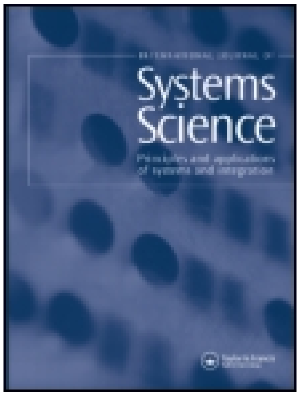

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State estimation for complex systems with randomly occurring nonlinearities and randomly missing measurements

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State estimation for complex systems with randomly occurring nonlinearities and randomly missing measurements

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This paper is concerned with the state estimation problem for the complex networked systems with randomly occurring nonlinearities and randomly missing measurements. The nonlinearities are included to describe the phenomena of nonlinear disturbances which exist in the network and may occur in a probabilistic way. Considering the fact that probabilistic data missing may occur in the process of information transmission, we introduce the randomly data missing into the sensor measurements. The aim of this paper is to design a state estimator to estimate the true states of the considered complex network through the available output measurements. By using a Lyapunov functional and some stochastic analysis techniques, sufficient criteria are obtained in the form of linear matrix inequalities under which the estimation error dynamics is globally asymptotically stable in the mean square. Furthermore, the state estimator gain is also obtained. Finally, a numerical example is employed to illustrate the effectiveness of the proposed state estimation conditions.

Keywords: complex networks; state estimation; coupling configuration matrix; randomly occurring nonlinearities

1. Introduction

Dynamic analysis for complex networks has been an active field of research in recent years due to their theoretical importance as well as the extensive applications of these systems in many areas (Bose, Buchberger, & Guiver, 2003; Ding, Wang, Shen, & Shu, 2012; Li, Ning, Yin, & Tang, 2012; Liang, Wang, Liu, & Liu, 2012; Liu, 2013). The main reason lies in the fact that complex networks serve as natural models for many practical systems such as social sciences, power grid networks, coupled mechanical systems, information networks, etc. (Liang, Wang, & Liu, 2011). As the topic has attracted an ever-increasing attention from a variety of communities such as mathematicians, computer scientists, statisticians and control engineers, a rich body of literature can be found on the general topic of complex networks and applications (Liang et al., 2012; Liu, Wang, Liang, & Liu, 2008). Different kinds of issues have been extensively investigated for complex networks, for example, the stability and stabilisation, synchronisation, pinning control and state estimation (Li et al., 2012; Liang et al., 2011; Song & Cao, 2010; Wang, Wang, & Liu, 2010). As one of the mostly investigated dynamical behaviours, the state estimation in complex networks has drawn significant research interest in recent years (see Liang et al., 2012; Ding et al., 2012; Liang et al., 2011; Liang, Wang, Shen, & Liu, 2012, and the references therein).

In complex networks, the connections between the nodes can be represented in terms of nodes, edges and coupling strengths. However, the weight of the connection between nodes are often affected by certain disturbance and capacitance values, which include uncertainties (modelling errors) subject to stochastic disturbances as well as limited communication constraints. Hence, there is a need to investigate ways to understand the stochastic influence, network-induced delay and stability.

It is well known that the complex networks are often subject to noisy environments and the random phenomena are often unavoidable because of connections over communication channels, such as random communication delay, random measurements and random packet losses. For complex networks, the nonlinear disturbances may go through a set of switches as a result of abrupt phenomena such as random failures and repairs of the components, changes in the interconnections of subsystems, sudden environment changes, etc. In other words, the nonlinear disturbances may appear in a probabilistic way and are randomly changeable. On the other hand, each sensor node has wireless communication capability as well as some level of intelligence for signal processing and disseminating data. However, the limited energy, computational power and communication resources of the sensor nodes will inevitably lead to communication constraints. That is, the data missing also may

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occur randomly. In order to reflect more realistic dynamical behaviours, many researchers have recently investigated the random phenomena (Ding et al., 2012; Liu, Huang, Shi, & Xu, 2013; Wang, Shen, Shu, & Wei, 2012; Wang, Wang, & Liang, 2009; Wei, Wang, Shen, & Li, 2011). For example, in Wang et al. (2009), the synchronisation problem for a new class of continuous-time-delayed complex networks with stochastic nonlinearities, interval time-varying delays, unbounded distributed delays and multiple stochastics is investigated; in Ding et al. (2012), the state estimation problem was investigated, by employing the Lyapunov stability theory, some sufficient conditions were established in the form of linear matrix inequalities (LMIs) and the explicit expression of the estimator gains was given. Unfortunately, to the best of the authors' knowledge, very little research attention has been paid to the continuous-time-delayed complex networks, including stochastic nonlinearities and randomly data missing. Therefore, the main purpose of this paper is to consider the state estimation problems for the continuous-time-delayed complex networks, including stochastic nonlinearities and randomly data missing. It is therefore the purpose of this paper to shorten such a gap.

Motivated by the above discussion, in this paper, we focused on the state estimation problems for the continuous-time-delayed complex networks, including stochastic nonlinearities and randomly data missing. The main contributions of this paper lie in the new research problem and can be summarised as follows. (1) The stochastic nonlinearities are described by the randomly switching sequences. (2) The probabilistic missing measurements occur in a probabilistic way, which account for the random data missing during the signal transmission or information collection among the sensor networks; specifically, our aim is to derive sufficient conditions for the addressed problem by employing a Lyapunov functional, the free-weighting approach and the stochastic analysis techniques. Then, the state estimate gain can be designed. (3) It should be pointed out that the sufficient conditions are in the form of LMIs that can be solved by using the standard numerical software.

The rest of this paper is organised in the following way. In Section 2, the problem addressed is presented and some preliminaries are briefly provided. In Section 3, a sufficient condition is established in terms of LMIs and the explicit expression of the estimator gains is given. In Section 3, a numerical example is provided to demonstrate the effectiveness of the main results obtained.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n \times m$ real matrices; the superscript 'T' stands for matrix transposition; I is the identity matrix of appropriate dimension; $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate; the notation $X > 0$ ($X \geq 0$), for $X \in \mathbb{R}^{n\times n}$, means that the matrix X is real symmetric positive definite (positive semi-definite). x is a stochastic variable. For a matrix B and two symmetric matrices A and C, $\left[\begin{smallmatrix} A & * \\ B & C \end{smallmatrix}\right]$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System description

Consider the following stochastic complex networks consisting of N coupled nodes with time-varying delays:

$$\begin{aligned}\dot{x}_i(t) &= \delta(t)Af_1(x_i(t)) + (1-\delta(t))Bf_2(x_i(t)) \\ &\quad + \sum_{j=1}^{N} g_{ij}\Gamma_1 x_j(t) + \sum_{j=1}^{N} g_{ij}\Gamma_2 x_j(t-\tau(t)), \quad (1)\end{aligned}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T$ is the state vector of the ith node, A and B are constant matrices with appropriate dimensions, $f_1(\cdot)$ and $f_2(\cdot)$ are nonlinear vector-valued functions satisfying certain conditions given later, and Γ_1 and Γ_2 are the inner coupling matrices of the network from the vertical and the horizontal directions, respectively. $G = (g_{ij}) \in \mathbb{R}^{N\times N}$ is the coupling configuration matrix denoting the topological structures of the complex network, if there is a connection between nodes i and j ($i \neq j$); then, $g_{ij} = g_{ji} = 1$, otherwise $g_{ij} = g_{ji} = 0$. The diagonal elements of matrix G are defined as $g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij} (i = 1, 2, \ldots, N)$. $\tau(t)$ denotes the time-varying delay that satisfies $\tau_1 \leq \tau(t) \leq \tau_2$ with $0 \leq \tau_1 \leq \tau_2$ being known positive integers. $\delta(t)$ are Bernoulli distributed white sequences governed by $\text{Prob}\{\delta(t) = 1\} = \delta_0$, $\text{Prob}\{\delta(t) = 0\} = 1 - \delta_0$, where δ_0 are known constants. Obviously, the stochastic variable $\delta(t)$ has the variance $\delta_0(1 - \delta_0)$.

