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Event-triggering in networked systems with probabilistic sensor and actuator faults

Jinliang Liu [a], Dong Yue [b,*]

[a] Department of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, PR China
[b] Institute of Automation, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu 210023, PR China

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ABSTRACT

This paper investigates the reliable control design for networked control system under event-triggered scheme. The key idea is that only the newly sampled sensor measurements that violate specified triggering condition will be transmitted to the controller, and the main attribute of our approach is that the proposed event-triggered scheme only needs a supervision of the system state in discrete instants and there is no need to retrofit the existing system. Considering the effect of the network transmission delay, event-triggered scheme and probabilistic sensor or actuator fault with different failure rates, a new fault model is proposed. Based on the newly built model, criteria for the exponential stability and criteria for co-designing both the feedback and the trigger parameters are derived by using Lyapunov functional. These criteria are obtained in the form of linear matrix inequalities. A simulation example is employed to show the effectiveness of the proposed method.

1. Introduction

Nowadays, networked control systems (NCSs) have recently received considerable research attention because of the rapid development of network technologies and their successful industrial applications [29,30,36,11,1]. The insertion of the network offers advantages including simplicity, scalability and cost-effectiveness. However, implementing a control network over a communication network induces packet dropout, multiple channel transmission and so on. The problem of stability analysis and control design for NCSs have been investigated and lots of outstanding results have been obtained [32,3,25,17]. However, only a few of them consider sensor failures, actuator failures and data distortion, which may inevitably degrade the performance and could be a source of instability [20,6,8,2,21]. Therefore, it is necessary and important to design a reliable controller which can tolerate actuator or sensor failures, data distortion and network-induced delay.

On the other hand, much attention has been paid to the issue that how to use the limited network bandwidth available for transmitting state information more effectively. Researchers have proposed different methods to deal with the problem. For example, in [5], a periodic triggered method for system modeling and analysis is used due to the easy implementation and analysis. In this triggered method, in order to guarantee a desired performance, a fixed sampling interval should be selected under worst conditions such as external disturbances and time delay. However, the worst cases are seldom occur in practical systems, which will result in transmitting many unnecessary sampling signals and cause high utilization of the communication bandwidth. Under a event triggered scheme which needed continuous supervision of the system state, the authors in [19] derived the methods for design and implementation of controllers. But if we implement such a kind of event triggered

* Corresponding author. Tel.: +86 25 85481180; fax:+86 25 85481197.
E-mail address: medongy@vip.163.com (D. Yue).

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method, the existing system should be retrofitted. In [23], a self-triggered method was proposed, which can save energy for the sensor and be less complexity in implementation compared with the event-triggered scheme in [19]. Because the release period is often smaller than the event-triggered scheme in [19], more constraints are needed for design or implementation of controllers under self-triggered scheme. In [9], the authors firstly proposed a event-triggered method which only needs supervision of the system state in discrete instants, and in [10], the authors proposed a novel event-triggering scheme to discussed the problem of event-based H_∞ filtering for networked systems with communication delay. However, they did not consider the case of sensor failures.

Considerable efforts have been invested toward the problem of stability analysis and control design for time delay systems and NCSs, see [21,9,19,13,12,4,16]. In [21], the reliable control design is considered for networked control systems (NCSs) against probabilistic actuator fault with different failure rates, measurements distortion,random network-induced delay and packet dropout, but they did not consider the insertion of the event-trigger scheme which used to reduce the communication load. In [9], the authors firstly proposed a event-triggered method, but they did not take sensor fault into consideration. Motivated by the literatures above, we focus on the reliable control for event-triggered networked control systems with probabilistic sensor and actuator faults in this paper. To reduce the computation load or to reduce the exchange of information between the control agents (sensors, controller, actuator), we firstly proposed a event-triggering sampling mechanism, which takes the probabilistic sensor and actuator faults in consideration. Under the proposed scheme, the sensor measurement is transmitted only when the expectation of a certain function of the current sensor value and the expectation of the previously transmitted one exceeds a threshold value. Unlike the cases in [19,9,10], considering the sensor faults, the key attribute of our approach is that the proposed event-triggered scheme only needs a supervision of the system state in discrete instants and there is no need to retrofit the existing system. The implementation of our event-triggering sampling scheme only monitors the system state in discrete instants.

This paper is organized as follows. In Section 2, under the proposed event-triggered scheme, a new fault model is proposed, which includes both sensor and actuator faults, network-induced delay, data distortion and so on. Based on the model, new criteria for the exponentially mean square stability and criteria for co-designing both the feedback and the trigger parameters are obtained in the form of linear matrix inequality in Section 3. In Section 4, a simulation example is given to illustrate the effectiveness of the proposed design procedures. The paper is concluded in Section 5.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n \times m$ real matrices; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$ (respectively, $X \geqslant 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively,positive semi-definite). When x is a stochastic variable, $\mathbb{E}\{x\}$ stands for the expectation of x. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System description

In this paper, we consider the following system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ denote the state vector and control vector, respectively; A, and B are parameter matrices with appropriate dimensions.

Throughout this paper, we assume the system (1) is controlled though an unreliable network which has probabilistic sensor and actuator faults.

As is well known, periodic sampling mechanism has been widely used in practical systems, however, it may often lead to transmitting many unnecessary signals through the network, which in turn will increase the load of network transmission and wastes the network bandwidth. Therefore, for the control of networked control systems shown in Fig. 1, in order to save network resources such as network bandwidth, it is significant to introduce an event triggered mechanism which decides whether the newly sampled state should be send out to the controller. As is shown in Fig. 1, an event generator is constructed between the sensor and the controller which decides when to transmit the state to the controller via a network medium by a specified trigger condition, the state are sampled regularly by the sampler of the smart sensor with period h and feeds into the Event Generator, which will be given in sequel. The following function of network architecture in Fig. 1 is expected:

1. The state are sampled at time kh by sampler with a given period h. The next state is at time $(k+1)h$.
2. As shown in Fig. 1, the event generator is constructed between the sensor and the controller which uses the sampled state to determine whether the newly sampled state will be sent out to the controller. Considering the unreliable network which has probabilistic sensor faults, we adopt the following judgement algorithm:

$$[\mathbb{E}\{\Xi x((k+j)h)\} - \mathbb{E}\{\Xi x(kh)\}]^T \Omega [\mathbb{E}\{\Xi x((k+j)h)\} - \mathbb{E}\{\Xi x(kh)\}] \leqslant \rho [\mathbb{E}\{\Xi x((k+j)h)\}]^T \Omega \mathbb{E}[\{\Xi x((k+j)h)\}] \tag{2}$$

where Ω is a symmetric positive definite matrix, $j = 1, 2, \ldots, \rho \in [0, 1)$ and $\Xi = diag\{\Xi_1, \Xi_2, \ldots, \Xi_n\}$ with $\Xi_i (i = 1, 2, \ldots, n)$ being n unrelated random variables taking values on the interval $[0, \theta]$, $\theta \geqslant 1$.

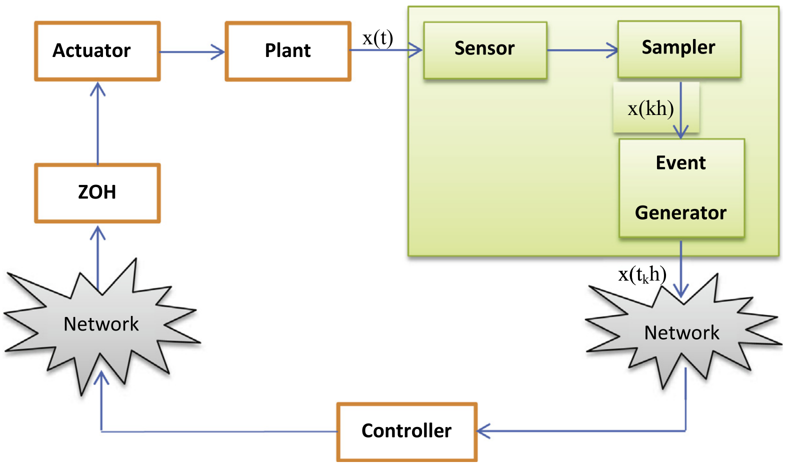

Fig. 1. The structure of an event-triggered networked control system.

3. Under the event triggered scheme (2), the release times are assumed to be $t_0h, t_1h, t_2h, \ldots$, where t_0 is the initial time. $s_ih = t_{i+1}h - t_ih$ denotes the release period of event generator in (2). Considering the effect of the transmission delay on the network system, we suppose the time-varying delay in the network communication is τ_k and $\tau_k \in [0, \bar{\tau})$, where $\bar{\tau}$ is a positive real number. Therefore, the states $x(t_0h), x(t_1h), x(t_2h), \ldots$ will arrive at the controller side at the instants $t_0h + \tau_0, t_1h + \tau_1, t_2h + \tau_2, \ldots$, respectively.

