10.1109/TSMC.2016.2531676.](http://dx.doi.org/10.1109/TSMC.2016.2531676)
- [11] H. Li, J. Wang, P. Shi, Output-feedback based sliding mode control for fuzzy systems with actuator saturation, IEEE Trans. Fuzzy Syst. (2016) [http://dx.doi.org/10.1109/TFUZZ.2015.2513085.](http://dx.doi.org/10.1109/TFUZZ.2015.2513085)
- [12] J. Li[u, S. Fei, E. Tian, et al., Co-design of event generator and filtering for a class of T-S fuzzy systems with stochastic sensor faults, Fuzzy Sets and](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref12) Systems 273 (2015) 124–140.
- [13] C. P[eng, M. Fei, E. Tian, Y. Guan, On hold or drop out of order packets in networked control systems, Inform. Sci. 268 \(2014\) 436–446.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref13)
- [14] H. Hu, B. Jiang, H. Yang, Non-fragile *H*<sup>2</sup> [reliable control for switched linear systems with actuator faults, Signal Process. 93 \(7\) \(2013\) 1804–1812.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref14)
- [15] H. Hu, B. Jiang, H. Yang, Robust *H*∞ [reliable control for uncertain switched systems with circular disk pole constraints, J. Franklin Inst. B 350 \(4\) \(2013\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref15) 802–817.
- <span id="page-12-2"></span>[16] S. [Hu, D. Yue, Event-triggered control design of linear networked systems with quantizations, ISA Trans. 51 \(1\) \(2012\) 153–162.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref16)
- <span id="page-12-3"></span>[17] S. Zhou, Y. Guan, *H*∞ [filter design for fuzzy systems with quantized measurements, Neurocomputing 166 \(2015\) 193–200.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref17)
- [18] H. [Yu, P.J. Antsaklis, Event-triggered output feedback control for networked control systems using passivity: Achieving image stability in the presence](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref18) of communication delays and signal quantization, Automatica 49 (1) (2013) 30–38.
- <span id="page-12-4"></span>[19] P. S[hi, H. Wang, C.C. Lim, Network-based event-triggered control for singular systems with quantizations, IEEE Trans. Ind. Electron. 63 \(2\) \(2016\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref19) 1230–1238.
- <span id="page-12-5"></span>[20] D. [Yue, E. Tian, Q.L. Han, A delay system method for designing event-triggered controllers of networked control systems, IEEE Trans. Automat. Control](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref20) 58 (2) (2013) 475–481.
- <span id="page-12-6"></span>[21] C. Peng, J. Zhang, Event-triggered output-feedback *H*∞ [control for networked control systems with time-varying sampling, IET Control Theory Appl.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref21) 9 (9) (2015) 1384–1391.
- <span id="page-12-7"></span>[22] H. [Wang, P. Shi, C.C. Lim, et al., Event-triggered control for networked Markovian jump systems, Internat. J. Robust Nonlinear Control 25 \(17\) \(2014\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref22)
- <span id="page-12-8"></span>3422–3438. [23] H. Li, Z. Chen, L. Wu, H.K. Lam, H. Du, Event-triggered fault detection of nonlinear networked systems, IEEE Trans. Cybern. (2016) [http://dx.doi.org/](http://dx.doi.org/10.1109/TCYB.2016.2536750) [10.1109/TCYB.2016.2536750.](http://dx.doi.org/10.1109/TCYB.2016.2536750)
- <span id="page-12-9"></span>[24] J. Li[u, D. Yue, Event-triggering in networked systems with probabilistic sensor and actuator faults, Inform. Sci. 240 \(2013\) 145–160.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref24)
- <span id="page-12-10"></span>[25] Y. [Abdi, T. Ristaniemi, Joint local quantization and linear cooperation in spectrum sensing for cognitive radio networks, IEEE Trans. Signal Process. 62](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref25) (17) (2014) 4349–4362.
- <span id="page-12-11"></span>[26] X. [Mao, Exponential stability of stochastic delay interval systems with markovian switching, IEEE Trans. Automat. Control 47 \(10\) \(2002\) 1604–1612.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref26)
- <span id="page-12-12"></span>[27] Y. [Wang, L. Xie, C. Souza, Robust control of a class of uncertain nonlinear systems, Systems Control Lett. 19 \(2\) \(1992\) 139–149.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref27)
- <span id="page-12-13"></span>[28] E. Tian, D. Yue, Y. Zhang, Delay-dependent robust *H*∞ [control for T–S fuzzy system with interval time-varying delay, Fuzzy Sets and Systems 160 \(12\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref28) (2009) 1708–1719.
- <span id="page-12-14"></span>[29] L. Xie, M. Fu, C. De Souza, *H*∞ [control and quadratic stabilization of system with parameter uncertainty via output feedback, IEEE Trans. Automat.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref29) Control 37 (8) (1992) 1253–1257.
- <span id="page-12-15"></span>[30] Y. [He, M. Wu, J. She, G. Liu, Parameter-dependent lyapunov functional for stability of time-delay systems with polytopic-type uncertainties, IEEE](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref30) Trans. Automat. Control 49 (5) (2004) 828–832.
- <span id="page-12-16"></span>[31] D. Yue, Q. Han, J. Lam, Network-based robust *H*∞ [control of systems with uncertainty, Automatica 41 \(6\) \(2005\) 999–1007.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref31)
- <span id="page-12-17"></span>[32] E. Tian, D. Yue, Reliable *H*∞ [filter design for T-S fuzzy model-based networked control systems with random sensor failure, Internat. J. Robust Nonlinear](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref32) Control 23 (1) (2013) 15–32.
- <span id="page-12-18"></span>[33] J. Li[u, D. Yue, Event-based fault detection for networked systems with communication delay and nonlinear perturbation, J. Franklin Inst. B 350 \(9\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref33) (2013) 2791–2807.
- <span id="page-12-19"></span>[34] J. Zhang, C. Peng, M.R. Fei, et al., Output feedback control of networked systems with a stochastic communication protocol, J. Franklin Inst. B (2016) [http://dx.doi.org/10.1016/j.jfranklin.2016.02.009.](http://dx.doi.org/10.1016/j.jfranklin.2016.02.009)
- <span id="page-12-20"></span>[35] E.K. [Boukas, Static output feedback control for stochastic hybrid systems: LMI approach, Automatica 42 \(1\) \(2006\) 183–188.](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref35)
- <span id="page-12-21"></span>[36] C. [Peng, Y. Tian, D. Yue, Output feedback control of discrete-time systems in networked environments, IEEE Trans. Syst. Man Cybern.-Part A 41 \(1\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref36) (2011) 185–190.
- <span id="page-12-22"></span>[37] X.H[. Chang, L. Zhang, H.P. Ju, Robust static output feedback \[formula omitted\] control for uncertain fuzzy systems, Fuzzy Sets and Systems 273 \(2015\)](http://refhub.elsevier.com/S1751-570X(16)30067-X/sbref37) 87–104.