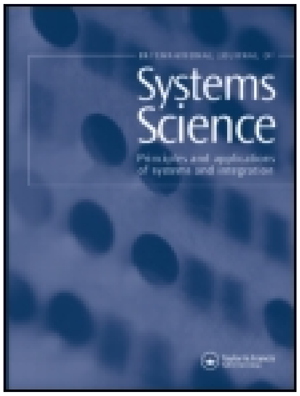

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New results on H_∞ filter design for nonlinear systems with time-delay through a T-S fuzzy model approach

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New results on H_∞ filter design for nonlinear systems with time-delay through a T-S fuzzy model approach

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H_∞ filter design for nonlinear systems with time-delay via a T-S fuzzy model approach is investigated based on a piecewise analysis method. Two cases of time-varying delays are fully considered: one is the time-varying delay being continuous uniformly bounded while the other is the time-varying delay being differentiable uniformly bounded with delay-derivative bounded by a constant. Based on a piecewise analysis method, the variation interval of the time delay is first divided into several subintervals, then the convexity property of the matrix inequality and the free weighting matrix method are fully used in this article. Some novel delay-dependent H_∞ filtering criteria are expressed as a set of linear matrix inequalities, which can lead to much less conservative analysis results. Finally, a numerical example is given to illustrate that the results in this article are more effective and less conservative than some existing ones.

Keywords: H_∞ filter; linear matrix inequalities; time delay; fuzzy systems

1. Introduction

The nonlinear filtering problem has long been one of the fundamental problems in signal processing, communication and control applications. The problem of filtering can be briefly described as the design of an estimator from the measured output to estimate the state of the given systems. During the past few decades, the H_∞ filtering technique introduced in Elsayed and Grimble (1989) has received increasing attention (for example Ariba and Gouaisbaut 2007; Peng and Tian 2008; Gao, Meng, and Chen 2008a,b; Zhang and Han 2008a,b; Gao, Zhao, Lam, and Chen 2009; Gao, Meng, and Chen 2009 and the references therein). One of its main advantages is that it is insensitive to the exact knowledge of the statistics of the noise signals.

Recently, the problem of H_∞ filtering of linear/nonlinear time-delay systems has been given much attention due to the fact that it has many practical applications. Time delays cannot be neglected in the procedure of filter design and their existence usually results in poor performance (Wang and Ho 2003; Wang, Ho, and Liu 2004; Nguang and Shi 2007; Xiao, Xi, Zhu, and Ji 2008). Some nice results on H_∞ filtering for time-delay systems have been reported in the literature and there are two kinds of results, namely delay-independent filtering (de Souza, Palhares, and Peres 2001) and delay-dependent filtering (Yue and Han 2006; Yue, Han, and Lam 2008; Zhang and Han 2008; Su, Chen, Lin, and Zhang 2009; Qiu, Feng, Yang, and Sun 2009; Zhang, Xia, and Tao 2009; Liu, Yu, Gu, and Hu 2010). The delay-dependent results are usually less conservative, especially when the time-delay is small. The main objective of the delay-dependent H_∞ filtering is to obtain a filter such that the filtering error system allows a maximum delay bound for a fixed H_∞ performance or achieves a minimum H_∞ performance for a given delay bound.

During the past two decades, the T-S fuzzy model (Takagi and Sugeno 1985) has been recognised as a powerful tool in approximating complex nonlinear systems by some simple local linear dynamic systems, and some analysis methods in the linear systems can be effectively extended to the T-S fuzzy systems. Consequently, much effort has been made to investigate T-S fuzzy systems and various techniques have been obtained (Liu and Zhang 2003; Guan and Chen 2004; Gao, Wang, and Wang 2005; Tian and Peng 2006; Montagner, Oliveira, and Peres 2009).

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For fuzzy H_∞ filtering, two main cases of time-varying delay should be considered:

(1) the time-varying delay being continuous uniformly bounded and
(2) the time-varying delay being differentiable uniformly bounded with delay derivative bounded by a constant.

For the first case, to the best of our knowledge, few results on fuzzy H_∞ filtering have been discussed in the literature. Therefore, it is significant to pay more attention to this case since this kind of time-varying delay exists in networked control systems (Yue, Qing-long, and Peng 2004), which is one motivation for this research. For the second case, when the bound of the time derivative of the time-varying delay is less than one, which does not allow the fast time-varying delay, some useful results on fuzzy H_∞ filtering have been obtained (Yang, Wang, and Lin 2007; Lin, Wang, Lee, and Chen 2008; Qiu et al. 2009; Su et al. 2009; Zhang et al. 2009).

In this article, we have studied the problem of H_∞ filter design for nonlinear systems with time delay via a T-S fuzzy model approach, where two cases of time-varying delay have been studied. Combining the piecewise analysis method in Yue, Han, and Lam (2005) and Yue, Tian, and Zhang (2009a) and employing the convexity property of the matrix inequality, novel criteria for the H_∞ performance analysis are derived. Based on the derived criteria for the H_∞ performance analysis of the filtering-error system, novel H_∞ filter design criteria are obtained. An example used in Lin et al. (2008), Qiu et al. (2009), Su et al. (2009) and Zhang et al. (2009) is employed to show the effectiveness and less conservativeness of the proposed method.

Notation $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices, the superscript 'T' stands for matrix transposition, I is the identity matrix of appropriate dimension. $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. The notation $X>0$ (respectively $X\geq 0$) for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C\end{bmatrix}$ denotes a symmetric matrix, where * denotes the entries implied by symmetry.

2. Systems description and preliminaries

Consider a nonlinear system with time delay which could be approximated by a time delay T-S fuzzy model with r plant rules.

Plant rule i: IF $\theta_1(t)$ is $W_1^i,\cdots$ and $\theta_g(t)$ is W_g^i, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di}x(t-\tau(t)) + A_{\omega i}\omega(t) \\ y(t) = C_i x(t) + C_{di}x(t-\tau(t)) + C_{\omega i}\omega(t) \\ z(t) = L_i x(t) + L_{di}x(t-\tau(t)) + L_{\omega i}\omega(t) \\ x(t) = \varphi(t),\, t\in[-\tau_M, 0], \end{cases} \tag{1}$$

where $\theta_1(t), \theta_2(t),\ldots,\theta_g(t)$ are the premise variables, and W_j^i $(i=1,2,\ldots,r, j=1,2,\ldots,g)$ are fuzzy sets, r is the number of IF-THEN rules, $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^m$ and $z(t)\in\mathbb{R}^p$ are the state vector, output vector and the signal to be estimated, A_i, A_{di}, $A_{\omega i}$, C_i, C_{di}, $C_{\omega i}$, L_i, L_{di} and $L_{\omega i}$ are parameter matrices with appropriate dimensions, $\omega(t)\in L_2[0,\infty)$ denotes the exogenous disturbance signal and $\varphi(t)$ is a continuous vector-valued initial function on $[-\tau_M, 0]$.

$\tau(t)$ is a time-varying delay which will be treated as the following two cases:

Case I: $\tau(t)$ is a continuous function satisfying

$$0\leq\tau_m\leq\tau(t)\leq\tau_M\leq\infty \quad \forall t\geq 0. \tag{2}$$

Case II: $\tau(t)$ is a continuous function satisfying

$$0\leq\tau_m\leq\tau(t)\leq\tau_M\leq\infty, \dot{\tau}(t)\leq d<\infty \quad \forall t\geq 0. \tag{3}$$

By using a centre-average defuzzifier, product interference and singleton fuzzifier, the global dynamics of (1) can be inferred as

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_d(t)x(t-\tau(t)) + A_\omega(t)\omega(t) \\ y(t) = C(t)x(t) + C_d(t)x(t-\tau(t)) + C_\omega(t)\omega(t) \\ z(t) = L(t)x(t) + L_d(t)x(t-\tau(t)) + L_\omega(t)\omega(t), \end{cases} \tag{4}$$

where

$$A(t)=\sum_{i=1}^r h_i A_i,\, A_d(t)=\sum_{i=1}^r h_i A_{di},\, A_\omega(t)=\sum_{i=1}^r h_i A_{\omega i},$$
$$C(t)=\sum_{i=1}^r h_i C_i,\, C_d(t)=\sum_{i=1}^r h_i C_{di},\, C_\omega(t)=\sum_{i=1}^r h_i C_{\omega i},$$
$$L(t)=\sum_{i=1}^r h_i L_i,\, L_d(t)=\sum_{i=1}^r h_i L_{di},\, L_\omega(t)=\sum_{i=1}^r h_i L_{\omega i},$$

h_i is the abbreviation for $h_i(\theta(t))$, and

$$h_i(\theta(t))=\frac{\alpha_i(\theta(t))}{\sum_{i=1}^r \alpha_i(\theta(t))},\quad \alpha_i(\theta(t))=\prod_{j=1}^g W_j^i(\theta_j(t)),$$

$W_j^i(\theta_j(t))$ is the grade membership value of $\theta_j(t)$ in W_j^i and $h_i(\theta(t))$ satisfies

$$h_i(\theta(t))\geq 0,\qquad \sum_{i=1}^r h_i(\theta(t))=1.$$

In this article, we will design the following H_∞ fuzzy filter,

Filter rule i: IF $\theta_1(t)$ is W_1^i, $\cdots$ and $\theta_g(t)$ is W_g^i, THEN

$$\begin{cases} \dot{x}_f(t) = A_{fi}x_f(t) + B_{fi}y(t) \\ z_f(t) = C_{fi}x_f(t) + D_{fi}y(t), \end{cases} \tag{5}$$

where $x_f(t) \in \mathbb{R}^n$ and $z_f(t) \in \mathbb{R}^p$ are the state and output of the filter, respectively. The matrices $A_{fi} \in \mathbb{R}^{n\times n}$, $B_{fi} \in \mathbb{R}^{n\times m}$, $C_{fi} \in \mathbb{R}^{p\times n}$ and $D_{fi} \in \mathbb{R}^{p\times m}$ are to be determined.

The defuzzified output of (5) is referred to by

$$\begin{cases} \dot{x}_f(t) = A_f(t)x_f(t) + B_f(t)\,y(t) \\ z_f(t) = C_f(t)x_f(t) + D_f(t)\,y(t), \end{cases} \tag{6}$$

where

$$A_f(t) = \sum_{i=1}^r h_i A_{fi}, \quad B_f(t) = \sum_{i=1}^r h_i B_{fi},$$
$$C_f(t) = \sum_{i=1}^r h_i C_{fi}, \quad D_f(t) = \sum_{i=1}^r h_i D_{fi}.$$

Defining the augmented state vector $e(t) = \left[\begin{smallmatrix} x(t) \\ x_f(t) \end{smallmatrix}\right]$ and $\tilde{z}(t) = z(t) - z_f(t)$, we can obtain the following filtering-error system:

$$\begin{cases} \dot{e}(t) = \hat{A}_{ij}e(t) + \hat{A}_{dij}x(t-\tau(t)) + \hat{A}_{\omega ij}\omega(t) \\ \tilde{z}(t) = \hat{L}_{ij}e(t) + \hat{L}_{dij}x(t-\tau(t)) + \hat{L}_{\omega ij}\omega(t), \end{cases} \tag{7}$$

where

$$\hat{A}_{ij} = \sum_{i=1}^r\sum_{j=1}^r h_ih_j \begin{bmatrix} A_i & 0 \\ B_{fj}C_i & A_{fj} \end{bmatrix},$$
$$\hat{A}_{dij} = \sum_{i=1}^r\sum_{j=1}^r h_ih_j \begin{bmatrix} A_{di} \\ B_{fj}C_{di} \end{bmatrix},$$
$$\hat{A}_{\omega ij} = \sum_{i=1}^r\sum_{j=1}^r h_ih_j \begin{bmatrix} A_{\omega i} \\ B_{fj}C_{\omega i} \end{bmatrix},$$
$$\hat{L}_{ij} = \sum_{i=1}^r\sum_{j=1}^r h_ih_j \begin{bmatrix} L_i - D_{fj}C_i & -C_{fj} \end{bmatrix},$$
$$\hat{L}_{dij} = \sum_{i=1}^r\sum_{j=1}^r h_ih_j\,(L_{di} - D_{fj}C_{di}),$$
$$\hat{L}_{\omega ij} = \sum_{i=1}^r\sum_{j=1}^r h_ih_j\,(L_{\omega i} - D_{fj}C_{\omega i}).$$

Remark 1: For $L_\omega(t) = 0$, the system (4) reduces to the system (3) in Lin et al. (2008), and the filter design problem (5), for $D_f(t) = 0$, reduces to the filter system (4) in Lin et al. (2008). Hence, our model includes the filter design problem in Lin et al. (2008) as a special case.

