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H_∞ filtering for event-based T-S fuzzy systems with stochastic sensor faults

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Abstract: This paper is firstly concerned with reliable H_∞ filter design for a class of T-S fuzzy systems with stochastic sensor faults under a event triggered scheme. Under the event-triggered scheme, the sensor data is transmitted only when the sampled measurements of the plant violate the specified event condition. Then, an event-based filter design model for T-S fuzzy systems is constructed by taking the effect of event-triggered scheme and the sensor faults into consideration. Sufficient conditions for the existence of the desired filter are established in terms of linear matrix inequalities and the explicit expression is given for the designed filter parameters. A numerical example is provided to illustrate the design method.

Key Words: Stochastic sensor failure, Event-triggered scheme, T-S fuzzy systems, Networked control systems

1 Introduction

The control problem for T-S fuzzy systems has received considerable attention in recent years. It has been proved that T-S fuzzy systems, which can be analyzed by many properties of conventional linear systems and described by a family of IF-THEN rules, can approximate any continuous functions [1–3]. Over the past few years, a great effort has been made on the stability analysis and synthesis for T-S fuzzy systems. More specifically, the filtering problems for T-S fuzzy systems have been widely investigated by many researchers via various methodologies [4–6]. In [4], a class of T-S fuzzy stochastic systems was investigated. In [5], the authors investigated the robust and reliable H_∞ filter design for a class of nonlinear NCSs with random sensor faults via T-S fuzzy model. The authors in [6] investigated the H_∞ filtering issue for nonlinear systems with time delay via T-S fuzzy systems.

The insertion of network in the control systems has many advantages such as low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability. However, it can also bring about new interesting and challenging issues as to the limited capacity of the network cable, for example, the transmission delay, packet dropout, etc. Recently, some work has been made to increase the energy efficiency and reduce the cost of sensor network. There has been many publications in the literature [7, 8]. Most of the available results employ time-triggered communication scheme. However, this might lead to insufficient utilization of limited resource and communication bandwidth. Especially when the system is close to its equilibrium point, there is little new information to be transmitted, thus, redundant communications have inevitability occurred [9]. Therefore, it is necessary to build a communication mechanism in a unified framework. Recently, event-triggered method has received considerable attention, which can reduce the burden of the network communication and the occupation of the sensor. Many outstanding results under event-triggered method have been available [9–13]. For example, In [10], the authors proposed a novel event-triggered scheme and constructed a delay system model for the analysis, then they derived the criteria for stability with an H_∞ norm bound and criteria for co-designing both the feedback gain and the trigger parameters. The authors in [11] proposed another event-triggered communication scheme and investigated a reliable control design for networked control system under event-triggered scheme. However, little attention has been paid to the filtering problem for T-S fuzzy systems under event-triggered scheme. Up to now, to the best of the authors' knowledge, event-triggered filtering for a class of T-S fuzzy systems with stochastic sensor faults has not been well addressed. This situation has motivated our current investigation.

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2 System description

Consider the following T-S fuzzy system with r plant rules

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_d(t)x(t-\tau(t)) + A_\omega(t)\omega(t) \\ y(t) = C(t)x(t) \\ z(t) = L(t)x(t) \end{cases} \tag{1}$$

where $A(t) = \sum_{i=1}^{r} h_i A_i$,$A_d(t) = \sum_{i=1}^{r} h_i A_{di}$, $A_\omega(t) = \sum_{i=1}^{r} h_i A_{\omega i}$, $C(t) = \sum_{i=1}^{r} h_i C_i$, $L(t) = \sum_{i=1}^{r} h_i L_i$. h_i is the abbreviation for $h_i(\theta(t))$, $h_i(\theta(t)) = \frac{\alpha_i(\theta(t))}{\sum_{i=1}^{r}\alpha_i(\theta(t))}$, $\alpha_i(\theta(t)) = \prod_{j=1}^{g} W_j^i(\theta_j(t))$, $W_j^i(\theta_j(t))$ is the grade membership value of $\theta_j(t)$ in W_j^i and $h_i(\theta(t))$ satisfies $h_i(\theta(t)) \geq 0, \sum_{i=1}^{r} h_i(\theta(t)) = 1$.

The defuzzified output of (1) is referred by

$$\begin{cases} \dot{x}_f(t) = A_f(t)x_f(t) + B_f(t)\hat{y}(t) \\ z_f(t) = C_f(t)x_f(t) \end{cases} \tag{2}$$

where $A_f(t) = \sum_{i=1}^{r} h_i A_{fi}$,$B_f(t) = \sum_{i=1}^{r} h_i B_{fi}$, $C_f(t) = \sum_{i=1}^{r} h_i C_{fi}$

Remark 1 *Under the traditional control system structure, the effect of the transmission delay can be neglected, that is $\hat{y}(t) = y(t)$. However, in networked control systems, the transmission delay should be take into account. In this paper, considering the existence of the network induced delay, $\hat{y}(t) \neq y(t)$.*

