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# Event-based fault detection for networked systems with communication delay and nonlinear perturbation

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#### Abstract

This paper is concerned with the event-based fault detection for the networked systems with communication delay and nonlinear perturbation. We propose an event-triggered scheme, which has some advantages over existing ones. The sensor data is transmitted only when the specified event condition involving the sampled measurements of the plant is violated. An event-based fault detection model is firstly constructed by taking the effect of event-triggered scheme and the network transmission delay into consideration. The main purpose of this paper is to design an event-based fault detection filter such that, for all unknown input, communication delay and nonlinear perturbation, the error between the residual signal and the fault signal is made as small as possible. Sufficient conditions for the existence of the desired fault detection filter are established in terms of linear matrix inequalities. Based on these conditions, the explicit expression is given for the designed fault detection filter parameters. A numerical example is employed to illustrate the advantage of the introduced event-triggered scheme and the effectiveness of the proposed method.

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# 1. Introduction

Fault detection and isolation (FDI) has been an active field of research over the past decades, in response to an increasing demand for higher performance, higher safety and reliability standards of modern dynamic systems. In general, the aim of FDI is to construct a residual signal

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and compare it with a predefined threshold. When the residual signal exceeds the threshold, an alarm is generated. Recently, the model-based approaches to FDI problems for dynamic systems have received more and more attention, as it makes use of the mathematical model for designing a fault detection filter/observer to detect the fault signal. So far, FDI problems have been widely investigated and lots of outstanding results have been made [1–5]. For example, in [1], the authors investigated the robust fault detection problem for a class of discrete-time networked systems with unknown input and multiple state delays. The authors in [2] are concerned with the fault detection problem for a class of discrete-time systems with randomly occurring nonlinearities, mixed stochastic time-delays as well as measurement quantization. In [3], the robust fault detection filter (RFDF) was designed for a class of linear systems with some nonlinear perturbations and mixed neutral and discrete time-varying delays. Different from robust control, the goal of robust fault detection is to make the error between the residual and the fault signal as small as possible.

On the other hand, networked control systems (NCSs) have received a great deal of research attention, which have many advantages such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. However, the insertion of network in the control systems can also bring about new interesting and challenging issues as to the limited capacity of the network cable, for example, the transmission delay, packet dropout, signal quantization, scheduling confusion, etc. Recently, many efforts have been made on NCSs. Due to the output signal of the plant is often measured at sampled points in many practical situations, most of the available results use a periodic triggered method (also called a timetriggered control). However, this might be a conservative choice. For example, the issues of limited resource and insufficient communication bandwidth and the case of inadequate computation power for fast systems are problems that often have to be dealt with. It is therefore of great need to build mechanisms for sampling that do not rely on periodicity or time-triggering techniques. Recently, event-triggered method, advocating the use of action only when some function of the system exceeds a threshold, has received considerable attention. Event-triggered method provides a useful way to determine when the sampling action is carried out. Compared with time-triggered method, it has the following advantages: (1) it only samples when necessary; (2) the burden of the network communication is reduced; (3) the computation cost of the controller and the occupation of the sensor and actuator are reduced. So far, many outstanding results under event-triggered method have been reported. In [6], the authors proposed an eventtriggered control for linear systems with an external disturbance and derived the criteria to guarantee the uniform boundedness of the system. The authors in [7] proposed event-triggered strategies for control of discrete-time systems, in which the plant was assumed input-to-state stable with respect to measurement errors and the control law was updated once a triggering condition involving the norm of a measurement error was violated. The methods for design or implementation of controllers in the event-triggered form based on dissipation inequalities were proposed for both linear and nonlinear systems in [8]. In [9], the authors were concerned with the problem of event-based  $H_{\infty}$  filtering for networked systems with communication delay under a novel event-triggered scheme upon which the sensor data transmitted only when the specified event condition involving the sampled measurements of the plant was violated. Up to now, to the best of the authors' knowledge, little attention has been paid to the FDI problem for networked control system under event-triggered scheme. This situation has motivated our current investigation with the hope to shorten such a gap by addressing the fault detection with transmission delay under the event-triggered scheme.

In this paper, the event-based fault detection problem is studied for networked systems with communication delay, unknown input and nonlinear perturbation. The event generator is used to determine whether the newly sampled sensor data to be carried out is constructed between the sensor and the fault detection filter. Unlike the cases in [6–8], the implementation of our event-triggered scheme only needs a supervision in discrete instants. Similar to [9], there is no need to retrofit the existing system by using our method. By augmenting the states of the original system and the fault detection filter, the fault detection problem addressed is converted into an auxiliary  $H_{\infty}$  filtering problem.

The paper is organized in the following way. Section 2 presents the system description, the event triggered scheme and the formation of the overall fault detection dynamic system are described. In Section 3, a sufficient condition for the existence of the desired fault detection filter is established in terms of linear matrix inequalities (LMIs) and a fault detection filter design method is provided. In the final part, a numerical example is provided to show the effectiveness and applicability of the proposed method.

Notation:  $\mathbb{R}^n$  and  $\mathbb{R}^{n \times m}$  denote the *n*-dimensional Euclidean space and the set of  $n \times m$  real matrices, respectively; the superscript "*T*" stands for matrix transposition; *I* is the identity matrix of appropriate dimension;  $\|\cdot\|$  stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation X > 0 (respectively,  $X \ge 0$ ), for  $X \in \mathbb{R}^{n \times n}$  means that the matrix *X* is real symmetric positive definite (respectively, positive semi-definite). For a matrix *B* and two

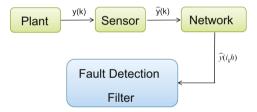


Fig. 1. The structure of a fault detection filtering system.

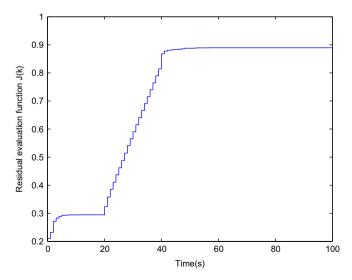


Fig. 2. Residual evaluation function J(k).

symmetric matrices A and C,  $\begin{bmatrix} A \\ B \\ C \end{bmatrix}$  denotes a symmetric matrix, where \* denotes the entries implied by symmetry.

# 2. System description

Consider a discrete-time networked system with the structure shown in Fig. 1.

