

# Event-Based Reliable $H_\infty$ Control for Networked Control System with Probabilistic Actuator Faults\*

LIU Jinliang

(Department of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing 210046, China)

**Abstract** — This paper investigates the reliable controller design for networked control system with probabilistic actuator faults under event-triggered scheme. The key idea is that only the newly states violating specified triggering condition will be transmitted to the controller. Considering the effect of the network transmission delay, event-triggered scheme and probabilistic actuator faults with different failure rates, a new actuator fault model is proposed. Criteria for the exponential stability and criteria for co-designing both the feedback and the trigger parameters are derived by using Lyapunov functional. These criteria are obtained in the form of linear matrix inequalities. A simulation example is employed to show the effectiveness of the proposed method.

**Key words** — Networked control system, Probabilistic actuator fault, Event-triggered scheme, Reliable control.

## I. Introduction

Networked control systems (NCSs) have received much attention in recent years. It is widely used in many practical applications, for example, automobiles, aircraft and manufacturing plants. The advantages of applying NCSs include simplicity, scalability and cost-effectiveness. However, the insertion of network are inherently prone to induce multiple channel transmission, time delay, packet dropout and so on. In this case, it is necessary to offer a unified approach to improve both the control and communication performances within NCSs. The temporary measurements failure and probabilistic distortion is usually unavoidable for variety of reasons, for example, networked delay, sensor/actuators aging, electromagnetic interference, zero shift, which may lead to intolerable system performance<sup>[1]</sup>. Therefore, from a safety as well as performance point of view, it is required to design a reliable controller that can tolerate actuators failures as well as networked delay. In recent, the fault model has received a lot of interest and lots of outstanding results have been obtained<sup>[2,3]</sup>. On the other hand, it is an important problem about how to reduce communication requirements. Many researches have proposed different methods to deal with this problem.

Recently, event-triggered scheme for control design has re-

ceived considerable attention and many important results have been reported<sup>[4-7]</sup>. Event-triggering method advocating the use of actuation only when some function of the system state exceeds a threshold, provides a useful way of determining when the sampling action is carried out. More specifically, a method for design or implication of controllers in the event-triggered form based on dissipation inequalities were proposed for both linear and nonlinear systems in Ref.[4]. The authors<sup>[5]</sup> studied event design in event-triggered feedback systems and a novel event-triggering scheme was presented to ensure exponential stability of the resulting sampled-data system. The authors<sup>[6]</sup> concerned with the control design problem of event-triggered networked systems with both state and control input quantizations. In Ref.[7], the authors studied the problem of event-based  $H_\infty$  filtering for networked systems with communication delay.

Up to now, to the best of authors knowledge, there are no papers to deal with the event-based reliable  $H_\infty$  control for networked control system with probabilistic actuator faults, which still remains as a challenging problem. In this paper, the event-based reliable  $H_\infty$  control for NCSs is investigated. The actuators in the closed-loop systems have different failure rates and the measurements distortion of every actuator is also take into consideration. By using Lyapunov functional, criteria for the exponential stability and criteria for co-designing both the feedback and the trigger parameters are derived in the form of linear matrix inequalities.

## II. System Description

Consider a discrete-time NCSs with the structure shown in Fig.1. The system can be described as the following discrete-time systems with nonlinearities:

$$\begin{cases} x(k+1) = \mathbf{A}x(k) + \mathbf{B}u(k) + \mathbf{B}_1w(k) + f(k, x(k)) \\ z(k) = \mathbf{C}x(k) + \mathbf{D}u(k) + \mathbf{B}_2w(k) \end{cases} \quad (1)$$

where  $x(k) \in \mathbb{R}^n$  is a state vector;  $u(k) \in \mathbb{R}^m$  is a control vector,  $w(k) \in \mathbb{R}^r$  is an unknown input belonging to  $L_2[0, \infty)$ ;  $f(k, x(k))$  is a nonlinear function,  $z(k) \in \mathbb{R}^m$  is an observed

---

\*Manuscript Received May 2012; Accepted Mar. 2013. This work is supported by the National Natural Science Foundation of China (No.11226240, No.61074025, No.60834002, No.60904013), the Natural Science Foundation of Jiangsu Province of China (No.BK2012469), the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (No.12KJD120001) and a project funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD).

vector, and  $A, B, B_1, C, D, B_2$  are all constant matrices with appropriate dimensions.

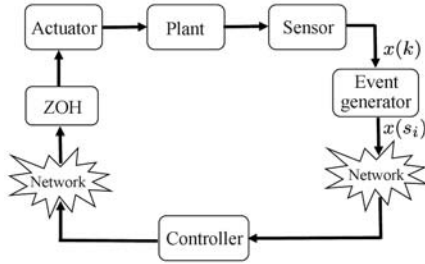


Fig. 1. The structure of an event-triggered networked control system

Throughout this paper, similar to Refs.[8,9], we make the following assumptions:

**Assumption 1**  $f(k, 0) = 0$ , for all  $k \in \mathbb{N}$

**Assumption 2**

$$[f(k, x) - f(k, y) - \Xi_1(x - y)]^T [f(k, x) - f(k, y) - \Xi_2(x - y)] \leq 0 \quad (2)$$

where  $\Xi_1$  and  $\Xi_2$  are known real constant matrices.

