T-S FUZZY MODEL-BASED MEMORY CONTROL FOR DISCRETE-TIME SYSTEM WITH RANDOM INPUT DELAY

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Abstract. A memory control for T-S fuzzy discrete-time systems with stochastic input delay is proposed in this paper. Different from the common assumptions on the time delay in the existing literatures, it is assumed in this paper that the delays vary randomly and satisfy some probabilistic distribution. A new state space model of the discrete-time T-S fuzzy system is derived by introducing some stochastic variables satisfying Bernoulli random binary distribution and using state augmentation method, some criterion for the stochastic stability analysis and stabilization controller design are derived for T-S fuzzy systems with stochastic time-varying input delay. Finally, a numerical example is given to demonstrate the effectiveness and the merit of the proposed method.

1. Introduction

It is well known that T-S fuzzy systems have been widely studied and have had many practical applications during the past decades. Much effort has been devoted to both theoretical research and implemental techniques for fuzzy control, see [19] and the references therein. Actually, large number of studies show that fuzzy control is a practical control approach for complex nonlinear systems. In many model-based fuzzy control methods, the well-known Takagi-Sugeno (T-S) fuzzy model, which had been first proposed in [19], has attracted much attention in the study of system analysis and synthesis of a class of nonlinear systems through membership functions, see [1, 6, 21] for some representative works. However, in the real world, nonlinear dynamic systems with time delay exist extensively in many industrial and engineering systems, such as rolling mill systems, chemical processes, communication networks, and it is well known that the T-S fuzzy model is qualified to represent a certain class of nonlinear dynamic systems(of course including time delay systems), and thus, it is natural to investigate nonlinear systems with time delay via the corresponding T-S fuzzy model. Some nice results on T-S fuzzy model have been reported in literature and there are two kinds of results, namely delay-independent T-S fuzzy system [1, 2, 15, 16] and delay-dependent T-S fuzzy system

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[7, 8, 11, 12, 20, 22]. The delay-dependent results are usually less conservative than the delay-independent ones, especially when the delay is small.

Furthermore, recent studies on the delay-dependent stability and stabilization for the T-S fuzzy systems are also fruitful, and many valuable results have been reported in the open literature, for constant delay, see [11, 22] and for time-varying delay, see [3, 4, 14, 17, 18]. However, it should be noted that most of the aforementioned results are about continuous systems with state delay, there is few results about input delay, especially discrete-time delay T-S fuzzy systems. In terms of the methods proposed in [23, 24], memory controller is designed, but it only consider continuous-time systems with time-varying input delay. For discrete-time systems with time-varying state delay, [9] proposed some new results on stability criteria. However, for discrete-time system with time-varying input delay which is often uncounted in practice. Since it is difficult to analyze by using current approach, such as state augmentation technique, which make the transformed systems usually have time-varying matrix coefficients, for this case, there is few available results to be used. To the best of the authors' knowledge, up to now, few results have been reported for the stability analysis and controller synthesis of discrete-time T-S fuzzy system with time-varying input delay when employing the information of the probabilities of the time delay taking values in finite set, which motivates the present study.

In this paper, we concerned with the stochastic stability analysis and controller synthesis for discrete-time T-S fuzzy systems with time-varying input delay. Different from the existing literatures [5, 10, 13],we assume that the occurrence probabilities of the time delays $\tau(k)$ taking values in two finite point sets are known. More specifically, the probability of $\tau(k)$ taking value in every point belonging to the finite set $\Omega_1 = \{1, 2, \cdots, h\}$ is known as $\alpha_i \in [0, 1]\,(i = 1, 2, \cdots, h)$, while the probability of $\tau(k)$ locating in $\Omega_2 = \{h + 1, h + 2, \cdots, h + \tau_M\}$ is also known as $\beta \in [0, 1]$, where α_n and β satisfy $\sum_{i=1}^{h} \alpha_i + \beta = 1, h$ and τ_M are positive integers. By introducing two stochastic variables satisfying Bernoulli random binary distribution, the original system is transformed into a new T-S fuzzy model with stochastic parameter matrices. Based on the new model, the delay-dependent stability criteria and controller synthesis are derived, which depend on not only the size of the delay steps, but also the probabilities of the delay taking values in one finite set. Since more information of the time delay is employed, it can be expected that the concluded results will be less conservative.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n \times m$ real matrices; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$(respectively, $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). When x is a stochastic variable, $\mathbb{E}\{x\}$ stands for the expectation of x. For a matrix B and two symmetric matrices A and

C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denote a symmetric matrix, where * denotes the entries implied by symmetry.