Remark 1: As is well known, the randomly occurring phenomena have been extensively investigated in recent years for discrete-time systems (Liang et al., 2011), complex networks (Ding et al., 2012), as well as Markovian jump systems (Dong, Wang, Ho, & Gao, 2011; Wang, Liu, Yu, & Liu, 2006; Ma & Jia, in press). Nevertheless, the interesting phenomenon studied in this paper has not been investigated. For the addressed complex networks (1), the nonlinear functions $f_1(x_i(t))$ and $f_2(x_i(t))$ affect the dynamics of the complex system through a probabilistic way described by Bernoulli random variables $\delta(t)$. Moreover, $\sum_{j=1}^{N} g_{ij}\Gamma_1 x_j(t)$ and $\sum_{j=1}^{N} g_{ij}\Gamma_2 x_j(t-\tau(t))$ are used to describe the complex network system.

Assumption 1:

$$\begin{aligned}&[f_1(u) - f_1(v) - \Xi_1(u-v)]^T[f_1(u) - f_1(v) \\ &\quad -\Xi_2(u-v)] \leq 0, \\ &[f_2(u) - f_2(v) - \Xi_3(u-v)]^T[f_2(u) - f_2(v) \\ &\quad -\Xi_4(u-v)] \leq 0,\end{aligned}$$

where $\Xi_i (i = 1, 2, 3, 4)$ *are known real constant matrices with appropriate dimensions.*

Remark 2: On the basis of Assumption 1, we can get

$$\begin{bmatrix} x_i(t) \\ f_1(x_i(t)) \end{bmatrix}^T \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & I_n \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_1(x_i(t)) \end{bmatrix} \leq 0,$$
$$\begin{bmatrix} x_i(t) \\ f_2(x_i(t)) \end{bmatrix}^T \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} \\ \bar{\Omega}_{21} & I_n \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_2(x_i(t)) \end{bmatrix} \leq 0,$$

where

$$\Omega_{11} = \frac{\Xi_1^T \Xi_2 + \Xi_2^T \Xi_1}{2}, \ \Omega_{21}^T = \Omega_{12} = -\frac{\Xi_1^T + \Xi_2^T}{2},$$
$$\bar{\Omega}_{11} = \frac{\Xi_3^T \Xi_4 + \Xi_4^T \Xi_3}{2}, \ \bar{\Omega}_{21}^T = \bar{\Omega}_{12} = -\frac{\Xi_3^T + \Xi_4^T}{2}.$$

In this paper, the sensor measurements of the complex networks (1) are modelled by

$$y(t) = \Xi C x(t) = \sum_{i=1}^{m} \xi_i E_i C x(t), \tag{2}$$

where $y(t) \in \mathbb{R}^m$ is the actual measured output vector and C is a known constant real matrix with appropriate dimensions. $\Xi = \text{diag}\{\xi_1, \xi_2, \ldots, \xi_m\}$, where $\xi_i (i = 1, 2, \ldots, m)$ are unrelated stochastic variables taking values on [0, 1]. The mathematical expectation and variance of ξ_i are $\bar{\xi}_i$ and σ_i^2, respectively. $E_i = \text{diag}\{\underbrace{0, \ldots, 0}_{i-1}, 1, \underbrace{0, \ldots, 0}_{m-i}\}$.

Remark 3: In the sensor measurement equation (2), random variables ξ_i are used to describe the phenomenon of probabilistic data missing occurring in the process of information transmission from the system to each sensor. Considering the case of data missing, we assume the variables $\xi_i (i = 1, 2, \ldots, m)$ take values in the interval [0, 1].

By using the Kronecker product, the stochastic complex networks (1) can be rewritten as

$$\begin{aligned} \dot{x}(t) &= \delta(t)(I_N \otimes A)F_1(x(t)) + (1 - \delta(t))(I_N \otimes B)F_2(x(t)) \\ &\quad + (G \otimes \Gamma_1)x(t) + (G \otimes \Gamma_2)x(t - \tau(t)), \end{aligned} \tag{3}$$

where

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix}, F_1(x(t)) = \begin{bmatrix} f_1(x_1(t)) \\ f_1(x_2(t)) \\ \vdots \\ f_1(x_N(t)) \end{bmatrix},$$
$$F_2(x(t)) = \begin{bmatrix} f_2(x_1(t)) \\ f_2(x_2(t)) \\ \vdots \\ f_2(x_N(t)) \end{bmatrix}.$$

In this paper, based on the measurement $y(t)$, we construct the following state estimator for the stochastic complex network (3):

$$\begin{aligned} \dot{\hat{x}}(t) &= (G \otimes \Gamma_1)\hat{x}(t) + (G \otimes \Gamma_2)\hat{x}(t - \tau(t)) \\ &\quad + K(y(t) - \bar{\Xi} C \hat{x}(t)), \end{aligned} \tag{4}$$

where $\dot{\hat{x}}(t) \in \mathbb{R}^{N \times n}$ is the estimation of the network state $x(t)$, $K \in \mathbb{R}^{(N \times n) \times m}$ is the estimate gain matrix to be designed and $\bar{\Xi} = \text{diag}\{\bar{\xi}_1, \bar{\xi}_2, \ldots, \bar{\xi}_m\}$ is the mathematical expectation of Ξ.

By setting the estimation error $e(t) = x(t) - \hat{x}(t)$, the error dynamics of the state estimation can be obtained from (2)–(4) as follows:

$$\begin{aligned} \dot{e}(t) &= \delta(t)(I_N \otimes A)F_1(x(t)) + (1 - \delta(t))(I_N \otimes B)F_2(x(t)) \\ &\quad + (G \otimes \Gamma_1 - K\bar{\Xi}C)e(t) + (G \otimes \Gamma_2)e(t - \tau(t)) \\ &\quad - K(\Xi - \bar{\Xi})Cx(t). \end{aligned} \tag{5}$$

Then, by setting

$$\begin{aligned} \eta^T(t) = \big[F_1^T(x(t)) \quad F_2^T(x(t)) \quad e^T(t) \quad e^T(t - \tau_1) \\ e^T(t - \tau(t)) \quad e^T(t - \tau_2) \; x^T(t) \big]^T \end{aligned}$$

we can rewrite (5) as follows:

$$\dot{e}(t) = \delta(t)\mathcal{A}_1\eta(t) + (1 - \delta(t))\mathcal{A}_2\eta(t) - K(\Xi - \bar{\Xi})Cx(t), \tag{6}$$

where

$$\mathcal{A}_1 = \left[I_N \otimes A \; 0 \; (G \otimes \Gamma_1) - K\bar{\Xi}C \; 0 \; (G \otimes \Gamma_2) \; 0 \; 0 \right],$$
$$\mathcal{A}_2 = \left[0 \; I_N \otimes B \; (G \otimes \Gamma_1) - K\bar{\Xi}C \; 0 \; (G \otimes \Gamma_2) \; 0 \; 0 \right].$$

The purpose of this paper is to choose a suitable estimator gain matrix K and establish some easy-to-verify criteria such that the state estimation approaches the state vector $x(t)$ of the network (3) globally asymptotically in the mean square.