**Remark 1** From Assumptions 1 and 2, we can obtain that

$$\begin{bmatrix} x(k) \\ f(k, x(k)) \end{bmatrix}^T \begin{bmatrix} \Omega_1 & * \\ \Omega_2 & I_n \end{bmatrix} \begin{bmatrix} x(k) \\ f(k, x(k)) \end{bmatrix} \leq 0 \quad (3)$$

where  $\Omega_1 = \frac{\Xi_1^T \Xi_2 + \Xi_2^T \Xi_1}{2}$ ,  $\Omega_2 = -\frac{\Xi_1 + \Xi_2}{2}$ , and \* denotes the entries implied by symmetry.

As is shown in Fig.1, the new state feeds into an event generator that decide when to transmit the state to the controller via a network medium by a specified trigger condition, which will be given in sequel. The following function of network architecture in Fig.1 is expected:

(1) As shown in Fig.1, the event generator is constructed between the sensor and the controller which is used to determine when the new state  $x(k)$  to be sent out to the controller by using the following judgment algorithm<sup>[10]</sup>:

$$[x(k) - x(s_i)]^T \Omega [x(k) - x(s_i)] \leq \sigma x^T(k) \Omega x(k) \quad (4)$$

where  $\Omega \in \mathbb{R}^{m \times m}$  is a positive matrix,  $\sigma \in [0, 1)$ ,  $x(s_i)$  is the previously transmitted state. If the current state  $x(k)$  satisfying the inequality Eq.(4), it will not be transmitted. Only the one that exceeds the threshold in Eq.(4) will be sent to the controller.

(2) When the sampled data has been transmitted (or released) by the event generator, it is forwarded to the ZOH (Zero-order holder) through network channel, introducing a communication delay  $d(k)$ .

**Assumption 3** The time-varying delay in the network communication is  $d(k)$  and  $d(k) \in [0, d^M]$ , where  $d^M$  is a positive real number.

Define

$$u(k) = Kx(k) \quad (5)$$

based on above analysis, considering the behavior of ZOH and the effect of the transmission delay, the controller can be described as

$$u(k) = Kx(s_i), t \in [s_i + d(s_i), s_{i+1} + d(s_{i+1}) - 1] \quad (6)$$

Similar to Refs.[6, 7], for technical convenience, we consider the following two cases:

**Case 1** if  $s_i + 1 + d^M \geq s_{i+1} + d(s_{i+1}) - 1$ , define a function  $\tau(k)$  as

$$\tau(k) = k - s_i, k \in [s_i + d(s_i), s_{i+1} + d(s_{i+1}) - 1] \quad (7)$$

clearly,

$$d(s_i) \leq \tau(k) \leq (s_{i+1} - s_i) + d(s_{i+1}) - 1 \leq 1 + d^M \quad (8)$$

**Case 2** If  $s_i + 1 + d^M \leq s_{i+1} + d(s_{i+1}) - 1$ , consider the following two intervals:

$$[s_i + d(s_i), s_i + d^M], [s_i + d^M + l, s_i + d^M + l + 1] \quad (9)$$

Since  $d(k) \leq d^M$ , it can be easily shown that there exists  $d$  such that

$$s_i + d + d^M < s_{i+1} + d(s_{i+1}) - 1 \leq s_i + d + 1 + d^M \quad (10)$$

Moreover,  $x(s_i)$  and  $x(s_i + l)$  with  $l = 1, 2, \dots, d$  satisfy Eq.(4).

Let

$$\begin{cases} I_0 = [s_i + d(s_i), s_i + d^M + 1) \\ I_l = [s_i + d^M + l, s_i + d^M + l + 1) \\ I_d = [s_i + d + d^M, s_{i+1} + d(s_{i+1}) - 1) \end{cases} \quad (11)$$

where  $l = 1, 2, \dots, d - 1$ . One can see that

$$[s_i + d(s_i), s_{i+1} + d(s_{i+1}) - 1] = \bigcup_{i=0}^{i=d} I_i \quad (12)$$

Define  $\tau(k)$  as

$$\tau(k) = \begin{cases} k - s_i, & k \in I_0 \\ k - s_i - l, & k \in I_l, l = 1, 2, \dots, d - 1 \\ k - s_i - d, & k \in I_d \end{cases} \quad (13)$$

Then, we have

$$\begin{cases} d(s_i) \leq \tau(k) \leq 1 + d^M \triangleq \tau_M, & k \in I_0 \\ d(s_i) \leq d^M \leq \tau(k) \leq \tau_M, & k \in I_l, l = 1, 2, \dots, d - 1 \\ d(s_i) \leq d^M \leq \tau(k) \leq \tau_M, & k \in I_d \end{cases} \quad (14)$$

where the third row in Eq.(14) holds because  $s_{i+1} + d(s_{i+1}) - 1 \leq s_i + d + 1 + d^M$ . Obviously,

$$0 \leq d(s_i) \leq \tau(k) \leq \tau_M, \quad k \in I_d \quad (15)$$

In Case 1, for  $k \in [s_i + d(s_i), s_{i+1} + d(s_{i+1}) - 1]$ , define  $e_i(k) = 0$ . In Case 2, define

$$e_i(k) = \begin{cases} 0, & k \in I_0 \\ x(s_i) - x(s_i + l), & k \in I_l, l = 1, 2, \dots, d - 1 \\ x(s_i) - x(s_i + d), & k \in I_d \end{cases} \quad (16)$$

From the definition of  $e_i(k)$  and the triggering algorithm of Eq.(4), it can be easily seen that for  $k \in [s_i + d(s_i), s_{i+1} + d(s_{i+1}) - 1]$ ,

$$e_i^T(k) \Omega e_i(k) \leq \sigma x^T(k - \tau(k)) \Omega x(k - \tau(k)) \quad (17)$$

**Remark 2** It should be noted that when  $\mu = 0$ , the event-triggered scheme reduces to a periodic time-triggered scheme. Thus, the event-triggered scheme considered is more general.