The H_∞ filtering problem addressed in this article is to design a filter of form (5) such that

- the filtering-error system (7) with $\omega(t) = 0$ is asymptotically stable,
- the H_∞ performance $\|\tilde{z}(t)\|_2 < \gamma\|\omega(t)\|_2$ is guaranteed for all non-zero $\omega(t) \in L_2\ [0,\infty)$ and a prescribed $\gamma > 0$ under the condition $e(t) = 0\ \forall t \in [-\tau_M, -\tau_m]$.

The following lemmas are needed in the proof of our main results.

Lemma 1 (Gu, Kharitonov, and Chen 2003)**:** *For any constant matrix $R \in \mathbb{R}$, $R = R^T > 0$, constant $\tau_M > 0$ and vector function $\dot{x}: [-\tau_M, 0] \to \mathbb{R}^n$ such that the following integration is well defined, it holds that*

$$-\tau_M\int_{t-\tau_M}^t \dot{x}^T(s)R\dot{x}(s)\mathrm{d}s \le \begin{bmatrix} x(t) \\ x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_M) \end{bmatrix}. \tag{8}$$

Lemma 2 (Yue, Tian, Zhang, and Peng 2009b)**:** *Suppose $0 \le \tau_m \le \tau(t) \le \tau_M$, Ξ_1, Ξ_2 and Ω are constant matrices of appropriate dimensions, then*

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \tag{9}$$

if and only if

$$(\tau_M - \tau_m)\Xi_1 + \Omega < 0 \tag{10}$$

and

$$(\tau_M - \tau_m)\Xi_2 + \Omega < 0 \tag{11}$$

hold.

3. H_∞ performance analysis

In this section, we will concentrate our attention on the performance analysis for the filtering-error system (7) for $\tau(t)$ satisfying Case I or Case II.

Similar to Yue et al. (2005) and Yue et al. (2009a), we divide the variation interval of the delay into l parts with equal length. Define

$$\tau_i = \tau_m + \frac{i(\tau_M - \tau_m)}{l}, \quad i = 1, 2, \ldots, l. \tag{12}$$

Then, $[\tau_m, \tau_M] = [\tau_m, \tau_1]\bigcup_{i=1}^{l-1}(\tau_i, \tau_{i+1}]$. In the proof of our main results, we discuss the cases when $l = 2$ and $l = 3$. From the following sections, it can be seen that the proposed method of this article can also be easily extended to the case with l being any finite integer.

In the following two sections, stability criteria of the filtering error system (7) for $l = 2$ and $l = 3$ will be derived, respectively, based on the Lyapunov functional method and the piecewise analysis method.

3.1. Stability criteria for $l = 2$

Define

$$\delta = \frac{\tau_M - \tau_m}{2}.$$

Then, $\tau_1 = \tau_m + \delta = \frac{(\tau_m + \tau_M)}{2}$ denotes the central point of variation of the delay $\tau(t)$.

Furthermore, define a new vector

$$\zeta^T(t) = \begin{bmatrix} e^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_m) & x^T(t-\tau_1) & x^T(t-\tau_M) & \omega^T(t) \end{bmatrix}$$

and two matrices

$$\Gamma_1 = [\hat{A}_{ij} \quad \hat{A}_{dij} \quad 0 \quad 0 \quad 0 \quad \hat{A}_{\omega ij}],$$
$$\Gamma_2 = [\hat{L}_{ij} \quad \hat{L}_{dij} \quad 0 \quad 0 \quad 0 \quad \hat{L}_{\omega ij}].$$

Rewrite (7) as

$$\begin{cases} \dot{e}(t) = \Gamma_1 \zeta(t) \\ \tilde{z}(t) = \Gamma_2 \zeta(t). \end{cases} \tag{13}$$

On the basis of (13), we get the following results.

3.1.1. H_∞ performance analysis for case I

Theorem 1: *Under Case* I, *for given constants τ_m, τ_M and γ, the system (7) is asymptotically stable with the H_∞-norm bound γ if there exist $P > 0$, $Q_0 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_0 > 0$, $R_1 > 0$, $R_2 > 0$, M_{ijk}, N_{ijk}, S_{ijk} and T_{ijk} ($i, j \in \mathbb{S}$, $k = 1, 2, \ldots, 6$) with appropriate dimensions such that*

$$\Psi_1^{ij}(l) + \Psi_1^{ji}(l) < 0, \tag{14}$$

$$\Psi_2^{ij}(l) + \Psi_2^{ji}(l) < 0, \quad l = 1, 2, \ i \le j \in \mathbb{S}, \tag{15}$$

where

$$\Psi_1^{ij}(l) = \begin{bmatrix} \Psi_{11} & * & * & * \\ \Psi_{21} & \Psi_{22} & * & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * \\ \Psi_{41}(l) & \Psi_{42}(l) & 0 & -R_1 \end{bmatrix}, \quad \Psi_2^{ij}(l) = \begin{bmatrix} \Psi_{11} & * & * & * \\ \hat{\Psi}_{21} & \hat{\Psi}_{22} & * & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * \\ \hat{\Psi}_{41}(l) & \hat{\Psi}_{42}(l) & 0 & -R_2 \end{bmatrix}$$

$$\Psi_{11} = P\hat{A}_{ij} + \hat{A}_{ij}^T P + H^T(Q_0 + Q_1 + Q_2 - R_0)H,$$

$$\Psi_{21} = \begin{bmatrix} \hat{A}_{dij}^T P - N_{ij1}^T + M_{ij1}^T \\ R_0 H + N_{ij1}^T \\ -M_{ij1}^T \\ 0 \\ \hat{A}_{\omega ij}^T P \end{bmatrix}, \quad \hat{\Psi}_{21} = \begin{bmatrix} \hat{A}_{dij}^T P - T_{ij1}^T + S_{ij1}^T \\ R_0 H \\ T_{ij1}^T \\ -S_{ij1}^T \\ \hat{A}_{\omega ij}^T P \end{bmatrix},$$

$$\Psi_{22} = \begin{bmatrix} -N_{ij2} - N_{ij2}^T + M_{ij2} + M_{ij2}^T & * & * & * & * \\ -N_{ij3} + N_{ij2}^T + M_{ij3} & -Q_0 - R_0 + N_{ij3} + N_{ij3}^T & * & * & * \\ -N_{ij4} + M_{ij4} - M_{ij2}^T & N_{ij4} - M_{ij3}^T & -Q_1 - M_{ij4} - M_{ij4}^T - \frac{R_2}{\delta} & * & * \\ -N_{ij5} + M_{ij5} & N_{ij5} & -M_{ij5} + \frac{R_2}{\delta} & -\frac{R_2}{\delta} - Q_2 & * \\ -N_{ij6} + M_{ij6} & N_{ij6} & -M_{ij6} & 0 & -\gamma^2 I \end{bmatrix},$$

$$\hat{\Psi}_{22} = \begin{bmatrix} -T_{ij2} - T_{ij2}^T + S_{ij2} + S_{ij2}^T & * & * & * & * \\ -T_{ij3} + S_{ij3} & -Q_0 - R_0 - \frac{R_1}{\delta} & * & * & * \\ -T_{ij4} + S_{ij4} + T_{ij2}^T & \frac{R_1}{\delta} + T_{ij3}^T & -Q_1 + T_{ij4} + T_{ij4}^T - \frac{R_1}{\delta} & * & * \\ -T_{ij5} + S_{ij5} - S_{ij2}^T & -S_{ij3}^T & T_{ij5} - S_{ij4}^T & -Q_2 - S_{ij5} - S_{ij5}^T & * \\ -T_{ij6} + S_{ij6} & 0 & T_{ij6} & -S_{ij6} & -\gamma^2 I \end{bmatrix}$$

$$\Psi_{31} = \begin{bmatrix} \hat{L}_{ij} \\ \tau_0 R_0 H\hat{A}_{ij} \\ \sqrt{\delta} R_1 H\hat{A}_{ij} \\ \sqrt{\delta} R_2 H\hat{A}_{ij} \end{bmatrix}, \quad \Psi_{32} = \begin{bmatrix} \hat{L}_{dij} & 0 & 0 & 0 & \hat{L}_{\omega ij} \\ \tau_0 R_0 H\hat{A}_{dij} & 0 & 0 & 0 & \tau_0 R_0 H\hat{A}_{\omega ij} \\ \sqrt{\delta} R_1 H\hat{A}_{dij} & 0 & 0 & 0 & \sqrt{\delta} R_1 H\hat{A}_{\omega ij} \\ \sqrt{\delta} R_2 H\hat{A}_{dij} & 0 & 0 & 0 & \sqrt{\delta} R_2 H\hat{A}_{\omega ij} \end{bmatrix},$$

$$\Psi_{33} = \mathrm{diag}\{-I,\ -R_0,\ -R_1,\ -R_2\},$$

$$\Psi_{41}(1) = \sqrt{\delta} N_{ij1}^T, \quad \Psi_{41}(2) = \sqrt{\delta} M_{ij1}^T, \quad \hat{\Psi}_{41}(1) = \sqrt{\delta} T_{ij1}^T, \quad \hat{\Psi}_{41}(2) = \sqrt{\delta} S_{ij1}^T,$$

$$\Psi_{42}(1) = \begin{bmatrix} \sqrt{\delta} N_{ij2}^T & \sqrt{\delta} N_{ij3}^T & \sqrt{\delta} N_{ij4}^T & \sqrt{\delta} N_{ij5}^T & 0 \end{bmatrix}, \quad \Psi_{42}(2) = \begin{bmatrix} \sqrt{\delta} M_{ij2}^T & \sqrt{\delta} M_{ij3}^T & \sqrt{\delta} M_{ij4}^T & \sqrt{\delta} M_{ij5}^T & 0 \end{bmatrix},$$

$$\hat{\Psi}_{42}(1) = \begin{bmatrix} \sqrt{\delta} T_{ij2}^T & \sqrt{\delta} T_{ij3}^T & \sqrt{\delta} T_{ij4}^T & \sqrt{\delta} T_{ij5}^T & 0 \end{bmatrix}, \quad \hat{\Psi}_{42}(2) = \begin{bmatrix} \sqrt{\delta} S_{ij2}^T & \sqrt{\delta} S_{ij3}^T & \sqrt{\delta} S_{ij4}^T & \sqrt{\delta} S_{ij5}^T & 0 \end{bmatrix},$$

$$H = \begin{bmatrix} I & 0 \end{bmatrix}, \quad \mathbb{S} = 1, 2, \ldots, r.$$

Proof: Construct a Lyapunov functional candidate as

$$\begin{aligned} V(t, e_t) = {} & e^T(t)Pe(t) + \int_{t-\tau_m}^{t} e^T(s)H^T Q_0 He(s)\mathrm{d}s \\ & + \int_{t-\tau_1}^{t} e^T(s)H^T Q_1 He(s)\mathrm{d}s \\ & + \int_{t-\tau_M}^{t} e^T(s)H^T Q_2 He(s)\mathrm{d}s \\ & + \int_{t-\tau_1}^{t-\tau_m} \int_{s}^{t} \dot{e}^T(v)H^T R_1 H\dot{e}(v)\mathrm{d}v\,\mathrm{d}s \\ & + \int_{t-\tau_M}^{t-\tau_1} \int_{s}^{t} \dot{e}^T(v)H^T R_2 H\dot{e}(v)\mathrm{d}v\,\mathrm{d}s \\ & + \tau_m \int_{t-\tau_m}^{t} \int_{s}^{t} \dot{e}^T(v)H^T R_0 H\dot{e}(v)\mathrm{d}v\,\mathrm{d}s, \end{aligned} \qquad (16)$$

where $P > 0$, $Q_0 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_0 > 0$, $R_1 > 0$, $R_2 > 0$ are to be determined. Then, the proof can be completed by using a similar way in Theorem 3, we omit it for brevity.

