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Brief Papers

Two channel event-triggering communication schemes for networked control systems

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ABSTRACT

This paper is concerned with event-triggered controller design for networked control systems. At first, novel model-based event-triggered transmission strategies for both the sensor-to-controller and the controller to actuator channels are proposed, which are capable of reducing the communication bandwidth utilization, while preserving the desired control performance. Second, considering the effect of the network transmission delay, a newly delay system model for the analysis is firstly constructed. Third, based on our newly proposed model, criteria for stability and criteria for co-designing both the feedback gain and the trigger parameters are derived. Finally, a numerical example is given to demonstrate the effectiveness of the proposed method.

1. Introduction

Due to the popularization and advantages of using network in control systems, networked control systems (NCSs) have received considerable attention in recent years [1–7]. In many practical systems, the insertion of the network can also bring about several challenging issues including network-induced delays, packet dropouts and the constrained bandwidth of the communication network. With these concerns, some effective methods have been proposed to improve these problems [8–12]. For example, in [9], the authors propose an existence theorem of the maximum packet dropout rate and show that the NCS is stabilizable if the network-induced delay and the packet dropout rate satisfy some simple algebraic inequalities. The authors in [10] and [12] propose new event triggered mechanisms to reduce the utilization rate of communication bandwidth.

More recently, in order to save the limited communication resource for NCSs, much attention has also been paid to design the reasonable communication scheme [13–16,18,19]. In the context of NCSs, the bandwidth of the communication network and the power in sensor nodes are inevitably constrained. Therefore, one needs to design a reasonable communication scheme to save the limited resources of communication capacity and energy supply while guaranteeing the control performance. A widely used method is time-triggered communication scheme, which is believed to be beneficial for resource saving. Although there have been publications about nonlinear NCSs in the literature [18,24], it should be pointed out that time-triggered communication scheme leads to inefficient utilization of the limited network resources. Especially when there is little new information in the transmission, such as when no disturbances are acting on the system and the system is operating desirably, inefficient or redundant communications have inevitably transmitted through communication networks. Therefore, it is necessary to find an alternative control paradigm to mitigate the unnecessary waste of communication resources.

Recently, event-triggered method has received considerable attention [20–23], which can reduce the burden of the network communication and the occupation of the sensor, while retaining a satisfactory closed-loop performance. Compared to time-triggered communication scheme, event-triggered method is a control strategy in which the control task is executed after the occurrence of an event. “Event” will be triggered by some well-designed event-triggering condition, rather than the elapse of a certain fixed period of time [25]. In this way, event-triggered method is capable of increase the energy efficiency and reduce the cost of sensor network. For example, in [2], a novel distributed event-triggered sampled-data transmission strategy is proposed and a sufficient condition on the consensus of the multi-agent system is derived. The authors in [13] proposed a novel event-triggering scheme and developed an event-triggered H_∞ control design method for networked control systems with network-induced delay. In [16], the authors proposed a discrete event-triggered communication scheme for a class of networked T–S

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fuzzy systems. In the aforementioned studies, different communication schemes are established mostly between the sensor and the controller, which can decide whether or not the sampled sensor measurements are to be transmitted. Only when the current sampled sensor measurements violate a special condition, they can be transmitted. However, the network resource between the controller and the actuator are also limited, only little attention has been paid to deal with this problem. Especially when the control signal is sent to plant over a lossy communication channel, where network-induced delay and packet dropout occur simultaneously, it is essential to design a transmission scheme to save the capacity of the network. The control output transmitted in practical NCSs should be event-based before they are sent to the actuator in order to achieve better performance. However, to the best of the authors' knowledge, little attention has been paid to this problem, which is another motivation of the current study.

In this paper, we will propose model-based event-based mechanisms for both the sensor-to-controller and the controller to actuator channels. The communication traffic will be significantly reduced while preserving the desired performance and without resorting to extra hardware. We only measure the state and compute the error at a constant sampling period. Notice that not all of the measured states are transmitted through the communication network, that is, only the error violates the prescribed threshold, then the measured state is transmitted to the controller. Moreover, not all of the output of controller can be sent to the actuator. Only when the error of the output of controller violate a special condition, they can be transmitted. The main contributions of this paper are as follows: (1) the event-based mechanisms for both the sensor-to-controller and the controller to actuator channels are firstly proposed. (2) Considering the effect of the network transmission delay and the properties of the event-triggering schemes, a novel model is firstly proposed for the use of system analysis and control design, which has not been considered in the existing references. (3) Based on the model, sufficient conditions for the stability and controller design are derived in terms of linear matrix inequalities.

The paper is organized as follows. Firstly, a novel two-channel event-triggered transmission strategy will be proposed in Section 2. Then, sufficient conditions for the stability of the addressed model are established in terms of linear matrix inequalities in Section 3. Finally, in Section 4, a numerical example is employed in the final part to demonstrate the effectiveness and applicability of our method.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n\times m$ real matrices, respectively; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X>0$ ($X\geq 0$), for $X\in\mathbb{R}^{n\times n}$, means that the matrix X is real symmetric positive definite (positive semi-definite). When x is a stochastic variable. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C\end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System description

In this section, we will study the networked control configuration as shown in Fig. 1, in which the system is described by

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{1}$$

where $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^m$ denote the state vector, control vector, respectively; A and B are parameter matrices with appropriate dimensions.

As is well known, all the sampled data are transmitted to the controller via the communication channel in network control systems, all the controller output can be transmitted to the actuator in the same way. Indeed, if the current data vary slightly compared with the previous one, we can still use the previous one. In this case, part of sampled signal need not be transmitted over network, thus, the transmission frequency can be reduced and the network bandwidth can be saved. Then, how to filter out those unuseful signal before transmitting to the controller and actuator through the network? In recent years, there are some interesting results as to how to choose those useful sampled data to be transmitted, see, e.g. [13,16,17]. Different from these studies, in this paper, we propose a two channel event triggered schemes to decide whether the current signal should be transmitted or not.

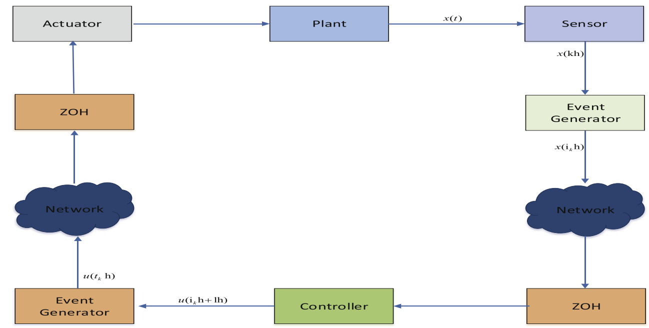

Fig. 1. The structure of an event-triggered networked control system.

