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Asymptotic and robust stability of T-S fuzzy genetic regulatory networks with time-varying delays

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SUMMARY

In this paper, we propose and investigate a new general model of fuzzy genetic regulatory networks described by the Takagi–Sugeno (T-S) fuzzy model with time-varying delays. By using a Lyapunov functional approach and linear matrix inequality (LMI) techniques, the stability criteria for the delayed fuzzy genetic regulatory networks are expressed as a set of LMIs, which can be solved numerically by LMI toolbox in Matlab. Two fuzzy genetic network example are given to verify the effectiveness and applicability of the proposed approach. Copyright © 2011 John Wiley & Sons, Ltd.

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KEY WORDS: T-S fuzzy model; fuzzy genetic regulatory network; time-varying delays; linear matrix inequality (LMI)

1. INTRODUCTION

During the past decades, genetic regulatory networks have received more attention in the biological and biomedical sciences, a few results have been done in this area [1–6]. Nowadays, one of the main challenges in systems biology is to understand the genetic regulatory networks, for example, how genes and proteins interact to form a complex network that performs complicated biological functions. Recently, mathematical modeling of genetic networks as dynamical system models provides a powerful tool for studying gene regulation processes in living organisms, genetic network models in literature can be roughly classified into two types, i.e. the Boolean model (or discrete model) and the differential equation model (or continuous model) [7, 8]. In Boolean models, the activity of each gene is expressed in one of two states: ON or OFF, and the state of a gene is determined by a Boolean function of the states of other related genes. In the differential equation models, the variables describe the concentrations of gene products, such as mRNAs and proteins, as continuous values of the gene regulation systems. Using continuous values, the second approach is considered more accurate, being able to provide more detailed understanding and insights into the dynamic behavior exhibited by biological systems. In this paper, we consider differential equation model of genetic network, in which the variables describe the concentrations of mRNAs and proteins, as continuous values of the gene regulation systems.

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Recent studies on genetic regulatory networks are fruitful, and many important results have been reported in the literature [9–12]. These results make significant contributions for discovering higher order structure of an organism and for gaining deep insights into both static and dynamic behaviors of genetic networks by extracting functional information from observation data. Based on the theoretical analysis, several simple genetic networks have been successfully constructed by means of experiments, for example, genetic switches [13], repressilator [14], and a single negative feedback loop network [15]. However, time delay should be considered in the biological systems or artificial genetic networks due to the slow processes of transcription, translation, and translocation or the finite switching speed of amplifiers, theoretical models without consideration delay may even provide wrong predictions [8, 14].

In reality, genetic regulatory networks may exhibit a special property called fuzzy reasoning. Among various fuzzy systems models, one of the most famous models is the Takagi–Sugeno (T-S) model [16]. Recently, the stability and stabilization problems for T-S fuzzy delayed systems have been investigated widely [17–21] and some delay-independent or delay-dependent conditions have been obtained using linear matrix inequalities (LMIs). However, to the best of the authors' knowledge, so far, the problem of stability for T-S fuzzy genetic regulatory networks has not been addressed in the literature, not to mention the time-varying delay is also involved. This work is the first attempt to explore the problem of asymptotic and robust stability for T-S fuzzy genetic regulatory networks.

This paper aims to investigate the robust stability of the fuzzy regulatory networks with time-varying delays, and the time delays are assumed to belong to the given intervals. Employing the convexity property of the matrix inequality, sufficient conditions of the asymptotic stability and robust stability are derived in terms of LMIs which are easy to be verified via the LMI toolbox. Two examples are employed to show the effectiveness and less conservativeness of the proposed method.

The remainder of this paper is organized as follows. In Section 2, genetic network model and preliminaries are given. In Section 3, asymptotic stability condition is derived for genetic networks with time-varying delays. In Section 4, robust stability condition is derived for genetic networks with time-varying delays and uncertainties. In Section 5, two genetic network examples are given to show the effectiveness of the proposed results. Finally, conclusions are given in Section 6.

Notation: $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of real $n\times m$ matrices, I is the identity matrix of appropriate dimensions, The notation $X>0$ (respectively, $X<0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive-definite square matrix (respectively, negative-definite square matrix). The asterisk $*$ in a matrix is used to denote term that is induced by symmetry.

2. MODEL AND PRELIMINARIES

The fuzzy system was proposed to represent a nonlinear system [16]. The system dynamics can be captured by a set of fuzzy rules which can characterize local correlations in the state space. Based on the T-S fuzzy model concept, a general class of T-S fuzzy genetic regulatory networks is considered here. The model of T-S fuzzy genetic regulatory networks is described as follows.

Plant rule i:

$$\text{IF } \theta_1(t) \text{ is } F_{i1}, \ldots, \theta_r(t) \text{ is } F_{ir},$$

$$\text{THEN } \begin{cases} \dot{M}(t) = -A_i M(t) + W_i f(P(t)) + B_i \\ \dot{P}(t) = -C_i P(t) + D_i M(t) \end{cases}$$

where $\theta_1(t), \ldots, \theta_r(t)$ are the premise variables; $F_{i1}, \ldots, F_{ir}$ are the fuzzy sets; $i \in \{1, 2, \ldots, r\} \overset{\Delta}{=} \mathbb{S}$, r is the number of IF-THEN rules; A_i, W_i, B_i, C_i, D_i are known constant matrices with appropriate dimensions.

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By using center-average defuzzifier, product interference and singleton fuzzifier, the T-S fuzzy systems can be inferred as

$$\begin{aligned}\dot{M}(t) &= -A(\mu_i)M(t)+W(\mu_i)f(P(t))+B(\mu_i)\\ \dot{P}(t) &= -C(\mu_i)P(t)+D(\mu_i)M(t)\end{aligned} \tag{1}$$

where $A(\mu_i)=\sum_{i=1}^{r}\mu_i A_i,\ W(\mu_i)=\sum_{i=1}^{r}\mu_i W_i,\ B(\mu_i)=\sum_{i=1}^{r}\mu_i B_i,\ C(\mu_i)=\sum_{i=1}^{r}\mu_i C_i,\ D(\mu_i)=\sum_{i=1}^{r}\mu_i D_i$ and

$$\mu_i(\theta(t))=\frac{\omega_i(\theta(t))}{\sum_{i=1}^{r}\omega_i(\theta(t))},\quad \omega_i(\theta(t))=\prod_{j=1}^{g}W_j^i(\theta_j(t))$$

$W_j^i(\theta_j(t))$ is the grade membership value of $\theta_j(t)$ in W_j^i and $\mu_i(\theta(t))$ satisfies

$$\mu_i(\theta(t))\geqslant 0,\quad \sum_{i=1}^{r}\mu_i(\theta(t))=1$$

For notational simplicity, we use μ_i to represent $\mu_i(\theta(t))$ in the following description.

