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Asymptotic Stability of Markovian Jumping Genetic Regulatory Networks with Random Delays*

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Abstract — This paper investigates the asymptotic stability of genetic regulatory networks with random delays and Markovian jumping parameters. The delay considered here is assumed to be satisfying a certain stochastic characteristic. Corresponding to the probability of the delay taking value in different intervals, stochastic variables satisfying Bernoulli random binary distribution are introduced and a new system model is established by employing the information of the probability distribution. By using a Lyapunov functional approach and linear matrix inequality techniques, the stability criteria for the delayed Markovian jumping genetic regulatory networks are expressed as a set of Linear matrix inequalities (LMIs), which can be solved numerically by LMI toolbox in MATLAB. A genetic network example is given to verify the effectiveness and the applicability of the proposed approach.

Key words — Genetic regulatory network, Random delays, Asymptotic stability, Markovian jumping parameters.

I. Introduction

In the past decades, genetic regulatory networks have received more and more attention in the biological and biomedical sciences, many valuable results have been reported in Refs.[1–4], and the references therein. However, the gap between a complete genome sequence and its functional understanding with respect to an organism is still very large. Many questions about gene functions, express mechanism, and the global integration of individual mechanism still remain open[5]. Recently, by constructing the Genetic regulatory network (GRN) models from time-series, that is, modeling genetic networks as a dynamical system provides a powerful tool for studying gene regulation processes in living organisms, dynamics analysis of GRNs has attracted increasing attention recently, see for example, Refs.[6, 7] and the references therein.

Since time-delays exist in transcription and translation processes and the modeling error is unavoidable in practice, when analyzing the dynamics behaviors of GRNs, the time delay and modeling uncertainty must be taken into account. In addition, most of gene networks include some kinds of switching mechanisms. For example, a bistable system can switch from one steady state to the other by increasing stimulation or inhibition or by changing other regulatory mechanisms[8]. Therefore, Markov chains can be used as a generic framework for modeling gene networks. Recent studies on the dynamics of the so-called Markovian genetic regulatory networks are fruitful, and many important results have been reported in the Refs.[5, 8], and the references therein. It is suggested that Markov chain models incorporating rule-based transitions between state are capable of mimicking biological phenomena.

It is worth mentioning that, to have the accurate predictions, time delay should be considered in the biological systems or artificial genetic networks due to the slow processes of transcription, translation, and translocation or the finite switching speed of amplifiers, theoretical models without consideration delay may even provide wrong predictions. However, though there are many results on the dynamic analysis of Markovian swithching systems with time delays[9–12], there are still no results considering quantitatively describing the gene regulation by using the Markovian swithching system from system point of view. Moreover, to the best of authors' knowledge, few contributions have addressed such stability problems involving random or stochastic delays, which are the focus of this work.

II. Model and Preliminaries

In this paper, based on the structure of the genetic regulatory network present in Ref.[8], we consider a functional differential equation model described by

$$\begin{cases} \dot{m}(t) = -\boldsymbol{A}(r(t))m(t) + \boldsymbol{W}(r(t))g(p(t-\sigma(t))) \\ \dot{p}(t) = -\boldsymbol{C}(r(t))p(t) + \boldsymbol{D}(r(t))m(t-\tau(t)) \end{cases} \tag{1}$$

where $m(t)$, $p(t)$ are state vector, $g(\cdot)$ is a monotonically increasing function and satisfies the sector condition $g(a)(g(a)-$

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$ka) \leq 0$, $\boldsymbol{A}(r(t))$, $\boldsymbol{W}(r(t))$, $\boldsymbol{C}(r(t))$, $\boldsymbol{D}(r(t))$ are known constant matrices for a fixed system mode.

Let $r(t)$, $t \geq 0$, be a right-continuous Markov chain on the probability space taking values in a finite space $\mathbb{S} = \{1, 2, \cdots, N\}$ with generator $\prod = (\pi_{ij})_{N\times N}$ given by

$$P\{r(t+\delta t) = j \| r(t) = i\} = \begin{cases} (\pi_{ij} + \Delta\pi_{ij})\delta t + o(\delta t), & \text{if } i \neq j \\ 1 + (\pi_{ij} + \Delta\pi_{ij})\delta t + o(\delta t), & \text{if } i = j \end{cases}$$

where $\delta t > 0$, $\pi_{ij} \geq 0$ is the known transition rate from i to j, if $i = j$ where $\pi_{ij} = -\sum_{j\neq i}\pi_{ij}$, $i, j \in \mathbb{S}$.

Then, one can rewrite Markovian gene network of Eq.(1) as

$$\begin{cases} \dot{m}(t) = -\boldsymbol{A}_i m(t) + \boldsymbol{W}_i g(p(t-\sigma(t))) \\ \dot{p}(t) = -\boldsymbol{C}_i p(t) + \boldsymbol{D}_i m(t-\tau(t)) \end{cases} \tag{2}$$

Assumption 1 Considering the information of probability distribution of the time delays $\tau(t)$, $\sigma(t)$, for some given scalars τ_1 and σ_1, two sets of functions are defined

$$\tau_1(t) = \begin{cases} \tau(t), & \text{for } t \in \Omega_1 \\ 0, & \text{for } t \in \Omega_2 \end{cases}, \quad \tau_2(t) = \begin{cases} \tau(t), & \text{for } t \in \Omega_2 \\ 0, & \text{for } t \in \Omega_1 \end{cases}$$

$$\sigma_1(t) = \begin{cases} \sigma(t), & \text{for } t \in \Omega_3 \\ 0, & \text{for } t \in \Omega_4 \end{cases}, \quad \sigma_2(t) = \begin{cases} \sigma(t), & \text{for } t \in \Omega_4 \\ 0, & \text{for } t \in \Omega_3 \end{cases}$$

where $\Omega_1 = \{t : \tau(t) \in [\tau_m, \tau_1)\}$, $\Omega_2 = \{t : \tau(t) \in [\tau_1, \tau_M]\}$, $\Omega_3 = \{t : \sigma(t) \in [\sigma_m, \sigma_1)\}$ and $\Omega_4 = \{t : \sigma(t) \in [\sigma_1, \sigma_M]\}$.