Throughout this paper, we assume the system (1) is controlled by a network and possibly wireless network, for which communication resources and energy sources, e.g., the batteries for the wireless devices, are limited. For this reason, it is desirable to propose new event-triggered communication schemes to reduce the number of transmitted packets over the sensor to controller and controller to actuator channels as much as possible, while preserving the stability and desired control performance.

In solving this problem, two mechanisms are proposed based on the event-triggered control for the configuration in Fig. 1. In order to save the limited resources in the sensor to controller channel, an event generator is constructed between the sensor and the controller to determine when information should be transmitted to the controller system. Similarly, the other event generator constructed between the controller and actuator is used to determine whether information should be transmitted to the actuator side, which can reduce the transmissions in the controller to actuator.

For ease of exposition, we make the following assumptions:

(i) The system states are sampled at a constant period h. The sampled instants is lh, $l\in\{0,1,2,\ldots\}$.

(ii) The logic ZOH (zero-order-hold) before the controller (or actuator) is used to hold the control input (or actuator input), when there is no latest control packet arrived at the controller (actuator).

(iii) The transmitted instant i_{kh} from the sensor to controller is determined by the sampled state $x(lh)$. The set of transmission instants is represented by i_{kh}, $i_k\in N$. The transmitted instant t_{kh} from the controller to actuator is determined by the state that arrived at the controller, which can be represented by t_{kh}, $t_k\in N$.

(iv) $\tau_{t_k}^{sc}$ and $\tau_{t_k}^{ca}$ are network-induced delays from the sensor to the controller and from the controller to the actuator, representatively. $\tau_{t_k}^{sc}$ and $\tau_{t_k}^{ca}$ and the computational and waiting delays are lumped together as τ_{t_k}, where $\tau_{t_k}\in(0,\overline{\tau}]$, $\overline{\tau}$ is the upper bound of τ_{t_k}.

The purpose of this paper is to design a linear controller $u(t)=Kx(t)$; where K is a gain matrix to be determined later, such that the resulting closed-loop system satisfies the required performance.

Based on the previous description, whether or not the sampled-data should be transmitted from sensor to actuator relies on two event triggered conditions rather than the elapse of a fixed time. The first event triggered mechanism from sensor to controller is designed as

$$e_k^T(i_kh)\Phi_1 e_k(i_kh) \le \sigma_1 x(i_kh+lh)^T \Phi_1 x(i_kh+lh) \tag{2}$$

where $e_k(i_kh) = x(i_kh) - x(i_kh+lh)$, Φ_1 is a positive-definite weighting matrix and σ_1 is a given scalar parameter, $\sigma_1 \in [0,1)$, $l \in \mathbb{N}$, $x(i_kh)$ is the latest sensor data transmitted to the controller, $x(i_kh+lh)$ is the current sensor data. Once the first triggering condition (2) is violated, the current sampled-data packet is immediately released and transmitted to the first zero-order-hold (ZOH) through a communication network; otherwise, it is discarded right away. Apparently, whether or not the newly sampled-data should be sent to the controller is dependent on state error $e_k(i_kh)$ and the latest transmitted sampled-data $x(i_kh)$. Clearly, the next release time instant $i_{k+1}h$ can be expressed as

$$\begin{aligned} i_{k+1}h = i_kh + \min_{l \ge 0}\Big\{&(l+1)h \,|\, e_k^T(i_kh)\Phi_1 e_k(i_kh) \\ &\le \sigma_1 x(i_kh+lh)^T \Phi_1 x(i_kh+lh)\Big\} \end{aligned} \tag{3}$$

The second event triggered mechanism between controller and actuator is given by

$$f^T(t_kh)\Phi_2 f(t_kh) \le \sigma_2 u(t_kh+(i_{t_k+j}-i_{t_k})h)^T \Phi_2 u(t_kh+(i_{t_k+j}-i_{t_k})h) \tag{4}$$

where $f(t_kh) = u(t_kh) - u(t_kh+(i_{t_k+j}-i_{t_k})h)$, Φ_2 is a positive-definite weighting matrix, and σ_2 is a given scalar parameter, $\sigma_2 \in [0,1)$, $j \in \mathbb{N}$, $u(t_kh)$ is the latest transmitted control data, $u(t_kh+(i_{t_k+j}-i_{t_k})h)$ is a current control data, Once the second triggering condition (4) is violated, the current computed control data is immediately released and transmitted to the second zero-order-hold (ZOH) through a communication network; otherwise, it is discarded off right away. Clearly, the next transmission instant determined by the second event generator can be expressed as

$$\begin{aligned} t_{k+1}h = t_kh + \min_{j \ge 1}\Big\{&(i_{t_k+j}-i_{t_k})h \,|\, f^T(t_kh)\Phi_2 f(t_kh) \\ &\le \sigma_2 u(t_kh+(i_{t_k+j}-i_{t_k})h)^T \Phi_2 u(t_kh+(i_{t_k+j}-i_{t_k})h)\Big\} \end{aligned} \tag{5}$$

Remark 1. Considering the effect of the communication delay in networked control systems, the sampled sensor data $x(t_kh)$, which satisfies (2) and (4), will arrive at the control side at the instants $t_kh + \tau_{t_k}^{sc}$ and reaches the actuator side at the instants $t_kh + \tau_{t_k}$.

Remark 2. It is easily seen from event-triggered conditions (2) and (4) that the set of the release instants $\{t_1h, t_2h, t_3h, \ldots\} \subseteq \{i_1h, i_2h, i_3h, \ldots\} \subseteq \{h, 2h, 3h, \ldots\}$ The amount of $\{i_1h, i_2h, i_3h, \ldots\}$ and $\{t_1h, t_2h, t_3h, \ldots\}$ depend not only on the variation of the sensor measurements and the control outputs but also on the value of σ_1 and σ_2, respectively.

