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H_∞ FILTERING FOR MARKOVIAN JUMP SYSTEMS WITH TIME-VARYING DELAYS

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Abstract. *This paper proposes a class of H_∞ filter design for Markovian jump systems with time-varying delays. Firstly, by exploiting a new Lyapunov function and using the convexity property of the matrix inequality, new criteria is derived for the H_∞ performance analysis of the filtering-error systems, which can lead to much less conservative analysis results. Secondly, based on the obtained conditions, the gain of filter can be obtained in terms of linear matrix inequalities (LMIs). Finally, numerical examples are given to demonstrate the effectiveness and the merit of the proposed method.*
Keywords: Time-delay systems, H_∞ filter, Markovian jump systems

1. **Introduction.** During the past few decades, Markovian Jump Systems (MJSs) have been attracted much attention [1, 2, 3, 4, 5, 6], which can be regarded as a special class of hybrid systems with finite operation modes whose structures are subject to random abrupt changes. The system parameters usually jump among finite modes, and the mode switching is governed by a Markov process. MJSs have many applications, such as failure prone manufacturing systems, power systems and economics, etc. A great number of results on estimation and control problems related to such systems have been reported in the literature [7, 8, 9, 10, 11].

Recently, the problem of H_∞ filtering of linear/nonlinear time-delay systems has also received much attention due to the fact that for many practical filtering applications, time-delays cannot be neglected in the procedure of filter design and their existence usually results in a poor performance [12, 13, 14]. Some nice results on H_∞ filtering for time-delay systems [15, 16] have been reported in the literature and there are two kinds of results, namely delay-independent filtering [17] and delay-dependent [18, 19, 20, 21, 22, 23]. The delay-dependent results are usually less conservative, especially when the time-delay is small. The main objective of the delay-dependent H_∞ filtering is to obtain a filter such that the filtering error system allows a maximum delay bound for a fixed H_∞ performance or achieves a minimum H_∞ performance for a given delay bound.

This paper addresses the problem of H_∞ filter design for MJSs with interval time-varying delay. To obtain less conservative results, a new Lyapunov function is constructed, which includes the lower and upper delay bound of interval time-varying delay. Based on this, one splits the item $\int_{t-\tau(t)}^{t} e^T(s)Q_2(\theta_t)e(s)ds$ into two parts to deal with, respectively, and uses the convexity of the matrix functions to avoid the conservative caused by enlarging $\tau(t)$ to τ_M in the deriving results. Compared with the existing method [24, 25], the conservativeness of the derived H_∞ performance analysis results is further reduced. Novel H_∞ filter design criteria are obtained. Examples used in [24, 25] are employed to show the effectiveness and less conservativeness of the proposed methods.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n\times m$ real matrices; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). When x is a stochastic variable, $E\{x\}$ stands for the expectation of x. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denote a symmetric matrix, where * denotes the entries implied by symmetry.

2. **Problem Statement and Preliminaries.**

Fix a probability space $(\Omega,\ \mathcal{F},\ \mathcal{P})$ and consider the following class of uncertain linear stochastic systems with markovian jump parameters and time-varying delays (Σ)

$$\begin{cases} \dot{x}(t) = A(\theta_t)x(t) + A_d(\theta_t)x(t-\tau(t)) + A_\omega(\theta_t)\omega(t) \\ y(t) = C(\theta_t)x(t) + C_d(\theta_t)x(t-\tau(t)) + C_\omega(\theta_t)\omega(t) \\ z(t) = L(\theta_t)x(t) + L_d(\theta_t)x(t-\tau(t)) + L_\omega(\theta_t)\omega(t) \\ x(t) = \phi(t), \quad \forall t \in [-\tau_M, -\tau_m] \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}^r$ is the measurement vector, $\omega(t) \in L_2[0,\infty)$ is the exogenous disturbance signal, $z(t) \in \mathbb{R}^p$ is the signal to be estimated, $\{\theta_t\}$ is a continuous-time Markovian process with right continuous trajectories and taking values in a finite set $\mathcal{S} = \{1, 2, \cdots, \mathcal{N}\}$ with stationary transition probabilities:

$$Prob\{\theta_{t+h} = j|\theta_t = i\} = \begin{cases} \pi_{ij}h + o(h), & i \neq j \\ 1 + \pi_{ii}h + o(h), & i = j \end{cases} \tag{2}$$

where $h > 0$, $\lim\limits_{h\to 0} \frac{o(h)}{h} = 0$ and $\pi_{ij} \geq 0$ for $j \neq i$ is the transition rate from mode i at time t to the mode j at time $t + h$ and

$$\pi_{ii} = -\sum_{j=1,\, j\neq i}^{N} \pi_{ij} \tag{3}$$

In the system (1), the time delay $\tau(t)$ is a time-varying continuous function satisfying the following assumption.

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M < \infty, \quad \dot{\tau}(t) \leq \mu, \quad \forall t > 0 \tag{4}$$

where τ_m is the lower bound and τ_M is the upper bound of the communication delay.

