H_∞ FILTERING FOR TIME-DELAY SYSTEMS WITH MARKOVIAN JUMPING PARAMETERS: DELAY PARTITIONING APPROACH

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ABSTRACT

This paper proposes an H_∞ filter design for Markovian jump systems with time delay. First, exploiting the delay partitioning-based Lyapunov function, new criteria are derived for the H_∞ performance analysis of the filtering-error systems, which can lead to much less conservative analysis results. Second, based on the obtained conditions, the filter gain can be obtained in terms of linear matrix inequalities (LMIs). Finally, numerical examples are given to demonstrate the effectiveness and the merit of the proposed method.

Key Words: Time delay, H_∞ filter, Markovian jump systems.

I. INTRODUCTION

The filtering problem has long been one of the fundamental problems in signal processing, communication and control applications. The problem of filtering can be briefly described as the design of an estimator from the measured output to estimate the state of the given systems. During the last few decades, the H_∞ filtering technique introduced in (Elsayed *et al*., 1989) has received increasing attention, for example (Gao *et al*., 2006; Peng *et al*., 2008; Zhang *et al*., 2005; Zhang *et al*., 2008) and the references therein. One of its main advantages is that it is insensitive to the exact knowledge of the statistics of the noise signals.

During the past few decades, Markovian Jump Systems (MJSs) have attracted much attention (Yue *et al*., 2005; Zhang *et al*., 2009; Shao *et al*., 2008; Xu *et al*., 2003). These can be regarded as a special class of hybrid systems with finite operation modes whose structures are subject to random abrupt changes. The system parameters usually jump among finite modes, and the mode switching is governed by a Markov process. MJSs have many applications, such as failure prone manufacturing systems, power systems and economics, etc. A great number of results on estimation and control problems related to such systems have been reported in the literature (Zhang *et al*., 2009; Wang *et al*., 2006; Wang *et al*., 2004; Xiong *et al*., 2009).

Recently, the problem of H_∞ filtering of linear / nonlinear time-delay systems has also received much attention due to the fact that for many practical filtering applications, time-delays cannot be neglected in the procedure of filter design and their existence usually results in poor performance (Wang *et al*., 2003; Nguang *et al*., 2007; Wang *et al*., 2004). Some useful results on H_∞ filtering for time-delay systems have been reported in the literature and there are two kinds of results, namely delay-independent filtering (Souza *et al*., 2001) and delay-dependent (Peng *et al*., 2009; Wang *et al*., 2006; Basin *et al*., 2007; Yue *et al*., 2006). The delay-dependent results are usually less conservative, especially when the time-delay is small. The main objective of the delay-dependent H_∞ filtering is to obtain a filter such that the filtering error system allows a maximum delay bound for a fixed H_∞ performance or achieves a minimum H_∞ performance for a given delay bound.

This paper has addressed the problem of H_∞ filter design for MJSs with time delay. To obtain less conservative results, a new Lyapunov function is

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constructed, which includes delay partitioning. Based on this, an LMI-based sufficient condition for the existence of the desired H_∞ filter has been derived, which can lead to much less conservative analysis results. Compared with the existing method (Hu *et al.*, 2007; Xu *et al.*, 2005; Lin *et al.*, 2006; Li *et al.*, 2007), the conservativeness of the derived H_∞ performance analysis results is further reduced, and novel H_∞ filter design criteria are obtained. Examples used in (Hu *et al.*, 2007; Xi *et al.*, 1997) are employed to show the effectiveness and reduced conservativeness of the proposed methods.

II. SYSTEMS DESCRIPTION AND PRELIMINARIES

Fix a probability space (Ω, F, P) and consider the following class of uncertain linear stochastic systems with markovian jump parameters and time delays (Σ)

$$\begin{cases} \dot{x}(t) = A(\theta_t)x(t) + A_d(\theta_t)x(t-\tau) + A_\omega(\theta_t)\omega(t) \\ y(t) = C(\theta_t)x(t) + C_d(\theta_t)x(t-\tau) + C_\omega(\theta_t)\omega(t) \\ z(t) = L(\theta_t)x(t) + L_d(\theta_t)x(t-\tau) + L_\omega(\theta_t)\omega(t) \\ x(t) = \phi(t),\ \forall t \in [-\tau, 0] \end{cases}, \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^r$ is the measurement vector, $\omega(t) \in L_2[0, \infty)$ is the exogenous disturbance signal, $z(t) \in \mathbb{R}^p$ is the signal to be estimated, τ is the constant delay time of the state in the system, $\{\theta_t\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $S = \{1, 2 \cdots, N\}$ with stationary transition probabilities:

$$Prob\{\theta_{t+h} = j \,|\, \theta_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ii}h + o(h), & i = j \end{cases} \tag{2}$$

where $h > 0$, $\lim_{h\to 0}\frac{o(h)}{h} = 0$, and $\pi_{ij} \geq 0$, for $j \neq i$ is the transition rate from mode i at time t to mode j at time $t + h$ and