For a detailed timing analysis, the holding interval of the first ZOH and the second ZOH are $[i_kh + \tau_{i_k}^{sc}, i_{k+1}h + \tau_{i_{k+1}}^{sc})$ and $[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$, respectively. From the previous assumption, one can see that

$$[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) = \bigcup_{j=1}^{d} \Omega_{l^j, j} \tag{6}$$

where $\Omega_{l^j, j} = [t_kh + (i_{t_k+j} - i_{t_k})h + l^jh + \tau_{t_k + i_{t_k+j} - i_{t_k} + l^j},\ t_kh + (i_{t_k+j+1} - i_{t_k})h + l^jh + h + \tau_{t_k + i_{t_k+j+1} - i_{t_k} + l^j})$, where $j = 0, 1, \ldots, d$, $l^j = 0, 1, \ldots, l_M^j$, $i_{t_k+d} - i_{t_k} + l_M^j = t_{k+1} - t_k - 1$, different $[i_{t_k+j}, i_{t_k+j+1})$ have different maximum of l^j, $l_M^j = i_{t_k+j+1} - i_{t_k+j} - 1$. In fact, $[t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) = [t_kh + (i_{t_k+j} - i_{t_k})h + \tau_{t_k + i_{t_k+j} - i_{t_k}}, t_kh + (i_{t_k+j} - i_{t_k})h + (i_{t_k+j+1} - i_{t_k+j})h + \tau_{t_k + i_{t_k+j+1} - i_{t_k}})$.

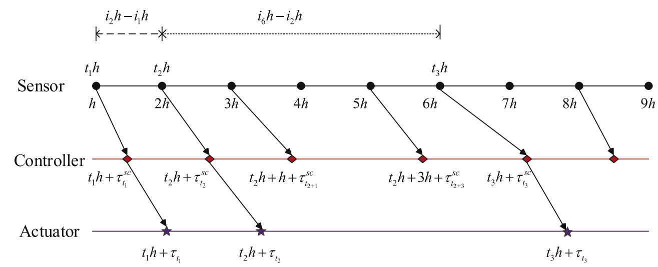

Fig. 2. Example of signal sampling and and release instants.

In order to understand the partition of the interval in (6), we give an illustrative example in Fig. 2.

Define $\eta(t) = t - t_kh - (i_{t_k+j} - i_{t_k})h - l^jh$, it is clear that

$$0 \le \tau_{t_k + i_{t_k+j} - i_{t_k} + l^j} \le \eta(t) \le h + \overline{\tau} \triangleq \overline{\eta} \tag{7}$$

Based on the two event triggered mechanisms, the actual input of the actuator is

$$u(t) = Kx(t_kh), \quad t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{8}$$

For $t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$, Define $g_k(t) = x(t_kh) - x(t_kh + (i_{t_k+j} - i_{t_k})h)$. $e_k(t) = x(t_kh + (i_{t_k+j} - i_{t_k})h) - x(t_kh + (i_{t_k+j} - i_{t_k})h + l^jh)$.

Remark 3. From the definition of $e_k(t)$, $\eta(t)$ and the first triggering algorithm (2), it can be seen that

$$\begin{aligned} &e_k^T(t)\Phi_1 e_k(t) \le \sigma_1 x(t-\eta(t))^T \Phi_1 x(t-\eta(t)), \\ &t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \end{aligned} \tag{9}$$

Combining the definition of $g_k(t)$ and the second triggering algorithm (4), it can be easily obtained

$$\begin{aligned} &g_k^T(t)K^T\Phi_2 K g_k(t) \le \sigma_2[(x(t-\eta(t)) + e_k(t)]^T K^T \Phi_2 K[(x(t-\eta(t)) + e_k(t)], \\ &t \in [t_kh + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \end{aligned} \tag{10}$$

Remark 4. It should be noted that not all of the measured states and the output of controller can be transmitted through the communication network. Only both of the state error and the control output error violate the prescribed thresholds in (9) and (10), then they can be transmitted, respectively. There is no doubt that our two-channel event-trigger based transmission strategy is capable to reduce the communication load. The effectiveness of our transmission strategy can be seen in Section 4.

From the above three definitions, $x(t_kh)$ can be rewritten as

$$x(t_kh) = g_k(t) + e_k(t) + x(t-\eta(t)) \tag{11}$$

Combining (1), (8) and (11), we obtain a closed-loop networked control model

$$\dot{x}(t) = Ax(t) + BKg_k(t) + BKe_k(t) + BKx(t-\eta(t)) \tag{12}$$

We supplement the initial condition of the state $x(t)$ on $[t_0 - \overline{\eta}, t_0]$ as $x(t_0 + \theta) = \varphi(\theta), \theta \in [-\overline{\eta}, 0]$, with $\varphi(0) = x_0$, where $\varphi(\theta)$ is a continuous function on $[t_0 - \overline{\eta}, t_0]$, and $-\overline{\eta}$ is given in (7).

Remark 5. When formulating the system (12), the two channel event triggering conditions (3) and (5) are taken into consideration. If (5) is not considered, that is $\sigma_2 = 0$, then the system (12) reduces to the case in [13], where an event-triggered H_∞ control design method has been developed for networked control systems with network-induced delay. when $\sigma_1 = 0$ and $\sigma_2 = 0$, the two channel event-triggered schemes reduce to a periodic time-triggered scheme.

Remark 6. Different from the existing publications, the proposed two channel event triggered scheme can save the network resources more significantly. Especially, condition (3) is used to reduce the burden of the network between the sensor and controller, condition (5) is to capture those necessary controller output to be transmitted to the actuator over the network.

The following lemmas are needed in the proof of our main results.

Lemma 1 (*Wang et al.* [26]). *For any vector* $x, y \in R^n$, *and positive definite matrix* $Q \in R^{n\times n}$, *the following inequality holds:*

$$2x^Ty \le x^TQx + y^TQ^{-1}y$$

Lemma 2 (*Tian et al.* [11]). $\Xi_i (i=1,2)$ *and* Ω *are matrices with appropriate dimensions,* $\eta(t)$ *is a function of t and* $0 \le \eta(t) \le \overline{\eta}$, *then*

$$\eta(t)\Xi_1 + (\overline{\eta}-\eta(t))\Xi_2 + \Omega < 0$$

if and only if

$$\overline{\eta}\Xi_1 + \Omega < 0 \overline{\eta}\Xi_2 + \Omega < 0$$

3. Main results

Theorem 1. *For given parameters* $\overline{\eta}$, σ_1, σ_2, *and matrix K, the system* (12) *is asymptotically stable, if there exist matrices* $P>0$, $Q>0$ $R>0$, Φ_1, Φ_2 *and M, N with appropriate dimensions such that for* $l=1,2$

$$\Xi^l = \begin{bmatrix} \Omega_{11}+\Gamma+\Gamma^T & * & * & * \\ \Omega_{21} & -PR^{-1}P & * & * \\ \Omega_{31} & 0 & -\sigma_2 K^T\Phi_2 K & * \\ \Omega_{41}^l & 0 & 0 & -R \end{bmatrix} < 0, \quad (l=1,2) \tag{13}$$