Suppose the time-varying delay in the network communication is d_k and $d_k \in [0, \bar{d})$, where $\bar{d}$ is a positive real number. Therefore, the sampled sensor measurements $y(t_0 h), y(t_1 h), y(t_2 h), \cdots$ will arrive at the filter side at the instants $t_0 h + d_0, t_1 h + d_1, t_2 h + d_2, \cdots$, respectively. $\hat{y}(t)$ in Equation (2) can be described as [5, 14]

$$\hat{y}(t) = \sum_{i=1}^{r} h_i C_i x(t_k h), t \in [t_k h + d_{t_k}, t_{k+1} h + d_{t_{k+1}}] \quad (3)$$

where h is the sampling period, $t_k \in \{1, 2, 3, \cdots\}$. d_{t_k} and $d_{t_{k+1}}$ are the network induced delays at the transmission instant $t_k h$ and $t_{k+1} h$, respectively.

Considering the possible sensor failure, (3) can be rewritten as

$$\hat{y}(t) = \sum_{i=1}^{r} h_i \Xi C_i x(t_k h) = \sum_{i=1}^{r} \sum_{l=1}^{m} h_i \Xi_l E_l C_i x(t_k h), \quad t \in [t_k h + d_{t_k}, t_{k+1} h + d_{t_{k+1}}] \quad (4)$$

where $\Xi = diag\{\Xi_1, \Xi_2, \cdots, \Xi_m\}$ with $\Xi_i (i = 1, 2, \cdots, m)$ being m unrelated random variables taking values on the interval $[0, \theta]$, $\theta \geq 1$ and $E_l = diag\{\underbrace{0, \cdots, 0}_{l-1}, 1, \underbrace{0, \cdots, 0}_{m-l}\}$. The mathematical expectation and variance of $\Xi_i (i = 1, 2, \cdots, m)$ are $\bar{\Xi}_i$ and δ_i^2 , respectively. $\bar{\Xi}_i$ and δ_i^2 can determined the failure rate and the distortion degree of the ith sensor.

Define $\bar{\Xi} = diag\{\bar{\Xi}_1, \bar{\Xi}_2, \cdots, \bar{\Xi}_m\}$, we can easily derive $\bar{\Xi} = \sum_{l=1}^{m} \bar{\Xi}_l E_l$. For a matrix $\Theta > 0$, we can get

$$\begin{cases} \mathbb{E}\{\Xi - \bar{\Xi}\} = 0 \\ \mathbb{E}\{(\Xi - \bar{\Xi})^T \Theta (\Xi - \bar{\Xi})\} = \sum_{l=1}^{m} \delta_i^2 E_l^T \Theta E_l \end{cases}$$

Remark 2 *When sensors have faults, the output signal may be larger or smaller than what it should be. Considering this case, we assume the variables $\Xi_i (i = 1, 2, \cdots, m)$ take values in the interval $[0, \theta]$, $\theta \geq 1$. When $\Xi_i \in \{0, 1\}$, it means the sensor i has completely failure or not. $\Xi_i = 1$ means the sensor i works normally, $\Xi_i = 0$ means signal sent by sensor i is lost during transmission. Moreover, $0 < \Xi_i < 1$ and $\Xi_i > 1$ means the case of data distortion happen, that is, the signal at the filter is smaller or greater than it actually is.*

As is well known, the widely used periodic sampling mechanism may lead to transmit many unnecessary signals, which reduces bandwidth utilization. In order to reduce the load of network transmission and save the network bandwidth, there is a great need to introduce an event triggered mechanism which decides whether the newly sampled data should be send out to the filter. As is shown in Fig.1, similar to [11], we introduce an event generator between the sensor and the filter. The sensor measurements are sampled regularly by the sampler of the smart sensor with period h, which will be given in sequel. Whether or not the newly sampled sensor measurements will be sent out to the filter is determined by the following judgement algorithm:

$$\left[\mathbb{E}\left\{\bar{\Xi} y((k+j)h)\right\} - \mathbb{E}\left\{\bar{\Xi} y(kh)\right\}\right]^T \Omega \left[\mathbb{E}\left\{\bar{\Xi} y((k+j)h)\right\} - \mathbb{E}\left\{\bar{\Xi} y(kh)\right\}\right] \leq \rho \left[\mathbb{E}\left\{\bar{\Xi} y((k+j)h)\right\}\right]^T \Omega \mathbb{E}\left[\left\{\bar{\Xi} y((k+j)h)\right\}\right] \quad (5)$$

where Ω is a symmetric positive definite matrix, $j = 1, 2, \cdots$, and $\rho \in [0, 1)$. Only when the current sampled sensor measurements $y((k+j)h)$ and the latest transmitted sensor measurements $y(kh)$ variate the specified threshold (5), the current sampled sensor measurements $y((k+j)h)$ can be transmitted by the event generator and sent into the filter.

