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Research on the model of rough set over dual-universes

Ruixia Yan [a], Jianguo Zheng [a], Jinliang Liu [b,*], Yuming Zhai [a]

[a] Glorious Sun School of Business Administration, Donghua University, Shanghai 200051, PR China
[b] College of Information Science and Technology, Donghua University, Shanghai 201620, PR China

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ABSTRACT

To tackle the problem of rough set on single-universe, we discuss the rough set model over dual-universes in aspect of building connection between single-universe model and dual-universes model. The rough set model over dual-universes denoted as RSMDU in this paper is built through inspecting the relation between the two universes. Firstly, we propose the RSMDU and study its property using character function and relation matrix. The algorithm for obtaining the lower and upper approximations is then presented. Secondly, we show that Pawlak rough set model can be induced using RSMDU. The theorem inferring the connection between Pawlak model induced by RSMDU and RSMDU is presented. Finally, the applications of RSMDU are studied. According to proposed model, we demonstrate that the existing models of rough set are special cases of RSMDU and that the set of conditional attribute and the set of decision attribute can be regarded as dual-universes in decision-making system, where the model can be utilized to handle decision processing.

1. Introduction

Rough set theory is an extension of set theory [1–4], in which a subset of a universe is described by a pair of ordinary sets called the lower and upper approximation. Rough set theory, a mathematical approach dealing with inexact, uncertain or vague knowledge, has recently received a lot of attention on the research areas in both of the real-life applications and the theory itself. On the other hand, the real-life applications promote the theory research about rough set. It is emerging as a powerful theory dealing with imperfect data and it is an expanding research area stimulating explorations on both real-world applications and the theory itself. It has found practical applications in many areas such as knowledge discovery [5,6], data analysis [7], approximate classification [8], conflict analysis [4].

There are mainly two methodologies for the development of this theory, the constructive and axiomatic approaches [9]. In constructive methods, lower and upper approximations are constructed from the primitive notions, such as equivalence relations on a universe and neighborhood systems. However, equivalence relations could not exist in every area which we are interested in, so extended models about the relation on universes were studied by many scholars. For example, the notions of approximation operators have been generalized by tolerance relation or similarity relation [10–13], dominance relation [14,15], general binary relation on the universe of discourse [12,16–18], partitions and coverings of the universe of discourse [19–24], neighborhood systems and Boolean algebras [25–27] and general approximation spaces [25,26,28]. Scholars have put forward many extended rough set models combining with other soft computing theories or relaxing the relation on the universe or broadening the boundary, such as statistical rough set [29], fuzzy rough set [30–33], probabilistic rough set [34], variable precision rough set [35,36], Bayesian rough set [37] and grey rough set [38].

Grzymala–Busse has demonstrated that rough set theory represents an objective approach to imperfections in data [39]. In rough set theory, all computations are performed directly on the dataset. In other words, there is no additional feedback needed from an external expert. Therefore, there is no need for any additional information about data, such as a probability distribution function as found in statistics, or a grade of membership as we find in fuzzy set theory, and so forth [40]. However, when processing a decision-making problem, rough set theory is widely adopted to deal with problems, which have only one decision attribute. Nevertheless dealing with the issue of decision-making often is the case of multiple attributes decision-making. In literatures, research communities have given countless efforts to deal with multiple attributes [41–44]. As a result, the subjective factors of researchers are added, which more or less affect the best decision-making. Then the objectivity of rough set cannot be made full use of. On the other hand, it is difficult to describe relations in our life using a unified model, as well as information rationally. Rough set models on two universal sets can be interpreted by both generalized approximation spaces and the notions of interval structures [45]. Many scholar have been done much research for these models [46–49].

* Corresponding author. Tel.: +86 25 85481180; fax: +86 25 85481197.
E-mail addresses: ljl9@163.com, liujinliang@vip.163.com (J. Liu).

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In this study, we model the rough set using two correlational universes: U and V, with the given name Rough Set Model over Dual-universes (RSMDU). In this paper, we discuss the property of RSMDU and Pawlak rough set induced by RSMDU. Conclusions are drawn by analyzing the model's practical applications. For example, the existing models of rough set are special cases of RSMDU. Specifically, the conditional attribute and the decision attribute regarded as dual-universes in decision-making system indicates that RSMDU is significant in the real life.