where

$$\Gamma = [N \;\; -N+M \;\; -M \;\; 0 \;\; 0]$$

$$\Omega_{11} = \begin{bmatrix} PA+A^TP+Q & * & * & * & * \\ K^TB^TP & \sigma_1\Phi_1 & * & * & * \\ 0 & 0 & -Q & * & * \\ K^TB^TP & 0 & 0 & -\Phi_1 & * \\ K^TB^TP & 0 & 0 & 0 & -K^T\Phi_2K \end{bmatrix}$$

$$\Omega_{21} = \left[\sqrt{\overline{\eta}}PA \;\; \sqrt{\overline{\eta}}PBK \;\; 0 \;\; \sqrt{\overline{\eta}}PBK \;\; \sqrt{\overline{\eta}}PBK\right]$$

$$\Omega_{31} = [0 \;\; I \;\; 0 \;\; I \;\; 0], \quad \Omega_{41}^1 = \sqrt{\overline{\eta}}N^T, \quad \Omega_{41}^2 = \sqrt{\overline{\eta}}M^T$$

Proof. Choose the following Lyapunov function for system (12)

$$V(t) = x^T(t)Px(t) + \int_{t-\overline{\eta}}^{t} x^T(s)Qx(s)\,ds + \int_{t-\overline{\eta}}^{t}\int_{s}^{t} \dot{x}^T(v)R\dot{x}(v)\,dv\,ds \tag{14}$$

where P, Q, R are symmetric positive definite matrices.

Then, taking the time derivative of $V(t)$ in (14) along the trajectory of system (12) yields

$$\dot{V}(t) = 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\overline{\eta})Qx(t-\overline{\eta}) + \overline{\eta}\dot{x}^T(t)R\dot{x}(t) - \int_{t-\overline{\eta}}^{t} \dot{x}^T(s)R\dot{x}(s)\,ds \tag{15}$$

Using the following free matrices

$$2\xi^T(t)N\left[x(t) - x(t-\eta(t)) - \int_{t-\eta(t)}^{t} \dot{x}(s)\,ds\right] = 0 \tag{16}$$

$$2\xi^T(t)M\left[x(t-\eta(t)) - x(t-s\overline{\eta}) - \int_{t-\overline{\eta}}^{t-\eta(t)} \dot{x}(s)\,ds\right] = 0 \tag{17}$$

where matrices M and N are appropriate dimensions and

$$\xi^T(t) = \left[x^T(t) \;\; x^T(t-\eta(t)) \;\; x^T(t-\overline{\eta}) \;\; e_k^T(t) \;\; g_k^T(t))\right]$$

Notice that, by Lemma 1, there exists real matrix $R>0$, such that

$$-2\xi^T(t)N\int_{t-\eta(t)}^{t} \dot{x}(s)\,ds \le \eta(t)\xi^T(t)NR^{-1}N^T\xi(t) + \int_{t-\eta(t)}^{t} \dot{x}^T(s)R\dot{x}(s)\,ds \tag{18}$$

$$-2\xi^T(t)M\int_{t-\overline{\eta}}^{t-\eta(t)} \dot{x}(s)\,ds \le (\overline{\eta}-\eta(t))\xi^T(t)MR^{-1}M^T\xi(t) + \int_{t-\overline{\eta}}^{t-\eta(t)} \dot{x}^T(s)R\dot{x}(s)\,ds \tag{19}$$

Then, combining (9)–(10) and (14)–(19), we derive that

$$\begin{aligned}\dot{V}(t) \le\; & 2x^T(t)P[Ax(t)+BKg_k(t)+BKe_k(t)+BKx(t-\eta(t))] + x^T(t)Qx(t) \\ & - x^T(t-\overline{\eta})Qx(t-\overline{\eta}) + \overline{\eta}\dot{x}^T(t)R\dot{x}(t) + \eta(t)\xi^T(t)NR^{-1}N^T\xi(t) \\ & + (\overline{\eta}-\eta(t))\xi^T(t)MR^{-1}M^T\xi(t) + 2\xi^T(t)N[x(t)-x(t-\eta(t))] \\ & + 2\xi^T(t)M[x(t-\eta(t))-x(t-\overline{\eta})] + \sigma_1 x(t-\eta(t))^T\Phi_1 x(t-\eta(t)) \\ & - e_k^T(t)\Phi_1 e_k(t) + \sigma_2[x(t-\eta(t))+e_k(t)]^TK^T\Phi_2K[x(t-\eta(t)) \\ & + e_k(t)] - g_k^T(t)K^T\Phi_2Kg_k(t) \\ =\; & \xi^T(t)\left[\Omega_{11}+\Gamma+\Gamma^T + (\overline{\eta}-\eta(t))MR^{-1}M^T + \eta(t)NR^{-1}N^T \right. \\ & \left. + \Omega_{21}^TR\Omega_{21} - \sigma_2\Omega_{31}^TK^T\Phi_2K\Omega_{31}\right]\xi(t)\end{aligned} \tag{20}$$

where Ω_{11}, Ω_{21} and Ω_{31} are defined in Theorem 1.

Using the Lyapunov–Krasovskii functional (14) and Lemma 2, from (13), one can readily derive that the system (12) is asymptotically stable under the zero initial condition.□

Based on Theorem 1, we are in a position to design the controller (8) under the two-channel event triggered scheme.

Theorem 2. *For some given positive constants* $\overline{\eta}$, σ_1, σ_2 *and* ε, *under the two-channel communication scheme* (2) *and* (4), *the system* (12) *is asymptotically stable with controller gain* $K = YX^{-1}$, *if there exist matrices* $X>0$, $\tilde{Q}>0$, $\tilde{R}>0$, $\tilde{\Phi}_1$, $\tilde{\Phi}_2$, $\tilde{N}$, $\tilde{M}$ *and Y with appropriate dimensions such that for* $l=1,2$

$$\tilde{\Xi}^l = \begin{bmatrix} \tilde{\Omega}_{11}+\tilde{\Gamma}+\tilde{\Gamma}^T & * & * & * \\ \tilde{\Omega}_{21} & -2\varepsilon X+\varepsilon^2\tilde{R} & * & * \\ \tilde{\Omega}_{31} & 0 & -\sigma_2\tilde{\Phi}_2 & * \\ \tilde{\Omega}_{41}^l & 0 & 0 & -\tilde{R} \end{bmatrix} < 0, \quad (l=1,2) \tag{21}$$

