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State estimation for Markovian jumping genetic regulatory networks with random delays

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A B S T R A C T

In this paper, the state estimation problem is investigated for stochastic genetic regulatory networks (GRNs) with random delays and Markovian jumping parameters. The delay considered is assumed to be satisfying a certain stochastic characteristic. Meantime, the delays of GRNs are described by a binary switching sequence satisfying a conditional probability distribution. The aim of this paper is to design a state estimator to estimate the true states of the considered GRNs through the available output measurements. By using Lyapunov functional and some stochastic analysis techniques, the stability criteria of the estimation error systems are obtained in the form of linear matrix inequalities under which the estimation error dynamics is globally asymptotically stable. Then, the explicit expression of the desired estimator is shown. Finally, a numerical example is presented to show the effectiveness of the proposed results.

1. Introduction

It is well known that genetic regulatory networks (GRNs) have become an important new area in biological and biomedical sciences and a large amount of outstanding results have been published in recent years [1–4]. Different kinds of computational models have been applied to investigate the behaviors of GRNs, for example, Bayesian network models [5], Petri net models [6], the Boolean models [7], and the differential equation models [8]. Among these models, the differential equation model describes the rate of change of the concentrations of gene products, such as mRNAs and proteins, as continuous values.

As one of the mostly investigated dynamical behaviors, the state estimation for GRNs has recently stirred increasing research interest, see [9,10] and the references therein. In fact, this is a difficult issue since GRNs are complex nonlinear systems. Due to the complexity, it is often the case that only partial information about the states of the nodes is available in the network outputs. In order to understand the GRNs better, it becomes necessary to estimate the states of the nodes through available measurements. In [9], the robust H_∞ state estimation problem has been investigated for a class of discrete-time stochastic genetic regulatory networks (GRNs) with probabilistic measurement delays. In [10], the robust H_∞ state

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estimation problem has been investigated for a general class of uncertain discrete-time stochastic neural networks with probabilistic measurement delays.

Recently, GRNs with time-delays in the form of differential equations have received particular research attention [9,11–13]. The main reason lies in the fact that the parameters and the saturation functions of GRNs cannot be measured exactly [9]. Time delays can occur inevitably in transcription, translation, and translocation processes because of the slow reaction process. Considering that time delays is inevitable in practice, we must take this case into account.

In practice, due to component failures or repairs and sudden environmental changes, the transition from one state to the next usually takes place in accordance with certain transition probabilities. GRNs may be subject to network mode switching, which is determined by a Markovian chain. It should be pointed out that, there are many results on the dynamic analysis of Markovian switching systems [14–16], In [14], the stability analysis problem is investigated for a class of Markovian jumping genetic regulatory networks (GRNs) with mixed time delays (discrete time delays and distributed time delays) and stochastic perturbations. In [16], the problem of parameter-dependent robust stability analysis has been studied for uncertain Markovian jump linear systems with time-varying delay. However, there are no papers to deal with the state estimation for stochastic genetic regulatory networks (GRNs) with random delays and Markovian jumping parameters.

Motivated by the above discussion, in this paper, we focused on the state estimation problems for Markovian jumping stochastic genetic regulatory networks (GRNs). Our aim is to derive sufficient conditions for the addressed problem by employing Lyapunov functional, the free-weighting approach and the stochastic analysis techniques. Then the state estimate gains can be designed. The rest of this paper is outlined in the following way. The problem addressed is presented and some preliminaries are briefly provided in Section 2. In Section 3, a sufficient criteria is established in terms of linear matrix in equalities (LMIs) and the explicit expression of the estimator gains is derived. In Section 3, a numerical example is provided to demonstrate the effectiveness of the main results obtained.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n \times m$ real matrices; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$ (respectively, $X \geqslant 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). When x is a stochastic variable. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System description

Consider the following genetic regulatory network with Markovian jumping parameters and time delays present in [17]:

$$\begin{cases} \dot{m}(t) = -A(r(t))m(t) + W(r(t))g(p(t-\sigma(t))) \\ \dot{p}(t) = -C(r(t)) + D(r(t))m(t-\tau(t)) \end{cases} \tag{1}$$

where $A(r(t)), W(r(t)), C(r(t)), D(r(t))$ are known constant matrices for a fixed system mode. The nonlinear function $g(y(t)) = (g_1(y_1(t)), g_2(y_2(t)), \ldots, g_n(y_n(t)),)^T$ denotes the feedback regulation of the protein on the transcription, which is usually taken as the Hill form, i.e., $g_i(y_i(t)) = \frac{y^{h_i}}{1+y^{h_i}}$, h_i is the Hill coefficient. $\sigma(t)$ and $\tau(t)$ are the time-varying delays;

Let $r(t)$ $(t \geqslant 0)$ be a right-continuous Markov chain on the probability space and take values in a finite space $\mathbb{S} = \{1, 2, \ldots, N\}$ with generator $\Pi = (\pi_{ij})_{N\times N}$ given by

$$P\{r(t+\delta(t)) = j \| r(t) = i\} = \begin{cases} (\pi_{ij} + \Delta\pi_{ij})\delta(t) + o(\delta t), & \mathit{if}\ i \neq j \\ 1 + (\pi_{ij} + \Delta\pi_{ij})\delta(t) + o(\delta t), & \mathit{if}\ i = j \end{cases}$$

where $\delta(t) > 0, \pi_{ij} \geqslant 0$ is the known transition rate from i to j, if $j \neq i$ while $\pi_{ii} = -\sum_{j\neq i}\pi_{ij}$. Then, the genetic regulatory networks (1) could be described by the following vector form:

$$\begin{cases} \dot{m}(t) = -A_i m(t) + W_i g(p(t-\delta(t))) \\ \dot{p}(t) = -C_i p(t) + D_i m(t-\tau(t)) \end{cases} \tag{2}$$

Assumption 1 [17]. Taking probability distribution of the time delays $\tau(t)$ and $\sigma(t)$, into account, for some given scalars τ_1 and σ_1, two sets of functions are defined as

$$\begin{aligned} &\Omega_1 = \{t : \tau(t) \in [\tau_m, \tau_1)\}, \quad \Omega_2 = \{t : \tau(t) \in [\tau_1, \tau_M]\} \\ &\Omega_3 = \{t : \sigma(t) \in [\sigma_m, \sigma_1)\}, \quad \Omega_4 = \{t : \sigma(t) \in [\sigma_1, \sigma_M]\} \\ &\tau_1(t) = \begin{cases} \tau(t) \text{ for } t \in \Omega_1 \\ 0 \text{ for } t \in \Omega_2 \end{cases}, \quad \tau_2(t) = \begin{cases} \tau(t) \text{ for } t \in \Omega_2 \\ 0 \text{ for } t \in \Omega_1 \end{cases} \\ &\sigma_1(t) = \begin{cases} \sigma(t) \text{ for } t \in \Omega_3 \\ 0 \text{ for } t \in \Omega_4 \end{cases}, \quad \sigma_2(t) = \begin{cases} \sigma(t) \text{ for } t \in \Omega_4 \\ 0 \text{ for } t \in \Omega_3 \end{cases} \end{aligned}$$

From the definitions of the $\Omega_1, \Omega_2, \Omega_3$ and Ω_4, it can be easily seen that $t \in \Omega_1$ means that the event $\tau(t) \in [\tau_m, \tau_1)$ occurs, $t \in \Omega_2$ means that the event $\tau(t) \in [\tau_1, \tau_M]$ occurs, $t \in \Omega_3$ means that the event $\sigma(t) \in [\sigma_m, \sigma_1)$ occurs and $t \in \Omega_4$ means that the event $\sigma(t) \in [\sigma_1, \sigma_M]$ occurs.

Therefore, the stochastic variables $\alpha(t), \beta(t)$ can be define as

$$\alpha(t) = \begin{cases} 1, & t \in \Omega_1 \\ 0, & t \in \Omega_2 \end{cases}, \quad \beta(t) = \begin{cases} 1, & t \in \Omega_3 \\ 0, & t \in \Omega_4 \end{cases}$$

Assumption 2. $\alpha(t), \beta(t)$ are Bernoulli distributed sequences with

$$Prob\{\alpha(t) = 1\} = \mathbb{E}\{\alpha(t)\} = \alpha_0, \quad Prob\{\alpha(t) = 0\} = 1 - \mathbb{E}\{\alpha(t)\} = 1 - \alpha_0.$$
$$Prob\{\beta(t) = 1\} = \mathbb{E}\{\beta(t)\} = \beta_0, \quad Prob\{\beta(t) = 0\} = 1 - \mathbb{E}\{\beta(t)\} = 1 - \beta_0.$$

where $0 \leqslant \alpha_0 \leqslant 1, 0 \leqslant \beta_0 \leqslant 1$ are constants and $\mathbb{E}\{\alpha(t)\}$ and $\mathbb{E}\{\beta(t)\}$ are the expectation of $\alpha(t), \beta(t)$ respectively.

Remark 1. It should be noticed from Assumption 2 that

$$\mathbb{E}\{\alpha(t)\} = \alpha_0, \quad \mathbb{E}\{(\alpha(t) - \alpha_0)^2\} = \alpha_0(1 - \alpha_0), \quad \mathbb{E}\{\beta(t)\} = \beta_0, \quad \mathbb{E}\{(\beta(t) - \beta_0)^2\} = \beta_0(1 - \beta_0)$$

Assumption 3. Since $g(\cdot)$ is a monotonically increasing function with saturation, from the definition of $g(\cdot)$, we can find that $g(\cdot)$ satisfies the following condition

$$g(y_i)(g_i(y_i) - ky_i) \leqslant 0 \quad (y_i \neq 0, i = 1, 2, \ldots, n) \tag{3}$$

By Assumptions 1 and 2, the system (2) can be rewritten as

$$\begin{cases} \dot{m}(t) = -A_i m(t) + \beta(t) W_i g(p(t - \sigma_1(t))) + (1 - \beta(t)) W_i g(p(t - \sigma_2(t))) \\ \dot{p}(t) = -C_i p(t) + \alpha(t) D_i m(t - \tau_1(t)) + (1 - \alpha(t)) D_i m(t - \tau_2(t)) \end{cases} \tag{4}$$

For the complexity of large-scale networks, only partial information about the gene states is available. Therefore, in order to obtain the true states of the GRNs, we need to estimate the gene states from available measurements. Similar to Refs. [9,18], we can assume the network measurements to be given as follows:

$$\begin{cases} z_m(t) = Mm(t) \\ z_p(t) = Np(t) \end{cases} \tag{5}$$

where $z_m(t), z_p(t) \in R^m$ are the actual measurement outputs and M, N are known constant matrices with appropriate dimensions.