Remark 2: Throughout the proof of Theorem 1, it can be seen that we need not enlarge $\tau(t)$ to τ_M, therefore the common existing conservatism caused by this kind of enlargement in Chen, Liu, and Tong (2006), Lien (2006), Tian and Peng (2006), Jiang and Han (2007) and Wu and Li (2007) can be avoided, which will reduce the conservative of the result.

3.1.2. *H_∞ performance analysis for case II*

For Case II, we chose a Lyapunov functional candidate as

$$\begin{aligned} V(t, e_t) = {} & e^T(t)Pe(t) + \int_{t-\tau_m}^{t} e^T(s)H^T Q_0 He(s)\mathrm{d}s \\ & + \int_{t-\tau(t)}^{t} e^T(s)H^T Q_1 He(s)\mathrm{d}s \\ & + \int_{t-\delta}^{t} \lambda^T(s) Q \lambda(s)\mathrm{d}s \\ & + \tau_m \int_{t-\tau_m}^{t} \int_{s}^{t} \dot{e}^T(v)H^T R_0 H\dot{e}(v)\mathrm{d}v\,\mathrm{d}s \\ & + \int_{t-\tau_1}^{t-\tau_m} \int_{s}^{t} \dot{e}^T(v)H^T R_1 H\dot{e}(v)\mathrm{d}v\,\mathrm{d}s \\ & + \int_{t-\tau_M}^{t-\tau_1} \int_{s}^{t} \dot{e}^T(v)H^T R_2 H\dot{e}(v)\mathrm{d}v\,\mathrm{d}s, \end{aligned} \qquad (17)$$

where

$$\lambda^T(t) = \begin{bmatrix} x^T(t-\tau_m) & x^T(t-\tau_1) \end{bmatrix}.$$

Then, similar to the proof of Theorem 1, we can conclude the following result.

Theorem 2: *Under Case* II, *for given constants τ_m, τ_M, d and γ, the system* (7) *is asymptotically stable with the H_∞ norm bound γ if there exist* $P > 0$, $Q_0 > 0$, $Q_1 > 0$, $R_0 > 0$, $R_1 > 0$, $R_2 > 0$, $Q = \left[\begin{smallmatrix} Q_{11} & Q_{21}^T \\ Q_{21} & Q_{22} \end{smallmatrix}\right] > 0$, M_{ijk}, N_{ijk}, S_{ijk} and T_{ijk} $(i,\ j \in \mathbb{S},\ k = 1, 2, \ldots, 5)$ *with appropriate dimensions such that*

$$\Phi_1^{ij}(l) + \Phi_1^{ji}(l) < 0, \qquad (18)$$

$$\Phi_2^{ij}(l) + \Phi_2^{ji}(l) < 0, \quad l = 1, 2,\ i \le j \in \mathbb{S}, \qquad (19)$$

where

$$\Phi_1^{ij}(l) = \begin{bmatrix} \Phi_{11} & * & * & * \\ \Psi_{21} & \Phi_{22} & * & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * \\ \Psi_{41}(l) & \Psi_{42}(l) & 0 & -R_1 \end{bmatrix},$$

$$\Phi_2^{ij}(l) = \begin{bmatrix} \Phi_{11} & * & * & * \\ \hat{\Psi}_{21} & \hat{\Phi}_{22} & * & * \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & * \\ \hat{\Psi}_{41}(l) & \hat{\Psi}_{42}(l) & 0 & -R_2 \end{bmatrix},$$

$$\Phi_{11} = P\hat{A}_{ij} + \hat{A}_{ij}^T P + H^T(Q_0 + Q_1 - R_0)H,$$

$$\Phi_{22} = \begin{bmatrix} \nu_1 & * & * & * & * \\ -N_{ij3} + N_{ij2}^T + M_{ij3} & \nu_2 & * & * & * \\ -N_{ij4} + M_{ij4} - M_{ij2}^T & Q_{21} + N_{ij4} - M_{ij3}^T & \nu_3 & * & * \\ -N_{ij5} + M_{ij5} & N_{ij5} & -Q_{21} - M_{ij5} + \frac{R_2}{\delta} & -Q_{22} - \frac{R_2}{\delta} & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\hat{\Phi}_{22} = \begin{bmatrix} \mu_1 & * & * & * & * \\ -T_{ij3} + S_{ij3} & \mu_2 & * & * & * \\ -T_{ij4} + S_{ij4} + T_{ij2}^T & Q_{21} + T_{ij3}^T + \frac{R_1}{\delta} & \mu_3 & * & * \\ -T_{ij5} + S_{ij5} - S_{ij2}^T & -S_{ij3}^T & -Q_{21} + T_{ij5} - S_{ij4}^T & -Q_{22} - S_{ij5} - S_{ij5}^T & * \\ 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\nu_1 = -(1-d)Q_1 - N_{ij2} - N_{ij2}^T + M_{ij2} + M_{ij2}^T, \quad \nu_2 = -Q_0 - R_0 + Q_{11} + N_{ij3} + N_{ij3}^T,$$

$$\nu_3 = Q_{22} - Q_{11} - M_{ij4} - M_{ij4}^T - \frac{R_2}{\delta}, \quad \mu_1 = -(1-d)Q_1 - T_{ij2} - T_{ij2}^T + S_{ij2} + S_{ij2}^T,$$

$$\mu_2 = -Q_0 - R_0 + Q_{11} - \frac{R_2}{\delta}, \quad \mu_3 = -Q_{11} + Q_{22} + T_{ij4} + T_{ij4}^T - \frac{R_1}{\delta},$$

and $\Psi_{21}, \Psi_{31}, \Psi_{32}, \Psi_{33}, \hat{\Psi}_{21}, \Psi_{41}(l), \Psi_{42}(l), \hat{\Psi}_{41}(l), \hat{\Psi}_{42}(l)$, $l = 1, 2$ *are as defined in Theorem* 1.

3.2. *Stability criteria for l = 3*

Define

$$\delta = \frac{\tau_M - \tau_m}{3}.$$

Then,

$$\tau_1 = \tau_m + \delta, \quad \tau_2 = \tau_m + 2\delta.$$

Furthermore, define a new vector

$$\zeta^T(t) = \begin{bmatrix} e^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_m) & x^T(t-\tau_1) & x^T(t-\tau_2) & x^T(t-\tau_M) & \omega^T(t) \end{bmatrix}$$

and two matrices

$$\Gamma_1 = \begin{bmatrix} \hat{A}_{ij} & \hat{A}_{dij} & 0 & 0 & 0 & 0 & \hat{A}_{\omega ij} \end{bmatrix},$$
$$\Gamma_2 = \begin{bmatrix} \hat{L}_{ij} & \hat{L}_{dij} & 0 & 0 & 0 & 0 & \hat{L}_{\omega ij} \end{bmatrix}.$$

Rewrite (7) as

$$\begin{cases} \dot{e}(t) = \Gamma_1 \zeta(t) \\ \tilde{z}(t) = \Gamma_2 \zeta(t). \end{cases} \tag{20}$$

On the basis of (20), we get the following results.

3.2.1. H_∞ *performance analysis for case I*

Theorem 3: *Under Case* I, *for given constants* τ_m, τ_M *and* γ, *the system* (7) *is asymptotically stable with the* H_∞*-norm bound* γ *if there exist* $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $R_4 > 0$, M_{ijk}, N_{ijk}, S_{ijk}, T_{ijk}, W_{ijk} and $V_{ijk}(i, j \in \mathbb{S}, k = 1, 2, \ldots, 6)$ *with appropriate dimensions such that*

$$\Pi_1^{ij}(l) + \Pi_1^{ji}(l) < 0, \tag{21}$$

$$\Pi_2^{ij}(l) + \Pi_2^{ji}(l) < 0, \tag{22}$$

$$\Pi_3^{ij}(l) + \Pi_3^{ji}(l) < 0, \quad l = 1, 2, i \le j \in \mathbb{S}, \tag{23}$$

where

$$\Pi_1^{ij}(l) = \begin{bmatrix} \Pi_{11} & * & * & * \\ \Pi_{21} & \Pi_{22} & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ \Pi_{41}(l) & \Pi_{42}(l) & 0 & -R_2 \end{bmatrix},$$

$$\Pi_2^{ij}(l) = \begin{bmatrix} \Pi_{11} & * & * & * \\ \bar{\Pi}_{21} & \bar{\Pi}_{22} & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ \bar{\Pi}_{41}(l) & \bar{\Pi}_{42}(l) & 0 & -R_3 \end{bmatrix},$$

$$\Pi_3^{ij}(l) = \begin{bmatrix} \Pi_{11} & * & * & * \\ \hat{\Pi}_{21} & \hat{\Pi}_{22} & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ \hat{\Pi}_{41}(l) & \hat{\Pi}_{42}(l) & 0 & -R_4 \end{bmatrix},$$

$$\Pi_{11} = P\hat{A}_{ij} + \hat{A}_{ij}^T P + H^T(Q_1 + Q_2 + Q_3 + Q_4 - R_1)H$$