In this paper, we consider the following filter for system (1)

$$\begin{cases} \dot{\hat{x}}(t) = A(\theta_t)\hat{x}(t) + A_d(\theta_t)\hat{x}(t-\tau(t)) + G(\theta_t)(\hat{y}(t) - y(t)) \\ \hat{y}(t) = C(\theta_t)\hat{x}(t) + C_d(\theta_t)\hat{x}(t-\tau(t)) \\ \hat{z}(t) = L(\theta_t)\hat{x}(t) + L_d(\theta_t)\hat{x}(t-\tau(t)) \end{cases} \tag{5}$$

The set $\mathcal{S}$ comprises the various operation modes of system (1) and for each possible value of $\theta_t = i$, $i \in \mathcal{S}$, the matrices associated with "*i-th* mode" will be denoted by

$$\begin{aligned} &A_i := A(\theta_t = i),\ A_{di} := A_d(\theta_t = i),\ A_{\omega i} := A_\omega(\theta_t = i) \\ &C_i := C(\theta_t = i),\ C_{di} := C_d(\theta_t = i),\ \ C_{\omega i} := C_\omega(\theta_t = i) \\ &L_i := L(\theta_t = i),\ \ L_{di} := L_d(\theta_t = i),\ \ L_{\omega i} := L_\omega(\theta_t = i) \end{aligned}$$

where A_i, A_{di}, $A_{\omega i}$, C_i, C_{di}, $C_{\omega i}$, L_i, L_{di}, $L_{\omega i}$ are constant matrices for any $i \in \mathcal{S}$. It is assumed that the jumping process $\{\theta_t\}$ is accessible, i.e., the operation mode of system (Σ) is known for every $t \geq 0$.

Let $e(t) = \hat{x}(t) - x(t)$ and $\tilde{z}(t) = \hat{z}(t) - z(t)$. Then we have the following filtering error system:

$$\begin{cases} \dot{e}(t) = \bar{A}_i e(t) + \bar{A}_{di} e(t - \tau(t)) + \bar{A}_{\omega i}\omega(t) \\ \tilde{z}(t) = L_i e(t) + L_{di} e(t - \tau(t)) - L_{\omega i}\omega(t) \end{cases} \tag{6}$$

where $\bar{A}_i = A_i + G_iC_i$, $\bar{A}_{di} = A_{di} + G_iC_{di}$, $\bar{A}_{\omega i} = -A_{\omega i} - G_iC_{\omega i}$.

The H_∞ filtering problem addressed in this paper is to design a filter of form (6) such that

- The filtering error systems (6) with $\omega(t) = 0$ is exponentially stable.
- The H_∞ performance $\|\tilde{z}(t)\|_2 < \gamma\|\omega(t)\|_2$ is guaranteed for all nonzero $\omega(t) \in L_2[0, \infty)$ and a prescribed $\gamma > 0$ under the condition $e(t) = 0$, $\forall t \in [-\tau_M, -\tau_m]$.

The following lemmas and definitions are needed in the proof of our main results.

Lemma 2.1. [26] *For any constant matrix $R \in \mathbb{R}$, $R = R^T > 0$, constant $\tau_M > 0$ and vector function $\dot{x} : [-\tau_M, 0] \to \mathbb{R}^n$ such that the following integration is well defined, it holds that*

$$-\tau_M \int_{t-\tau_M}^{t} \dot{x}^T(s)R\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_M) \end{bmatrix} \tag{7}$$

Lemma 2.2. [27] *Suppose $0 \leq \tau_m \leq \tau(t) \leq \tau_M$, Ξ_1, Ξ_2 and Ω are constant matrices of appropriate dimensions, then*

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \tag{8}$$

if and only if $(\tau_M - \tau_m)\Xi_1 + \Omega < 0$ and $(\tau_M - \tau_m)\Xi_2 + \Omega < 0$ hold.

Definition 2.1. *The system (6) is said to be exponentially stable in the mean-square sense (EMSS), if there exist constants $\alpha > 0$, $\lambda > 0$, such that $t > 0$*

$$E\{\|x(t)\|^2\} \leq \alpha e^{-\lambda t} \sup_{-\tau_M < s < 0} \{\|\phi(s)\|^2\} \tag{9}$$

Definition 2.2. *For a given function $V : C^b_{F_0}([-\tau_M, 0], R^n) \times S \to R$, its infinitesimal operator $\mathcal{L}$ [28] is defined as*

$$\mathcal{L}V(x_t) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}[E(V(x_{t+\Delta}|x_t) - V(x_t))] \tag{10}$$

3. Main Results.

Theorem 3.1. *For some given constants $0 \leq \tau_m \leq \tau_M$ and γ, the systems (6) is EMSS with a prescribed H_∞ performance γ, if there exist $P_i > 0$, $Q_0 > 0$, $Q_1 > 0$, $Q_{2i} > 0$,*