Remark 1. Notice that the sensor measurement is transmitted only when the expectation of a certain function of the current sensor value and the expectation of the previously transmitted one exceeds the threshold in (2), that is, only some of the sampled sensor measurements that violate (2) can be sent out to the controller.

Remark 2. It is worth noting that the set of the release instants, i.e. $\{t_0, t_1, t_2, \ldots\}$ is a subset of $\{0, 1, 2, \ldots\}$ in event-triggering (2). The amount of $\{t_0, t_1, t_2, \ldots\}$ depends on not only the value of ρ, but also the variation of the state. When $\rho = 0$, $\{t_0, t_1, t_2, \ldots\} = \{0, 1, 2, \ldots\}$, the event-triggered scheme reduces to periodic time-triggered scheme.

Remark 3. The similar event-triggering scheme was firstly proposed in [34,35], however, [34,35] only considered the controller design problem and did not discuss the case when the sensor and actuator have faults. we update the event-triggering scheme and investigate the above problem in this paper.

If we consider the case of sensor and actuator both in good condition and the effect of the transmission delay, under the event generator with (2), the controller of the system model (1) can be described as

$$u(t) = Kx(t_kh), t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) \tag{3}$$

Considering the unreliable channel from controller to actuator (3) can be rewritten as

$$u_1(t) = \Xi_1 Kx(t_kh), t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) \tag{4}$$

where $\Xi_1 = diag\{\Xi_{11}, \Xi_{12}, \ldots, \Xi_{1m}\}$ with $\Xi_{1i}(i = 1, 2, \ldots, m)$ being m unrelated random variables taking values on the interval $[0, \theta_1]$, where $\theta_1 \geqslant 1$. The mathematical expectation and variance of $\Xi_{1i}(i = 1, 2, \ldots, m)$ are β_i and δ_i^2, respectively.

Based on (4), if we consider the unreliable channel from sensor to controller, (4) can be described as

$$u_2(t) = \Xi_1 K \Xi_2 x(t_kh), t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1}) \tag{5}$$

where $\Xi_2 = diag\{\Xi_{21}, \Xi_{22}, \ldots, \Xi_{2n}\}$, $\Xi_{2i}(i = 1, 2, \ldots, n)$ are n unrelated random variables taking values on the interval $[0, \theta_2]$, where $\theta_2 \geqslant 1$. Also, $\Xi_{2i}(i = 1, 2, \ldots, n)$ are unrelated with $\Xi_{1i}(i = 1, 2, \ldots, m)$. The mathematical expectation and variance of $\Xi_{2i}(i = 1, 2, \ldots, n)$ are α_i and γ_i^2, respectively.

Remark 4. It should be pointed out that $\beta_i = 1$ does not mean the *ith* sensor is always in good condition, it represent the expectation of Ξ_{2j} is 1. when $\Xi_1 = I$ and $\Xi_2 = I$, (5) degenerates into the general controller [18,26–28].

Remark 5. When sensors or actuators have faults, the output signal may be larger or smaller than what it should be. Considering this case, we assume the variables Ξ_{1i} and Ξ_{2i} take values in the interval $[0, \theta_1]$ and $[0, \theta_2]$, where $\theta_i \geqslant 1(i = 1, 2)$. If Ξ_{1i} and Ξ_{2i} taking values in $\{0, 1\}$, it means the sensor and actuator have completely failure or not. If $0 < \Xi_{1i} < 1$, $0 < \Xi_{2j} < 1$ and $\Xi_{1i} > 1$, $\Xi_{2j} > 1$, it means the case of data distortion happen.

Under the control (5), for $t \in [t_kh + \tau_k, t_{k+1}h + \tau_{k+1})$, (1) can be rewritten as:

$$\dot{x}(t) = Ax(t) + B\Xi_1 K \Xi_2 x(t_kh) = Ax(t) + B\overline{\Xi}_1 K \overline{\Xi}_2 x(t_kh) + \mathcal{V}x(t_kh) \tag{6}$$

where

$$\mathcal{V} = B\overline{\Xi}_1 K(\Xi_2 - \overline{\Xi}_2) + B(\Xi_1 - \overline{\Xi}_1)K\overline{\Xi}_2 + B(\Xi_1 - \overline{\Xi}_1)K(\Xi_2 - \overline{\Xi}_2) \tag{7}$$

$$\overline{\Xi}_1 = diag\{\beta_1, \beta_2, \ldots, \beta_m\} = \sum_{i=1}^{m} \beta_i L_1^i, \quad \overline{\Xi}_2 = diag\{\alpha_1, \alpha_2, \ldots, \alpha_n\} = \sum_{i=1}^{n} \alpha_i L_2^j$$

$$\mathbb{E}\{(\Xi_1 - \overline{\Xi}_1)^2\} = diag\{\delta_1^2, \ldots, \delta_m^2\}, \quad \mathbb{E}\{(\Xi_2 - \overline{\Xi}_2)^2\} = diag\{\gamma_1^2, \ldots, \gamma_m^2\}$$

$$L_1^i = diag\{\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{m-i}\}, \quad L_2^j = diag\{\underbrace{0, \ldots, 0}_{j-1}, 1, \underbrace{0, \ldots, 0}_{n-j}\}$$

For technical convenience, consider the following two cases:

Case 1: If $t_k h + h + \bar{\tau} \geqslant t_{k+1}h + \tau_{k+1}$, where $\bar{\tau} = \max \tau_k$, define a function $\tau(t)$ as

$$\tau(t) = t - t_k h, t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}) \tag{8}$$

Clearly

$$t_k \leqslant \tau(t) \leqslant (t_{k+1} - t_k)h + \tau_{k+1} \leqslant h + \bar{\tau} \tag{9}$$

Case 2: If $t_k h + h + \bar{\tau} < t_{k+1}h + \tau_{k+1}$, consider the following two intervals:

$$[t_k h + \tau_k, t_k h + h + \bar{\tau}), [t_k h + ih + \bar{\tau}, t_k h + ih + h + \bar{\tau})$$

Since $\tau_k \leqslant \bar{\tau}$, it can be easily shown that there exists d_M such that

$$t_k h + d_M h + \bar{\tau} < t_{k+1}h + \tau_{k+1} \leqslant t_k h + d_M h + h + \bar{\tau}$$

Moreover, $x(t_k h)$ and $t_k h + ih$ with $i = 1, 2, \ldots, d_M$ satisfy (2). Let

$$\begin{cases} I_0 = [t_k h + \tau_k, t_k h + h + \bar{\tau}) \\ I_i = [t_k h + ih + \bar{\tau}, t_k h + ih + h + \bar{\tau}) \\ I_{d_M} = [t_k h + d_M h + \bar{\tau}, t_{k+1}h + \tau_{k+1}) \end{cases} \tag{10}$$

where $i = 1, 2, \ldots, d_M - 1$. One can see that

$$[t_k h + \tau_k, t_{k+1}h + \tau_{k+1}) = \bigcup_{i=0}^{i=d_M} I_i \tag{11}$$

Define

$$\tau(t) = \begin{cases} t - t_k h, & t \in I_0 \\ t - t_k h - ih, & t \in I_i, i = 1, 2, \ldots, d_M - 1 \\ t - t_k h - d_M h, & t \in I_{d_M} \end{cases} \tag{12}$$

Then, we have

$$\begin{cases} t_k \leqslant \tau(t) < h + \bar{\tau}, & t \in I_0 \\ t_k \leqslant \bar{\tau} \leqslant \tau(t) < h + \bar{\tau}, & t \in I_i, i = 1, 2, \ldots, d_M - 1 \\ t_k \leqslant \bar{\tau} \leqslant \tau(t) < h + \bar{\tau}, & t \in I_{d_M} \end{cases} \tag{13}$$

where the third row in (10) holds because $t_{k+1}h + \tau_{k+1} \leqslant t_k h + (d_M + 1)h + \bar{\tau}$. Obviously,

$$0 \leqslant \tau_k \leqslant \tau(t) \leqslant h + \bar{\tau} \triangleq \tau_M, t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1}) \tag{14}$$

In Case 1, for $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$, define $e_k(t) = 0$. In Case 2, define

$$\overline{\Xi}_2 e_k(t) = \begin{cases} 0, & t \in I_0 \\ \overline{\Xi}_2 x(t_k h) - \overline{\Xi}_2 x(t_k h + ih), & t \in I_i,\ i = 1, 2, \ldots, d_M - 1 \\ \overline{\Xi}_2 x(t_k h) - \overline{\Xi}_2 x(t_k h + d_M h), & t \in I_{d_M} \end{cases} \tag{15}$$