$$\Pi_{21} = \begin{bmatrix} \hat{A}_{dij}^T P - N_{ij1}^T + M_{ij1}^T \\ R_1 H + N_{ij1}^T \\ -M_{ij1}^T \\ 0 \\ 0 \\ \hat{A}_{\omega ij}^T P \end{bmatrix}, \quad \bar{\Pi}_{21} = \begin{bmatrix} \hat{A}_{dij}^T P - T_{ij1}^T + S_{ij1}^T \\ R_1 H \\ T_{ij1}^T \\ -S_{ij1}^T \\ 0 \\ \hat{A}_{\omega ij}^T P \end{bmatrix}, \hat{\Pi}_{21} = \begin{bmatrix} \hat{A}_{dij}^T P - W_{ij1}^T + V_{ij1}^T \\ R_1 H \\ 0 \\ W_{ij1}^T \\ -V_{ij1}^T \\ \hat{A}_{\omega ij}^T P \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} \Upsilon_1 & * & * & * & * & * \\ -N_{ij3} + M_{ij3} + N_{ij2}^T & \Upsilon_2 & * & * & * & * \\ -N_{ij4} + M_{ij4} - M_{ij2}^T & N_{ij4} - M_{ij3}^T & \Upsilon_3 & * & * & * \\ -N_{ij5} + M_{ij5} & N_{ij5} & -M_{ij5} + \frac{R_3}{\delta} & -Q_3 - \frac{R_3}{\delta} - \frac{R_4}{\delta} & * & * \\ -N_{ij6} + M_{ij6} & N_{ij6} & -M_{ij6} & \frac{R_4}{\delta} & -Q_4 - \frac{R_4}{\delta} & * \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\bar{\Pi}_{22} = \begin{bmatrix} \Upsilon_4 & * & * & * & * & * \\ -T_{ij3} + S_{ij3} & -Q_1 - R_1 - \frac{R_2}{\delta} & * & * & * & * \\ -T_{ij4} + S_{ij4} + T_{ij2}^T & T_{ij3}^T + \frac{R_2}{\delta} & \Upsilon_5 & * & * & * \\ -T_{ij5} + S_{ij5} - S_{ij2}^T & -S_{ij3}^T & -S_{ij4}^T + T_{ij5} & \Upsilon_6 & * & * \\ -T_{ij6} + S_{ij6} & 0 & T_{ij6} & -S_{ij6} + \frac{R_4}{\delta} & -Q_4 - \frac{R_4}{\delta} & * \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\hat{\Pi}_{22} = \begin{bmatrix} \Upsilon_7 & * & * & * & * & * \\ -W_{ij3} + V_{ij3} & -Q_1 - R_1 - \frac{R_2}{\delta} & * & * & * & * \\ -W_{ij4} + V_{ij4} & \frac{R_2}{\delta} & -Q_2 - \frac{R_2}{\delta} - \frac{R_3}{\delta} & * & * & * \\ -W_{ij5} + V_{ij5} + W_{ij2}^T & W_{ij3}^T & W_{ij4}^T + \frac{R_3}{\delta} & \Upsilon_8 & * & * \\ -W_{ij6} + V_{ij6} - V_{ij2}^T & -V_{ij3}^T & -V_{ij4}^T & W_{ij6} - V_{ij5}^T & \Upsilon_9 & * \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Pi_{31} = \begin{bmatrix} \hat{L}_{ij} \\ \tau_m R_1 H \hat{A}_{ij} \\ \sqrt{\delta} R_2 H A_{ij} \\ \sqrt{\delta} R_3 H A_{ij} \\ \sqrt{\delta} R_4 H A_{ij} \end{bmatrix}, \quad \Pi_{32} = \begin{bmatrix} \hat{L}_{dij} & 0 & 0 & 0 & 0 & \hat{L}_{\omega ij} \\ \tau_m R_1 H \hat{A}_{dij} & 0 & 0 & 0 & 0 & \tau_m R_1 H \hat{A}_{\omega ij} \\ \sqrt{\delta} R_2 H A_{dij} & 0 & 0 & 0 & 0 & \sqrt{\delta} R_2 H A_{\omega ij} \\ \sqrt{\delta} R_3 H A_{dij} & 0 & 0 & 0 & 0 & \sqrt{\delta} R_3 H A_{\omega ij} \\ \sqrt{\delta} R_4 H A_{dij} & 0 & 0 & 0 & 0 & \sqrt{\delta} R_4 H A_{\omega ij} \end{bmatrix},$$

$$\Pi_{33} = \mathrm{diag}\{-I, -R_1, -R_2, -R_3, -R_4\},$$

$$\Pi_{41}(1) = \sqrt{\delta} N_{ij1}^T, \quad \Pi_{42}(1) = \begin{bmatrix} \sqrt{\delta} N_{ij2}^T & \sqrt{\delta} N_{ij3}^T & \sqrt{\delta} N_{ij4}^T & \sqrt{\delta} N_{ij5}^T & \sqrt{\delta} N_{ij6}^T & 0 \end{bmatrix},$$

$$\Pi_{41}(2) = \sqrt{\delta} M_{ij1}^T, \quad \Pi_{42}(2) = \begin{bmatrix} \sqrt{\delta} M_{ij2}^T & \sqrt{\delta} M_{ij3}^T & \sqrt{\delta} M_{ij4}^T & \sqrt{\delta} M_{ij5}^T & \sqrt{\delta} M_{ij6}^T & 0 \end{bmatrix},$$

$$\bar{\Pi}_{41}(1) = \sqrt{\delta} T_{ij1}^T, \quad \bar{\Pi}_{42}(1) = \begin{bmatrix} \sqrt{\delta} T_{ij2}^T & \sqrt{\delta} T_{ij3}^T & \sqrt{\delta} T_{ij4}^T & \sqrt{\delta} T_{ij5}^T & \sqrt{\delta} T_{ij6}^T & 0 \end{bmatrix},$$

$$\bar{\Pi}_{41}(2) = \sqrt{\delta} S_{ij1}^T, \quad \bar{\Pi}_{42}(2) = \begin{bmatrix} \sqrt{\delta} S_{ij2}^T & \sqrt{\delta} S_{ij3}^T & \sqrt{\delta} S_{ij4}^T & \sqrt{\delta} S_{ij5}^T & \sqrt{\delta} S_{ij6}^T & 0 \end{bmatrix},$$

$$\hat{\Pi}_{41}(1) = \sqrt{\delta} W_{ij1}^T, \quad \hat{\Pi}_{42}(1) = \begin{bmatrix} \sqrt{\delta} W_{ij2}^T & \sqrt{\delta} W_{ij3}^T & \sqrt{\delta} W_{ij4}^T & \sqrt{\delta} W_{ij5}^T & \sqrt{\delta} W_{ij6}^T & 0 \end{bmatrix},$$

$$\hat{\Pi}_{41}(2) = \sqrt{\delta} V_{ij1}^T, \quad \hat{\Pi}_{42}(2) = \begin{bmatrix} \sqrt{\delta} V_{ij2}^T & \sqrt{\delta} V_{ij3}^T & \sqrt{\delta} V_{ij4}^T & \sqrt{\delta} V_{ij5}^T & \sqrt{\delta} V_{ij6}^T & 0 \end{bmatrix},$$

$$\Upsilon_1 = -N_{ij2} - N_{ij2}^T + M_{ij2} + M_{ij2}^T,$$
$$\Upsilon_2 = -Q_1 - R_1 + N_{ij3} + N_{ij3}^T,$$
$$\Upsilon_3 = -Q_2 - M_{ij4} - M_{ij4}^T - \frac{R_3}{\delta},$$
$$\Upsilon_4 = -T_{ij2} - T_{ij2}^T + S_{ij2} + S_{ij2}^T,$$
$$\Upsilon_5 = -Q_2 + T_{ij4} + T_{ij4}^T - \frac{R_2}{\delta},$$
$$\Upsilon_6 = -Q_3 - S_{ij5} - S_{ij5}^T - \frac{R_4}{\delta}$$
$$\Upsilon_7 = -W_{ij2} - W_{ij2}^T + V_{ij2} + V_{ij2}^T,$$
$$\Upsilon_8 = -Q_3 + W_{ij5} + W_{ij5}^T - \frac{R_3}{\delta}$$
$$\Upsilon_9 = -Q_4 - V_{ij6} - V_{ij6}^T.$$

Proof: Construct a Lyapunov functional candidate as

$$\begin{aligned} V(t, e_t) &= e^T(t)Pe(t) + \int_{t-\tau_m}^{t} x^T(s)Q_1x(s)\mathrm{d}s \\ &+ \int_{t-\tau_1}^{t} x^T(s)Q_2x(s)\mathrm{d}s + \int_{t-\tau_2}^{t} x^T(s)Q_3x(s)\mathrm{d}s \\ &+ \int_{t-\tau_M}^{t} x^T(s)Q_4x(s)\mathrm{d}s \\ &+ \tau_m \int_{t-\tau_m}^{t}\int_{s}^{t} \dot{x}^T(v)R_1\dot{x}(v)\mathrm{d}v\,\mathrm{d}s \\ &+ \int_{t-\tau_1}^{t-\tau_m}\int_{s}^{t} \dot{x}^T(v)R_2\dot{x}(v)\mathrm{d}v\,\mathrm{d}s \\ &+ \int_{t-\tau_2}^{t-\tau_1}\int_{s}^{t} \dot{x}^T(v)R_3\dot{x}(v)\mathrm{d}v\,\mathrm{d}s \\ &+ \int_{t-\tau_M}^{t-\tau_2}\int_{s}^{t} \dot{x}^T(v)R_4\dot{x}(v)\mathrm{d}v\,\mathrm{d}s, \end{aligned} \tag{24}$$

where $P > 0$, $Q_i > 0$ and $R_i > 0$ $(i = 1, 2, 3, 4)$.

Taking the time derivative of $V(t)$ with respect to t along the trajectory of (7) yields

$$\begin{aligned} \dot{V}(t, e_t) &= 2e^T(t)P\Gamma_1\zeta(t) + e^T(t)H^T(Q_1 + Q_2 + Q_3 + Q_4) \\ &\quad He(t) - x^T(t-\tau_m)Q_1x(t-\tau_m) \\ &\quad - x^T(t-\tau_1)Q_2x(t-\tau_1) \\ &\quad - x^T(t-\tau_2)Q_3x(t-\tau_2) - x^T(t-\tau_M)Q_4x(t-\tau_M) \\ &\quad + \delta\dot{e}^T(t)H^T(R_2 + R_3 + R_4)H\dot{e}(t) \\ &\quad + \tau_m^2\dot{e}^T(t)H^TR_1H\dot{e}(t) - \tau_m\int_{t-\tau_m}^{t} \dot{x}^T(s)R_1\dot{x}(s)\mathrm{d}s \\ &\quad - \int_{t-\tau_1}^{t-\tau_m} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s \\ &\quad - \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s - \int_{t-\tau_M}^{t-\tau_2} \dot{x}^T(s)R_4\dot{x}(s)\mathrm{d}s. \end{aligned} \tag{25}$$

Applying Lemma 1, we have

$$\begin{aligned} &-\tau_m\int_{t-\tau_m}^{t} \dot{x}^T(s)R_1\dot{e}(s)\mathrm{d}s \\ &\leq \begin{bmatrix} He(t) \\ x(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} He(t) \\ x(t-\tau_m) \end{bmatrix}. \end{aligned} \tag{26}$$

It is noted that for any $t \in R_+$, $\tau(t) \in [\tau_m, \tau_1]$ or $\tau(t) \in (\tau_1, \tau_2]$ or $\tau(t) \in (\tau_2, \tau_M]$. Define three sets

$$\Omega_1 = \{t : \tau(t) \in [\tau_m, \tau_1]\}, \tag{27}$$

$$\Omega_2 = \{t : \tau(t) \in (\tau_1, \tau_2]\}, \tag{28}$$

$$\Omega_3 = \{t : \tau(t) \in (\tau_2, \tau_M]\}. \tag{29}$$

In the following, we will discuss the variation of $\dot{V}(t)$ for three cases, that is, $t \in \Omega_1$ or $t \in \Omega_2$ or $t \in \Omega_3$.