In this paper, based on the available network outputs in (5), we construct the following state estimator for the GRNs (4):

$$\begin{cases} \dot{\hat{m}}(t) = -A_i \hat{m}(t) + K_{1i}[z_m(t) - \hat{z}_m(t)] \\ \dot{\hat{p}}(t) = -C_i \hat{p}(t) + K_{2i}[z_p(t) - \hat{z}_p(t)] \end{cases} \tag{6}$$

and

$$\begin{cases} \hat{z}_m(t) = M\hat{m}(t) \\ \hat{z}_p(t) = N\hat{p}(t) \end{cases} \tag{7}$$

where $\hat{z}_m(t), \hat{z}_p(t) \in \mathbb{R}^{n \times n}$ are the estimations of $m(t), p(t)$ and $K_{1i}, K_{2i} \in \mathbb{R}^{n \times m}$ is the estimate gain matrix to be designed later.

The main objective of this paper is to find suitable observer gains K_{1i} and K_{2i}, so that $\hat{z}_m(t)$ and $\hat{z}_p(t)$, respectively, approach to $m(t)$ and $p(t)$.

By setting the estimation error $\bar{m}(t) = m(t) - \hat{m}(t), \bar{p}(t) = p(t) - \hat{p}(t)$ and the output errors be $\bar{z}_m(t) = z_m(t) - \hat{z}_m(t)$, $\bar{z}_p(t) = z_p(t) - \hat{z}_p(t)$, the error dynamics of the state estimation can be obtained from (4)–(7) as follows:

$$\begin{cases} \dot{\bar{m}}(t) = -(A_i + K_{1i}M)\bar{m}(t) + \beta(t) W_i g(p(t - \sigma_1(t))) + (1 - \beta(t)) W_i g(p(t - \sigma_2(t))) \\ \dot{\bar{p}}(t) = -(C_i + K_{2i}N)\bar{p}(t) + \alpha(t) D_i m(t - \tau_1(t)) + (1 - \alpha(t)) D_i m(t - \tau_2(t)) \end{cases} \tag{8}$$

Remark 2. $\alpha(t)$ and $\beta(t)$ are introduced to describe the distribution information of the random delay, from which we can derive less conservative conditions For example, the piecewise analysis method for delayed systems has been employed in [19,20]. Furthermore, we could use the delay-partitioning approach to further reduce conservatism of the system analysis.

Before giving the main result, we will firstly introduce the following definition and lemmas, which will help us in deriving the main results.

Definition 1 [21]. *For a given function* $V : C^b_{F_0}([-\tau_M, 0], \mathbb{R}^n) \times S$, *its infinitesimal operator* $\mathcal{L}$ *is defined as*

$$\mathcal{L}(V\eta(t)) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}[\mathbb{E}(V(\eta_t + \Delta)|\eta_t) - V(\eta_t)]$$

Lemma 1 [22]. *For any vectors* $x, y \in R^n$, *and positive definite matrix* $Q \in R^{n\times n}$, *the following inequality holds:*

$$2x^T y \leqslant x^T Qx + y^T Q^{-1} y$$

Lemma 2 [23]. Ξ_1, Ξ_2 *and* Ω *are matrices with appropriate dimensions,* $\tau(t)$ *is a function of* t *and* $\tau_1 \leqslant \tau(t) \leqslant \tau_2$, *then*
$$[(\tau(t) - \tau_1)\Xi_1 + (\tau_2 - \tau(t))\Xi_2] + \Omega < 0$$
if and only if the following two inequalities hold

$$(\tau_2 - \tau_1)\Xi_1 + \Omega < 0$$
$$(\tau_2 - \tau_1)\Xi_2 + \Omega < 0$$

3. Main results

In this section, we will invest the estimation problem for the GRNs (8). A sufficient condition is established, such that the estimation error system to be globally asymptotically stable. Then, according to the analysis results, the schemes to design the estimator gain matrix K_{1i} and K_{2i} are derived in terms of the solution to certain matrix inequalities.

Theorem 1. *The system (8) is asymptotically stable for given scalars* $0 \leqslant \tau_m \leqslant \tau(t) \leqslant \tau_M, 0 \leqslant \sigma_m \leqslant \sigma(t) \leqslant \sigma_M, \tau_1, \sigma_1, k$, *and the estimator gain matrix* K_{1i} *and* K_{2i} *in (8) if there exist positive definite matrices* $\bar{Q}_{1i} > 0, \bar{R}_{1i} > 0, Q_{1i} > 0, R_{1i} > 0$ $(i \in \mathbb{S}), \bar{Q}_i > 0, \bar{R}_i > 0, Q_i > 0, R_i > 0 (i = 2, 3, \ldots, 6), \Lambda_i = diag(\lambda_{i1}, \lambda_{i2}, \ldots, \lambda_{in}) > 0 \quad (i = 1, 2)$ *and* $\bar{M}_i, \bar{N}_i, \bar{T}_i, \bar{S}_i, \bar{V}_i, \bar{W}_i, \bar{G}_i, \bar{F}_i, M_i, N_i, T_i, S_i, V_i, W_i, G_i, F_i \in \mathbb{R}^{6\times 1}$, *such that the following LMIs hold:*

$$\Phi(l,s) = \begin{bmatrix} \Phi_{11} & * & * & * & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * & * & * & * \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & * & * & * & * \\ \Phi_{41}(l) & 0 & 0 & \Phi_{44} & * & * & * \\ 0 & \Phi_{52}(s) & 0 & 0 & \Phi_{55} & * & * \\ 0 & 0 & \Phi_{63}(l) & 0 & 0 & \Phi_{66} & * \\ 0 & 0 & 0 & \Phi_{74}(s) & 0 & 0 & \Phi_{77} \end{bmatrix} < 0, \quad (l, s = 1, 2, 3, 4) \tag{9}$$

where

$$\Phi_{11} = \begin{bmatrix} \Pi_{11} + \bar{\Gamma}_{1i} + \bar{\Gamma}_{1i}^T & * & * & * \\ \Pi_{21} & \Pi_{22} + \bar{\Gamma}_{2i} + \bar{\Gamma}_{2i}^T & * & * \\ 0 & 0 & \Pi_{33} + \Gamma_{1i} + \Gamma_{1i}^T & * \\ \Pi_{41} & 0 & 0 & \Pi_{44} + \Gamma_{2i} + \Gamma_{2i}^T \end{bmatrix}$$

$$\Phi_{21} = [\Pi_{211} \quad 0 \quad \Pi_{213} \quad \Pi_{214}], \quad \Phi_{22} = diag\{-2\Lambda_1, -2\Lambda_2\}$$

$$\Phi_{31} = \begin{bmatrix} \Phi_{311} & 0 & 0 & 0 \\ 0 & \Phi_{322} & \Phi_{323} & 0 \\ 0 & 0 & \Phi_{333} & 0 \\ 0 & 0 & \Phi_{343} & \Phi_{344} \end{bmatrix}, \quad \Phi_{32} = [\Delta_1 \quad 0_{2\times 4} \quad \Delta_2 \quad 0_{2\times 4}]$$

$$\Phi_{33} = diag\{-\bar{Q}_5, -\bar{Q}_6, -\bar{Q}_5, -\bar{Q}_6, -\bar{R}_5, -\bar{R}_6, -\bar{R}_5, -\bar{R}_6, -Q_5, -Q_6, -Q_5, -Q_6, -R_5, -R_6, -R_5, -R_6\}$$

$$\Phi_{41}(1) = \begin{bmatrix} \sqrt{\delta_{10}}\bar{N}_i^T \\ \sqrt{\delta_{11}}\bar{S}_i^T \end{bmatrix}, \quad \Phi_{41}(2) = \begin{bmatrix} \sqrt{\delta_{10}}\bar{N}_i^T \\ \sqrt{\delta_{11}}\bar{T}_i^T \end{bmatrix}, \quad \Phi_{41}(3) = \begin{bmatrix} \sqrt{\delta_{10}}\bar{M}^T \\ \sqrt{\delta_{11}}\bar{S}_i^T \end{bmatrix}, \quad \Phi_{41}(4) = \begin{bmatrix} \sqrt{\delta_{10}}\bar{M}_i^T \\ \sqrt{\delta_{11}}\bar{T}_i^T \end{bmatrix}$$

$$\Phi_{52}(1)=\begin{bmatrix}\sqrt{\delta_{20}}\bar{W}_i^T\\ \sqrt{\delta_{21}}\bar{F}_i^T\end{bmatrix},\quad \Phi_{52}(2)=\begin{bmatrix}\sqrt{\delta_{20}}\bar{W}_i^T\\ \sqrt{\delta_{21}}\bar{G}_i^T\end{bmatrix},\quad \Phi_{52}(3)=\begin{bmatrix}\sqrt{\delta_{20}}\bar{V}_i^T\\ \sqrt{\delta_{21}}\bar{F}^T\end{bmatrix},\quad \Phi_{52}(4)=\begin{bmatrix}\sqrt{\delta_{20}}\bar{V}_i^T\\ \sqrt{\delta_{21}}\bar{G}_i^T\end{bmatrix}$$

$$\Phi_{63}(1)=\begin{bmatrix}\sqrt{\delta_{10}}N_i^T\\ \sqrt{\delta_{11}}S_i^T\end{bmatrix},\quad \Phi_{63}(2)=\begin{bmatrix}\sqrt{\delta_{10}}N_i^T\\ \sqrt{\delta_{11}}T_i^T\end{bmatrix},\quad \Phi_{63}(3)=\begin{bmatrix}\sqrt{\delta_{10}}M_i^T\\ \sqrt{\delta_{11}}S_i^T\end{bmatrix},\quad \Phi_{63}(4)=\begin{bmatrix}\sqrt{\delta_{10}}M_i^T\\ \sqrt{\delta_{11}}T_i^T\end{bmatrix}$$

$$\Phi_{74}(1)=\begin{bmatrix}\sqrt{\delta_{20}}W_i^T\\ \sqrt{\delta_{21}}F_i^T\end{bmatrix},\quad \Phi_{74}(2)=\begin{bmatrix}\sqrt{\delta_{20}}W_i^T\\ \sqrt{\delta_{21}}G_i^T\end{bmatrix},\quad \Phi_{74}(3)=\begin{bmatrix}\sqrt{\delta_{20}}V_i^T\\ \sqrt{\delta_{21}}F_i^T\end{bmatrix},\quad \Phi_{74}(4)=\begin{bmatrix}\sqrt{\delta_{20}}V_i^T\\ \sqrt{\delta_{21}}G_i^T\end{bmatrix}$$