The system consists of a plant with smart sensor, a fault detection filter and network channel, which can be described as the following discrete-time systems with nonlinearities and time-delay:

$$\begin{cases} x(k+1) = Ax(k) + A_{d1}x(k-d_1(k)) + D_1w(k) + Gf(k) + g(k, x(k)) \\ y(k) = Cx(k) \end{cases}$$
(1)

where  $x(k) \in \mathbb{R}^n$  is the state vector;  $d_1(k)$  is time-varying delay;  $w(k) \in \mathbb{R}^n$  is the unknown input belonging to  $L_2[0, \infty)$ ;  $f(k) \in \mathbb{R}^l$  is the fault signal to be detected; g(k, x(k)) is a known nonlinear function,  $y(k) \in \mathbb{R}^m$  is the process output, and A,  $A_{d1}$ ,  $D_1$ , G, C are all constant matrices with appropriate dimensions. Throughout this paper, similar to [10,11], we make the following assumptions:

Assumption 1. g(k, 0) = 0; for all  $k \in N$ .

## Assumption 2.

$$[g(k,x)-g(k,y)-\Xi_1(x-y)]^T[g(k,x)-g(k,y)-\Xi_2(x-y)] \le 0$$

where  $\Xi_1$  and  $\Xi_2$  are known real constant matrices.

Remark 1. From Assumptions 1 and 2, we can obtain that

$$\begin{bmatrix} x(k) \\ g(k, x(k)) \end{bmatrix}^T \begin{bmatrix} \Omega_1 & * \\ \Omega_2 & I_n \end{bmatrix} \begin{bmatrix} x(k) \\ g(k, x(k)) \end{bmatrix} \le 0$$
(2)

where

$$\Omega_1 = \frac{\Xi_1^T \Xi_2 + \Xi_2^T \Xi_1}{2}, \quad \Omega_2 = -\frac{\Xi_1 + \Xi_2}{2}$$

Assumption 3. The time-varying delay  $d_1(k)$  satisfies  $d_1^m \le d_1(k) \le d_1^M$ , where  $d_1^m$  and  $d_1^M$  are constant positive scalars representing the lower and upper bounds, respectively.

Consider the following discrete-time full-order fault detection filter:

$$\begin{cases} \hat{x}(k+1) = A_F \hat{x}(k) + B_F \hat{y}(k) \\ r(k) = C_F \hat{x}(k) + D_F \hat{y}(k) \end{cases}$$
(3)

where  $\hat{x}(k) \in \mathbb{R}^n$  is the state of the fault detection filter;  $r(k) \in \mathbb{R}^l$  is the so-called residual that is compatible with f(k);  $\hat{y}(k)$  is the real input of the fault detection filter.  $A_F$ ,  $B_F$ ,  $C_F$  and  $D_F$  are appropriately the dimensioned filter matrices to be determined.

**Remark 2.** For traditional fault detection filtering problem, the effect of the communication network is neglected. However, due to the existence of the network-induced delays, it is quite common that  $\hat{y}(k) \neq y(k)$ .

As is well known, periodic sampling mechanism has been widely used in many practical systems, however, it may often lead to transmitting many unnecessary signals through the network, which in turn will increase the load of network transmission and wastes in the network bandwidth. As stated in [12], the event-triggered sampling scheme is an effective way for network systems. Therefore, for networked systems shown in Fig. 1, in order to save network resources such as network bandwidth, it is significant to introduce an event triggered mechanism which decides whether the newly sampled sensor data should be sent out to the fault detection filter. As is shown in Fig. 1, the sensor data feeds into an Event Generator that decides when to transmit the sensor data to the fault detection filter via a network medium by a specified trigger condition, which will be given in sequel. The following function of network architecture in Fig. 1 is expected:

1. As shown in Fig. 1, the event generator is constructed between the sensor and the fault detection filter which is used to determine whether the newly sampled sensor data y(k) should be sent out to the fault detection filter by using the following judgement algorithm [13]:

$$[y(k) - y(s_i)]^T \Omega[y(k) - y(s_i)] \le \sigma y^T(k) \Omega y(k)$$
(4)

where  $\Omega \in \mathbb{R}^{m \times m}$  is a positive matrix,  $\sigma \in [0, 1)$ ,  $y(s_i)$  is the previously transmitted sensor data. If the current sensor data y(k) satisfy the inequality (4), it will not be transmitted. Only the one that exceeds the threshold in Eq. (4) will be sent to the fault detection filter.

2. When the sampled data has been transmitted (or released) by the event generator, it is forwarded to the ZOH through network channel, introducing a communication delay  $\tau(k)$ .

Under the event triggered (4), the release times are assumed to be  $s_0, s_1, s_2, ...$ , where  $s_0 = 0$  is the initial time.  $t_i = s_{i+1} - s_i$  denotes the release period of event generator in Eq. (4). Considering the effect of the transmission delay on the network system, the released signals will arrive at the fault detection filter at the instants  $s_0 + \tau(s_0), s_1 + \tau(s_1), s_2 + \tau(s_2), ...$ , where  $\tau(0) = 0$ .

**Remark 3.** From event triggering (4), it is easy to see that the set of the release instants, i.e.,  $\{s_0, s_1, s_2, ...\}$  is a subset of  $\{0, 1, 2, ...\}$ . The amount of  $\{s_0, s_1, s_2, ...\}$  depends on not only the value of  $\sigma$ , but also the variation of the state. When  $\sigma = 0$ ,  $\{s_0, s_1, s_2, ...\} = \{0, 1, 2, ...\}$ , it reduces to the case with periodic release times.

Assumption 4. The time-varying delay in the network communication is  $\tau(k)$  and  $\tau(k) \in [0, \tau^M)$ , where  $\tau^M$  is apositive real number.