Case 1 For $t \in \Omega_1$, i. e. $\tau(t) \in [\tau_m, \tau_1]$

Combining (25) and (26) and introducing some free-weighting matrices M_{ij}, N_{ij}, i, $j = 1, 2 \ldots 6$, we obtain

$$\begin{aligned} &\dot{V}(t) - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \\ &= 2e^T(t)P\Gamma_1\zeta(t) + e^T(t)H^T(Q_1 + Q_2 + Q_3 + Q_4)He(t) \\ &\quad - x^T(t-\tau_m)Q_1x(t-\tau_m) - x^T(t-\tau_1)Q_2x(t-\tau_1) \\ &\quad - x^T(t-\tau_2)Q_3x(t-\tau_2) - x^T(t-\tau_M)Q_4x(t-\tau_M) \\ &\quad + \delta\dot{e}^T(t)H^T(R_2 + R_3 + R_4)H\dot{e}(t) + \tau_m^2\dot{e}^T(t)H^TR_1H\dot{e}(t) \\ &\quad - \int_{t-\tau_1}^{t-\tau_m} \dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s - \int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R_3\dot{x}(s)\mathrm{d}s \\ &\quad - \int_{t-\tau_M}^{t-\tau_2} \dot{x}^T(s)R_4\dot{x}(s)\mathrm{d}s - \gamma^2\omega^T(t)\omega(t) + \zeta(t)^T\Gamma_2^T\Gamma_2\zeta(t) \\ &\quad + \begin{bmatrix} He(t) \\ x(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R_1 & R_1 \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} He(t) \\ x(t-\tau_m) \end{bmatrix} \\ &\quad + 2\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\zeta^T(t)N_{ij}\Big[x(t-\tau_m) - x(t-\tau(t)) \\ &\quad - \int_{t-\tau(t)}^{t-\tau_m} \dot{x}(s)\mathrm{d}s\Big] \\ &\quad + 2\sum_{i=1}^{r}\sum_{j=1}^{r} h_ih_j\zeta^T(t)M_{ij}\Big[x(t-\tau(t) - x(t-\tau_1) \\ &\quad - \int_{t-\tau_1}^{t-\tau(t)} \dot{x}(s)\mathrm{d}s\Big] \end{aligned} \tag{30}$$

where

$$M_{ij}^T = \begin{bmatrix} M_{ij1}^T & M_{ij2}^T & M_{ij3}^T & M_{ij4}^T & M_{ij5}^T & M_{ij6}^T & 0 \end{bmatrix},$$
$$N_{ij}^T = \begin{bmatrix} N_{ij1}^T & N_{ij2}^T & N_{ij3}^T & N_{ij4}^T & N_{ij5}^T & N_{ij6}^T & 0 \end{bmatrix}.$$

Applying Lemma 1, we have

$$-\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_3\dot{x}(s) \le \frac{1}{\delta}\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2)\end{bmatrix}^T\begin{bmatrix} -R_3 & R_3 \\ R_3 & -R_3\end{bmatrix}\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2)\end{bmatrix}, \quad (31)$$

$$-\int_{t-\tau_M}^{t-\tau_2}\dot{x}^T(s)R_4\dot{x}(s) \le \frac{1}{\delta}\begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_M)\end{bmatrix}^T\begin{bmatrix} -R_4 & R_4 \\ R_4 & -R_4\end{bmatrix}\begin{bmatrix} x(t-\tau_2) \\ x(t-\tau_M)\end{bmatrix}. \quad (32)$$

Note that

$$\begin{aligned} -2\sum_{i=1}^r\sum_{j=1}^r h_ih_j\zeta^T(t)N_{ij}\int_{t-\tau(t)}^{t-\tau_m}\dot{x}(s)\mathrm{d}s &\le \int_{t-\tau(t)}^{t-\tau_m}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s \\ &\quad + \sum_{i=1}^r\sum_{j=1}^r h_ih_j[\tau(t)-\tau_m]\zeta^T(t)N_{ij}R_2^{-1}N_{ij}^T\zeta(t), \end{aligned} \quad (33)$$

$$\begin{aligned} -2\sum_{i=1}^r\sum_{j=1}^r h_ih_j\zeta^T(t)M_{ij}\int_{t-\tau_1}^{t-\tau(t)}\dot{x}(s)\mathrm{d}s &\le \int_{t-\tau_1}^{t-\tau(t)}\dot{x}^T(s)R_2\dot{x}(s)\mathrm{d}s \\ &\quad + \sum_{i=1}^r\sum_{j=1}^r h_ih_j[\tau_1-\tau(t)]\zeta^T(t)M_{ij}R_2^{-1}M_{ij}^T\zeta(t). \end{aligned} \quad (34)$$

Substituting (31)–(34) into (30) and using Lemma 2 and Schur complement, it is easy to see that (21) with l=1, 2 are sufficient conditions to guarantee

$$\dot{V}(t) - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \le 0.$$

Case 2 For $t \in \Omega_2$, i. e. $\tau(t) \in (\tau_1, \tau_2]$

By using Lemma 1, we have

$$-\int_{t-\tau_1}^{t-\tau_m}\dot{x}^T(s)R_2\dot{x}(s) \le \frac{1}{\delta}\begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_1)\end{bmatrix}^T\begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2\end{bmatrix}\begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_1)\end{bmatrix}. \quad (35)$$

Combining (25), (26), (32) and (35) and introducing some free-weighting matrices T_{ij}, S_{ij}, $i, j = 1, 2, \ldots, 6$,

$$2\sum_{i=1}^r\sum_{j=1}^r h_ih_j\zeta^T(t)T_{ij}\Big[x(t-\tau_1) - x(t-\tau(t) - \int_{t-\tau(t)}^{t-\tau_1}\dot{x}(s)\mathrm{d}s\Big] = 0, \quad (36)$$

$$2\sum_{i=1}^r\sum_{j=1}^r h_ih_j\zeta^T(t)S_{ij}\Big[x(t-\tau(t) - x(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)}\dot{x}(s)\mathrm{d}s\Big] = 0. \quad (37)$$

It is easy to see that (22) with $l=1$, 2 are sufficient conditions to guarantee

$$\dot{V}(t) - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \le 0.$$

Case 3 For $t \in \Omega_3$, i. e. $\tau(t) \in (\tau_2, \tau_M]$

Combining (25), (26), (31) and 35) and introducing some free-weighting matrices T_{ij}, S_{ij}, $i, j = 1, 2\cdots 6$,

$$2\sum_{i=1}^r\sum_{j=1}^r h_ih_j\zeta^T(t)W_{ij}\Big[x(t-\tau_2) - x(t-\tau(t) - \int_{t-\tau(t)}^{t-\tau_2}\dot{x}(s)\mathrm{d}s\Big] = 0, \quad (38)$$

$$2\sum_{i=1}^r\sum_{j=1}^r h_ih_j\zeta^T(t)V_{ij}\Big[x(t-\tau(t) - x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot{x}(s)\mathrm{d}s\Big] = 0. \quad (39)$$

It is easy to see that (23) with $l=1$, 2 are sufficient conditions to guarantee

$$\dot{V}(t) - \gamma^2\omega^T(t)\omega(t) + \tilde{z}^T(t)\tilde{z}(t) \le 0.$$

Next, simlar to the proof of Theorem 1, we can get Theorem 2. This completes the proof.

3.2.2. *H_∞ performance analysis for Case II*

For Case II, we chose a Lyapunov functional candidate as

$$\begin{aligned} V(t, e_t) &= e^T(t)Pe(t) + \int_{t-\tau_m}^t x^T(s)Q_1x(s)\mathrm{d}s \\ &\quad + \int_{t-\tau_1}^t x^T(s)Q_2x(s)\mathrm{d}s + \int_{t-\tau_2}^t x^T(s)Q_3x(s)\mathrm{d}s \\ &\quad + \int_{t-\tau_M}^t x^T(s)Q_4x(s)\mathrm{d}s + \int_{t-\tau(t)}^t x^T(s)Q_5x(s)\mathrm{d}s \\ &\quad + \tau_m\int_{t-\tau_m}^t\int_s^t \dot{x}^T(v)R_1\dot{x}(v)\mathrm{d}v\,\mathrm{d}s \\ &\quad + \int_{t-\tau_1}^{t-\tau_m}\int_s^t \dot{x}^T(v)R_2\dot{x}(v)\mathrm{d}v\,\mathrm{d}s \\ &\quad + \int_{t-\tau_2}^{t-\tau_1}\int_s^t \dot{x}^T(v)R_3\dot{x}(v)\mathrm{d}v\,\mathrm{d}s \\ &\quad + \int_{t-\tau_M}^{t-\tau_2}\int_s^t \dot{x}^T(v)R_4\dot{x}(v)\mathrm{d}v\,\mathrm{d}s, \end{aligned} \quad (40)$$

where $P>0$, $Q_i>0$ $(i=1,2,3,4,5)$ and $R_j>0$ $(j=1,2,3,4)$.

Then, similar to the proof of Theorem 3, we can conclude the following result.

Theorem 4: *Under Case* II, *for given constants* τ_m, τ_M, d *and* γ, *the system* (7) *is asymptotically stable with the* H_∞*-norm bound* γ *if there exist* $P>0$, $Q_i>0$ $(i=1,2,3,4,5)$, $Q_i>0$ $(i=1,2,3,4)$, M_{ijk}, N_{ijk}, S_{ijk}, T_{ijk}, W_{ijk} *and* $V_{ijk}(i,j\in\mathbb{S},k=1,2,\ldots,6)$ *with appropriate dimensions such that*

$$\Omega_1^{ij}(l)+\Omega_1^{ji}(l)<0, \tag{41}$$

$$\Omega_2^{ij}(l)+\Omega_2^{ji}(l)<0, \tag{42}$$

$$\Omega_3^{ij}(l)+\Omega_3^{ji}(l)<0, \quad l=1,2, i\le j\in\mathbb{S}, \tag{43}$$

where

$$\Omega_1^{ij}(l)=\begin{bmatrix}\Omega_{11} & * & * & * \\ \Pi_{21} & \Omega_{22} & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ \Pi_{41}(l) & \Pi_{42}(l) & 0 & -R_2\end{bmatrix}, \quad \Omega_2^{ij}(l)=\begin{bmatrix}\Omega_{11} & * & * & * \\ \bar{\Pi}_{21} & \bar{\Omega}_{22} & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ \bar{\Pi}_{41}(l) & \bar{\Pi}_{42}(l) & 0 & -R_3\end{bmatrix},$$

$$\Omega_3^{ij}(l)=\begin{bmatrix}\Omega_{11} & * & * & * \\ \hat{\Pi}_{21} & \hat{\Omega}_{22} & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ \hat{\Pi}_{41}(l) & \hat{\Pi}_{42}(l) & 0 & -R_4\end{bmatrix},$$

$$\Omega_{11}=P\hat{A}_{ij}+\hat{A}_{ij}^TP+H^T(Q_1+Q_2+Q_3+Q_4+Q_5-R_1)H,$$

$$\Omega_{22}=\begin{bmatrix}\vartheta_1 & * & * & * & * & * \\ -N_{ij3}+M_{ij3}+N_{ij2}^T & \Upsilon_2 & * & * & * & * \\ -N_{ij4}+M_{ij4}-M_{ij2}^T & N_{ij4}-M_{ij3}^T & \Upsilon_3 & * & * & * \\ -N_{ij5}+M_{ij5} & N_{ij5} & -M_{ij5}+\frac{R_3}{\delta} & -Q_3-\frac{R_3}{\delta}-\frac{R_4}{\delta} & * & * \\ -N_{ij6}+M_{ij6} & N_{ij6} & -M_{ij6} & \frac{R_4}{\delta} & -Q_4-\frac{R_4}{\delta} & * \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I\end{bmatrix}$$

$$\bar{\Omega}_{22}=\begin{bmatrix}\vartheta_2 & * & * & * & * & * \\ -T_{ij3}+S_{ij3} & -Q_1-R_1-\frac{R_2}{\delta} & * & * & * & * \\ -T_{ij4}+S_{ij4}+T_{ij2}^T & T_{ij3}^T+\frac{R_2}{\delta} & \Upsilon_5 & * & * & * \\ -T_{ij5}+S_{ij5}-S_{ij2}^T & -S_{ij3}^T & -S_{ij4}^T+T_{ij5} & \Upsilon_6 & * & * \\ -T_{ij6}+S_{ij6} & 0 & T_{ij6} & -S_{ij6}+\frac{R_4}{\delta} & -Q_4-\frac{R_4}{\delta} & * \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I\end{bmatrix}$$

$$\hat{\Omega}_{22}=\begin{bmatrix}\vartheta_3 & * & * & * & * & * \\ -W_{ij3}+V_{ij3} & -Q_1-R_1-\frac{R_2}{\delta} & * & * & * & * \\ -W_{ij4}+V_{ij4} & \frac{R_2}{\delta} & -Q_2-\frac{R_2}{\delta}-\frac{R_3}{\delta} & * & * & * \\ -W_{ij5}+V_{ij5}+W_{ij2}^T & W_{ij3}^T & W_{ij4}^T+\frac{R_3}{\delta} & \Upsilon_8 & * & * \\ -W_{ij6}+V_{ij6}-V_{ij2}^T & -V_{ij3}^T & -V_{ij4}^T & W_{ij6}-V_{ij5}^T & \Upsilon_9 & * \\ 0 & 0 & 0 & 0 & 0 & -\gamma^2I\end{bmatrix}$$

$$\vartheta_1=-N_{ij2}-N_{ij2}^T+M_{ij2}+M_{ij2}^T-(1-d)Q_5,$$

$$\vartheta_2=-T_{ij2}-T_{ij2}^T+S_{ij2}+S_{ij2}^T-(1-d)Q_5,$$

$$\vartheta_3=-W_{ij2}-W_{ij2}^T+V_{ij2}+V_{ij2}^T-(1-d)Q_5,$$

and $\Pi_{21}, \Pi_{31}, \Pi_{32}, \Pi_{33}, \Pi_{41}(l), \Pi_{42}(l)$, $\bar{\Pi}_{21}, \hat{\Pi}_{21}, \bar{\Pi}_{41}(l), \bar{\Pi}_{42}(l), \hat{\Pi}_{41}(l), \hat{\Pi}_{42}(l)$ $(l=1,\ 2)$, Υ_2, Υ_3, Υ_5, Υ_6, Υ_8 and Υ_9 are as defined in Theorem 3.