$$\Phi_{44}=diag\{-\bar{Q}_5,-\bar{Q}_6\},\Phi_{55}=diag\{-\bar{R}_5,-\bar{R}_6\},\Phi_{66}=diag\{-Q_5,-Q_6\},\Phi_{77}=diag\{-R_5,-R_6\}$$

$$\Pi_{21}=\begin{bmatrix}0&0&\alpha_0\bar{R}_{1i}D_i&0&(1-\alpha_0)\bar{R}_{1i}D_i&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix},\quad \Pi_{41}=\begin{bmatrix}0&0&\alpha_0R_{1i}D_i&0&(1-\alpha_0)R_{1i}D_i&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\\0&0&0&0&0&0\end{bmatrix}$$

$$\Pi_{11}=diag\{\hat{\Upsilon}_1,-\bar{Q}_2,0,-\bar{Q}_3,0,-\bar{Q}_4\},\quad \Pi_{22}=diag\{\hat{\Upsilon}_2,-\bar{R}_2,0,-\bar{R}_3,0,-\bar{R}_4\}$$

$$\Pi_{33}=diag\{\Upsilon_1,-Q_2,0,-Q_3,0,-Q_4\},\quad \Pi_{44}=diag\{\Upsilon_2,-R_2,0,-R_3,0,-R_4\}$$

$$\hat{\Upsilon}_1=-\bar{Q}_{1i}A_i-A_i^T\bar{Q}_{1i}-\bar{Q}_{1i}K_{1i}M-M^TK_{1i}^T\bar{Q}_{1i}^T+\bar{Q}_2+\bar{Q}_3+\bar{Q}_4+\sum_{j=1}^{N}\pi_{ij}\bar{Q}_{1j}$$

$$\hat{\Upsilon}_2=-\bar{R}_{1i}C_i-C_i^T\bar{R}_{1i}-\bar{R}_{1i}K_{2i}N-N^TK_{2i}^T\bar{R}_{1i}+\bar{R}_2+\bar{R}_3+\bar{R}_4+\sum_{j=1}^{N}\pi_{ij}\bar{R}_{1j}$$

$$\Upsilon_1=-Q_{1i}A_i-A_i^TQ_{1i}^T-Q_{1i}K_{1i}M-M^TK_{1i}^TQ_{1i}^T+Q_2+Q_3+Q_4+\sum_{j=1}^{N}\pi_{ij}Q_{1j}$$

$$\Upsilon_2=-R_{1i}C_i-C_i^TR_{1i}-R_{1i}K_{2i}N-N^TK_{2i}^TR_{1i}^T+R_2+R_3+R_4+\sum_{j=1}^{N}\pi_{ij}R_{1j}$$

$$\Pi_{211}=\begin{bmatrix}\beta_0W_i^T\bar{Q}_{1i}&0&0&0&0&0\\(1-\beta_0)W_i^T\bar{Q}_{1i}&0&0&0&0&0\end{bmatrix},\quad \Pi_{213}=\begin{bmatrix}\beta_0W_i^TQ_{1i}&0&0&0&0&0\\(1-\beta_0)W_i^TQ_{1i}&0&0&0&0&0\end{bmatrix}$$

$$\Pi_{214}=\begin{bmatrix}0&0&k\Lambda_1&0&0&0\\0&0&0&0&k\Lambda_2&0\end{bmatrix},\quad \Phi_{311}=\begin{bmatrix}-\sqrt{\beta_0\delta_{10}}\bar{Q}_5(A_i+K_{1i}M)&0&0&0&0&0\\-\sqrt{\beta_0\delta_{11}}\bar{Q}_6(A_i+K_{1i}M)&0&0&0&0&0\\-\sqrt{(1-\beta_0)\delta_{10}}\bar{Q}_5(A_i+K_{1i}M)&0&0&0&0&0\\-\sqrt{(1-\beta_0)\delta_{11}}\bar{Q}_6(A_i+K_{1i}M)&0&0&0&0&0\end{bmatrix}$$

$$\Phi_{322}=\begin{bmatrix}-\sqrt{\alpha_0\delta_{20}}\bar{R}_5(C_i+K_{2i}N)&0&0&0&0&0\\-\sqrt{\alpha_0\delta_{21}}\bar{R}_6(C_i+K_{2i}N)&0&0&0&0&0\\-\sqrt{(1-\alpha_0)\delta_{20}}\bar{R}_5(C_i+K_{2i}N)&0&0&0&0&0\\-\sqrt{(1-\alpha_0)\delta_{21}}\bar{R}_6(C_i+K_{2i}N)&0&0&0&0&0\end{bmatrix}$$

$$\Phi_{323}=\begin{bmatrix}0&0&\sqrt{\alpha_0\delta_{20}}\bar{R}_5D_i&0&0&0\\0&0&\sqrt{\alpha_0\delta_{21}}\bar{R}_6D_i&0&0&0\\0&0&0&0&\sqrt{(1-\alpha_0)\delta_{20}}\bar{R}_5D_i&0\\0&0&0&0&\sqrt{(1-\alpha_0)\delta_{21}}\bar{R}_6D_i&0\end{bmatrix}$$

$$\Phi_{333}=\begin{bmatrix}-\sqrt{\beta_0\delta_{10}}Q_5A_i&0&0&0&0&0\\-\sqrt{\beta_0\delta_{11}}Q_6A_i&0&0&0&0&0\\-\sqrt{(1-\beta_0)\delta_{10}}Q_5A_i&0&0&0&0&0\\-\sqrt{(1-\beta_0)\delta_{11}}Q_6A_i&0&0&0&0&0\end{bmatrix},\quad \Phi_{344}=\begin{bmatrix}-\sqrt{\alpha_0\delta_{20}}R_5C_i&0&0&0&0&0\\-\sqrt{\alpha_0\delta_{21}}R_6C_i&0&0&0&0&0\\-\sqrt{(1-\alpha_0)\delta_{20}}R_5C_i&0&0&0&0&0\\-\sqrt{(1-\alpha_0)\delta_{21}}R_6C_i&0&0&0&0&0\end{bmatrix}$$

$$\Phi_{343}=\begin{bmatrix}0&0&\sqrt{\alpha_0\delta_{20}}R_5D_i&0&0&0\\0&0&\sqrt{\alpha_0\delta_{21}}R_6D_i&0&0&0\\0&0&0&0&\sqrt{(1-\alpha_0)\delta_{20}}R_5D_i&0\\0&0&0&0&\sqrt{(1-\alpha_0)\delta_{21}}R_6D_i&0\end{bmatrix}$$

$$\Delta_1 = \begin{bmatrix} \sqrt{\beta_0\delta_{10}}\bar{Q}_5W_i & 0 \\ \sqrt{\beta_0\delta_{11}}\bar{Q}_6W_i & 0 \\ 0 & \sqrt{(1-\beta_0)\delta_{10}}\bar{Q}_5W_i \\ 0 & \sqrt{(1-\beta_0)\delta_{11}}\bar{Q}_6W_i \end{bmatrix}, \quad \Delta_2 = \begin{bmatrix} \sqrt{\beta_0\delta_{10}}Q_5W_i & 0 \\ \sqrt{\beta_0\delta_{11}}Q_6W_i & 0 \\ 0 & \sqrt{(1-\beta_0)\delta_{10}}Q_5W_i \\ 0 & \sqrt{(1-\beta_0)\delta_{11}}Q_6W_i \end{bmatrix}$$

$$\bar{\Gamma}_{1i} = [\,0 \quad \bar{M}_i \quad -\bar{M}_i+\bar{N}_i \quad -\bar{N}_i+\bar{T}_i \quad -\bar{T}_i+\bar{S}_i \quad -\bar{S}_i\,]$$
$$\bar{\Gamma}_{2i} = [\,0 \quad \bar{V}_i \quad -\bar{V}_i+\bar{W}_i \quad -\bar{W}_i+\bar{G}_i \quad -\bar{G}_i+\bar{F}_i \quad -\bar{F}_i\,]$$
$$\Gamma_{1i} = [\,0 \quad M_i \quad -M_i+N_i \quad -N_i+T_i \quad -T_i+S_i \quad -S_i\,]$$
$$\Gamma_{2i} = [\,0 \quad V_i \quad -V_i+W_i \quad -W_i+G_i \quad -G_i+F_i \quad -F_i\,]$$
$$\delta_1 = \tau_M-\tau_m, \delta_{10} = \tau_1-\tau_m, \delta_{11} = \tau_M-\tau_1, \delta_2 = \sigma_M-\sigma_m, \delta_{20} = \sigma_1-\sigma_m, \delta_{21} = \sigma_M-\sigma_1.$$

Proof. Choose the following Lyapunov function for system (7):

$$V(t) = V_1(t)+V_2(t)+V_3(t) \tag{10}$$

where

$$\begin{aligned} V_1(t) =& \bar{m}^T(t)\bar{Q}_1(r(t))\bar{m}(t)+\bar{p}^T(t)\bar{R}_1(r(t))\bar{p}(t)+m^T(t)Q_1(r(t))m(t)+p^T(t)R_1(r(t))p(t) \\ V_2(t) =& \int_{t-\tau_m}^t \bar{m}^T(s)\bar{Q}_2\bar{m}(s)ds+\int_{t-\tau_1}^t \bar{m}^T(s)\bar{Q}_3\bar{m}(s)ds+\int_{t-\tau_M}^t \bar{m}^T(s)\bar{Q}_4\bar{m}(s)ds+\int_{t-\sigma_m}^t \bar{p}^T(s)\bar{R}_2\bar{p}(s)ds \\ &+\int_{t-\sigma_1}^t \bar{p}^T(s)\bar{R}_3\bar{p}(s)ds+\int_{t-\sigma_M}^t \bar{p}^T(s)\bar{R}_4\bar{p}(s)ds+\int_{t-\tau_m}^t m^T(s)Q_2m(s)ds+\int_{t-\tau_1}^t m^T(s)Q_3m(s)ds \\ &+\int_{t-\tau_M}^t m^T(s)Q_4m(s)ds+\int_{t-\sigma_m}^t p^T(s)R_2p(s)ds+\int_{t-\sigma_1}^t p^T(s)R_3p(s)ds+\int_{t-\sigma_M}^t p^T(s)R_4p(s)ds \\ V_3(t) =& \int_{t-\tau_1}^{t-\tau_m}\int_s^t \dot{\bar{m}}^T(v)\bar{Q}_5\dot{\bar{m}}(v)dvds+\int_{t-\tau_M}^{t-\tau_1}\int_s^t \dot{\bar{m}}^T(v)\bar{Q}_6\dot{\bar{m}}(v)dvds+\int_{t-\sigma_1}^{t-\sigma_m}\int_s^t \dot{\bar{p}}^T(v)\bar{R}_5\dot{\bar{p}}(v)dvds \\ &+\int_{t-\sigma_M}^{t-\sigma_1}\int_s^t \dot{\bar{p}}^T(v)\bar{R}_6\dot{\bar{p}}(v)dvds+\int_{t-\tau_1}^{t-\tau_m}\int_s^t \dot{m}^T(v)Q_5\dot{m}(v)dvds+\int_{t-\tau_M}^{t-\tau_1}\int_s^t m^T(v)Q_6\dot{m}(v)dvds \\ &+\int_{t-\sigma_1}^{t-\sigma_m}\int_s^t \dot{p}^T(v)R_5\dot{p}(v)dvds+\int_{t-\sigma_M}^{t-\sigma_1}\int_s^t \dot{p}^T(v)R_6\dot{p}(v)dvds \end{aligned}$$