where

$$\tilde{\Gamma} = \left[\tilde{N} \;\; -\tilde{N}+\tilde{M} \;\; -\tilde{M} \;\; 0 \;\; 0\right]$$

$$\tilde{\Omega}_{11} = \begin{bmatrix} AX+XA^T+\tilde{Q} & * & * & * & * \\ Y^TB^T & \sigma_1\tilde{\Phi}_1 & * & * & * \\ 0 & 0 & -\tilde{Q} & * & * \\ Y^TB^T & 0 & 0 & -\tilde{\Phi}_1 & * \\ Y^TB^T & 0 & 0 & 0 & -\tilde{\Phi}_2 \end{bmatrix}$$

$$\tilde{\Omega}_{21} = \left[\sqrt{\eta_1}AX \;\; \sqrt{\eta_1}BY \;\; 0 \;\; \sqrt{\eta_1}BY \;\; \sqrt{\eta_1}BY\right]$$

$$\tilde{\Omega}_{31} = [0 \;\; I \;\; 0 \;\; I \;\; 0]$$

$$\tilde{\Omega}_{41}^1 = \sqrt{\eta_1}\tilde{N}^T, \quad \tilde{\Omega}_{41}^2 = \sqrt{\eta_1}\tilde{M}^T, \quad \eta_1 = \overline{\eta}$$

Proof. Define $Y = KX$, $X = P^{-1}$, $\tilde{Q} = XQX$, $\tilde{R} = XRX, \tilde{N} = XNX$, $\tilde{M} = XMX$, $\tilde{\Phi}_1 = X\Phi_1 X$, $\tilde{\Phi}_2 = XK^T\Phi_2 KX$

Due to $(R - \varepsilon^{-1}P)R^{-1}(R - \varepsilon^{-1}P) \geq 0$, we have $-PR^{-1}P \leq -2\varepsilon P + \varepsilon^2 R$.

Substituting $-PR^{-1}P$ with $-2\varepsilon P + \varepsilon^2 R$ into (13), we can get

$$\Xi^l = \begin{bmatrix} \Omega_{11} + \Gamma + \Gamma^T & * & * & * \\ \Omega_{21} & -2\varepsilon P + \varepsilon^2 R & * & * \\ \Omega_{31} & 0 & -\sigma_2 K^T \Phi_2 K & * \\ \Omega_{41}^l & 0 & 0 & -R \end{bmatrix} < 0, \quad (l = 1, 2) \tag{22}$$

Then, pre and post-multiplying (13) with $diag\{X, X, \dots, X\}$. Eq. (21) can be obtained.□

Remark 7. Compared with the time-triggered transmission scheme in [27], the proposed two-channel-based event-triggered transmission schemes in this paper rely on discrete supervision of the sampled measurement rather than continuous supervision of measurement output. Therefore, the special hardware for continuous measurement and calculation mentioned in [28] is no longer needed.

Remark 8. From Theorem 2, we can obtain that $\overline{\eta}$, σ_1, σ_2, ε, the corresponding trigger matrix Φ_1, Φ_2 and the controller gain K are coupled together, the control performance and the network resource usage are related to these parameters.

If there is only one event triggered generator constructed between the sensor and the controller, that is $\sigma_1 \neq 0$ in (2) and $\sigma_2 = 0$ in (4), respectively, then the system (12) is recasted into the one below:

$$\dot{x}(t) = Ax(t) + BKe_k(t) + BKx(t - \eta(t)) \tag{23}$$

which has been studied in [13]. Otherwise, if there is only one event triggered communication scheme (4) inserted between the controller and the actuator, that is $\sigma_1 = 0$ in (2) and $\sigma_2 \neq 0$ in (4), respectively, then the system (12) is rewritten as:

$$\dot{x}(t) = Ax(t) + BKg_k(t) + BKx(t - \eta(t)) \tag{24}$$

If we only consider the event triggered scheme (2), similar to the derivation of Theorem 2, we can obtain the system (23) is asymptotically stable by the following Corollary 1.

Corollary 1. *For given constants $\overline{\eta}$, σ_1 and matrix K, the system (23) under the event-triggered communication scheme (2) is asymptotically stable with the controller $K = YX^{-1}$, if there exist matrices $X > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{\Phi}_1$, $\tilde{N}$, $\tilde{M}$ and Y with appropriate dimensions such that for $l = 1, 2$*

$$\hat{\Xi}^l = \left[\hat{\Omega}_{11} + \hat{\Gamma} + \hat{\Gamma}^T ** \hat{\Omega}_{21} - 2\varepsilon X + \varepsilon^2 \tilde{R} * \hat{\Omega}_{31}^l 0 - \tilde{R} \right] < 0, \quad (l = 1, 2) \tag{25}$$

where

$$\hat{\Gamma} = \begin{bmatrix} \tilde{N} & -\tilde{N} + \tilde{M} & -\tilde{M} & 0 \end{bmatrix}$$

$$\hat{\Omega}_{11} = \begin{bmatrix} AX + XA^T + \tilde{Q} & * & * & * \\ Y^T B^T & \sigma_1 \tilde{\Phi}_1 & * & * \\ 0 & 0 & -\tilde{Q} & * \\ Y^T B^T & 0 & 0 & -\tilde{\Phi}_1 \end{bmatrix}$$

$$\hat{\Omega}_{21} = \begin{bmatrix} \sqrt{\eta_1} AX & \sqrt{\eta_1} BY & 0 & \sqrt{\eta_1} BY \end{bmatrix}$$

$$\hat{\Omega}_{31}^1 = \sqrt{\eta_1} N^T, \quad \hat{\Omega}_{31}^2 = \sqrt{\eta_1} M^T, \quad \eta_1 = \overline{\eta}$$

and the other symbols are defined in Theorem 2.

If we consider event triggered scheme (4), we can get the system (24) is asymptotically stable by the following corollary.