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}. \tag{3}$$

In this paper, we consider the following filter for system Eq. (1)

$$\begin{cases} \dot{\hat{x}}(t) = A(\theta_t)\hat{x}(t) + A_d(\theta_t)\hat{x}(t-\tau) + G(\theta_t)(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C(\theta_t)\hat{x}(t) + C_d(\theta_t)\hat{x}(t-\tau) \\ \hat{z}(t) = L(\theta_t)\hat{x}(t) + L_d(\theta_t)\hat{x}(t-\tau) \end{cases} \tag{4}$$

The set S comprises the various operation modes of system Eq. (1) and for each possible value of $\theta_t = i$, $i \in S$, the matrices associated with "$i - th$ mode" will be denoted by

$$A_i := A(\theta_t = i),\ A_{di} := A_d(\theta_t = i),\ A_{\omega i} := A_\omega(\theta_t = i),$$

$$C_i := C(\theta_t = i),\ C_{di} := C_d(\theta_t = i),\ C_{\omega i} := C_\omega(\theta_t = i),$$

$$L_i := L(\theta_t = i),\ L_{di} := L_d(\theta_t = i),\ L_{\omega i} := L_\omega(\theta_t = i),$$

where A_i, A_{di}, $A_{\omega i}$, C_i, C_{di}, $C_{\omega i}$, L_i, L_{di}, $L_{\omega i}$ are constant matrices for any $i \in S$. It is assumed that the jumping process $\{\theta_t\}$ is accessible, i.e. the operation mode of system (Σ) is known for every $t \geq 0$.

Let $e(t) = \hat{x}(t) - x(t)$ and $\tilde{z}(t) = \hat{z}(t) - z(t)$. Then we have the following filtering error system:

$$\begin{cases} \dot{e}(t) = \bar{A}_i e(t) + \bar{A}_{di}e(t-\tau) + \bar{A}_{\omega i}\omega(t) \\ \tilde{z}(t) = L_i e(t) + L_{di}e(t-\tau) - L_{\omega i}\omega(t) \end{cases}, \tag{5}$$

where

$$\bar{A}_i = A_i + G_iC_i,\ \bar{A}_{di} = A_{di} + G_iC_{di},$$

$$\bar{A}_{\omega i} = -A_{\omega i} - G_iC_{\omega i}.$$

The following lemma and definitions are needed in the proof of our main results.

Lemma 1. (Gu *et al.*, 2003) For any constant matrix $Q \in \mathbb{R}^{n\times n}$, $Q > 0$, scalar τ, and vector function $\dot{x}$: $[-\tau, 0] \to \mathbb{R}^n$ such that the following integration is well defined, then it holds that

$$-\tau\int_{t-\tau}^{t} \dot{x}^T(t)Q\dot{x}(t)dt \leq \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix}^T \begin{bmatrix} -Q & * \\ Q & -Q \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau) \end{bmatrix} \tag{6}$$

Definition 1. The system Eq. (5) is said to be exponentially stable in the mean-square sense (EMSS), if there exist constants $\alpha > 0$, $\lambda > 0$, such that $t > 0$

$$E\{\| e(t) \|^2\} \leq \alpha e^{-\lambda t} \sup_{-\tau < s < 0}\{\| \phi(s) \|^2\} \tag{7}$$

Definition 2. For a given function V: $C_{F_0}^b([-\tau, 0], R^n) \times S \to R$, its infinitesimal operator L (Mao *et al.*, 2002) is defined as

$$\mathcal{L}V(x_t) = \lim_{\Delta\to 0^+}\frac{1}{\Delta}[E(V(x_{t+\Delta}|x_t) - V(x_t))] \tag{8}$$

The H_∞ filtering problem addressed in this paper is to design a filter of form Eq. (5) such that

- The filtering error system Eq. (5) with $\omega(t) = 0$ is exponentially stable

- The $\boldsymbol{H}_\infty$ performance $\|\tilde{\boldsymbol{z}}(\boldsymbol{t})\|_2 < \gamma \|\omega(\boldsymbol{t})\|_2$ is guaranteed for all nonzero $\omega(\boldsymbol{t}) \in \boldsymbol{L}_2[0, \infty)$ and a prescribed $\gamma > 0$ under the condition $\boldsymbol{e}(\boldsymbol{t}) = 0, \forall \boldsymbol{t} \in [-\tau, 0]$.