Taking the time derivative of $V(t)$ along the trajectory of system (8), and taking expectation on it, we have

$$\begin{aligned} \mathbb{E}\{\mathbb{L}V_1(t)\} =& 2\bar{m}^T(t)\bar{Q}_{1i}[-(A_i+K_{1i}M)\bar{m}(t)+\beta_0W_ig(p(t-\sigma_1(t)))+(1-\beta_0)W_ig(p(t-\sigma_2(t)))] \\ &+\sum_{j=1}^N \pi_{ij}\bar{m}^T(t)\bar{Q}_{1j}\bar{m}(t)+2\bar{p}^T(t)\bar{R}_{1i}[-(C_i+K_{2i}N)\bar{p}(t)+\alpha_0D_im(t-\tau_1(t))+(1-\alpha_0)D_im(t-\tau_2(t))] \\ &+\sum_{j=1}^N \pi_{ij}\bar{p}^T(t)\bar{R}_{1j}\bar{p}(t)+2m^T(t)Q_{1i}[-A_im(t)+\beta_0W_ig(p(t-\sigma_1(t)))+(1-\beta_0)W_ig(p(t-\sigma_2(t)))] \\ &+\sum_{j=1}^N \pi_{ij}m^T(t)Q_{1j}m(t)+2p^T(t)R_{1i}[-C_ip(t)+\alpha_0D_im(t-\tau_1(t))+(1-\alpha_0)D_im(t-\tau_2(t))] \\ &+\sum_{j=1}^N \pi_{ij}p^T(t)R_{1j}p(t) \end{aligned} \tag{11}$$

$$\begin{aligned} \mathbb{E}\{\mathbb{L}V_2(t)\} =& \bar{m}^T(t)[\bar{Q}_2+\bar{Q}_3+\bar{Q}_4]\bar{m}(t)+\bar{p}^T(t)[\bar{R}_2+\bar{R}_3+\bar{R}_4]\bar{p}(t)-\bar{m}^T(t-\tau_m)\bar{Q}_2\bar{m}(t-\tau_m)-\bar{m}^T(t-\tau_M)\bar{Q}_4\bar{m}(t \\ &-\tau_M)-\bar{m}^T(t-\tau_1)\bar{Q}_3\bar{m}(t-\tau_1)-\bar{p}^T(t-\sigma_m)\bar{R}_2\bar{p}(t-\sigma_m)-\bar{p}^T(t-\sigma_M)\bar{R}_4\bar{p}(t-\sigma_M)-\bar{p}^T(t-\sigma_1)\bar{R}_3\bar{p}(t \\ &-\sigma_1)+m^T(t)[Q_2+Q_3+Q_4]m(t)+p^T(t)[R_2+R_3+R_4]p(t)-m^T(t-\tau_m)Q_2m(t-\tau_m)-m^T(t \\ &-\tau_1)Q_3m(t-\tau_1)-m^T(t-\tau_M)Q_4m(t-\tau_M)-p^T(t-\sigma_m)R_2p(t-\sigma_m)-p^T(t-\sigma_1)R_3p(t-\sigma_1) \\ &-p^T(t-\sigma_M)R_4p(t-\sigma_M) \end{aligned} \tag{12}$$

$$\begin{aligned}\mathbb{E}\{\mathbb{L}V_3(t)\} &= \dot{\bar{m}}^T(t)\left[\delta_{10}\bar{Q}_5+\delta_{11}\bar{Q}_6\right]\dot{\bar{m}}(t)+\dot{\bar{p}}^T(t)\left[\delta_{20}\bar{R}_5+\delta_{21}\bar{R}_6\right]\dot{\bar{p}}(t)+\dot{m}^T(t)[\delta_{10}Q_5+\delta_{11}Q_6]\dot{m}(t)\\ &\quad+\dot{p}^T(t)[\delta_{20}R_5+\delta_{21}R_6]\dot{p}(t)-\int_{t-\tau_1}^{t-\tau_m}\dot{\bar{m}}^T(s)\bar{Q}_5\dot{\bar{m}}(s)ds-\int_{t-\tau_M}^{t-\tau_1}\dot{\bar{m}}^T(s)\bar{Q}_6\dot{\bar{m}}(s)ds-\int_{t-\sigma_1}^{t-\sigma_m}\dot{\bar{p}}^T(s)\bar{R}_5\dot{\bar{p}}(s)ds\\ &\quad-\int_{t-\sigma_M}^{t-\sigma_1}\dot{\bar{p}}^T(s)\bar{R}_6\dot{\bar{p}}(s)ds-\int_{t-\tau_1}^{t-\tau_m}\dot{m}^T(s)Q_5\dot{m}(s)ds-\int_{t-\tau_M}^{t-\tau_1}\dot{m}^T(s)Q_6\dot{m}(s)ds-\int_{t-\sigma_1}^{t-\sigma_m}\dot{p}^T(s)R_5\dot{p}(s)ds\\ &\quad-\int_{t-\sigma_M}^{t-\sigma_1}\dot{p}^T(s)R_6\dot{p}(s)ds \end{aligned} \tag{13}$$

Notice that

$$\begin{aligned}\mathbb{E}\left\{\mathcal{L}\left[\dot{\bar{m}}^T(t)\bar{Q}\dot{\bar{m}}(t)\right]\right\} &= \beta_0[-(A_i+K_{1i}M)\bar{m}(t)+W_ig(p(t-\sigma_1(t)))]^T\bar{Q}[-(A_i+K_{1i}M)\bar{m}(t)+W_ig(p(t-\sigma_1(t)))]+(1\\ &\quad-\beta_0)\Theta_1^T\bar{Q}\Theta_1\end{aligned} \tag{14}$$

$$\mathbb{E}\left\{\mathcal{L}\left[\dot{\bar{p}}^T(t)\bar{R}\dot{\bar{p}}(t)\right]\right\} = \alpha_0[-(C_i+K_{2i}N)\bar{p}(t)+D_im(t-\tau_1(t))]^T\bar{R}[-(C_i+K_{2i}N)\bar{p}(t)+D_im(t-\tau_1(t))]+(1-\alpha_0)\Theta_2^T\bar{R}\Theta_2 \tag{15}$$

$$\begin{aligned}\mathbb{E}\left\{\mathcal{L}\left[\dot{m}^T(t)Q\dot{m}(t)\right]\right\} &= \beta_0[-A_im(t)+W_ig(p(t-\sigma_1(t)))]^TQ[-A_im(t)+W_ig(p(t-\sigma_1(t)))]+(1\\ &\quad-\beta_0)[-A_im(t)+W_ig(p(t-\sigma_2(t)))]^TQ[-A_im(t)+W_ig(p(t-\sigma_2(t)))]\end{aligned} \tag{16}$$

$$\begin{aligned}\mathbb{E}\left\{\mathcal{L}\left[\dot{p}^T(t)R\dot{p}(t)\right]\right\} &= \alpha_0[-C_ip(t)+D_im(t-\tau_1(t))]^TR[-C_ip(t)+D_im(t-\tau_1(t))]+(1\\ &\quad-\alpha_0)[-C_ip(t)+D_im(t-\tau_2(t))]^TR[-C_ip(t)+D_im(t-\tau_2(t))]\end{aligned} \tag{17}$$

where $\bar{Q}=\delta_{10}\bar{Q}_5+\delta_{11}\bar{Q}_6, \bar{R}=\delta_{20}\bar{R}_5+\delta_{21}\bar{R}_6, Q=\delta_{10}Q_5+\delta_{11}Q_6, R=\delta_{20}R_5+\delta_{21}R_6, \Theta=-(A_i+K_{1i}M)\bar{m}(t)+W_ig(p(t-\sigma_2(t))),$ $\Theta_2=-(C_i+K_{2i}N)\bar{p}(t)+D_im(t-\tau_2(t))$.