Let M_i^* and $P_i^*(i\in\mathbb{S})$ be an equilibrium of (1), that is $(M_i^*,\ P_i^*)$ is the solution of equation

$$\begin{aligned}-A(\mu_i)M_i^*+W(\mu_i)f(P_i^*)+B(\mu_i)&=0\\ -C(\mu_i)P_i^*+D(\mu_i)M_i^*&=0\end{aligned} \tag{2}$$

For convenience, we will always shift an intended equilibrium point (M_i^*, P_i^*) of the system (1) to the origin by letting

$$m(t)=M(t)-M_i^*,\quad p(t)=P(t)-P_i^*$$

then, we have

$$\begin{aligned}\dot{m}(t) &= -A(\mu_i)m(t)+W(\mu_i)g_i(p(t))\\ \dot{p}(t) &= -C(\mu_i)p(t)+D(\mu_i)m(t)\end{aligned} \tag{3}$$

where $g_i(p(t))=f(p(t)+P_i^*)-f(P_i^*)$, since $f(x)$ represents the feedback regulation of the protein on the transcription, which is generally a monotonically increasing function with saturation [7, 8], it satisfies, for all $a,b\in R$, with $a\neq b$

$$0\leqslant\frac{f(a)-f(b)}{a-b}<k$$

where $f(x)$ is the differentiable, the above inequality is equivalent to $0\leqslant(\mathrm{d}f(a)/\mathrm{d}a)\leqslant k$, from the relationship of $f(\cdot)$ and $g_i(\cdot)$, we know that $g_i(\cdot)$ satisfies the sector condition $0\leqslant(g_i(a)/a)\leqslant k$, or equivalently

$$g_i(a)(g_i(a)-ka)\leqslant 0\quad (a\neq 0) \tag{4}$$

Recall that a Lur'e system is linear dynamic system, feedback interconnected to a static nonlinearity $f(\cdot)$ that satisfies a sector condition [22]. Hence, the genetic network (3) can be seen as a kind of Lur'e system, and can be investigated by using fruitful Lur'e system theory.

To have the accurate predictions, time delay should be considered in the biological systems due to the slow processes of transcription, translation, and translocation or the finite switching speed of amplifiers. It should be noted delay may play an important role in dynamics of genetic networks, and theoretical models without consideration of these factors may even provide wrong predictions [23]. In the following, we consider asymptotic stability of genetic networks with time-varying delays

$$\begin{aligned}\dot{m}(t) &= -A(\mu_i)m(t)+W(\mu_i)g_i(p(t-\sigma(t)))\\ \dot{p}(t) &= -C(\mu_i)P(t)+D(\mu_i)m(t-\tau(t))\end{aligned} \tag{5}$$

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where $\tau(t)$, $\sigma(t)$ are the time-varying delays, which satisfy the following conditions:

$$0 \leqslant \tau_m \leqslant \tau(t) \leqslant \tau_M, \tag{6}$$

$$0 \leqslant \sigma_m \leqslant \sigma(t) \leqslant \sigma_M. \tag{7}$$

Remark 1
To the best of the authors' knowledge, the genetic regulatory networks model is described by T-S fuzzy logic for the first time. When $r=1$, system (1) deduces into general genetic regulatory networks. It is also worth mentioning that, compared with the well-studied works [2, 6, 10, 23], the delayed coupling is also investigated here.

Remark 2
T-S fuzzy model can represent the local dynamics in different state–space regions by many linear models, which present a way to utilize the mature theories and analysis methods to study the nonlinear systems. As a useful approach, in the past few decades, the T-S fuzzy model has been demonstrated to be effective in dealing with a variety of complex nonlinear systems, which has therefore received a great deal of attention in the literature [17–21].

To obtain the main results, the following lemmas are needed.

Lemma 1 ([*24*])
Suppose $\tau_m \leqslant \tau(t) \leqslant \tau_M$, and $x(t) \in R^n$, for any positive matrix $R \in R^{n\times n}$, $R = R^T > 0$, then

$$-(\tau_M - \tau_m)\int_{t-\tau_M}^{t-\tau_m} \dot{x}^T(s) R \dot{x}(s)\,\mathrm{d}s \leqslant \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_M) \end{bmatrix} \tag{8}$$

Lemma 2 (*Han and Yue* [*25*])
Suppose $0 \leqslant \tau_m \leqslant \tau(t) \leqslant \tau_M$, Ξ_1, Ξ_2 and Ω are constant matrices of appropriate dimensions, then

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \tag{9}$$

if and only if the following inequalities hold:

$$(\tau_M - \tau_m)\Xi_1 + \Omega < 0 \tag{10}$$

$$(\tau_M - \tau_m)\Xi_2 + \Omega < 0 \tag{11}$$

Lemma 3 (*Hale and Verduyn Lunel* [*26*])
Suppose that μ, v, ω: $\mathbb{R}^+ \to \mathbb{R}^+$ are continuous, strictly monotonically increase functions, and that $\mu(s)$, $v(s)$, $\omega(s)$ are positive for $s>0$, $\mu(0)=v(0)=0$, with $\mu(s)\to\infty$ as $s\to\infty$. If there exists a continuous function $V:\mathbb{R}\times\mathbb{R}^n \to \mathbb{R}$ and a positive number $q>1$ such that

$$\mu(\|x\|) \leqslant V(t,x) \leqslant v(\|x\|) \quad \forall t \in \mathbb{R}, \quad x \in \mathbb{R}^n \tag{12}$$

and that

$$\dot{v}(t,x(t)) \leqslant -\omega(\|x\|) \quad \text{if } \|x(t+\theta)\| < q\|x\| \ \ \forall -\tau \leqslant \theta \leqslant 0 \tag{13}$$

where $\dot{v}(t,x(t))$ is the derivative of V along the solutions of system (5), τ is the upper bound of the time delay, then system (5) is asymptotically stable.

3. ASYMPTOTIC STABILITY CONDITION OF GENETIC NETWORKS WITH TIME-VARYING DELAYS

In this section, by using convexity property of the matrix inequality and the Lyapunov stability theorem, we analyze the stability of the fuzzy genetic network with time-varying delays.