2. Systems Description and Preliminaries

Consider the following discrete-time T-S fuzzy model with time varying input delay:

Plant rule i :

IF $z_1(k)$ is W_1^i and $\cdots$ and $z_g(k)$ is W_g^i,

THEN

$$x(k+1) = A_i x(k) + B_i u(k - \tau(k)) \tag{1}$$

where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ denote the state vector and control input vector, respectively, A_i, B_i are known constant matrices with compatible dimensions; W_j^i is the fuzzy set; $z_j(k)$ is the premise variable, $j = 1, 2, \cdots, g, i = 1, 2, \cdots, r$, which denotes the number of IF-THEN rules, $\tau(k)$ is a bounded time-varying delay.

By using center-average defuzzifier, product interference and singleton fuzzifier, the global dynamics of (1) can be described as

$$\begin{cases} x(k+1) = \sum_{i=1}^{r} \mu_i(z(k)) A_i(k) + \sum_{i=1}^{r} \mu_i(z(k)) B_i(k) u(k - \tau(k)) \\ \qquad\qquad x(k) = \phi(k), k = -d, -d+1, \cdots, 0 \end{cases} \tag{2}$$

where

$$\mu_i(z(k)) \quad = \quad \frac{w_i(z(k))}{\sum_{i=1}^{r} w_i(z(k))}, w_i(z(k)) = \Pi_{j=1}^{g} W_j^i(z_j(k)), 0 \le \tau(k) \le d$$

$W_j^i(z_j(k))$ is the membership function of $z_j(k)$ in W_j^i, and $\mu_i(z(k)) \ge 0$, $\sum_{i=1}^{r} \mu_i(z(k)) = 1$, d is the upper bound of the time-delay $\tau(k)$.

In the following, for the sake of brevity, we use μ_i representing $\mu_i(z(k))$.

Before proceeding further, we first introduce the following assumption.

Assumption 2.1. The time varying delay $\tau(k) \in \Omega_1$ or Ω_2, without loss of generality, for a given positive integer h, we assume $\Omega_1 = \{1, 2, \cdots, h\}$ and $\Omega_2 = \{h+1, h+2, \cdots, h+\tau_M\}$, τ_M is a finite positive integer. The probability of $\tau(k)$ taking value in every point belonging to the finite set Ω_1 is known as $\alpha_i \in [0, 1]$ $(i = 1, 2, \cdots, h)$, while the probability of $\tau(k)$ locating in Ω_2 is also known as $\beta \in [0, 1]$, where α_n and β satisfy $\sum_{i=1}^{h} \alpha_i + \beta = 1$.

Remark 2.2. From Assumption 2.1, we can only see that the probability of $\tau(k)$ taking each point in a finite set Ω_1 is assumed to be known, but for Ω_2, we only need to know the probability of $\tau(k)$ locating in Ω_2 rather than to know the probability of $\tau(k)$ taking every point in Ω_2. of course, the information of probability distribution can be measured by lots of statistical experiments.