This paper is organized as follows. In Section 2, we build RSMDU and study its properties. In Section 3, Pawlak rough set derived by RSMDU is discussed. Section 4 presents the relation between RSMDU and Pawlak rough set. In Section 5, the conclusions and future works are described.

Notation: A^C denotes complementation of the set A.

Let p and q be propositions. The conjunction of p and q, denoted $p \wedge q$, is the proposition p and q.

The disjunction of p and q, denoted $p \vee q$, is the proposition p or q.

2. The model and properties of rough set over dual-universes

In this paper, we use the following mark unless there are special requirements. Let U and V denote the universe of discourse. Let $R \subseteq U \times V$ denote a given relation depending on the universes U and V, $R' \subseteq V \times U$ be the inverse relation of R. Let $R(x) = \{y \in V | (x,y) \in R\}$ and $R'(y) = \{x \in U | (x,y) \in R\}$ where $x \in U$, $y \in V$.

In order to discuss the relation between U and V distinctly, we use matrix to describe the relation under the help of characteristic function.

Definition 1. Let U and V be the universes of discourse. $R \subseteq U \times V$ and $R' \subseteq V \times U$ be the inverse relation of R. $\forall x \in U$, $\forall y \in V$, the characteristic function of R and the characteristic function of R' are defined respectively as

$$\chi_R(x,y) = \begin{cases} 1 & (x,y) \in R \\ 0 & (x,y) \notin R \end{cases} \quad \chi_{R'}(y,x) = \begin{cases} 1 & (x,y) \in R \\ 0 & (x,y) \notin R \end{cases}$$

Using characteristic function, we define relation matrix of R denoted by $A = [a_{ij}]_{m \times n}$, where

$$a_{ij} = \begin{cases} 1 & \chi_R(x_i,y_j) = 1 \\ 0 & \chi_R(x_i,y_j) = 0 \end{cases}$$

while the relation matrix of R' denoted by A' is the transpose of matrix A.

If A is a matrix with none row filled with zeros, then A is called information matrix.

For the sake of illustration, we consider the following example.

Example 1. Let U and V denote the universe of discourse. Let $U = \{x_1,x_2,\ldots,x_6\}$, $V = \{y_1,y_2,\ldots,y_7\}$ and $R \subseteq U \times V$. $R(x_1) = \{y_1,y_4\}$, $R(x_2) = \{y_1,y_2,y_3,y_4\}$, $R(x_3) = \{y_1,y_4,y_5,y_6,y_7\}$, $R(x_4) = \{y_1,y_2,y_3,y_4\}$, $R(x_5) = \{y_5,y_6,y_7\}$, $R(x_6) = \{y_5,y_6,y_7\}$.

Then the relation matrix A of R is denoted by

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

And $R'(y_1) = \{x_1,x_2,x_3,x_4\}$, $R'(y_2) = \{x_2,x_4\}$, $R'(y_3) = \{x_2,x_4\}$, $R'(y_4) = \{x_1,x_2,x_3,x_4\}$, $R'(y_5) = \{x_3,x_5,x_6\}$, $R'(y_6) = \{x_3,x_5,x_6\}$, $R'(y_7) = \{x_3,x_5,Sx_6\}$.

The relation matrix of R' from V to U is A' which is the transpose of matrix A.

Definition 2. Let U and V be the universe of discourse. $R \subseteq U \times V$ and R' the inverse relation of R. $\forall Y \subseteq V$, the lower approximation and upper approximation of Y over dual-universes under R are defined as

$$\begin{aligned} \underline{R'}Y &= \{x | x \in U, R(x) \subseteq Y \wedge R(x) \neq \emptyset\}, \\ \overline{R'}Y &= \{x | x \in U, R(x) \cap Y \neq \emptyset \vee R(x) = \emptyset\} \end{aligned} \tag{1}$$

If

$$\underline{R'}Y = \overline{R'}Y$$

then Y is a crisp set on V over dual-universes. If

$$\underline{R'}Y \neq \overline{R'}Y$$

then Y is called a rough set over dual-universes on V denoted as RSDU in this paper.

Operators

$$\underline{R'}, \overline{R'} : P(V) \to P(U)$$

are referred to as approximate operators from $P(V)$ to $P(U)$ where $P(V)$ and $P(U)$ are power sets of U and V respectively.