Utilizing  $\tau(k)$  and  $e_i(k)$ , the control can be expressed as

$$u(k) = \mathbf{K}x(s_i) = \mathbf{K}x(k - \tau(k)) + \mathbf{K}e_i(k),$$

$$k \in [s_i + d(s_i), s_{i+1} + d(s_{i+1}) - 1] \quad (18)$$

**Assumption 4** The actuators in the closed-loop systems have different failure rates because of different working conditions. Furthermore, the measurements distortion of every actuator is also take into consideration.

Under Assumption 4, the control can be described as

$$u^F(k) = \mathbf{E}Kx(s_i) = \sum_{i=1}^m \xi_i L_i Kx(s_i) \quad (19)$$

where  $\mathbf{E} = \text{diag}\{\xi_1, \dots, \xi_m\}$ , and  $\xi_i$  ( $i = 1, 2, \dots, m$ ) are  $m$  unrelated variables taking values on the interval  $[0, \theta]$ , where  $\theta \geq 1$ , the mathematical expectation and variance of  $\xi_i$  are  $\mu_i$  and  $\sigma_i^2$  ( $i = 1, 2, \dots, m$ ),  $L_i = \text{diag}\{0, \dots, 0, 1, 0, \dots, 0\}$ . De-

fine  $\bar{\mathbf{E}} = \text{diag}\{\mu_1, \dots, \mu_m\} = \sum_{i=1}^m \mu_i L_i$ , obviously,  $\mathcal{E}(\mathbf{E}) = \bar{\mathbf{E}}$ ,  $\mathcal{E}(\mathbf{E} - \bar{\mathbf{E}}) = 0$ ,  $\mathcal{E}(\xi_i - \mu_i)^2 = \sigma_i^2$ , where  $\mathcal{E}\{x\}$  stands for the expectation of  $x$ .

Combining Eqs.(16–19), Eq.(1) can be rewritten as

$$\begin{cases} x(k+1) = \mathbf{A}x(k) + \mathbf{B}\bar{\mathbf{E}}K(x(k - \tau(k)) + e_i(k)) \\ \quad + \mathbf{B}(\mathbf{E} - \bar{\mathbf{E}})K(x(k - \tau(k)) + e_i(k)) + \mathbf{B}_1w(k) \\ \quad + f(k, x(k)) \\ z(k) = \mathbf{C}x(k) + \mathbf{D}\bar{\mathbf{E}}K(x(k - \tau(k)) + e_i(k)) \\ \quad + \mathbf{D}(\mathbf{E} - \bar{\mathbf{E}})K(x(k - \tau(k)) + e_i(k)) + \mathbf{B}_2w(k) \end{cases} \quad (20)$$

In the following, we need to introduce a lemma, which will help us in deriving the main results.

**Lemma 1**<sup>[11]</sup>  $\mathbf{E}_1$ ,  $\mathbf{E}_2$  and  $\mathbf{\Omega}$  are matrices with appropriate dimensions,  $\tau(k)$  is a function of  $k$  and  $0 \leq \tau(k) \leq \tau_M$ , then

$$\tau(k)\mathbf{E}_1 + (\tau_M - \tau(k))\mathbf{E}_2 + \mathbf{\Omega} < 0$$

if and only if the following inequalities hold

$$\tau_M \mathbf{E}_1 + \mathbf{\Omega} < 0$$

$$\tau_M \mathbf{E}_2 + \mathbf{\Omega} < 0$$

### III. Main Results

In this section, we will give a sufficient condition for the reliable  $H_\infty$  control problem and design a reliable controller for system of Eq.(20).

**Theorem 1** For given  $\gamma$ ,  $\sigma$  and matrix  $\mathbf{K}$ , the nominal system of Eq.(20) is exponentially stable in the mean square under the event trigger scheme of Eq.(20) if there exist matrices  $\mathbf{P} > 0$ ,  $\mathbf{Q} > 0$ ,  $\mathbf{R} > 0$ ,  $\mathbf{N}$ , and  $\mathbf{M}$  with appropriate dimensions satisfying

$$\mathbf{\Sigma}(s) = \begin{bmatrix} \mathbf{\Sigma}_{11} + \mathbf{\Gamma} + \mathbf{\Gamma}^T & * & * & * & * \\ \mathbf{\Sigma}_{21} & \mathbf{\Sigma}_{22} & * & * & * \\ \mathbf{\Sigma}_{31} & \mathbf{0} & \mathbf{\Sigma}_{33} & * & * \\ \mathbf{\Sigma}_{41} & \mathbf{0} & \mathbf{0} & \mathbf{\Sigma}_{44} & * \\ \mathbf{\Sigma}_{51}(s) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{R} \end{bmatrix}$$