From the definition of $\overline{\Xi}_2 e_k(t)$ and the triggering algorithm (2), it can be easily seen that for $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$

$$e_k^T(t)\overline{\Xi}_2^T \Omega \overline{\Xi}_2 e_k(t) \leqslant \rho x^T(t - \tau(t))\overline{\Xi}_2^T \Omega \overline{\Xi}_2 x(t - \tau(t)), \tag{16}$$

Remark 6. From (15), we can deduce that

$$e_k(t) = \begin{cases} 0, & t \in I_0 \\ x(t_k h) - x(t_k h + ih), & t \in I_i,\ i = 1, 2, \ldots, d_M - 1 \\ x(t_k h) - x(t_k h + d_M h), & t \in I_{d_M} \end{cases} \tag{17}$$

Utilizing $\tau(t)$ and $e_k(t)$, (6) can be rewritten as

$$\dot{x}(t) = Ax(t) + B\overline{\Xi}_1 K\overline{\Xi}_2[x(t-\tau(t)) + e_k(t)] + \mathcal{V}[x(t-\tau(t)) + e_k(t)] \tag{18}$$

where $t \in [t_k h + \tau_k, t_{k+1}h + \tau_{k+1})$. For the system (14), we supplement the initial condition of the state $x(t)$ on $[-\tau_M, 0]$ as

$$x(t) = \phi(t), t \in [-\tau_M, 0] \tag{19}$$

where $\phi(t)$ is a continuous function on $[-\tau_M, 0]$.

Remark 7. The model (18) can describe a large number of well-known dynamical systems with time-delays, such as unmanned aerial vehicles, vehicular control networks, power systems and teleoperation systems. The proposed event-triggered scheme (2), which can reduce the communication load, can deal with the case when the sensor and actuator have probability failures.

Remark 8. If $\tau_k = 0$, it means no transmission delay exists or transmission delay can be ignored, the maximum sampling period is τ_M. Note that $\tau_M = h + \bar{\tau}$, If $\tau_k > 0$, the selecting sampling period $h < \tau_M$, $\bar{\tau} = \tau_M - h$ is the allowable maximum transmission delay.

Remark 9. When formulating the system (18) and (19), the signal transmission delay and event triggering condition (2) are taken into consideration. If the event-triggering scheme is not considered, that is $\overline{\Xi}_2 e_k(t) = 0$, then the system (18) and (19) reduces to the case in [21], where the reliable controller design for networked control system against both probabilistic sensor and actuator faults are studied.

In the following, we need to introduce the notion of the infinitesimal operator $\mathcal{L}(\cdot)$, stochastic stability in the mean-square sense and two lemmas, which will help us in deriving the main results.

Definition 1 [15]. For a given function $V : C^b_{F_0}([-\tau_M, 0], \mathbb{R}^n) \times S$, its infinitesimal operator $\mathcal{L}$ is defined as

$$\mathcal{L}(V\eta(t)) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}[\mathbb{E}(V(\eta_t + \Delta)|\eta_t) - V(\eta_t)] \tag{20}$$

Definition 2 [14]. System (18) and (19) is said to be exponentially mean square stability (EMSS) if there exist constants $\alpha > 0$ and $\beta > 0$ such that for $t \geqslant 0$

$$\mathbb{E}(\|x(t)\|^2) \leqslant \alpha e^{-\beta t}\mathbb{E}\{\sup_{-\tau_M \leqslant s \leqslant 0} \|\phi(s)\|^2\} \tag{21}$$

Lemma 1 [24]. *For any vectors x, $y \in R^n$, and positive definite matrix $Q \in R^{n \times n}$, the following inequality holds:*

$$2x^T y \leqslant x^T Q x + y^T Q^{-1} y \tag{22}$$

Lemma 2 [22]. *Ξ_1, Ξ_2 and Ω are matrices with appropriate dimensions, $\tau(t)$ is a function of t and $0 \leqslant \tau(t) \leqslant \tau_M$, then*

$$\tau(t)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \tag{23}$$

if and only if

$$\tau_M \Xi_1 + \Omega < 0 \tag{24}$$

$$\tau_M \Xi_2 + \Omega < 0 \tag{25}$$

3. Main results

In this section, we will give the EMSS criteria for system (18), (19) with the reliable controller (5) under the event trigger (2).

Theorem 1. *For given parameters τ_M, $\alpha_i (i = 1, \ldots, n)$, $\beta_j (j = 1, \ldots, m)$, $\gamma_i (i = 1, \ldots, n)$, $\delta_j (j = 1, \ldots, m)$, $\rho \in [0, 1)$ and feedback gain K, the system described by (18) is EMSS, if there exists matrices $P > 0$, $Q > 0$, $R > 0$, $\Omega > 0$, N and M with appropriate dimensions such that for s = 1,2*

$$\Sigma(s) \triangleq \begin{bmatrix} \Sigma_{11}+\Gamma+\Gamma^T & \Sigma_{12}^s & \sqrt{\tau_M}\mathcal{A}^T & \Sigma_{14} & \Sigma_{15} \\ * & -R & 0 & 0 & 0 \\ * & * & -R^{-1} & 0 & 0 \\ * & * & * & \Sigma_{44} & 0 \\ * & * & * & 0 & \Sigma_{55} \end{bmatrix} < 0, \quad s=1,2. \tag{26}$$

where

$$\Sigma_{11} = \begin{bmatrix} PA+A^TP+Q & * & * & * \\ \overline{\Xi}_2K^T\overline{\Xi}_1B^TP & \sum_{i=1}^{n}\rho\alpha_i^2(L_2^i)^T\Omega L_2^i & * & * \\ 0 & 0 & -Q & * \\ \overline{\Xi}_2K^T\overline{\Xi}_1B^TP & 0 & 0 & -\sum_{i=1}^{n}\alpha_i^2(L_2^i)^T\Omega L_2^i \end{bmatrix}$$

$$\Sigma_{12}^1 = \sqrt{\tau_M}N,\ \Sigma_{12}^2 = \sqrt{\tau_M}M$$
$$\Gamma = [N \quad -N+M \quad -M \quad 0]$$
$$\mathcal{A} = \begin{bmatrix} A & B\overline{\Xi}_1K\overline{\Xi}_2 & 0 & B\overline{\Xi}_1K\overline{\Xi}_2 \end{bmatrix}$$
$$w_{ij} = 2\tau_M\left(\beta_i^2\gamma_j^2+\delta_i^2\alpha_j^2+\delta_i^2\gamma_j^2\right)$$
$$\Sigma_{14} = [\Lambda_1 \quad \Lambda_2 \quad \cdots \quad \Lambda_n]$$
$$\Lambda_i = \begin{bmatrix} \Lambda_{i1}^T & \Lambda_{i2}^T & \cdots & \Lambda_{in}^T \end{bmatrix}, \quad i=1,\cdots,m$$
$$\Lambda_{ij} = \sqrt{w_{ij}}\begin{bmatrix} 0 & BL_1^iKL_2^j & 0 & 0 \end{bmatrix}$$
$$\Sigma_{15} = [\Upsilon_1 \quad \Upsilon_2 \quad \cdots \quad \Upsilon_n]$$
$$\Upsilon_i = \begin{bmatrix} \Upsilon_{i1}^T & \Upsilon_{i2}^T & \cdots & \Upsilon_{in}^T \end{bmatrix}, \quad i=1,\cdots,m$$
$$\Upsilon_{ij} = \sqrt{w_{ij}}\begin{bmatrix} 0 & 0 & 0 & BL_1^iKL_2^j \end{bmatrix}$$
$$\Sigma_{44} = diag\{-R^{-1},-R^{-1},\cdots,-R^{-1}\}$$
$$\Sigma_{55} = diag\{-R^{-1},-R^{-1},\cdots,-R^{-1}\}$$

Proof. Choose the following Lyapunov functional candidate as

$$V(x_t) = x^T(t)Px(t) + \int_{t-\tau_M}^{t} x^T(s)Qx(s)ds + \int_{t-\tau_M}^{t}\int_{s}^{t} \dot{x}^T(v)R\dot{x}(v)dvds \tag{27}$$

in which P, Q and R are symmetric positive definite matrices.