4. Fuzzy H_∞ filter design

In this section, we seek to design the H_∞ filtering based on Theorems 1–4.

4.1. *H_∞ filter design for l = 2*

4.1.1. *H_∞ filter design for case I*

For Case I, based on Theorem 1, we obtain a criterion for the H_∞ filter design.

Theorem 5: *Under Case* I, *for some given constants* $0 \le \tau_m \le \tau_M$ *and* γ, *the augmented systems* (7) *is asymptotically stable with a prescribed* H_∞ *performance* γ *if there exist* $P_1 > 0$, $\bar{P}_3 > 0$, $Q_0 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_0 > 0$, $R_1 > 0$, $R_2 > 0$, $\bar{M}_{ij10}$, $\bar{M}_{ij11}$, $\bar{N}_{ij10}$, $\bar{N}_{ij11}$, $\bar{T}_{ij10}$, $\bar{T}_{ij11}$, $\bar{S}_{ij10}$, $\bar{S}_{ij11}$, M_{ijk}, N_{ijk}, S_{ijk}, T_{ijk}, A_{fj}, $\bar{B}_{fj}$, $\bar{C}_{fj}$ and $\bar{D}_{fj}$ $(k=2,\ 3,\ldots,6,\ i,\ j\in\mathbb{S})$ with appropriate dimensions such that the following linear matrix linequalities (LMIs) hold:

$$\hat{\Psi}_1^{ij}(l) + \hat{\Psi}_1^{ji}(l) < 0, \tag{44}$$

$$\hat{\Psi}_2^{ij}(l) + \hat{\Psi}_2^{ji}(l) < 0, \tag{45}$$

$$P_1 - \bar{P}_3 > 0, \quad l = 1, 2, i \le j \in \mathbb{S}, \tag{46}$$

where

$$\hat{\Psi}_1^{ij}(l) = \begin{bmatrix} \acute{\Psi}_{11} & * & * & * \\ \acute{\Psi}_{21} & \Psi_{22} & * & * \\ \acute{\Psi}_{31} & \Psi_{32} & \Psi_{33} & * \\ \acute{\Psi}_{41}(l) & \Psi_{42}(l) & \Psi_{43} & \Psi_{44} \end{bmatrix}, \quad \hat{\Psi}_2^{ij}(l) = \begin{bmatrix} \acute{\Psi}_{11} & * & * & * \\ \acute{\hat{\Psi}}_{21} & \hat{\Psi}_{22} & * & * \\ \acute{\Psi}_{31} & \Psi_{32} & \Psi_{33} & * \\ \acute{\hat{\Psi}}_{41}(l) & \hat{\Psi}_{42}(l) & \Psi_{43} & \hat{\Psi}_{44} \end{bmatrix}$$

$$\acute{\Psi}_{11} = \begin{bmatrix} \Lambda & * \\ \bar{P}_3 A_i + \bar{B}_{fj} C_i + \bar{A}_{fj}^T & \bar{A}_{fj} + \bar{A}_{fj}^T \end{bmatrix},$$

$$\acute{\Psi}_{21} = \begin{bmatrix} A_i^T P_1 + C_i^T \bar{B}_f^T - \bar{N}_{ij10}^T + \bar{M}_{ij10}^T & A_i^T \bar{P}_3 + C_i^T \bar{B}_{fj}^T - N_{ij11} + M_{ij11}^T \\ R_0 + \bar{N}_{ij10}^T & \bar{N}_{ij11}^T \\ -\bar{M}_{ij10}^T & -\bar{M}_{ij11}^T \\ 0 & 0 \\ A_{\omega i}^T P_1 + C_{\omega i}^T \bar{B}_{fj}^T & A_{\omega i}^T \bar{P}_3 + C_{\omega i}^T \bar{B}_{fj}^T \end{bmatrix},$$

$$\acute{\hat{\Psi}}_{21} = \begin{bmatrix} A_i^T P_1 + C_i^T \bar{B}_{fj}^T - \bar{T}_{ij10}^T + \bar{S}_{ij10}^T & A_i^T \bar{P}_3 + C_i^T \bar{B}_{fj}^T - T_{ij11} + S_{ij11}^T \\ R_0 & 0 \\ \bar{T}_{ij10}^T & \bar{T}_{ij11}^T \\ -\bar{S}_{ij10}^T & -\bar{S}_{ij11}^T \\ A_{\omega i}^T P_1 + C_{\omega i}^T \bar{B}_{fj}^T & A_{\omega i}^T \bar{P}_3 + C_{\omega i}^T \bar{B}_{fj}^T \end{bmatrix},$$

$$\acute{\Psi}_{31} = \begin{bmatrix} L_i - \bar{D}_{fj} C_i & -\bar{C}_{fj} \\ \tau_0 R_0 A_i & 0 \\ \sqrt{\delta} R_1 A_i & 0 \\ \sqrt{\delta} R_2 A_i & 0 \end{bmatrix},$$

$$\acute{\Psi}_{41}(1) = \begin{bmatrix} \sqrt{\delta}\bar{N}_{ij10}^T & \sqrt{\delta}\bar{N}_{ij11}^T \end{bmatrix}, \quad \acute{\Psi}_{41}(2) = \begin{bmatrix} \sqrt{\delta}\bar{M}_{ij10}^T & \sqrt{\delta}\bar{M}_{ij11}^T \end{bmatrix},$$

$$\acute{\hat{\Psi}}_{41}(1) = \begin{bmatrix} \sqrt{\delta}\bar{T}_{ij10}^T & \sqrt{\delta}\bar{T}_{ij11}^T \end{bmatrix}, \quad \acute{\hat{\Psi}}_{41}(2) = \begin{bmatrix} \sqrt{\delta}\bar{S}_{ij10}^T & \sqrt{\delta}\bar{S}_{ij11}^T \end{bmatrix},$$

$$\Lambda = P_1 A_i + A_i^T P_1 + \bar{B}_{fj} C_i + C_i^T \bar{B}_{fj}^T + Q_0 + Q_1 + Q_2 - R_0$$

and $\Psi_{22}, \hat{\Psi}_{22}, \Psi_{32}, \Psi_{33}, \Psi^{s}_{42}, \hat{\Psi}^{l}_{42}, \Psi_{43}, \Psi_{44}$ and $\hat{\Psi}_{44}$ are as defined in Theorem 1. Moreover, a suitable filter of the form (6) is given as

$$\begin{bmatrix} A_{fj} & B_{fj} \\ C_{fj} & D_{fj} \end{bmatrix} = \begin{bmatrix} *20c\bar{A}_{fj}\bar{P}_3^{-1} & \bar{B}_{fj} \\ \bar{C}_{fj}\bar{P}_3^{-1} & \bar{D}_{fj} \end{bmatrix}. \quad (47)$$

Proof: Since $\bar{P}_3 > 0$, there exist nonsingular matrices P_2 and $P_3 > 0$ such that $\bar{P}_3 = P_2^T P_3^{-1} P_2$. Defining

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}, \quad J = \begin{bmatrix} I & 0 \\ 0 & P_2^T P_3^{-1} \end{bmatrix}. \quad (48)$$

It is easy to see that $P > 0$ is equivalent to $P_1 - \bar{P}_3 = P_1 - P_2^T P_3^{-1} P_2 > 0$.

Pre-and post-multiplying (14) and (15) with $\Pi = \mathrm{diag}\{J, \underbrace{I, I, \cdots I}_{10}\}$ and its transpose and letting

$$\begin{aligned} &\bar{A}_{fj} = \hat{A}_{fj}\bar{P}_3, \quad \hat{A}_{fj} = P_2^T A_{fj} P_2^{-T}, \\ &\bar{B}_{fj} = P_2^T B_{fj}, \\ &\bar{C}_{fj} = \hat{C}_{fj}\bar{P}_3, \quad \hat{C}_{fj} = C_{fj}P_2^{-T}, \\ &\bar{D}_{fj} = D_{fj}, \\ &N_{ij1}^T J^T = \begin{bmatrix} \bar{N}_{ij10}^T & \bar{N}_{ij11}^T \end{bmatrix}, \quad M_{ij1}^T J^T = \begin{bmatrix} \bar{M}_{ij10}^T & \bar{M}_{ij11}^T \end{bmatrix} \\ &T_{ij1}^T J^T = \begin{bmatrix} \bar{T}_{ij10}^T & \bar{T}_{ij11}^T \end{bmatrix}, \quad S_{ij1}^T J^T = \begin{bmatrix} \bar{S}_{ij10}^T & \bar{S}_{ij11}^T \end{bmatrix}, \end{aligned} \quad (49)$$

we can conclude (44) and (45).

Next, we will show that, if (44) and (45) are solvable for $\bar{A}_{fj}$, $\bar{B}_{fj}$, $\bar{C}_{fj}$, $\bar{D}_{fj}$ and $\bar{P}_3$, then the parameter matrices of the filter (6) can be chosen as in (47).

Replacing $(A_{fj}, B_{fj}, C_{fj}, D_{fj})$ by $(P_2^{-T}\hat{A}_{fj}P_2^T,\ P_2^{-T}\bar{B}_{fj},\ \hat{C}_{fj}P_2^T,\ \bar{D}_{fj})$ in (6) and then pre-and post-multiplying them with Π and its transpose, we can also obtain (44) and (45). Obviously $(P_2^{-T}\hat{A}_{fj}P_2^T,\ P_2^{-T}\bar{B}_{fj},\ \hat{C}_{fj}P_2^T,\ \bar{D}_{fj})$ can be chosen as the filter parameters. That is, the following filter

$$\begin{cases} \dot{\bar{x}}_f(t) = P_2^{-T}\hat{A}_{fj}P_2^T\bar{x}_f(t) + P_2^{-T}\bar{B}_{fj}y(t) \\ \bar{z}_f(t) = \hat{C}_{fj}P_2^T\bar{x}_f(t) + \bar{D}_{fj}y(t) \end{cases} \quad (50)$$

can guarantee that the filtering-error system (7) is asymptotically stable with the H_∞ performance bound γ. Defining $x_f(t) = P_2^T\bar{x}_f(t)$, (50) becomes

$$\begin{cases} \dot{x}_f(t) = \hat{A}_{fj}x_f(t) + \bar{B}_{fj}y(t) \\ z_f(t) = \hat{C}_{fj}x_f(t) + \bar{D}_{fj}y(t). \end{cases} \quad (51)$$

Then, from (49) and (51) we can obtain (47). This completes the proof.

4.1.2. *H_∞ filter design for Case* II

For Case II, based on Theorem 2, similar to the proof of Theorem 5, we obtain a criterion for the H_∞ filter design.