$$< 0, \quad s = 1, 2 \quad (21)$$

where

$$\mathbf{\Sigma}_{11} = \begin{bmatrix} \beta & * & * & * & * & * \\ \mathbf{K}^T \bar{\mathbf{E}}^T \mathbf{B}^T \mathbf{P} & \mu \mathbf{\Omega} & * & * & * & * \\ \mathbf{0} & \mathbf{0} & -\mathbf{Q} & * & * & * \\ \mathbf{K}^T \bar{\mathbf{E}}^T \mathbf{B}^T \mathbf{P} & \mathbf{0} & \mathbf{0} & -\mathbf{\Omega} & * & * \\ \mathbf{B}_1^T \mathbf{P} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\gamma^2 \mathbf{I} & * \\ \mathbf{P} - \mathbf{\Omega}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mathbf{I} \end{bmatrix},$$

$$\beta = \mathbf{P}(\mathbf{A} - \mathbf{I}) + (\mathbf{A} - \mathbf{I})^T \mathbf{P} + \mathbf{Q} - \mathbf{\Omega}_1,$$

$$\mathbf{\Sigma}_{21} = \begin{bmatrix} \mathbf{P}(\mathbf{A} - \mathbf{I}) & \mathbf{P}\mathbf{B}\bar{\mathbf{E}}\mathbf{K} & \mathbf{0} & \mathbf{P}\mathbf{B}\bar{\mathbf{E}}\mathbf{K} & \mathbf{P}\mathbf{B}_1 & \mathbf{P} \\ \mathbf{0} & \mathbf{\Pi}_1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Pi}_1 & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{\Pi}_1 = \begin{bmatrix} \sigma_1 \mathbf{P}\mathbf{B}\mathbf{L}_1 \mathbf{K} \\ \vdots \\ \sigma_m \mathbf{P}\mathbf{B}\mathbf{L}_1 \mathbf{K} \end{bmatrix},$$

$$\mathbf{\Sigma}_{22} = \text{diag}\{-\mathbf{P}, \dots, -\mathbf{P}\},$$

$$2m+1$$

$$\mathbf{\Sigma}_{33} = \text{diag}\{-\mathbf{R}, \dots, -\mathbf{R}\},$$

$$2m+1$$

$$\mathbf{\Sigma}_{44} = \text{diag}\{-\mathbf{I}, \dots, -\mathbf{I}\},$$

$$2m+1$$

$$\mathbf{\Sigma}_{31} = \begin{bmatrix} \varphi(\mathbf{A} - \mathbf{I}) & \varphi\mathbf{B}\bar{\mathbf{E}}\mathbf{K} & \mathbf{0} & \varphi\mathbf{B}\bar{\mathbf{E}}\mathbf{K} & \varphi\mathbf{B}_1 & \varphi \\ \mathbf{0} & \sqrt{\tau_M} \mathbf{\Pi}_2 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \sqrt{\tau_M} \mathbf{\Pi}_2 & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\varphi = \sqrt{\tau_M} \mathbf{R}, \quad \mathbf{\Pi}_2 = \begin{bmatrix} \sigma_1 \mathbf{R}\mathbf{B}\mathbf{L}_1 \mathbf{K} \\ \vdots \\ \sigma_m \mathbf{R}\mathbf{B}\mathbf{L}_1 \mathbf{K} \end{bmatrix},$$

$$\mathbf{\Sigma}_{41} = \begin{bmatrix} \mathbf{C} & \mathbf{D}\bar{\mathbf{E}}\mathbf{K} & \mathbf{0} & \mathbf{D}\bar{\mathbf{E}}\mathbf{K} & \mathbf{B}_2 & \mathbf{0} \\ \mathbf{0} & \mathbf{\Pi}_3 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{\Pi}_3 & \mathbf{0} & \mathbf{0} \end{bmatrix},$$

$$\mathbf{\Pi}_3 = \begin{bmatrix} \sigma_1 \mathbf{D}\mathbf{L}_1 \mathbf{K} \\ \vdots \\ \sigma_m \mathbf{D}\mathbf{L}_1 \mathbf{K} \end{bmatrix},$$

$$\mathbf{\Sigma}_{51}(1) = \sqrt{\tau_M} \mathbf{N}^T, \quad \mathbf{\Sigma}_{51}(2) = \sqrt{\tau_M} \mathbf{M}^T,$$

$$\mathbf{\Gamma} = [\mathbf{N} \quad -\mathbf{N} + \mathbf{M} \quad -\mathbf{M} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}],$$

$$\mathbf{N} = [\mathbf{N}_1 \quad \mathbf{N}_2 \quad \mathbf{N}_3 \quad \mathbf{N}_4 \quad \mathbf{0} \quad \mathbf{0}],$$

$$\mathbf{M} = [\mathbf{M}_1 \quad \mathbf{M}_2 \quad \mathbf{M}_3 \quad \mathbf{M}_4 \quad \mathbf{0} \quad \mathbf{0}]$$

and \* denotes the entries implied by symmetry.