Using the infinitesimal operator (20) for $V(x_t)$ and taking expectation on it, we obtain

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(x_t)\} &= 2x^T(t)P[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))+B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)] + x^T(t)Qx(t) - x^T(t-\tau_M)Qx(t-\tau_M) \\ &\quad + \mathbb{E}\{\tau_M\dot{x}^T(t)R\dot{x}(t)\} - \int_{t-\tau_M}^{t}\dot{x}^T(s)R\dot{x}(s)ds + \Gamma_1+\Gamma_2\end{aligned} \tag{28}$$

where Γ_1 and Γ_2 are introduced by employing free weight matrix method [31,7]

$$\Gamma_1 = 2\zeta^T(t)N[x(t)-x(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{x}(s)ds] = 0 \tag{29}$$

$$\Gamma_2 = 2\eta^T(t)M[x(t-\tau(t))-x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot{x}(s)ds] = 0 \tag{30}$$

where N and M are matrices with appropriate dimensions, and

$$\zeta^T(t) = \begin{bmatrix} x^T(t) & x^T(t-\tau(t)) & x^T(t-\tau_M) & e_k^T(t) \end{bmatrix}$$

By Lemma 1, we have

$$-2\zeta^T(t)N\int_{t-\tau(t)}^{t}\dot{x}(s)ds \leqslant \tau(t)\zeta^T(t)NR^{-1}N^T\zeta(t) + \int_{t-\tau(t)}^{t}\dot{x}^T(s)R\dot{x}(s)ds \tag{31}$$

$$-2\zeta^T(t)M\int_{t-\tau_M}^{t-\tau(t)}\dot{x}(s)ds \leqslant (\tau_M-\tau(t))\zeta^T(t)MR^{-1}M^T\zeta(t) + \int_{t-\tau_M}^{t-\tau(t)}\dot{x}^T(s)R\dot{x}(s)ds \tag{32}$$

Notice that

$$\begin{aligned}\mathbb{E}[\tau_M\dot{x}^T(t)R\dot{x}(t)] = \mathbb{E}\{\tau_M[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2(x(t-\tau(t))+e_k(t))+\mathcal{V}(x(t-\tau(t))+e_k(t))]^TR[Ax(t)\\ +B\overline{\Xi}_1K\overline{\Xi}_2(x(t-\tau(t))+e_k(t))+\mathcal{V}(x(t-\tau(t))+e_k(t))]\} = \tau_M[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))\\ +B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)]^TR[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))+B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)]+\mathbb{E}\{\tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}x(t-\tau(t))\}\\ +\mathbb{E}\{2\tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}e_k(t)\}+\mathbb{E}\{\tau_Me_k^T(t)\mathcal{V}^TR\mathcal{V}e_k(t)\}\end{aligned} \tag{33}$$

in which there is a term of $2\tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}e_k(t)$, by using Lemma 2, we have

$$2\tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}e_k(t) \leqslant \tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}x(t-\tau(t)) + \tau_Me_k^T(t)\mathcal{V}^TR\mathcal{V}e_k(t) \tag{34}$$

Then, we can obtain that

$$\begin{aligned}\mathbb{E}[\tau_M\dot{x}^T(t)R\dot{x}(t)] \leqslant \tau_M[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))+B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)]^TR[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))+B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)]\\ +\mathbb{E}\{2\tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}x(t-\tau(t))\}+\mathbb{E}\{2\tau_Me_k^T(t)\mathcal{V}^TR\mathcal{V}e_k(t)\}\end{aligned} \tag{35}$$

Recalling (7), we obtain

$$\begin{aligned}\mathbb{E}\{2\tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}x(t-\tau(t))\} =& 2\tau_M\mathbb{E}\{x^T(t-\tau(t))[B\overline{\Xi}_1K(\Xi_2-\overline{\Xi}_2)+B(\Xi_1-\overline{\Xi}_1)K\overline{\Xi}_2\\ &+B(\Xi_1-\overline{\Xi}_1)K(\Xi_2-\overline{\Xi}_2)]^TR[B\overline{\Xi}_1K(\Xi_2-\overline{\Xi}_2)+B(\Xi_1-\overline{\Xi}_1)K\overline{\Xi}_2\\ &+B(\Xi_1-\overline{\Xi}_1)K(\Xi_2-\overline{\Xi}_2)]x(t-\tau(t))\}\\ =& 2\tau_Mx^T(t-\tau(t))[(B\overline{\Xi}_1K(\Xi_2-\overline{\Xi}_2))^TR(B\overline{\Xi}_1K(\Xi_2-\overline{\Xi}_2))\\ &+(B(\Xi_1-\overline{\Xi}_1)K\overline{\Xi}_2)^TR(B(\Xi_1-\overline{\Xi}_1)K\overline{\Xi}_2)\\ &+(B(\Xi_1-\overline{\Xi}_1)K(\Xi_2-\overline{\Xi}_2))^TR(B(\Xi_1-\overline{\Xi}_1)K(\Xi_2-\overline{\Xi}_2))]x(t-\tau(t))\end{aligned} \tag{36}$$

Notice that

$$\mathbb{E}\{2\tau_M(B\overline{\Xi}_1K(\Xi_2-\overline{\Xi}_2))^TR(B\overline{\Xi}_1K(\Xi_2-\overline{\Xi}_2))\} = \sum_{i=1}^{m}\sum_{j=1}^{n}2\tau_M\beta_i^2\gamma_j^2(BL_1^iKL_2^j)^TR\left(BL_1^iKL_2^j\right) \tag{37}$$

$$\mathbb{E}\{2\tau_M(B(\Xi_1-\overline{\Xi}_1)K\overline{\Xi}_2)^TR(B(\Xi_1-\overline{\Xi}_1)K\overline{\Xi}_2)\} = \sum_{i=1}^{m}\sum_{j=1}^{n}2\tau_M\delta_i^2\alpha_j^2\left(BL_1^iKL_2^j\right)^TR\left(BL_1^iKL_2^j\right) \tag{38}$$

$$\mathbb{E}\{2\tau_M(B(\Xi_1-\overline{\Xi}_1)K(\Xi_2-\overline{\Xi}_2))^TR(B(\Xi_1-\overline{\Xi}_1)K(\Xi_2-\overline{\Xi}_2))\} = \sum_{i=1}^{m}\sum_{j=1}^{n}2\tau_M\delta_i^2\gamma_j^2\left(BL_1^iKL_2^j\right)^TR\left(BL_1^iKL_2^j\right) \tag{39}$$

Combining (36)–(39), we have

$$\mathbb{E}\{2\tau_Mx^T(t-\tau(t))\mathcal{V}^TR\mathcal{V}x(t-\tau(t))\} = \sum_{i=1}^{m}\sum_{j=1}^{n}w_{ij}x^T(t-\tau(t))\left(BL_1^iKL_2^j\right)^TR\left(BL_1^iKL_2^j\right)x(t-\tau(t)) \tag{40}$$

where $w_{ij} = 2\tau_M\left(\beta_i^2\gamma_j^2+\delta_i^2\alpha_j^2+\delta_i^2\gamma_j^2\right)$.

Using the same method as (40), we have

$$\mathbb{E}\{2\tau_Me_k^T(t)\mathcal{V}^TR\mathcal{V}e_k(t)\} = \sum_{i=1}^{m}\sum_{j=1}^{n}w_{ij}e_k^T(t)\left(BL_1^iKL_2^j\right)^TR\left(BL_1^iKL_2^j\right)e_k(t) \tag{41}$$

Substituting (29)–(33) and (40) and (41) into (28) and combining (16), we can obtain that

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(x_t)\} \leqslant& 2x^T(t)P[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))+B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)]+x^T(t)Qx(t)-x^T(t-\tau_M)Qx(t-\tau_M)+\tau_M[Ax(t)\\ &+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))+B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)]^TR[Ax(t)+B\overline{\Xi}_1K\overline{\Xi}_2x(t-\tau(t))+B\overline{\Xi}_1K\overline{\Xi}_2e_k(t)]+\sum_{i=1}^{m}\sum_{j=1}^{n}w_{ij}x^T(t\\ &-\tau(t))\left(BL_1^iKL_2^j\right)^TR\left(BL_1^iKL_2^j\right)x(t-\tau(t))+\sum_{i=1}^{m}\sum_{j=1}^{n}w_{ij}e_k^T(t)\left(BL_1^iKL_2^j\right)^TR\left(BL_1^iKL_2^j\right)e_k(t)+2\zeta^T(t)N[x(t)\\ &-x(t-\tau(t))]+2\zeta^T(t)M[x(t-\tau(t))-x(t-\tau_M)]+\tau(t)\zeta^T(t)NR^{-1}N^T\zeta(t)+(\tau_M\\ &-\tau(t))\zeta^T(t)MR^{-1}M^T\zeta(t)+\rho\sum_{i=1}^{n}\alpha_i^2x^T(t-\tau(t))\left(L_2^i\right)^T\Omega L_2^ix(t-\tau(t))-\sum_{i=1}^{n}\alpha_i^2e_k^T(t)(L_2^i)^T\Omega L_2^ie_k(t)\\ =& \zeta^T(t)(\Pi+\tau(t)NR^{-1}N^T+(\tau_M-\tau(t))MR^{-1}M^T)\zeta(t)\end{aligned} \tag{42}$$

where

$$\Pi = \Upsilon_{11} + \Gamma + \Gamma^T + \tau_M \mathcal{A}^T R \mathcal{A}$$

with

$$\Upsilon_{11} = \begin{bmatrix} PA + A^T P + Q & * & * & * \\ \overline{\Xi}_2 K^T \overline{\Xi}_1 B^T P & \Theta + \sum_{i=1}^{n} \rho \alpha_i^2 \left(L_2^i\right)^T \Omega L_2^i & * & * \\ 0 & 0 & -Q & * \\ \overline{\Xi}_2 K^T \overline{\Xi}_1 B^T P & 0 & 0 & \Theta - \sum_{i=1}^{n} \alpha_i^2 (L_2^i)^T \Omega L_2^i \end{bmatrix}$$

$$\Theta = \sum_{i=1}^{m} \sum_{j=1}^{n} w_{ij} \left(B L_1^i K L_2^j\right)^T R \left(B L_1^i K L_2^j\right), \Gamma = [N \quad -N + M \quad -M \quad 0]$$