III. STOCHASTIC STABILITY ANALYSIS

Theorem 1. For some given constants τ, $\boldsymbol{d}$ and γ, the system Eq. (5) is exponentially mean-square stable (EMSS) with a prescribed $\boldsymbol{H}_\infty$ performance γ, if there exist $\boldsymbol{P}_i > 0$, $\boldsymbol{Q}_1 > 0$, $\boldsymbol{Q}_2 > 0$, $\boldsymbol{R}_i > 0$ and $\boldsymbol{R} > 0$ ($\boldsymbol{i} \in S$) with appropriate dimensions such that the following matrix inequalities hold.

$$\boldsymbol{\Psi} = \begin{bmatrix} \Psi_{11} & * & * \\ \Psi_{21} & \Psi_{22} & * \\ \Psi_{31} & \Psi_{32} & -\boldsymbol{Q} \end{bmatrix} < 0 , \tag{9}$$

$$\sum_{j=1}^{N} \pi_{ij} \boldsymbol{R}_j \le \boldsymbol{R} , \tag{10}$$

where

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{11} & \boldsymbol{R}_{12} & \cdots & \boldsymbol{R}_{1n} \\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} & \cdots & \boldsymbol{R}_{2n} \\ \cdots & \cdots & \ddots & \cdots \\ \boldsymbol{R}_{n1} & \boldsymbol{R}_{n2} & \cdots & \boldsymbol{R}_{nn} \end{bmatrix},$$

$$\boldsymbol{R}_i = \begin{bmatrix} \boldsymbol{R}_{i11} & \boldsymbol{R}_{i12} & \cdots & \boldsymbol{R}_{i1n} \\ \boldsymbol{R}_{i21} & \boldsymbol{R}_{i22} & \cdots & \boldsymbol{R}_{i2n} \\ \cdots & \cdots & \ddots & \cdots \\ \boldsymbol{R}_{in1} & \boldsymbol{R}_{in2} & \cdots & \boldsymbol{R}_{inn} \end{bmatrix},$$

$$\boldsymbol{\Psi}_{11} = \begin{bmatrix} \Lambda_1 & * & * & \cdots & * & * \\ \Lambda_2 & \Lambda_3 & * & \cdots & * & * \\ \boldsymbol{R}_{i31} + \frac{\tau}{\boldsymbol{d}}\boldsymbol{R}_{31} & \Lambda_4 & \Lambda_5 & \cdots & * & * \\ \vdots & \vdots & \vdots & \ddots & * & * \\ \boldsymbol{R}_{id1} + \frac{\tau}{\boldsymbol{d}}\boldsymbol{R}_{d1} & \Lambda_6 & \Lambda_7 & \cdots & \Lambda_8 & * \\ \overline{\boldsymbol{A}}_{di}^T\boldsymbol{P}_i + \boldsymbol{Q}_2 & -\boldsymbol{R}_{id1} & -\boldsymbol{R}_{id2} & \cdots & -\boldsymbol{R}_{id(d-1)} & -\boldsymbol{R}_{idd} - \boldsymbol{Q}_2 \end{bmatrix},$$

$$\boldsymbol{\Psi}_{21} = \begin{bmatrix} \overline{\boldsymbol{A}}_{wi}^T\boldsymbol{P}_i & 0 & 0 & \cdots & 0 & 0 \\ \boldsymbol{L}_i & 0 & 0 & \cdots & 0 & \boldsymbol{L}_{di} \end{bmatrix},$$

$$\boldsymbol{\Psi}_{22} = \begin{bmatrix} -\gamma^2\boldsymbol{I} & * \\ -\boldsymbol{L}_{wi} & \boldsymbol{I} \end{bmatrix},$$

$$\boldsymbol{\Psi}_{31} = [\boldsymbol{Q}\overline{\boldsymbol{A}}_i \quad 0 \quad 0 \quad \cdots \quad 0 \quad \boldsymbol{Q}\overline{\boldsymbol{A}}_{di}],$$

$$\boldsymbol{\Psi}_{32} = [\boldsymbol{Q}\overline{\boldsymbol{A}}_{wi} \quad 0],$$

$$\Lambda_1 = \boldsymbol{P}_i\overline{\boldsymbol{A}}_i + \overline{\boldsymbol{A}}_i^T\boldsymbol{P}_i + \sum_{j=1}^{N} \pi_{ij}\boldsymbol{P}_j + \boldsymbol{R}_{i11} - \boldsymbol{d} * \boldsymbol{Q}_1 - \boldsymbol{Q}_2 + \frac{\tau}{d}\boldsymbol{R}_{11} ,$$

$$\Lambda_2 = \boldsymbol{R}_{i21} + \boldsymbol{d} * \boldsymbol{Q}_1 + \frac{\tau}{d}\boldsymbol{R}_{21},$$

$$\Lambda_3 = \boldsymbol{R}_{i22} - \boldsymbol{R}_{i11} - \boldsymbol{d} * \boldsymbol{Q}_1 + \frac{\tau}{d}\boldsymbol{R}_{22},$$

$$\Lambda_4 = \boldsymbol{R}_{i32} - \boldsymbol{R}_{i21} + \frac{\tau}{d}\boldsymbol{R}_{32},$$