$R_0 > 0$, $R_1 > 0$, $Z_1 > 0$, $Z_2 > 0$, M_{ik} and N_{ik} ($i \in \mathcal{S}$, $k = 1, 2, \cdots, 5$) with appropriate dimensions such that the following matrix inequalities hold

$$\Psi = \begin{bmatrix} \Psi_{11} & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * \\ \Psi_{41}(s) & \Psi_{42}(s) & 0 & -R_1 \end{bmatrix} < 0, \quad s = 1, 2 \tag{11}$$

$$\sum_{j=1}^{N} \pi_{ij} Q_{2j} \leq Z_k, \quad k = 1, 2 \tag{12}$$

where

$$\Psi_{11} = P_i \bar{A}_i + \bar{A}_i^T P_i + Q_0 + Q_1 + Q_{2i} - R_0 + \tau_m Z_1 + \delta Z_2 + \sum_{j=1}^{N} \pi_{ij} P_j$$

$$\Psi_{21} = \begin{bmatrix} \bar{A}_{di}^T P_i - M_{i1}^T + N_{i1}^T \\ R_0 + M_{i1}^T \\ -N_{i1}^T \\ \bar{A}_{\omega i}^T P_i \end{bmatrix}$$

$$\Psi_{22} = \begin{bmatrix} \Gamma & * & * & * \\ -M_{i3} + M_{i2}^T + N_{i3} & -Q_0 - R_0 + M_{i3} + M_{i3}^T & * & * \\ -M_{i4} + N_{i4} - N_{i2}^T & M_{i4} - N_{i3}^T & -Q_1 - N_{i4} - N_{i4}^T & * \\ -M_{i5} + N_{i5} & M_{i5} & -N_{i5} & -\gamma^2 I \end{bmatrix}$$

$$\Psi_{31} = \begin{bmatrix} \tau_m R_0 \bar{A}_i \\ \sqrt{\delta} R_1 \bar{A}_i \\ L_i \end{bmatrix}, \Psi_{32} = \begin{bmatrix} \tau_m R_0 \bar{A}_{di} & 0 & 0 & \tau_m R_0 \bar{A}_{\omega i} \\ \sqrt{\delta} R_1 \bar{A}_{di} & 0 & 0 & \sqrt{\delta} R_1 \bar{A}_{\omega i} \\ L_{di} & 0 & 0 & -L_{\omega i} \end{bmatrix}$$

$$\Psi_{33} = diag\{-R_0, -R_1, -I\}$$

$$\Psi_{41}(1) = \sqrt{\delta} M_{i1}^T, \Psi_{41}(2) = \sqrt{\delta} N_{i1}^T$$

$$\Psi_{42}(1) = \begin{bmatrix} \sqrt{\delta} M_{i2}^T & \sqrt{\delta} M_{i3}^T & \sqrt{\delta} M_{i4}^T & \sqrt{\delta} M_{i5}^T \end{bmatrix}$$

$$\Psi_{42}(2) = \begin{bmatrix} \sqrt{\delta} N_{i2}^T & \sqrt{\delta} N_{i3}^T & \sqrt{\delta} N_{i4}^T & \sqrt{\delta} N_{i5}^T \end{bmatrix}$$

$$\Gamma = -(1 - \mu) Q_{2i} - M_{i2} - M_{i2}^T + N_{i2} + N_{i2}^T$$

$$\delta = \tau_M - \tau_m.$$

Proof: Introduce a new vector

$$\zeta^T(t) = \begin{bmatrix} e^T(t) & e^T(t - \tau(t)) & e^T(t - \tau_m) & e^T(t - \tau_M) & \omega^T(t) \end{bmatrix}$$

and two matrices $\Gamma_1 = \begin{bmatrix} \bar{A}_i & \bar{A}_{di} & 0 & 0 & \bar{A}_{\omega i} \end{bmatrix}$, $\Gamma_2 = \begin{bmatrix} L_i & L_{di} & 0 & 0 & -L_{\omega i} \end{bmatrix}$.

Rewrite (6) as

$$\begin{cases} \dot{e}(t) = \Gamma_1 \zeta(t) \\ \tilde{z}(t) = \Gamma_2 \zeta(t) \end{cases} \tag{13}$$

Let $x_t(s) = x(t + s)$ ($-\tau(t) \leq s \leq 0$). Then, similar to [29], $\{(x_t, \theta_t), t \geq 0\}$ is a Markov process. Construct a Lyapunov functional candidate as

$$V(x_t, \theta_t) = \sum_{i=1}^{4} V_i(x_t, \theta_t) \tag{14}$$

where

$$V_1(x_t, \theta_t) = e^T(t) P(\theta_t) e(t)$$

$$V_2(x_t,\theta_t) = \int_{t-\tau_m}^{t} e^T(s)Q_0e(s)ds + \int_{t-\tau_M}^{t} e^T(s)Q_1e(s)ds + \int_{t-\tau(t)}^{t} e^T(s)Q_2(\theta_t)e(s)ds$$

$$V_3(x_t,\theta_t) = \tau_m \int_{t-\tau_m}^{t}\int_s^t \dot{e}^T(v)R_0\dot{e}(v)dvds + \int_{t-\tau_M}^{t-\tau_m}\int_s^t \dot{e}^T(v)R_1\dot{e}(v)dvds$$

$$V_4(x_t,\theta_t) = \int_{t-\tau_m}^{t}\int_s^t e^T(v)Z_1e(v)dvds + \int_{t-\tau_M}^{t-\tau_m}\int_s^t e^T(v)Z_2e(v)dvds$$