In order to describe the above Assumption, we define two stochastic variables $\alpha_n(k)$ and $\beta(k)$ that if $\tau(k)$ is within a finite set Ω_1 or not. More specifically, $\alpha_n(k)$ and $\beta(k)$ are defined as follows:

$$\alpha_n(k) = \begin{cases} 1, \tau(k) = n \\ 0, \tau(k) \neq n \end{cases}, n = 1, 2, \cdots, h\ , \beta(k) = \begin{cases} 1, \tau(k) > h \\ 0, \tau(k) \leq h \end{cases}, q \in \mathbb{Z}^+$$

Remark 2.3. According to the above analysis, we can obtain $E\{\alpha_n(k)\} = E\{\alpha_n^2(k)\} = \alpha_n$, $E\{\beta(k)\} = E\{\beta^2(k)\} = \beta$, $\sum_{n=1}^{h} \alpha_n + \beta = 1$, α_n and β is the probability of Prob$\{\tau(k) = n\}, n = 1, 2, \cdots, h$ and prob$\{\tau(k) > h\}$, respectively.

Based on a PDC approach, the following fuzzy rules for fuzzy system (2) are employed in this paper.

Control rule i :

IF $z_1(k)$ is W_1^i and $\cdots$ and $z_g(k)$ is W_g^i,

THEN $u(k) = K_i X(k)$.

The final state feedback controller is expressed as

$$u(k) = \sum_{i=1}^{r} \mu_i K_i X(k) \tag{3}$$

where $K_i \in \mathbb{R}^{m \times n}$ is to be determined later, and

$$X^T(k) = \begin{bmatrix} x^T(k) & u^T(k-h) & \cdots & u^T(k-1) \end{bmatrix}$$

Due to introducing the augmented vector $X(k)$, the system (2) under the memory controller (3) can be rewritten as follows, if we take the input delay $\tau(k) = 1$, $\tau(k) = 2, \cdots, \tau(k) = h$, and $\tau(k) = d_k + h$, where $d_k = \tau(k) - h$ when $\tau(k) > h$.

$$\begin{cases} \tau(k) = 1 & X(k+1) = \sum_{i=1}^{r} \mu_i \bar{A}_{i1} X(k) + \bar{B} u(k) \\ \tau(k) = 2 & X(k+1) = \sum_{i=1}^{r} \mu_i \bar{A}_{i2} X(k) + \bar{B} u(k) \\ \quad\vdots & \quad\vdots \\ \tau(k) = h & X(k+1) = \sum_{i=1}^{r} \mu_i \bar{A}_{ih} X(k) + \bar{B} u(k) \\ \tau(k) > h & X(k+1) = \sum_{i=1}^{r} \mu_i \hat{A}_i X(k) + \sum_{i=1}^{r} \mu_i \hat{B}_i X(k - d_k) \end{cases} \tag{4}$$

where

$$\bar{A}_{ij} = \begin{bmatrix} A_i & 0 & 0 & \overbrace{B_i \quad 0 \quad \cdots \quad 0}^{j} \\ 0 & 0 & I & 0 \quad 0 \quad \cdots \quad 0 \\ 0 & 0 & 0 & I \quad 0 \quad \cdots \quad 0 \\ 0 & 0 & 0 & 0 \quad I \quad \cdots \quad 0 \\ \cdots & \cdots & \cdots & \cdots \quad \cdots \quad \ddots \quad \cdots \\ 0 & 0 & 0 & 0 \quad 0 \quad \cdots \quad I \\ 0 & 0 & 0 & 0 \quad 0 \quad \cdots \quad 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \\ I \end{bmatrix}$$

$$\hat{A}_i = \begin{bmatrix} A_i & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & I & 0 & \cdots & 0 \\ 0 & 0 & 0 & I & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & I \\ 0 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \hat{B}_i = \begin{bmatrix} 0 & B_i & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \ddots & \cdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

In terms of the definitions of the random variables $\alpha_n(k)$ and $\beta(k)$, system (4) can be expressed as

$$\begin{aligned} X(k+1) &= \sum_{j=1}^{h} \alpha_j(k) \left[\sum_{i=1}^{r} \mu_i \bar{A}_{ij} X(k) + \bar{B} u(k) \right] \\ &+ \beta(k) \left[\sum_{i=1}^{r} \mu_i \hat{A}_i X(k) + \sum_{i=1}^{r} \mu_i \hat{B}_i X(k-d_k) \right] \end{aligned} \tag{5}$$