Then, by employing free weight matrix method [24,25], we have

$$2\bar{\xi}_1^T(t)\bar{M}_i\left[\bar{m}(t-\tau_m)-\bar{m}(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t-\tau_m}\dot{\bar{m}}(s)ds\right]=0 \tag{18}$$

$$2\bar{\xi}_1^T(t)\bar{N}_i\left[\bar{m}(t-\tau_1(t))-\bar{m}(t-\tau_1)-\int_{t-\tau_1}^{t-\tau_1(t)}\dot{\bar{m}}(s)ds\right]=0 \tag{19}$$

$$2\bar{\xi}_1^T(t)\bar{T}_i\left[\bar{m}(t-\tau_1)-\bar{m}(t-\tau_2(t))-\int_{t-\tau_2(t)}^{t-\tau_1}\dot{\bar{m}}(s)ds\right]=0 \tag{20}$$

$$2\bar{\xi}_1^T(t)\bar{S}_i\left[\bar{m}(t-\tau_2(t))-\bar{m}(t-\tau_M)-\int_{t-\tau_M}^{t-\tau_2(t)}\dot{\bar{m}}(s)ds\right]=0 \tag{21}$$

$$2\bar{\xi}_2^T(t)\bar{V}_i\left[\bar{p}(t-\sigma_m)-\bar{p}(t-\sigma_1(t))-\int_{t-\sigma_1(t)}^{t-\sigma_m}\dot{\bar{p}}(s)ds\right]=0 \tag{22}$$

$$2\bar{\xi}_2^T(t)\bar{W}_i\left[\bar{p}(t-\sigma_1(t))-\bar{p}(t-\sigma_1)-\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot{\bar{p}}(s)ds\right]=0 \tag{23}$$

$$2\bar{\xi}_2^T(t)\bar{G}_i\left[\bar{p}(t-\sigma_1)-\bar{p}(t-\sigma_2(t))-\int_{t-\sigma_2(t)}^{t-\sigma_1}\dot{\bar{p}}(s)ds\right]=0 \tag{24}$$

$$2\bar{\xi}_2^T(t)\bar{F}_i\left[\bar{p}(t-\sigma_2(t))-\bar{p}(t-\sigma_M)-\int_{t-\sigma_M}^{t-\sigma_2(t)}\dot{\bar{p}}(s)ds\right]=0 \tag{25}$$

$$2\xi_1^T(t)M_i\left[m(t-\tau_m)-m(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t-\tau_m}\dot{m}(s)ds\right]=0 \tag{26}$$

$$2\xi_1^T(t)N_i\left[m(t-\tau_1(t))-m(t-\tau_1)-\int_{t-\tau_1}^{t-\tau_1(t)}\dot{m}(s)ds\right]=0 \tag{27}$$

$$2\xi_1^T(t)T_i\left[m(t-\tau_1)-m(t-\tau_2(t))-\int_{t-\tau_2(t)}^{t-\tau_1}\dot{m}(s)ds\right]=0 \tag{28}$$

$$2\xi_1^T(t)S_i\left[m(t-\tau_2(t))-m(t-\tau_M)-\int_{t-\tau_M}^{t-\tau_2(t)}\dot{m}(s)ds\right]=0 \tag{29}$$

$$2\xi_2^T(t)V_i\left[p(t-\sigma_m)-p(t-\sigma_1(t))-\int_{t-\sigma_1(t)}^{t-\sigma_m}\dot{p}(s)ds\right]=0 \tag{30}$$

$$2\xi_2^T(t)W_i\left[p(t-\sigma_1(t))-p(t-\sigma_1)-\int_{t-\sigma_1}^{t-\sigma_1(t)}\dot{p}(s)ds\right]=0 \tag{31}$$

$$2\xi_2^T(t)G_i\left[p(t-\sigma_1)-p(t-\sigma_2(t))-\int_{t-\sigma_2(t)}^{t-\sigma_1}\dot{p}(s)ds\right]=0 \tag{32}$$

$$2\xi_2^T(t)F_i\left[p(t-\sigma_2(t))-p(t-\sigma_M)-\int_{t-\sigma_M}^{t-\sigma_2(t)}\dot{p}(s)ds\right]=0 \tag{33}$$

where

$$\bar{\xi}_1^T(t)=\begin{bmatrix}\bar{m}^T(t) & \bar{m}^T(t-\tau_m) & \bar{m}^T(t-\tau_1(t)) & \bar{m}^T(t-\tau_1) & \bar{m}^T(t-\tau_2(t)) & \bar{m}^T(t-\tau_M)\end{bmatrix}^T$$
$$\bar{\xi}_2^T(t)=\begin{bmatrix}\bar{p}^T(t) & \bar{p}^T(t-\sigma_m) & \bar{p}^T(t-\sigma_1(t)) & \bar{p}^T(t-\sigma_1) & \bar{p}^T(t-\sigma_2(t)) & \bar{p}^T(t-\sigma_M)\end{bmatrix}^T$$
$$\xi_1^T(t)=\begin{bmatrix}m^T(t) & m^T(t-\tau_m) & m^T(t-\tau_1(t)) & m^T(t-\tau_1) & m^T(t-\tau_2(t)) & m^T(t-\tau_M)\end{bmatrix}^T$$
$$\xi_2^T(t)=\begin{bmatrix}p^T(t) & p^T(t-\sigma_m) & p^T(t-\sigma_1(t)) & p^T(t-\sigma_1) & p^T(t-\sigma_2(t)) & p^T(t-\sigma_M)\end{bmatrix}^T$$

On the other hand, by sector condition (3), it follows that

$$-2g^T(p(t-\sigma_i(t)))\Lambda_i g(p(t-\sigma_i(t)))+2kg^T(p(t-\sigma_i(t)))\Lambda_i p(t-\sigma_i(t))\geqslant 0 \tag{34}$$

where $\Lambda_i=diag(\lambda_{i1},\lambda_{i2},\ldots,\lambda_{in})>0$

From (11)–(34) and by Lemma 1, we can easily obtain that

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(t)\}=&\,\zeta^T(t)\begin{bmatrix}\Phi_{11} & *\\ \Phi_{21} & \Phi_{22}\end{bmatrix}\zeta(t)+\mathbb{E}\{\mathcal{L}[\dot{\bar{m}}^T(t)\bar{Q}\dot{\bar{m}}(t)]\}+\mathbb{E}\{\mathcal{L}[\dot{\bar{p}}^T(t)\bar{R}\dot{\bar{p}}(t)]\}+\mathbb{E}\{\mathcal{L}[\dot{m}^T(t)Q\dot{m}(t)]\}\\
&+\mathbb{E}\{\mathcal{L}[\dot{p}^T(t)R\dot{p}(t)]\}+(\tau_1(t)-\tau_m)\bar{\xi}_1^T(t)\bar{M}_i\bar{Q}_5^{-1}\bar{M}_i^T\bar{\xi}_1(t)+(\tau_1-\tau_1(t))\bar{\xi}_1^T(t)\bar{N}_i\bar{Q}_5^{-1}N_i^T\bar{\xi}_1(t)\\
&+(\tau_2(t)-\tau_1)\bar{\xi}_1^T(t)\bar{T}_i\bar{Q}_6^{-1}T_i^T\bar{\xi}_1(t)+(\tau_M-\tau_2(t))\bar{\xi}_1^T(t)\bar{S}_i\bar{Q}_6^{-1}S_i^T\bar{\xi}_1(t)+(\sigma_1(t)-\sigma_m)\bar{\xi}_2^T(t)\bar{V}_i\bar{R}_5^{-1}\bar{V}_i^T\xi_2(t)\\
&+(\sigma_1-\sigma_1(t))\bar{\xi}_2^T(t)\bar{W}_i\bar{R}_5^{-1}\bar{W}_i^T\bar{\xi}_2(t)+(\sigma_2(t)-\sigma_1)\bar{\xi}_2^T(t)\bar{G}_i\bar{R}_6^{-1}\bar{G}_i^T\bar{\xi}_2(t)+(\sigma_M-\sigma_2(t))\bar{\xi}_2^T(t)\bar{F}_i\bar{R}_6^{-1}\bar{F}_i^T\bar{\xi}_2(t)\\
&+(\tau_1(t)-\tau_m)\xi_1^T(t)M_iQ_5^{-1}M_i^T\xi_1(t)+(\tau_1-\tau_1(t))\xi_1^T(t)N_iQ_5^{-1}N_i^T\xi_1(t)+(\tau_2(t)-\tau_1)\xi_1^T(t)T_iQ_6^{-1}T_i^T\xi_1(t)\\
&+(\tau_M-\tau_2(t))\xi_1^T(t)S_iQ_6^{-1}S_i^T\xi_1(t)+(\sigma_1(t)-\sigma_m)\xi_2^T(t)V_iR_5^{-1}V_i^T\xi_2(t)+(\sigma_1-\sigma_1(t))\xi_2^T(t)W_iR_5^{-1}W_i^T\xi_2(t)\\
&+(\sigma_2(t)-\sigma_1)\xi_2^T(t)G_iR_6^{-1}G_i^T\xi_2(t)+(\sigma_M-\sigma_2(t))\xi_2^T(t)F_iR_6^{-1}F_i^T\xi_2(t)\end{aligned} \tag{35}$$

where $\zeta^T(t)=\begin{bmatrix}\xi_1^T(t) & \xi_2^T(t) & g^T(p(t-\sigma_1(t))) & g^T(p(t-\sigma_2(t)))\end{bmatrix}^T$.

Subsequently, by Lemma 2 and the well-known Schur complement, from (9), we can conclude that

$$\mathbb{E}\{\mathcal{L}V(t)\}\leqslant 0 \tag{36}$$

Then, by Lyapunov stability theory, the system (8) is globally asymptotic stable.

Remark 3. As mentioned in the Introduction section, GRNs have received a great deal of attention, and many results on the topic have been available. However, the methods cannot be applied to state estimation problem with randomly occurring probability distribution of the time delays. Instead of only one variable, there are two variables in the GRNs (8), which increase the difficulty and take us great effort. After some rigorous and complex deducing process, the criteria are obtained which are used to guaranteed the dynamics of the estimation error system (8) globally asymptotic stable in the mean square.

Based on Theorem 1, we are now in a position to design the state estimator for the complex networks (1). The following Theorem 2 gives the explicit expression of the estimator gain matrix K_{1i} and K_{2i} $(i\in\mathbb{S})$.