Theorem 1

The origin of system (5) is asymptotically stable for any given $0\leqslant\tau_m\leqslant\tau(t)\leqslant\tau_M$, $0\leqslant\sigma_m\leqslant\sigma(t)\leqslant\sigma_M$ and k, if there exist positive-definite matrices $Q_i>0$, $R_i>0 (i=1,2,\ldots,5)$, $\Lambda_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\ldots,\lambda_{in})>0$ $(i\in\mathbb{S})$, M_i, N_i, T_i, V_i $(i\in\mathbb{S})$ and U_i, $S_i (i=1,2,3,4,5)$ of appropriate dimensions such that the following LMIs hold:

$$\Xi(i,j,l)=\begin{bmatrix}\Xi_{11}+\Omega+\Omega^{\mathrm{T}} & * & * & * & * \\ \Xi_{21} & \Xi_{22}+\Gamma+\Gamma^{\mathrm{T}} & * & * & * \\ \Xi_{31} & \Xi_{32} & -2\Lambda_i & * & * \\ \Xi_{41}(j) & 0 & 0 & -Q_5 & * \\ 0 & \Xi_{52}(l) & 0 & 0 & -R_5\end{bmatrix}<0 \quad (j,l=1,2) \qquad (14)$$

where

$$\Xi_{11}=\begin{bmatrix}\Upsilon_1 & * & * & * & * \\ -S_2A_i & 0 & * & * & * \\ Q_4-S_3A_i & 0 & -Q_2-Q_4 & * & * \\ -S_4A_i & 0 & 0 & -Q_3 & * \\ -S_5A_i-S_1^{\mathrm{T}} & -S_2^{\mathrm{T}} & -S_3^{\mathrm{T}} & -S_4^{\mathrm{T}} & \tau_m^2Q_4+\delta_1Q_5-S_5-S_5^{\mathrm{T}}\end{bmatrix}$$

$$\Xi_{21}=\begin{bmatrix}0 & R_1D_i+U_1D_i & 0 & 0 & 0 \\ 0 & U_2D_i & 0 & 0 & 0 \\ 0 & U_3D_i & 0 & 0 & 0 \\ 0 & U_4D_i & 0 & 0 & 0 \\ 0 & U_5D_i & 0 & 0 & 0\end{bmatrix}$$

$$\Xi_{22}=\begin{bmatrix}\Upsilon_2 & * & * & * & * \\ -U_2C_i & 0 & * & * & * \\ R_4-U_3C_i & 0 & -R_2-R_4 & * & * \\ -U_4C_i & 0 & 0 & -R_3 & * \\ -U_1^{\mathrm{T}}-U_5C_i & -U_2^{\mathrm{T}} & -U_3^{\mathrm{T}} & -U_4^{\mathrm{T}} & \sigma_m^2R_4+\delta_2R_5-U_5-U_5^{\mathrm{T}}\end{bmatrix}$$

$$\Xi_{31}=[W_i^TQ_1+W_i^TS_1^{\mathrm{T}} \;\; W_i^TS_2^{\mathrm{T}} \;\; W_i^TS_3^{\mathrm{T}} \;\; W_i^TS_4^{\mathrm{T}} \;\; W_i^TS_5^{\mathrm{T}}]$$

$$\Xi_{32}=[0 \;\; k\Lambda_i \;\; 0 \;\; 0 \;\; 0]$$

$$\Xi_{41}(1)=\sqrt{\delta_1}M_i^{\mathrm{T}}, \quad \Xi_{41}(2)=\sqrt{\delta_1}N_i^{\mathrm{T}}$$

$$\Xi_{52}(1)=\sqrt{\delta_2}T_i^{\mathrm{T}}, \quad \Xi_{52}(2)=\sqrt{\delta_2}V_i^{\mathrm{T}}$$

$$\Upsilon_1=-Q_1A_i-A_i^TQ_1+Q_2+Q_3-Q_4-S_1A_i-A_i^TS_1^{\mathrm{T}}$$

$$\Upsilon_2=-R_1C_i-C_i^TR_1-U_1C_i-C_i^TU_1^{\mathrm{T}}+R_2+R_3-R_4$$

$$\Omega=[0 \;\; -M_i+N_i \;\; M_i \;\; -N_i \;\; 0]$$

$$\Gamma=[0 \;\; -T_i+V_i \;\; T_i \;\; -V_i \;\; 0]$$

$$\delta_1=\tau_M-\tau_m, \quad \delta_2=\sigma_M-\sigma_m$$

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Proof
Construct a Lyapunov–Krasovskii candidate as

$$V(t)=V_1(t)+V_2(t)+V_3(t) \tag{15}$$

where

$$\begin{aligned}
V_1(t) &= m^{\mathrm{T}}(t)Q_1 m(t)+p^{\mathrm{T}}(t)R_1 p(t)\\
V_2(t) &= \int_{t-\tau_m}^{t} m^{\mathrm{T}}(s)Q_2 m(s)\,\mathrm{d}s+\int_{t-\tau_M}^{t} m^{\mathrm{T}}(s)Q_3 m(s)\,\mathrm{d}s+\int_{t-\sigma_m}^{t} p^{\mathrm{T}}(s)R_2 p(s)\,\mathrm{d}s\\
&\quad+\int_{t-\sigma_M}^{t} p^{\mathrm{T}}(s)R_3 p(s)\,\mathrm{d}s\\
V_3(t) &= \tau_m\int_{t-\tau_m}^{t}\int_{s}^{t}\dot m^{\mathrm{T}}(v)Q_4\dot m(v)\,\mathrm{d}v\,\mathrm{d}s+\int_{t-\tau_M}^{t-\tau_m}\int_{s}^{t}\dot m^{\mathrm{T}}(v)Q_5\dot m(v)\,\mathrm{d}v\,\mathrm{d}s\\
&\quad+\sigma_m\int_{t-\sigma_m}^{t}\int_{s}^{t}\dot p^{\mathrm{T}}(v)R_4\dot p(v)\,\mathrm{d}v\,\mathrm{d}s+\int_{t-\sigma_M}^{t-\sigma_m}\int_{s}^{t}\dot p^{\mathrm{T}}(v)R_5\dot p(v)\,\mathrm{d}v\,\mathrm{d}s
\end{aligned}$$