Before giving the main result, we will first introduce the following definition and lemmas, which will help us in deriving the main results.

Definition 1: The system (6) is said to be a globally asymptotic state estimator in the mean square for the stochastic complex networks (3) with sensor measurements (2) if

$$\lim_{t \to +\infty} \mathbb{E}\{\| x(t) - \hat{x}(t) \|^2\} = 0. \tag{7}$$

Lemma 1 (Wang, Xie, & de Souza, 1992)**:** *For any vectors $x, y \in R^n$, and positive-definite matrix $Q \in R^{n \times n}$, the following inequality holds:*

$$2x^T y \leq x^T Q x + y^T Q^{-1} y.$$

Lemma 2 (Han & Yue, 2007)**:** *For any constant positive matrix $R \in \mathbb{R}^{n\times n}$, $\tau_1 \le \tau(t) \le \tau_2$ and vector function $x(t) \in \mathbb{R}^n$, such that the following integration is well defined, the following inequality holds:*

$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1} \dot{x}^T(s)R\dot{x}(s)ds \le \begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}^T \times \begin{bmatrix} -R & * \\ R & -R \end{bmatrix}\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau_2) \end{bmatrix}.$$

$$\Phi_{11} = \begin{bmatrix} -\Lambda_1\otimes I_n & * & * & * & * & * & * \\ 0 & -\Lambda_2\otimes I_n & * & * & * & * & * \\ \delta_0 P I_A & \delta_{10} P I_B & \Delta_1 & * & * & * & * \\ 0 & 0 & R_1 & -Q_1-R_1 & * & * & * \\ 0 & 0 & G_{\Gamma_2}^T P & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q_2 & * \\ -(\Lambda_1\otimes\Omega_{21})^T & -(\Lambda_2\otimes\bar{\Omega}_{21})^T & 0 & 0 & 0 & 0 & -\Lambda_1\otimes\Omega_{11}-\Lambda_2\otimes\bar{\Omega}_{11} \end{bmatrix},$$

$$\Phi_{21} = \begin{bmatrix} \tau_1\sqrt{\delta_0}R_1 I_A & 0 & \tau_1\sqrt{\delta_0}R_1\Pi_1 & 0 & \tau_1\sqrt{\delta_0}R_1 G_{\Gamma_2} & 0 & 0 \\ \sqrt{\delta_0\tau_{21}}R_2 I_A & 0 & \sqrt{\delta_0\tau_{21}}R_2\Pi_1 & 0 & \sqrt{\delta_0\tau_{21}}R_2 G_{\Gamma_2} & 0 & 0 \\ 0 & \tau_1\sqrt{\delta_{10}}R_1 I_B & \tau_1\sqrt{\delta_{10}}R_1\Pi_1 & 0 & \tau_1\sqrt{\delta_{10}}R_1 G_{\Gamma_2} & 0 & 0 \\ 0 & \sqrt{\delta_{10}}\sqrt{\tau_{21}}R_2 I_B & \sqrt{\delta_{10}}\sqrt{\tau_{21}}R_2\Pi_1 & 0 & \sqrt{\delta_{10}}\sqrt{\tau_{21}}R_2 G_{\Gamma_2} & 0 & 0 \end{bmatrix},$$

Lemma 3 (Yue, Tian, Zhang, & Peng, 2009)**:** *Ξ_1, Ξ_2 and Ω are matrices with appropriate dimensions, $\tau(t)$ is a function of t and $\tau_1 \le \tau(t) \le \tau_2$, then*

$$[(\tau(t)-\tau_1)\Xi_1 + (\tau_2-\tau(t))\Xi_2] + \Omega < 0$$

if and only if the following two inequalities hold:

$$(\tau_2-\tau_1)\Xi_1 + \Omega < 0,$$
$$(\tau_2-\tau_1)\Xi_2 + \Omega < 0.$$

3. Main results

In this section, we will invest the estimation problem for the stochastic complex networks (1) with sensor measurements (2). A sufficient condition is established to ensure the estimation error to be globally asymptotically stable in the mean square. Then, according to the analysis results, the methods to design the estimator gain matrix K are derived in terms of the solution to certain matrix inequalities.

Theorem 1: *Let some scalars $0 \le \tau_1 \le \tau_2$ and the estimator gain matrix K in* (4) *be given, then the estimation error dynamics* (6) *is globally asymptotically stable in the mean square if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$ and N, M, Λ_1, Λ_2 with appropriate dimensions, such that the following matrix inequalities hold for $s = 1, 2$:*

$$\begin{bmatrix} \Phi_{11}+\Upsilon+\Upsilon^T & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ \Phi_{31} & 0 & \Phi_{33} & * \\ \Phi_{41}(s) & 0 & 0 & -R_2 \end{bmatrix} < 0, \quad (8)$$

where

$$\Phi_{31} = \begin{bmatrix} \Theta_{11}^T & \Theta_{12}^T & \cdots & \Theta_{1m}^T & \Theta_{21}^T & \Theta_{22}^T & \cdots & \Theta_{2m}^T \end{bmatrix}^T,$$
$$\Phi_{41}(1) = \sqrt{\tau_{21}}N^T,\ \Phi_{41}(2) = \sqrt{\tau_{21}}M^T,$$
$$\Phi_{22} = \mathrm{diag}\{-R_1, -R_2, -R_1, -R_2\},$$
$$\Phi_{33} = \mathrm{diag}\{\underbrace{-R_1,\ldots,-R_1}_{m}, \underbrace{-R_2,\ldots,-R_2}_{m}\},$$
$$N = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T & N_7^T \end{bmatrix}^T,$$
$$M = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T & M_7^T \end{bmatrix}^T,$$
$$\Delta_1 = -R_1 + Q_1 + Q_2 + P\Pi_1 + \Pi_1^T P,$$
$$\Upsilon = \begin{bmatrix} 0 & 0 & 0 & N & -N+M & -M & 0 & 0 \end{bmatrix},$$
$$\Theta_{1i} = \begin{bmatrix} 0_{1\times 6} & \tau_1\sigma_i R_1 K E_i C & 0 \end{bmatrix},$$
$$\Theta_{2i} = \begin{bmatrix} 0_{1\times 6} & \sqrt{\tau_{21}}\sigma_i R_2 K E_i C & 0 \end{bmatrix} (i = 1,\ldots,m),$$
$$I_A = I_N\otimes A,\ I_B = I_N\otimes B,\ G_{\Gamma_2} = G\otimes\Gamma_2,$$
$$\tau_{21} = \tau_2-\tau_1,\ \delta_{10} = 1-\delta_0,\ \Pi_1 = (G\otimes\Gamma_1) - K\bar{\Xi}C.$$

Proof: Choose the following Lyapunov function for the system (6):

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (9)$$

with

$$V_1(t) = e^T(t)Pe(t),$$
$$V_2(t) = \int_{t-\tau_1}^{t} e^T(s)Q_1 e(s)ds + \int_{t-\tau_2}^{t} e^T(s)Q_2 e(s)ds,$$

$$V_3(t) = \tau_1 \int_{t-\tau_1}^{t} \int_{s}^{t} \dot{e}^T(v) R_1 \dot{e}(v) dv ds + \int_{t-\tau_2}^{t-\tau_1} \int_{s}^{t} \dot{e}^T(v) R_2 \dot{e}(v) dv ds,$$

where P, Q_1, Q_2, R_1, R_2 are symmetric positive-definite matrices.