Theorem 2. *For given scalars* $0\leqslant\tau_m\leqslant\tau(t)\leqslant\tau_M, 0\leqslant\sigma_m\leqslant\sigma(t)\leqslant\sigma_M,\tau_1,\sigma_1,k,\varepsilon_i(i=1,2,3,4)$, *the augmented system* (8) *is exponentially stable, if there exist positive definite matrices* $\bar{Q}_{1i}>0,\bar{R}_{1i}>0,Q_{1i}>0,R_{1i}>0$ $(i\in\mathbb{S}),\bar{Q}_i>0,\bar{R}_i>0,Q_i>0,R_i>0(i=2,3,\ldots,6),\Lambda_i=diag(\lambda_{i1},\lambda_{i2},\ldots,\lambda_{in})>0$ $(i=1,2)$ *and* $\bar{M}_i,\bar{N}_i,\bar{T}_i,\bar{S}_i,\bar{V}_i,\bar{W}_i,\bar{G}_i,\bar{F}_i,M_i,N_i,T_i,S_i,V_i,W_i,G_i,F_i\in\mathbb{R}^{6\times 1}$ *satisfying the following LMIs hold:*

$$\bar{\Phi}(l,s)=\begin{bmatrix}\bar{\Phi}_{11} & * & * & * & * & * & * \\ \bar{\Phi}_{21} & \bar{\Phi}_{22} & * & * & * & * & * \\ \bar{\Phi}_{31} & \bar{\Phi}_{32} & \bar{\Phi}_{33} & * & * & * & * \\ \Phi_{41}(l) & 0 & 0 & \Phi_{44} & * & * & * \\ 0 & \Phi_{52}(s) & 0 & 0 & \Phi_{55} & * & * \\ 0 & 0 & \Phi_{63}(l) & 0 & 0 & \Phi_{66} & * \\ 0 & 0 & 0 & \Phi_{74}(s) & 0 & 0 & \Phi_{77}\end{bmatrix}<0, \quad (l,s=1,2,3,4) \tag{37}$$

where

$$\bar{\Phi}_{11}=\begin{bmatrix}\bar{\Pi}_{11}+\bar{\Gamma}_{1i}+\bar{\Gamma}_{1i}^T & * & * & * \\ \Pi_{21} & \bar{\Pi}_{22}+\bar{\Gamma}_{2i}+\bar{\Gamma}_{2i}^T & * & * \\ 0 & 0 & \Pi_{33}+\Gamma_{1i}+\Gamma_{1i}^T & * \\ \Pi_{41} & 0 & 0 & \Pi_{44}+\Gamma_{2i}+\Gamma_{2i}^T\end{bmatrix}$$

$$\bar{\Phi}_{31}=\begin{bmatrix}\bar{\Phi}_{311} & 0 & 0 & 0 \\ 0 & \bar{\Phi}_{322} & \bar{\Phi}_{323} & 0 \\ 0 & 0 & \Phi_{333} & 0 \\ 0 & 0 & \Phi_{343} & \Phi_{344}\end{bmatrix}, \quad \bar{\Phi}_{32}=\bar{\Delta}_1 0_{2\times 4}\Delta_2 0_{2\times 4}$$

$$\bar{\Phi}_{33}=diag\{\Xi_1,\Xi_2,-Q_5,-Q_6,-Q_5,-Q_6,-R_5,-R_6,-R_5,-R_6\}$$

$$\Xi_1=diag\{-2\varepsilon_1\bar{Q}_{1i}+\varepsilon_1^2\bar{Q}_5,-2\varepsilon_2\bar{Q}_{1i}+\varepsilon_2^2\bar{Q}_6,-2\varepsilon_1\bar{Q}_{1i}+\varepsilon_1^2\bar{Q}_5,-2\varepsilon_2\bar{Q}_{1i}+\varepsilon_2^2\bar{Q}_6\}$$

$$\Xi_2=diag\{-2\varepsilon_3\bar{R}_{1i}+\varepsilon_3^2\bar{R}_5,-2\varepsilon_4\bar{R}_{1i}+\varepsilon_4^2\bar{R}_6,-2\varepsilon_3\bar{R}_{1i}+\varepsilon_3^2\bar{R}_5,-2\varepsilon_4\bar{R}_{1i}+\varepsilon_4^2\bar{R}_6\}$$

$$\bar{\Pi}_{11}=diag\{\hat{\bar{\Upsilon}}_1,-\bar{Q}_2,0,-\bar{Q}_3,0,-\bar{Q}_4\},\bar{\Pi}_{22}=diag\{\hat{\bar{\Upsilon}}_2,-\bar{R}_2,0,-\bar{R}_3,0,-\bar{R}_4\}$$

$$\hat{\bar{\Upsilon}}_1=-\bar{Q}_{1i}A_i-A_i^T\bar{Q}_{1i}-Y_{1i}M-M^TY_{1i}^T+\bar{Q}_2+\bar{Q}_3+\bar{Q}_4+\sum_{j=1}^N\pi_{ij}\bar{Q}_{1j}$$

$$\hat{\bar{\Upsilon}}_2=-\bar{R}_{1i}C_i-C_i^T\bar{R}_{1i}-Y_{2i}N-N^TY_{2i}^T+\bar{R}_2+\bar{R}_3+\bar{R}_4+\sum_{j=1}^N\pi_{ij}\bar{R}_{1j}$$

$$\bar{\Phi}_{311}=\begin{bmatrix}-\sqrt{\beta_0\delta_{10}}\bar{Q}_{1i}A_i-\sqrt{\beta_0\delta_{10}}Y_{1i}M & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\beta_0\delta_{11}}\bar{Q}_{1i}A_i-\sqrt{\beta_0\delta_{11}}Y_{1i}M & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\beta_0)\delta_{10}}\bar{Q}_{1i}A_i-\sqrt{(1-\beta_0)\delta_{10}}Y_{1i}M & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\beta_0)\delta_{11}}\bar{Q}_{1i}A_i-\sqrt{(1-\beta_0)\delta_{11}}Y_{1i}M) & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\bar{\Phi}_{322}=\begin{bmatrix}-\sqrt{\alpha_0\delta_{20}}\bar{R}_{1i}C_i-\sqrt{\alpha_0\delta_{20}}Y_{2i}N & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\alpha_0\delta_{21}}\bar{R}_{1i}C_i-\sqrt{\alpha_0\delta_{21}}Y_{2i}N & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\alpha_0)\delta_{20}}\bar{R}_{1i}C_i-\sqrt{(1-\alpha_0)\delta_{20}}Y_{2i}N & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\alpha_0)\delta_{21}}\bar{R}_{1i}C_i-\sqrt{(1-\alpha_0)\delta_{21}}Y_{2i}N & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\bar{\Phi}_{323}=\begin{bmatrix}0 & 0 & \sqrt{\alpha_0\delta_{20}}\bar{R}_{1i}D_i & 0 & 0 & 0 \\ 0 & 0 & \sqrt{\alpha_0\delta_{21}}\bar{R}_{1i}D_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{(1-\alpha_0)\delta_{20}}\bar{R}_{1i}D_i & 0 \\ 0 & 0 & 0 & 0 & \sqrt{(1-\alpha_0)\delta_{21}}\bar{R}_{1i}D_i & 0\end{bmatrix}, \quad \bar{\Delta}_1=\begin{bmatrix}\sqrt{\beta_0\delta_{10}}\bar{Q}_{1i}W_i & 0 \\ \sqrt{\beta_0\delta_{11}}\bar{Q}_{1i}W_i & 0 \\ 0 & \sqrt{(1-\beta_0)\delta_{10}}\bar{Q}_{1i}W_i \\ 0 & \sqrt{(1-\beta_0)\delta_{11}}\bar{Q}_{1i}W_i\end{bmatrix}$$

and the other symbols are defined in Theorem 1. Moreover, if (37) is true, the desired state estimator gain in (8) can be determined by $K_{1i}=\bar{Q}_{1i}^{-1}Y_{1i}, K_{2i}=\bar{R}_{1i}^{-1}Y_{2i}$.

Proof. Combining (9) and (35) and applying Schur complement, we can obtain

$$\bar{\Psi}(l,s)=\begin{bmatrix}\bar{\Psi}_{11} & * & * & * & * & * & * \\ \Phi_{21} & \bar{\Phi}_{22} & * & * & * & * & * \\ \Psi_{31} & \bar{\Phi}_{32} & \bar{\Psi}_{33} & * & * & * & * \\ \Phi_{41}(l) & 0 & 0 & \Phi_{44} & * & * & * \\ 0 & \Phi_{52}(s) & 0 & 0 & \Phi_{55} & * & * \\ 0 & 0 & \Phi_{63}(l) & 0 & 0 & \Phi_{66} & * \\ 0 & 0 & 0 & \Phi_{74}(s) & 0 & 0 & \Phi_{77}\end{bmatrix}<0, \quad (l,s=1,2,3,4) \tag{38}$$

where

$$\bar{\Psi}_{11} = \begin{bmatrix} \bar{\Omega}_{11} + \bar{\Gamma}_{1i} + \bar{\Gamma}_{1i}^T & * & * & * \\ \Pi_{21} & \bar{\Omega}_{22} + \bar{\Gamma}_{2i} + \bar{\Gamma}_{2i}^T & * & * \\ 0 & 0 & \Pi_{33} + \Gamma_{1i} + \Gamma_{1i}^T & * \\ \Pi_{41} & 0 & 0 & \Pi_{44} + \Gamma_{2i} + \Gamma_{2i}^T \end{bmatrix}$$

$$\bar{\Psi}_{31} = \begin{bmatrix} \bar{\Psi}_{311} & 0 & 0 & 0 \\ 0 & \bar{\Psi}_{322} & \bar{\Phi}_{323} & 0 \\ 0 & 0 & \bar{\Phi}_{333} & 0 \\ 0 & 0 & \bar{\Phi}_{343} & \bar{\Phi}_{344} \end{bmatrix},$$

$$\bar{\Psi}_{33} = diag\{\bar{\Xi}_1, \bar{\Xi}_2, -Q_5, -Q_6, -Q_5, -Q_6, -R_5, -R_6, -R_5, -R_6\}$$

$$\bar{\Xi}_1 = diag\{-\bar{Q}_{1i}\bar{Q}_5^{-1}\bar{Q}_{1i}, -\bar{Q}_{1i}\bar{Q}_6^{-1}\bar{Q}_{1i}, -\bar{Q}_{1i}\bar{Q}_5^{-1}\bar{Q}_{1i}, -\bar{Q}_{1i}\bar{Q}_6^{-1}\bar{Q}_{1i}\}$$

$$\bar{\Xi}_2 = diag\{-\bar{R}_{1i}\bar{R}_5^{-1}\bar{R}_{1i}, -\bar{R}_{1i}\bar{R}_6^{-1}\bar{R}_{1i}, -\bar{R}_{1i}\bar{R}_5^{-1}\bar{R}_{1i}, -\bar{R}_{1i}\bar{R}_6^{-1}\bar{R}_{1i}\}$$

$$\bar{\Omega}_{11} = diag\{\Omega_{\Upsilon 1}, -\bar{Q}_2, 0, -\bar{Q}_3, 0, -\bar{Q}_4\}, \bar{\Omega}_{22} = diag\{\Omega_{\Upsilon 2}, -\bar{R}_2, 0, -\bar{R}_3, 0, -\bar{R}_4\}$$

$$\Omega_{\Upsilon 1} = -\bar{Q}_{1i}A_i - A_i^T\bar{Q}_{1i} - \bar{Q}_{1i}K_{1i}M - M^TK_{1i}^T\bar{Q}_{1i} + \bar{Q}_2 + \bar{Q}_3 + \bar{Q}_4 + \sum_{j=1}^{N}\pi_{ij}\bar{Q}_{1j}$$