From the definitions of the $\Omega_1, \Omega_2, \Omega_3$ and Ω_4, it can be seen that $t \in \Omega_1$ means that the event $\tau(t) \in [\tau_m, \tau_1)$ occurs, $t \in \Omega_2$ means that the event $\tau(t) \in [\tau_1, \tau_M]$ occurs, $t \in \Omega_3$ means that the event $\sigma(t) \in [\sigma_m, \sigma_1)$ occurs and $t \in \Omega_4$ means that the event $\sigma(t) \in [\sigma_1, \sigma_M]$ occurs.

Therefore, the stochastic variables $\alpha(t)$, $\beta(t)$ can be define as

$$\alpha(t) = \begin{cases} 1, & t \in \Omega_1 \\ 0, & t \in \Omega_2 \end{cases}, \quad \beta(t) = \begin{cases} 1, & t \in \Omega_3 \\ 0, & t \in \Omega_4 \end{cases} \tag{3}$$

Assumption 2 $\alpha(t)$, $\beta(t)$ are Bernoulli distributed sequences with

$$\begin{aligned} Prob\{\alpha(t) = 1\} &= \mathcal{E}\{\alpha(t)\} = \alpha_0 \\ Prob\{\alpha(t) = 0\} &= 1 - \mathcal{E}\{\alpha(t)\} = 1 - \alpha_0 \\ Prob\{\beta(t) = 1\} &= \mathcal{E}\{\beta(t)\} = \beta_0 \\ Prob\{\beta(t) = 0\} &= 1 - \mathcal{E}\{\beta(t)\} = 1 - \beta_0 \end{aligned}$$

where $0 \leq \alpha_0 \leq 1$, $0 \leq \beta_0 \leq 1$ are constants and $\mathcal{E}\{\alpha(t)\}$ and $\mathcal{E}\{\beta(t)\}$ are the expectation of $\alpha(t)$, $\beta(t)$ respectively.

Remark 1 From Assumption 2, we can see that

$$\begin{aligned} &\mathcal{E}\{\alpha(t)\} = \alpha_0, \quad \mathcal{E}\{(\alpha(t) - \alpha_0)^2\} = \alpha_0(1 - \alpha_0) \\ &\mathcal{E}\{\beta(t)\} = \beta_0, \quad \mathcal{E}\{(\beta(t) - \beta_0)^2\} = \beta_0(1 - \beta_0) \end{aligned}$$

By using Assumptions 1 and 2, the system of Eq.(2) can be rewritten as

$$\begin{cases} \dot{m}(t) = -\boldsymbol{A}_i m(t) + \beta(t)\boldsymbol{W}_i g(p(t-\sigma_1(t))) \\ \qquad\quad +(1-\beta(t))\boldsymbol{W}_i g(p(t-\sigma_2(t))) \\ \dot{p}(t) = -\boldsymbol{C}_i p(t) + \alpha(t)\boldsymbol{D}_i m(t-\tau_1(t)) \\ \qquad\quad +(1-\alpha(t))\boldsymbol{D}_i m(t-\tau_2(t)) \end{cases} \tag{4}$$

Remark 2 $\alpha(t)$ and $\beta(t)$ are introduced to describe the distribution information of the random delay. Since more delayed information has been employed, less conservative results can be expected to obtain. Furthermore, piecewise analysis method for delayed systems is widely used in Refs.[13, 14], also we could use the delay-partitioning approach to further reduce conservatism of the system analysis. For the sake of brevity and simplicity, we omit here this.

To obtain the main results, the following lemmas are needed.

Lemma 1[15] Suppose $\tau_m \leq \tau(t) \leq \tau_M$, and $x(t) \in \mathbb{R}^n$, for any positive matrix $\boldsymbol{R} \in \mathbb{R}^{n\times n}$

$$-(\tau_M - \tau_m)\int_{t-\tau_M}^{t-\tau_m} \dot{x}^T(s)\boldsymbol{R}\dot{x}(s)ds \leq \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -\boldsymbol{R} & \boldsymbol{R} \\ \boldsymbol{R} & -\boldsymbol{R} \end{bmatrix} \begin{bmatrix} x(t-\tau_m) \\ x(t-\tau_M) \end{bmatrix} \tag{5}$$

Lemma 2[16] Suppose $0 \leq \tau_m \leq \tau(t) \leq \tau_M$, $\boldsymbol{\Xi}_1, \boldsymbol{\Xi}_2$ and $\boldsymbol{\Omega}$ are constant matrices of appropriate dimensions, then

$$(\tau(t) - \tau_m)\boldsymbol{\Xi}_1 + (\tau_M - \tau(t))\boldsymbol{\Xi}_2 + \boldsymbol{\Omega} < 0 \tag{6}$$

if and only if the following inequalities hold

$$(\tau_M - \tau_m)\boldsymbol{\Xi}_1 + \boldsymbol{\Omega} < 0 \tag{7}$$

$$(\tau_M - \tau_m)\boldsymbol{\Xi}_2 + \boldsymbol{\Omega} < 0 \tag{8}$$

III. Main Results

In this section, by using convexity property of the matrix inequality and the Lyapunov stability theory, we analyze the stability for Markovian jumping gene regulatory networks of Eq.(4).