$$\Lambda_5 = \boldsymbol{R}_{i33} - \boldsymbol{R}_{i22} + \frac{\tau}{d}\boldsymbol{R}_{33},$$

$$\Lambda_6 = \boldsymbol{R}_{id2} + \frac{\tau}{d}\boldsymbol{R}_{d2} - \boldsymbol{R}_{i(d-1)1},$$

$$\Lambda_7 = \boldsymbol{R}_{id3} + \frac{\tau}{d}\boldsymbol{R}_{d3} - \boldsymbol{R}_{i(d-1)2} ,$$

$$\Lambda_8 = \boldsymbol{R}_{idd} - \boldsymbol{R}_{i(d-1)(d-1)} + \frac{\tau}{d}\boldsymbol{R}_{dd},$$

$$\boldsymbol{Q} = \frac{\tau^2}{d}\boldsymbol{Q}_1 + \tau^2\boldsymbol{Q}_2.$$

Proof. Introduce a new vector

$$\boldsymbol{\zeta}^T(\boldsymbol{t}) = [\boldsymbol{e}^T(\boldsymbol{t}) \;\; \boldsymbol{e}^T(\boldsymbol{t} - \frac{\tau}{\boldsymbol{d}}) \;\; \boldsymbol{e}^T(\boldsymbol{t} - \frac{2\tau}{\boldsymbol{d}}) \;\; \cdots \;\; \boldsymbol{e}^T(\boldsymbol{t} - \frac{(d-1)\tau}{\boldsymbol{d}})] .$$

Let $\boldsymbol{x}_t(\boldsymbol{s}) = \boldsymbol{x}(\boldsymbol{t} + \boldsymbol{s})$, $-\tau \le \boldsymbol{s} \le 0$. Then, similar to (Boukas, *et al.*, 2001), $\{(\boldsymbol{x}_t, \theta_t), \boldsymbol{t} \ge 0\}$ is a Markov process. Construct a Lyapunov functional candidate as

$$\boldsymbol{V}(x_t , \theta_t) = \sum_{i=1}^{4} \boldsymbol{V}_i(\boldsymbol{x}_t , \theta_t) , \tag{11}$$

where

$$\boldsymbol{V}_1(\boldsymbol{x}_t, \theta_t) = \boldsymbol{e}^T(\boldsymbol{t})\boldsymbol{P}(\theta_t)\boldsymbol{e}(\boldsymbol{t}) ,$$

$$\boldsymbol{V}_2(\boldsymbol{x}_t , \theta_t) = \int_{t-\frac{\tau}{d}}^{t} \zeta^T(\boldsymbol{s})\boldsymbol{R}(\theta_t)\zeta(\boldsymbol{s})\boldsymbol{ds} ,$$

$$V_3(x_t, \theta_t) = \tau\int_{-\frac{\tau}{d}}^{0}\int_{t+s}^{t} \dot{e}^T(v)Q_1\dot{e}(v)dvds + \tau\int_{-\tau}^{0}\int_{t+s}^{t} \dot{e}^T(v)Q_2\dot{e}(v)dvds\,,$$

$$V_4(x_t, \theta_t) = \int_{-\frac{\tau}{d}}^{0}\int_{t+s}^{t} \dot{\zeta}^T(v)R\dot{\zeta}(v)dvds\,.$$

Let L be the weak infinite generator of the random process $\{x_t, \theta_t\}$. Then, for each $\theta_t = i$, $i \in S$, we have

$$\begin{aligned}
&\mathcal{L}[V(x_t, \theta_t)] \\
&\le e^T(t)(2P_i\overline{A}_i + \sum_{j=1}^{N}\pi_{ij}P_j)e(t) \\
&\quad + 2e^T(t)P_i\overline{A}_{di}e(t-\tau) + 2e^T(t)P_i\overline{A}_{\omega i}\omega(t) \\
&\quad + \zeta^T(t)R_i\zeta(t) - \zeta^T(t-\tfrac{\tau}{d})R_i\zeta(t-\tfrac{\tau}{d}) \\
&\quad + \int_{t-\frac{\tau}{d}}^{t}\zeta^T(s)(\sum_{j=1}^{N}\pi_{ij}R_j)\zeta(s)ds \\
&\quad + \dot{e}^T(t)(\frac{\tau^2}{d}Q_1 + \tau^2Q_2)\dot{e}(t) \\
&\quad - \tau\int_{t-\frac{\tau}{d}}^{t}\dot{e}^T(s)Q_1\dot{e}(s)ds - \tau\int_{t-\tau}^{t}\dot{e}^T(s)Q_2\dot{e}(s)ds \\
&\quad + \frac{\tau}{d}\dot{\zeta}^T(t)R\dot{\zeta}(t) - \int_{t-\frac{\tau}{d}}^{t}\zeta^T(s)R\zeta(s)ds\,.
\end{aligned} \tag{12}$$