From the definition of the fuzzy controller (3), the system (5) can be further rewritten as

$$\begin{aligned} X(k+1) &= \sum_{i=1}^{r} \mu_i \left\{ \left[\sum_{j=1}^{h} \alpha_j(k)(\bar{A}_{ij} + \bar{B}K_i) + \beta(k)\hat{A}_i \right] X(k) \right. \\ &+ \left. \beta(k)\hat{B}_i X(k-d_k) \right\} \end{aligned} \tag{6}$$

where $d_k = \tau(k) - h$ and $0 \le d_k \le d_M$

Remark 2.4. The augmented state variable technique, of course, is not always implemental as the dimension of the augmented system increases with the delay size. That is, when the delay is too large, the augmented system will become much complicated and thus difficult to analyze and synthesize. So, if delay is small, then this method is a good way, or else it is not feasible, especially in time varying delay case.

Remark 2.5. From the definition of $X(k)$, it can be seen that, $u(k)$ is dependent on not only the current state $x(k)$, but also the past $h-$step information of the control signal $u(k)$. specifically, $u(k-h), \cdots, u(k-1)$. Therefore, $u(k)$ in (3) is memory.

Before stating our main results, we first introduce the following definition and Lemma.

Definition 2.6. System (2) is said to be stochastic mean-square stable(SMSS) if, for any initial condition $\phi(i) \in \mathbb{R}^n$, $i = -d_M, -d_M+1, \cdots, 0$, there exists a positive scalar μ such that the following condition holds,

$$E\left\{\sum_{k=0}^{\infty} X^T(k)X(k)\right\} \leq \mu \sup_{-d_M \leq i \leq 0} E\left\{\phi^T(i)\phi(i)\right\}$$

Lemma 2.7. *Suppose* $\Omega_{1,}\Omega_2$*, and* Ω *are constant matrices of appropriate dimensions. Then*

$$(\tau(k)-\tau_m)\Omega_1 + (\tau_M - \tau(k))\Omega_2 + \Omega < 0 \tag{7}$$

is true for any $\tau(k) \in [\tau_m, \tau_M]$ *if and only if*

$$(\tau_M - \tau_m)\Omega_1 + \Omega < 0 \tag{8}$$

$$(\tau_M - \tau_m)\Omega_2 + \Omega < 0. \tag{9}$$

Proof. (1) Sufficiency: Setting $\tau(k) \equiv \tau_m$ and $\tau(k) \equiv \tau_M$ in (7), respectively, we can obtain (8) and (9).

(2) Necessity: Define a function as follows:

$$f(\tau(k)) = (\tau(k)-\tau_m)\Omega_1 + (\tau_M - \tau(k))\Omega_2 + \Omega \tag{10}$$

Note that $\tau(k) \in [\tau_m, \tau_M]$, then (10) can be rewritten as

$$\begin{aligned} & f(\tau(k)) = (\tau(k)-\tau_m)\Omega_1 + (\tau_M - \tau(k))\Omega_2 + \Omega \\ = \ & \frac{\tau(k)-\tau_m}{\tau_M-\tau_m}((\tau_M-\tau_m)\Omega_1+\Omega) + \frac{\tau_M-\tau(k)}{\tau_M-\tau_m}((\tau_M-\tau_m)\Omega_2+\Omega) \end{aligned} \tag{11}$$

Combining (8) and (9), we can deduce that $f(\tau(k)) < 0$, that is,

$$(\tau(k)-\tau_m)\Omega_1 + (\tau_M - \tau(k))\Omega_2 + \Omega < 0$$

This completes the proof. □

The purpose of this paper is to derive the stochastic stability criteria for system (6) and design a memory state-feedback fuzzy controller in the form of (3) such that the resulting closed-loop system (6) is SMSS.