Theorem 6: *Under Case* II, *for given constants τ_m, τ_M, d and γ, the system* (7) *is asymptotically stable with the H_∞-norm bound γ if there exist* $P_1 > 0$, $\bar{P}_3 > 0$, $Q_0 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_0 > 0$, $R_1 > 0$, $R_2 > 0$, $Q = \begin{bmatrix} Q_{11} & Q_{21}^T \\ Q_{21} & Q_{22} \end{bmatrix} > 0$, $\bar{M}_{ij10}$, $\bar{M}_{ij11}$, $\bar{N}_{ij10}$, $\bar{N}_{ij11}$, $\bar{T}_{ij10}$, $\bar{T}_{ij11}$, $\bar{S}_{ij10}$, $\bar{S}_{ij11}$, M_{ijk}, N_{ijk}, S_{ijk}, T_{ijk}, $\bar{A}_{fj}$, $\bar{B}_{fj}$, $\bar{C}_{fj}$ and $\bar{D}_{fj}$ $(k = 2, 3, \ldots, 5,\ i,\ j \in \mathbb{S})$ *with appropriate dimensions such that the following LMIs hold*:

$$\hat{\Phi}_1^{ij}(l) + \hat{\Phi}_1^{ji}(l) < 0, \quad (52)$$

$$\hat{\Phi}_2^{ij}(l) + \hat{\Phi}_2^{ji}(l) < 0, \quad (53)$$

$$\begin{aligned} &P_1 - \bar{P}_3 > 0, \\ &l = 1, 2, i \le j \in \mathbb{S}, \end{aligned} \quad (54)$$

where

$$\hat{\Phi}_1^{ij}(l) = \begin{bmatrix} \acute{\Psi}_{11} & * & * & * \\ \acute{\Psi}_{21} & \Phi_{22} & * & * \\ \acute{\Psi}_{31} & \Psi_{32} & \Psi_{33} & * \\ \acute{\Psi}_{41}(l) & \Psi_{42}(l) & \Psi_{43} & \Psi_{44} \end{bmatrix},$$

$$\hat{\Phi}_2^{ij}(l) = \begin{bmatrix} \acute{\Psi}_{11} & * & * & * \\ \hat{\acute{\Psi}}_{21} & \hat{\Phi}_{22} & * & * \\ \acute{\Psi}_{31} & \Psi_{32} & \Psi_{33} & * \\ \hat{\acute{\Psi}}_{41}(l) & \hat{\Psi}_{42}(l) & \Psi_{43} & \hat{\Psi}_{44} \end{bmatrix},$$

$\acute{\Psi}_{11}, \acute{\Psi}_{21}, \hat{\acute{\Psi}}_{21}, \acute{\Psi}_{31}, \acute{\Psi}_{41}(l)$ and $\hat{\acute{\Psi}}_{41}(l)$ are as defined in Theorem 5 and $\Phi_{22}, \hat{\Phi}_{22}, \Psi_{32}, \Psi_{33}, \Psi_{42}(l), \hat{\Psi}_{42}(l), \Psi_{43}, \Psi_{44}$ and $\hat{\Psi}_{44}$ are as defined in Theorems 1 and 2. Moreover, a suitable filter of the form (6) is given as (47).

4.2. H_∞ *filter design for l = 3*

4.2.1. *H_∞ filter design for Case* I

For Case I, based on Theorem 3, similar to the proof of Theorem 5, we obtain a criterion for the H_∞ filter design.

Theorem 7: *Under Case* I, *for some given constants $0 \le \tau_m \le \tau_M$ and γ, the augmented system* (7) *is asymptotically stable with a prescribed H_∞ performance γ if there exist* $P_1 > 0$, $\bar{P}_3 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $R_4 > 0$, $\bar{M}_{ij10}$, $\bar{M}_{ij11}$, $\bar{N}_{ij10}$, $\bar{N}_{ij11}$, $\bar{T}_{ij10}$, $\bar{T}_{ij11}$, $\bar{S}_{ij10}$, $\bar{S}_{ij11}$, $\bar{W}_{ij10}$, $\bar{W}_{ij11}$, $\bar{V}_{ij10}$, $\bar{V}_{ij11}$, M_{ijk}, N_{ijk}, S_{ijk}, T_{ijk}, W_{ijk}, V_{ijk}, A_{fj}, $\bar{B}_{fj}$, $\bar{C}_{fj}$ *and* $\bar{D}_{fj}$ $(k = 2, 3, \ldots, 6,\ i,\ j \in \mathbb{S})$ *with appropriate dimensions* *such that the following LMIs hold*:

$$\hat{\Pi}_1^{ij}(l) + \hat{\Pi}_1^{ji}(l) < 0, \tag{55}$$

$$\hat{\Pi}_2^{ij}(l) + \hat{\Pi}_2^{ji}(l) < 0, \tag{56}$$

$$\hat{\Pi}_3^{ij}(l) + \hat{\Pi}_3^{ji}(l) < 0, \tag{57}$$

$$\begin{aligned} &P_1 - \bar{P}_3 > 0, \\ &l = 1, 2, i \le j \in \mathbb{S}, \end{aligned} \tag{58}$$

where

$$\hat{\Pi}_1^{ij}(l) = \begin{bmatrix} \Delta_1 & * & * & * \\ \Delta_2 & \Pi_{22} & * & * \\ \Delta_5 & \Delta_6 & \Pi_{33} & * \\ \Delta_7(l) & \Pi_{42}(l) & 0 & -R_2 \end{bmatrix}, \quad \hat{\Pi}_2^{ij}(l) = \begin{bmatrix} \Delta_1 & * & * & * \\ \Delta_3 & \bar{\Pi}_{22} & * & * \\ \Delta_5 & \Delta_6 & \Pi_{33} & * \\ \Delta_8(l) & \bar{\Pi}_{42}(l) & 0 & -R_3 \end{bmatrix},$$

$$\hat{\Pi}_3^{ij}(l) = \begin{bmatrix} \Delta_1 & * & * & * \\ \Delta_4 & \hat{\Pi}_{22} & * & * \\ \Delta_5 & \Delta_6 & \Pi_{33} & * \\ \Delta_9(l) & \hat{\Pi}_{42}(l) & 0 & -R_4 \end{bmatrix}, \quad \Delta_1 = \begin{bmatrix} \Delta_{11} & * \\ \bar{P}_3 A_i + \bar{B}_{fj} C_i + \bar{A}_{fj}^T & \bar{A}_{fj} + \bar{A}_{fj}^T \end{bmatrix},$$

$$\Delta_2 = \begin{bmatrix} A_{di}^T P_1 + C_{di}^T \bar{B}_{fj}^T - \bar{N}_{ij10}^T + \bar{M}_{ij10}^T & A_{di}^T \bar{P}_3 + C_{di}^T \bar{B}_{fj}^T - \bar{N}_{ij11}^T + \bar{M}_{ij11}^T \\ R_1 + \bar{N}_{ij10}^T & \bar{N}_{ij11}^T \\ -\bar{M}_{ij10}^T & -\bar{M}_{ij11}^T \\ 0 & 0 \\ 0 & 0 \\ A_{\omega i}^T P_1 + C_{\omega i}^T \bar{B}_{fj}^T & A_{\omega i}^T \bar{P}_3 + C_{\omega i}^T \bar{B}_{fj}^T \end{bmatrix},$$

$$\Delta_3 = \begin{bmatrix} A_{di}^T P_1 + C_{di}^T \bar{B}_{fj}^T - \bar{T}_{ij10}^T + \bar{S}_{ij10}^T & A_{di}^T \bar{P}_3 + C_{di}^T \bar{B}_{fj}^T - \bar{T}_{ij11}^T + \bar{S}_{ij11}^T \\ R_1 & 0 \\ \bar{T}_{ij10}^T & \bar{T}_{ij11}^T \\ -\bar{S}_{ij10}^T & -\bar{S}_{ij11}^T \\ 0 & 0 \\ A_{\omega i}^T P_1 + C_{\omega i}^T \bar{B}_{fj}^T & A_{\omega i}^T \bar{P}_3 + C_{\omega i}^T \bar{B}_{fj}^T \end{bmatrix},$$

$$\Delta_4 = \begin{bmatrix} A_{di}^T P_1 + C_{di}^T \bar{B}_{fj}^T - \bar{W}_{ij10}^T + \bar{V}_{ij10}^T & A_{di}^T \bar{P}_3 + C_{di}^T \bar{B}_{fj}^T - \bar{W}_{ij11}^T + \bar{V}_{ij11}^T \\ R_1 & 0 \\ 0 & 0 \\ \bar{W}_{ij10}^T & \bar{W}_{ij11}^T \\ -\bar{V}_{ij10}^T & -\bar{V}_{ij11}^T \\ A_{\omega i}^T P_1 + C_{\omega i}^T \bar{B}_{fj}^T & A_{\omega i}^T \bar{P}_3 + C_{\omega i}^T \bar{B}_{fj}^T \end{bmatrix},$$

$$\Delta_5 = \begin{bmatrix} L_i - D_{fj} C_i & -\bar{C}_{fj} \\ \tau_m R_1 A_i & 0 \\ \sqrt{\delta} R_2 A_i & 0 \\ \sqrt{\delta} R_3 A_i & 0 \\ \sqrt{\delta} R_4 A_i & 0 \end{bmatrix}, \quad \Delta_6 = \begin{bmatrix} L_{di} - D_{fj} C_{di} & 0 & 0 & 0 & 0 & L_{\omega i} - D_{fj} C_{\omega i} \\ \tau_m R_1 A_{di} & 0 & 0 & 0 & 0 & \tau_m R_1 A_{\omega i} \\ \tau_m R_2 A_{di} & 0 & 0 & 0 & 0 & \tau_m R_2 A_{\omega i} \\ \tau_m R_3 A_{di} & 0 & 0 & 0 & 0 & \tau_m R_3 A_{\omega i} \\ \tau_m R_4 A_{di} & 0 & 0 & 0 & 0 & \tau_m R_4 A_{\omega i} \end{bmatrix},$$

$$\Delta_7(1) = \begin{bmatrix} \sqrt{\delta} \bar{N}_{ij10}^T & \sqrt{\delta} \bar{N}_{ij11}^T \end{bmatrix}, \quad \Delta_7(2) = \begin{bmatrix} \sqrt{\delta} \bar{M}_{ij10}^T & \sqrt{\delta} \bar{M}_{ij11}^T \end{bmatrix}, \quad \Delta_8(1) = \begin{bmatrix} \sqrt{\delta} \bar{T}_{ij10}^T & \sqrt{\delta} \bar{T}_{ij11}^T \end{bmatrix},$$

$$\Delta_8(2) = \begin{bmatrix} \sqrt{\delta} \bar{S}_{ij10}^T & \sqrt{\delta} \bar{S}_{ij11}^T \end{bmatrix}, \quad \Delta_9(1) = \begin{bmatrix} \sqrt{\delta} \bar{W}_{ij10}^T & \sqrt{\delta} \bar{W}_{ij11}^T \end{bmatrix}, \quad \Delta_9(2) = \begin{bmatrix} \sqrt{\delta} \bar{V}_{ij10}^T & \sqrt{\delta} \bar{V}_{ij11}^T \end{bmatrix},$$

$$\Delta_{11} = P_1 A_i + A_i^T P_1 + \bar{B}_{fj} C_i + C_i^T \bar{B}_{fj}^T + Q_1 + Q_2 + Q_3 + Q_4 - R_1,$$

and $\Pi_{22}, \bar{\Pi}_{22}, \hat{\Pi}_{22}, \Pi_{42}(l), \bar{\Pi}_{42}(l), \hat{\Pi}_{42}(l)$ and Π_{33} are as defined in Theorem 3. Moreover, a suitable filter of the form (6) is given as (47).

4.2.2. *H_∞ filter design for Case* II

For Case II, based on Theorem 4, similar to the proof of Theorem 5 we obtain a criterion for the H_∞ filter design.