$$\Omega_{\Upsilon 2} = -\bar{R}_{1i}C_i - C_i^T\bar{R}_{1i} - \bar{R}_{1i}K_{2i}N - N^TK_{2i}^T\bar{R}_{1i} + \bar{R}_2 + \bar{R}_3 + \bar{R}_4 + \sum_{j=1}^{N}\pi_{ij}\bar{R}_{1j}$$

$$\bar{\Psi}_{311} = \begin{bmatrix} -\sqrt{\beta_0\delta_{10}}\bar{Q}_{1i}A_i - \sqrt{\beta_0\delta_{10}}\bar{Q}_{1i}K_{1i}M & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\beta_0\delta_{11}}\bar{Q}_{1i}A_i - \sqrt{\beta_0\delta_{11}}\bar{Q}_{1i}K_{1i}M & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\beta_0)\delta_{10}}\bar{Q}_{1i}A_i - \sqrt{(1-\beta_0)\delta_{10}}\bar{Q}_{1i}K_{1i}M & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\beta_0)\delta_{11}}\bar{Q}_{1i}A_i - \sqrt{(1-\beta_0)\delta_{11}}\bar{Q}_{1i}K_{1i}M & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Psi}_{322} = \begin{bmatrix} -\sqrt{\alpha_0\delta_{20}}\bar{R}_{1i}C_i - \sqrt{\alpha_0\delta_{20}}Y_{2i}N & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{\alpha_0\delta_{21}}\bar{R}_{1i}C_i - \sqrt{\alpha_0\delta_{21}}Y_{2i}N & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\alpha_0)\delta_{20}}\bar{R}_{1i}C_i - \sqrt{(1-\alpha_0)\delta_{20}}Y_{2i}N & 0 & 0 & 0 & 0 & 0 \\ -\sqrt{(1-\alpha_0)\delta_{21}}\bar{R}_{1i}C_i - \sqrt{(1-\alpha_0)\delta_{21}}Y_{2i}N & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and the other symbols are defined in Theorem 2.

Due to

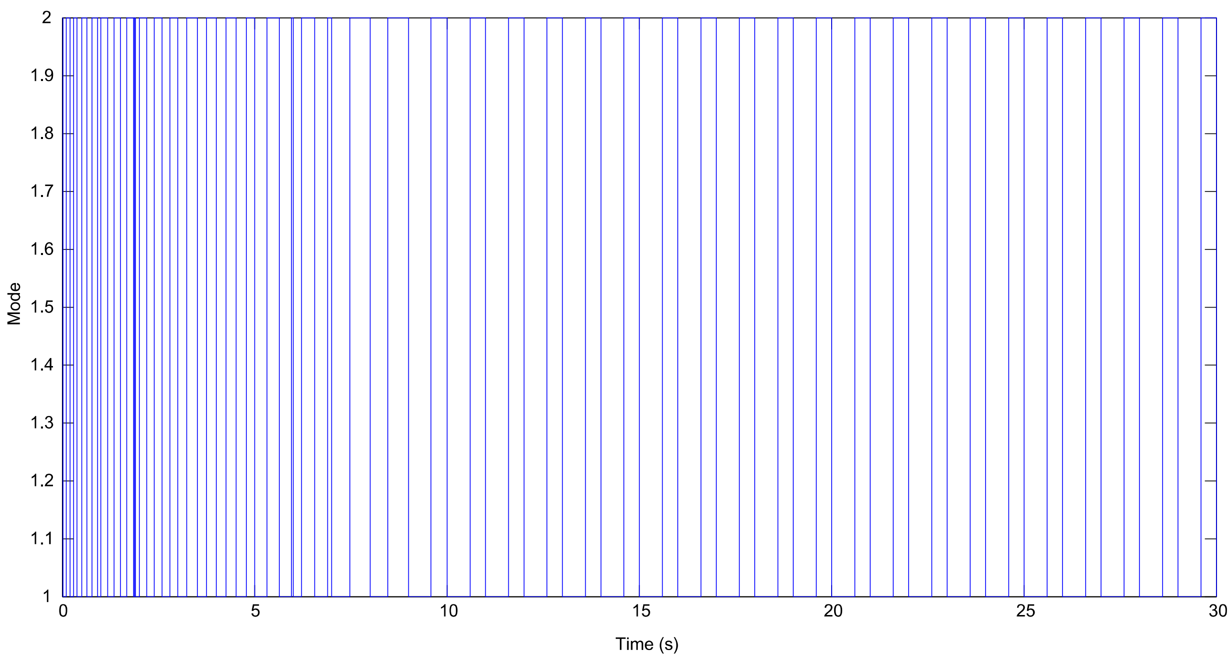

Fig. 1. The probabilities of switching between modes.

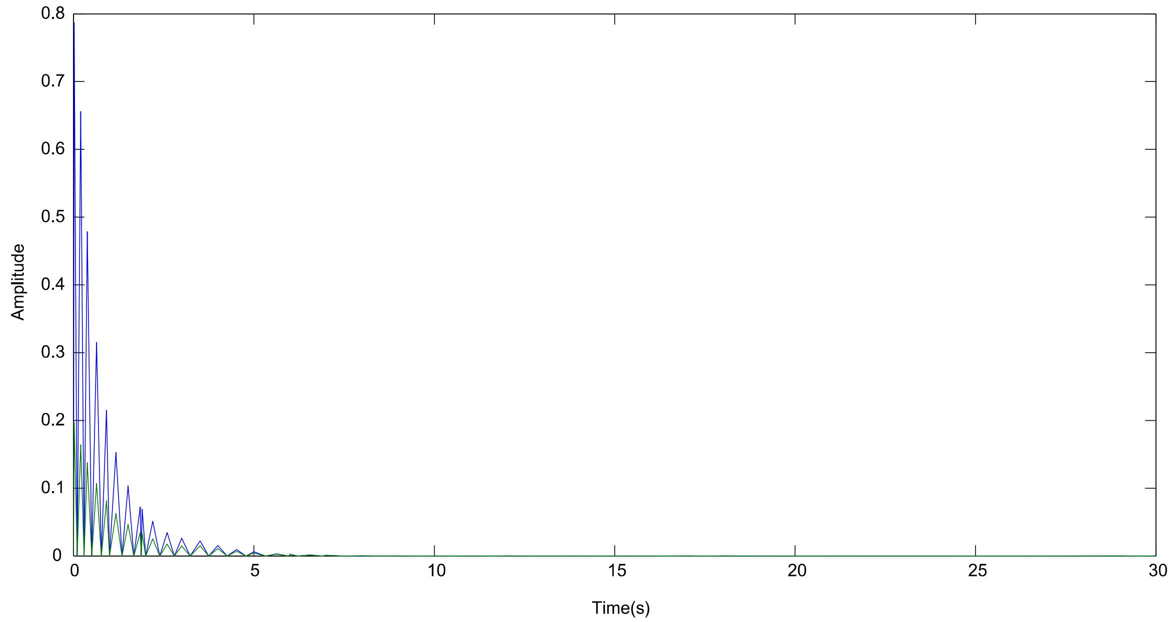

Fig. 2. State trajectory $m(t)$ in (4).

$$(R - \varepsilon^{-1}P)R^{-1}(R - \varepsilon^{-1}P) \geqslant 0,$$

we can have

$$-PR^{-1}P \leqslant -2\varepsilon P + \varepsilon^2 R$$

Substituting $-\bar{Q}_{1i}\bar{Q}_5^{-1}\bar{Q}_{1i}, -\bar{Q}_{1i}\bar{Q}_6^{-1}\bar{Q}_{1i}, -\bar{R}_{1i}\bar{R}_5^{-1}\bar{R}_{1i}$ and $-\bar{R}_{1i}\bar{R}_6^{-1}\bar{R}_{1i}$ with $-2\varepsilon_1\bar{Q}_{1i} + \varepsilon_1^2\bar{Q}_5, -2\varepsilon_2\bar{Q}_{1i} + \varepsilon_2^2\bar{Q}_6, -2\varepsilon_3\bar{R}_{1i} + \varepsilon_3^2\bar{R}_5$ and $-2\varepsilon_4\bar{R}_{1i} + \varepsilon_4^2\bar{R}_6$ into (38), respectively, we obtain

$$\bar{\Psi}(l,s) = \begin{bmatrix} \bar{\Psi}_{11} & * & * & * & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * & * & * & * \\ \bar{\Psi}_{31} & \bar{\Phi}_{32} & \bar{\Phi}_{33} & * & * & * & * \\ \Phi_{41}(l) & 0 & 0 & \Phi_{44} & * & * & * \\ 0 & \Phi_{52}(s) & 0 & 0 & \Phi_{55} & * & * \\ 0 & 0 & \Phi_{63}(l) & 0 & 0 & \Phi_{66} & * \\ 0 & 0 & 0 & \Phi_{74}(s) & 0 & 0 & \Phi_{77} \end{bmatrix} < 0, \quad (l,s = 1,2,3,4) \tag{39}$$

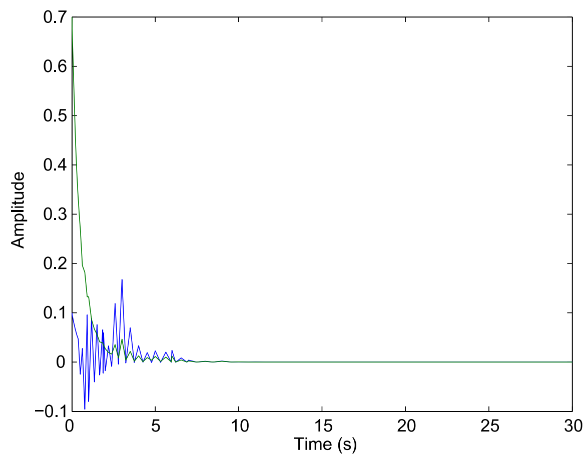

Fig. 3. State trajectory $p(t)$ in (4).

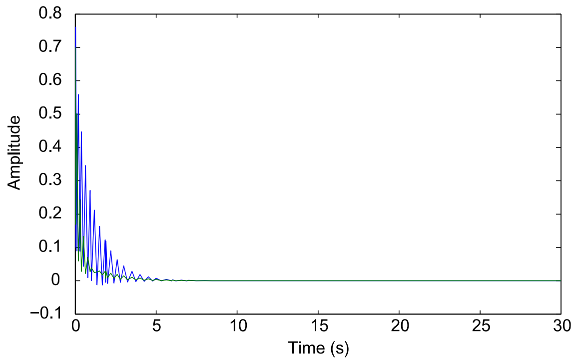

Fig. 4. State estimator $\hat{m}(t)$ in (6).