3. Stochastic Stability Analysis

Theorem 3.1. *For given integers $h \geq 1$, $d_M \geq 0$ and matrices K_i, the system (6) is SMSS if there exist symmetric matrices $P > 0, Q > 0$, $R > 0$, M, N with appropriate dimensions such that the following matrix inequalities hold.*

$$\begin{bmatrix} \Omega_{11} & * & * \\ \Omega_{21} & \Omega_{22} & * \\ \sqrt{d_M}M^T & 0 & -R \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} \Omega_{11} & * & * \\ \Omega_{21} & \Omega_{22} & * \\ \sqrt{d_M}N^T & 0 & -R \end{bmatrix} < 0 \tag{13}$$

where

$$\begin{aligned}
\Omega_{11} &= \begin{bmatrix} \Upsilon & * & * \\ \beta\hat{B}_i^T P & 0 & * \\ 0 & 0 & -Q \end{bmatrix} + \Psi + \Psi^T \\
\Omega_{21} &= \begin{bmatrix} \sqrt{\alpha_1}\tilde{R}(\bar{A}_{i1} + \bar{B}K_i - I) & 0 & 0 \\ \sqrt{\alpha_2}\tilde{R}(\bar{A}_{i2} + \bar{B}K_i - I) & 0 & 0 \\ \vdots & \vdots & \vdots \\ \sqrt{\alpha_h}\tilde{R}(\bar{A}_{ih} + \bar{B}K_i - I) & 0 & 0 \\ \sqrt{\beta}\tilde{R}(\hat{A}_i - I) & \sqrt{\beta}\tilde{R}\hat{B}_i & 0 \end{bmatrix} \\
\Omega_{22} &= diag(\overbrace{\begin{matrix} -\tilde{R} & -\tilde{R} & \cdots & -\tilde{R} \end{matrix}}^{h+1}) \\
\Upsilon &= \sum_{j=1}^{h} \alpha_j \left[P\bar{A}_{ij} + \bar{A}_{ij}^T P + P\bar{B}K_i + K_i^T \bar{B}^T P\right] + \beta P\hat{A}_i + \beta\hat{A}_i^T P - 2P + Q \\
\Psi &= \begin{bmatrix} N_i & -N_i + M_i & -M_i \end{bmatrix} \\
\tilde{R} &= P + d_M R \\
M_i^T &= \begin{bmatrix} M_{i1}^T & M_{i2}^T & M_{i3}^T \end{bmatrix} \\
N_i^T &= \begin{bmatrix} N_{i1}^T & N_{i2}^T & N_{i3}^T \end{bmatrix}
\end{aligned}$$

Proof. The Lyapunov functional is constructed as

$$V(k) = X^T(k)PX(k) + \sum_{i=k-d_M}^{k-1} X^T(i)QX(i) + \sum_{i=-d_M}^{-1} \sum_{j=k+i}^{k-1} y^T(j)Ry(j) \tag{14}$$

where $P > 0, Q > 0, R > 0$ and $y(k) = X(k+1) - X(k)$. Defining $\mathbb{H}(k) = \{X(k), X(k-1), \cdots, X(k-h)\}$, then calculating the difference of V(k) along the system (6) and taking the mathematical expectation, we have

$$
\begin{aligned}
& \mathbb{E}\{\triangle V(k)|\mathbb{H}(k)\} \\
= & \sum_{i=1}^{r}\mu_i\mathbb{E}\left\{y^T(k)Py(k)+2X^T(k)P\left[\sum_{j=1}^{h}\alpha_j(k)(\bar{A}_{ij}+\bar{B}K_i)+\beta(k)\hat{A}_i-I\right]X(k)\right. \\
+ & 2\beta(k)X^T(k)P\hat{B}_iX(k-d_k)+X^T(k)QX(k)-X^T(k-d_M)QX(k-d_M) \\
+ & \left. d_My^T(k)Ry(k)-\sum_{i=k-d_M}^{k-1}y^T(i)Ry(i)\right\}
\end{aligned} \tag{15}
$$