Theorem 1 System Eq.(4) is asymptotically stable for any given $0 \leq \tau_m \leq \tau(t) \leq \tau_M$, $0 \leq \sigma_m \leq \sigma(t) \leq \sigma_M$, τ_1, σ_1 and k, if there exist positive definite matrices $\boldsymbol{Q}_{1i} > 0$, $\boldsymbol{R}_{1i} > 0$ $(i \in \mathbb{S})$, $\boldsymbol{Q}_i > 0$, $\boldsymbol{R}_i > 0$ $(i = 2, 3, \cdots, 6)$, $\boldsymbol{\Lambda}_i = \mathrm{diag}(\lambda_{i1}, \lambda_{i2}, \cdots, \lambda_{in}) > 0$ $(i = 1, 2)$, $\boldsymbol{M}$, $\boldsymbol{N}$, $\boldsymbol{T}$, $\boldsymbol{S}$, $\boldsymbol{V}$, $\boldsymbol{W}$, $\boldsymbol{G}$ and $\boldsymbol{F}$ of appropriate dimensions such that the following LMIs hold for l, $s = 1, 2, 3, 4$:

$$\boldsymbol{\Xi}(l,s) = \begin{bmatrix} \boldsymbol{\Xi}_{11} + \overline{\boldsymbol{\Omega}} & * & * & * & * & * \\ \boldsymbol{\Xi}_{21} & \boldsymbol{\Xi}_{22} + \overline{\boldsymbol{\Gamma}} & * & * & * & * \\ \boldsymbol{\Xi}_{31} & \boldsymbol{\Xi}_{32} & \boldsymbol{\Xi}_{33} & * & * & * \\ \boldsymbol{\Xi}_{41} & \boldsymbol{\Xi}_{42} & \boldsymbol{\Xi}_{43} & \boldsymbol{\Xi}_{44} & * & * \\ \boldsymbol{\Xi}_{51}(l) & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Q}_6 & * \\ \boldsymbol{0} & \boldsymbol{\Xi}_{62}(s) & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{R}_6 \end{bmatrix} < 0 \tag{9}$$

where

$$\boldsymbol{\Xi}_{11} = \mathrm{diag}\{\boldsymbol{Y}_1, -\boldsymbol{Q}_2, \boldsymbol{0}, -\boldsymbol{Q}_3, \boldsymbol{0}, -\boldsymbol{Q}_4\}$$

$$\boldsymbol{\Xi}_{21} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \alpha_0\boldsymbol{R}_{1i}\boldsymbol{D}_i & \boldsymbol{0} & \alpha_{10}\boldsymbol{R}_{1i}\boldsymbol{D}_i & \boldsymbol{0} \\ \boldsymbol{0}_{5n\times n} & \boldsymbol{0}_{5n\times n} & \boldsymbol{0}_{5n\times n} & \boldsymbol{0}_{5n\times n} & \boldsymbol{0}_{5n\times n} & \boldsymbol{0}_{5n\times n} \end{bmatrix}$$

$$\boldsymbol{\Xi}_{22} = \mathrm{diag}\{Y_2, -\boldsymbol{R}_2, 0, -\boldsymbol{R}_3, 0, -\boldsymbol{R}_4\}$$

$$\boldsymbol{\Xi}_{31} = \begin{bmatrix} \beta_0\boldsymbol{W}_i^T\boldsymbol{Q}_{1i} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ (1-\beta_0)\boldsymbol{W}_i^T\boldsymbol{Q}_{1i} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{\Xi}_{32} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & k\boldsymbol{\Lambda}_1 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & k\boldsymbol{\Lambda}_2 & \boldsymbol{0} \end{bmatrix}$$

$$\boldsymbol{\Xi}_{33} = \mathrm{diag}\{-2\boldsymbol{\Lambda}_1, -2\boldsymbol{\Lambda}_2\}$$

$$\boldsymbol{\Xi}_{41}=\begin{bmatrix}-\varphi_1\boldsymbol{Q}_5\boldsymbol{A}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -\varphi_2\boldsymbol{Q}_5\boldsymbol{A}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -\varphi_3\boldsymbol{Q}_6\boldsymbol{A}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ -\varphi_4\boldsymbol{Q}_6\boldsymbol{A}_i & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \psi_1\boldsymbol{R}_5\boldsymbol{D}_i & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \psi_2\boldsymbol{R}_5\boldsymbol{D}_i & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \psi_3\boldsymbol{R}_6\boldsymbol{D}_i & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \psi_4\boldsymbol{R}_6\boldsymbol{D}_i & \mathbf{0}\end{bmatrix}$$

$$\boldsymbol{\Xi}_{42}=\begin{bmatrix}\mathbf{0}_{4n\times n} & \mathbf{0}_{4n\times 5n}\\ -\psi_1\boldsymbol{R}_5\boldsymbol{C}_i & \mathbf{0}_{n\times 5n}\\ -\psi_2\boldsymbol{R}_5\boldsymbol{C}_i & \mathbf{0}_{n\times 5n}\\ -\psi_3\boldsymbol{R}_6\boldsymbol{C}_i & \mathbf{0}_{n\times 5n}\\ -\psi_4\boldsymbol{R}_6\boldsymbol{C}_i & \mathbf{0}_{n\times 5n}\end{bmatrix},\ \boldsymbol{\Xi}_{43}=\begin{bmatrix}\varphi_1\boldsymbol{Q}_5\boldsymbol{W}_i & \mathbf{0}\\ \mathbf{0} & \varphi_2\boldsymbol{Q}_5\boldsymbol{W}_i\\ \varphi_3\boldsymbol{Q}_6\boldsymbol{W}_i & \mathbf{0}\\ \mathbf{0} & \varphi_4\boldsymbol{Q}_6\boldsymbol{W}_i\\ \mathbf{0}_{4n\times n} & \mathbf{0}_{4n\times n}\end{bmatrix}$$

$$\boldsymbol{\Xi}_{44}=\mathrm{diag}\{-\boldsymbol{Q}_5,-\boldsymbol{Q}_5,-\boldsymbol{Q}_6,-\boldsymbol{Q}_6,-\boldsymbol{R}_5,-\boldsymbol{R}_5,-\boldsymbol{R}_6,-\boldsymbol{R}_6\}$$