Applying Lemma 1, we have

$$-\tau\int_{t-\frac{\tau}{d}}^{t}\dot{e}^T(s)Q_1\dot{e}(s)ds \le d\begin{bmatrix} e(t) \\ e(t-\frac{\tau}{d})\end{bmatrix}^T\begin{bmatrix} -Q_1 & * \\ Q_1 & -Q_1\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\frac{\tau}{d})\end{bmatrix} \tag{13}$$

$$-\tau\int_{t-\tau}^{t}\dot{e}^T(s)Q_2\dot{e}(s)ds \le \begin{bmatrix} e(t) \\ e(t-\tau)\end{bmatrix}^T\begin{bmatrix} -Q_2 & * \\ Q_2 & -Q_2\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau)\end{bmatrix} \tag{14}$$

Combining Eq. (12), Eq. (13) and Eq. (14), it is easy to see that

$$\begin{aligned}
&\mathcal{L}[V(x_t, \theta_t)] - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)z(t) \\
&\le e^T(t)(2P_i\overline{A}_i + \sum_{j=1}^{N}\pi_{ij}P_j)e(t) + 2e^T(t)P_i\overline{A}_{di}e(t-\tau) \\
&\quad + 2e^TP_i\overline{A}_{\omega i}\omega(t) - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \\
&\quad + \zeta^T(t)R_i\zeta(t) - \zeta^T(t-\tfrac{\tau}{d})R_i\zeta(t-\tfrac{\tau}{d}) \\
&\quad + \dot{e}^T(t)(\frac{\tau^2}{d}Q_1 + \tau^2Q_2)\dot{e}(t) + \frac{\tau}{d}\dot{\zeta}^T(t)R\dot{\zeta}(t) \\
&\quad + d\begin{bmatrix} e(t) \\ e(t-\frac{\tau}{d})\end{bmatrix}^T\begin{bmatrix} -Q_1 & * \\ Q_1 & -Q_1\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\frac{\tau}{d})\end{bmatrix} \\
&\quad + \begin{bmatrix} e(t) \\ e(t-\tau)\end{bmatrix}^T\begin{bmatrix} -Q_2 & * \\ Q_2 & -Q_2\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau)\end{bmatrix}.
\end{aligned} \tag{15}$$

From Eq. (15) and using Schur complement, it is easy to see that $L[V(x_t, \theta_t)] - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) < 0$ if Eq. (9) and Eq. (10) hold for any delay smaller than τ. Define a new function as

$$W(x_t, i, t) = e^{\epsilon t}V(x_t, i, t). \tag{16}$$

Its infinitesimal operator L is given by

$$W(x_t, i, t) = \epsilon e^{\epsilon t}V(x_t, i, t) + e^{\epsilon t}LV(x_t, i, t). \tag{17}$$

By the generalized ***Itô*** formula (Gahinet *et al.*, 1995), we can obtain from Eq. (17) that

$$E\{W(x_t, i, t)\} - E\{W(x_0, i)\} = \int_0^t \epsilon e^{\epsilon s}E\{V(x_s, i)\}ds + \int_0^t e^{\epsilon s}E\{LV(x_s, i)\}ds\,. \tag{18}$$

Then, using the method similar to (Yue *et al.*, 2005), we can see that there exists a positive number α such that for $t > 0$

$$E\{V(x_t, i, t)\} \le \alpha \sup_{-\tau\le s\le 0}\{\|\phi(s)\|^2\}e^{-\epsilon t} \tag{19}$$

Since $V(x_t, i, t) \ge \{\lambda_{min}(P_i)\}x^T(t)x(t)$, it can be shown from Eq. (19) that for $t \ge 0$

$$E\{x^T(t)x(t)\} \le \bar{\alpha}^{-\epsilon t}\sup_{-\tau\le s\le 0}\{\|\phi(s)\|^2\}\,, \tag{20}$$

where $\bar{\alpha} = \alpha/(\lambda_{min}P_i)$. Recalling Definition 1, the proof can be completed.

Remark 1. As mentioned above, the interval delay $[0, \tau]$ is segmented into d intervals, and from the example below, we can see that the more intervals are segmented, the less conservative the results.

Remark 2. In the proof of Theorem 1, by segmentalizing the state-delay into several continuous equivalent subintervals in constructing the Lyapunov function, a new analysis method is proposed to address the problem of stability, which leads to much less conservative results than those in the existing (Yue *et al.*, 2005; Zhang *et al.*, 2009; Hu *et al.*, 2007).

IV. H_∞ FILTER DESIGN

In the following, we are seeking to design the H_∞ filtering based on Theorem 1.