Definition 3. Let U and V be the universe of discourse. $R \subseteq U \times V$ and R' the inverse relation of R. $\forall X \subseteq U$, the lower approximation and upper approximation of X over dual-universes under R are defined as

$$\begin{aligned} \underline{R}X &= \{y | y \in V, R'(y) \subseteq X \wedge R'(y) \neq \emptyset\}, \\ \overline{R}X &= \{y | y \in V, R'(y) \cap X \neq \emptyset \vee R'(y) = \emptyset\} \end{aligned} \tag{2}$$

If

$$\underline{R}X = \overline{R}X$$

then X is a crisp set on U over dual-universes. If

$$\underline{R}X \neq \overline{R}X$$

then X is called a rough set over dual-universes on U denoted as RSDU in this paper.

Operators

$$\underline{R}, \overline{R} : P(U) \to P(V)$$

are referred to as approximate operators from $P(U)$ to $P(V)$ where $P(U)$ and $P(V)$ are power sets of U and V respectively.

Remark 1. Definition 2 and Definition 3 are relative and which is adopted depending on objectives researched. In the following paper, the properties based on Definition 2 are just studied. The properties based on Definition 3 can be got by analogy.

Example 2. Let U and V denote the universe of discourse. Let $U = \{x_1,x_2,x_3,x_4\}$, $V = \{y_1,y_2,y_3\}$. $R \subseteq U \times V$. $R(x_1) = \{y_1,y_2\}$, $R(x_2) = \{y_2,y_3\}$, $R(x_3) = \{y_3\}$, $R(x_4) = \{y_1,y_3\}$. Then the relation matrix of R is

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Suppose $Y = \{y_1,y_3\}$, we have $\underline{R'}Y = \{x_3,x_4\}$, $\overline{R'}Y = \{x_1,x_2,x_3,x_4\}$.

Let U and V denote the universe of discourse. Let $U = \{x_1, x_2, \ldots, x_m\}$, $V = \{y_1, y_2, \ldots, y_n\}$ and $Y \subseteq V$ where $Y = \{y_{j1}, y_{j2}, \ldots, y_{jp}\}$. When the universes U and V have a fixed number elements, the lower and upper approximations of Y over dual-universe can be drawn following these three steps.

(1) Obtain the relation matrix $A = [a_{ij}]_{m\times n}$ depending on the relation R. The algorithm is as follows.
S1 Set $A = []_{m\times n}$;
S2 For $i = 1,\ldots,m$, for $j = 1,\ldots,n$, if $y_j \in R(x_i)$, $a_{ij} = 1$; else $a_{ij} = 0$.
(2) Obtain the lower approximation of Y over dual-universes under the relation R from Definition 2. This can be done by three steps. Firstly, we compute the sum of the elements in the i-th row of matrix A and denote the sum as p_i. Secondly, we compute the sum of the elements $a_{ij_1}, a_{ij_2}, \cdots, a_{ij_p}$ of matrix A and denoted the sum as q_i. Thirdly, we compare p_i and q_i. If $p_i == q_i \neq 0$, then the element x_i should be an element of the lower approximation of Y over dual-universes denoted as $\underline{R'}Y$. The algorithm is as follows.
S1 Set $\underline{R'}Y = \emptyset$;
S2 For $i = 1,\ldots,m$, $p_i = 0$, for $j = 1,\ldots,n$, $p_i = p_i + a_{ij}$;
S3 For $i = 1,\ldots,m$, $q_i = 0$, for $k = 1,\ldots,p$, $q_i = q_i + a_{ij_k}$;
S4 If $p_i == q_i$ and $p_i \neq 0$, then $\underline{R'}Y = \underline{R'}Y \cup \{x_i\}$.
(3) Obtain the upper approximation over dual-universes of Y under R from Definition 2. We can look at the issue at two levels:
- If the i-th row filled with zeros, then x_i should be an element of the upper approximation of Y over dual-universes denoted as $\overline{R'}Y$;
- Considering the elements $a_{ij_1}, a_{ij_2}, \ldots, a_{ij_p}$ of the matrix, if there exists an element $a_{ij_k} (k = 1, 2, \ldots, p)$, whose at least one value equals 1, then the element x_i should be an element of the upper approximation of Y over dual-universes denoted as $\overline{R'}Y$. The algorithm is as follows.