Remark 3 *From event-triggered algorithm (5), it is easily seen that the sensor measurement are sampled at time kh by sampler with a given period h, the next sensor measurement is at time $(k+1)h$. Suppose that the release times are $t_0 h, t_1 h, t_2 h, \cdots$, it is easily seen that $s_i h = t_{i+1} h - t_i h$ denotes the release period of event generator in (5), $s_i h$ means that the sampling between the two conjoint transmitted instant.*

Remark 4 *It is easily seen from event-triggered algorithm (5) that the set of the release instants $\{t_0 h, t_1 h, t_2 h, \cdots\} \subseteq \{0, 1, 2, \cdots\}$. The amount of $\{t_0 h, t_1 h, t_2 h, \cdots\}$ depends on the value of ρ and the variation of the sensor measurements.*

Similar to [10–12] for technical convenience, consider the following two cases:

Case 1: If $t_k h + h + \bar{d} \geq t_{k+1} h + d_{k+1}$, where $\bar{d} = \max d_k$, define a function $d(t)$ as

$$d(t) = t - t_k h, t \in [t_k h + d_k, t_{k+1} h + d_{k+1}) \quad (6)$$

It can easily be obtained that

$$d_k \leq d(t) \leq (t_{k+1} - t_k)h + d_{k+1} \leq h + \bar{d} \quad (7)$$

Case 2: If $t_k h + h + \bar{d} < t_{k+1} h + d_{k+1}$, consider the following two intervals:

$$[t_k h + d_k, t_k h + h + \bar{d}), [t_k h + ih + \bar{d}, t_k h + ih + h + \bar{d})$$

Since $d_k \leq \bar{d}$, it can be easily shown that there exists a positive integer $\delta_M \geq 1$ such that

$$t_k h + \delta_M h + \bar{d} < t_{k+1} h + d_{k+1} \leq t_k h + \delta_M h + h + \bar{d}$$

Moreover, $x(t_k h)$ and $t_k h + ih$ with $i = 1, 2, \cdots, \delta_M$ satisfy (5). Let

$$\begin{cases} I_0 = [t_k h + d_k, t_k h + h + \bar{d}) \\ I_i = [t_k h + ih + \bar{d}, t_k h + ih + h + \bar{d}) \\ I_{d_M} = [t_k h + \delta_M h + \bar{d}, t_{k+1} h + d_{k+1}) \end{cases} \quad (8)$$

where $i = 1, 2, \cdots, \delta_M - 1$. It can be easily shown that

$$[t_k h + d_k, t_{k+1} h + d_{k+1}) = \bigcup_{i=0}^{i=\delta_M} I_i \tag{9}$$

Define

$$d(t) = \begin{cases} t - t_k h, t \in I_0 \\ t - t_k h - ih, t \in I_i, i = 1, 2, \cdots, \delta_M - 1 \\ t - t_k h - \delta_M h, t \in I_{\delta_M} \end{cases} \tag{10}$$

From the definition of $d(t)$, we have

$$\begin{cases} t_k \leq d(t) < h + \bar{d}, t \in I_0 \\ t_k \leq \bar{d} \leq d(t) < h + \bar{d}, t \in I_i \\ t_k \leq \bar{d} \leq d(t) < h + \bar{d}, t \in I_{d_M} \end{cases} \tag{11}$$

where the third row in (8) holds because $t_{k+1}h + d_{k+1} \leq t_k h + (d_M + 1)h + \bar{d}$. Obviously,

$$0 \leq d_k \leq d(t) \leq h + \bar{d} \triangleq d_M \tag{12}$$

In Case 1, for $t \in [t_k h + d_k, t_{k+1} h + d_{k+1})$, define a error vector $e_k(t) = 0$. In Case 2, define the mathematical expectation of the sensor measurement error between the current sampling instant and the latest transmission instant

$$\bar{\Xi} e_k(t) = \begin{cases} 0, t \in I_0 \\ \bar{\Xi} y(t_k h) - \bar{\Xi} y(t_k h + ih), t \in I_i \\ \bar{\Xi} y(t_k h) - \bar{\Xi} y(t_k h + \delta_M h), t \in I_{\delta_M} \end{cases} \tag{13}$$

From the definition of $\bar{\Xi} e_k(t)$ and the triggering algorithm (5), it can be easily seen that for $t \in [t_k h + d_k, t_{k+1} h + d_{k+1})$

$$e_k^T(t) \bar{\Xi}^T \Omega \bar{\Xi} e_k(t) \leq \rho y^T(t - d(t)) \bar{\Xi}^T \Omega \bar{\Xi} y(t - d(t)), \tag{14}$$

Here, the measurement output is sampled before it enters the filter; based on the sampling technique and zero-order hold, the actual output can be described as

$$y(t_k h) = \sum_{i=1}^{r} h_i \Xi C_i x(t_k h) \tag{15}$$

Remark 5 *Notice that the relation of $t_k h + h + \bar{d} \geq t_{k+1} h + d_{k+1}$ in case 1 means the newly sampled sensor measurement $y(t_k h + h)$ will be transmitted and arrive at the filter side at the instant $t_k h + h + d_{k+1}$; $t_k h + h + \bar{d} < t_{k+1} h + d_{k+1}$ in case 2 means the newly sampled sensor measurement $y(t_k h + h)$ and the latest sensor measurement $y(t_k h)$ variate the judgement algorithm (5), and $y(t_k h + h)$ will not be transmitted to the filter side.*