Theorem 2. For some given constants τ, d and γ, the augmented system Eq. (5) is exponentially mean-square stable (EMSS) with a prescribed H_∞ performance γ if there exist $P_i > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_i > 0$, $R > 0$ and $\bar{G}_i$ ($i \in S$) with appropriate dimensions such that the following LMIs hold for a given $\varepsilon > 0$

$$\hat{\Psi} = \begin{bmatrix} \hat{\Psi}_{11} & * & * \\ \hat{\Psi}_{21} & \Psi_{22} & * \\ \hat{\Psi}_{31} & \Psi_{32} & \hat{\Psi}_{33} \end{bmatrix} < 0\,, \tag{21}$$

$$\sum_{j=1}^{N} \pi_{ij} R_j \le R\,, \tag{22}$$

where

$$\hat{\Psi}_{11} = \begin{bmatrix} \Gamma_1 & * & * & \cdots & * & * \\ \Lambda_2 & \Lambda_3 & * & \cdots & * & * \\ R_{i31} + \frac{\tau}{d} R_{31} & \Lambda_4 & \Lambda_5 & \cdots & * & * \\ \vdots & \vdots & \vdots & \ddots & * & * \\ R_{id1} + \frac{\tau}{d} R_{d1} & \Lambda_6 & \Lambda_7 & \cdots & \Lambda_8 & * \\ \Gamma_2 & -R_{id1} & -R_{id2} & \cdots & -R_{id(d-1)} & -R_{idd} - Q_2 \end{bmatrix},$$

$$\hat{\Psi}_{21} = \begin{bmatrix} -A_{wi}^T P_i - C_{wi} \bar{G}_i^T & 0 & 0 & \cdots & 0 & 0 \\ L_i & 0 & 0 & \cdots & 0 & L_{di} \end{bmatrix},$$

$$\hat{\Psi}_{31} = [P_i A_i + \bar{G}_i C_i \;\; 0 \;\; 0 \;\; \cdots \;\; 0 \;\; P_i A_{di} + \bar{G}_i C_{di}],$$

$$\hat{\Psi}_{33} = -2\varepsilon P_i + \varepsilon^2 Q,$$

$$\Gamma_1 = P_i A_i + A_i^T P_i + \bar{G}_i C_i + C_i^T \bar{G}_i^T + \sum_{j=1}^{N} \pi_{ij} P_j + R_{i11} - d * Q_1 - Q_2 + \frac{\tau}{d} R_{11}\,,$$

$$\Gamma_2 = A_{di}^T P_i + C_{di}^T \bar{G}_i^T + Q_2$$

and Q, Λ_3, Λ_4, Λ_5, Λ_6, Λ_7, Λ_8, Ψ_{22}, Ψ_{32} are as defined in Theorem 1.

Moreover, the filter gain in the form of Eq. (4) is as follows:

$$G_i = P_i^{-1} \bar{G}_i \tag{23}$$

Proof. Defining $\bar{G}_i = P_i G_i$, from Eq. (5), Eq. (9) and using Schur complement, the matrix inequality Eq. (9) holds if and only if

$$\begin{bmatrix} \hat{\Psi}_{11} & * & * \\ \hat{\Psi}_{21} & \Psi_{22} & * \\ \hat{\Psi}_{31} & \Psi_{31} & -P_i Q^{-1} P_i \end{bmatrix} < 0\,. \tag{24}$$

Due to

$$(\varepsilon Q - P_i) Q^{-1} (\varepsilon Q - P_i) \ge 0 \tag{25}$$

which gives

$$-P_i Q^{-1} P_i \le -2\varepsilon P_i + \varepsilon^2 Q, \quad i = 1, 2. \tag{26}$$

Substituting $-P_i Q^{-1} P_i$ with $-2\varepsilon P_i + \varepsilon^2 Q$ into Eq. (24), we obtain Eq. (21), so if Eq. (21) holds, we have Eq. (9) holds.

From the above proof, we have $G_i = P_i^{-1} \bar{G}_i$. This completes the proof.

Remark 3. The inequality Eq. (26) is used to bound the term $-P_i Q^{-1} P_i$ in Eq. (24). This step can be improved by adopting the cone complementary algorithm (El *et al.*, 1997), which is popular in current control designs. But the cone complementary algorithm carries much computational burden due to its complexity. Here the scaling parameter $\varepsilon > 0$ can be used to improve conservatism in Theorem 2.

Remark 4. From Theorem 2, we can get the upper bound of time delay τ through solving the following maximum problem by using LMI SOLVER FEASP in MATLAB LMI tool box (Gahinet *et al.*, 1995)

$$\max \quad \tau$$

$$\text{subject to} \quad \text{LMIs} \quad Eq(21),\ Eq(22)$$

If the system mode set $S = \{1\}$, the jump system is just a general linear system. From the proof of

Table 1 Max. value of τ

Method	Delay bound τ
(Xu *et al*., 2005; Lin *et al*., 2006)	4.4721
(Li *et al*., 2007)	4.54
Corollary 1 ($\boldsymbol{d}$ = 4)	6.0568

Table 2 Max. value of τ for different d

$\boldsymbol{d}$	$\boldsymbol{d}$ = 2	$\boldsymbol{d}$ = 3	$\boldsymbol{d}$ = 4	⋯
τ	5.7175	5.9677	6.0568	⋯

Theorem 1, we can conclude the following corollary.