Theorem 8: *Under Case* II, *for some given constants* $0 \le \tau_m \le \tau_M$, *d and* γ, *the augmented system* (7) *is asymptotically stable with a prescribed* H_∞ *performance* γ *if there exist* $P_1 > 0$, $\bar{P}_3 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $Q_4 > 0$, $Q_5 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $R_4 > 0$, $\bar{M}_{ij10}$, $\bar{M}_{ij11}$, $\bar{N}_{ij10}$, $\bar{N}_{ij11}$, $\bar{T}_{ij10}$, $\bar{T}_{ij11}$, $\bar{S}_{ij10}$, $\bar{S}_{ij11}$, $\bar{W}_{ij10}$, $\bar{W}_{ij11}$, $\bar{V}_{ij10}$, $\bar{V}_{ij11}$, M_{ijk},N_{ijk}, S_{ijk}, T_{ijk}, W_{ijk}, V_{ijk}, $\bar{A}_{fj}$, $\bar{B}_{fj}$, *and* $\bar{C}_{fj}$, $\bar{D}_{fj}$ $(k = 2, 3, \ldots, 6,\ i,\ j \in \mathbb{S})$ *with appropriate dimensions such that the following LMIs hold*:

$$\Omega_1^{ij}(l) + \Omega_1^{ji}(l) < 0, \tag{59}$$

$$\Omega_2^{ij}(l) + \Omega_2^{ji}(l) < 0, \tag{60}$$

$$\Omega_3^{ij}(l) + \Omega_3^{ji}(l) < 0, \tag{61}$$

$$\begin{aligned} &P_1 - \bar{P}_3 > 0, \\ &l = 1, 2, i \le j \in \mathbb{S}, \end{aligned} \tag{62}$$

where

$$\Omega_1^{ij}(l) = \begin{bmatrix} \bar{\Omega}_{11} & * & * & * \\ \Delta_2 & \Omega_{22} & * & * \\ \Delta_5 & \Delta_6 & \Pi_{33} & * \\ \Delta_7(l) & \Pi_{42}(l) & 0 & -R_2 \end{bmatrix},$$

$$\Omega_2^{ij}(l) = \begin{bmatrix} \bar{\Omega}_{11} & * & * & * \\ \Delta_3 & \bar{\Omega}_{22} & * & * \\ \Delta_5 & \Delta_6 & \Pi_{33} & * \\ \Delta_8(l) & \bar{\Pi}_{42}(l) & 0 & -R_3 \end{bmatrix},$$

$$\Omega_3^{ij}(l) = \begin{bmatrix} \bar{\Omega}_{11} & * & * & * \\ \Delta_4 & \hat{\Omega}_{22} & * & * \\ \Delta_5 & \Delta_6 & \Pi_{33} & * \\ \Delta_9(l) & \hat{\Pi}_{42}(l) & 0 & -R_4 \end{bmatrix},$$

$$\bar{\Omega}_{11} = \begin{bmatrix} \Delta_{11} + Q_5 & * \\ \bar{P}_3 A_i + \bar{B}_{fj} C_i + \bar{A}_{fj}^T & \bar{A}_{fj} + \bar{A}_{fj}^T \end{bmatrix},$$

and $\Delta_2, \Delta_3, \Delta_4, \Delta_5, \Delta_6, \Delta_7(l), \Delta_8(l), \Delta_9(l), \Pi_{33}, \Omega_{22}, \hat{\Omega}_{22}, \bar{\Omega}_{22}, \Pi_{42}(l), \hat{\Pi}_{42}(l)$ and $\bar{\Pi}_{42}(l)$ are as defined in Theorems 3, 4 and 7. Moreover, a suitable filter of the form (6) is given as (47).

5. Example

Example 1: Consider the H_∞ filtering design for the system (4) with parameters (Lin et al. 2008; Qiu et al. 2009; Su et al. 2009; Zhang et al. 2009)

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix},$$

$$A_{\omega 1} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, \quad A_{\omega 2} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}, \quad C_{d1} = \begin{bmatrix} -0.8 & 0.6 \end{bmatrix},$$

$$C_{d2} = \begin{bmatrix} -0.2 & 1 \end{bmatrix}, \quad C_{\omega 1} = 0.3, \quad C_{\omega 2} = -0.6,$$

$$L_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix}, \quad L_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix},$$

$$L_{d1} = \begin{bmatrix} 0.1 & 0 \end{bmatrix}, \quad L_{d2} = \begin{bmatrix} 0 & 0.2 \end{bmatrix}, \quad L_{\omega 1} = L_{\omega 2} = 0,$$

$$\omega(t) = \begin{cases} 0.1, & 5 < t < 10 \\ -0.1, & 15 < t < 20 \\ 0, & \text{otherwise.} \end{cases}, \quad \begin{aligned} &\mu_1(\theta(t)) = \sin^2(t), \\ &\mu_2(\theta(t)) = \cos^2(t). \end{aligned}$$

It needs to be pointed out that the H_∞ filter design problem was discussed in Lin et al. (2008), Qiu et al. (2009), Su et al. (2009) and Zhang et al. (2009) for this example, and some computation results were given, whose results are affected by a predefined scalar δ. However, no method was given in Lin et al. (2008), Qiu et al. (2009), Su et al. (2009) and Zhang et al. (2009) on how to achieve the best δ.

To compare with the recently developed fuzzy H_∞ filter, we consider different τ_m and d to find the minimum index γ. For several values of τ_m and d, the computation results of $\gamma_{\min}$ are listed in Tables 1 and 2.

Table 1. Minimum index γ for $\tau_m = 0$, $d = 0.2$.

Reference	$\tau_M = 0.5$	$\tau_M = 0.6$	$\tau_M = 0.8$	$\tau_M = 1$
Su et al. (2009)	0.24	0.24	0.25	0.26
Zhang et al. (2009)	0.24	0.24	0.25	0.26
Lin et al. (2008)	0.34	0.34	0.35	0.37
Theorem 8	0.21	0.21	0.22	0.24

Table 2. Minimum index γ for $\tau_M = 1.25$.

	Methods	$d = 0.4$	$d = 0.6$	$d = 0.8$
$\tau_m = 0$	Qiu et al. (2009)	0.32	0.49	0.84
	Theorem 8	0.27	0.29	0.30
$\tau_m = 0.8$	Qiu et al. (2009)	0.32	0.40	0.40
	Theorem 8	0.26	0.27	0.27
$\tau_m = 1.0$	Qiu et al. (2009)	0.28	0.28	0.28
	Theorem 8	0.25	0.25	0.25

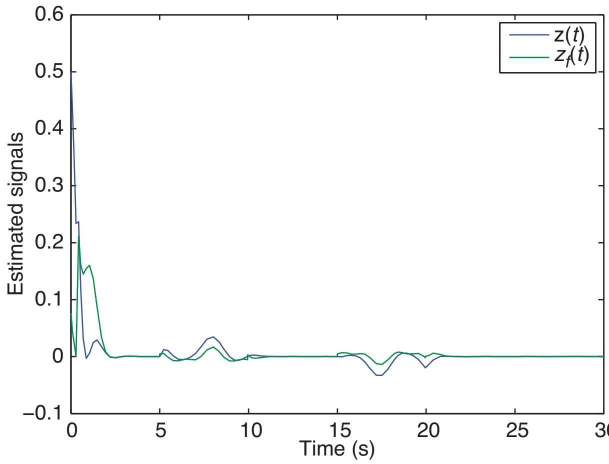

Figure 1. Estimated signals $z(t)$ and $z_f(t)$.

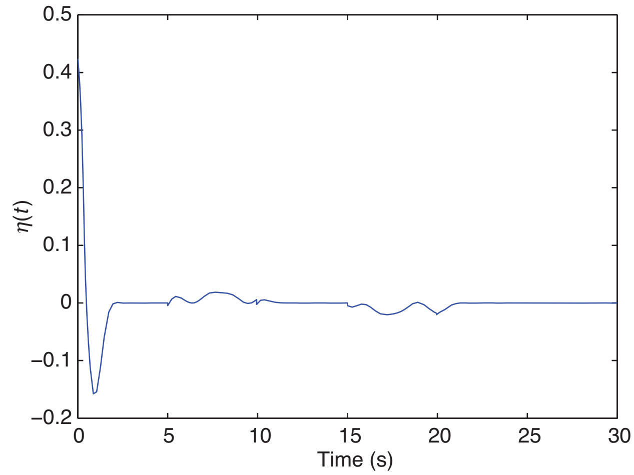

Figure 2. Estimated signals error $\eta(t) = z(t) - z_f(t)$.

Remark 3: In Lin et al. (2008), Qiu et al. (2009), Su et al. (2009) and Zhang et al. (2009) the authors list sets of $\gamma_{\min}$ for various δ. To illustrate our results are with less conservativeness, we chose their best results.

According to Theorem 8, we can get the minimum attenuation level $\gamma_{\min} = 0.21$ for $\tau_m = 0.1$, $\tau_M = 0.5$, $d = 0.2$, and a set of feasible solutions as follows:

$$\bar{P}_3 = \begin{bmatrix} 0.1326 & -0.1013 \\ -0.1013 & 0.1287 \end{bmatrix},$$
$$\bar{A}_{f1} = \begin{bmatrix} -0.4107 & 0.1412 \\ 0.4648 & -0.7985 \end{bmatrix}, \quad \bar{B}_{f1} = \begin{bmatrix} -0.2405 \\ 0.2261 \end{bmatrix},$$
$$\bar{C}_{f1} = \begin{bmatrix} -0.5978 & 0.4464 \end{bmatrix}, \quad \bar{D}_{f1} = 0.1974,$$
$$\bar{A}_{f2} = \begin{bmatrix} -0.3764 & 0.2150 \\ 0.2174 & -0.7640 \end{bmatrix}, \quad \bar{B}_{f2} = \begin{bmatrix} -0.1953 \\ 0.1924 \end{bmatrix},$$
$$\bar{C}_{f2} = \begin{bmatrix} 0.3081 & -0.4531 \end{bmatrix}, \quad \bar{D}_{f2} = 0.2329.$$

Furthermore, the H_∞ filter parameter matrices are computed from (47) as

$$\left[\begin{array}{c|c} A_{f1} & B_{f1} \\ \hline C_{f1} & D_{f1} \end{array}\right] = \left[\begin{array}{cc|c} -5.6597 & -3.3561 & -0.2405 \\ -3.0873 & -8.6327 & 0.2261 \\ \hline -4.6568 & -0.1956 & 0.1974 \end{array}\right]$$
$$\left[\begin{array}{c|c} A_{f2} & B_{f2} \\ \hline C_{f2} & D_{f2} \end{array}\right] = \left[\begin{array}{cc|c} -3.9146 & -1.4095 & -0.1953 \\ -7.2475 & -11.6377 & 0.1924 \\ \hline -0.9146 & -4.2395 & 0.2329 \end{array}\right].$$

With this filter, for an initial condition $x(0) = \left[\begin{smallmatrix} -1 \\ 1 \end{smallmatrix}\right]$, $x_f(0) = \left[\begin{smallmatrix} 0.1 \\ -0.1 \end{smallmatrix}\right]$, the time delay $\tau(t) = 0.3 + 0.2\sin(t)$, the simulation results are shown in Figures 1 and 2.

6. Conclusion

In this article, we have studied the problem of H_∞ filter design for nonlinear systems with time-delay through the T-S Fuzzy model approach, where two cases of time-varying delay have been studied. To analyse the H_∞ performance of the filtering-error system, a piecewise analysis method is used by using the convexity of the matrix function. Based on the new H_∞ performance analysis results, we have derived several criteria for the filter design. An example with simulation results has been carried out to demonstrate the effectiveness of the proposed method.

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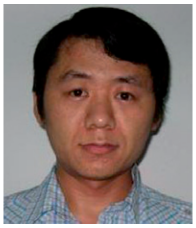

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