Corollary 2. *For given constants $\overline{\eta}$, σ_1 and matrix K, the system (24) under the event-triggered communication scheme (4) is asymptotically stable with the controller $K = YX^{-1}$, if there exist matrices $X > 0$, $\tilde{Q} > 0$, $\tilde{R} > 0$, $\tilde{\Phi}_1$, $\tilde{N}$, $\tilde{M}$ and Y with appropriate dimensions such that for $l = 1, 2$*

$$\overline{\Xi}^l = \left[\overline{\Omega}_{11} + \hat{\Gamma} + \hat{\Gamma}^T ** \hat{\Omega}_{21} - 2\varepsilon X + \varepsilon^2 \tilde{R} * \hat{\Omega}_{31}^l 0 - \tilde{R} \right] < 0, \quad (l = 1, 2) \tag{26}$$

where

$$\overline{\Omega}_{11} = \begin{bmatrix} AX + XA^T + \tilde{Q} & * & * & * \\ Y^T B^T & \sigma_2 \tilde{\Phi}_2 & * & * \\ 0 & 0 & -\tilde{Q} & * \\ Y^T B^T & 0 & 0 & -\tilde{\Phi}_2 \end{bmatrix}$$

and the other symbols are defined in Corollary 2.

Remark 9. Setting $\sigma_i \to 0_+ (i = 1, 2)$ in (2) and (4), our method is simplified as time triggered communication scheme. Letting $\sigma_2 \to 0_+ (i = 1, 2)$ in (4), our method is simplified as event triggered communication scheme in [13].

In the following, we will make a fair comparison by simulation and show the advantage of the new two channel event-triggering communication schemes (2) and (4).

4. Simulation examples

Consider a special system of (12), in which the system parameters are given as follows [7]:

$$A = \begin{bmatrix} -2 & -0.1 \\ -0.1 & 0.01 \end{bmatrix}, \quad B = \begin{bmatrix} 0.05 \\ 0.02 \end{bmatrix} \tag{27}$$

We can easily see that the system is unstable without a controller. The initial state is given as $x_0 = [-0.3 \ 0.3]^T$.

In the following, we will consider three possible cases, which can illustrate the effectiveness of the proposed two channel communication schemes.

Case 1: When the system (12) is under the proposed two channel event triggered schemes (2) and (4), we assume $\overline{\eta} = 0.12$, $\sigma_1 = 0.02$, $\sigma_2 = 0.01$ and $\varepsilon = 1$, based on Matlab/LMIs toolbox and applying Theorem 2, we can get the controller feedback gain

$$K = [-0.7036 \quad -12.8768] \tag{28}$$

the corresponding trigger matrix $\Phi_1 = \left[\begin{smallmatrix} 20.4174 & -0.1792 \\ -0.1792 & 22.0463 \end{smallmatrix}\right]$ and $\Phi_2 = \left[\begin{smallmatrix} 21.4744 & -0.1593 \\ -0.1593 & 24.4777 \end{smallmatrix}\right]$, respectively. With the feedback gain (28), the maximum allowable bound of $\overline{\eta}$ is 1.995 s. Suppose that $\tau_k = 0$, since $\overline{\eta} = h + \overline{\tau}$, it can be known that the maximum allowable sampling period is 1.995 s. The state trajectories of (12) and communication instants and communication intervals are shown in Figs. 3–5, respectively.

Case 2: When corresponding triggered parameter $\sigma_2 = 0$, that is, the system (12) reduces to one channel event triggered system (23). For given $\overline{\eta} = 0.12$, $\sigma_1 = 0.02$, and $\varepsilon = 1$, we can obtain

$$K = [0.5370 \quad -10.7954] \tag{29}$$

the corresponding trigger matrix $\Phi_1 = \left[\begin{smallmatrix} 1.1618 & -0.003 \\ -0.0031 & 1.2168 \end{smallmatrix}\right]$, respectively. the state trajectories of (23) and communication instants and communication intervals are shown in Figs. 6 and 7, respectively.

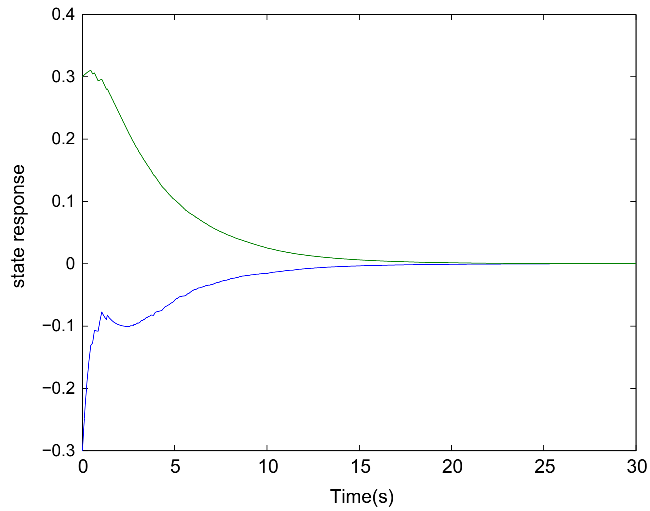

Fig. 3. The state responses under feedback gain (28) for (12).

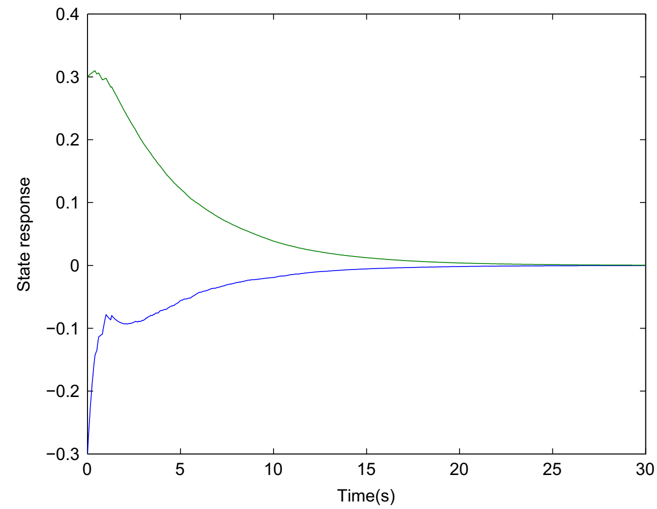

Fig. 6. The state responses under feedback gain (29) for (23).

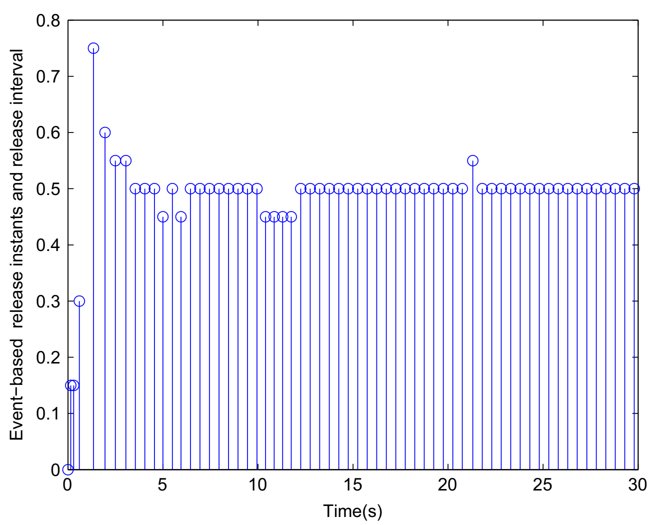

Fig. 4. The release instants and release interval with feedback gain (28) in (12) from sensor to controller channel.