Remark 6 *From (13), we can deduce that the sensor measurement error between the current sampling instant and the latest transmission instant can be calculated as*

$$e_k(t) = \begin{cases} 0, t \in I_0 \\ y(t_k h) - y(t_k h + ih), t \in I_i \\ y(t_k h) - y(t_k h + \delta_M h), t \in I_{\delta_M} \end{cases} \tag{16}$$

Remark 7 *It is seen from the definition of $d(t)$ that $d(t)$ is different from the traditional time-varying delay. $d(t)$ depends ont only the release times, but also on the network induce delay d_k and the sampling period h.*

Remark 8 *Since there is a communication network between the sensor and the filter, the premises in the system and the ones in the filter should be asynchronous. That is, at the same instant $t \in [t_k h + d_k, t_{k+1} h + d_{k+1})$, when $\theta_i(t)$ is available in (1), only $\theta_i(t_k h)$ is available in the filter. In this paper, we assume the mechanical model of the studied system is known a priori, when the initial condition is given and the state of the studied system can be calculated based on the known mechanical model. Since $\theta_i(t_k h)$ is available at the filter, $\theta_i(t)$ can be calculated for $t \in [t_k h, t_{k+1} h)$. Therefore, the synchronous premise variables $\theta_i(t)$ can be derived in the filter side.*

Based on the above description, combining (15)and (11), from (4) and (13), the filter input can be rewritten as

$$\begin{aligned} \hat{y}(t) &= \sum_{i=1}^{r} h_i \bar{\Xi} C_i x(t - d(t)) + \bar{\Xi} e_k(t) \\ &\quad + \sum_{i=1}^{r} h_i ((\Xi - \bar{\Xi}) C_i x(t - d(t)) \end{aligned} \tag{17}$$

Combining (2) and (17), the defuzzified value of the filter can be rewritten as

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j (A_{fi} x_f(t) + \bar{\Xi} e_k(t) \\ + B_{fi} (\bar{\Xi} C_i x(t - d(t)) + (\Xi - \bar{\Xi}) C_i x(t - d(t))) \\ z_f(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j C_{fi} x_f(t) \end{cases} \tag{18}$$

Define $e(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}$, $\tilde{z}(t) = z(t) - z_f(t)$, the following filtering-error system based on Equations (1) and (18) can be obtained as

$$\begin{cases} \dot{e}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \{ \bar{A} e(t) + \bar{A}_d H e(t - \tau(t)) \\ + \bar{B} H e(t - d(t)) + \bar{B}_1 e_k(t) + \bar{A}_w w(t) \\ + \bar{B}_t H e(t - d(t)) \} \\ \tilde{z}(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i h_j \bar{L} e(t) \end{cases} \tag{19}$$

where

$$\bar{A} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fj} \end{bmatrix}, \bar{A}_d = \begin{bmatrix} A_{di} \\ 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} 0 \\ B_{fj} \bar{\Xi} C_i \end{bmatrix},$$

$$\bar{B}_1 = \begin{bmatrix} 0 \\ B_{fj} \bar{\Xi} \end{bmatrix}, \bar{A}_w = \begin{bmatrix} A_{wi} \\ 0 \end{bmatrix}, \bar{B}_t = \begin{bmatrix} 0 \\ B_{fj} (\Xi - \bar{\Xi}) C_i \end{bmatrix},$$

$$\bar{L} = \begin{bmatrix} L_i & -C_{fj} \end{bmatrix}, H = \begin{bmatrix} I & 0 \end{bmatrix}$$

3 Main results

In this section, with a given filter with form (18), we will first propose the following performance analysis for the augmented systems (19). Then we deal with the design problem of the filter design for system (1).

Theorem 1 *For given parameters γ, τ_m, τ_M, d_M and ρ , system (19) is exponentially stable in the mean square with an H_∞ disturbance attenuation level γ under the event trigger scheme (5) if there exist matrices $P > 0$, $Q_k > 0$,*

$R_k > 0 (k = 1, 2, 3)$, $\Omega > 0$, *and* M_{ij}, N_{ij}, T_{ij}, S_{ij} *with appropriate dimensions satisfying*