Let $\mathcal{L}$ be the weak infinite generator of the random process $\{x_t,\theta_t\}$. Then, for each $\theta_t = i$ $(i \in \mathcal{S})$, we have

$$\begin{aligned}
\mathcal{L}[V(x_t,\theta_t)] \le\ & e^T(t)\left(2P_i\bar{A}_i + \sum_{j=1}^{N}\pi_{ij}P_j + Q_0 + Q_1 + Q_{2i} + \tau_m Z_1 + \delta Z_2\right)e(t) \\
& +2e^T(t)P_i\bar{A}_{di}e(t-\tau(t)) + 2e^T(t)P_i\bar{A}_{\omega i}\omega(t) - e^T(t-\tau_m)Q_0e(t-\tau_m) \\
& +\delta\dot{e}^T(t)R_1\dot{e}(t) - (1-\mu)e^T(t-\tau(t))Q_{2i}e(t-\tau(t)) \\
& +\int_{t-\tau(t)}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{2j}\right)e(s)ds + \tau_m^2\dot{e}^T(t)R_0\dot{e}(t) \\
& -e^T(t-\tau_M)Q_1e(t-\tau_M) - \tau_m\int_{t-\tau_m}^{t}\dot{e}^T(s)R_0\dot{e}(s)ds \\
& -\int_{t-\tau_M}^{t-\tau_m}\dot{e}^T(s)R_1\dot{e}(s)ds - \int_{t-\tau_m}^{t} e^T(s)Z_1e(s)ds - \int_{t-\tau_M}^{t-\tau_m} e^T(s)Z_2e(s)ds \qquad (15)
\end{aligned}$$

Note that

$$\begin{aligned}
& \int_{t-\tau(t)}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{2j}\right)e(s)ds \\
= & \int_{t-\tau(t)}^{t-\tau_m} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{2j}\right)e(s)ds + \int_{t-\tau_m}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{2j}\right)e(s)ds \qquad (16)
\end{aligned}$$

From (12) and (16), we have

$$\begin{aligned}
& \int_{t-\tau(t)}^{t} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{2j}\right)e(s)ds - \int_{t-\tau_m}^{t} e^T(s)Z_1e(s)ds - \int_{t-\tau_M}^{t-\tau_m} e^T(s)Z_2e(s)ds \\
= & \int_{t-\tau_m}^{t} e^T(s)\left[\sum_{j=1}^{N}\pi_{ij}Q_{2j} - Z_1\right]e(s)ds + \int_{t-\tau(t)}^{t-\tau_m} e^T(s)\left(\sum_{j=1}^{N}\pi_{ij}Q_{2j}\right)e(s)ds \\
& -\int_{t-\tau_M}^{t-\tau_m} e^T(s)Z_2e(s)ds \\
\le & \int_{t-\tau_m}^{t} e^T(s)\left[\sum_{j=1}^{N}\pi_{ij}Q_{2j} - Z_1\right]e(s)ds \\
& +\int_{t-\tau(t)}^{t-\tau_m} e^T(s)\left[\sum_{j=1}^{N}\pi_{ij}Q_{2j} - Z_2\right]e(s)ds \le 0 \qquad (17)
\end{aligned}$$

Applying Lemma 2.1, we have

$$-\tau_m\int_{t-\tau_m}^{t}\dot{e}^T(s)R_0\dot{e}(s)ds \le \begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix}^T \begin{bmatrix} -R_0 & R_0 \\ R_0 & -R_0\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix} \tag{18}$$

Combining (15), (17), (18) and introducing slack matrices M_i, N_i, $i \in \mathcal{S}$, we obtain

$$\begin{aligned}
&\mathcal{L}[V(x_t,\theta_t)] - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \\
\le\ & e^T(t)\left(2P_i\bar{A}_i + \sum_{j=1}^{N}\pi_{ij}P_j + Q_0 + Q_1 + Q_{2i} + \tau_m Z_1 + \delta Z_2\right)e(t) \\
&+2e^T(t)P_i\bar{A}_{\omega i}\omega(t) - e^T(t-\tau_m)Q_0e(t-\tau_m) - e^T(t-\tau_M)Q_1e(t-\tau_M) \\
&-(1-\mu)e^T(t-\tau(t))Q_{2i}e(t-\tau(t)) + \tau_m^2\dot{e}^T(t)R_0\dot{e}(t) + \delta\dot{e}^T(t)R_1\dot{e}(t) \\
&+\begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix}^T \begin{bmatrix} -R_0 & R_0 \\ R_0 & -R_0\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix} - \int_{t-\tau_M}^{t-\tau_m}\dot{e}^T(s)R_1\dot{e}(s)ds \\
&+2e^T(t)P_i\bar{A}_{di}e(t-\tau(t)) - \gamma^2\omega^T(t)\omega(t) + \zeta^T(t)\Gamma_2^T\Gamma_2\zeta(t) \\
&+2\zeta^T(t)M_i\left[e(t-\tau_m) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_m}\dot{e}(s)ds\right] \\
&+2\zeta^T(t)N_i\left[e(t-\tau(t) - e(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)ds\right]
\end{aligned} \tag{19}$$

where

$$\begin{aligned} M_i^T &= \begin{bmatrix} M_{i1}^T & M_{i2}^T & M_{i3}^T & M_{i4}^T & M_{i5}^T \end{bmatrix} \\ N_i^T &= \begin{bmatrix} N_{i1}^T & N_{i2}^T & N_{i3}^T & N_{i4}^T & N_{i5}^T \end{bmatrix} \end{aligned}$$