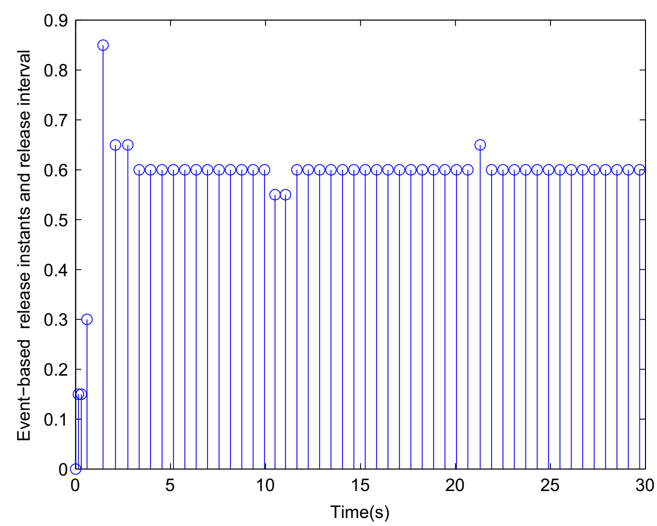

Fig. 7. The release instants and release interval with feedback gain (29) in (23) under event triggered mechanism (2).

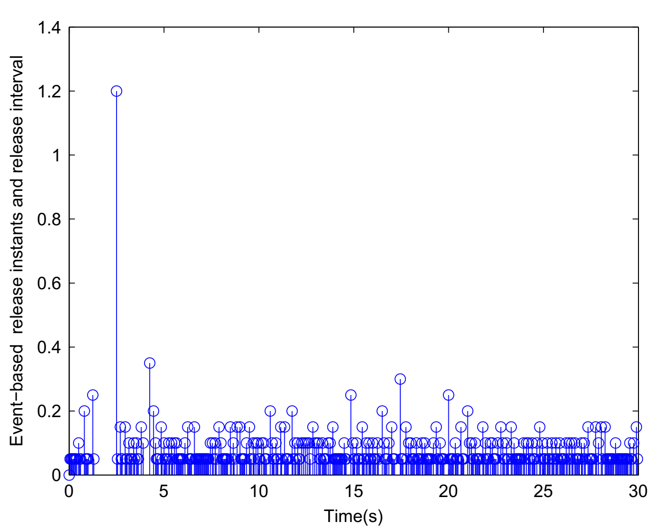

Fig. 5. The release instants and release interval with feedback gain (28) in (23) from controller to actuator channel.

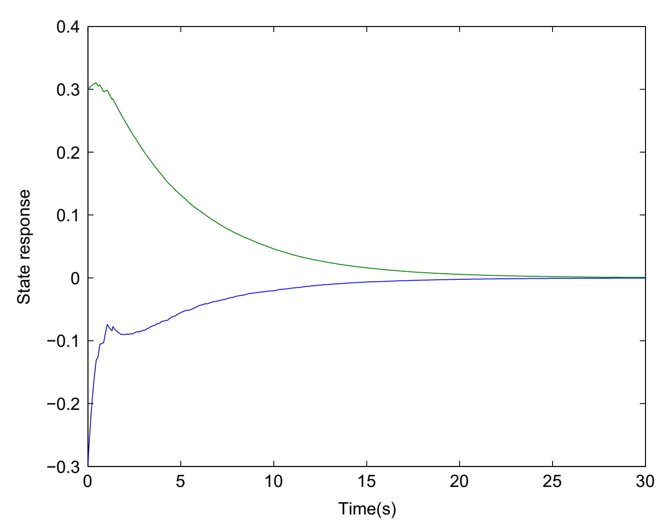

Fig. 8. The state responses under feedback gain (30) for (24).

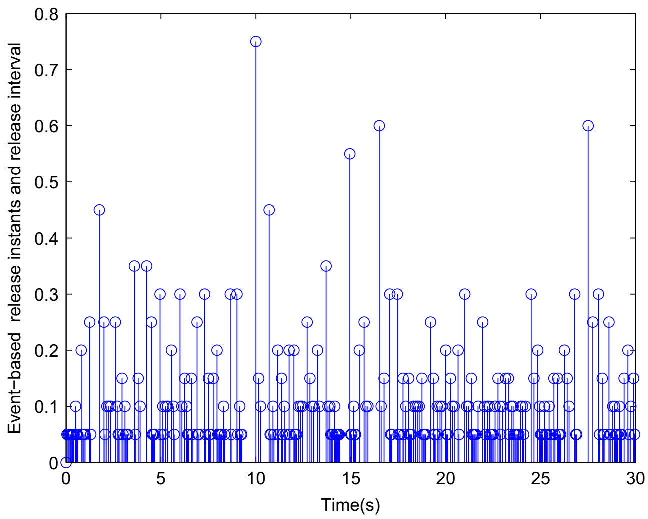

Fig. 9. The release instants and release interval with feedback gain (30) in (24) under event triggered mechanism (4).

Case 3: When the triggered parameter $\sigma_2 = 0$ in (4), that is, the system (12) reduces to one channel event triggered system (24). Letting $\overline{\eta} = 0.12$, $\sigma_2 = 0.01$, and $\varepsilon = 1$, we can obtain

$$K = [0.5648 \quad -10.7697] \tag{30}$$

the corresponding trigger matrix $\Phi_2 = \begin{bmatrix} 1.1706 & -0.0045 \\ -0.0045 & 1.2342 \end{bmatrix}$, respectively. The simulation result for the responses of $x(t)$ are shown in Fig. 8. The event-triggering release instants and intervals are shown in Fig. 9.