$\varphi_1=\sqrt{\beta_0\delta_{10}}$, $\varphi_2=\sqrt{\beta_{10}\delta_{10}}$, $\varphi_3=\sqrt{\beta_0\delta_{11}}$, $\varphi_4=\sqrt{\beta_{10}\delta_{11}}$
$\psi_1=\sqrt{\alpha_0\delta_{20}}$, $\psi_2=\sqrt{\alpha_{10}\delta_{20}}$, $\psi_3=\sqrt{\alpha_0\delta_{21}}$, $\psi_4=\sqrt{\alpha_{10}\delta_{21}}$

$$\boldsymbol{Y}_1=-\boldsymbol{Q}_{1i}\boldsymbol{A}_i-\boldsymbol{A}_i^T\boldsymbol{Q}_{1i}+\boldsymbol{Q}_2+\boldsymbol{Q}_3+\boldsymbol{Q}_4+\sum_{j=1}^N\pi_{ij}\boldsymbol{Q}_{1j}$$
$$\boldsymbol{Y}_2=-\boldsymbol{R}_{1i}\boldsymbol{C}_i-\boldsymbol{C}_i^T\boldsymbol{R}_{1i}+\boldsymbol{R}_2+\boldsymbol{R}_3+\boldsymbol{R}_4+\sum_{j=1}^N\pi_{ij}\boldsymbol{R}_{1j}$$
$$\overline{\boldsymbol{\Omega}}=\boldsymbol{\Omega}+\boldsymbol{\Omega}^T$$
$$\boldsymbol{\Omega}=[\mathbf{0}\quad \boldsymbol{M}\quad -\boldsymbol{M}+\boldsymbol{N}\quad -\boldsymbol{N}+\boldsymbol{T}\quad -\boldsymbol{T}+\boldsymbol{S}\quad -\boldsymbol{S}]$$
$$\overline{\boldsymbol{\Gamma}}=\boldsymbol{\Gamma}+\boldsymbol{\Gamma}^T$$
$$\boldsymbol{\Gamma}=[\mathbf{0}\quad \boldsymbol{V}\quad -\boldsymbol{V}+\boldsymbol{W}\quad -\boldsymbol{W}+\boldsymbol{G}\quad -\boldsymbol{G}+\boldsymbol{F}\quad -\boldsymbol{F}]$$
$$\boldsymbol{\Xi}_{51}(1)=\begin{bmatrix}\sqrt{\delta_{10}}\boldsymbol{M}^T\\ \sqrt{\delta_{11}}\boldsymbol{T}^T\end{bmatrix},\quad \boldsymbol{\Xi}_{51}(2)=\begin{bmatrix}\sqrt{\delta_{10}}\boldsymbol{M}^T\\ \sqrt{\delta_{11}}\boldsymbol{S}^T\end{bmatrix}$$
$$\boldsymbol{\Xi}_{51}(3)=\begin{bmatrix}\sqrt{\delta_{10}}\boldsymbol{N}^T\\ \sqrt{\delta_{11}}\boldsymbol{T}^T\end{bmatrix},\quad \boldsymbol{\Xi}_{51}(4)=\begin{bmatrix}\sqrt{\delta_{10}}\boldsymbol{N}^T\\ \sqrt{\delta_{11}}\boldsymbol{S}^T\end{bmatrix}$$
$$\boldsymbol{\Xi}_{62}(1)=\begin{bmatrix}\sqrt{\delta_{20}}\boldsymbol{V}^T\\ \sqrt{\delta_{21}}\boldsymbol{G}^T\end{bmatrix},\quad \boldsymbol{\Xi}_{62}(2)=\begin{bmatrix}\sqrt{\delta_{20}}\boldsymbol{V}^T\\ \sqrt{\delta_{21}}\boldsymbol{F}^T\end{bmatrix}$$
$$\boldsymbol{\Xi}_{62}(3)=\begin{bmatrix}\sqrt{\delta_{20}}\boldsymbol{W}^T\\ \sqrt{\delta_{21}}\boldsymbol{G}^T\end{bmatrix},\quad \boldsymbol{\Xi}_{62}(4)=\begin{bmatrix}\sqrt{\delta_{20}}\boldsymbol{W}^T\\ \sqrt{\delta_{21}}\boldsymbol{F}^T\end{bmatrix}$$

$\delta_1=\tau_M-\tau_m$, $\delta_{10}=\tau_1-\tau_m$, $\delta_{11}=\tau_M-\tau_1$, $\delta_2=\sigma_M-\sigma_m$, $\delta_{20}=\sigma_1-\sigma_m$, $\delta_{21}=\sigma_M-\sigma_1$, $\alpha_{10}=1-\alpha_0$, $\beta_{10}=1-\beta_0$.

In Eq.(9) "*" denotes the entries implied by symmetry.