Corollary 1. For some given constants τ, $\boldsymbol{d}$, the augmented systems Eq. (5) with $\boldsymbol{i} \in S = \{1\}$ are exponentially mean-square stable (EMSS) if there exist $P > 0$, $\boldsymbol{Q}_1 > 0$, $\boldsymbol{Q}_2 > 0$ and $\boldsymbol{R} > 0$ with appropriate dimensions such that the following LMIs hold for a given $\varepsilon > 0$

$$\begin{bmatrix} \boldsymbol{\Psi} & * \\ \Lambda & -\boldsymbol{Q} \end{bmatrix} < 0\,, \tag{27}$$

where

$$\boldsymbol{\Psi} = \begin{bmatrix} \boldsymbol{\Psi}_{11} + \boldsymbol{\Phi} + \boldsymbol{\Phi}^T & * & * \\ -\boldsymbol{M}^T & -\boldsymbol{dQ}_1 & * \\ -\boldsymbol{N}^T & 0 & -\boldsymbol{Q}_2 \end{bmatrix},$$

$$\boldsymbol{\Psi}_{11} = \begin{bmatrix} \boldsymbol{P}\overline{\boldsymbol{A}}_1 + \overline{\boldsymbol{A}}_1^T P + \boldsymbol{R}_{11} & * & \cdots & * & * \\ \boldsymbol{R}_{21} & \boldsymbol{R}_{22} - \boldsymbol{R}_{11} & \cdots & * & * \\ \vdots & \vdots & \ddots & * & * \\ \boldsymbol{R}_{d1} & \boldsymbol{R}_{d2} - \boldsymbol{R}_{(d-1)1} & \cdots & \boldsymbol{R}_{dd} - \boldsymbol{R}_{(d-1)(d-1)} & * \\ \overline{\boldsymbol{A}}_{d1}^T \boldsymbol{P} & -\boldsymbol{R}_{d1} & \cdots & -\boldsymbol{R}_{d(d-1)} & -\boldsymbol{R}_{dd} \end{bmatrix},$$

$$\boldsymbol{\Phi} = [\boldsymbol{M} + \boldsymbol{N} \quad -\boldsymbol{M} \quad \underbrace{0 \cdots 0}_{d-2} \quad -\boldsymbol{N}],$$

$$\boldsymbol{M} = [\boldsymbol{M}_1 \quad \ldots \quad \boldsymbol{M}_{d+1}],\ \boldsymbol{N} = [\boldsymbol{N}_1 \quad \ldots \quad \boldsymbol{N}_{d+1}],$$

$$\Lambda = [\boldsymbol{Q}\bar{\boldsymbol{A}} \quad \underbrace{0 \cdots 0}_{d-1} \quad \boldsymbol{Q}\bar{\boldsymbol{A}}_{d1} \quad 0 \quad 0],$$

$$\boldsymbol{Q} = \frac{\tau^2}{d}\boldsymbol{Q}_1 + \tau^2\boldsymbol{Q}_2\,.$$

V. EXAMPLES

Example 1. Consider a Markovian jump system in Eq. (5) with one modes and the following parameters (Xi *et al*., 1997):

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}\boldsymbol{x}(\boldsymbol{t}) + \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}\boldsymbol{x}(\boldsymbol{t} - \tau)\,.$$

For several methods (Xu *et al*., 2005; Lin *et al*., 2006; Li *et al*., 2007) and different values of $\boldsymbol{d}$, the computation results of τ are listed in Table 1, Table2. Obviously, for the same conditions for the time delay, using delay partitioning can lead to less conservative results.

To illustrate the proposed method of filtering design, another example is considered as follows.

Example 2. Consider linear Markovian jump systems in the form Eq. (1) with two modes. For mode 1 and 2, the dynamics of the system are described as

$$\boldsymbol{A}_1 = \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix},$$

$$\boldsymbol{A}_{d1} = \begin{bmatrix} -0.2 & 0.1 & 0.6 \\ 0.5 & -1 & -0.8 \\ 0 & 1 & -2.5 \end{bmatrix},$$

$$\boldsymbol{A}_{\omega 1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$

$C_1 = [0.8 \quad 0.3 \quad 0]$, $C_{d1} = [0.2 \quad -0.3 \quad -0.6]$,

$C_{\omega 1} = 0.2$,

$L_1 = [0.5 \quad -0.1 \quad 1]$, $L_{d1} = [0 \quad 0 \quad 0]$, $L_{\omega 1} = 0$,

$$\boldsymbol{A}_2 = \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix},$$