Recalling (26) and using Lemma 2, we can conclude from (26) that there exists a constant λ such that

$$\mathbb{E}\{\mathcal{L}V(x_t)\} \leqslant -\lambda \mathbb{E}\{\|x(t)\|^2\} \tag{43}$$

where $\lambda = min\{\lambda_{min}\Sigma(i)\}(i = 1, 2)$. Define a new function as

$$W(x_t) = e^{\epsilon t} V(x_t) \tag{44}$$

Its infinitesimal operator $\mathcal{L}$ is given by

$$\mathcal{L}W(x_t) = \epsilon e^{\epsilon t} V(x_t) + e^{\epsilon t} \mathcal{L}V(x_t) \tag{45}$$

From (45), we can obtain that

$$\mathbb{E}W(x_t) - \mathbb{E}W(x_0) = \int_0^t \epsilon e^{\epsilon s} \mathbb{E}\{V(x_s)\} ds + \int_0^t e^{\epsilon s} \mathbb{E}\{\mathcal{L}V(x_s)\} ds \tag{46}$$

Then using the similar method of [33], we can observe that there exists a positive number α such that for $t \geqslant 0$

$$\mathbb{E}\{V(x_t)\} \leqslant \alpha \sup_{-\tau_M \leqslant s \leqslant 0} e^{-\epsilon s} \mathbb{E}\{\|\psi(s)\|^2\} \tag{47}$$

Since $V(x_t) \geqslant \lambda_{min}(P) x^T(t) x(t)$, it can be shown from (47) that for $t \geqslant 0$

$$\mathbb{E}\{x^T(t) x(t)\} \leqslant \bar{\alpha} e^{-\epsilon t} \sup_{-\tau_M \leqslant s \leqslant 0} \mathbb{E}\{\|\psi(s)\|^2\} \tag{48}$$

where $\bar{\alpha} = \frac{\alpha}{\lambda_{min}(P)}$. Recalling Definition 2, the proof can be completed. □

Based on Theorem 1, we are in a position to design the reliable controller (5) under the event trigger (2). Select a constant α to minimize $\sum_{i=1}^{n} (\alpha_i - \alpha)$, the reliable controller (5) can be designed.

Theorem 2. *For given parameters τ_M, $\alpha_i (i = 1, \ldots, n)$, β_j $(j = 1, \ldots, m)$, γ_i $(i = 1, \ldots, n)$, δ_j $(j = 1, \ldots, m)$, $\varepsilon > 0$ and $\rho \in [0, 1)$, the system described by (18) with controller gain $K = YX^{-1}$ is EMSS, if there exists matrices $X > 0$, $\widetilde{Q} > 0$, $\widetilde{R} > 0$, $\widetilde{N}$, $\widetilde{M}$ and Y with appropriate dimensions such that for s = 1,2*

$$\begin{bmatrix} \widetilde{\Sigma}_{11} + \widetilde{\Gamma} + \widetilde{\Gamma}^T & \widetilde{\Sigma}_{12}^s & \sqrt{\tau_M} \widetilde{\mathcal{A}}^T & 0 & \widetilde{\Sigma}_{15} & 0 & 0 & \widetilde{\Sigma}_{18} & \widetilde{\Sigma}_{19} \\ * & -\widetilde{R} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2\varepsilon X + \varepsilon^2 \widetilde{R} & 0 & \widetilde{\Sigma}_{35} & 0 & 0 & 0 & 0 \\ * & * & * & \widetilde{\Sigma}_{44} & 0 & \widetilde{\Sigma}_{46} & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Sigma}_{55} & 0 & \widetilde{\Sigma}_{57} & 0 & 0 \\ * & * & * & * & * & \widetilde{\Sigma}_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \widetilde{\Sigma}_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \widetilde{\Sigma}_{88} & 0 \\ * & * & * & * & * & * & * & * & \widetilde{\Sigma}_{99} \end{bmatrix} < 0 \tag{49}$$

where

$$\widetilde{\Sigma}_{11} = \begin{bmatrix} AX + XA^T + \widetilde{Q} & \alpha B\overline{\Xi}_1 Y & 0 & \alpha B\overline{\Xi}_1 Y \\ * & \rho\tilde{\Omega} & 0 & 0 \\ * & * & -\widetilde{Q} & 0 \\ * & * & * & -\widetilde{\Omega} \end{bmatrix}$$

$$\widetilde{\Sigma}_{12}^1 = \sqrt{\tau_M}\widetilde{N}, \widetilde{\Sigma}_{12}^2 = \sqrt{\tau_M}\widetilde{M}$$

$$\widetilde{\Gamma} = \begin{bmatrix} \widetilde{N} & -\widetilde{N} + \widetilde{M} & -\widetilde{M} & 0 \end{bmatrix}$$

$$\widetilde{\mathcal{A}}^T = \begin{bmatrix} AX & \alpha B\overline{\Xi}_1 Y & 0 & \alpha B\overline{\Xi}_1 Y \end{bmatrix}$$

$$\widetilde{\Sigma}_{15} = \begin{bmatrix} B\overline{\Xi}_1 Y & B\overline{\Xi}_1 Y & 0 & 0 \\ 0 & 0 & \overline{\Xi}_2 - \alpha I & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{\Xi}_2 - \alpha I \end{bmatrix}$$

$$\widetilde{\Sigma}_{18} = \begin{bmatrix} 0 & \cdots & 0 \\ L_2^1 & \cdots & L_2^n \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{bmatrix}, \quad \widetilde{\Sigma}_{19} = \begin{bmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ L_2^1 & \cdots & L_2^n \end{bmatrix}$$

$$\widetilde{\Sigma}_{35} = \begin{bmatrix} \sqrt{\tau_M} B\overline{\Xi}_1 Y & 0 & 0 & 0 \end{bmatrix}$$

$$\widetilde{\Sigma}_{44} = diag\{\underbrace{-2\varepsilon X + \varepsilon^2\widetilde{R}, \cdots, -2\varepsilon X + \varepsilon^2\widetilde{R}}_{2mn}\}, \quad \widetilde{\Sigma}_{55} = diag\{-X, -X, -X, -X\}$$

$$\widetilde{\Sigma}_{46} = \begin{bmatrix} \widetilde{\Lambda}_1 & \widetilde{\Lambda}_2 & \cdots & \widetilde{\Lambda}_n \end{bmatrix}$$

$$\widetilde{\Lambda}_j = \begin{bmatrix} 0_{1\times(j-1)m} & \widetilde{C}_{ij}^T & 0_{(n-j)m\times 1} \end{bmatrix}^T$$

$$\widetilde{\Sigma}_{57} = \begin{bmatrix} \tilde{\Upsilon}_1 & \widetilde{\Upsilon}_2 & \cdots & \widetilde{\Upsilon}_n \end{bmatrix}$$

$$\widetilde{\Upsilon}_j = \begin{bmatrix} 0_{1\times(j-1)m} & \widetilde{C}_{ij}^T & 0_{(n-j)m\times 1} \end{bmatrix}^T$$

$$\widetilde{C}_{ij}^T = \sqrt{w_{ij}}\begin{bmatrix} Y^T L_1^1 B^T & \cdots & Y^T L_1^m B^T \end{bmatrix}$$

$$\widetilde{\Sigma}_{66} = \Pi_{77} = \Pi_{88} = \Pi_{99} = diag\{\underbrace{-X, \ldots, -X}_{mn}\}$$