Based on the above analysis, considering the behavior of ZOH, the input of the fault detection filter can be described as

$$\hat{y}(k) = y(s_i), \quad t \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]$$
(5)

Similar to [14], for technical convenience, we consider the following two cases: Case 1: If  $s_i + 1 + \tau^M \ge s_{i+1} + \tau(s_{i+1}) - 1$ , define a function  $d_2(k)$  as

$$d_2(k) = k - s_i, \quad k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]$$
(6)

clearly,

$$\tau(s_i) \le d_2(k) \le (s_{i+1} - s_i) + \tau(s_{i+1}) - 1 \le 1 + \tau^M$$
(7)

*Case* 2: If  $s_i + 1 + \tau^M \leq s_{i+1} + \tau(s_{i+1}) - 1$ , consider the following two intervals:

$$[s_i + \tau(s_i), s_i + \tau^M], \quad [s_i + \tau^M + l, s_i + \tau^M + l + 1]$$
(8)

Since  $\tau(k) \leq \tau^M$ , it can be easily shown that there exists *d* such that

$$s_i + d + \tau^M < s_{i+1} + \tau(s_{i+1}) - 1 \le s_i + d + 1 + \tau^M$$
(9)

Moreover,  $y(s_i)$  and  $y(s_i + l)$  with l = 1, 2, ..., d satisfy Eq. (4). Let

$$\begin{cases}
I_0 = [s_i + \tau(s_i), s_i + \tau^M + 1) \\
I_l = [s_i + \tau^M + l, s_i + \tau^M + l + 1) \\
I_d = [s_i + d + \tau^M, s_{i+1} + \tau(s_{i+1}) - 1]
\end{cases}$$
(10)

where l = 1, 2, ..., d-1. One can see that

$$[s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1] = \bigcup_{i=0}^{i=d} I_i$$
(11)

Define  $d_2(k)$  as

$$d_2(k) = \begin{cases} k - s_i, & k \in I_0 \\ k - s_i - l, & k \in I_l, & l = 1, 2, ..., d - 1 \\ k - s_i - d, & k \in I_d \end{cases}$$
(12)

Then, we have

$$\begin{cases} \tau(s_i) \le d_2(k) \le 1 + \tau^M \triangleq d_2^M, \quad k \in I_0 \\ \tau(s_i)) \le \tau^M \le d_2(k) \le d_2^M, \quad k \in I_l, \quad l = 1, 2, ..., d-1 \\ \tau(s_i) \le \tau^M \le d_2(k) \le d_2^M, \quad k \in I_d \end{cases}$$
(13)

where the third row in Eq. (13) holds because  $s_{i+1} + \tau(s_{i+1}) - 1 \leq s_i + d + 1 + \tau^M$ . Obviously,

$$\tau(s_i) \le \tau^M \le d_2(k) \le d_2^M, \quad k \in I_d \tag{14}$$

In Case 1, for  $k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]$ , define  $e_i(k) = 0$ . In Case 2, define

$$e_i(k) = \begin{cases} 0, & k \in I_0 \\ y(s_i) - y(s_i + l), & k \in I_l, \ l = 1, 2, \dots, d-1 \\ y(s_i) - y(s_i + d), & k \in I_d \end{cases}$$
(15)

From the definition of  $e_i(k)$  and the triggering algorithm (4), it can be easily seen that for  $k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]$ ,

$$e_i^T(k)\Omega e_i(k) \le \sigma y^T(k - d_2(k))\Omega y(k - d_2(k))$$
(16)

Utilizing  $d_2(k)$  and  $e_i(k)$ , the input of the fault detection filter  $\tilde{y}(k)$  can be expressed as

$$\hat{y}(k) = y(s_i) = y(k - d_2(k)) + e_i(k), \quad k \in [s_i + \tau(s_i), s_{i+1} + \tau(s_{i+1}) - 1]$$
(17)

Then, combining Eqs. (1) and (17), Eq. (3) can be rewritten as

$$\begin{cases} \hat{x}(k+1) = A_F \hat{x}(k) + B_F C x(k-d_2(k)) + B_F e_i(k) \\ r(k) = C_F \hat{x}(k) + D_F C x(k-d_2(k)) + D_F e_i(k) \end{cases}$$
(18)

From Eqs. (1) and (18), we have the overall fault detection dynamics governed by the following system:

$$\begin{cases} \overline{x}(k+1) = \overline{A}\overline{x}(k) + \overline{A}_{d1}x(k-d_1(k)) + \overline{A}_{d2}x(k-d_2(k)) + \overline{D}_1v(k) + \tilde{B}_Fe_i(k) + H_1^Tg(k,x(k)) \\ \overline{r}(k) = \tilde{C}_F\overline{x}(k) + D_FCx(k-d_2(k)) + D_Fe_i(k) + H_2u(k) \end{cases}$$

where

$$\overline{x}(k) = \begin{bmatrix} x^{T}(k) \ \hat{x}^{T}(k) \end{bmatrix}^{T}, \quad \overline{r}(k) = r(k) - f(k), \quad v(k) = \begin{bmatrix} w^{T}(k) \ f^{T}(k) \end{bmatrix}^{T}$$
$$\overline{A} = \begin{bmatrix} A & 0 \\ 0 & A_{F} \end{bmatrix}, \quad \overline{A}_{d1} = \begin{bmatrix} A_{d1} \\ 0 \end{bmatrix}, \quad \overline{A}_{d2} = \begin{bmatrix} 0 \\ B_{F}C \end{bmatrix}, \quad \overline{D}_{1} = \begin{bmatrix} D_{1} & G \\ 0 & 0 \end{bmatrix}$$
$$\tilde{B}_{F} = \begin{bmatrix} 0 \\ B_{F} \end{bmatrix}, \quad H_{1} = \begin{bmatrix} I & 0 \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} 0 & C_{F} \end{bmatrix}, \quad H_{2} = \begin{bmatrix} 0 & -I \end{bmatrix}$$

After the above manipulations, the problem of event-based fault detection filter design can now be formulated as an auxiliary  $H_{\infty}$  filtering problem: design a filter of the type (3) that makes the error between residual and fault signal as small as possible. The aim of this paper can be restated as finding the filter parameters  $A_F$ ,  $B_F$ ,  $C_F$  and  $D_F$  such that the following two requirements are satisfied: (i) the overall fault detection dynamics (19) is exponentially stable in the mean square [15]; (ii) under zero initial condition, the infimum of  $\gamma$  is made as small as possible in the feasibility of

$$\sup_{\nu(k)\neq 0} \frac{\|\overline{r}(k)\|^2}{\|\overline{\nu}(k)\|^2} < \gamma^2, \quad \gamma > 0$$

$$\tag{20}$$

We further adopt a residual evaluation function J(k) and a threshold  $J_{ih}$  of the following form:

$$J(k) = \left\{ \sum_{h=0}^{k} r^{T}(h)r(h) \right\}^{1/2}, \quad J_{th} = \sup_{w(k) \in l_{2}, f(k) = 0} J(k)$$
(21)

Based on Eq. (21), the occurrence of faults can be detected by comparing J(k) with  $J_{th}$  according to the following rules:

 $J(k) > J_{th} \Rightarrow$  with faults  $\Rightarrow$  *alarm*  $J(k) \leq J_{th} \Rightarrow$  no faults

In the following, we need to introduce two lemmas, which will help us in deriving the main results.