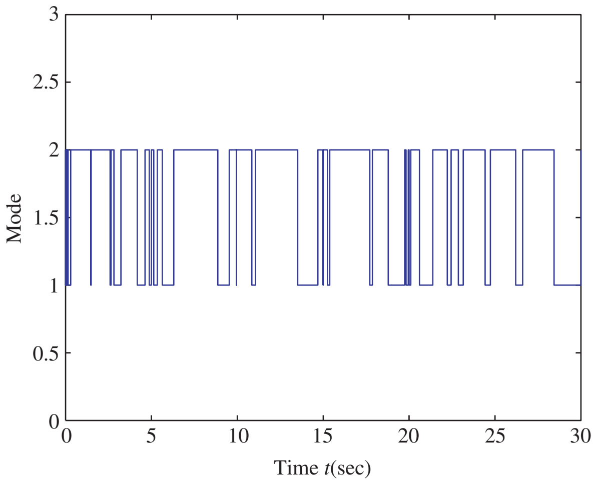

Fig. 1 Operation modes

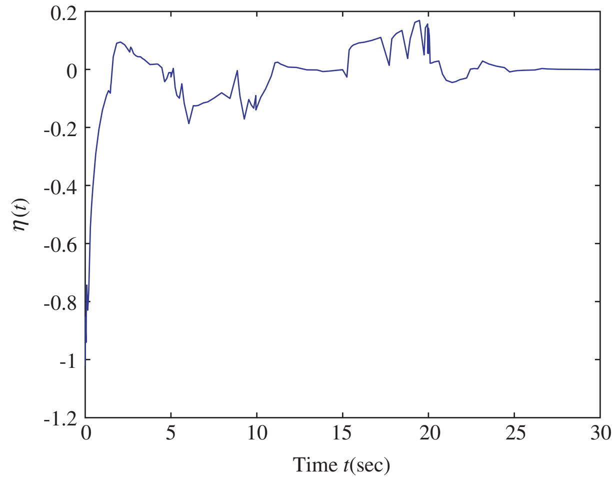

Fig. 2 Estimated signals error $\eta(t) = z(t) - \tilde{z}(t)$

$$A_{d2} = \begin{bmatrix} 0 & -0.3 & 0.6 \\ 0.1 & 0.5 & 0 \\ -0.6 & 1 & -0.8 \end{bmatrix},$$

$$A_{\omega 2} = \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix},$$

$C_2 = [0.5 \quad 0.2 \quad 0.3]$, $C_{d2} = [0 \quad -0.6 \quad 0.2]$,

$C_{\omega 2} = 0.5$

$L_2 = [0 \quad 1 \quad 0.6]$, $L_{d2} = [0 \quad 0 \quad 0]$, $L_{\omega 2} = 0$.

Suppose the transition probability matrix is given by $\pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}$ and the initial conditions $x(0) = [0.8 \quad 0.2 \quad -0.9]^T$, $\hat{x}(0) = [0 \quad 0.2 \quad 0]^T$.

This system is nominally the same one considered in (Hu *et al*., 2007). By Theorem 2, we get the maximum time delay $\tau = 12.2864$ for $\varepsilon = 10$ and $\gamma = 1.2$. This upper bound is much larger than the one $\tau = 1.9195$ given by (Hu *et al*., 2007), which shows our method is less conservative than that of (Hu *et al*., 2007).

The corresponding filter are given by

$$G_1 = \begin{bmatrix} -4.9979 \\ -0.0004 \\ -4.9979 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 1.2000 \\ -0.9998 \\ -0.0000 \end{bmatrix}.$$

To illustrate the performance of the designed filter, choose the disturbance function as follows

$$\omega(t) = \begin{cases} -0.5, & 5 < t < 10 \\ 0.5, & 15 < t < 20 \\ 0, & \text{otherwise} \end{cases}.$$

With this filter, the simulation results are shown in Figs. 1-2.

VI. CONCLUSIONS

In this paper, we have studied a class of H_∞ filter design for Markovian jump systems with time delay via manipulating the delay partitioning-based Lyapunov-Krasovskii Functionals. With the proposed method, an LMI-based sufficient condition for the existence of the desired H_∞ filter has been derived, which can lead to much less conservative analysis results. Finally, Numerical examples have been carried out to demonstrate the effectiveness of the proposed method.

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NOMEMCLATURE

$\boldsymbol{I}$	the identity matrix of appropriate dimension
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{R}^{n \times m}$	the set of $n \times m$ real matrices
$\boldsymbol{T}$	matrix transposition
$\boldsymbol{X} > 0$	the matrix $\boldsymbol{X}$ is real symmetric positive definite
$\boldsymbol{X} \geq 0$	the matrix $\boldsymbol{X}$ is real symmetric positive semi-definite
$\|\|\cdot\|\|$	the Euclidean vector norm
$E\{x\}$	the expectation of stochastic variable x
$\begin{bmatrix} \boldsymbol{A} & * \\ \boldsymbol{B} & \boldsymbol{C} \end{bmatrix}$	a symmetric matrix

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