Calculating the derivative of $V(t)$ leads to the following equality:

$$\begin{aligned}
\dot V(t) &= 2m^{\mathrm{T}}(t)Q_1\dot m(t)+2p^{\mathrm{T}}(t)R_1\dot p(t)+m^{\mathrm{T}}(t)(Q_2+Q_3)m(t)-m^{\mathrm{T}}(t-\tau_m)Q_2 m(t-\tau_m)\\
&\quad-m^{\mathrm{T}}(t-\tau_M)Q_3 m(t-\tau_M)+p^{\mathrm{T}}(t)(R_2+R_3)p(t)-p^{\mathrm{T}}(t-\sigma_m)R_2 p(t-\sigma_m)\\
&\quad-p^{\mathrm{T}}(t-\sigma_M)R_3 p(t-\sigma_M)+\dot m^{\mathrm{T}}(t)(\tau_m^2 Q_4+\delta_1 Q_5)\dot m(t)+\dot p^{\mathrm{T}}(t)(\sigma_m^2 R_4+\delta_2 R_5)\dot p(t)\\
&\quad-\tau_m\int_{t-\tau_m}^{t}\dot m^{\mathrm{T}}(s)Q_4\dot m(s)\,\mathrm{d}s-\sigma_m\int_{t-\sigma_m}^{t}\dot p^{\mathrm{T}}(s)R_4\dot p(s)\,\mathrm{d}s-\int_{t-\tau_M}^{t-\tau_m}\dot m^{\mathrm{T}}(s)Q_5\dot m(s)\,\mathrm{d}s\\
&\quad-\int_{t-\sigma_M}^{t-\sigma_m}\dot p^{\mathrm{T}}(s)R_5\dot p(s)\,\mathrm{d}s
\end{aligned} \tag{16}$$

Using Lemma 1, we can obtain

$$-\tau_m\int_{t-\tau_m}^{t}\dot m^{\mathrm{T}}(s)Q_4\dot m(s)\,\mathrm{d}s\leqslant\begin{bmatrix} m(t)\\ m(t-\tau_m)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-Q_4 & Q_4\\ Q_4 & -Q_4\end{bmatrix}\begin{bmatrix} m(t)\\ m(t-\tau_m)\end{bmatrix} \tag{17}$$

$$-\sigma_m\int_{t-\sigma_m}^{t}\dot p^{\mathrm{T}}(s)R_4\dot p(s)\,\mathrm{d}s\leqslant\begin{bmatrix} p(t)\\ p(t-\sigma_m)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-R_4 & R_4\\ R_4 & -R_4\end{bmatrix}\begin{bmatrix} p(t)\\ p(t-\sigma_m)\end{bmatrix} \tag{18}$$

Noting the sector condition, for any $\lambda_{ij}>0(i\in\mathbb{S},\ j=1,2,\ldots,n)$, we have

$$-2\sum_{j=1}^{n}\lambda_{ij}g_i(p_j(t-\sigma(t)))[g_i(p_j(t-\sigma(t)))-kp_j(t-\sigma(t))]\geqslant 0 \tag{19}$$

Rewriting the above inequalities into a compact matrix form, we obtain

$$-2g_i^{\mathrm{T}}(p(t-\sigma(t)))\Lambda_i g_i(p(t-\sigma(t)))+2kg_i^{\mathrm{T}}(p(t-\sigma(t)))\Lambda_i p(t-\sigma(t))\geqslant 0 \tag{20}$$

where $\Lambda_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\ldots,\lambda_{in})>0$

From (16)–(20), we can get

$$\begin{aligned}\dot{V}(t) \leqslant & 2m^{\mathrm{T}}(t)Q_1\left\{\sum_{i=1}^{r}\mu_i[-A_i m(t)+W_i g_i(p(t-\sigma(t)))]\right\}\\ &+2p^{\mathrm{T}}(t)R_1\left\{\sum_{i=1}^{r}\mu_i[-C_i p(t)+D_i m(t-\tau(t))]\right\}+m^{\mathrm{T}}(t)(Q_2+Q_3)m(t)\\ &-m^{\mathrm{T}}(t-\tau_m)Q_2 m(t-\tau_m)-m^{\mathrm{T}}(t-\tau_M)Q_3 m(t-\tau_M)+p^{\mathrm{T}}(t)(R_2+R_3)p(t)\\ &-p^{\mathrm{T}}(t-\sigma_m)R_2 p(t-\sigma_m)-p^{\mathrm{T}}(t-\sigma_M)R_3 p(t-\sigma_M)-2g_i^{\mathrm{T}}(p(t-\sigma(t)))\Lambda_i g_i(p(t-\sigma(t)))\\ &+2kg_i^{\mathrm{T}}(p(t-\sigma(t)))\Lambda_i p(t-\sigma(t))+\begin{bmatrix} m(t)\\ m(t-\tau_m)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-Q_4 & Q_4\\ Q_4 & -Q_4\end{bmatrix}\begin{bmatrix} m(t)\\ m(t-\tau_m)\end{bmatrix}\\ &+\begin{bmatrix} p(t)\\ p(t-\sigma_m)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-R_4 & R_4\\ R_4 & -R_4\end{bmatrix}\begin{bmatrix} p(t)\\ p(t-\sigma_m)\end{bmatrix}-\int_{t-\tau_M}^{t-\tau_m}\dot{m}^{\mathrm{T}}(s)Q_5\dot{m}(s)\,\mathrm{d}s\\ &-\int_{t-\sigma_M}^{t-\sigma_m}\dot{p}^{\mathrm{T}}(s)R_5\dot{p}(s)\,\mathrm{d}s \end{aligned} \tag{21}$$

By using Lemma 1, we have

$$-\int_{t-\tau_M}^{t-\tau_m}\dot{m}^{\mathrm{T}}(s)Q_5\dot{m}(s)\leqslant\frac{1}{\delta_1}\begin{bmatrix} m(t-\tau_m)\\ m(t-\tau_M)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-Q_5 & Q_5\\ Q_5 & -Q_5\end{bmatrix}\begin{bmatrix} m(t-\tau_m)\\ m(t-\tau_M)\end{bmatrix} \tag{22}$$

$$-\int_{t-\sigma_M}^{t-\sigma_m}\dot{p}^{\mathrm{T}}(s)R_5\dot{p}(s)\leqslant\frac{1}{\delta_2}\begin{bmatrix} p(t-\sigma_m)\\ p(t-\sigma_M)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}-R_5 & R_5\\ R_5 & -R_5\end{bmatrix}\begin{bmatrix} p(t-\sigma_m)\\ p(t-\sigma_M)\end{bmatrix} \tag{23}$$

Employing the free matrix method, we have

$$\sum_{i=1}^{r}\mu_i\left\{2\xi_1^{\mathrm{T}}(t)M_i\left[m(t-\tau_m)-m(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_m}\dot{m}(v)\mathrm{d}v\right]\right\}=0 \tag{24}$$

$$\sum_{i=1}^{r}\mu_i\left\{2\xi_1^{\mathrm{T}}(t)N_i\left[m(t-\tau(t))-m(t-\tau_M)-\int_{t-\tau_1}^{t-\tau(t)}\dot{m}(v)dv\right]\right\}=0 \tag{25}$$

$$\sum_{i=1}^{r}\mu_i\left\{2\xi_2^{\mathrm{T}}(t)T_i\left[p(t-\sigma_m)-p(t-\sigma(t))-\int_{t-\sigma(t)}^{t-\sigma_m}\dot{p}(v)dv\right]\right\}=0 \tag{26}$$

$$\sum_{i=1}^{r}\mu_i\left\{2\xi_2^{\mathrm{T}}(t)V_i\left[p(t-\sigma(t))-p(t-\sigma_M)-\int_{t-\sigma_1}^{t-\sigma(t)}\dot{p}(v)\mathrm{d}v\right]\right\}=0 \tag{27}$$

$$\sum_{i=1}^{r}\mu_i\{2\xi_1^{\mathrm{T}}(t)S[-A_i m(t)+W_i g_i(p(t-\sigma(t)))-\dot{m}(t)]\}=0 \tag{28}$$

$$\sum_{i=1}^{r}\mu_i\{2\xi_2^{\mathrm{T}}(t)U[-C_i p(t)+D_i m(t-\tau(t)))-\dot{p}(t)]\}=0 \tag{29}$$