Proof. Separating $\overline{\Xi}_2$ with $\overline{\Xi}_2 - \alpha I$ and αI and defining $X = P^{-1}$, from (26), we can obtain that

$$\begin{aligned} &\Sigma_1(s) + \begin{bmatrix} PB\overline{\Xi}_1 K \\ 0_{4\times 1} \\ \sqrt{\tau_M} B\overline{\Xi}_1 K \\ 0_{2mn\times 1} \end{bmatrix} \begin{bmatrix} 0 & \overline{\Xi}_2 - \alpha I & 0_{1\times(2mn+4)} \end{bmatrix} + \begin{bmatrix} 0 \\ \overline{\Xi}_2 - \alpha I \\ 0_{(2mn+4)\times 1} \end{bmatrix} \begin{bmatrix} K^T\overline{\Xi}_1 B^T P & 0_{1\times 4} & \sqrt{\tau_M} K^T \overline{\Xi}_1 B^T & 0_{1\times 2mn} \end{bmatrix} + \begin{bmatrix} PB\overline{\Xi}_1 K \\ 0_{(2mn+5)\times 1} \end{bmatrix} \\ &\begin{bmatrix} 0_{1\times 3} & \overline{\Xi}_2 - \alpha I & 0_{1\times(2mn+2)} \end{bmatrix} + \begin{bmatrix} 0_{3\times 1} \\ \overline{\Xi}_2 - \alpha I \\ 0_{(2mn+2)\times 1} \end{bmatrix} \begin{bmatrix} K^T\overline{\Xi}_1 B^T P & 0_{1\times(2mn+5)} \end{bmatrix} \leqslant \begin{bmatrix} PB\overline{\Xi}_1 K \\ 0_{4\times 1} \\ \sqrt{\tau_M} B\overline{\Xi}_1 K \\ 0_{2mn\times 1} \end{bmatrix} X \begin{bmatrix} PB\overline{\Xi}_1 K \\ 0_{4\times 1} \\ \sqrt{\tau_M} B\overline{\Xi}_1 K \\ 0_{2mn\times 1} \end{bmatrix}^T \\ &+ \begin{bmatrix} 0 \\ \overline{\Xi}_2 - \alpha I \\ 0_{(2mn+4)\times 1} \end{bmatrix} X^{-1} \begin{bmatrix} 0 \\ \overline{\Xi}_2 - \alpha I \\ 0_{(2mn+4)\times 1} \end{bmatrix}^T + \begin{bmatrix} PB\overline{\Xi}_1 K \\ 0_{(2mn+5)\times 1} \end{bmatrix} X \begin{bmatrix} PB\overline{\Xi}_1 K \\ 0_{(2mn+5)\times 1} \end{bmatrix}^T + \begin{bmatrix} 0_{3\times 1} \\ \overline{\Xi}_2 - \alpha I \\ 0_{(2mn+2)\times 1} \end{bmatrix} X^{-1} \begin{bmatrix} 0_{3\times 1} \\ \overline{\Xi}_2 - \alpha I \\ 0_{(2mn+2)\times 1} \end{bmatrix}^T \end{aligned} \tag{50}$$

where $\Sigma_1(s)$ is obtained from $\Sigma(s)(s = 1, 2)$ by replacing $PB\overline{\Xi}_1 K\overline{\Xi}_2, \overline{\Xi}_2 K^T\overline{\Xi}_1 B^T P$, $\sqrt{\tau_M} B\overline{\Xi}_1 K\overline{\Xi}_2$, $\sqrt{\tau_M}\overline{\Xi}_2 K^T\overline{\Xi}_1 B^T$ by $\alpha PB\overline{\Xi}_1 K, \alpha K^T\overline{\Xi}_1 B^T P$, $\alpha\sqrt{\tau_M} B\overline{\Xi}_1 K, \alpha\sqrt{\tau_M} K^T\overline{\Xi}_1 B^T$, respectively.

$$\begin{aligned}
&\Sigma_2(s)+\sum_{j=1}^{n}\left\{\begin{bmatrix}0_{(6+mn+(j-1)m)\times 1}\\ \mathcal{C}_{ij}\\ 0_{(n-j)m\times 1}\end{bmatrix}\begin{bmatrix}0_{1\times 3} & L_2^j & 0_{1\times(2mn+2)}\end{bmatrix}+\begin{bmatrix}0_{3\times 1}\\ L_2^j\\ 0_{(2mn+2)\times 1}\end{bmatrix}\begin{bmatrix}0_{1\times(6+mn+(j-1)m)} & \mathcal{C}_{ij}^T & 0_{1\times(n-j)m}\end{bmatrix}\right\}\\
&\quad+\sum_{j=1}^{n}\left\{\begin{bmatrix}0_{(6+(j-1)m)\times 1}\\ \mathcal{C}_{ij}\\ 0_{(mn+(n-j)m)\times 1}\end{bmatrix}\begin{bmatrix}0 & L_2^j & 0_{1\times(4+2mn)}\end{bmatrix}+\begin{bmatrix}0\\ L_2^j\\ 0_{(4+2mn)\times 1}\end{bmatrix}\begin{bmatrix}0_{1\times(6+(j-1)m)} & \mathcal{C}_{ij}^T & 0_{1\times(2mn-jm)}\end{bmatrix}\right\}\\
&\quad\leqslant \Sigma_2(s)+\sum_{j=1}^{n}\left\{\begin{bmatrix}0_{(6+mn+(j-1)m)\times 1}\\ \mathcal{C}_{ij}\\ 0_{(n-j)m\times 1}\end{bmatrix}X\begin{bmatrix}0_{(6+mn+(j-1)m)\times 1}\\ \mathcal{C}_{ij}\\ 0_{(n-j)m\times 1}\end{bmatrix}^T+\begin{bmatrix}0_{3\times 1}\\ L_2^j\\ 0_{(2mn+2)\times 1}\end{bmatrix}X^{-1}\begin{bmatrix}0_{3\times 1}\\ L_2^j\\ 0_{(2mn+2)\times 1}\end{bmatrix}^T\right\}\\
&\quad+\sum_{j=1}^{n}\left\{\begin{bmatrix}0_{(6+(j-1)m)\times 1}\\ \mathcal{C}_{ij}\\ 0_{(mn+(n-j)m)\times 1}\end{bmatrix}X\begin{bmatrix}0_{(6+(j-1)m)\times 1}\\ \mathcal{C}_{ij}\\ 0_{(mn+(n-j)m)\times 1}\end{bmatrix}^T+\begin{bmatrix}0\\ L_2^j\\ 0_{(4+2mn)\times 1}\end{bmatrix}X^{-1}\begin{bmatrix}0\\ L_2^j\\ 0_{(4+2mn)\times 1}\end{bmatrix}^T\right\} \qquad (51)
\end{aligned}$$

where $\Sigma_2(s)$ is obtained from $\Sigma_1(s)(s = 1, 2)$ by deleting $BL_1^iKL_2^j$ and its transposes from the last mn columns and rows, and

$$\mathcal{C}_{ij}=\sqrt{w_{ij}}\begin{bmatrix}BL_1^1K\\ \vdots\\ BL_1^mK\end{bmatrix}$$

Combining (26), (50), and (51) and applying Schur complement, we can obtain

$$\begin{bmatrix}
\Pi_{11}+\Gamma+\Gamma^T & \Sigma_{12}^s & \sqrt{\tau_M}\widehat{\mathcal{A}}^T & 0 & \Pi_{15} & 0 & 0 & \Pi_{18} & \Pi_{19}\\
* & -R & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
* & * & -PR^{-1}P & 0 & \Pi_{35} & 0 & 0 & 0 & 0\\
* & * & * & \Pi_{44} & 0 & \Pi_{46} & 0 & 0 & 0\\
* & * & * & * & \Pi_{55} & 0 & \Pi_{57} & 0 & 0\\
* & * & * & * & * & \Pi_{66} & 0 & 0 & 0\\
* & * & * & * & * & * & \Pi_{77} & 0 & 0\\
* & * & * & * & * & * & * & \Pi_{88} & 0\\
* & * & * & * & * & * & * & * & \Pi_{99}
\end{bmatrix}<0,\quad s=1,2 \qquad (52)$$

where

$$\Pi_{11}=\begin{bmatrix}
PA+A^TP+Q & \alpha PB\overline{\Xi}_1K & 0 & \alpha PB\overline{\Xi}_1K\\
* & \rho\sum_{i=1}^{n}\alpha_i^2\left(L_2^i\right)^T\Omega L_2^i & 0 & 0\\
* & * & -Q & 0\\
* & * & * & \sum_{i=1}^{n}\alpha_i^2\left(L_2^i\right)^T\Omega L_2^i
\end{bmatrix}$$

$$\widehat{\mathcal{A}}=\begin{bmatrix}PA & \alpha\sqrt{\tau_M}PB\overline{\Xi}_1K & 0 & \alpha\sqrt{\tau_M}PB\overline{\Xi}_1K\end{bmatrix}$$

$$\Pi_{15}=\begin{bmatrix}
PB\overline{\Xi}_1KX & PB\overline{\Xi}_1KX & 0 & 0\\
0 & 0 & \overline{\Xi}_2-\alpha I & 0\\
0 & 0 & 0 & 0\\
0 & 0 & 0 & \overline{\Xi}_2-\alpha I
\end{bmatrix}$$