$$\Xi^{ij} + \Xi^{ji} < 0, i \le j \in \mathcal{S} \tag{20}$$

where

$$\begin{aligned}
\Xi^{ij} &= \begin{bmatrix} \Omega_{11}^{ij} & * & * & * \\ \Omega_{21}^{ij} & \Omega_{22}^{ij} & * & * \\ \Omega_{31}^{ij} & 0 & \Omega_{33}^{ij} & * \\ \Omega_{41}^{ij}(s) & 0 & 0 & \Omega_{44}^{ij} \end{bmatrix}, (s = 1, 2, 3, 4) \\
\Omega_{11}^{ij} &= \begin{bmatrix} \Upsilon_1 & * \\ \Upsilon_2 & \Upsilon_3 \end{bmatrix} \\
\Upsilon_1 &= \begin{bmatrix} \Gamma_{ij1} & * & * & * \\ R_2 & \Gamma_{ij2} & * & * \\ H^T \bar{A}_d^T P & M_{ij3} - M_{ij2}^T & \Gamma_{ij3} & * \\ 0 & 0 & N_{ij4} - N_{ij3}^T & \Gamma_{ij4} \end{bmatrix} \\
\Upsilon_2 &= \begin{bmatrix} H^T \bar{B}^T P + T_{ij5} - T_{ij1}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{B}_1^T P & 0 & 0 & 0 \\ \bar{A}_w^T P & 0 & 0 & 0 \end{bmatrix} \\
\Upsilon_3 &= \begin{bmatrix} \Gamma_{ij5} & * & * & * \\ S_{ij6} - S_{ij5}^T & \Gamma_{ij6} & * & * \\ 0 & 0 & -\bar{\Xi}^T \Omega \bar{\Xi} & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \\
\Gamma_{ij1} &= P\bar{A} + \bar{A}^T P + Q_1 + Q_2 + Q_3 - R_2 + T_{ij1} \\
&\quad + T_{ij1}^T, \Gamma_{ij2} = -Q_1 - R_2 + M_{ij2} + M_{ij2}^T \\
\Gamma_{ij3} &= -M_{ij3} - M_{ij3}^T + N_{ij3} + N_{ij3}^T \\
\Gamma_{ij4} &= -Q_2 - N_{ij4} - N_{ij4}^T \\
\Gamma_{ij5} &= \rho H^T C_i^T \bar{\Xi}^T \Omega \bar{\Xi} C_i H - T_{ij5} - T_{ij5}^T + S_{ij5} \\
&\quad + S_{ij5}^T, \Gamma_{ij6} = -Q_3 - S_{ij6} - S_{ij6}^T \\
\Omega_{21}^{ij} &= \begin{bmatrix} \Upsilon_4 & \Upsilon_5 \end{bmatrix} \\
\Upsilon_4 &= \begin{bmatrix} \bar{L} & 0 & 0 & 0 \\ \sqrt{\tau_{21}} P\bar{A} & 0 & \sqrt{\tau_{21}} P\bar{A}_d H & 0 \\ \tau_m P\bar{A} & 0 & \tau_m P\bar{A}_d H & 0 \\ \sqrt{d_M} P\bar{A} & 0 & \sqrt{d_M} P\bar{A}_d H & 0 \end{bmatrix} \\
\Upsilon_5 &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{\tau_{21}} P\bar{B}H & 0 & \sqrt{\tau_{21}} P\bar{B}_1 & \sqrt{\tau_{21}} P\bar{A}_w \\ \tau_m P\bar{B}H & 0 & \tau_m P\bar{B}_1 & \tau_m P\bar{A}_w \\ \sqrt{d_M} P\bar{B}H & 0 & \sqrt{d_M} P\bar{B}_1 & \sqrt{d_M} P\bar{A}_w \end{bmatrix} \\
\Omega_{22}^{ij} &= diag\{-I, -PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\}, \\
\Omega_{33}^{ij} &= diag\{\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3\}, \\
\mathfrak{R}_k &= diag\{\underbrace{-PR_k^{-1}P, \cdots, -PR_k^{-1}P}_{m}\}, k = 1, 2, 3. \\
\Omega_{44}^{ij} &= diag\{-R_1, -R_3\}, \sqrt{\tau_{21}} = \sqrt{\tau_M - \tau_m} \\
\Omega_{31}^{ij} &= \begin{bmatrix} 0 & 0 & 0 & 0 & \breve{D}_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \breve{D}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \breve{D}_3 & 0 & 0 & 0 \end{bmatrix}, \\
\breve{D}_1 &= \begin{bmatrix} \sqrt{\tau_{21}} \delta_1 P\hat{D}_1 H \\ \vdots \\ \sqrt{\tau_{21}} \delta_m P\hat{D}_m H \end{bmatrix}, \hat{D}_l = \begin{bmatrix} 0 \\ B_{fj} E_l C_i \end{bmatrix} \\
\breve{D}_2 &= \begin{bmatrix} \delta_1 \tau_m P\hat{D}_1 H \\ \vdots \\ \delta_1 \tau_m \delta_m P\hat{D}_m H \end{bmatrix}, \breve{D}_3 = \begin{bmatrix} \sqrt{d_M} \delta_1 P\hat{D}_1 H \\ \vdots \\ \sqrt{d_M} \delta_m P\hat{D}_m H \end{bmatrix} \\
\Omega_{41}^{ij}(1) &= \begin{bmatrix} \sqrt{\tau_{21}} M_{ij}^T \\ \sqrt{d_M} T_{ij}^T \end{bmatrix}, \Omega_{41}^{ij}(2) = \begin{bmatrix} \sqrt{\tau_{21}} M_{ij}^T \\ \sqrt{d_M} S_{ij}^T \end{bmatrix}, \\
\Omega_{41}^{ij}(3) &= \begin{bmatrix} \sqrt{\tau_{21}} N_{ij}^T \\ \sqrt{d_M} S_{ij}^T \end{bmatrix}, \Omega_{41}^{ij}(4) = \begin{bmatrix} \sqrt{\tau_{21}} N_{ij}^T \\ \sqrt{d_M} T_{ij}^T \end{bmatrix} \\
M_{ij}^T &= \begin{bmatrix} 0 & M_{ij2}^T & M_{ij3}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
N_{ij}^T &= \begin{bmatrix} 0 & 0 & N_{ij3}^T & N_{ij4}^T & 0 & 0 & 0 & 0 \end{bmatrix} \\
T_{ij}^T &= \begin{bmatrix} T_{ij1}^T & 0 & 0 & 0 & T_{ij5}^T & 0 & 0 & 0 \end{bmatrix} \\
S_{ij}^T &= \begin{bmatrix} 0 & 0 & 0 & 0 & S_{ij5}^T & S_{ij6}^T & 0 & 0 \end{bmatrix}
\end{aligned}$$