Proof Construct a Lyapunov-Krasovskii candidate as

$$V(t)=V_1(t)+V_2(t)+V_3(t) \tag{10}$$

where

$$\begin{aligned}V_1(t)=&m^T(t)\boldsymbol{Q}_1(r(t))m(t)+p^T(t)\boldsymbol{R}_1(r(t))p(t)\\ V_2(t)=&\int_{t-\tau_m}^t m^T(s)\boldsymbol{Q}_2m(s)ds+\int_{t-\tau_1}^t m^T(s)\boldsymbol{Q}_3m(s)ds\\ &+\int_{t-\tau_M}^t m^T(s)\boldsymbol{Q}_4m(s)ds+\int_{t-\sigma_m}^t p^T(s)\boldsymbol{R}_2p(s)ds\\ &+\int_{t-\sigma_1}^t p^T(s)\boldsymbol{R}_3p(s)ds+\int_{t-\sigma_M}^t p^T(s)\boldsymbol{R}_4p(s)ds\\ V_3(t)=&\int_{t-\tau_1}^{t-\tau_m}\int_s^t \dot m^T(v)\boldsymbol{Q}_5\dot m(v)dvds\\ &+\int_{t-\tau_M}^{t-\tau_1}\int_s^t \dot m^T(v)\boldsymbol{Q}_6\dot m(v)dvds\\ &+\int_{t-\sigma_1}^{t-\sigma_m}\int_s^t \dot p^T(v)\boldsymbol{R}_5\dot p(v)dvds\\ &+\int_{t-\sigma_M}^{t-\sigma_1}\int_s^t \dot p^T(v)\boldsymbol{R}_6\dot p(v)dvds\end{aligned}$$

The infinitesimal operator $\mathcal{L}$ of $V(t)$ is defined as follows[17]:

$$\mathcal{L}V(t)=\lim_{\Delta\to 0^+}\frac{1}{\Delta}\{\mathcal{E}(V(x_{t+\Delta})|x_t)-V(x_t)\} \tag{11}$$

Then, using the infinitesimal operator for $V(t)$ in Eq.(10) and taking expectation on it, we have

$$\begin{aligned}\mathcal{E}\{\mathcal{L}V(t)\}=&2m^T(t)\boldsymbol{Q}_{1i}[-\boldsymbol{A}_im(t)+\beta_0g(p(t-\tau_1(t)))\\ &+\beta_{10}\boldsymbol{W}_ig(p(t-\tau_2(t)))]+\sum_{j=1}^N\pi_{ij}m^T(t)\boldsymbol{Q}_{1j}m(t)\\ &+2p^T(t)\boldsymbol{R}_{1i}[-\boldsymbol{C}_ip(t)+\alpha_0\boldsymbol{D}_im(t-\tau_1(t))\\ &+\alpha_{10}\boldsymbol{D}_im(t-\tau_2(t))]+\sum_{j=1}^N\pi_{ij}p^T(t)\boldsymbol{R}_{1j}p(t)\\ &+m^T(t)(\boldsymbol{Q}_2+\boldsymbol{Q}_3+\boldsymbol{Q}_4)m(t)\\ &+p^T(t)(\boldsymbol{R}_2+\boldsymbol{R}_3+\boldsymbol{R}_4)p(t)\\ &-m^T(t-\tau_m)\boldsymbol{Q}_2m(t-\tau_m)\\ &-m^T(t-\tau_M)\boldsymbol{Q}_4m(t-\tau_M)\\ &-p^T(t-\sigma_m)\boldsymbol{R}_2p(t-\sigma_m)\\ &-p^T(t-\sigma_1)\boldsymbol{R}_3p(t-\sigma_1)\\ &-p^T(t-\sigma_M)\boldsymbol{R}_4p(t-\sigma_M)\\ &-m^T(t-\tau_1)\boldsymbol{Q}_3m(t-\tau_1)\\ &+\dot m^T(t)(\delta_{10}\boldsymbol{Q}_5+\delta_{11}\boldsymbol{Q}_6)\dot m(t)\\ &+\dot p^T(t)(\delta_{20}\boldsymbol{R}_5+\delta_{21}\boldsymbol{R}_6)\dot p(t)\\ &-\int_{t-\tau_1}^{t-\tau_m}\dot m^T(s)\boldsymbol{Q}_5\dot m(s)ds\\ &-\int_{t-\tau_M}^{t-\tau_1}\dot m^T(s)\boldsymbol{Q}_6\dot m(s)ds\\ &-\int_{t-\sigma_1}^{t-\sigma_m}\dot p^T(s)\boldsymbol{R}_5\dot p(s)ds\\ &-\int_{t-\sigma_M}^{t-\sigma_1}\dot p^T(s)\boldsymbol{R}_6\dot p(s)ds\end{aligned} \tag{12}$$

By introducing the slack matrix method, we have

$$2\xi_1^T(t)\boldsymbol{M}\left[m(t-\tau_m)-m(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t-\tau_m}\dot m^T(s)ds\right]=0 \tag{13}$$

$$2\xi_1^T(t)\boldsymbol{N}\left[m(t-\tau_1(t))-m(t-\tau_1)-\int_{t-\tau_1}^{t-\tau_1(t)}\dot m^T(s)ds\right]=0 \tag{14}$$

$$2\xi_1^T(t)\boldsymbol{T}\left[m(t-\tau_1)-m(t-\tau_2(t))-\int_{t-\tau_2(t)}^{t-\tau_M}\dot m^T(s)ds\right]=0 \tag{15}$$

$$2\xi_1^T(t)\boldsymbol{S}\left[m(t-\tau_2(t))-m(t-\tau_M)-\int_{t-\tau_M}^{t-\tau_2(t)}\dot m^T(s)ds\right]=0 \tag{16}$$

$$2\xi_2^T(t)\boldsymbol{V}\left[p(t-\sigma_m)-p(t-\sigma_1(t))-\int_{t-\sigma_1(t)}^{t-\sigma_m}\dot p(s)ds\right]=0 \tag{17}$$

$$2\xi_2^T(t)\boldsymbol{W}\left[p(t-\sigma_1(t))-p(1-\sigma_1)-\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot p(s)ds\right]=0 \tag{18}$$

$$2\xi_2^T(t)\boldsymbol{G}\left[p(t-\sigma_1)-p(t-\sigma_2(t))-\int_{t-\sigma_2(t)}^{t-\sigma_1}\dot{p}(s)ds\right]=0 \tag{19}$$