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where

$$\xi_1^{\mathrm{T}}(t)=[m^{\mathrm{T}}(t)\ \ m^{\mathrm{T}}(t-\tau(t))\ \ m^{\mathrm{T}}(t-\tau_m)\ \ m^{\mathrm{T}}(t-\tau_M)\ \ \dot{m}^{\mathrm{T}}(t)]$$
$$\xi_2^{\mathrm{T}}(t)=[p^{\mathrm{T}}(t)\ \ p^{\mathrm{T}}(t-\sigma(t))\ \ p^{\mathrm{T}}(t-\sigma_m)\ \ p^{\mathrm{T}}(t-\sigma_M)\ \ \dot{p}^{\mathrm{T}}(t)]$$
$$M_i^{\mathrm{T}}=[M_{i1}^{\mathrm{T}}\ \ M_{i2}^{\mathrm{T}}\ \ M_{i3}^{\mathrm{T}}\ \ M_{i4}^{\mathrm{T}}\ \ M_{i5}^{\mathrm{T}}]$$
$$N_1^{\mathrm{T}}=[N_{i1}^{\mathrm{T}}\ \ N_{i2}^{\mathrm{T}}\ \ N_{i3}^{\mathrm{T}}\ \ N_{i4}^{\mathrm{T}}\ \ N_{i5}^{\mathrm{T}}]$$
$$T_1^{\mathrm{T}}=[T_{i1}^{\mathrm{T}}\ \ T_{i2}^{\mathrm{T}}\ \ T_{i3}^{\mathrm{T}}\ \ T_{i4}^{\mathrm{T}}\ \ T_{i5}^{\mathrm{T}}]$$
$$V_1^{\mathrm{T}}=[V_{i1}^{\mathrm{T}}\ \ V_{i2}^{\mathrm{T}}\ \ V_{i3}^{\mathrm{T}}\ \ V_{i4}^{\mathrm{T}}\ \ V_{i5}^{\mathrm{T}}]$$
$$S^{\mathrm{T}}=[S_1^{\mathrm{T}}\ \ S_2^{\mathrm{T}}\ \ S_3^{\mathrm{T}}\ \ S_4^{\mathrm{T}}\ \ S_5^{\mathrm{T}}]$$
$$U^{\mathrm{T}}=[U_1^{\mathrm{T}}\ \ U_2^{\mathrm{T}}\ \ U_3^{\mathrm{T}}\ \ U_4^{\mathrm{T}}\ \ U_5^{\mathrm{T}}]$$

There exist Q_5, R_5 such that

$$-2\sum_{i=1}^{r}\mu_i\xi_1^{\mathrm{T}}(t)M_i\int_{t-\tau(t)}^{t-\tau_m}\dot{m}(v)\,\mathrm{d}v$$
$$\leqslant(\tau(t)-\tau_m)\sum_{i=1}^{r}\mu_i\xi_1^{\mathrm{T}}(t)M_iQ_5^{-1}M_i^{\mathrm{T}}\xi_1(t)+\int_{t-\tau(t)}^{t-\tau_m}\dot{m}^{\mathrm{T}}(v)Q_5\dot{m}(v)\,\mathrm{d}v \tag{30}$$

$$-2\sum_{i=1}^{r}\mu_i\xi_1^{\mathrm{T}}(t)N_i\int_{t-\tau_M}^{t-\tau(t)}\dot{m}(v)\,\mathrm{d}v$$
$$\leqslant(\tau_M-\tau(t))\sum_{i=1}^{r}\mu_i\xi_1^{\mathrm{T}}(t)N_iQ_5^{-1}N_i^{\mathrm{T}}\xi_1(t)+\int_{t-\tau_1}^{t-\tau(t)}\dot{m}^{\mathrm{T}}(v)Q_5\dot{m}(v)\,\mathrm{d}v \tag{31}$$

$$-2\sum_{i=1}^{r}\mu_i\xi_2^{\mathrm{T}}(t)T_i\int_{t-\sigma(t)}^{t-\sigma_m}\dot{p}(v)\,\mathrm{d}v$$
$$\leqslant(\sigma(t)-\sigma_m)\sum_{i=1}^{r}\mu_i\xi_2^{\mathrm{T}}(t)T_iR_5^{-1}T_i^{\mathrm{T}}\xi_2(t)+\int_{t-\sigma(t)}^{t-\sigma_m}\dot{p}^{\mathrm{T}}(v)R_5\dot{p}(v)\,\mathrm{d}v \tag{32}$$

$$-2\sum_{i=1}^{r}\mu_i\xi_2^{\mathrm{T}}(t)V_i\int_{t-\sigma_M}^{t-\sigma(t)}\dot{p}(v)\,\mathrm{d}v$$
$$\leqslant(\sigma_M-\sigma(t))\sum_{i=1}^{r}\mu_i\xi_2^{\mathrm{T}}(t)V_iR_5^{-1}V_i^{\mathrm{T}}\xi_2(t)+\int_{t-\sigma_1}^{t-\sigma(t)}\dot{p}^{\mathrm{T}}(v)R_5\dot{p}(v)\,\mathrm{d}v \tag{33}$$

Adding (24)–(29) to the right of (21) and substituting (22), (23), and (30)–(33) into (21), we have

$$\begin{aligned}\dot{V}(t)\leqslant\sum_{i=1}^{r}\mu_i\Bigg\{&\xi^{\mathrm{T}}(t)\begin{bmatrix}\Xi_{11}+\Omega+\Omega^{\mathrm{T}} & *\\ \Xi_{21} & \Xi_{22}+\Gamma+\Gamma^{\mathrm{T}}\end{bmatrix}\xi(t)+(\tau(t)-\tau_m)\xi_1^{\mathrm{T}}(t)M_iQ_5^{-1}M_i^{\mathrm{T}}\xi_1(t)\\ &+(\tau_M-\tau(t))\xi_1^{\mathrm{T}}(t)N_iQ_5^{-1}N_i^{\mathrm{T}}\xi_1(t)+(\sigma(t)-\sigma_m)\xi_2^{\mathrm{T}}(t)T_iR_5^{-1}T_i^{\mathrm{T}}\xi_2(t)\\ &+(\sigma_M-\sigma(t))\xi_2^{\mathrm{T}}(t)V_iR_5^{-1}V_i^{\mathrm{T}}\xi_2(t)\Bigg\}\end{aligned} \tag{34}$$

where $\xi^{\mathrm{T}}(t)=[\xi_1^{\mathrm{T}}(t)\ \ \xi_2^{\mathrm{T}}(t)\ \ g_i^{\mathrm{T}}(p(t-\sigma(t)))]$.