Taking the time derivative of $V(t)$ along the trajectory of the system (6), and taking expectation on it, we obtain

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(t)\} &= 2e^T(t)P[\delta_0 \mathcal{A}_1 \eta(t) + (1-\delta_0)\mathcal{A}_2 \eta(t)] \\ &+ e^T(t)(Q_1+Q_2)e(t) - e^T(t-\tau_1)Q_1 e(t-\tau_1) \\ &- e^T(t-\tau_2)Q_2 e(t-\tau_2) + \mathbb{E}\{\dot{e}^T(t)(\tau_1^2 R_1 + \tau_{21} R_2)\dot{e}(t)\} \\ &- \tau_1 \int_{t-\tau_1}^{t} \dot{e}^T(s) R_1 \dot{e}(s) ds - \int_{t-\tau_2}^{t-\tau_1} \dot{e}^T(s) R_2 \dot{e}(s) ds. \end{aligned} \tag{10}$$

By Lemma 2, we can get

$$-\tau_1 \int_{t-\tau_1}^{t} \dot{e}^T(s) R_1 \dot{e}(s) ds \le \begin{bmatrix} e(t) \\ e(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & * \\ R_1 & -R_1 \end{bmatrix} \times \begin{bmatrix} e(t) \\ e(t-\tau_1) \end{bmatrix}. \tag{11}$$

Notice that

$$\begin{aligned}\mathbb{E}\{\dot{e}^T(t)\tilde{R}\dot{e}(t)\} &= \delta_0 \eta^T(t)\mathcal{A}_1^T \tilde{R} \mathcal{A}_1 \eta(t) \\ &+ (1-\delta_0)\eta^T(t)\mathcal{A}_2^T \tilde{R}\mathcal{A}_2 \eta(t) \\ &+ \sum_{i=1}^{n} \sigma_i^2 x^T(t) C^T E_i^T K^T \tilde{R} K E_i C x(t), \end{aligned} \tag{12}$$

where $\tilde{R} = \tau_1^2 R_1 + \tau_{21} R_2$.

Based on Assumption 1 and Remark 2, we can know there exist Λ_1 and Λ_2, such that

$$\begin{bmatrix} x(t) \\ F_1(x(t)) \end{bmatrix}^T \begin{bmatrix} \Lambda_1 \otimes \Omega_{11} & * \\ \Lambda_1 \otimes \Omega_{21} & \Lambda_1 \otimes I_n \end{bmatrix} \begin{bmatrix} x(t) \\ F_1(x(t)) \end{bmatrix} \le 0, \tag{13}$$

$$\begin{bmatrix} x(t) \\ F_2(x(t)) \end{bmatrix}^T \begin{bmatrix} \Lambda_2 \otimes \bar{\Omega}_{11} & * \\ \Lambda_2 \otimes \bar{\Omega}_{21} & \Lambda_2 \otimes I_n \end{bmatrix} \begin{bmatrix} x(t) \\ F_2(x(t)) \end{bmatrix} \le 0. \tag{14}$$

Then, by employing the free weight matrix method (He, Wu, She, & Liu, 2004; Yue, Han, & Lam, 2005), we have

$$2\eta^T(t) N \left[e(t-\tau_1) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_1} \dot{e}(s) ds \right] = 0, \tag{15}$$

$$2\eta^T(t) M \left[e(t-\tau(t)) - e(t-\tau_2) - \int_{t-\tau_2}^{t-\tau(t)} \dot{e}(s) ds \right] = 0. \tag{16}$$

Notice that, by Lemma 1, we can obtain

$$-2\eta^T(t) N \int_{t-\tau(t)}^{t-\tau_1} \dot{e}(s) ds \le (\tau(t)-\tau_1)\eta^T(t) N R_2^{-1} N^T \eta(t) + \int_{t-\tau(t)}^{t-\tau_1} \dot{e}^T(s) R_2 \dot{e}(s) ds, \tag{17}$$

$$-2\eta^T(t) M \int_{t-\tau_2}^{t-\tau(t)} \dot{e}(s) ds \le (\tau_2-\tau(t))\eta^T(t) M R_2^{-1} M^T \eta(t) + \int_{t-\tau_2}^{t-\tau(t)} \dot{e}^T(s) R_2 \dot{e}(s) ds. \tag{18}$$

Combining (10) and (11)–(18), we can obtain that

$$\begin{aligned}&\mathbb{E}\{\mathcal{L}V(t)\} \\ &\le 2e^T(t)P[\delta_0 \mathcal{A}_1 \eta(t) + (1-\delta_0)\mathcal{A}_2 \eta(t)] \\ &\quad + e^T(t)(Q_1+Q_2)e(t) - e^T(t-\tau_1)Q_1 e(t-\tau_1) \\ &\quad - e^T(t-\tau_2)Q_2 e(t-\tau_2) + \delta_0 \eta^T(t)\mathcal{A}_1^T \tilde{R}\mathcal{A}_1 \eta(t) \\ &\quad + (1-\delta_0)\eta^T(t)\mathcal{A}_2^T \tilde{R}\mathcal{A}_2 \eta(t) \\ &\quad + \sum_{i=1}^{n} \sigma_i^2 x^T(t) C^T E_i^T K^T \tilde{R} K E_i C x(t) \\ &\quad + \begin{bmatrix} e(t) \\ e(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R_1 & * \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} e(t) \\ e(t-\tau_1) \end{bmatrix} \\ &\quad + 2\eta^T(t) N [e(t-\tau_1) - e(t-\tau(t))] \\ &\quad + 2\eta^T(t) M [e(t-\tau(t)) - e(t-\tau_2)] \\ &\quad + (\tau(t)-\tau_1)\eta^T(t) N R_2^{-1} N^T \eta(t) \\ &\quad + (\tau_2-\tau(t))\eta^T(t) M R_2^{-1} M^T \eta(t) \\ &\quad + \begin{bmatrix} x(t) \\ F_2(x(t)) \end{bmatrix}^T \begin{bmatrix} -\Lambda_2 \otimes \bar{\Omega}_{11} & * \\ -\Lambda_2 \otimes \bar{\Omega}_{21} & -\Lambda_2 \otimes I_n \end{bmatrix} \begin{bmatrix} x(t) \\ F_2(x(t)) \end{bmatrix} \\ &\quad + \begin{bmatrix} x(t) \\ F_1(x(t)) \end{bmatrix}^T \begin{bmatrix} -\Lambda_1 \otimes \Omega_{11} & * \\ -\Lambda_1 \otimes \Omega_{21} & -\Lambda_1 \otimes I_n \end{bmatrix} \begin{bmatrix} x(t) \\ F_1(x(t)) \end{bmatrix} \\ &\le \eta^T(t)[\Phi_{11} + \Upsilon + \Upsilon^T]\eta(t) + \delta_0 \eta^T(t)\mathcal{A}_1^T \tilde{R}\mathcal{A}_1 \eta(t) \\ &\quad + (1-\delta_0)\eta^T(t)\mathcal{A}_2^T \tilde{R}\mathcal{A}_2 \eta(t) \\ &\quad + \sum_{i=1}^{n} \sigma_i^2 x^T(t) C^T E_i^T K^T \tilde{R} K E_i C x(t) \\ &\quad + (\tau(t)-\tau_1)\eta^T(t) N R_2^{-1} N^T \eta(t) \\ &\quad + (\tau_2-\tau(t))\eta^T(t) M R_2^{-1} M^T \eta(t). \end{aligned} \tag{19}$$