S1 Set $\overline{R'}Y = \emptyset$;
S2 For $i = 1,\ldots,m$, for $k = 1,\ldots,p$, if $a_{ij_k} == 1$, then $\overline{R'}Y = \overline{R'}Y \cup \{x_i\}$.

Theorem 1. *Let U and V be the universe of discourse. $R \subseteq U \times V$ and R' the inverse relation of R. Y, Y_1, $Y_2 \subseteq V$, we have*

(1) $\underline{R'}Y \subseteq \overline{R'}Y$;
(2) $Y_1 \subseteq Y_2 \Rightarrow \underline{R'}Y_1 \subseteq \underline{R'}Y_2, \overline{R'}Y_1 \subseteq \overline{R'}Y_2$;
(3) $\underline{R'}(Y_1 \cap Y_2) = \underline{R'}Y_1 \cap \underline{R'}Y_2$;
(4) $\overline{R'}(Y_1 \cup Y_2) = \overline{R'}Y_1 \cup \overline{R'}Y_2$;
(5) $(\underline{R'}Y)^C = \overline{R'}(Y^C), \underline{R'}(Y^C) = (\overline{R'}Y)^C$.

Proof

(1) The proof is straightforward from Definition 2;
(2) The proof is straightforward from Definition 2;
(3) $x \in \underline{R'}Y_1 \cap \underline{R'}Y_2 \Longleftrightarrow R(x) \in Y_1, R(x) \in Y_2$

$$\Longleftrightarrow R(x) \in Y_1 \cap Y_2$$
$$\Longleftrightarrow x \in \underline{R'}(Y_1 \cap Y_2)$$

Then $\underline{R'}(Y_1 \cap Y_2) = \underline{R'}\, Y_1 \cap \underline{R'}Y_2$.
(4) $x \in \overline{R'}(Y_1 \cup Y_2) \Longleftrightarrow R(x) \cap (Y_1 \cup Y_2) \neq \emptyset$

$$\Longleftrightarrow (R(x) \cap Y_1) \cup (R(x) \cap Y_2) \neq \emptyset$$
$$\Longleftrightarrow R(x) \cap Y_1 \neq \emptyset \vee R(x) \cap Y_2 \neq \emptyset$$
$$\Longleftrightarrow x \in \overline{R'}Y_1 \vee x \in \overline{R'}Y_2$$
$$\Longleftrightarrow x \in \overline{R'}Y_1 \cup \overline{R'}Y_2$$

Then $\overline{R'}(Y_1 \cup Y_2) = \overline{R'}Y_1 \cup \overline{R'}Y_2$.

(5) $y \in \underline{R'}Y \Longleftrightarrow R(y) \subseteq Y$

$$\Longleftrightarrow R(y) \cap (Y^C) = \emptyset$$
$$\Longleftrightarrow y \notin \overline{R'}(Y^C)$$
$$\Longleftrightarrow y \in (\overline{R'}(Y^C)^C.$$

That is $(\underline{R'}Y)^C = \overline{R'}(Y^C)$.

$$y \in \underline{R'}(Y^C) \Longleftrightarrow R(y) \subseteq Y^C$$
$$\Longleftrightarrow R(y) \cap Y = \emptyset$$
$$\Longleftrightarrow y \notin \overline{R'}(Y)$$
$$\Longleftrightarrow y \in (\overline{R'}(Y))^C.$$

Then $\underline{R'}(Y^C) = (\overline{R'}Y)^C$. □

Remark 2. We can obtain $\overline{R'}(Y_1 \cap Y_2) \subseteq \overline{R'}Y_1 \cap \overline{R'}Y_2$ from Theorem 1(2). However, $\overline{R'}(Y_1 \cap Y_2) = \overline{R'}Y_1 \cap \overline{R'}Y_2$ is not always correct. As

$$x \in \overline{R'}Y_1 \cap \overline{R'}Y_2 \Longleftrightarrow x \in \overline{R'}Y_1 \wedge x \in \overline{R'}Y_2$$
$$\Longleftrightarrow R(x) \cap Y_1 \neq \emptyset \wedge R(x) \cap Y_2 \neq \emptyset$$
$$\Leftarrow (R(x) \cap Y_1) \cap (R(x) \cap Y_2) \neq \emptyset$$
$$\Longleftrightarrow R(x) \cap (Y_1 \cap Y_2) \neq \emptyset$$
$$\Longleftrightarrow x \in \overline{R'}(Y_1 \cap Y_2)$$