**Lemma 1** (Wang et al. [16]). For any vectors  $x, y \in \mathbb{R}^n$ , and positive definite matrix  $Q \in \mathbb{R}^{n \times n}$ , the following inequality holds:

$$2x^T y \le x^T Q x + y^T Q^{-1} y$$

(19)

**Lemma 2** (Yue et al. [17]).  $\Xi_{1i}, \Xi_{2i}(i = 1, 2)$  and  $\Omega$  are matrices with appropriate dimensions,  $d_i(k)$  is a function of k and  $d_1^m \leq d_1(k) \leq d_1^M$ ,  $0 \leq d_2(k) \leq d_2^M$ , then

$$[(d_1(k)-d_1^m)\Xi_{11}+(d_1^M-d_1(k))\Xi_{21}]+[d_2(k)\Xi_{12}+(d_2^M-d_2(k))\Xi_{22}]+\Omega<0$$

if and only if the following four inequalities hold:

$$\begin{split} & (d_1^M - d_1^m) \varXi_{11} + d_2^M \varXi_{22} + \varOmega < 0 \\ & (d_1^M - d_1^m) \varXi_{21} + d_2^M \varXi_{22} + \varOmega < 0 \\ & (d_1^M - d_1^m) \varXi_{11} + d_2^M \varXi_{12} + \varOmega < 0 \\ & (d_1^M - d_1^m) \varXi_{21} + d_2^M \varXi_{12} + \varOmega < 0 \end{split}$$

## 3. Main results

In this section, we will investigate both the analysis and synthesis problems for the fault detection filter design of system (1) under the event trigger (4).

**Theorem 1.** Consider system (19) and suppose the parameters  $\gamma$ ,  $d_1^m$ ,  $d_1^M$ ,  $d_2^M$  and  $\sigma$  are given, the fault detection dynamics is exponentially stable in the mean square under the event trigger scheme (4) if there exist matrices P > 0,  $Q_i > 0$  (i = 1, 2, 3),  $R_i > 0$  (i = 1, 2, 3),  $\Omega > 0$ ,  $N_2$ ,  $N_3 M_3$ ,  $M_4$ ,  $S_1$ ,  $S_5$ ,  $T_5$  and  $T_6$  with appropriate dimensions satisfying

$$\Sigma(s) = \begin{bmatrix} \Sigma_{11} & * & * & * & * \\ \Sigma_{21} & \Sigma_{22} & * & * & * \\ \Sigma_{31} & 0 & \Sigma_{33} & * & * \\ \Sigma_{41} & \Sigma_{42} & \Sigma_{43} & \Sigma_{44} & * \\ \Sigma_{51}(s) & 0 & 0 & 0 & \Sigma_{55} \end{bmatrix} < 0, \quad s = 1, 2, 3, 4$$

$$(22)$$

where

$$\Sigma_{11} = -P + H_1^T (Q_1 + Q_2 + Q_3 - R_1 - \Omega_1) H_1 + S_1 H_1 + H_1^T S_1^T$$

$$\Sigma_{21} = \begin{bmatrix} R_1 H_1 \\ 0 \\ 0 \\ S_5 H_1 - S_1^T \\ 0 \end{bmatrix}, \quad \Sigma_{22} = \begin{bmatrix} \Gamma_1 & \ast & \ast & \ast & \ast \\ N_3 - N_2^T & \Gamma_2 & \ast & \ast & \ast \\ 0 & M_4 - M_3^T & \Gamma_3 & \ast & \ast \\ 0 & 0 & 0 & \Gamma_4 & \ast \\ 0 & 0 & 0 & T_6 - T_5^T & -Q_3 - T_6 - T_6^T \end{bmatrix}$$

 $\Sigma_{31} = [0 \ 0 \ -H_1^T \Omega_2^T]^T, \quad \Sigma_{33} = diag\{-\gamma^2 I, -\Omega, -I\}$ 

$$\Sigma_{41} = \begin{bmatrix} P\overline{A} \\ d_1^m R_1(A-I)H_1 \\ \sqrt{d_1^M - d_1^m} R_2(A-I)H_1 \\ \sqrt{d_2^M} R_3(A-I)H_1 \\ \tilde{C}_F \end{bmatrix}, \quad \Sigma_{42} = \begin{bmatrix} 0 & P\overline{A}_{d1} & 0 & P\overline{A}_{d2} & 0 \\ 0 & d_1^m R_1 R_1 A_{d1} & 0 & 0 & 0 \\ 0 & \sqrt{d_1^m - d_1^m} R_2 A_{d1} & 0 & 0 & 0 \\ 0 & \sqrt{d_2^M} R_3 A_{d1} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_F C & 0 \end{bmatrix}$$

$$\Sigma_{43} = \begin{bmatrix} P\overline{D}_1 & P\tilde{B}_F & PH_1^T \\ d_1^m R_1 \overline{D}_2 & 0 & d_1^m R_1 \\ \sqrt{d_1^M - d_1^m} R_2 \overline{D}_2 & 0 & \sqrt{d_1^M - d_1^m} R_2 \\ \sqrt{d_2^M} R_3 \overline{D}_2 & 0 & \sqrt{d_2^M} R_3 \\ H_2 & D_F & 0 \end{bmatrix},$$