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Using Lemma 2 and Schur complement, it is easy to see that (14) with $j, l=1,2$ can lead $\dot{V}(t)\leqslant 0$. This implies that $\dot{V}(t)\leqslant -\rho\|x(t)\|^2$ for a sufficiently small $\rho>0$, thereby revealing Lemma 3, the origin of system (5) is asymptotically stable. □

4. ROBUSTLY ASYMPTOTIC STABILITY CONDITION OF FUZZY GENETIC NETWORKS WITH TIME-VARYING DELAYS

Consider robust stability for fuzzy genetic networks with time-varying delays

$$\begin{aligned}\dot{m}(t) &= -(A_i+\triangle A_i(t))m(t)+(W_i+\triangle W_i(t))g_i(P(t-\sigma(t)))\\ \dot{p}(t) &= -(C_i+\triangle C_i(t))P(t)+(D_i+\triangle D_i(t))m(t-\tau(t))\end{aligned} \tag{35}$$

where the time-varying delay $\tau(t)$, $\sigma(t)$ satisfy (11), (12). The time-varying uncertain matrices $\triangle A_i(t)$, $\triangle W_i(t)$, $\triangle C_i(t)$, $\triangle D_i(t)$ are defined as follows:

$$\begin{aligned}&\triangle A_i(t)=E_{1i}F_{1i}(t)T_{1i}, \quad \triangle W_i(t)=E_{2i}F_{2i}(t)T_{2i}, \quad \triangle C_i(t)=E_{3i}F_{3i}(t)T_{3i},\\ &\triangle D_i(t)=E_{4i}F_{4i}(t)T_{4i}\end{aligned} \tag{36}$$

where E_{1i}, E_{2i}, E_{3i}, E_{4i}, T_{1i}, T_{2i}, T_{3i} and T_{4i} are known constant real matrices with appropriate dimensions, $F_{1i}(t)$, $F_{2i}(t)$, $F_{3i}(t)$, and $F_{4i}(t)$ are unknown time-varying matrices satisfying

$$F_{1i}^{\mathrm{T}}(t)F_{1i}(t)\leqslant I, \quad F_{2i}^{\mathrm{T}}(t)F_{2i}(t)\leqslant I, \quad F_{3i}^{\mathrm{T}}(t)F_{3i}(t)\leqslant I, \quad F_{4i}^{\mathrm{T}}(t)F_{4i}(t)\leqslant I \tag{37}$$

Based on (35)–(37), a sufficient condition for delay-dependent asymptotical stability of the system (35) is given in the following theorem.

Theorem 2

The origin of system (35) is robustly asymptotically stable for any given $0\leqslant\tau_m\leqslant\tau(t)\leqslant\tau_M$, $0\leqslant\sigma_m\leqslant\sigma(t)\leqslant\sigma_M$ and k, if there exist positive-definite matrices $Q_i>0, R_i>0(i=1,2,\ldots,5)$, $\Lambda_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\ldots,\lambda_{in})>0$, M_i, N_i, T_i, V_i $(i\in\mathbb{S})$, U_i, $S_i(i=1,2,3,4,5)$ and scalars $l_{ij}(i=1,2,3,4, j\in\mathbb{S})$ of appropriate dimensions such that the following LMIs hold:

$$\Xi(i,j,l)=\begin{bmatrix}\Xi_{11}+\Omega+\Omega^{\mathrm{T}}+\Phi_1 & * & * & * & * & *\\ \Xi_{21} & \Xi_{22}+\Gamma+\Gamma^{\mathrm{T}}+\Phi_2 & * & * & * & *\\ \Xi_{31} & \Xi_{32} & -2\Lambda_i+l_{4i}T_{2i}^T T_{2i} & * & * & *\\ \Phi_3 & \Phi_4 & 0 & \Phi_5 & * & *\\ \Xi_{41}(j) & 0 & 0 & 0 & -Q_5 & *\\ 0 & \Xi_{52}(l) & 0 & 0 & 0 & -R_5\end{bmatrix}<0$$

$$(j,l=1,2) \tag{38}$$

where

$$\Phi_1=\mathrm{diag}\{l_{1i}T_{1i}^T T_{1i}, l_{2i}T_{4i}^T T_{4i}, 0,0,0\}$$

$$\Phi_2=\mathrm{diag}\{l_{3i}T_{3i}^T T_{3i}, 0,0,0,0\}$$

$$\Phi_3=\begin{bmatrix}-E_{1i}^T Q_1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ E_{2i}^T Q_1 & 0 & 0 & 0 & 0\end{bmatrix}$$

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$$\Phi_4=\begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ E_{4i}^T R_1 & 0 & 0 & 0 & 0\\ -E_{3i}^T R_1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_5=\mathrm{diag}\{-l_{1i}I,-l_{2i}I,-l_{3i}I,-l_{4i}I\}$$

and Ξ_{11}, Ξ_{21}, Ξ_{22}, Ξ_{31}, Ξ_{32}, $\Xi_{41}(j)$, $\Xi_{52}(l)$, Ω, Γ are defined in Theorem 1.