Recalling (8), from (19), we have

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(t)\} \le\ & \eta^T(t)[\Phi_{11} + \Upsilon + \Upsilon^T]\eta(t) \\ & + \delta_0\eta^T(t)\mathcal{A}_1^T\tilde{R}\mathcal{A}_1\eta(t) \\ & + (1-\delta_0)\eta^T(t)\mathcal{A}_2^T\tilde{R}\mathcal{A}_2\eta(t) \\ & + \sum_{i=1}^{n}\sigma_i^2 x^T(t)C^TE_i^TK^T\tilde{R}KE_iCx(t) \\ & + (\tau_2-\tau_1)\eta^T(t)NR_2^{-1}N^T\eta(t) < 0, \end{aligned} \tag{20}$$

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(t)\} \le\ & \eta^T(t)[\Phi_{11} + \Upsilon + \Upsilon^T]\eta(t) \\ & + \delta_0\eta^T(t)\mathcal{A}_1^T\tilde{R}\mathcal{A}_1\eta(t) \\ & + (1-\delta_0)\eta^T(t)\mathcal{A}_2^T\tilde{R}\mathcal{A}_2\eta(t) \\ & + \sum_{i=1}^{n}\sigma_i^2 x^T(t)C^TE_i^TK^T\tilde{R}KE_iCx(t) \\ & + (\tau_2-\tau_1)\eta^T(t)MR_2^{-1}M^T\eta(t) < 0. \end{aligned} \tag{21}$$

Subsequently, by Lemma 3 and the well-known Schur complement, we can conclude that

$$\mathbb{E}\{\mathcal{L}V(t)\} \le 0, \tag{22}$$

that is,

$$\lim_{t\to+\infty}\mathbb{E}\{\| x(t)-\hat{x}(t)\|^2\} = 0. \tag{23}$$

Based on Definition 1, the system (6) is globally asymptotically stable in the mean square.

Remark 4: As mentioned in the Introduction section, complex dynamical networks have received a great deal of attention, and many results on the topic have been available that require the symmetry and the zero-row-sum properties for the configuration matrix G. However, the methods cannot be applied to the state estimation problem with randomly occurring nonlinearities and missing measurements. To overcome this difficulty, we investigated a new more general model, including both randomly occurring nonlinearities and missing measurements. After some rigorous and complex deducing process, the criteria which are used to guarantee the dynamics of the estimation error system (6) to be globally asymptotically stable in the mean square.

Based on Theorem 1, we are now in a position to design the state estimator for the complex networks (1). The following Theorem 2 gives the explicit expression of the estimator gain matrix K.

Theorem 2: *For given scalars $0 \le \tau_1 \le \tau_2$ and ε, the augmented system (6) is globally asymptotically stable in the mean square, if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$ and N, M, Λ_1, Λ_2 with appropriate dimensions satisfying the following LMIs:*

$$\begin{bmatrix} \tilde{\Phi}_{11}+\Upsilon+\Upsilon^T & * & * & * \\ \tilde{\Phi}_{21} & \tilde{\Phi}_{22} & * & * \\ \tilde{\Phi}_{31} & 0 & \tilde{\Phi}_{33} & * \\ \Phi_{41}(s) & 0 & 0 & -R_2 \end{bmatrix} < 0, \quad s = 1, 2, \tag{24}$$

where

$$\tilde{\Phi}_{11} = \begin{bmatrix} -\Lambda_1\otimes I_n & * & * & * & * & * & * \\ 0 & -\Lambda_2\otimes I_n & * & * & * & * & * \\ \delta_0 PI_A & \delta_{10}PI_B & \bar{\Delta}_1 & * & * & * & * \\ 0 & 0 & R_1 & -Q_1-R_1 & * & * & * \\ 0 & 0 & G_{\Gamma_2}^TP & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q_2 & * \\ -(\Lambda_1\otimes\Omega_{21})^T & -(\Lambda_2\otimes\bar{\Omega}_{21})^T & 0 & 0 & 0 & 0 & -\Lambda_1\otimes\Omega_{11}-\Lambda_2\otimes\bar{\Omega}_{11} \end{bmatrix},$$

$$\tilde{\Phi}_{21} = \begin{bmatrix} \tau_1\sqrt{\delta_0}PI_A & 0 & \tau_1\sqrt{\delta_0}\bar{\Pi}_1 & 0 & \tau_1\sqrt{\delta_0}PG_{\Gamma_2} & 0 & 0 \\ \sqrt{\delta_0\tau_{21}}PI_A & 0 & \sqrt{\delta_0\tau_{21}}\bar{\Pi}_1 & 0 & \sqrt{\delta_0\tau_{21}}PG_{\Gamma_2} & 0 & 0 \\ 0 & \tau_1\sqrt{\delta_{10}}PI_B & \tau_1\sqrt{\delta_{10}}\bar{\Pi}_1 & 0 & \tau_1\sqrt{\delta_{10}}PG_{\Gamma_2} & 0 & 0 \\ 0 & \sqrt{\delta_{10}\tau_{21}}PI_B & \sqrt{\delta_{10}\tau_{21}}\bar{\Pi}_1 & 0 & \sqrt{\delta_{10}\tau_{21}}PG_{\Gamma_2} & 0 & 0 \end{bmatrix},$$