 $\Sigma_{44} = diag\{-P, -R_1, -R_2, -R_3, -I\}, \quad H_1 = [I \ 0]$ 

$$\begin{split} \Sigma_{51}(1) &= \begin{bmatrix} 0\\ \sqrt{d_2^M} S_1^T \end{bmatrix}, \quad \Sigma_{51}(2) = 0, \quad \Sigma_{51}(3) = \begin{bmatrix} 0\\ \sqrt{d_2^M} S_1^T \end{bmatrix}, \quad \Sigma_{51}(4) = 0, \\ \Sigma_{52}(1) &= \begin{bmatrix} \sqrt{d_1^M - d_1^m} N_2^T & \sqrt{d_1^M - d_1^m} N_3^T & 0 & 0 & 0\\ 0 & 0 & \sqrt{d_2^M} S_5^T & 0 \end{bmatrix} \\ \Sigma_{52}(2) &= \begin{bmatrix} \sqrt{d_1^M - d_1^m} N_2^T & \sqrt{d_1^M - d_1^m} N_3^T & 0 & 0 & 0\\ 0 & 0 & 0 & \sqrt{d_2^M} T_5^T & \sqrt{d_2^M} T_6^T \end{bmatrix} \\ \Sigma_{52}(3) &= \begin{bmatrix} 0 & \sqrt{d_1^M - d_1^m} M_3^T & \sqrt{d_1^M - d_1^m} M_4^T & 0 & 0\\ 0 & 0 & 0 & \sqrt{d_2^M} T_5^T & \sqrt{d_2^M} T_6^T \end{bmatrix} \\ \Sigma_{52}(4) &= \begin{bmatrix} 0 & \sqrt{d_1^M - d_1^m} M_3^T & \sqrt{d_1^M - d_1^m} M_4^T & 0 & 0\\ 0 & 0 & 0 & \sqrt{d_2^M} S_5^T & 0 \end{bmatrix} \\ \Sigma_{55} &= diag\{-R_2, -R_3\} \\ \Gamma_1 &= -Q_1 - R_1 + N_2 + N_2^T, \quad \Gamma_2 = -N_3 - N_3^T + M_3 + M_3^T, \quad \Gamma_3 = -Q_2 - M_4 - M_4^T, \end{split}$$

 $\Gamma_4 = -S_5 - S_5^T + T_5 + T_5^T + \sigma \Omega$ 

Proof. Define

$$\delta(k) = x(k+1) - x(k) \tag{23}$$

$$= (A - I)H_1\overline{x}(k) + A_{d1}x(k - d_1(k)) + \overline{D}_2v(k) + g(k, x(k))$$
(24)

where  $\overline{D}_2 = [D_1 \ G]].$ 

Choose the following Lyapunov functional candidate as:

$$V(k) = V_1(k) + V_2(k) + V_3(k)$$
(25)

where

$$V_{1}(k) = \overline{x}^{T}(k)P\overline{x}(k)$$

$$V_{2}(k) = \sum_{i=k-d_{1}^{m}}^{k-1} x^{T}(i)Q_{1}x(i) + \sum_{i=k-d_{1}^{M}}^{k-1} x^{T}(i)Q_{2}x(i) + \sum_{i=k-d_{2}^{M}}^{k-1} x^{T}(i)Q_{3}x(i)$$

$$V_{3}(k) = d_{1}^{m} \sum_{i=-d_{1}^{m}}^{-1} \sum_{j=k+i}^{k-1} \delta^{T}(i)R_{1}\delta(i) + \sum_{i=-d_{1}^{M}}^{-d_{1}^{m}-1} \sum_{j=k+i}^{k-1} \delta^{T}(i)R_{2}\delta(i) + \sum_{i=-d_{2}^{M}}^{-1} \sum_{j=k+i}^{k-1} \delta^{T}(i)R_{3}\delta(i)$$

Let 
$$\Delta V_i(k) = V_i(k+1) - V_i(k), i = 1, 2, 3$$
, then along the system (19), we have  
 $\Delta V_1(k) = \overline{x}^T(k+1)P\overline{x}(k+1) - \overline{x}^T(k)P\overline{x}(k)$ 

$$\Delta V_2(k) = x^T(k)H_1^T(Q_1 + Q_2 + Q_3)H_1x(k) - x^T(k - d_1^m)Q_1x(k - d_1^m) -x^T(k - d_1^M)Q_2x(k - d_1^M) - x^T(k - d_2^M)Q_3x(k - d_2^M)$$
(27)

(26)

$$\Delta V_{3}(k) = (d_{1}^{m})^{2} \delta^{T}(k) R_{1} \delta(k) - d_{1}^{m} \sum_{i=k-d_{1}^{m}}^{k-1} \delta^{T}(i) R_{1} \delta(i) + (d_{1}^{M} - d_{1}^{m}) \delta^{T}(k) R_{2} \delta(k) - \sum_{i=k-d_{1}^{M}}^{k-d_{1}^{m}-1} \delta^{T}(i) R_{2} \delta(i) + d_{2}^{M} \delta^{T}(k) R_{3} \delta(k) - \sum_{i=k-d_{2}^{M}}^{k-1} \delta^{T}(i) R_{3} \delta(i)$$
(28)

where P > 0,  $Q_i > 0$ , and  $R_i > 0$  (i = 1, 2, 3).

By using Jessen's inequality [18], we have

$$-d_{1}^{m}\sum_{i=k-d_{1}^{m}}^{k-1}\delta^{T}(i)R_{1}\delta(i) \leq \begin{bmatrix} \overline{x}(k) \\ x(k-d_{1}^{m}) \end{bmatrix}^{T} \begin{bmatrix} -H_{1}^{T}R_{1}H_{1} & H_{1}^{T}R_{1} \\ R_{1}H_{1} & -R_{1} \end{bmatrix} \begin{bmatrix} \overline{x}(k) \\ x(k-d_{1}^{m}) \end{bmatrix}$$
(29)

Employing the free-weighting matrices method [19,20] and combining Eqs. (25)–(28), we have

$$\Delta V(k) = \Delta V_1(k) + \Delta V_2(k) + \Delta V_3(k) + 2\zeta^T(k) N \left[ x(k-d_1^m) - x(k-d_1(k)) - \sum_{i=k-d_1(k)}^{k-d_1^m-1} \delta(i) \right] + 2\zeta^T(k) M \left[ x(k-d_1(k)) - x(k-d_1^M) - \sum_{i=k-d_1^M}^{k-d_1(k)-1} \delta(i) \right] + 2\zeta^T(k) S \left[ H_1 \overline{x}(k) - x(k-d_2(k)) - \sum_{i=k-d_2(k)}^{k-1} \delta(i) \right] + 2\zeta^T(k) T \left[ x(k-d_2(k)) - x(k-d_2^M)) - \sum_{i=k-d_2^M}^{k-d_2(k)-1} \delta(i) \right]$$
(30)

where

$$\zeta^{T}(k) = [\overline{x}^{T}(k) \ x^{T}(k-d_{1}^{m}) \ x^{T}(k-d_{1}(k)) \ x^{T}(k-d_{1}^{M}) \ x^{T}(k-d_{2}(k))$$
$$x^{T}(k-d_{2}^{M}) \ v^{T}(k) \ e_{i}^{T}(k) \ g^{T}(k,x(k))]$$