Proof
Take the same Lyapunov functional as that in the proof of Theorem 1, and replace A_i, W_i, C_i, and D_i by $A_i+E_{1i}F_{1i}(t)T_{1i}$, $W_i+E_{2i}F_{2i}(t)T_{2i}$, $C_i+E_{3i}F_{3i}(t)T_{3i}$, and $D_i+E_{4i}F_{4i}(t)T_{4i}$. Note that

$$l_{1i}m^{\mathrm{T}}(t)T_{1i}^{\mathrm{T}}T_{1i}m(t)-l_{1i}[F_{1i}(t)T_{1i}m(t)]^{\mathrm{T}}[F_{1i}(t)T_{1i}m(t)]\geqslant 0 \tag{39}$$

$$l_{2i}m^{\mathrm{T}}(t-\tau(t))T_{4i}^{\mathrm{T}}T_{4i}m(t-\tau(t))-l_{2i}[F_{4i}(t)T_{4i}m(t-\tau(t))]^{\mathrm{T}}[F_{4i}(t)T_{4i}m(t-\tau(t))]\geqslant 0 \tag{40}$$

$$l_{3i}p^{\mathrm{T}}(t)T_{3i}^{\mathrm{T}}T_{3i}p(t)-l_{3i}[F_{3i}(t)T_{3i}p(t)]^{\mathrm{T}}[F_{3i}(t)T_{3i}p(t)]\geqslant 0 \tag{41}$$

$$l_{4i}g^{\mathrm{T}}(p(t-\sigma(t)))T_{2i}^{\mathrm{T}}T_{2i}g_i(p(t-\sigma(t)))-l_{4i}[F_{2i}(t)T_{2i}g_i(p(t-\sigma(t)))]^{\mathrm{T}}[F_{2i}(t)T_{2i}g(p(t-\sigma(t)))]\geqslant 0 \tag{42}$$

Using the above inequalities, we have

$$\begin{aligned}\dot V(t)\leqslant \eta^{\mathrm{T}}(t)&\begin{bmatrix} \Xi_{11}+\Omega+\Omega^{\mathrm{T}}+\Phi_1 & * & * & *\\ \Xi_{21} & \Xi_{22}+\Gamma+\Gamma^{\mathrm{T}}+\Phi_2 & * & *\\ \Xi_{31} & \Xi_{32} & -2\Lambda_i+l_{4i}T_{2i}^{\mathrm{T}}T_{2i} & *\\ \Phi_3 & \Phi_4 & 0 & \Phi_5 \end{bmatrix}\eta(t)\\ &+(\tau(t)-\tau_m)\xi_1^{\mathrm{T}}(t)M_1Q_5^{-1}M_1^{\mathrm{T}}\xi_1(t)+(\tau_1-\tau(t))\xi_1^{\mathrm{T}}(t)N_1Q_5^{-1}N_1^{\mathrm{T}}\xi_1(t)\\ &+(\sigma(t)-\sigma_m)\xi_2^{\mathrm{T}}(t)T_1R_5^{-1}T_1^{\mathrm{T}}\xi_2(t)+(\sigma_1-\sigma(t))\xi_2^{\mathrm{T}}(t)V_1R_5^{-1}V_1^{\mathrm{T}}\xi_2(t)\end{aligned} \tag{43}$$

where

$$\eta^{\mathrm{T}}(t)=[\xi^{\mathrm{T}}(t)\ \ [F_{1i}(t)T_{1i}m(t)]^{\mathrm{T}}\ \ [F_{4i}(t)T_{4i}m(t-\tau(t))]\ \ [F_{3i}(t)T_{3i}p(t)]^{\mathrm{T}}\ \ [F_{2i}(t)T_{2i}g_i(p(t-\sigma(t)))]^{\mathrm{T}}],$$

From (38) and (43), using Lemma 2 and the Schur complement, it is easy to see that the conditions (38) can lead $\dot V(t)\leqslant 0$. This implies that $\dot V(t)\leqslant -\rho\|x(t)\|^2$ for a sufficiently small $\rho>0$, thereby revealing Lemma 3, the origin of system (35) is asymptotically stable. □

5. EXAMPLE

In this section, we will present two examples to illustrate our theoretical results.

Example 1
We consider the following T-S fuzzy genetic regulatory networks with time-varying delays consisting of two modes, the deterministic parameters of (5) are given as: $\mu_1(t)=\sin^2(t)$,

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$\mu_2(t)=\cos^2(t)$, $A_1=C_1=I_5$, $D_1=0.8I_5$, $A_2=C_2=D_2=I_5$,

$$W_1=0.5\times\begin{bmatrix}0 & -1 & 1 & 0 & 0\\ -1 & 0 & 0 & 1 & 1\\ 0 & 1 & 0 & 0 & 0\\ 1 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\end{bmatrix},\quad W_2=0.3\times\begin{bmatrix}0 & -1 & 1 & 0 & 0\\ -1 & 0 & 0 & 1 & 1\\ 0 & 1 & 0 & 0 & 0\\ 1 & -1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\end{bmatrix}$$

The time-varying delays are chosen as $\tau_m=0.10$, $\sigma(t)=0.5+0.1sin(t)$, which means $\sigma_m=0.40$, $\sigma_M=0.60$; The nonlinear regulatory functions $g_i(\cdot)$ are taken as the usually used Hill form, that is, $g_1(x)=g_2(x)=x^2/(1+x^2)$.

According to Theorem 1, by using the MATLAB LMI Toolbox, the T-S fuzzy genetic regulatory networks with time-varying delays can achieve asymptotic stability under the allowable maximum delay of $\tau_M=4.19$, when $\tau_m=0.10$.

The computational simulation results of trajectories $p_i(t)$ and $m_i(t)$ are shown in Figure 1, Figure 2 with the initial values $m(0)=[0.1\ 0.2\ 0.3\ 0.4\ 0.5]^{\mathrm{T}}$, $p(0)=[0.5\ 0.4\ 0.3\ 0.2\ 0.1]^{\mathrm{T}}$ when $\tau_m=0.1$, $\tau_M=4.19$.

Example 2

Based on Example 1, we consider the following uncertain parameters:

$$E_{11}=E_{21}=\begin{bmatrix}0.4 & 0.1 & -0.2 & -0.1 & 0.1\\ 0.1 & 0.4 & -0.1 & 0.1 & 0.2\\ -0.2 & -0.1 & 0.3 & 0.1 & 0\\ -0.1 & 0.1 & 0.1 & 0.4 & 0.1\\ 0.1 & 0.2 & 0 & 0.1 & 0.4\end{bmatrix}$$

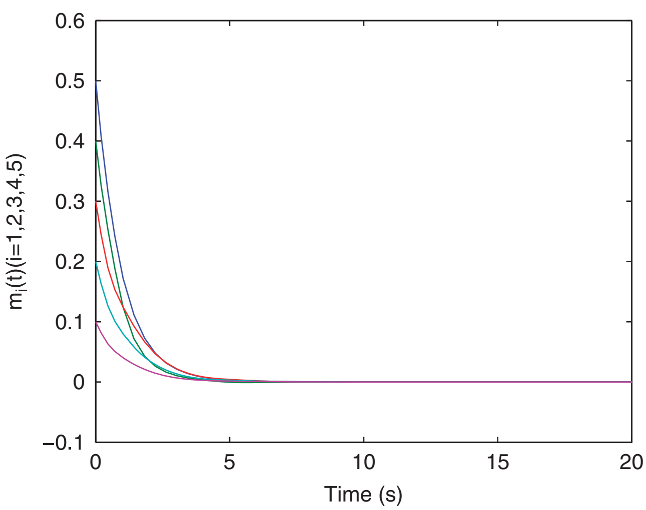

Figure 1. Transient response of $m_i(t)$ $(i=1,2,3,4,5)$.