$$\begin{aligned} \tilde{\Phi}_{31} &= \begin{bmatrix} \bar{\Theta}_{11}^T & \bar{\Theta}_{12}^T & \cdots & \bar{\Theta}_{1m}^T & \bar{\Theta}_{21}^T & \bar{\Theta}_{22}^T & \cdots & \bar{\Theta}_{2m}^T \end{bmatrix}^T, \\ \bar{\Delta}_1 &= -R_1 + Q_1 + Q_2 + (\bar{\Pi}_1 + \bar{\Pi}_1^T), \\ \bar{\Pi}_1 &= P(G\otimes\Gamma_1) - YC, \\ \bar{\Theta}_{1i} &= \begin{bmatrix} 0_{1\times 6} & \tau_1\sigma_i YE_iC & 0 \end{bmatrix}, \\ \bar{\Theta}_{2i} &= \begin{bmatrix} 0_{1\times 6} & \sqrt{\tau_{21}}\sigma_i YE_iC & 0 \end{bmatrix} (i = 1,\ldots,m), \\ \tilde{\Phi}_{22} &= \mathrm{diag}\{-2\varepsilon_1P+\varepsilon_1^2R_1, -2\varepsilon_2P+\varepsilon_2^2R_2, -2\varepsilon_1P \\ &\quad + \varepsilon_1^2R_1, -2\varepsilon_2P+\varepsilon_2^2R_2\}, \\ \tilde{\Phi}_{33} &= \mathrm{diag}\{\underbrace{-2\varepsilon_1P+\varepsilon_1^2R_1,\ldots,-2\varepsilon_1P+\varepsilon_1^2R_1}_{m}, \\ &\quad \underbrace{-2\varepsilon_2P+\varepsilon_2^2R_2,\ldots,-2\varepsilon_2P+\varepsilon_2^2R_2}_{m}\}, \end{aligned}$$

and the other symbols are defined in Theorem 1. Moreover, if (24) *is true, the desired state estimator gain in* (4) *can be determined by* $K = P^{-1}Y$.

Proof: Combining (8) and (20)–(21), and applying the Schur complement, we can obtain

$$\begin{bmatrix} \tilde{\Phi}_{11K} + \Upsilon + \Upsilon^T & * & * & * \\ \tilde{\Phi}_{21K} & \tilde{\Phi}_{22K} & * & * \\ \tilde{\Phi}_{31K} & 0 & \tilde{\Phi}_{33K} & * \\ \Phi_{41}(s) & 0 & 0 & -R_2 \end{bmatrix} < 0, \quad s = 1, 2, \tag{25}$$

where

$$\tilde{\Phi}_{11K} = \begin{bmatrix} -\Lambda_1 \otimes I_n & * & * & * & * & * & * \\ 0 & -\Lambda_2 \otimes I_n & * & * & * & * & * \\ \delta_0 PI_A & \delta_{10} PI_B & \Delta_{1K} & * & * & * & * \\ 0 & 0 & R_1 & -Q_1 - R_1 & * & * & * \\ 0 & 0 & G_{\Gamma_2}^T P & 0 & 0 & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q_2 & * \\ -(\Lambda_1 \otimes \Omega_{21})^T & -(\Lambda_2 \otimes \bar{\Omega}_{21})^T & 0 & 0 & 0 & 0 & -\Lambda_1 \otimes \Omega_{11} - \Lambda_2 \otimes \bar{\Omega}_{11} \end{bmatrix},$$

$$\tilde{\Phi}_{21K} = \begin{bmatrix} \tau_1\sqrt{\delta_0} PI_A & 0 & \tau_1\sqrt{\delta_0}\bar{\Pi}_{1K} & 0 & \tau_1\sqrt{\delta_0} PG_{\Gamma_2} & 0 & 0 \\ \sqrt{\delta_0\tau_{21}} PI_A & 0 & \sqrt{\delta_0\tau_{21}}\bar{\Pi}_{1K} & 0 & \sqrt{\delta_0\tau_{21}} PG_{\Gamma_2} & 0 & 0 \\ 0 & \tau_1\sqrt{\delta_{10}} PI_B & \tau_1\sqrt{\delta_{10}}\bar{\Pi}_{1K} & 0 & \tau_1\sqrt{\delta_{10}} PG_{\Gamma_2} & 0 & 0 \\ 0 & \sqrt{\delta_{10}\tau_{21}} PI_B & \sqrt{\delta_{10}\tau_{21}}\bar{\Pi}_{1K} & 0 & \sqrt{\delta_{10}\tau_{21}} PG_{\Gamma_2} & 0 & 0 \end{bmatrix},$$

$$\begin{aligned}
\tilde{\Phi}_{31K} &= \begin{bmatrix} \bar{\Theta}_{11K}^T & \bar{\Theta}_{12K}^T & \cdots & \bar{\Theta}_{1mK}^T & \bar{\Theta}_{21K}^T & \bar{\Theta}_{22K}^T & \cdots & \bar{\Theta}_{2mK}^T \end{bmatrix}^T, \\
\Delta_{1K} &= -R_1 + Q_1 + Q_2 + \bar{\Pi}_{1K} + \bar{\Pi}_{1K}^T, \\
\bar{\Pi}_{1K} &= P(G \otimes \Gamma_1) - PKC, \\
\bar{\Theta}_{1iK} &= \begin{bmatrix} 0_{1\times 6} & \tau_1\sigma_i PKE_iC & 0 \end{bmatrix}, \\
\bar{\Theta}_{2iK} &= \begin{bmatrix} 0_{1\times 6} & \sqrt{\tau_{21}}\sigma_i PKE_iC & 0 \end{bmatrix} (i = 1, \ldots, m), \\
\tilde{\Phi}_{22K} &= \mathrm{diag}\{-PR_1^{-1}P, -PR_2^{-1}P, -PR_1^{-1}P, -PR_2^{-1}P\}, \\
\tilde{\Phi}_{33K} &= \mathrm{diag}\{\underbrace{-PR_1^{-1}P, \ldots, -PR_1^{-1}P}_{m}, \underbrace{-PR_2^{-1}P, \ldots, -PR_2^{-1}P}_{m}\}.
\end{aligned}$$

Due to

$$(R - \varepsilon^{-1}P)R^{-1}(R - \varepsilon^{-1}P) \geq 0,$$

we can have

$$-PR^{-1}P \leq -2\varepsilon P + \varepsilon^2 R.$$

Substituting $-PR_i^{-1}P$ with $-2\varepsilon_i P + \varepsilon_i^2 R (i = 1, 2)$ into (25), we obtain

$$\begin{bmatrix} \tilde{\Phi}_{11K} + \Upsilon + \Upsilon^T & * & * & * \\ \tilde{\Phi}_{21K} & \tilde{\Phi}_{22} & * & * \\ \tilde{\Phi}_{31K} & 0 & \tilde{\Phi}_{33} & * \\ \Phi_{41}(s) & 0 & 0 & -R_2 \end{bmatrix} < 0, s = 1, 2. \tag{26}$$

Denoting $Y = PK$, Equation (24) can be obtained. Furthermore, the explicit expression of the desired state estimator gain matrix is $K = P^{-1}Y$.

Remark 5: In this paper, three phenomena, namely time delays, randomly occurring nonlinearities and data missing, have been taken into account. The main results contain all the information of the complex networks, including physical parameters, bounds of the state delays, occurrence probabilities of the nonlinear disturbances and data missing. In the next section, a numerical example is provided to show the usefulness of the proposed design procedure for the desired state estimations.