There exist  $R_2$  and  $R_3$ , such that

$$-2\zeta^{T}(k)N\sum_{i=k-d_{1}(k)}^{k-d_{1}^{m}-1}\delta(i) \leq (d_{1}(k)-d_{1}^{m})\zeta^{T}(k)NR_{2}^{-1}N^{T}\zeta(k) + \sum_{i=k-d_{1}(k)}^{k-d_{1}^{m}-1}\delta^{T}(i)R_{2}\delta(i)$$
(31)

$$-2\zeta^{T}(k)M\sum_{i=k-d_{1}^{M}}^{k-d_{1}(k)-1}\delta(i) \leq (d_{1}^{M}-d_{1}(k))\zeta^{T}(k)MR_{2}^{-1}M^{T}\zeta(k) + \sum_{i=k-d_{1}^{M}}^{k-d_{1}(k)-1}\delta^{T}(i)R_{2}\delta(i)$$
(32)

$$-2\zeta^{T}(k)S\sum_{i=k-d_{2}(k)}^{k-1}\delta(i) \leq d_{2}(k)\zeta^{T}(k)SR_{3}^{-1}S^{T}\zeta(k) + \sum_{i=k-d_{2}(k)}^{k-1}\delta^{T}(i)R_{3}\delta(i)$$
(33)

$$-2\zeta^{T}(k)T\sum_{i=k-d_{3}^{M}}^{k-d_{2}(k)-1}\delta(i) \leq (d_{2}^{M}-d_{2}(k))\zeta^{T}(k)TR_{3}^{-1}T^{T}\zeta(k) + \sum_{i=k-d_{2}^{M}}^{k-d_{2}(k)-1}\delta^{T}(i)R_{3}\delta(i)$$
(34)

Also, notice that  $x(k) = H_1 \overline{x}(k)$ , it follows from Eq. (2) that

$$\begin{bmatrix} \overline{x}(k) \\ g(k, x(k)) \end{bmatrix}^{T} \begin{bmatrix} -H_{1}^{T} \Omega_{1} H_{1} & \ast \\ -\Omega_{2} H_{1} & -I_{n} \end{bmatrix} \begin{bmatrix} \overline{x}(k) \\ g(k, x(k)) \end{bmatrix} \ge 0$$
(35)

Combining Eqs. (26)–(35) and the relation (4), we have

$$\begin{split} \Delta V(k) &-\gamma^2 v^T(k) v(k) + \overline{r}^T(k) \overline{r}(k) \\ \leq \overline{x}^T(k+1) P \overline{x}(k+1) - \overline{x}^T(k) P \overline{x}(k) + x^T(k) H_1^T(Q_1 + Q_2 + Q_3) H_1 x(k) \\ &-x^T(k-d_1^m) Q_1 x(k-d_1^m) - x^T(k-d_1^M) Q_2 x(k-d_1^M) - x^T(k-d_2^M) Q_3 x(k-d_2^M) \\ &+ (d_1^m)^2 \delta^T(k) R_1 \delta(k) + \begin{bmatrix} \overline{x}(k) \\ x(k-d_1^m) \end{bmatrix}^T \begin{bmatrix} -H_1^T R_1 H_1 & H_1^T R_1 \\ R_1 H_1 & -R_1 \end{bmatrix} \begin{bmatrix} \overline{x}(k) \\ x(k-d_1^m) \end{bmatrix} \\ &+ (d_1^M - d_1^m) \delta^T(k) R_2 \delta(k) + d_2^M \delta^T(k) R_3 \delta(k) + 2\zeta^T(k) N[x(k-d_1^m) - x(k-d_1(k))] \\ &+ 2\zeta^T(k) M[x(k-d_1(k)) - x(k-d_1^M)] + 2\zeta^T(k) S[H_1 \overline{x}(k) - x(k-d_2(k))] \\ &+ 2\zeta^T(k) T[x(k-d_2(k)) - x(k-d_2^M)] + (d_1(k) - d_1^m) \zeta^T(k) N R_2^{-1} N^T \zeta(k) \\ &+ (d_1^M - d_1(k)) \zeta^T(k) M R_2^{-1} M^T \zeta(k) + d_2(k) \zeta^T(k) S R_2^{-1} S^T \zeta(k) \\ &+ (d_2^M - d_2(k)) \zeta^T(k) T R_3^{-1} T^T \zeta(k) \\ &+ \left[ \overline{x}(k) \\ g(k, x(k)) \right]^T \begin{bmatrix} -H_1^T \Omega_1 H_1 & \ast \\ -\Omega_2 H_1 & -I_n \end{bmatrix} \begin{bmatrix} \overline{x}(k) \\ g(k, x(k)) \end{bmatrix} \\ &= \zeta^T(k) P \zeta(k) + \overline{x}^T(k+1) P \overline{x}(k+1) + (d_1^m)^2 \delta^T(k) R_1 \delta(k) + (d_1^M - d_1^m) \delta^T(k) R_2 \delta(k) \\ &+ d_2^M \delta^T(k) R_3 \delta(k) + \overline{r}^T(k) \overline{r}(k) \end{split}$$

where

$$\Psi = \begin{bmatrix} \Sigma_{11} & * & * \\ \Sigma_{21} & \Sigma_{22} & * \\ \Sigma_{31} & 0 & \Sigma_{33} \end{bmatrix} + (d_1(k) - d_1^m) N R_2^{-1} N^T + (d_1^M - d_1(k)) M R_2^{-1} M^T$$

$$+d_2(k)SR_2^{-1}S^T + (d_2^M - d_2(k))TR_3^{-1}T^T$$

Then, by using well-known Schur complement and Lemma 2, from Eq. (36), one can easily see that Eq. (22) with s = 1, 2, 3, 4 can lead  $\Delta V(k) - \gamma^2 v^T(k)v(k) + \overline{r}^T(k)\overline{r}(k) < 0$ . The remaining part of the proof is similar to those in [10,21] and so omitted here for simplicity. The proof is complete.  $\Box$ 

**Remark 4.** It is worth to point out that some other methods such as Monte Carlo Simulation Methods can be used to choose the threshold. Notice that the threshold should be adopted suitably. If the threshold is selected larger, some alarms may miss in the fault detection system; otherwise more false alarms may occur.