Note that

$$
\begin{aligned}
& \mathbb{E}\{y^T(k)\tilde{R}y(k)\} \\
& = X^T(k)\left\{\sum_{j=1}^{h}\alpha_j(\bar{A}_{ij}+\bar{B}K_i-I)^T\tilde{R}(\bar{A}_{ij}+\bar{B}K_i-I)\right. \\
& \left.+\beta(\hat{A}_i-I)^T\tilde{R}(\hat{A}_i-I)\right\}X(k)+2\beta X^T(k)(\hat{A}_i-I)^T\tilde{R}\hat{B}_iX(k-d_k) \\
& +\beta X^T(k-d_k)\hat{B}_i^T\tilde{R}\hat{B}_iX(k-d_k)
\end{aligned} \tag{16}
$$

For matrices N and M with appropriate dimensions, the following equations hold obviously

$$
\sum_{i=1}^{r}\mu_i\left\{2\zeta^T(k)N\left[X(k)-X(k-d_k)-\sum_{i=k-d_k}^{k-1}y(i)\right]\right\}=0 \tag{17}
$$

$$
\sum_{i=1}^{r}\mu_i\left\{2\zeta^T(k)M\left[X(k-d_k)-X(k-d_M)-\sum_{i=k-d_M}^{k-d_k-1}y(i)\right]\right\}=0 \tag{18}
$$

where $\zeta^T(k)=\begin{bmatrix}X^T(k) & X^T(k-d_k) & X^T(k-d_M)\end{bmatrix}$
We can obtain

$$
-2\zeta^T(k)N\sum_{i=k-d_k}^{k-1}y(i)\leq d_k\zeta^T(k)NR^{-1}N^T\zeta(k)+\sum_{i=k-d_k}^{k-1}y^T(i)Ry(i) \tag{19}
$$

$$
-2\zeta^T(k)M\sum_{i=k-d_M}^{k-d_k-1}y(i)\leq (d_M-d_k)\zeta^T(k)MR^{-1}M^T\zeta(k)+\sum_{i=k-d_M}^{k-d_k-1}y^T(i)Ry(i) \tag{20}
$$

Substituting (16) into (15) and according to (17)-(20), we obtain

$$\mathbb{E}\{\triangle V(k)|\mathbb{H}(k)\} \leq \sum_{i=1}^{r}\mu_i \left\{ 2X^T(k)P\left[\sum_{j=1}^{h}\alpha_j(\bar{A}_{ij}+\bar{B}K_i)+\beta\hat{A}_i - I\right]X(k)\right.$$
$$+2\beta X^T(k)P\hat{B}_iX(k-d_k)+X^T(k)QX(k)-X^T(k-d_M)QX(k-d_M)$$
$$+X^T(k)\left[\sum_{j=1}^{h}\alpha_j(\bar{A}_{ij}+\bar{B}K_i-I)^T\tilde{R}(\bar{A}_{ij}+\bar{B}K_i-I)\right.$$
$$\left.+\beta(\hat{A}_i-I)^T\tilde{R}(\hat{A}_i-I)\right]X(k)+2\beta X^T(k)(\hat{A}_i-I)^T\tilde{R}\hat{B}_iX(k-d_k)$$
$$+\beta X^T(k-d_k)\hat{B}_i^T\tilde{R}\hat{B}_iX(k-d_k)+2\zeta^T(k)M\left[X(k-d_k)-X(k-d_M)\right]$$
$$+2\zeta^T(k)N\left[X(k)-X(k-d_k)\right]+d_k\zeta^T(k)NR^{-1}N^T\zeta(k)$$
$$\left.+(d_M-d_k)\zeta^T(k)MR^{-1}M^T\zeta(k)\right\} \tag{21}$$

By using Schur complements, from (21) we can obtain that

$$\mathbb{E}\{\triangle V(k)|\mathbb{H}(k)\} \leq \sum_{i=1}^{r}\mu_i\zeta^T(k)\Pi\zeta(k) \tag{22}$$

where $\Pi = \Omega_{11}+\Omega_{21}^T\Omega_{22}^{-1}\Omega_{21}+d_kNR^{-1}N^T+(d_M-d_k)MR^{-1}M^T$