$$2\xi_2^T(t)\boldsymbol{F}\left[p(t-\sigma_2(t))-p(t-\sigma_M)-\int_{t-\sigma_M}^{t-\sigma_2(t)}\dot{p}(s)ds\right]=0 \tag{20}$$

where

$$\begin{aligned}\xi_1^T(t)=&[\,m^T(t)\quad m^T(t-\tau_m)\quad m^T(t-\tau_1(t))\\&m^T(t-\tau_1)\quad m^T(t-\tau_2(t))\quad m^T(t-\tau_M)\,]\\\xi_2^T(t)=&[\,p^T(t)\quad p^T(t-\sigma_m)\quad p^T(t-\sigma_1(t))\\&p^T(t-\sigma_1)\quad p^T(t-\sigma_2(t))\quad p^T(t-\sigma_M)\,]\\\boldsymbol{M}^T=&[\,\boldsymbol{M}_1^T\quad \boldsymbol{M}_2^T\quad \boldsymbol{M}_3^T\quad \boldsymbol{M}_4^T\quad \boldsymbol{M}_5^T\quad \boldsymbol{M}_6^T\,]\\\boldsymbol{N}^T=&[\,\boldsymbol{N}_1^T\quad \boldsymbol{N}_2^T\quad \boldsymbol{N}_3^T\quad \boldsymbol{N}_4^T\quad \boldsymbol{N}_5^T\quad \boldsymbol{N}_6^T\,]\\\boldsymbol{T}^T=&[\,\boldsymbol{T}_1^T\quad \boldsymbol{T}_2^T\quad \boldsymbol{T}_3^T\quad \boldsymbol{T}_4^T\quad \boldsymbol{T}_5^T\quad \boldsymbol{T}_6^T\,]\\\boldsymbol{S}^T=&[\,\boldsymbol{S}_1^T\quad \boldsymbol{S}_2^T\quad \boldsymbol{S}_3^T\quad \boldsymbol{S}_4^T\quad \boldsymbol{S}_5^T\quad \boldsymbol{S}_6^T\,]\\\boldsymbol{V}^T=&[\,\boldsymbol{V}_1^T\quad \boldsymbol{V}_2^T\quad \boldsymbol{V}_3^T\quad \boldsymbol{V}_4^T\quad \boldsymbol{V}_5^T\quad \boldsymbol{V}_6^T\,]\\\boldsymbol{W}^T=&[\,\boldsymbol{W}_1^T\quad \boldsymbol{W}_2^T\quad \boldsymbol{W}_3^T\quad \boldsymbol{W}_4^T\quad \boldsymbol{W}_5^T\quad \boldsymbol{W}_6^T\,]\\\boldsymbol{G}^T=&[\,\boldsymbol{W}_1^T\quad \boldsymbol{W}_2^T\quad \boldsymbol{W}_3^T\quad \boldsymbol{W}_4^T\quad \boldsymbol{W}_5^T\quad \boldsymbol{W}_6^T\,]\\\boldsymbol{F}^T=&[\,\boldsymbol{F}_1^T\quad \boldsymbol{F}_2^T\quad \boldsymbol{F}_3^T\quad \boldsymbol{F}_4^T\quad \boldsymbol{F}_5^T\quad \boldsymbol{F}_6^T\,]\end{aligned}$$

It can be shown that

$$\begin{aligned}\mathcal{E}\{\mathcal{L}[\dot{m}^T(t)\boldsymbol{Q}\dot{m}(t)]\}=&\beta_0[-\boldsymbol{A}_im(t)+\boldsymbol{W}_ig(p(t-\sigma_1(t)))]^T\\&\cdot\boldsymbol{Q}[-\boldsymbol{A}_im(t)+\boldsymbol{W}_ig(p(t-\sigma_1(t)))]\\&+\beta_{10}[-\boldsymbol{A}_im(t)+\boldsymbol{W}_ig(p(t-\sigma_2(t)))]^T\\&\cdot\boldsymbol{Q}[-\boldsymbol{A}_im(t)+\boldsymbol{W}_ig(p(t-\sigma_2(t)))]\end{aligned} \tag{21}$$

$$\begin{aligned}\mathcal{E}\{\mathcal{L}[\dot{p}(t)\boldsymbol{R}\dot{p}(t)]\}=&\alpha_0[-\boldsymbol{C}_ip(t)+\boldsymbol{D}_im(t-\tau_1(t))]^T\\&\cdot\boldsymbol{R}[-\boldsymbol{C}_ip(t)+\boldsymbol{D}_im(t-\tau_1(t))]\\&+\alpha_{10}[-\boldsymbol{C}_ip(t)+\boldsymbol{D}_im(t-\tau_2(t))]^T\\&\cdot\boldsymbol{R}[-\boldsymbol{C}_ip(t)+\boldsymbol{D}_im(t-\tau_2(t))]\end{aligned} \tag{22}$$

where $\boldsymbol{Q}=\delta_{10}\boldsymbol{Q}_5+\delta_{11}\boldsymbol{Q}_6$, $\boldsymbol{R}=\delta_{20}\boldsymbol{R}_5+\delta_{21}\boldsymbol{R}_6$.