**Proof** Define

$$y(k) = x(k+1) - x(k)$$

$$= (\mathbf{A} - \mathbf{I})x(k) + \mathbf{B}\bar{\mathbf{E}}Kx(k - \tau(k)) + \mathbf{B}\bar{\mathbf{E}}Ke_i(k)$$

$$+ \mathbf{B}_1w(k) + \mathbf{B}(\mathbf{E} - \bar{\mathbf{E}})Kx(k - \tau(k))$$

$$+ \mathbf{B}(\mathbf{E} - \bar{\mathbf{E}})Ke_i(k) + f(k, x(k)) \quad (22)$$

Choose the following Lyapunov functional candidate as

$$V(k) = x^T(k) \mathbf{P}x(k) + \sum_{i=k-\tau_M}^{k-1} x^T(i) \mathbf{Q}x(i)$$

$$+ \sum_{i=-\tau_M}^{-1} \sum_{j=k+i}^{k-1} y^T(i) \mathbf{R}y(i) \quad (23)$$

Let  $\Delta V(k) = V(k+1) - V(k)$ , then along the system of

Eq.(20), we have

$$\begin{aligned} \mathcal{E}\Delta V(k) &= 2x^T(k)P\left[(A-I)x(k) + B\bar{E}Kx(k-\tau(k))\right. \\ &\quad \left.+ B\bar{E}Ke_i(k) + B_1w(k) + f(k, x(k))\right] + \vartheta^T P\vartheta \\ &\quad + x^T(k-\tau(k))\sum_{i=1}^m \sigma_i^2 K^T \bar{E}^T B^T P B \bar{E} K x(k-\tau(k)) \\ &\quad + e_i^T(k)\sum_{i=1}^m \sigma_i^2 K^T \bar{E}^T B^T P B \bar{E} K e_i(k) \\ &\quad + x^T(k)Qx(k) - x^T(k-\tau_M)Qx(k-\tau_M) \\ &\quad + \mathcal{E}\left\{\tau_M y^T(k)Ry(k) - \sum_{i=k-\tau_M}^{k-1} y^T(i)Ry(i)\right\} \end{aligned} \quad (24)$$

where

$$\begin{aligned} \vartheta &= \left[(A-I)x(k) + B\bar{E}Kx(k-\tau(k))\right. \\ &\quad \left.+ B\bar{E}Ke_i(k) + B_1w(k) + f(k, x(k))\right], \\ P &> 0, \quad Q > 0, \quad R > 0 \end{aligned}$$

Employing the free-weighting matrices method, we have

$$\begin{aligned} 2\zeta^T(k)N\left[x(k) - x(k-\tau(k)) - \sum_{i=k-\tau(k)}^{k-1} y(i)\right] &= 0 \\ 2\zeta^T(k)M\left[x(k-\tau(k)) - x(k-\tau_M) - \sum_{i=k-\tau_M}^{k-\tau(k)-1} y(i)\right] &= 0 \end{aligned} \quad (25)$$

where

$$\begin{aligned} \zeta^T(k) &= \left[x^T(k) \quad x^T(k-\tau(k)) \quad x^T(k-\tau_M)\right. \\ &\quad \left. e_i^T(k) \quad w^T(k) \quad f^T(k, x(k))\right] \end{aligned}$$

There exists  $R$ , such that

$$\begin{aligned} -2\zeta^T(k)N\sum_{i=k-\tau(k)}^{k-1} y(i) &\leq \tau(k)\zeta^T(k)NR^{-1}N^T\zeta(k) \\ &\quad + \sum_{i=k-\tau(k)}^{k-1} y^T(i)Ry(i) \end{aligned} \quad (26)$$

$$\begin{aligned} -2\zeta^T(k)M\sum_{i=k-\tau_M}^{k-\tau(k)-1} y(i) &\leq (\tau_M - \tau(k))\zeta^T(k)MR^{-1}M^T\zeta(k) \\ &\quad + \sum_{i=k-\tau_M}^{k-\tau(k)-1} y^T(i)Ry(i) \end{aligned} \quad (27)$$

Note that

$$\begin{aligned} \mathcal{E}\{\tau_M y^T(k)Ry(k)\} &= \tau_M \vartheta^T R \vartheta + \tau_M x^T(k-\tau(k)) \\ &\quad \cdot \sum_{i=1}^m \sigma_i^2 K^T \bar{E}^T B^T R B \bar{E} K x(k-\tau(k)) \\ &\quad + \tau_M e_i^T(k)\sum_{i=1}^m \sigma_i^2 K^T \bar{E}^T B^T R B \bar{E} K e_i(k) \end{aligned} \quad (28)$$

Also, it follows from Eq.(3) that

$$\begin{bmatrix} x(k) \\ f(k, x(k)) \end{bmatrix}^T \begin{bmatrix} -\Omega_1 & * \\ -\Omega_2 & -I \end{bmatrix} \begin{bmatrix} x(k) \\ f(k, x(k)) \end{bmatrix} \geq 0 \quad (29)$$

Combining Eq.(22) and Eqs.(24-29) and the relation of Eq.(4), by using well-known Schur complement and Lemma 1, one can easily see that Eq.(30) with  $s = 1, 2$  can lead  $\mathcal{E}\{\Delta V(k)\} - \gamma^2 w^T(k)w(k) + z^T(k)z(k) < 0$ . The remaining part of the proof is similar to those in Refs.[8, 12] and so omitted here for simplicity. The proof is complete.

Based on analysis results in Theorem 1, we are in position to design the feedback gain  $K$  under the event trigger of Eq.(4).