**Remark 5.** The possible conservatism in Theorem 1 is twofold: firstly, Jessen's inequality, convexity of the matrix function and the free-weighting matrices are used in our derivation, which will bring conservatism in the design procedure; secondly, we firstly introduce the event-triggered mechanism we proposed in Ref. [13] to fault detection filter in this paper. Also, it can be seen that the proposed mechanism is superior to some existing event-triggered mechanism in the literature by comparison (see [13]). From the above, we can see that our results can provide less conservation.

Based on analysis results in Theorem 1, sufficient conditions for the existence of the desired fault detection filters are provided in Theorem 2.

**Theorem 2.** Consider the nominal system (19), and let  $\gamma > 0$ ,  $d_1^m$ ,  $d_1^M$ ,  $d_2^M$  and  $\sigma > 0$  be given parameters. There exists a desired full-order fault detection filter in the form of Eq. (18) if there exist matrices  $P_1$ ,  $\overline{P}_3$ ,  $Q_i$ ,  $R_i$  (i = 1, 2, 3) and  $\overline{A}_F$ ,  $\overline{B}_F$ ,  $\overline{C}_F$ ,  $D_F$ ,  $\Omega > 0$ ,  $N_2$ ,  $N_3$ ,  $M_3$ ,  $M_4$ ,  $\overline{S}_{10}$ ,  $\overline{S}_{11}$ ,  $S_5$ ,  $T_5$  and  $T_6$  with appropriate dimensions such that

$$\begin{bmatrix} \Phi_{11} & * & * & * & * \\ \Phi_{21} & \Sigma_{22} & * & * & * \\ \Phi_{31} & 0 & \Sigma_{33} & * & * \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} & * \\ \Phi_{51}(s) & 0 & 0 & 0 & \Sigma_{55} \end{bmatrix} < 0, \quad s = 1, 2, 3, 4.$$

$$(37)$$

where

$$\Phi_{11} = \begin{bmatrix} \Pi & * \\ -\overline{P}_3 + \overline{S}_{11} & -\overline{P}_3 \end{bmatrix}, \quad \Pi = -P_1 + Q_1 + Q_2 + Q_3 - R_1 - \Omega_1 + \overline{S}_{10} + \overline{S}_{10}^T \\
\Phi_{21} = \begin{bmatrix} R_1 & 0 \\ 0 & 0 \\ 0 & 0 \\ S_5 - \overline{S}_{10} & -\overline{S}_{11} \\ 0 & 0 \end{bmatrix}, \quad \Phi_{31} = \begin{bmatrix} 0 \\ 0 \\ -\Omega_2 \end{bmatrix}$$

$$\Phi_{41} = \begin{bmatrix}
P_1A & \overline{A}_F \\
\overline{P}_3A & A_F \\
d_1^m R_1 A - d_1^m R_1 & 0 \\
\sqrt{d_1^M - d_1^m} R_2 A - \sqrt{d_1^M - d_1^m} R_2 & 0 \\
\sqrt{d_2^M} R_3 A - \sqrt{d_2^M} R_3 & 0 \\
0 & \overline{C}_F
\end{bmatrix}$$

$$\Phi_{51}(1) = \Phi_{51}(3) = \begin{bmatrix}
0 & P_1 A_{d_1} & 0 & \overline{B}_F C & 0 \\
0 & \overline{P}_3 A_{d_1} & 0 & \overline{B}_F C & 0 \\
0 & d_1^m R_1 R_1 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_2^M} R_3 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_2^M} R_3 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_2^M} R_3 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_2^M} R_3 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_1^M - d_1^m} R_2 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_1^M - d_1^m} R_2 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_1^M - d_1^m} R_2 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_1^M - d_1^m} R_2 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_1^M - d_1^m} R_2 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_1^M - d_1^m} R_2 A_{d_1} & 0 & 0 & 0 \\
0 & \sqrt{d_1^M - d_1^m} R_2 A_{d_1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & D_F C & 0
\end{bmatrix}$$

$$\Phi_{43} = \begin{bmatrix}
P_1 D_1 & P_1 G & \overline{B}_F & P_1 \\
\overline{P}_3 D_1 & \overline{P}_3 G & \overline{B}_F & \overline{P}_3 \\
d_1^m R_1 D_1 & d_1^m R_1 G & 0 & d_1^m R_1 \\
\sqrt{d_1^M - d_1^m} R_2 D_1 & \sqrt{d_1^M - d_1^m} R_2 G & 0 & \sqrt{d_1^M - d_1^m} R_2 \\
\sqrt{d_2^M} R_3 D_1 & \sqrt{d_2^M} R_3 G & 0 & \sqrt{d_2^M} R_3 \\
0 & -I & D_F & 0
\end{bmatrix}$$

Moreover, if the above conditions are feasible, the parameter matrices of the filter are given by

$$\begin{cases}
A_F = \overline{A}_F \overline{P}_3^{-1} \\
B_F = \overline{B}_F \\
C_F = \overline{C}_F \overline{P}_3^{-1} \\
D_F = \overline{D}_F
\end{cases}$$
(39)

**Proof.** Since  $\overline{P}_3 > 0$ , there exist  $P_2$  and  $P_3 > 0$  satisfying  $\overline{P}_3 = P_2^T P_3^{-1} P_2$ . Define

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}, \quad J = \begin{bmatrix} I & 0 \\ 0 & P_2^T P_3^{-1} \end{bmatrix}, \quad \Upsilon = diag\{J, \underbrace{I, \dots, I}_{8}, J, \underbrace{I, \dots, I}_{6}\}$$
(40)

By Schur complement, P > 0 is equivalent to  $P_1 - \overline{P}_3 = P_1 - P_2^T P_3^{-1} P_2 > 0$ .

(38)

Pre- and post-multiplying Eq. (22) by  $\Upsilon$  and  $\Upsilon^T$ , respectively, and define variables

$$\begin{cases} \overline{A}_F = \hat{A}_F \overline{P}_3^{-1}, \hat{A}_F = P_2^T A_F P_2^{-T} \\ \overline{B}_F = P_2^T B_F \\ \overline{C}_F = \hat{C}_F \overline{P}_3^{-1}, \hat{C}_F = C_F P_2^{-T} \\ \overline{D}_F = D_F \\ S_1^T J^T = [\overline{S}_{10}^T \ \overline{S}_{11}^T] \end{cases}$$

$$\tag{41}$$

Then, Eq. (22) for s = 1, 2, 3, 4 is equivalent to Eq. (37) for s = 1, 2, 3, 4, respectively.