We can also obtain $\underline{R'}(Y_1 \cup Y_2) \supseteq \underline{R'}Y_1 \cup \underline{R'}Y_2$ from Theorem 1(2). Whereas, $\underline{R'}(Y_1 \cup Y_2) = \underline{R'}Y_1 \cup \underline{R'}Y_2$ is not always correct. Since

$$x \in \underline{R'}(Y_1 \cup Y_2) \Longleftrightarrow R(x) \subseteq Y_1 \cup Y_2$$
$$\Leftarrow R(x) \subseteq Y_1 \vee R(x) \subseteq Y_2$$
$$\Longleftrightarrow x \in \underline{R'}Y_1 \vee x \in \underline{R'}Y_2$$
$$\Longleftrightarrow x \in \underline{R'}Y_1 \cup \underline{R'}Y_2$$

Property 1. *Let U and V be the universe of discourse. $R \subseteq U \times V$ and R' the inverse relation of R. $Y \subseteq V$. If the relation matrix of R is an information matrix, then $\underline{R'}\emptyset = \overline{R'}\emptyset = \emptyset, \underline{R'}V = \overline{R'}V = U$.*

The proof is straightforward from Definition 2.

Theorem 2. *Let U and V be the universe of discourse. $R \subseteq U \times V$ and R' the reverse relation of R. $Y \subseteq V$. If the relation R is monic, then $\underline{R'}Y = \overline{R'}Y$.*

Proof. The proof $\underline{R'}Y \subseteq \overline{R'}Y$ is straightforward. Let us consider $\underline{R'}Y \supseteq \overline{R'}Y$. $\forall x_i \in \overline{R'}Y,\ R(x_i) \cap Y \neq \emptyset$. There must be at least one element $y_j \in Y$ such that $R(x_i) = \{y_j\}$ for the relation R is monic. Then $x_i \in \underline{R'}Y$. That is $\underline{R'}Y \supseteq \overline{R'}Y$. To sum up, $\underline{R'}Y = \overline{R'}Y$. □

Theorem 3. *Let U and V be the universe of discourse, where $U = \{x_1, x_2, \ldots, x_m\}$. $R \subseteq U \times V$ and R' the inverse relation of R. Consider $R(x_1) \subseteq R(x_2) \subseteq R(x_3) \subseteq \cdots \subseteq R(x_m)$. For the subset $Y \subseteq V$, we have*

(1) *If $x_i \in \underline{R'}Y$, then $x_1, x_2, \ldots, x_{i-1} \in \underline{R'}Y (i = 2, 3, \ldots, m)$;*
(2) *If $x_i \in \overline{R'}Y$, then $x_{i+1}, x_{i+2}, \ldots, x_m \in \overline{R'}Y\ (i = 1, 2, \ldots, m-1)$.*

Proof

(1) For $x_i \in \underline{R'}Y$, then $R(x_i) \subseteq Y$. For $R(x_1) \subseteq R(x_2) \subseteq \cdots \subseteq R(x_{i-1}) \subseteq R(x_i) \subseteq Y$, then $x_1, x_2, \ldots, x_{i-1} \in \underline{R'}Y$.
(2) For $x_i \in \overline{R'}Y$, then $R(x_i) \cap Y \neq \emptyset$. For $R(x_i) \subseteq R(x_{i+1}) \subseteq R(x_{i+2}) \subseteq \cdots \subseteq R(x_m)$, then $R(x_{i+1}) \cap Y \neq \emptyset$, $R(x_{i+2}) \cap Y \neq \emptyset$, $\cdots$, $R(x_m) \cap Y \neq \emptyset$. Therefore, $x_{i+1}, x_{i+2}, \ldots, x_m \in \overline{R'}Y$. □