Note that

$$-2\zeta^T(t)M_i\int_{t-\tau(t)}^{t-\tau_m}\dot{e}(s)ds \le \int_{t-\tau(t)}^{t-\tau_m}\dot{e}^T(s)R_1\dot{e}(s)ds + (\tau(t)-\tau_m)\,\zeta^T(t)M_iR_1^{-1}M_i^T\zeta(t) \tag{20}$$

$$-2\zeta^T(t)N_i\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)ds \le \int_{t-\tau_M}^{t-\tau(t)}\dot{e}^T(s)R_1\dot{e}(s)ds + (\tau_M-\tau(t))\zeta^T(t)N_iR_1^{-1}N_i^T\zeta(t) \tag{21}$$

From (19) – (21), we can obtain

$$\begin{aligned}
&\mathcal{L}[V(x_t,\theta_t)] - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \\
\le\ & e^T(t)\left(2P_i\bar{A}_i + \sum_{j=1}^{N}\pi_{ij}P_j + Q_0 + Q_1 + Q_{2i} + \tau_m Z_1 + \delta Z_2\right)e(t) \\
&+2e^T(t)P_i\bar{A}_{di}e(t-\tau(t)) - \gamma^2\omega^T(t)\omega(t) + \zeta^T(t)\Gamma_2^T\Gamma_2\zeta(t) \\
&+2e^T(t)P_i\bar{A}_{\omega i}\omega(t) - e^T(t-\tau_m)Q_0e(t-\tau_m) - e^T(t-\tau_M)Q_1e(t-\tau_M) \\
&-(1-\mu)e^T(t-\tau(t))Q_{2i}e(t-\tau(t)) + \tau_m^2\dot{e}^T(t)R_0\dot{e}(t) + \delta\dot{e}^T(t)R_1\dot{e}(t) \\
&+\begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix}^T \begin{bmatrix} -R_0 & R_0 \\ R_0 & -R_0\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix} \\
&+2\zeta^T(t)M_i[e(t-\tau_m) - e(t-\tau(t))] + 2\zeta^T(t)N_i[e(t-\tau(t) - e(t-\tau_M)] \\
&+(\tau(t)-\tau_m)\zeta^T(t)M_iR_1^{-1}M_i^T\zeta(t) + (\tau_M-\tau(t))\zeta^T(t)N_iR_1^{-1}N_i^T\zeta(t)
\end{aligned} \tag{22}$$

Using Lemma 2.2 and Schur complement, it is easy to see that (11) with $s = 1, 2$ are sufficient conditions to guarantee

$$\mathcal{L}[V(x_t,\theta_t)] - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) < 0 \tag{23}$$

Then, the following inequality can be concluded

$$E\{\mathcal{L}V(x_t,i,t)\} < -\lambda_{\min}(\Psi)E\{\zeta^T(t)\zeta(t)\} \tag{24}$$

Define a new function as $W(x_t,i,t) = e^{\epsilon t}V(x_t,i,t)$, Its infinitesimal operator $\mathcal{L}$ is given by

$$\mathcal{W}(x_t,i,t) = \epsilon e^{\epsilon t}V(x_t,i,t) + e^{\epsilon t}\mathcal{L}V(x_t,i,t) \tag{25}$$

By the generalized *Itô* formula [28], we can obtain from (25) that

$$E\{W(x_t,i,t)\} - E\{W(x_0,i)\} = \int_0^t \epsilon e^{\epsilon s}E\{V(x_s,i)\}ds + \int_0^t e^{\epsilon s}E\{\mathcal{L}V(x_s,i)\}ds \tag{26}$$

Then, using the similar method of [2], we can see that there exists a positive number α such that for $t>0$

$$E\{V(x_t,i,t)\} \le \alpha \sup_{-\tau_M\le s\le 0}\left\{\|\phi(s)\|^2\right\}e^{-\epsilon t} \tag{27}$$

since $V(x_t,i,t) \ge \{\lambda_{\min}(P_i)\}x^T(t)x(t)$, it can be shown from (27) that for $t \ge 0$

$$E\{x^T(t)x(t)\} \le \bar{\alpha}^{-\epsilon t} \sup_{-\tau_M\le s\le 0}\left\{\|\phi(s)\|^2\right\} \tag{28}$$

where $\bar{\alpha} = \alpha/(\lambda_{\min}P_i)$. Recalling Definition 2.1, the proof can be completed. □

Remark 3.1. *In the above proof, it should be noted that a new Lyapunov function is constructed, and* $\int_{t-\tau(t)}^t x^T(s)\left(\sum_{j=1}^N \pi_{ij}Q_{2j}\right)x(s)ds$ *in (17) are separated into two parts. Moreover, examples below show that this method has less conservative than the existing ones* [6, 25].