Proof:
Choose the following Lyapunov functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{21}$$

where

$$\begin{aligned}
V_1(t) &= e^T(t) P e(t) \\
V_2(t) &= \int_{t-\tau_m}^{t} e^T(s) Q_1 e(s) ds + \int_{t-\tau_M}^{t} e^T(s) Q_2 e(s) ds \\
&\quad + \int_{t-d_M}^{t} e^T(s) Q_3 e(s) ds \\
V_3(t) &= \int_{t-\tau_M}^{t-\tau_m} \int_{s}^{t} \dot{e}^T(v) R_1 \dot{e}(v) dv ds \\
&\quad + \tau_m \int_{t-\tau_m}^{t} \int_{s}^{t} \dot{e}^T(v) R_2 \dot{e}(v) dv ds \\
&\quad + \int_{t-d_M}^{t} \int_{s}^{t} \dot{e}^T(v) R_3 \dot{e}(v) dv ds
\end{aligned}$$

and $P > 0$, $Q_k > 0$, $R_k > 0$ $(k = 1, 2, 3)$.

Similar to the method in [11**?**], one can easily obtain the results above. Due to limited space, we omit the details here.

Based on Theorem 1, we are in position to design a filter in the form of (2). The explicit expression of the parameters of the designed filter are given in the following Theorem.

Theorem 2 *For given positive scalars* γ, τ_m, τ_M, d_M, ε_1, ε_2, ε_3 *and* σ, *system (19) is exponentially stable in the mean square under the event trigger scheme (5) if there exist matrices* $P_1 > 0$, $\bar{P}_3 > 0$, $\bar{Q}_1$, $\bar{Q}_2$, $\bar{Q}_3$, $\bar{R}_1$, $\bar{R}_2$, $\bar{R}_3$, $\Omega > 0$, $\bar{A}_{fj}$, $\bar{B}_{fj}$, $\bar{C}_{fj}$, *and* $\bar{M}_{ij}$, $\bar{N}_{ij}$, $\bar{T}_{ij}$, $\bar{S}_{ij}$ *with appropriate dimensions, such that the following LMIs hold:*

$$\Sigma^{ij} + \Sigma^{ji} < 0, i \le j \in \mathcal{S} \tag{22}$$

$$P_1 - \bar{P}_3 > 0 \tag{23}$$

where

$$\Sigma^{ij} = \begin{bmatrix} \Phi_{11}^{ij} & * & * & * \\ \Phi_{21}^{ij} & \Phi_{22}^{ij} & * & * \\ \Phi_{31}^{ij} & 0 & \Phi_{33}^{ij} & * \\ \Phi_{41}^{ij}(s) & 0 & 0 & \Phi_{44}^{ij} \end{bmatrix}, (s = 1, 2, 3, 4)$$

$$\Phi_{11}^{ij} = \begin{bmatrix}\Upsilon_6 & * \\ \Upsilon_7 & \Upsilon_8\end{bmatrix}, \tilde{\Upsilon}_{ij1} = \begin{bmatrix}P_1A_i & \bar{A}_{fj} \\ \bar{P}_3A_i & \bar{A}_{fj}\end{bmatrix}$$

$$\Upsilon_6 = \begin{bmatrix}\bar{\Gamma}_{ij1} & * & * & * \\ \bar{R}_2 & \bar{\Gamma}_{ij2} & * & * \\ \tilde{\Upsilon}_{ij31} & \bar{M}_{ij3} - \bar{M}_{ij2}^T & \bar{\Gamma}_{ij3} & * \\ 0 & 0 & \bar{N}_{ij4} - \bar{N}_{ij3}^T & \bar{\Gamma}_{ij4}\end{bmatrix}$$

$$\Upsilon_7 = \begin{bmatrix}\tilde{\Upsilon}_{ij51} + \bar{T}_{ij5} - \bar{T}_{ij1}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \tilde{\Upsilon}_{ij71} & 0 & 0 & 0 \\ \tilde{\Upsilon}_{ij81} & 0 & 0 & 0\end{bmatrix}$$

$$\Upsilon_8 = \begin{bmatrix}\bar{\Gamma}_{ij5} & * & * & * \\ S_{ij6} - S_{ij5}^T & \bar{\Gamma}_{ij6} & * & * \\ 0 & 0 & -\bar{\Xi}^T\Omega\bar{\Xi} & * \\ 0 & 0 & 0 & -\gamma^2 I\end{bmatrix}$$