Compared with the results above, from Figs. 3, 6 and 8, we can see that the studied system under the proposed two channel event triggering mechanism can be stabilized more quickly while preserving the desired control performance. Moreover, the number of the sampled-data transmission is reduced by using the two-channel event-triggered transmission strategy and the better performance can be obtained based on our method, which can be seen from Figs. 4, 5, 7 and 9. Based on the above figures, it can be seen that the introduction of two channel event triggering mechanism can result in larger maximum allowable sampling period $h = 1.2$ s between the controller and actuator, at the cost of higher transmission frequency between the sensor and the controller. In other words, the two channel event triggering mechanism can reduce the amount of transmission in some sense. From the above analysis, the superiority of the proposed two channel event triggering scheme is that the settling time is shorter compared with the system under one event triggered scheme.

5. Conclusion

In this paper, in order to reduce the computation load, we proposed two-channel event triggered transmission strategies for both the sensor-to-controller and the controller-to-actuator channels. Under the event triggered transmission strategies, a new event-triggered controller design method is obtained. By using Lyapunov functional, criteria for the asymptotical stabilization of the NCSs and criteria for co-designing both the feedback and the trigger parameters are derived in the form of linear matrix inequalities. Simulation results also show that the discussed system under our event-triggering schemes can be stabilized more quickly that under one event-triggering scheme between sensor and controller (or between controller and actuator). An interesting problem for our future work is to find an adaptive co-design method of controller for large scale systems with simultaneous consideration of the sensors and/or the actuators competing for communication channels randomly.

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References

[1] Liu Kun, Fridman Emilia, Henrik Johansson Karl, Dynamic quantization of uncertain linear networked control systems, Automatica 59 (2015) 248–255.
[2] G. Guo, L. Ding, Q.L. Han, A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems, Automatica 50 (5) (2014) 1489–1496.
[3] Y. Tang, H. Gao, J. Lu, et al., Pinning distributed synchronization of stochastic dynamical networks: a mixed optimization approach, IEEE Trans. Neural Netw. Learn. Syst. 25 (10) (2014) 1804–1815.
[4] Y. Tang, H. Gao, Z. Wei, et al., Distributed synchronization in networks of agent systems with nonlinearities and random switchings, IEEE Trans. Cybern. 43 (1) (2013) 358–370.
[5] H. Yu, P.J. Antsaklis, Event-triggered output feedback control for networked control systems using passivity: achieving L_2 stability in the presence of communication delays and signal quantization, Automatica 49 (1) (2013) 30–38.
[6] E. Tian, D. Yue, C. Peng, Reliable control for networked control systems with probabilistic sensors and actuators faults, IET Control Theory Appl. 4 (8) (2010) 1478–1488.
[7] J. Liu, D. Yue, Event-triggering in networked systems with probabilistic sensor and actuator faults, Inf. Sci. 240 (2013) 145–160.
[8] Y. Liu, X. He, Z. Wang, et al., Optimal filtering for networked systems with stochastic sensor gain degradation, Automatica 50 (5) (2014) 1521–1525.
[9] Ch. Tan, L. Li, H. Zhang, Stabilization of networked control systems with both network-induced delay and packet dropout, Automatica 59 (2015) 194–199.
[10] S. Hu, D. Yue, Event-triggered control design of linear networked systems with quantizations, ISA Trans. 51 (1) (2011) 153–162.
[11] E. Tian, D. Yue, Y. Zhang, Delay-dependent robust H_∞ control for ts fuzzy system with interval time-varying delay, Fuzzy Sets Syst. 160 (12) (2009) 1708–1719.
[12] J. Liu, S. Fei, E. Tian, et al., Co-design of event generator and filtering for a class of *T–S* fuzzy systems with stochastic sensor faults, Fuzzy Sets Syst. 273 (2015) 124–140.
[13] D. Yue, E. Tian, Q.L. Han, A delay system method for designing event-triggered controllers of networked control systems, IEEE Trans. Autom. Control 58 (2) (2013) 475–481.
[14] M. Mazo, A. Anta, P. Tabuada, I.S.S. An, Self-triggered implementation of linear controllers, Automatica 46 (8) (2010) 1310–1314.
[15] S. Hu, D. Yue, Event-based H_∞ filtering for networked system with communication delay, Signal Process. 92 (9) (2012) 2029–2039.
[16] C. Peng, Q.L. Han, D. Yue, To transmit or not to transmit: a discrete event-triggered communication scheme for networked Takagi-Sugeno fuzzy systems, IEEE Trans. Fuzzy Syst. 21 (1) (2013) 164–170.
[17] S. Hu, D. Yue, X. Xie, Z. Du, Event-triggered H_∞ stabilization for networked stochastic systems with multiplicative noise and network-induced delays, Inf. Sci. 299 (2015) 178–197.
[18] W. Yu, W.X. Zheng, G. Chen, et al., Second-order consensus in multi-agent dynamical systems with sampled position data, Automatica 47 (7) (2011) 1496–1503.
[19] Solmaz S. Kia, Jorge Corts, Sonia Martłnez, Distributed event-triggered communication for dynamic average consensus in networked systems, Automatica 59 (2015) 112–119.
[20] D. Zhang, Q.L. Han, X. Jia, Network-based output tracking control for *T–S* fuzzy systems using an event-triggered communication scheme, Fuzzy Sets Syst. 273 (2015) 26–48.
[21] S. Hu, D. Yue, X. Xie, Zh. Du, Event-triggered image stabilization for networked stochastic systems with multiplicative noise and network-induced delays, Inf. Sci. 299 (2015) 178 19.

[22] C. Peng, et al., Decentralized event-triggering communication scheme for large-scale systems under network environments, Inf. Sci. (2015) 1–13.
[23] D. Du, et al., Multiple event-triggered H_2/H_∞ filtering for hybrid wired-wireless networked systems with random network-induced delays, Inf. Sci. 325 (2015) 393–408.
[24] Y. Zhao, H. Gao, Fuzzy-model-based control of an overhead crane with input delay and actuator saturation, IEEE Trans. Fuzzy Syst. 20 (1) (2012) 181–186.
[25] W. Heemels, M.C.F. Donkers, Model-based periodic event-triggered control for linear systems, Automatica 49 (3) (2013) 698–711.
[26] Y. Wang, L. Xie, C.E. de Souza, Robust control of a class of uncertain nonlinear systems, Syst. Control Lett. 19 (2) (1992) 139–149.
[27] X. Zhang, G. Lu, Y. Zheng, Stabilization of networked stochastic time-delay fuzzy systems with data dropout, IEEE Trans. Fuzzy Syst. 16 (3) (2008) 798–807.
[28] X. Wang, M. Lemmon, Event-triggering in distributed networked control systems, IEEE Trans. Autom. Control 56 (3) (2011) 586–601.

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