Theorem 4. *Let U and V be the universe of discourse, where $U = \{x_1, x_2, \ldots, x_m\}$. $R \subseteq U \times V$ and R' the reverse relation of R. Consider $R(x_1) \subseteq R(x_2) \subseteq R(x_3) \subseteq \cdots \subseteq R(x_m)$, $\underline{R'}Y \neq \emptyset$ ($\forall Y \subseteq V$). If the relation matrix of R is an information matrix, then $\overline{R'}Y = U$.*

Proof. For $\underline{R'}Y \neq \emptyset$, then there exist at least an element $x_i \in \underline{R'}Y$. For $R(x_1) \subseteq R(x_2) \subseteq R(x_3) \subseteq \cdots \subseteq R(x_m)$, then $x_1, x_2, \ldots, x_{i-1} \in \underline{R'}Y$ from Theorem 3(1). Then $x_i \in \overline{R'}Y$ for $\underline{R'}Y \subseteq \overline{R'}Y$. For $R(x_1) \subseteq R(x_2) \subseteq R(x_3) \subseteq \cdots \subseteq R(x_i)$ and the relation matrix of R is an information matrix, then $x_{i+1}, x_{i+2}, \ldots, x_m \in \overline{R'}Y$ from Theorem 3(2). For $x_1, x_2, \ldots, x_{i-1} \in \underline{R'}Y \subseteq \overline{R'}Y$, therefore, $x_1, x_2, \ldots, x_m \in \overline{R'}Y$. That is $\overline{R'}Y = U$. □

3. Properties of Pawlak rough set induced by RSMDU

Let U and V be the universe of discourse. $R \subseteq U \times V$ and R' the inverse relation of R. The relation $E_U \subseteq U \times U$ induced by the relation R can be defined as

$$E_U : xE_Ux' \Longleftrightarrow R(x) = R(x'), \quad x, x' \in U \tag{3}$$

E_U is an equivalence relation on $U \times U$, because E_U meets with reflexivity, symmetry and transitivity straightforwardly. Then the equivalence relation E_U partitions the set U into disjoint subsets. Let U/E_U denotes the equivalence classes of E_U. Let $[x]_{E_U}$ denotes the equivalence class containing x, where $x \in U$. In this paper $[x]_{E_U}$ is replaced by $[x]_U$ for short. Then operators

$$\underline{E_U}, \overline{E_U} : P(U) \to P(U)$$

are just Pawlak approximate operators. $\forall X \subseteq U$, the lower approximation and upper approximation of X under E_U are defined as

$$\underline{E_U}X = \{x | x \in U, [x]_U \subseteq X\}, \quad \overline{E_U}X = \{x | x \in U, [x]_U \cap X \neq \emptyset\} \tag{4}$$

We can also obtain the equivalence relation E_V induced by R', which is defined as

$$E_V : yE_Vy' \Longleftrightarrow R'(y) = R'(y'), \quad \forall y, \ y' \in V$$

and Pawlak approximate operators

$$\underline{E_V}, \overline{E_V} : P(V) \to P(V)$$

Example 3. Following with Example 1, we can obtain

$$U/E_U = \{\{x_1\}, \{x_2, x_4\}, \{x_3\}, \{x_5, x_6\}\}$$
$$V/E_V = \{\{y_1, y_4\}, \{y_2, y_3\}, \{y_5, y_6, y_7\}\}$$

If

$$X = \{x_1, x_2, x_3\}$$

then

$$\underline{E_U}X = \{x_1, x_3\}, \quad \overline{E_U}X = \{x_1, x_2, x_3, x_4\}$$

If

$$Y = \{y_1, y_2, y_3\}$$

then

$$\underline{E_V}Y = \{y_2, y_3\}, \quad \overline{E_V}Y = \{y_1, y_2, y_3, y_4\}$$

Let E_U denote an equivalence relation on U, and $U/E_U = \{X_1, X_2, \ldots, X_P\}$. Let $W \subseteq U$. The steps obtaining the lower approximation of W are presented as

L1 Set $j = 1$, $L = \emptyset$;
L2 For $j = 1, \ldots, p$, if $X_j \subseteq W$, then $L = L \cup X_j$.

The steps obtaining the upper approximation of W are presented as

R1 Set $j = 1$, $R = \emptyset$;
R2 For $j = 1, \ldots, p$, if $X_j \cap W \neq \emptyset$, then $R = R \cup X_j$.