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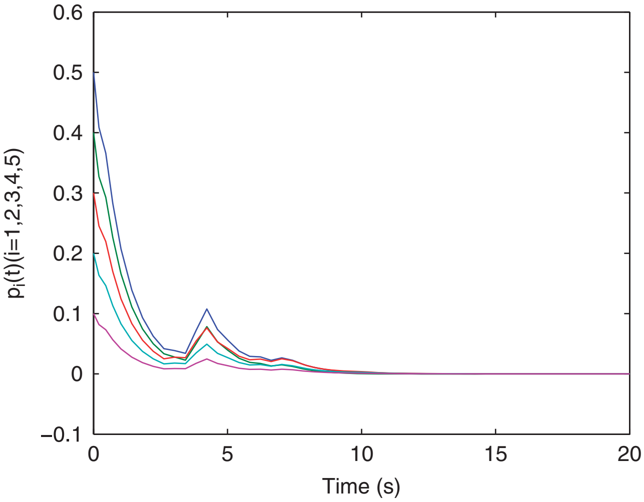

Figure 2. Transient response of $p_i(t)$ $(i=1,2,3,4,5)$.

$$E_{31}=E_{41}=\begin{bmatrix} 0.04 & 0.01 & -0.02 & -0.01 & 0.01 \\ 0.01 & 0.04 & -0.01 & 0.01 & 0.02 \\ -0.02 & -0.01 & 0.03 & 0.01 & 0 \\ -0.1 & 0.1 & 0.1 & 0.4 & 0.1 \\ 0.01 & 0.02 & 0 & 0.01 & 0.04 \end{bmatrix}$$

$$E_{12}=E_{22}=\begin{bmatrix} 0.2 & 0.2 & -0.1 & -0.2 & 0.1 \\ 0.2 & 0.3 & -0.1 & 0.1 & 0.2 \\ -0.1 & -0.1 & 0.5 & 0.1 & 0 \\ -0.2 & 0.1 & 0.1 & 0.4 & 0.2 \\ 0.1 & 0.2 & 0 & 0.2 & 0.4 \end{bmatrix}$$

$$E_{32}=E_{42}=\begin{bmatrix} 0.02 & 0.02 & -0.01 & -0.02 & 0.01 \\ 0.02 & 0.03 & -0.01 & 0.01 & 0.02 \\ -0.01 & -0.01 & 0.05 & 0.01 & 0 \\ -0.02 & 0.01 & 0.01 & 0.04 & 0.02 \\ 0.01 & 0.02 & 0 & 0.02 & 0.04 \end{bmatrix}$$

$$T_{11}=E_{11}, T_{12}=E_{12}, T_{21}=E_{21}, T_{22}=E_{22}, T_{31}=E_{31}, T_{32}=E_{32}, T_{41}=E_{41}, T_{42}=E_{42}$$

$$\begin{aligned} F_{11}(t)&=F_{12}(t)=F_{21}(t)=F_{22}(t)=F_{31}(t)=F_{32}(t)=F_{41}(t)=F_{42}(t) \\ &=\mathrm{diag}\{\sin(t), \cos(2t), \cos(t), -\sin(t), \sin(t)\} \end{aligned}$$

From Theorem 2, by using the Matlab LMI toolbox, we can easily obtain that the uncertain fuzzy genetic regulatory networks (35) is robustly asymptotically stable when $\tau_m=0.1$, $\tau_M=4.19$.

With the given initial condition $m(0)=[1\ 0.8\ 0.6\ 0.4\ 0.2]^{\mathrm{T}}$ and $p(0)=[0.2\ 0.4\ 0.6\ 0.8\ 1]^{\mathrm{T}}$ for fuzzy genetic regulatory networks, the simulation results are presented in Figures 3 and 4.

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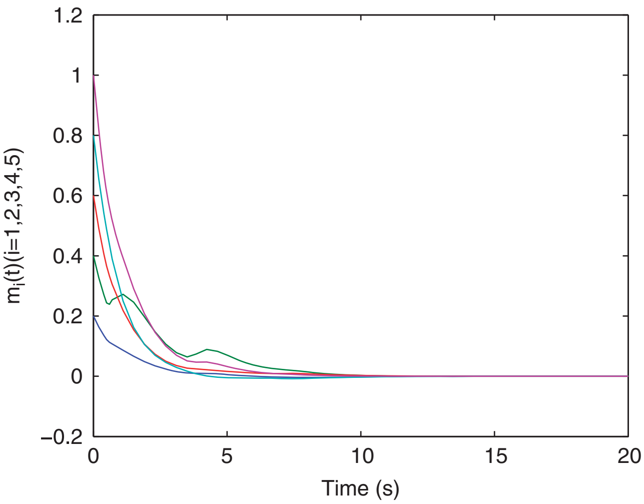

Figure 3. Transient response of $m_i(t)$ $(i=1,2,3,4,5)$.

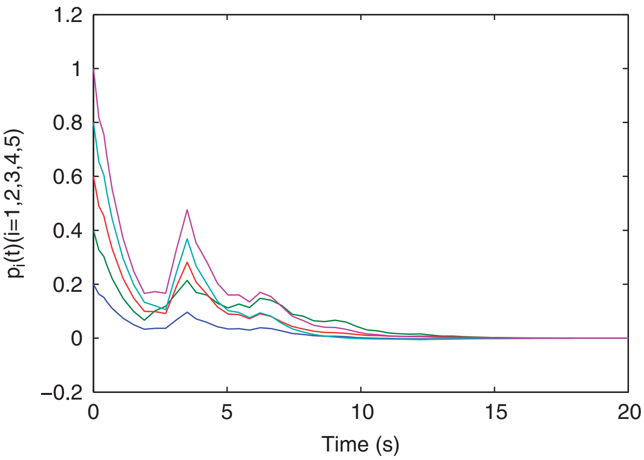

Figure 4. Transient response of $p_i(t)$ $(i=1,2,3,4,5)$.

6. CONCLUSION

In this paper, we have proposed a model of T-S fuzzy genetic networks with time-varying delays and investigated the robustly asymptotic stability of the proposed fuzzy genetic networks. To analyze the asymptotic stability of the fuzzy genetic network systems, the convexity of the matrix function technique has been used. Based on the free-weighting matrix method and the LMI techniques, stability conditions have been developed in terms of LMIs. Two examples with simulation results have been carried out to demonstrate the effectiveness of the proposed method.

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