4. Simulation examples

Consider the following continuous complex network with three coupled nodes:

$$\begin{aligned}
\dot{x}_i(t) &= \delta(t)Af_1(x_i(t)) + (1 - \delta(t))Bf_2(x_i(t)) \\
&\quad + \sum_{j=1}^{N} g_{ij}\Gamma_1 x_j(t) + \sum_{j=1}^{N} g_{ij}\Gamma_2 x_j(t - \tau(t)) \\
&\quad \times (i = 1, 2, 3, 4, 5),
\end{aligned} \tag{27}$$

where

$$x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix}, A = \begin{bmatrix} 0.2 & -0.01 \\ 0 & -0.3 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.2 & -0.1 \\ -0.35 & -0.3 \end{bmatrix},$$

$$f_1(x_i(t)) = \begin{bmatrix} 0.4x_{i1}(t) - \tanh(0.3x_{i2}(t)) \\ 0.9x_{i2}(t) - \tanh(0.7x_{i1}(t)) \end{bmatrix},$$

$$f_2(x_i(t)) = \begin{bmatrix} 0.3x_{i1}(t) - \tanh(0.2x_{i2}(t)) \\ 0.8x_{i2}(t) - \tanh(0.6x_{i1}(t)) \end{bmatrix}.$$

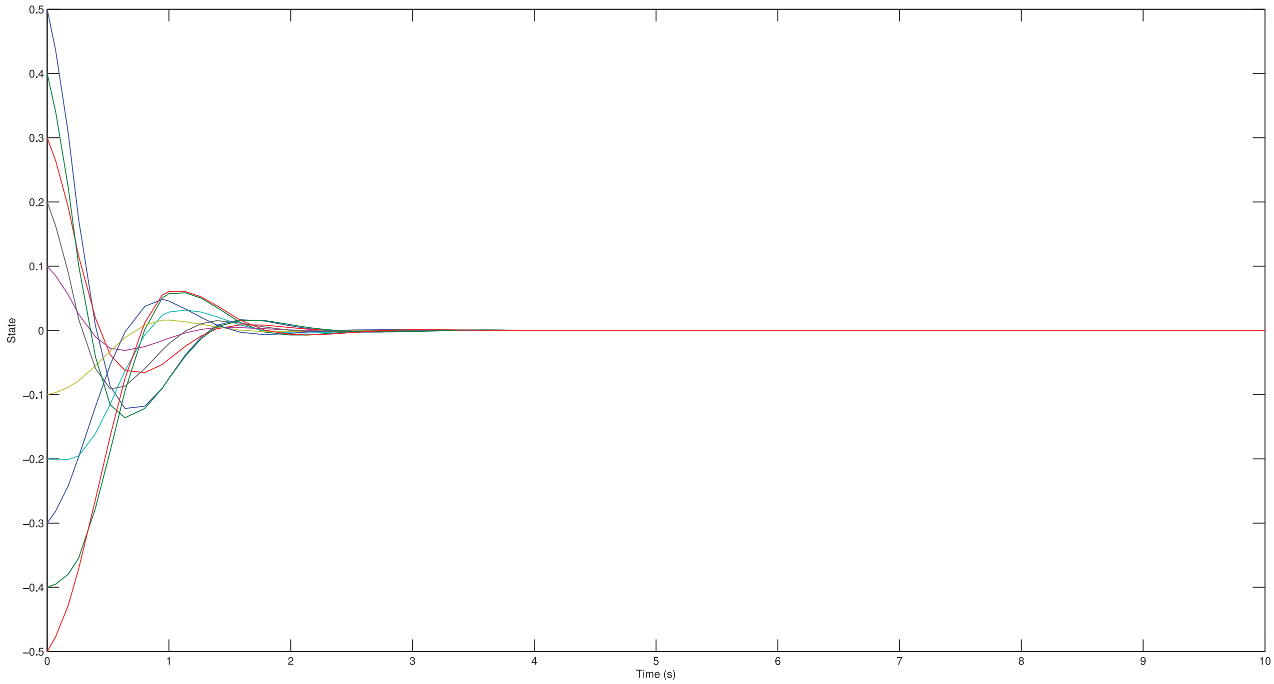

Figure 1. State trajectory $x(t)$ of (3).

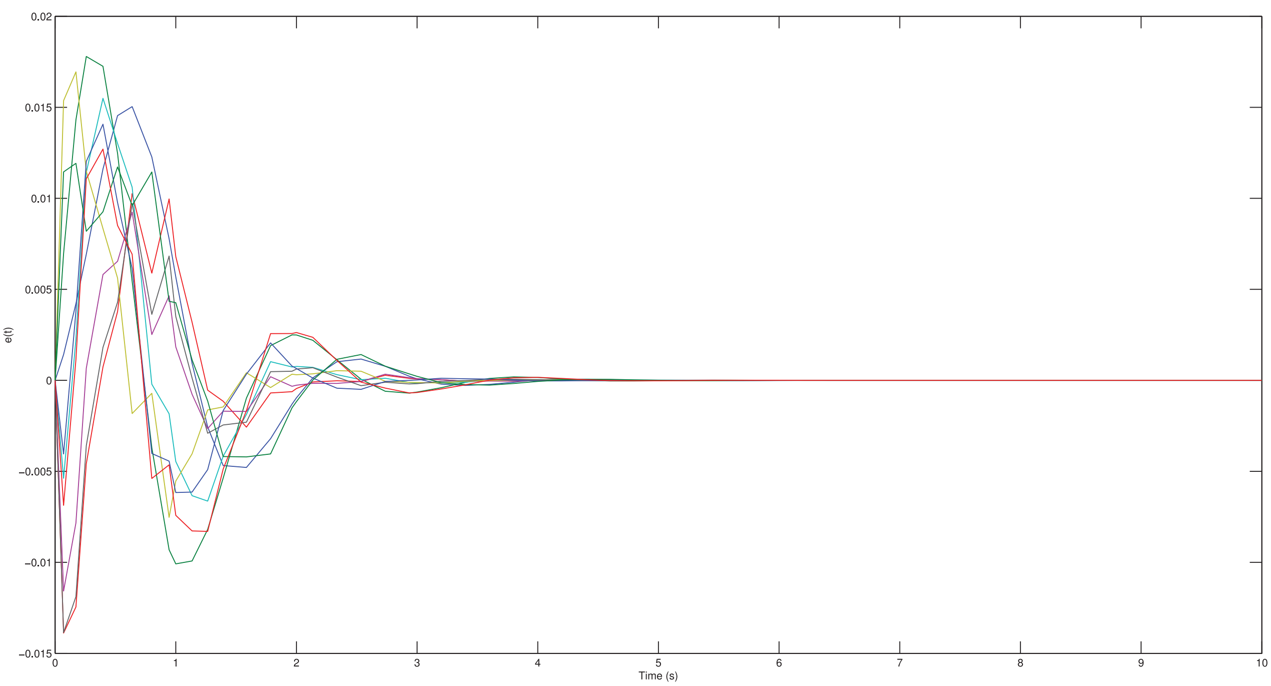

Figure 2. Estimation error trajectory $e(t)$ of (6).