$$\Pi_{35}=\begin{bmatrix}\sqrt{\tau_M}B\overline{\Xi}_1KX & 0 & 0 & 0\end{bmatrix}$$

$$\Pi_{44}=diag\{\underbrace{-PR^{-1}P,\cdots,-PR^{-1}P}_{2mn}\},\ \ \Pi_{55}=diag\{-X,-X,-X,-X\}$$

$$\Pi_{66}=\Pi_{77}=\Pi_{88}=\Pi_{99}=diag\underbrace{-X,\cdots,-X}_{mn}\}$$

$$\Pi_{18}=\begin{bmatrix}0 & \cdots & 0\\ L_2^1 & \cdots & L_2^n\\ 0 & \cdots & 0\\ 0 & \cdots & 0\end{bmatrix},\quad \Pi_{19}=\begin{bmatrix}0 & \cdots & 0\\ 0 & \cdots & 0\\ 0 & \cdots & 0\\ L_2^1 & \cdots & L_2^n\end{bmatrix}$$

$$\Pi_{46} = \begin{bmatrix} \widehat{\Lambda}_1 & \widehat{\Lambda}_2 & \cdots & \widehat{\Lambda}_n \end{bmatrix}$$

$$\widehat{\Lambda}_j = \begin{bmatrix} 0_{1\times(j-1)m} & XC_{ij}^T & 0_{(n-j)m\times 1} \end{bmatrix}^T$$

$$\Pi_{57} = \begin{bmatrix} \widehat{\Upsilon}_1 & \widehat{\Upsilon}_2 & \cdots & \widehat{\Upsilon}_n \end{bmatrix}$$

$$\widehat{\Upsilon}_j = \begin{bmatrix} 0_{1\times(j-1)m} & XC_{ij}^T & 0_{(n-j)m\times 1} \end{bmatrix}^T$$

Due to $(R-\varepsilon^{-1}P)R^{-1}(R-\varepsilon^{-1}P) \geqslant 0$, we have

$$-PR^{-1}P \leqslant -2\varepsilon P + \varepsilon^2 R \tag{53}$$

Substituting $-PR^{-1}P$ with $-2\varepsilon P+\varepsilon^2 R$ into (52), we obtain

$$\begin{bmatrix} \Pi_{11}+\Gamma+\Gamma^T & \Sigma_{12}^s & \sqrt{\tau_M}\widehat{\mathcal{A}}^T & 0 & \Pi_{15} & 0 & 0 & \Pi_{18} & \Pi_{19} \\ * & -R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -2\varepsilon P+\varepsilon^2 R & 0 & \Pi_{35} & 0 & 0 & 0 & 0 \\ * & * & * & \widehat{\Pi}_{44} & 0 & \Pi_{46} & 0 & 0 & 0 \\ * & * & * & * & \Pi_{55} & 0 & \Pi_{57} & 0 & 0 \\ * & * & * & * & * & \Pi_{66} & 0 & 0 & 0 \\ * & * & * & * & * & * & \Pi_{77} & 0 & 0 \\ * & * & * & * & * & * & * & \Pi_{88} & 0 \\ * & * & * & * & * & * & * & * & \Pi_{99} \end{bmatrix} < 0, \quad s=1,2 \tag{54}$$

where

$$\widehat{\Pi}_{44} = diag\{\underbrace{-2\varepsilon P+\varepsilon^2 R, \cdots, -2\varepsilon P+\varepsilon^2 R}_{2mn}\}$$

Denoting $\widetilde{\Omega} = X[\sum_{i=1}^n \alpha_i^2 (L_2^i)^T \Omega L_2^i]X$, $\widetilde{Q} = XQX$, $\widetilde{R} = XRX$, $\widetilde{N} = XNX$, $\widetilde{M} = XMX$ and $Y = KX$, then pre- and post-multiplying (54) with

$$diag\{\underbrace{X,\ldots,X}_{2mn+6}, I, I, \ldots, I\}$$

Eq. (49) can be obtained. □

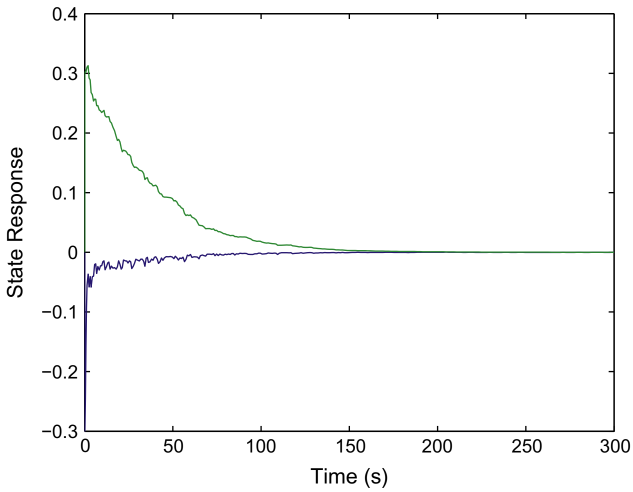

Fig. 2. The state responses under feedback gain (56) for Case 1.

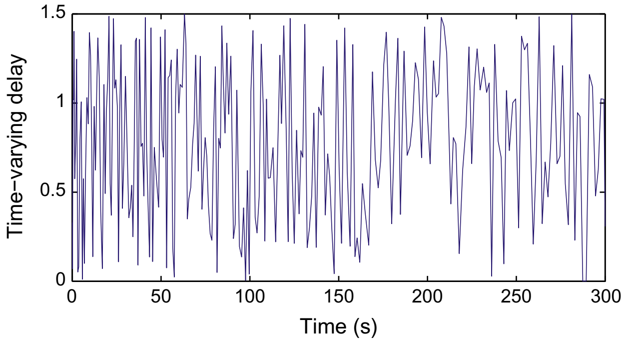

Fig. 3. The random delay $\tau(t)$ in Case 1.

Remark 10. From Theorem 2, we can obtain the admissible upper bounds τ_M of the time delay, the corresponding trigger matrix Ω, and the controller gain K through solving the following procedure by using LMI SOLVER FEASP in MATLAB LMI tool box:

1. If LMIs (49) for $s = 1, 2$ are feasible for matrix variables $X, \widetilde{Q}, \widetilde{R}, \widetilde{N}, \widetilde{M}, Y, \Omega$, we can get the corresponding trigger matrix Ω and the controller gain $K = YX^{-1}$
2. By choosing different ε, make sure that LMIs (49) for $s = 1, 2$ are feasible, we can derive the upper bound τ_M of the time delay.

4. Simulation examples

In this section, an example is presented to illustrate the validity of proposed event-triggering communication scheme.

Example 1. Consider the system (1) is described as:

$$\dot{x}(t) = \begin{bmatrix} -2 & -0.1 \\ -0.1 & 0.01 \end{bmatrix} x(t) + \begin{bmatrix} 0.05 \\ 0.02 \end{bmatrix} u(t) \tag{55}$$

In the following, we will discuss the reliable controller design for system (55) under the following four cases

Case 1: Firstly, the system (55) is time-triggered, setting $\rho = 0$ in (2), and $\overline{\Xi}_1 = 0.8$, $\overline{\Xi}_2 = diag\{0.7, 0.9\}$, $\delta_1 = 0.2$, $\gamma_1 = \gamma_2 = 0.1$, $\alpha = 1.6$, and $\varepsilon = 1$, based on Matlab/LMIs toolbox and applying Theorem 2, we can get the upper bound value $\tau_M = 1.9950$. When $\tau_M = 1.5$, the controller feedback gain K is

$$K = [\,1.5999 \quad -2.5873\,] \tag{56}$$

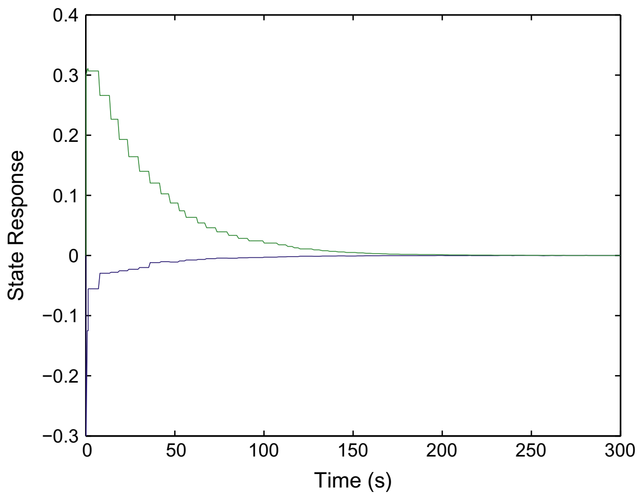

Fig. 4. The state responses under feedback gain (58) for Case 2.