Note that

$$\begin{aligned}&-2\xi_1^T(t)\boldsymbol{M}\int_{t-\tau_1(t)}^{t-\tau_m}\dot{m}(s)ds\\&\le(\tau_1(t)-\tau_m)\xi_1^T(t)\boldsymbol{M}\boldsymbol{Q}_5^{-1}\boldsymbol{M}^T\xi_1(t)+\int_{t-\tau_1(t)}^{t-\tau_m}\dot{m}^T(s)\boldsymbol{Q}_t\dot{m}(s)ds\end{aligned} \tag{23}$$

$$\begin{aligned}&-2\xi_1^T(t)\boldsymbol{N}\int_{t-\tau_1}^{t-\tau_1(t)}\dot{m}(s)ds\\&\le(\tau_1-\tau_1(t))\xi_1^T(t)\boldsymbol{N}\boldsymbol{Q}_5^{-1}\boldsymbol{N}^T\xi_1(t)+\int_{t-\tau_1}^{t-\tau_1(t)}\dot{m}^T(s)\boldsymbol{Q}_5\dot{m}(s)ds\end{aligned} \tag{24}$$

$$\begin{aligned}&-2\xi_1^T(t)\boldsymbol{T}\int_{t-\tau_2(t)}^{t-\tau_1}\dot{m}(s)ds\\&\le(\tau_2(t)-\tau_1)\xi_1^T(t)\boldsymbol{T}\boldsymbol{Q}_6^{-1}\boldsymbol{T}^T\xi_1(t)+\int_{t-\tau_2(t)}^{t-\tau_1}\dot{m}^T(s)\boldsymbol{Q}_6\dot{m}(s)ds\end{aligned} \tag{25}$$

$$\begin{aligned}&-2\xi_1^T(t)\boldsymbol{S}\int_{t-\tau_M}^{t-\tau_2(t)}\dot{m}(s)ds\\&\le(\tau_M-\tau_2(t))\xi_1^T(t)\boldsymbol{S}\boldsymbol{Q}_6^{-1}\boldsymbol{S}^T\xi_1(t)+\int_{t-\tau_M}^{t-\tau_2(t)}\dot{m}^T(s)\boldsymbol{Q}_6\dot{m}(s)ds\end{aligned} \tag{26}$$

$$\begin{aligned}&-2\xi_2^T(t)\boldsymbol{V}\int_{t-\sigma_1(t)}^{t-\sigma_m}\dot{p}(s)ds\\&\le(\sigma_1(t)-\sigma_m)\xi_2^T(t)\boldsymbol{V}\boldsymbol{R}_5^{-1}\boldsymbol{V}^T\xi_2(t)+\int_{t-\sigma_1(t)}^{t-\sigma_m}\dot{p}^T(s)\boldsymbol{R}_5\dot{p}(s)ds\end{aligned} \tag{27}$$

$$\begin{aligned}&-2\xi_2^T(t)\boldsymbol{W}\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot{p}(s)ds\\&\le(\sigma_1-\sigma_1(t))\xi_2^T(t)\boldsymbol{W}\boldsymbol{R}_5^{-1}\boldsymbol{W}^T\xi_2(t)+\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot{p}^T(s)\boldsymbol{R}_5\dot{p}(s)ds\end{aligned} \tag{28}$$

$$\begin{aligned}&-2\xi_2^T(t)\boldsymbol{G}\int_{t-\sigma_2(t)}^{t-\sigma_1}\dot{p}(s)ds\\&\le(\sigma_2(t)-\sigma_1)\xi_2^T(t)\boldsymbol{G}\boldsymbol{R}_6^{-1}\boldsymbol{G}^T\xi_2(t)+\int_{t-\sigma_2(t)}^{t-\sigma_1}\dot{p}^T(s)\boldsymbol{R}_6\dot{p}(s)ds\end{aligned} \tag{29}$$

$$\begin{aligned}&-2\xi_2^T(t)\boldsymbol{F}\int_{t-\sigma_M}^{t-\sigma_2(t)}\dot{p}(s)ds\\&\le(\sigma_M-\sigma_2(t))\xi_2^T(t)\boldsymbol{F}\boldsymbol{R}_6^{-1}\boldsymbol{F}^T\xi_2(t)+\int_{t-\sigma_M}^{t-\sigma_2(t)}\dot{p}^T(s)\boldsymbol{R}_6\dot{p}(s)ds\end{aligned} \tag{30}$$

Noting the sector condition, for any $\lambda_{ij}>0$ ($i=1,2$, $j=1,2,\cdots,n$), we have

$$\begin{aligned}&\left\{-2\sum_{j=1}^n\lambda_{ij}g(p_j(t-\sigma_i(t)))[g(p_j(t-\sigma_i(t)))-kp_j(t-\sigma_i(t))]\right\}\\&\quad\ge 0\end{aligned} \tag{31}$$

Rewriting above inequalities into compact matrix form, we obtain

$$\begin{aligned}&\Big\{-2g^T(p(t-\sigma_i(t)))\boldsymbol{\Lambda}_ig(p(t-\sigma_i(t)))\\&\quad+2kg^T(p(t-\sigma_i(t)))\boldsymbol{\Lambda}_ip(t-\sigma_i(t))\Big\}\ge 0\end{aligned} \tag{32}$$

where $\boldsymbol{\Lambda}_i=\mathrm{diag}(\lambda_{i1},\lambda_{i2},\cdots,\lambda_{in})>0$