$$\bar{\Gamma}_{ij1} = \tilde{\Upsilon}_{ij1} + \tilde{\Upsilon}_{ij1}^T + \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 - \bar{R}_2 + \bar{T}_{ij1} + \bar{T}_{ij1}^T,$$

$$\tilde{\Upsilon}_{ij31} = \begin{bmatrix}A_{di}^TP_1 & A_{di}^T\bar{P}_3 \\ 0 & 0\end{bmatrix}$$

$$\bar{\Gamma}_{ij2} = -\bar{Q}_1 - \bar{R}_2 + \bar{M}_{ij2} + \bar{M}_{ij2}^T$$

$$\bar{\Gamma}_{ij3} = -\bar{M}_{ij3} - \bar{M}_{ij3}^T + \bar{N}_{ij3} + \bar{N}_{ij3}^T$$

$$\bar{\Gamma}_{ij4} = -\bar{Q}_2 - \bar{N}_{ij4} - \bar{N}_{ij4}^T$$

$$\bar{\Gamma}_{ij5} = \tilde{\Upsilon}_{ij5} - \bar{T}_{ij5} - \bar{T}_{ij5}^T + \bar{S}_{ij5} + \bar{S}_{ij5}^T$$

$$\bar{\Gamma}_{ij6} = -\bar{Q}_3 - \bar{S}_{ij6} - \bar{S}_{ij6}^T$$

$$\tilde{\Upsilon}_{ij51} = = \begin{bmatrix}C_i^T\bar{\Xi}^T\bar{B}_{fj}^T & C_i^T\bar{\Xi}^T\bar{B}_{fj}^T \\ 0 & 0\end{bmatrix},$$

$$\tilde{\Upsilon}_{ij5} = \begin{bmatrix}\rho C_i^T\bar{\Xi}^T\Omega\bar{\Xi}C_i & 0 \\ 0 & 0\end{bmatrix},$$

$$\tilde{\Upsilon}_{ij71} = \begin{bmatrix}\bar{\Xi}^T\bar{B}_{fj}^T & \bar{\Xi}^T\bar{B}_{fj}^T\end{bmatrix},$$

$$\tilde{\Upsilon}_{ij81} = \begin{bmatrix}A_{wi}^TP_1 & A_{wi}^T\bar{P}_3\end{bmatrix}, \Omega_{21}^{ij} = \begin{bmatrix}\Upsilon_9 & \Upsilon_{10}\end{bmatrix}$$

$$\Upsilon_9 = \begin{bmatrix}\tilde{L}_{ij} & 0 & 0_{p\times 2n} & 0 \\ \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij1} & 0 & \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij31}^T & 0 \\ \tau_m\tilde{\Upsilon}_{ij1} & 0 & \tau_m\tilde{\Upsilon}_{ij31}^T & 0 \\ \sqrt{d_M}\tilde{\Upsilon}_{ij1} & 0 & \sqrt{d_M}\tilde{\Upsilon}_{ij31}^T & 0\end{bmatrix}$$

$$\Upsilon_{10} = \begin{bmatrix}0 & 0 & 0 & 0 \\ \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij51}^T & 0 & \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij71}^T & \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij81}^T \\ \tau_m\tilde{\Upsilon}_{ij51}^T & 0 & \tau_m\tilde{\Upsilon}_{ij71}^T & \tau_m\tilde{\Upsilon}_{ij81}^T \\ \sqrt{d_M}\tilde{\Upsilon}_{ij51}^T & 0 & \sqrt{d_M}\tilde{\Upsilon}_{ij71}^T & \sqrt{d_M}\tilde{\Upsilon}_{ij81}^T\end{bmatrix}$$

$$\Phi_{22}^{ij} = diag\{-I, -2\varepsilon_1\bar{P} + \varepsilon_1^2\bar{R}_1, -2\varepsilon_2\bar{P} + \varepsilon_2^2\bar{R}_2, -2\varepsilon_3\bar{P} + \varepsilon_3^2\bar{R}_3\}$$

$$\bar{P} = \begin{bmatrix}P_1 & \bar{P}_3 \\ \bar{P}_3 & \bar{P}_3\end{bmatrix}, \tilde{L}_{ij} = \begin{bmatrix}L_i & -\bar{C}_{fj}\end{bmatrix}$$

$$\mathfrak{R}_k = diag\{\underbrace{-2\varepsilon_k\bar{P} + \varepsilon_k^2\bar{R}_k, \cdots, -2\varepsilon_k\bar{P} + \varepsilon_k^2\bar{R}_k}_{m}\}$$

$$\Phi_{33}^{ij} = diag\{\mathfrak{R}_1, \mathfrak{R}_2, \mathfrak{R}_3\}, \Omega_{44}^{ij} = diag\{-\bar{R}_1, -\bar{R}_3\}$$