Next, we will show that if Eq. (37) is solvable for  $\overline{A}_F$ ,  $\overline{B}_F$ ,  $\overline{C}_F$ ,  $\overline{D}_F$  and  $\overline{P}_3$ , then the parameter matrices of the fault detection filter (3) can be chosen as in Eq. (39).

Replacing  $(\overline{A}_F, \overline{B}_F, \overline{C}_F, \overline{D}_F)$  by  $(P_2^{-T} \hat{A}_F P_2^T, P_2^{-T} \overline{B}_F, \hat{C}_F P_2^T, \overline{D}_F)$  in Eq. (3) and then pre-and post-multiplying them with  $\Upsilon$  and  $\Upsilon^T$ , we can also obtain Eq. (39). Obviously,  $(P_2^{-T} \hat{A}_F P_2^T, P_2^{-T} \overline{B}_F, \hat{C}_F P_2^T, \overline{D}_F)$  can be chosen as the fault detection filter parameters. That is, the following fault detection filter:

$$\begin{cases} \tilde{x}(k+1) = P_2^{-T} \hat{A}_F P_2^T \tilde{x}(k) + P_2^{-T} \overline{B}_F \tilde{y}(k) \\ \tilde{r}(k) = \hat{C}_F P_2^T \tilde{x}(k) + \overline{D}_F \tilde{y}(k)) \end{cases}$$

$$\tag{42}$$

can guarantee that the fault detection filter system (19) is asymptotically stable with the  $H_{\infty}$  performance bound  $\gamma$ . Defining  $\hat{x}(k) = P_2^T \tilde{x}(k)$ , Eq. (42) becomes

$$\begin{cases} \hat{x}(k+1) = \hat{A}_F \hat{x}(k) + \overline{B}_F \tilde{y}(k) \\ r(k) = \hat{C}_F \hat{x}(k) + \overline{D}_F \tilde{y}(k) \end{cases}$$
(43)

Then, from Eqs. (41) and (43) we can obtain Eq. (39). This completes the proof.  $\Box$ 

**Remark 6.** In most cases, we can know that the issues of limited resource and insufficient communication bandwidth and the case of inadequate computation power are problems that often have to be dealt with. Therefore, it is necessary to build a mechanism which provides a useful way to determine when the sampling action is carried out. In this sense, we propose an event-triggered mechanism which only samples when some function of the system exceeds a threshold, thus it has the main advantage to reduce the burden of the network communication and the computation cost of the controller and the occupation of the sensor and actuator. Under the event-triggered mechanism, Theorem 2 provides us with a fault detection filter design method.

## 4. Simulation examples

Consider the discrete-time systems (1) with nonlinearities and time-delay, in which the system parameters are given as follows:

$$A = \begin{bmatrix} 0.1 & 0.1 \\ -0.3 & 0.1 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.2 & 0 \\ 0.7 & 0.1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0.2 \\ -0.1 \end{bmatrix}, \quad G = \begin{bmatrix} -1 \\ 2 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 1 \end{bmatrix}$$
(44)

Let the time-varying communication delays satisfy  $1 \le d_1(k) \le 3$ ,  $0 \le d_2(k) \le 2$  and assume  $\sigma = 0.1$ ,  $\Xi_1 = diag\{0.1, 0.2\}$ ,  $\Xi_2 = diag\{0.2, 0.1\}$ , and  $\gamma = 4$ . Then, by using Theorem 2, the

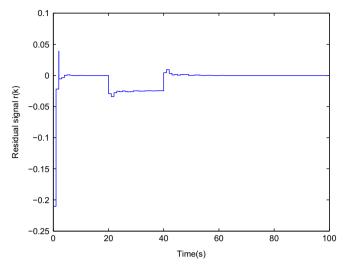


Fig. 3. Residual signal r(k).

event-based fault detection filter parameters can be obtained as follows:

$$A_F = \begin{bmatrix} 0.5256 & 1.3573 \\ -0.2788 & -0.3282 \end{bmatrix}, \quad B_F = \begin{bmatrix} -0.0026 \\ 0.0004 \end{bmatrix}, \\ C_F = \begin{bmatrix} 0.1400 & 0.8473 \end{bmatrix}, \quad D_F = -0.0019$$
(45)

and the minimal performance index of  $\gamma$  is  $\gamma_{opt} = 1.0399$ .

To illustrate the effectiveness of the designed event-based fault detection filter, assume that the fault signal f(k) is satisfied as

$$f(k) = \begin{cases} 5, & 20 \le k \le 40\\ 0 & \text{else} \end{cases}$$
(46)

for k = 0, 1, 2, ..., 100.

When w(k) = 0, choose the initial conditions to be  $x(0) = [0.5 - 0.5]^T$ ,  $\hat{x}(0) = [0.3 - 0.3]^T$ , chose the event-based fault detection filter parameters of Eq. (45), the residual evaluation function J(k) and residual signal r(k) are shown in Figs. 2 and 3, respectively. From Figs. 2 and 3, we can see that the fault can be easily detected. Select a threshold as  $J_{th} = \{sup_{f(k)} = 0 \sum_{h=0}^{k} r^T(h)r(h)\}^{1/2}$ , after 200 runs of the simulations, we can get an average value of  $J_{th} = 0.3325$ . From Fig. 2, we can see that  $0.3238 = J(23) < J_{th} < J(24) = 0.3577$ , which means that the fault can be detected in 4 time steps after its occurrence.

## 5. Conclusion

This paper has investigated the event-based fault detection for networked systems with communication delay and nonlinear perturbation. In order to reduce the communication load in the network and gear up its efficiency, we introduce a novel event-triggered scheme, which can determine when the sensor data is transmitted. Under this event-triggered scheme, the eventbased fault detection system has been developed into a networked control system with networkinduced delay. A fault detection filter has been designed such that the overall fault detection dynamics is exponentially stable in the mean square and the error between the residual signal and the fault signal is made as small as possible. Sufficient conditions for the existence of the desired fault detection have been derived. Then the explicit expression of the desired filter parameters has been obtained. A numerical example has been provided to show the usefulness and effectiveness of the proposed method.

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