Remark 3.2. *Theorem 3.1 provides a delay-dependent stochastic stability condition for MJS with interval time-varying delays. Throughout the proof of Theorem 3.1, it can be seen that the convexity property of the matrix inequality is treated in terms of Lemma 2.2, which need not enlarge* $\tau(t)$ *to* τ_M*, therefore, the common existed conservatism caused by this kind of enlargement in* [30, 31, 32, 33] *can be avoided, which will reduce the conservative of the result.*

In the following, we are seeking to design the H_∞ filtering based on Theorem 3.1.

Theorem 3.2. *For some given constants* $0 \le \tau_m \le \tau_M$ *and* γ*, the augmented systems (6) is EMSS with a prescribed* H_∞ *performance* γ *if there exist* $P_i > 0$, $Q_0 > 0$, $Q_1 > 0$, $Q_{2i} > 0$, $R_0 > 0$, $R_1 > 0$, $Z_1 > 0$, $Z_2 > 0$, $\bar{G}_i$, M_{ik} *and* N_{ik} $(i \in \mathcal{S},\ k = 1,2\cdots,5)$ *with appropriate dimensions such that the following LMIs hold for a given* $\varepsilon > 0$

$$\hat{\Psi} = \begin{bmatrix} \hat{\Psi}_{11} & * & * & * \\ \hat{\Psi}_{21} & \Psi_{22} & * & * \\ \hat{\Psi}_{31} & \hat{\Psi}_{32} & \hat{\Psi}_{33} & * \\ \Psi_{41}(s) & \Psi_{42}(s) & 0 & -R_1 \end{bmatrix} < 0, \quad s = 1,2 \tag{29}$$

$$\sum_{j=1}^N \pi_{ij}Q_{2j} \le Z_k, \quad k = 1,2 \tag{30}$$

where

$$\hat{\Psi}_{11} = P_iA_i + A_i^TP_i + \bar{G}_iC_i + C_i^T\bar{G}_i^T + Q_0 + Q_1 + Q_{2i} - R_0 + \tau_mZ_1 + \delta Z_2 + \sum_{j=1}^N \pi_{ij}P_j$$

$$\hat{\Psi}_{21} = \begin{bmatrix} A_{di}^T P_i + C_{di}^T \bar{G}_i^T - M_{i1}^T + N_{i1}^T \\ R_0 + M_{i1}^T \\ -N_{i1}^T \\ -A_{\omega i}^T P_i - C_{\omega i}^T \bar{G}_i^T \end{bmatrix}, \quad \hat{\Psi}_{31} = \begin{bmatrix} \tau_m P_i A_i + \tau_m \bar{G}_i C_i \\ \sqrt{\delta} P_i A_i + \sqrt{\delta} \bar{G}_i C_i \\ L_i \end{bmatrix}$$

$$\hat{\Psi}_{32} = \begin{bmatrix} \tau_m P_i A_{di} + \tau_m \bar{G}_i C_{di} & 0 & 0 & -\tau_m P_i A_{\omega i} - \tau_m \bar{G}_i C_{\omega i} \\ \sqrt{\delta} P_i A_{di} + \sqrt{\delta} \bar{G}_i C_{di} & 0 & 0 & -\sqrt{\delta} P_i A_{\omega i} - \sqrt{\delta} \bar{G}_i C_{\omega i} \\ L_{di} & 0 & 0 & -L_{\omega i} \end{bmatrix}$$

$$\hat{\Psi}_{33} = diag\{-2\varepsilon P_i + \varepsilon^2 R_0, -2\varepsilon P_i + \varepsilon^2 R_1, -I\}$$

and Ψ_{22}, $\Psi_{41}(s)$, $\Psi_{42}(s)$ *and* δ *are as defined in Theorem 3.1.*

Moreover, the filter gain in the form of (5) as following:

$$G_i = P_i^{-1} \bar{G}_i \tag{31}$$

Proof: Defining $\bar{G}_i = P_i G_i$, from (11) and using schur complement, the matrix inequality (11) holds if and only if

$$\breve{\Psi} = \begin{bmatrix} \hat{\Psi}_{11} & * & * & * \\ \hat{\Psi}_{21} & \Psi_{22} & * & * \\ \hat{\Psi}_{31} & \hat{\Psi}_{32} & \breve{\Psi}_{33} & * \\ \Psi_{41}(s) & \Psi_{42}(s) & 0 & -R_1 \end{bmatrix} < 0, \quad s = 1, 2 \tag{32}$$

where

$$\breve{\Psi}_{33} = diag\{-P_i R_0^{-1} P_i, -P_i R_1^{-1} P_i, -I\}$$

Due to

$$(R_k - \varepsilon^{-1} P_i)^{-1} (R_k - \varepsilon^{-1} P_i) \geq 0, \quad k = 0, 1, \quad i \in \mathcal{S} \tag{33}$$

which gives

$$-P_i R_k^{-1} P_i \leq -2\varepsilon P_i + \varepsilon^2 R_k, \quad k = 0, 1, \quad i \in \mathcal{S} \tag{34}$$

Substituting $-P_i R_k^{-1} P_i$ with $-2\varepsilon P_i + \varepsilon^2 R_k$ ($k = 0, 1$) in (32), we obtain (29), so, if (29) holds, we have (11) holds, and from above proof, we have $G_i = P_i^{-1} \bar{G}_i$. This completes the proof. □

Remark 3.3. *The inequality (34) is used to bound the term* $-P_i R_k^{-1} P_i$*. This step also can be improved by adopting the cone complementary algorithm* [34]*, which is popular in recent control designs. Here the scaling parameter* $\varepsilon > 0$ *can be used to improve conservatism in Theorem 3.2.*

4. **Numerical Example.**

In this section, well-studied examples are used to illustrate the the effectiveness of the approaches proposed in this paper.