Denoting $Y_{1i} = \bar{Q}_{1i}K_{1i},\ Y_{2i} = \bar{R}_{1i}K_{2i}$, then Eq. (37) can be obtained. Furthermore, the explicit expression of the desired state estimator gain matrix is $K_{1i} = \bar{Q}_{1i}^{-1}Y_{1i},\ K_{2i} = \bar{R}_{1i}^{-1}Y_{2i}$.

4. Simulation examples

In this section, we present an example to illustrate the effectiveness of the state estimator for the Markovian jumping genetic networks with time-varying delays.

Consider the following uncertain Markovian genetic regulatory networks (8) with two modes [17]:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\quad W_1 = \begin{bmatrix} 1 & -2 \\ 0.8 & 0 \end{bmatrix},\quad C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\quad D_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix},\quad W_2 = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix},\quad C_1 = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix},\quad D_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$
$$M = [-1 \quad 2],\quad N = [-1 \quad 1]$$

In this example, the regulation function is taken as $g(x) = \frac{x^2}{1+x^2}$, one can get $k = 0.65$. The time-varying delays are assumed to be $\tau_m = 0.2, \tau_M = 3.8525, \tau_1 = 0.5,\ \ \sigma_1 = 0.3, \sigma_m = 0.1, \sigma_M = 0.5$. The transmission probability is assumed to be $\Pi = \begin{bmatrix} -3 & 3 \\ 1 & -1 \end{bmatrix}$.

Set $\alpha_0 = 0.2, \beta_0 = 0.7, e_1 = 1; e_2 = 1$. Then, combine (37) and $K_{1i} = \bar{Q}_{1i}^{-1}Y_{1i}, K_{2i} = \bar{R}_{1i}^{-1}Y_{2i}$, the desired estimator parameters can be designed as

$$K_{11} = \begin{bmatrix} 1.2468 \\ 1.3479 \end{bmatrix},\quad K_{21} = \begin{bmatrix} 1.1427 \\ 1.7902 \end{bmatrix},\quad K_{12} = \begin{bmatrix} -1.1452 \\ 1.1136 \end{bmatrix},\quad K_{22} = \begin{bmatrix} -1.0148 \\ 1.2103 \end{bmatrix} \tag{40}$$

Choose the initial conditions $m(0) = \hat{m}(0) = \begin{bmatrix} 0.8 \\ 0.2 \end{bmatrix},\ p(0) = \hat{p}(0) = \begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix}$, the probabilities of switching between modes can be seen from Figs. 1. The state trajectory $m(t)$ and $p(t)$ are shown in Figs. 2 and 3, respectively. The State estimator $\hat{m}(t)$ and $\hat{p}(t)$ are shown in Figs. 4 and 5, respectively. The output errors $\bar{z}_m(t)$ and $\bar{z}_p(t)$ are shown in Figs. 6 and 7, respectively. From Figs. 6 and 7, we can see the designed state estimator performs well.

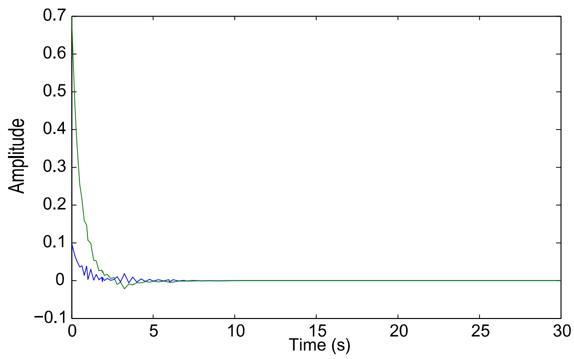

Fig. 5. State estimator $\hat{p}(t)$ in (6).

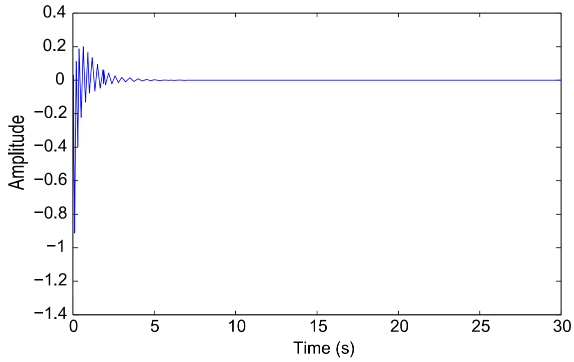

Fig. 6. Estimation error trajectory $\bar{z}_m(t)$.

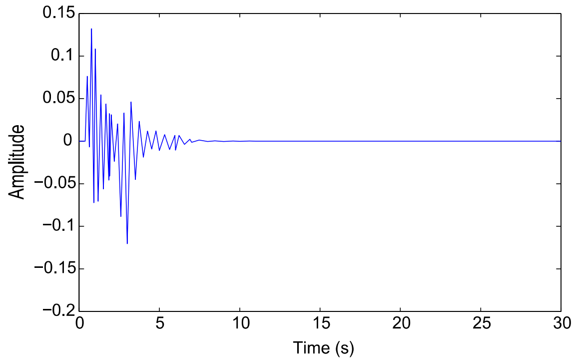

Fig. 7. Estimation error trajectory $\bar{z}_p(t)$.

5. Conclusion

In this paper, we have studied the state estimation problem of a Markovian jumping genetic networks with time-varying delays. By using the free-weighting matrix method and the LMI techniques, stability conditions have been developed in terms of LMIs which guarantee the estimation error dynamics to be asymptotically stable. Then,the explicit expression of the desired estimate gains are shown. Finally, a numerical example is given to demonstrate the effectiveness of the proposed designed method.

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References

[1] Zhang W, Fang J, Tang Y. Robust stability for genetic regulatory networks with linear fractional uncertainties. Commun Nonlinear Sci Numer Simul 2012;17(4):1753–65.
[2] Wang W, Zhong Sh. Delay-dependent stability criteria for genetic regulatory networks with time-varying delays and nonlinear disturbance. Commun Nonlinear Sci Numer Simul 2012;17(9):3597–611.
[3] Koo JH, Ji DH, Won SC, Park Ju H. An improved robust delay-dependent stability criterion for genetic regulatory networks with interval time delays. Commun Nonlinear Sci Numer Simul 2012;17(8):3399–405.
[4] Liu J, Yue D. Asymptotic and robust stability of *t–s* fuzzy genetic regulatory networks with time-varying delays. Int J Robust Nonlinear Control 2012;22(8):827–40.
[5] Friedman N, Linial M, Nachman I, Pe'er D. Using bayesian networks to analyze expression data. J Comput Biol 2000;7(3–4):601–20.
[6] Hardy S, Robillard PN. Modeling and simulation of molecular biology systems using petri nets: modeling goals of various approaches. J Bioinf Comput Biol 2004;2(04):619–37.
[7] Somogyi R, Sniegoski CA. Modeling the complexity of genetic networks: understanding multigenic and pleiotropic regulation. Complexity 1996;1(6):45–63.

[8] Smolen P, Baxter DA, Byrne JH. Mathematical modeling of gene networks. Neuron 2000;26(3):567–80.
[9] Wang T, Ding Y, Zhang L, Hao K. Robust state estimation for discrete-time stochastic genetic regulatory networks with probabilistic delays. Neurocomputing 2013;111:1–12.
[10] Wang Z, Liu Y, Liu X, Shi Y. Robust state estimation for discrete-time stochastic neural networks with probabilistic measurement delays. Neurocomputing 2010;74(1):256–64.
[11] Zhang X, Yu A, Zhang G. M-matrix-based delay-range-dependent global asymptotical stability criterion for genetic regulatory networks with time-varying delays. Neurocomputing 2013;113:8–15.
[12] Wang W, Zhong S, Kiong Nguang S, Liu F. Novel delay-dependent stability criterion for uncertain genetic regulatory networks with interval time-varying delays. Neurocomputing 2013;121:170–8.
[13] Zhu Y, Zhang Q, Wei Z, Zhang L. Robust stability analysis of markov jump standard genetic regulatory networks with mixed time delays and uncertainties. Neurocomputing 2013;110:44–50.
[14] Zhang W, Fang J-A, Tang Y. Stochastic stability of markovian jumping genetic regulatory networks with mixed time delays. Appl Math Comput 2011;217(17):7210–25.
[15] Liu A, Yu L, Zhang D, Zhang W-A. Finite-time H_∞ control for discrete-time genetic regulatory networks with random delays and partly unknown transition probabilities. J Franklin Inst 2013;350(7):1944–61.
[16] Li H, Zhou Q, Chen B, Liu H. Parameter-dependent robust stability for uncertain markovian jump systems with time delay. J Franklin Inst 2011;348(4):738–48.
[17] Liu J, Yue D. Asymptotic stability of markovian jumping genetic regulatory networks with random delays. Chin J Electron 2013;22(2):263–8.
[18] Lakshmanan S, Park JH, Jung H, Balasubramaniam P, Lee S. Design of state estimator for genetic regulatory networks with time-varying delays and randomly occurring uncertainties. Biosystems 2013;111(1):51–70.
[19] Yue D, Tian E, Zhang Y. A piecewise analysis method to stability analysis of linear continuous/discrete systems with time-varying delay. Int J Robust Nonlinear Control 2009;19(13):1493–518.
[20] Liu J, Yao B, Gu Z. Delay-dependent H_∞ filtering for markovian jump time-delay systems: a piecewise analysis method. Circuits Syst Signal Process 2011;30(6):1253–73.
[21] Mao X. Exponential stability of stochastic delay interval systems with markovian switching. IEEE Trans Autom Control 2002;47(10):1604–12.
[22] Wang Y, Xie L, de Souza C. Robust control of a class of uncertain nonlinear systems. Syst Control Lett 1992;19(2):139–49.
[23] Yue D, Tian E, Zhang Y, Peng C. Delay-distribution-dependent stability and stabilization of $T-S$ fuzzy systems with probabilistic interval delay. IEEE Trans Syst Man Cybern Part B: Cybern 2009;39(2):503–16.
[24] Yue D, Han Q, Lam J. Network-based robust H_∞ control of systems with uncertainty. Automatica 2005;41(6):999–1007.
[25] He Y, Wu M, She J, Liu G. Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties. IEEE Trans Autom Control 2004;49(5):828–32.