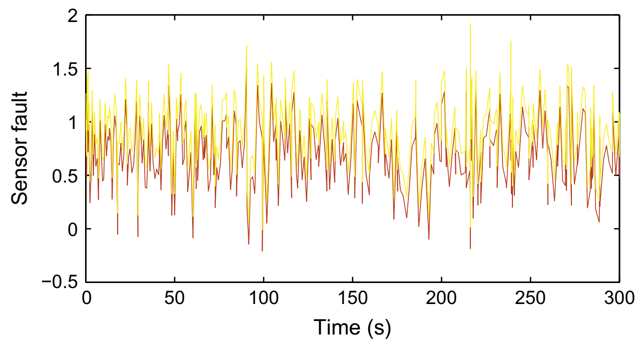

Fig. 5. The probabilistic sensor faults in Case 2.

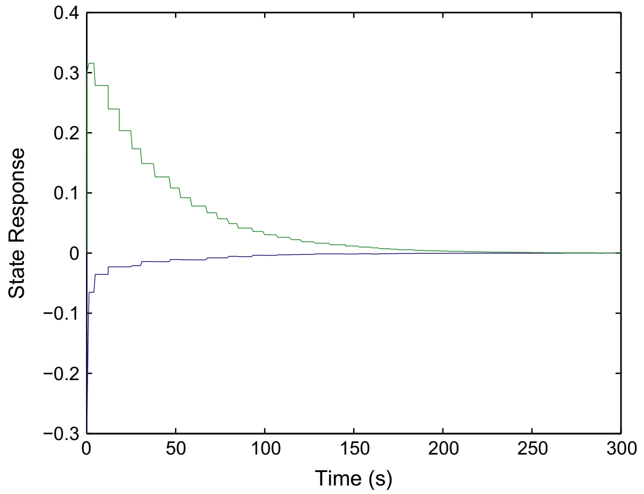

Fig. 6. The state responses under feedback gain (60) for Case 3.

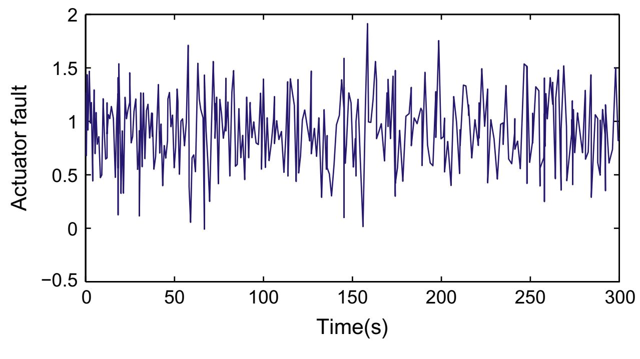

Fig. 7. The probabilistic actuator faults in Case 3.

When the initial condition is chosen as $x(0) = [-0.3 \quad 0.3]^T$, the state responses and time-varying delays are shown in Figs. 2 and 3, respectively.

Case 2: When the system (55) without actuators fault, by setting $\overline{\Xi}_1 = 1$, $\delta_1 = 0$, $\tau_M = 1.5$, $\overline{\Xi}_2 = diag\{0.7, 0.9\}$, $\gamma_1 = \gamma_2 = 0.1$, $\alpha = 1.6$, $\varepsilon = 1$, and the corresponding trigger parameter $\rho = 0.03$, based on the Theorem 2, the corresponding trigger matrix Ω is obtained by

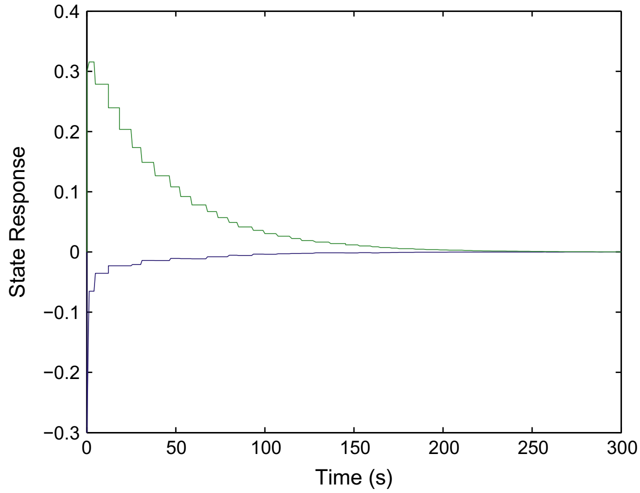

Fig. 8. The state responses under feedback gain (62) for Case 4.

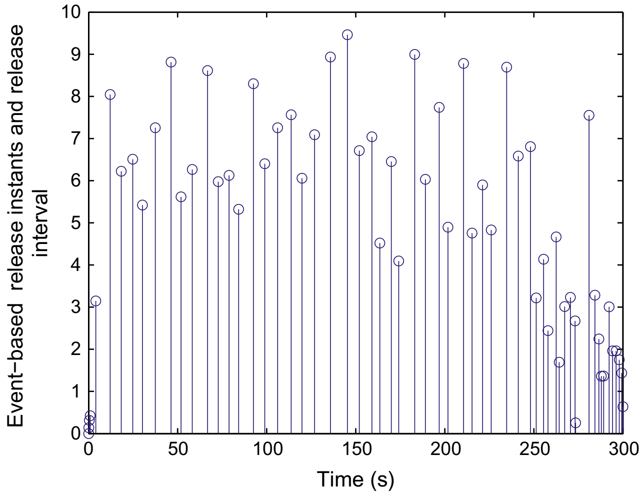

Fig. 9. The release instants and release interval with feedback gain (62) in Case 4.

$$\Omega = \begin{bmatrix} 23.0724 & -1.5899 \\ -1.5899 & 22.3669 \end{bmatrix} \tag{57}$$

and the controller feedback gain K is

$$K = [\,1.3455 \quad -2.2679\,] \tag{58}$$

When the initial condition is chosen as $x(0) = [\,-0.3 \quad 0.3\,]^T$, the state responses and probabilistic sensor faults are shown in Figs. 4 and 5, respectively.

Case 3: When the system (55) without sensors fault, by setting $\overline{\Xi}_1 = 0.9$, $\delta_1 = 0.1$, $\tau_M = 1.5$, $\overline{\Xi}_2 = diag\{1, 1\}$, $\gamma_1 = \gamma_2 = 0$, $\alpha = 2$, $\varepsilon = 1$, and the corresponding trigger parameter $\rho = 0.03$, based on the Theorem 2, the corresponding trigger matrix Ω is obtained by

$$\Omega = \begin{bmatrix} 596.7646 & -41.4499 \\ -41.4499 & 578.0689 \end{bmatrix} \tag{59}$$

and the controller feedback gain K is

$$K = [\,1.2804 \quad -2.0322\,] \tag{60}$$

When the initial condition is chosen as $x(0) = [-0.3 \quad 0.3]^T$, the state responses and probabilistic actuator faults are shown in Figs. 6 and 7, respectively.

Case 4: When the system (55) with actuators and sensors fault, setting $\rho = 0.03$ in (2), and $\overline{\Xi}_1 = 0.7,\ \overline{\Xi}_2 = diag\{0.8, 0.7\},\ \delta_1 = 0.2,\ \gamma_1 = \gamma_2 = 0.1,\ \alpha = 1.5$, and $\varepsilon = 1$, by using Theorem 2, we can get the upper bound value $\tau_M = 1.9950$. When $\tau_M = 1.5$, the corresponding trigger matrix Ω is obtained by

$$\Omega = \begin{bmatrix} 21.2240 & -1.3545 \\ -1.3545 & 20.0559 \end{bmatrix} \tag{61}$$

and the controller feedback gain K is

$$K = [1.9813 \quad -3.3253] \tag{62}$$

When the initial condition is chosen as $x(0) = [-0.3 \quad 0.3]^T$, the state responses are shown in Fig. 8, and the Fig. 9 describes the release instants and release interval. From Fig. 9, it is easy to see that the max release interval is 9.4616 s.

5. Conclusion

In this paper, in order to reduce the computation load, we propose a event-triggering sampling strategy when we take probabilistic sensor and actuator fault into consideration. Under the event-triggering scheme, the sampled sensor measurements information will be transmitted to the controller only when it violates specified triggering condition. Secondly, based on the proposed scheme, in terms of different failure rates and the measurements distortion of every sensor and actuator, a new probabilistic sensor and actuator fault model for event-triggered networked control systems is proposed. By using Lyapunov functional, criteria for the exponential stability and criteria for co-designing both the feedback and the trigger parameters are derived in the form of linear matrix inequalities. A simulation example is given to illustrate the effectiveness of the proposed method. Future research work will include the following: (1) extension of the proposed method to nonlinear networked control systems, T-S fuzzy systems, and complex networks. (2) Apply our proposed event-triggered scheme in fault estimation, sliding mode control, distributed state estimation, fault detection filters and reliable filtering design.

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