Theorem 5. *Let U and V be the universe of discourse. $R \subseteq U \times V$ and R' the reverse relation of R. $\forall Y \subseteq V$,*

(1) $\overline{E_U}(\overline{R'}Y) = \overline{R'}(\overline{E_V}Y)$;
(2) $\underline{E_U}(\underline{R'}Y) = \underline{R'}(\underline{E_V}Y)$.

Proof

(1) $\forall x \in \overline{E_U}(\overline{R'}Y), [x]_U \cap \overline{R'}Y \neq \emptyset$ from Definition 3. Let

$$x_i \in [x]_U \cap \overline{R'}Y$$

then

$$[x_i]_U = [x]_U \wedge x_i \in \overline{R'}Y$$

According to Definition 3,

$$\exists y \in Y \wedge x_iRy$$

Then xRy. Therefore

$$y \in R(x) \cap \overline{E_V}Y \quad \text{and } R(x) \cap \overline{E_V}Y \neq \emptyset$$

That is

$$x \in \overline{R'}(\overline{E_V}Y)$$

Then we get $\overline{E_U}(\overline{R'}Y) \subseteq \overline{R'}(\overline{E_V}Y)$.
Otherwise, if $x \in \overline{R'}(\overline{E_V}Y)$, then $R(x) \cap \overline{E_V}Y \neq \emptyset$. Let

$$y \in R(x) \cap \overline{E_V}Y$$

We have

$$y \in R(x) \wedge y \in \overline{E_V}Y$$

That is

$$(x, y) \in R \wedge [y]_V \cap Y \neq \emptyset$$

So there is

$$y_i \in V \wedge y_i \in [y]_V \cap Y$$

That is

$$(x, y) \in R, \quad [y]_V = [y_i]_V, \quad y_i \in Y$$

Therefore

$$(x, y_i) \in R \wedge y_i \in Y$$

That is

$$y_i \in R(x) \cap Y$$

Then

$$R(x) \cap Y \neq \emptyset \wedge x \in \overline{R'}(Y)$$

As

$$x \in [x]_U \cap \overline{R'}(Y)$$

there is

$$[x]_U \cap \overline{R'}(Y) \neq \emptyset$$

That is

$$x \in \overline{E_V}(\overline{R'}Y)$$

Then $\overline{R'}(\overline{E_V}Y) \subseteq \overline{E_U}(\overline{R'}Y)$. Hence $\overline{E_U}(\overline{R'}Y) = \overline{R'}(\overline{E_V}Y)$.

(2) For

$$\overline{E_U}(\overline{R'}Y) = \overline{R'}(\overline{E_V}Y)$$

then

$$\left(\overline{E_U}(\overline{R'}Y)\right)^C = \left(\overline{R'}(\overline{E_V}Y)\right)^C$$

And

$$\left(\overline{E_U}(\overline{R'}Y)\right)^C = \underline{E_U}\left((\overline{R'}Y)^C\right) = \underline{E_U}(\underline{R'}(Y^C)),$$
$$\left(\overline{R'}(\overline{E_V}Y)\right)^C = \underline{R'}((\overline{E_V}Y)^C) = \underline{R'}(\underline{E_V}(Y^C))$$

grounding on duality of Pawlak rough set and Theorem 1(5). Then

$$\underline{E_U}(\underline{R'}(Y^C)) = \underline{R'}(\underline{E_V}(Y^C))$$

For $\forall Y \subseteq V$, there is $\underline{E_U}(\underline{R'}Y) = \underline{R'}(\underline{E_V}Y)$. □

4. The relation between RSDU and Pawlak rough set

Rough set theory has been studied further increasingly. Practical applications also conduce more to the development of rough set theory, such as the extended research on the equivalence relation. The model of rough set over dual-universes is an extended model of Pawlak rough set and this model depending on the relation themselves. The similarities and differences between two models are as follows.

The similarities:

(1) Both two models deal with the approximation of sets constructed from descriptive data elements.
(2) Both Pawlak rough set and RSDU create three regions when they analyze data, namely, the positive, negative and boundary regions.
(3) In decision-making table, the set of condition attributes and the set of decision attributes in Pawlak rough set model are considered to be two universes in RSMDU.