Example 4.1. *Consider a Markovian jump system in (6) with two modes and the following parameters* [25]*:*

$$\bar{A}_1 = \begin{bmatrix} -2.2460 & -1.4410 \\ -1.5937 & -2.9289 \end{bmatrix}, \ \bar{A}_2 = \begin{bmatrix} -1.8999 & 0.8156 \\ 0.6900 & -0.7881 \end{bmatrix}, \ \bar{A}_{d1} = \begin{bmatrix} -0.7098 & 1.1908 \\ 0.6686 & -3.2025 \end{bmatrix},$$

$$\bar{A}_{d2} = \begin{bmatrix} -1.5198 & -1.6041 \\ -0.1567 & -1.2427 \end{bmatrix}, \ \bar{A}_{\omega 1} = \begin{bmatrix} 0.0403 \\ 0.6771 \end{bmatrix}, \ \bar{A}_{\omega 2} = \begin{bmatrix} 0.5689 \\ -0.2556 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -0.3775 & -0.2959 \end{bmatrix}, \ L_2 = \begin{bmatrix} -1.4751 & -0.2340 \end{bmatrix}, \ L_{d1} = \begin{bmatrix} 0 & 0 \end{bmatrix},$$

$$L_{d2} = \begin{bmatrix} 0 & 0 \end{bmatrix}, \ L_{\omega 1} = -0.1184, \ L_{\omega 2} = -0.3148.$$

Suppose the transition probability matrix is given by $\pi_{11} = -3$.

For several values of μ *and* π_{22}*, the computation results of* τ_M *are listed in Tables 1 – 3.* *Tables 1 – 3 show the maximum allowable values of* τ_M *for* $\mu = 0.5$ *and* $\pi_{22} = -0.6$, $\mu = 1$ *and* $\pi_{22} = -0.6$, $\mu = 1$ *and* $\pi_{22} = -1$*, respectively. From Tables 1 – 3, we can see our method can lead to less conservative results.*

TABLE 1. Maximum allowable values of τ_M for $\mu = 0.5$ and $\pi_{22} = -0.6$

γ	0.5	1	1.5	2
τ_M by [25]	0.3657	0.4062	0.4168	0.4219
τ_M by Theorem 3.1	0.3968	0.4388	0.4510	0.4569

TABLE 2. Maximum allowable values of τ_M for $\mu = 1$ and $\pi_{22} = -0.6$

γ	0.5	1	1.5	2
τ_M by [25]	0.2772	0.2846	0.2857	0.2861
τ_M by Theorem 3.1	0.3966	0.4355	0.4468	0.4520

TABLE 3. Maximum allowable values of τ_M for $\mu = 1$ and $\pi_{22} = -1$

γ	0.5	1	1.5	2
τ_M by [25]	0.2773	0.2846	0.2857	0.2861
τ_M by Theorem 3.1	0.3896	0.4296	0.4408	0.4463

To illustrate the proposed method on filtering design, another example is considered as follows.

Example 4.2. *Consider linear Markovian jump systems in the form (1) with two mode. For modes 1 and 2, the dynamics of system are described as*

$$A_1 = \begin{bmatrix} -3 & 1 & 0 \\ 0.3 & -2.5 & 1 \\ -0.1 & 0.3 & -3.8 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -0.2 & 0.1 & 0.6 \\ 0.5 & -1 & -0.8 \\ 0 & 1 & -2.5 \end{bmatrix}, \quad A_{\omega 1} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},$$
$$C_1 = \begin{bmatrix} 0.8 & 0.3 & 0 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} 0.2 & -0.3 & -0.6 \end{bmatrix}, \quad C_{\omega 1} = 0.2,$$
$$L_1 = \begin{bmatrix} 0.5 & -0.1 & 1 \end{bmatrix}, \quad L_{d1} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad L_{\omega 1} = 0,$$
$$A_2 = \begin{bmatrix} -2.5 & 0.5 & -0.1 \\ 0.1 & -3.5 & 0.3 \\ -0.1 & 1 & -2 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0 & -0.3 & 0.6 \\ 0.1 & 0.5 & 0 \\ -0.6 & 1 & -0.8 \end{bmatrix}, \quad A_{\omega 2} = \begin{bmatrix} -0.6 \\ 0.5 \\ 0 \end{bmatrix},$$
$$C_2 = \begin{bmatrix} 0.5 & 0.2 & 0.3 \end{bmatrix}, \quad C_{d2} = \begin{bmatrix} 0 & -0.6 & 0.2 \end{bmatrix}, \quad C_{\omega 2} = 0.5,$$
$$L_2 = \begin{bmatrix} 0 & 1 & 0.6 \end{bmatrix}, \quad L_{d2} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}, \quad L_{\omega 2} = 0.$$