Combining (12) and (13) and using Lemma 2.7, we can obtain that $\Pi < 0$. Therefore, there exists an scalar $c > 0$ such that

$$\mathbb{E}\left\{V(X(k+1))\right\}-\mathbb{E}\left\{V(X(k))\right\} \leq -c\mathbb{E}\left\{X^T(k)X(k)\right\} \tag{23}$$

Summing up both sides of the (23) from 0 to ∞ with respect to k gives

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}X^T(k)X(k)\right\} \leq \frac{1}{c}(\mathbb{E}\left\{V(0)\right\}-\mathbb{E}\left\{V(\infty)\right\}) \leq \frac{1}{c}\mathbb{E}\left\{V(0)\right\}$$

According to the definition of $V(X(k))$, we know that there always exists an scalar $\mu > 0$ such that

$$\mathbb{E}\left\{V(0)\right\} \leq \mu c \max_{-d_M\leq i\leq 0}\mathbb{E}\left\{\phi^T(i)\phi(i)\right\}$$

Thus,

$$\mathbb{E}\left\{\sum_{k=0}^{\infty}X^T(k)X(k)\right\} \leq \mu \max_{-d_M\leq i\leq 0}\mathbb{E}\left\{\phi^T(i)\phi(i)\right\}. \tag{24}$$

Then, from Definition 2.6 the augmented systems (6) is SMSS. This completes the proof. □

4. **Feedback Gain Design**

In this section, based on the Theorem 3.1, the criteria for the design of feedback gain K_i will be derived.

Theorem 4.1. *For given integers $h \geq 1$, $d_M \geq 0$, scalar $\rho > 0$, the system (6) is SMSS under the controller $K_i = Y_i X^{-1}$ if there exist symmetric matrices $\hat{X} > 0, \hat{Q} > 0$, $\hat{M}, \hat{N}, Y_i (i = 1, 2, \cdots, r)$ with appropriate dimensions such that the following linear matrix inequalities hold.*

$$\begin{bmatrix} \hat{\Omega}_{11} & * & * \\ \hat{\Omega}_{21} & \hat{\Omega}_{22} & * \\ \sqrt{d_M}\hat{M}^T & 0 & -\rho\hat{X} \end{bmatrix} < 0 \tag{25}$$

$$\begin{bmatrix} \hat{\Omega}_{11} & * & * \\ \hat{\Omega}_{21} & \hat{\Omega}_{22} & * \\ \sqrt{d_M}\hat{N}^T & 0 & -\rho\hat{X} \end{bmatrix} < 0 \tag{26}$$

where

$$\hat{\Omega}_{11} = \begin{bmatrix} \hat{\Upsilon} & * & * \\ \beta_0 \hat{X}\hat{B}_i^T & 0 & * \\ 0 & 0 & -\hat{Q} \end{bmatrix} + \hat{\Psi} + \hat{\Psi}^T$$

$$\hat{\Omega}_{21} = \begin{bmatrix} \sqrt{\alpha_1}(\bar{A}_{i1}\hat{X} + \bar{B}Y_i - \hat{X}) & 0 & 0 \\ \sqrt{\alpha_2}(\bar{A}_{i2}\hat{X} + \bar{B}Y_i - \hat{X}) & 0 & 0 \\ \vdots & \vdots & \vdots \\ \sqrt{\alpha_h}(\bar{A}_{ih}\hat{X} + \bar{B}Y_i - \hat{X}) & 0 & 0 \\ \sqrt{\beta}(\hat{A}_i\hat{X} - \hat{X}) & \sqrt{\beta}\hat{B}_i\hat{X} & 0 \end{bmatrix}$$

$$\Omega_{22} = diag(\overbrace{-\hat{R} \quad -\hat{R} \quad \cdots \quad -\hat{R}}^{h+1})$$

$$\hat{\Upsilon} = \sum_{j=1}^{h} \alpha_j \left[\bar{A}_{ij}\hat{X} + \hat{X}\bar{A}_{ij}^T + \bar{B}Y_i + Y_i^T \bar{B}^T \right] + \beta\hat{A}_i\hat{X} + \beta\hat{X}\hat{A}_i^T - 2\hat{X} + \hat{Q}$$