**Theorem 2** Suppose  $\mu > 0$ ,  $\gamma$  and  $\varepsilon > 0$  are given parameters. The system described by Eq.(20) with the feedback gain  $K = YX^{-1}$  under the event trigger condition of Eq.(4) is exponentially stable with an  $H_\infty$  performance index  $\gamma$  if there exist matrices  $X, \tilde{Q}, \tilde{R}, \tilde{\Omega} > 0, \tilde{N}, \tilde{M}$  and  $Y$  with appropriate dimensions such that

$$\begin{aligned} \Sigma(s) &= \begin{bmatrix} \tilde{\Sigma}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * & * \\ \tilde{\Sigma}_{21} & \tilde{\Sigma}_{22} & * & * & * \\ \tilde{\Sigma}_{31} & 0 & \tilde{\Sigma}_{33} & * & * \\ \tilde{\Sigma}_{41} & 0 & 0 & \Sigma_{44} & * \\ \tilde{\Sigma}_{51}(s) & 0 & 0 & 0 & -\tilde{R} \end{bmatrix} \\ &< 0, \quad s = 1, 2 \end{aligned} \quad (30)$$

where

$$\begin{aligned} \tilde{\Sigma}_{11} &= \begin{bmatrix} \tilde{G} & * & * & * & * & * \\ Y^T \bar{E}^T B^T & \mu \tilde{\Omega} & * & * & * & * \\ 0 & 0 & -\tilde{Q} & * & * & * \\ Y^T \bar{E}^T B^T & 0 & 0 & -\tilde{\Omega} & * & * \\ B_1^T & 0 & 0 & 0 & -\gamma^2 I & * \\ I - \Omega_2 X & 0 & 0 & 0 & 0 & -I \end{bmatrix}, \\ \tilde{G} &= (A-I)X + X(A-I)^T + \tilde{Q} - \tilde{\Omega}_1, \\ \tilde{\Sigma}_{21} &= \begin{bmatrix} (A-I)X & B\bar{E}Y & 0 & B\bar{E}Y & B_1 & I \\ 0 & \tilde{\Pi}_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\Pi}_1 & 0 & 0 \end{bmatrix}, \\ \tilde{\Pi}_1 &= \begin{bmatrix} \sigma_1 B L_1 Y \\ \vdots \\ \sigma_m B L_1 Y \end{bmatrix}, \\ \tilde{\Sigma}_{22} &= \underbrace{\text{diag}\{-X, \dots, -X\}}_{2m+1}, \\ \tilde{\Sigma}_{31} &= \begin{bmatrix} \lambda(A-I)X & \lambda B\bar{E}Y & 0 & \lambda B\bar{E}Y & \lambda B_1 & \lambda I \\ 0 & \lambda \tilde{\Pi}_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \lambda \tilde{\Pi}_1 & 0 & 0 \end{bmatrix}, \\ \lambda &= \sqrt{\tau_M}, \\ \tilde{\Sigma}_{33} &= \underbrace{\text{diag}\{-2\varepsilon X + \varepsilon^2 \tilde{R}, \dots, -2\varepsilon X + \varepsilon^2 \tilde{R}\}}_{2m+1} \\ \tilde{\Sigma}_{41} &= \begin{bmatrix} CX & D\bar{E}Y & 0 & D\bar{E}Y & B_2 & 0 \\ 0 & \tilde{\Pi}_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\Pi}_3 & 0 & 0 \end{bmatrix}, \\ \tilde{\Pi}_3 &= \begin{bmatrix} \sigma_1 D L_1 Y \\ \vdots \\ \sigma_m D L_1 Y \end{bmatrix}, \\ \tilde{\Sigma}_{51}(1) &= \sqrt{\tau_M} \tilde{N}^T, \quad \Sigma_{51}(2) = \sqrt{\tau_M} \tilde{M}^T \end{aligned}$$

$$\begin{aligned}\tilde{\Gamma} &= [\tilde{N} \quad -\tilde{N} + \tilde{M} \quad -\tilde{M} \quad \mathbf{0} \quad \mathbf{0} \quad \mathbf{0}], \\ \tilde{N} &= [\tilde{N}_1 \quad \tilde{N}_2 \quad \tilde{N}_3 \quad \tilde{N}_4 \quad \mathbf{0} \quad \mathbf{0}], \\ \tilde{M} &= [\tilde{M}_1 \quad \tilde{M}_2 \quad \tilde{M}_3 \quad \tilde{M}_4 \quad \mathbf{0} \quad \mathbf{0}]\end{aligned}$$

**Proof** By using Schur complement, we can obtain that Eq.(31) are equivalent to Eq.(30),

$$\Sigma(s) = \begin{bmatrix} \Sigma_{11} + \Gamma + \Gamma^T & * & * & * & * \\ \Sigma_{21} & \Sigma_{22} & * & * & * \\ \Psi_{31} & \mathbf{0} & \Psi_{33} & * & * \\ \Sigma_{41} & \mathbf{0} & \mathbf{0} & \Sigma_{44} & * \\ \Sigma_{51}(s) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -R \end{bmatrix} < 0, \quad s = 1, 2 \quad (31)$$

where

$$\Psi_{33} = \text{diag}\{-PR^{-1}P, \dots, -PR^{-1}P\}, \quad \Psi_{31} = \sqrt{\tau_M} \Sigma_{21}$$