Suppose the transition probability matrix is given by $\pi = \begin{bmatrix} -0.5 & 0.5 \\ 0.3 & -0.3 \end{bmatrix}$ *and the initial conditions* $x(0) = \begin{bmatrix} 0.8 & 0.2 & -0.9 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} 0 & 0.2 & 0 \end{bmatrix}^T$. *This system is nominal one consider in* [24]. *By Theorem 3.2, we get the maximum time delay* $\tau_M = 6.4763$ *for* $\tau_m = 0$, $\mu = 0$, $\varepsilon = 10$ *and* $\gamma = 1.2$. *This upper bound is much larger than the one* $\tau_M = 1.9195$ *given by* [24]*, which shows our method has less conservative than that of* [24].

The corresponding filter are given by

$$G_1 = \begin{bmatrix} 0.5399 \\ -1.1596 \\ -3.5158 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.4684 \\ 0.3923 \\ -1.3604 \end{bmatrix}$$

To illustrate the performance of the designed filter, choose the disturbance function as follows

$$\omega(t) = \begin{cases} -0.5, & 5 < t < 10 \\ 0.5, & 15 < t < 20 \\ 0, & \text{otherwise} \end{cases}$$

With this filter, Figures 1 – 4 show the operation modes of the MJS, interval time-varying delay, estimated signal $z(t)$, $\tilde{z}(t)$ and estimated signals error $\eta(t) = z(t) - \tilde{z}(t)$, respectively.

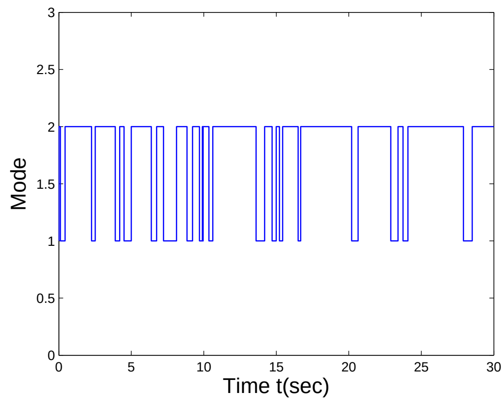

FIGURE 1. Operation modes of MJS

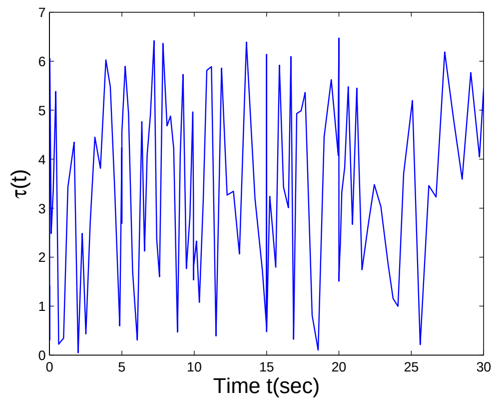

FIGURE 2. Interval time-varying delay

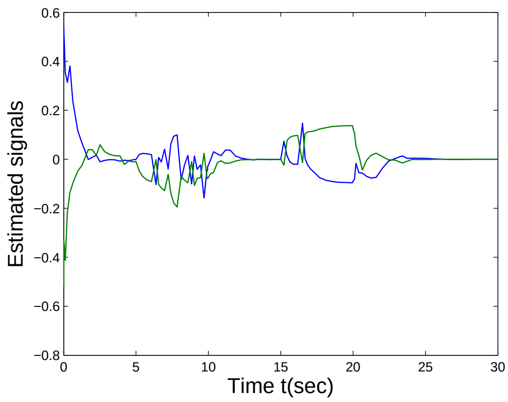

FIGURE 3. Estimated signal $z(t)$ and $\tilde{z}(t)$

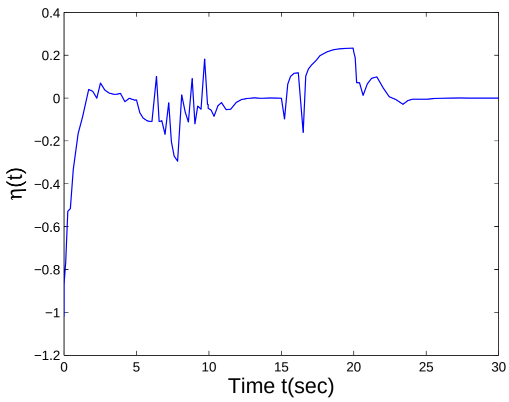

FIGURE 4. Estimated signals error $\eta(t) = z(t) - \tilde{z}(t)$

5. **Conclusions.** In this paper, we have studied a class of H_∞ filter design for Markovian jump systems with time-varying delays via manipulating a new Lyapunov function and using the convexity property of the matrix inequality. With the proposed method, an LMI-based sufficient condition for the existence of the desired H_∞ filter has been derived,

which can lead to much less conservative analysis results. Finally, Numerical examples have been carried out to demonstrate the effectiveness of the proposed method.

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