$$\hat{\Psi} = \begin{bmatrix} \hat{N}_i & -\hat{N}_i + \hat{M}_i & -\hat{M}_i \end{bmatrix}$$

$$\hat{R} = (1 + d_M\rho)^{-1}\hat{X}$$

$$\hat{M}_i^T = \begin{bmatrix} \hat{M}_{i1}^T & \hat{M}_{i2}^T & \hat{M}_{i3}^T \end{bmatrix}$$

$$\hat{N}_i^T = \begin{bmatrix} \hat{N}_{i1}^T & \hat{N}_{i2}^T & \hat{N}_{i3}^T \end{bmatrix}$$

Then, a controller is given by $K_i = Y_i\hat{X}^{-1}$.

Proof. Defining $\hat{X} = P^{-1}$, $R = \rho P$, $\hat{Q} = \hat{X}Q\hat{X}$, $\hat{M}_i = \hat{X}M_i\hat{X}$, $\hat{N}_i = \hat{X}N_i\hat{X}$ and $Y_i = K_i\hat{X}$

Pre, post-multiplying both side of (12) and (13) respectively with $diag\{X, \cdots, X\}$, we can obtain (25) and (26). □

5. **A Numerical Example**

In this section, a practical system is utilized to illustrate the effectiveness of the proposed stabilization method.

Example 5.1. Consider the following inverted pendulum T-S fuzzy system with input delay:

$$x(k+1) = \sum_{i=1}^{2} \mu_i \left[A_i x(k) + B_i u(k-\tau(k))\right] \tag{27}$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} 1.01 & 0 \\ 0 & -0.1 \end{bmatrix}, A_2 = \begin{bmatrix} 1.03 & 0 \\ 0 & 0.1 \end{bmatrix}, B_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix} B_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\ \mu_1 &= sin^2(x(k)), \mu_2 = cos^2(x(k)). \end{aligned}$$

For simplicity, we first set $\rho = 1, h = 3$, the initial conditions $X(0) = \begin{bmatrix} -0.4 & -0.3 & 0.2 & -0.2 & 0.1 \end{bmatrix}^T$, the probabilities of the delay $\tau(k)$ taking values in finite set $\Omega_1 = \{1, 2, \cdots, h\} = \{1, 2, 3\}$ and locating $\Omega_2 = \{h+1, h+2, \cdots, h+d_M\}$ $= \{4, 5, \cdots, 3+d_M\}$ is assumed to be known. Specifically, we assume that Prob$\{\tau(k) = 1\} = \alpha_1 = 0.2$, Prob$\{\tau(k) = 2\} = \alpha_2 = 0.3$, Prob$\{\tau(k) = 3\} = \alpha_3 = 0.4$, Prob$\{\tau(k) \in \Omega_2\} = \beta = 0.1$. By this assumption, and using Theorem 4.1, we can compute that $d_M = 3$ and the corresponding feedback gains are

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.0434 & -0.0142 & -0.0098 & -0.1582 & 0.4690 \end{bmatrix} \\ K_2 &= \begin{bmatrix} -0.0440 & -0.0054 & 0.0326 & -0.1249 & 0.4925 \end{bmatrix} \end{aligned} \tag{28}$$

Therefore, the upper bound of $\tau(k)$ is $d_M + 3 = 6$.

6. **Conclusion**

The problem of stability analysis and controller synthesis of the T-S fuzzy systems with time-varying stochastic input delay has been investigated based on some new assumptions on time-varying input delay, which describes the informations on the delay more comprehensive and more accurate. In terms of the informations of the occurrence probabilities of the time delays taking values in two finite point sets, a new state space model of the discrete fuzzy system has been derived by introducing two stochastic variables satisfying Bernoulli random binary distribution and using state augmentation method. Then new criteria for the stochastic stability and stabilization of the fuzzy systems have been derived based on the convexity of the matrix equations and the Lyapunov-Krasovskii functional method.

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