Concerning decision problems, we set the set of condition attributes as U, the set of decision attributes as V. Then decision problems can be dealt with as RSMDU. When the universe V has only one attribute, the relation matrix can be written in the way the same as decision table which is a common way to deal with information. For example, let $U = \{x_1, x_2, x_3\}$, $V = \{y_1\}$. $R(x_1) = \emptyset$, $R(x_2) = y_1$, $R(x_3) = \emptyset$. The relation matrix of U and V is

$$A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Then the decision matrix is

$$D = [d_{ij}] = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

where $d_{ij} = 1$ if the element x_i and x_j have the same value in the universe V, else $d_{ij} = 0$.

The differences are embodied in restrictions on universes and relation. Following is the differences in detail:

(1) RSMDU uses two distinct but related universal sets. However, Pawlak rough set just discusses questions in the same universe.
(2) Pawlak rough set is based on equivalence relation, whereas RSMDU is just based on intrinsic relation between the two universes.
(3) RSMDU's application is flexible for it's depending on intrinsic relation between the two universes. However, equivalence relations, which is a basic composition of Pawlak rough set model, impose restrictions on applications for equivalence relations could not exist in every area which we are interested in.

From the above, we draw the conclusion that RSMDU is a general rough set model. We can build rough set models grounding on their sensible plan. The set of conditional attributes and the set of decision attributes can be regarded as dual-universes. The expert system can be set up.

The expert system is comprised of Experts in systems, Users, man–machine interface, knowledge acquisition, knowledge database, reasoning machine. Let us take disease diagnosis as an example.

(1) Knowledge database: knowledge database corresponds to the relation between diseases and symptoms. Knowledge database is built depending on the experiences from experts.
(2) Reasoning machine: the function of reasoning machine is drawing an inference from the fact values that were imputed by users and the knowledge of repository, and returning a reference to page layout of outputting inference.
(3) User interface: by above knowledge, we know there is an user input interface which is the interface for the set of diseases or symptoms.

Example 4. Let U be the set of symptoms, V the set of diseases, where $U = \{x_1, x_2, \ldots, x_{19}\}$={adiaphoresis, panicky, night sweat, nausea, headache, tired, general malaise, chills, anorexia, a sore throat, dry cough, sneeze, a stuffed-up nose, low grade fever, high fever, afternoon fever, phlegm, heart beat fast, abdominal pain}, $V = \{y_1, y_2, \ldots, y_9\}$={common cold, influenza, myocarditis, tuberculosis, viral hepatitis, acute bronchitis, meningitis, pneumonia, acute tonsillitis}. If the disease y_j has the symptom x_i, then $a_{ij} = 1$ else $a_{ij} = 0$, where a_{ij} is an element of relation matrix $A = [a_{ij}]$. The matrix depending on the relation from U to V is as follows.

$$A = [a_{ij}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $X \subseteq U$ and $X = \{x_5, x_{11}, x_{15}, x_{17}\}$. Then $\underline{R}X = \{y_8\}$, $\overline{R}X = \{y_1, y_2, y_4, y_6, y_7, y_8, y_9\}$ and $(\overline{R}X)^c = V - \overline{R}X = \{y_3, y_5\}$. The explanations are as follows.

(1) If a person has symptoms headache, dry cough, high fever and phlegm, then he must suffer disease pneumonia.
(2) If a person has symptoms headache, dry cough, high fever and phlegm, then he might suffer disease common cold, influenza, tuberculosis, acute bronchitis, meningitis, pneumonia or acute tonsillitis
(3) If a person just has symptoms headache, dry cough, high fever and phlegm, then he does not suffer from disease myocarditis and viral hepatitis.

5. Conclusions and future work

The main contributions of the paper are:

(1) We discussed properties of RSMDU by introducing character function and relation matrix, and proposed algorithms for obtaining lower and upper approximation of RSDU.
(2) We studied Pawlak rough set induced by RSMDU, and advanced a theorem about the relation between Pawlak rough set and RSMDU.
(3) We discussed the relation between Pawlak rough set and RSMDU further, and pointed that RSMDU has general applications in real life.

However, RSMDU research is only in start and is still being worked up. Future work will be performed in researching the reduction of attributes and the extraction of the rules from the relation matrix. Furthermore, due to the complexity, uncertainty of information, the extended researches about RSMDU have to be studied, for example, fuzzy rough set over dual-universes, Bayesian rough set over dual-universes and so on.

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