From Eqs.(12–32), we can get that

$$\begin{aligned}\mathcal{E}\{\mathcal{L}V(t)\}\le&\xi^T(t)\begin{bmatrix}\boldsymbol{\Xi}_{11}+\overline{\boldsymbol{\Omega}} & * & * & *\\ \boldsymbol{\Xi}_{21} & \boldsymbol{\Xi}_{22}+\overline{\boldsymbol{\Gamma}} & * & *\\ \boldsymbol{\Xi}_{31} & \boldsymbol{\Xi}_{32} & \boldsymbol{\Xi}_{33} & *\\ \boldsymbol{\Xi}_{41} & \boldsymbol{\Xi}_{42} & \boldsymbol{\Xi}_{43} & \boldsymbol{\Xi}_{44}\end{bmatrix}\xi(t)\\&+(\tau_1(t)-\tau_m)\xi_1^T(t)\boldsymbol{M}\boldsymbol{Q}_5^{-1}\boldsymbol{M}^T\xi_1(t)\\&+(\tau_1-\tau_1(t))\xi_1^T(t)\boldsymbol{N}\boldsymbol{Q}_5^{-1}\boldsymbol{N}^T\xi_1(t)\\&+(\tau_2(t)-\tau_1)\xi_1^T(t)\boldsymbol{T}\boldsymbol{Q}_6^{-1}\boldsymbol{T}^T\xi_1(t)\\&+(\tau_M-\tau_2(t))\xi_1^T(t)\boldsymbol{S}\boldsymbol{Q}_6^{-1}\boldsymbol{S}^T\xi_1(t)\\&+(\sigma_1(t)-\sigma_m)\xi_2^T(t)\boldsymbol{V}\boldsymbol{R}_5^{-1}\boldsymbol{V}^T\xi_2(t)\\&+(\sigma_1-\sigma_1(t))\xi_2^T(t)\boldsymbol{W}\boldsymbol{R}_5^{-1}\boldsymbol{W}^T\xi_2(t)\\&+(\sigma_2(t)-\sigma_1)\xi_2^T(t)\boldsymbol{G}\boldsymbol{R}_6^{-1}\boldsymbol{G}^T\xi_2(t)\\&+(\sigma_M-\sigma_2(t))\xi_2^T(t)\boldsymbol{F}\boldsymbol{R}_6^{-1}\boldsymbol{F}^T\xi_2(t)\end{aligned}$$

where $\xi^T(t)=[\,\xi_1^T(t)\quad \xi_2^T(t)\quad g^T(p(t-\sigma_1(t)))\quad g^T(p(t-\sigma_2(t)))\,]$.

Then, using Lemma 2 and Schur complement, it is easy to see that Eq.(9) with l, $s=1,2,3,4$ can lead $\mathcal{E}\{\mathcal{L}V(t)\}\le 0$. Then, by Lyapunov stability theory, the system of Eq.(4) is asymptotically stable, which completes the proof.

IV. Example

In this section, we will present an example to illustrate our theoretical results.

Example 1 Consider the following uncertain Markovian genetic regulatory networks of Eq.(4) with two modes as:

$$\boldsymbol{A}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{W}_1 = \begin{bmatrix} 1 & -2 \\ 0.8 & 0 \end{bmatrix}$$
$$\boldsymbol{C}_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \boldsymbol{D}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$\boldsymbol{A}_2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}, \quad \boldsymbol{W}_2 = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$$
$$\boldsymbol{C}_2 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad \boldsymbol{D}_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

The regulation function in this example is taken as $g(x) = x^2/(1+x^2)$, one can get $k = 0.65$. The time-varying delays are chosen as $\tau_m = 0.2$, $\sigma(t) = 0.3 + 0.2\sin(t)$, $\tau_1 = 0.5$, $\sigma_1 = 0.3$. The transmission probability is assumed to be $\prod = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}$.

According to Theorem 1, by using the MATLAB LMI Toolbox, we can easily obtain feasible solutions of the LMIs (9). Thus, the network is globally asymptotic stable under the allowable maximum delay of $\tau_M = 3.8525$, when $\tau_m = 0.20$, $\alpha_0 = 0.2$, $\beta_0 = 0.7$.

With the given initial conditions $m(0) = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix}$, $p(0) = \begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix}$, the computational simulation results of trajectories $p(t)$ and $m(t)$ are shown in Figs.1 and 2, when $\alpha_0 = 0.2$, $\beta_0 = 0.7$, $\tau_m = 0.1$, $\tau_M = 3.8525$.

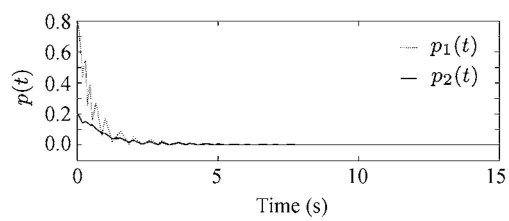

Fig. 1. Transient response of $p_i(t)$ $(i = 1, 2)$

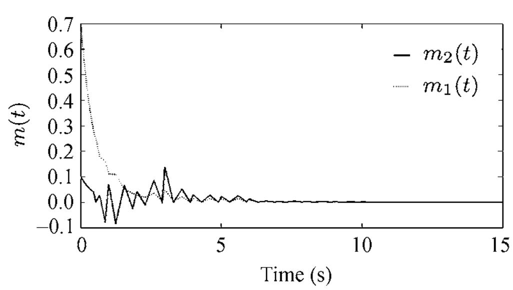

Fig. 2. Transient response of $m_i(t)$ $(i = 1, 2)$

To show the merit of proposed method, we set $N = 1$, the upper bound of the delay for different α_0, β_0 are listed in Table 1 when $\alpha_0 = 0.8$, $\tau_1 = 0.5$, $\sigma_1 = 0.3$ and $\sigma(t) = 0.3 + 0.2\sin(t)$.

From Table 1, we can clearly see that our results are significantly better than those in Ref.[4].

Table 1. Allowable upper bound of τ_M

τ_m	0.10	0.20	0.30	0.40
Ref.[4]	3.19	3.28	3.38	3.48
$\beta_0 = 0.7$	3.74	3.85	3.96	4.05
$\beta_0 = 0.9$	3.80	3.91	4.02	4.11

V. Conclusions

In this paper, we have studied the asymptotical stability of the proposed Markovian jumping genetic networks with time-varying delays when the information of the probability distributions of the delay is known a priori. To analyze the robust asymptotical stability of the propoded genetic networks system, the convexity of the matrix function technique is used. Based on the free-weighting matrix method and the LMI techniques, stability conditions have been developed in terms of LMIs. An example with simulation results have been carried out to demonstrate the effectiveness of the proposed method.

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