The coupling configuration matrix and the inner-coupling matrix are assumed to be

$$G = (g_{ij})_{N\times N} = \begin{bmatrix} -15 & 0.01 & 0 & 0 & 0.01 \\ 0.01 & -15 & 0 & 0 & 0 \\ 0.01 & 0.02 & -16 & 0 & 0.02 \\ 0.02 & 0.01 & 0 & -16 & 0.01 \\ 0 & 0 & 0.01 & 0.01 & -14 \end{bmatrix},$$

$$\Gamma_1 = \Gamma_2 = \begin{bmatrix} 0.070 & -0.075 \\ 0.090 & 0.100 \end{bmatrix}.$$

The sensor measurements with data missing are described as $y(t) = \Xi C x(t)$, in which the mathematical expectation and variance of ξ_i are $\bar{\xi}_1 = 0.6$ and $\sigma_1 = 0.02$, respectively, and $C = [0.2 \;\; -0.2 \;\; 0.2 \;\; 0 \;\; 0.2 \;\; -0.2 \;\; 0.2 \;\; 0$

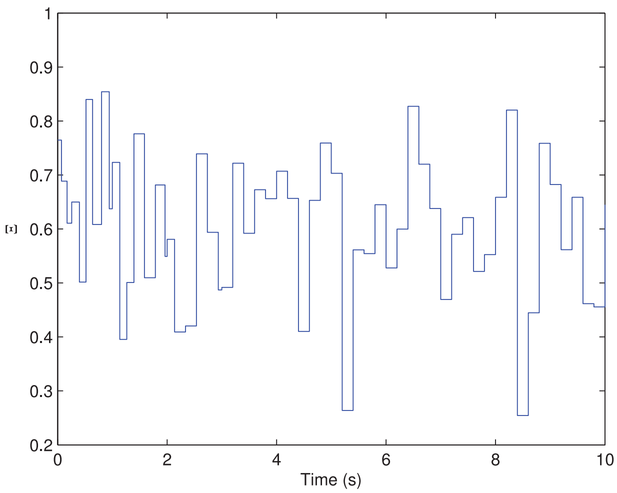

Figure 3. The probabilistic data missing of the output measurements.

$-0.2\ \ 0.2]$; Ξ_i $(i = 1, 2, 3, 4)$ in Assumption 1 are chosen as follows:

$$\Xi_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \Xi_2 = \begin{bmatrix} 0.15 & 0 \\ 0 & 0.25 \end{bmatrix},$$
$$\Xi_3 = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.3 \end{bmatrix}, \Xi_4 = \begin{bmatrix} -0.25 & 0 \\ 0 & -0.35 \end{bmatrix}.$$

In this example, the communication delays satisfy $\tau_1 \leq \tau(t) \leq \tau_2$, and assume $\tau_1 = 0.1$, $\tau_2 = 0.2$, $\delta_0 = 0.2$, $e_1 = 1$; $e_2 = 1$. Then, combining (24) and $K = P^{-1}Y$, the desired estimator parameters can be designed as

$$K^T = [0.0060\ 0.0051\ -12.7852\ -9.0336\ -11.3543\ 12.0988\ -13.0446\ -8.3119\ 9.3627\ -12.6306]^T.$$

Choose the initial conditions $x^T(0) = \hat{x}^T(0) = \left[0.5\ -0.4\ 0.3\ -0.2\ 0.1\ -0.1\ 0.2\ -0.3\ 0.4\ -0.5\right]^T$, the actual measurements $x(t)$ and the output errors $e(t)$ are shown in Figures 1 and 2, respectively. From Figures 1 and 2, we can see the designed state estimator performs well. Moreover, the probabilistic sensor faults in (2) and the probabilistic occurrence of the nodes $\delta(t)$ in (1) can be seen from Figures 3 and 4.

5. Conclusion

This paper has investigated the state estimation problem for the complex networked systems with randomly occurring nonlinearities and randomly missing measurements. The randomly occurring nonlinearities may go through switches in a probabilistic way. The missing measurements are assumed to occur randomly in the process of information transmission. Considering the two random phenomena, we construct a new state estimator for the stochastic complex network. By utilising stochastic analysis, sufficient conditions in terms of matrix inequalities have been given which guarantee the estimation error dynamics to be exponentially stable in the mean square. Furthermore, the expression of the state estimator has been derived. Finally, a numerical example has been provided to show the usefulness and effectiveness of the obtained results.

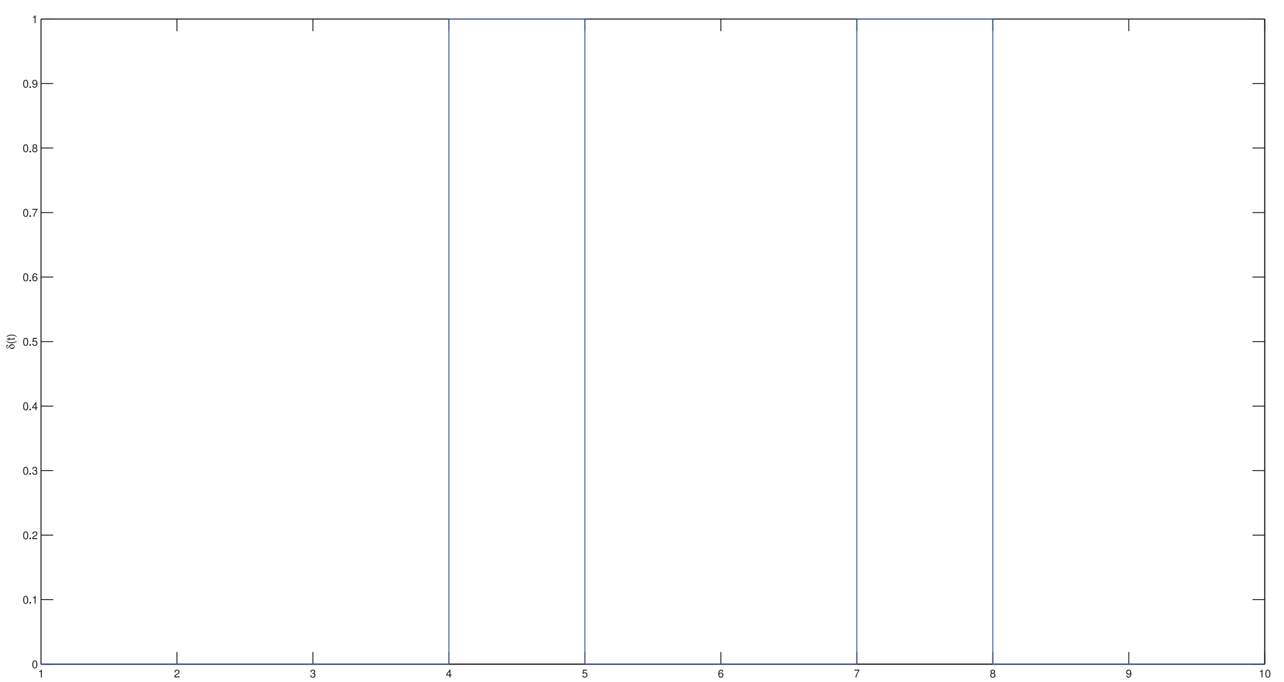

Figure 4. The probabilistic occurrence of the nodes.

We would like to point out that it is possible to extend our main results to the dynamical systems such as those with randomly occurring information (e.g. the stochastic Brownian motions) and disturbances. This will also be one of our future research issues.

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