Due to  $(R - \varepsilon^{-1}P)R^{-1}(R - \varepsilon^{-1}P) \geq 0$ , we have

$$-PR^{-1}P \leq -2\varepsilon P + \varepsilon^2 R \quad (32)$$

Substituting  $-PR^{-1}P$  with  $-2\varepsilon P + \varepsilon^2 R$  into Eq.(31), we obtain

$$\Sigma(s) = \begin{bmatrix} \Sigma_{11} + \Gamma + \Gamma^T & * & * & * & * \\ \Sigma_{21} & \Sigma_{22} & * & * & * \\ \Psi_{31} & \mathbf{0} & \tilde{\Psi}_{33} & * & * \\ \Sigma_{41} & \mathbf{0} & \mathbf{0} & \Sigma_{44} & * \\ \Sigma_{51}(s) & \mathbf{0} & \mathbf{0} & \mathbf{0} & -R \end{bmatrix} < 0, \quad s = 1, 2 \quad (33)$$

where  $\tilde{\Psi}_{33} = \text{diag}\{-2\varepsilon P + \varepsilon^2 R, \dots, -2\varepsilon P + \varepsilon^2 R\}$ .

Denoting  $X = P^{-1}$ ,  $\tilde{Q} = XQX$ ,  $\tilde{R} = XRX$ ,  $\tilde{N} = XNX$ ,  $\tilde{M} = XMX$ ,  $\tilde{\Omega} = X\Omega X$  and  $Y = KX$ , then pre- and post-multiplying Eq.(33) with  $\text{diag}\{X, X, X, X, I, I, X, \dots, X, I, \dots, I, X\}$ , Eq.(30) can be obtained.

#### IV. A Simulation Example

To demonstrate the effectiveness of our method, we consider system of Eq.(1) with parameters as follows:

$$\begin{aligned}A &= \begin{bmatrix} 0.1 & 0 \\ 0 & 1.01 \end{bmatrix}, \quad B = \begin{bmatrix} -0.1 \\ -0.01 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \\ C &= [0.1 \quad 0], \quad D = 0, \quad B_2 = 0 \end{aligned} \quad (34)$$

and the parameters of Assumptions 2 in Eq.(2) are given as:

$$\Xi_1 = \text{diag}\{-0.1, -0.2\}, \quad \Xi_2 = \text{diag}\{-0.2, -0.1\}$$

By simply calculation, we find the system of Eq.(1) with above parameters is unstable, our purpose is design to the reliable  $H_\infty$  controller.

We now consider the following three cases with different parameters:

**Case 1** When the system of Eq.(1) is under event-triggered scheme and the actuators are all in good condition, let  $\bar{\Xi} = 1$ ,  $\mu = 0.2$ , by using Theorem 2 with  $\tau_M = 3$ ,  $\gamma = 18$  and  $\varepsilon = 10$ , we can obtain the feedback gain and the trigger matrix are

$$K = [-0.1992 \quad 2.0369], \quad \Omega = \begin{bmatrix} 1.8267 & -0.1076 \\ -0.1076 & 0.3462 \end{bmatrix} \quad (35)$$

For the initial condition  $x^T(0) = [-0.3 \quad -1]$ , with the above feedback gain  $K$ , the state response of system of Eq.(1) with parameters in Eq.(34) and the release instants and release interval are shown in Fig.2 and Fig.3, respectively.

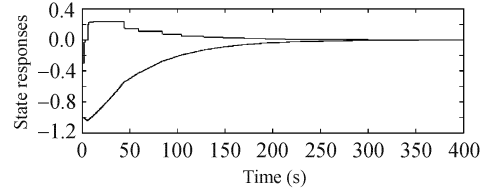


Fig. 2. State response under event-triggered scheme without actuator failures

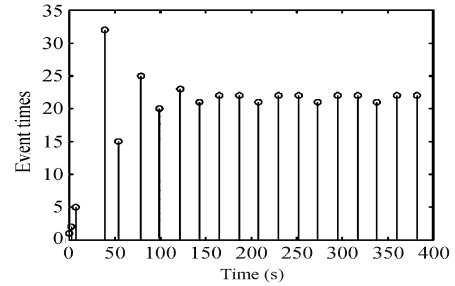


Fig. 3. The release instants and release interval without actuator failures

**Case 2** Suppose the system of Eq.(1) reduces to time-triggered scheme and the actuators have different failure rates, that is,  $\mu = 0$  and  $\bar{\Xi} = 0.8$ , set  $\sigma = 0.1$ ,  $\gamma = 18$ ,  $\varepsilon = 10$  and  $\tau_M = 3$ , by Theorem 2, the feedback gain is

$$K = [-0.4079 \quad 2.7808] \quad (36)$$

The state response and the probabilistic actuator failures are illustrated in Fig.4 and Fig.5, respectively.

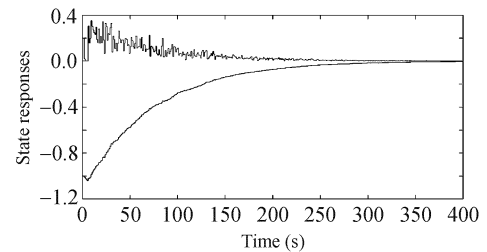


Fig. 4. State response under time-triggering scheme with actuator failures

**Case 3** Under the event-triggered scheme, suppose the actuators have different failure rates, under the condition of  $\bar{\Xi} = 0.8$ ,  $\sigma = 0.1$ ,  $\varepsilon = 10$ ,  $\mu = 0.2$ ,  $\gamma = 18$ , by Theorem 2, we can have the upper bound of  $\tau_M$  is 7.

when  $\mu = 0.1$ ,  $\gamma = 18$  and  $\tau_M = 3$ , the feedback gain and the trigger matrix are

$$K = [-0.3512 \quad 2.4146], \quad \Omega = \begin{bmatrix} 5.2160 & -0.2300 \\ -0.2300 & 0.5774 \end{bmatrix} \quad (37)$$

The state response of our event-triggered scheme and the release instants and release interval are illustrated in Fig.6 and Fig.7.

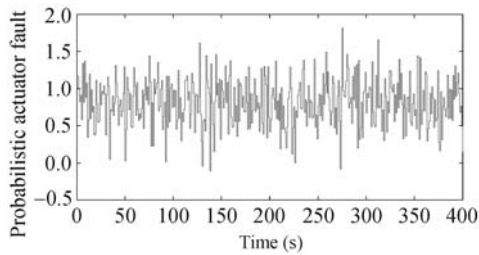


Fig. 5. The probabilistic actuator failures

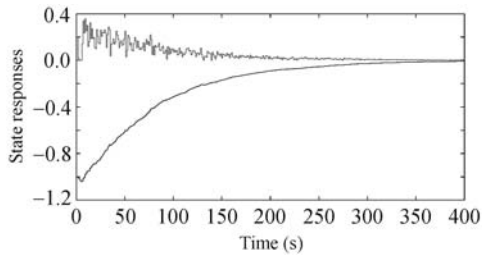


Fig. 6. State response under the event triggering scheme with actuator failures

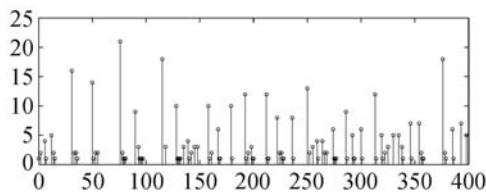


Fig. 7. The release instants and release interval with actuator failures

## V. Conclusion

This paper considers event-triggering in networked system with probabilistic actuator faults. Under the event-triggering scheme, the newly state information will be transmitted to the controller only when it violates specified triggering condition. In terms of different failure rates and the measurements distortion of every actuator, a new probabilistic actuator fault model for event-triggered networked control systems is proposed. By using Lyapunov functional, criteria for the exponential stability and criteria for co-designing both the feedback and the trigger parameters are derived in the form of linear matrix inequalities. A simulation example is given to illustrate the effectiveness of the proposed method.

## References

- [1] D. Yue, J. Lam and D. Ho, "Reliable  $H_\infty$  control of uncertain descriptor systems with multiple time delays", *Control Theory and Applications, IEE Proceedings, IET*, Vol.150, No.6, pp.557–564, 2003.
- [2] Y. Zhang, H. Zhou, S. Joeqin and T. Chai, "Decentralized fault diagnosis of large-scale processes using multiblock kernel partial least squares", *IEEE Transactions on Industrial Informatics*, Vol.6, No.1, pp.3–10, 2010.
- [3] Y. Zhang, T. Chai and Z. Li, "Modelling and monitoring of dynamic processes", *IEEE Transactions on Neural Networks and Learning System*, Vol.23, No.12, pp.277–284, 2012.
- [4] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks", *IEEE Transactions on Automatic Control*, Vol.52, No.9, pp.1680–1685, 2007.
- [5] X. Wang and M. Lemmon, "On event design in event-triggered feedback systems", *Automatica*, Vol.47, No.10, pp.2319–2322, 2011.
- [6] S. Hu and D. Yue, "Event-triggered control design of linear networked systems with quantizations", *ISA Transactions*, Vol.51, No.1, pp.153–162, 2012.
- [7] S. Hu and D. Yue, "Event-based  $H_\infty$  filtering for networked system with communication delay", *Signal Processing*, Vol.92, No.9, pp.2029–2039, 2012.
- [9] Z. Wang, Y. Liu, X. Liu and Y. Shi, "Robust state estimation for discrete-time stochastic neural networks with probabilistic measurement delays", *Neurocomputing*, Vol.74, No.1, pp.256–264, 2010.
- [10] J. Liang, Z. Wang and X. Liu, "Distributed state estimation for discrete-time sensor networks with randomly varying nonlinearities and missing measurements", *IEEE Transactions on Neural Networks*, Vol.22, No.3, pp.486–496, 2011.
- [11] D. Yue, E. Tian, Y. Zhang and Q. Han, "A delay system method for designing event-triggered controllers of networked control systems", *IEEE Transactions on Automatic Control*, Vol.58, No.2, pp.475–481, 2013.
- [12] D. Yue, E. Tian, Y. Zhang and C. Peng, "Delay-distribution-dependent stability and stabilization of T–S fuzzy systems with probabilistic interval delay", *IEEE Transactions on Systems, Man, and Cybernetics, Part B: Cybernetics*, Vol.39, No.2, pp.503–516, 2009.
- [13] Z. Wang, D. Ho, Y. Liu and X. Liu, "Robust  $H_\infty$  control for a class of nonlinear discrete time-delay stochastic systems with missing measurements", *Automatica*, Vol.45, No.3, pp.684–691, 2009.



**LIU Jinliang** was born in Shandong Province, China, in 1980. He received the Ph.D. degree from Donghua University of Information Science and Technology. Since 2011, he has been with the Department of Applied Mathematics, Nanjing University of Finance and Economics. His research interests include networked control systems, genetic regulatory networks, T-S fuzzy systems and time delay systems.