$$\Omega_{31}^{ij} = \begin{bmatrix}0_{2n\times 8n} & \breve{D}_{12} & 0_{2n\times(3n+m)} \\ 0_{2n\times 8n} & \breve{D}_{22} & 0_{2n\times(3n+m)} \\ 0_{2n\times 8n} & \breve{D}_{32} & 0_{2n\times(3n+m)}\end{bmatrix}$$

$$\breve{D}_{12} = \begin{bmatrix}\sqrt{\tau_{21}}\delta_1\hat{D}_{12} \\ \vdots \\ \sqrt{\tau_{21}}\delta_m\hat{D}_{m2}\end{bmatrix} \breve{D}_{22} = \begin{bmatrix}\delta_1\tau_m\hat{D}_{12} \\ \vdots \\ \delta_1\tau_m\delta_m\hat{D}_{m2}\end{bmatrix}$$

$$\breve{D}_{32} = \begin{bmatrix}\sqrt{d_M}\delta_1\hat{D}_{12} \\ \vdots \\ \sqrt{d_M}\delta_m\hat{D}_{m2}\end{bmatrix}, \hat{D}_{l2} = \begin{bmatrix}\bar{B}_{fj}E_lC_i & 0 \\ \bar{B}_{fj}E_lC_i & 0\end{bmatrix}$$

$$\Phi_{41}^{ij}(1) = \begin{bmatrix}\sqrt{\tau_{21}}\bar{M}_{ij}^T \\ \sqrt{d_M}\bar{T}_{ij}^T\end{bmatrix}, \Phi_{41}^{ij}(2) = \begin{bmatrix}\sqrt{\tau_{21}}\bar{M}_{ij}^T \\ \sqrt{d_M}\bar{S}_{ij}^T\end{bmatrix}$$

$$\Phi_{41}^{ij}(3) = \begin{bmatrix}\sqrt{\tau_{21}}\bar{N}_{ij}^T \\ \sqrt{d_M}\bar{S}_{ij}^T\end{bmatrix}, \Phi_{41}^{ij}(4) = \begin{bmatrix}\sqrt{\tau_{21}}\bar{N}_{ij}^T \\ \sqrt{d_M}\bar{T}_{ij}^T\end{bmatrix}$$

$$\bar{M}_{ij}^T = \begin{bmatrix}0_{2n\times 2n} & \bar{M}_{ij2}^T & \bar{M}_{ij3}^T & 0_{2n\times(7n+m)}\end{bmatrix}$$

$$\bar{N}_{ij}^T = \begin{bmatrix}0_{2n\times 4n} & \bar{N}_{ij3}^T & \bar{N}_{ij4}^T & 0_{2n\times(5n+m)}\end{bmatrix}$$

$$\bar{T}_{ij}^T = \begin{bmatrix}\bar{T}_{ij1}^T & 0_{2n\times 6n} & \bar{T}_{ij5}^T & 0_{2n\times(3n+m)}\end{bmatrix}$$

$$\bar{S}_{ij}^T = \begin{bmatrix}0_{2n\times 8n} & \bar{S}_{ij5}^T & \bar{S}_{ij6}^T & 0_{2n\times(n+m)}\end{bmatrix}$$

Moreover, if the above conditions are feasible, the parameter matrices of the filter are given by

$$\begin{cases} A_{fj} = \bar{A}_{fj}\bar{P}_3^{-1} \\ B_{fj} = \bar{B}_{fj} \\ C_{fj} = \bar{C}_{fj}\bar{P}_3^{-1}\end{cases}, j \in \mathbb{S} \quad (24)$$

Proof:

Since $\bar{P}_3 > 0$, there exist P_2 and $P_3 > 0$ satisfying $\bar{P}_3 = P_2^TP_3^{-1}P_2$.

Define

$$P = \begin{bmatrix}P_1 & P_2^T \\ P_2 & P_3\end{bmatrix}, J = \begin{bmatrix}I & 0 \\ 0 & P_2^TP_3^{-1}\end{bmatrix}$$

$$F = dig\{\underbrace{J, \cdots, J}_{6}, I, I, I, \underbrace{J, \cdots, J}_{3m+5}\}$$

and $\bar{P} = JPJ^T = \begin{bmatrix}P_1 & \bar{P}_3 \\ \bar{P}_3 & \bar{P}_3\end{bmatrix}$, $\bar{Q}_k = JQ_kJ^T$, $\bar{R}_k = JR_kJ^T, (k = 1,2,3)$ $\bar{M}_{ijv_1} = JM_{ijv_1}J^T$, $\bar{N}_{ijv_2} = JN_{ijv_2}J^T$, $\bar{T}_{ijv_3} = JT_{ijv_3}J^T$,$\bar{S}_{ijv_4} = JS_{ijv_4}J^T$, $(v_1 = 2,3.\ v_2 = 3,4.\ v_3 = 1,5.\ v_4 = 5,6)$ we can obtain that the parameter matrices of the filter are given by (24). Due to limited space, we omit the details here..

4 Simulation examples

Consider a specific T-S fuzzy system of Equation (1) under an network control system structure, in which the system parameters are given as follows:

$$A_1 = \begin{bmatrix}-2.1 & 0.1 \\ 1 & -2\end{bmatrix}, A_2 = \begin{bmatrix}-1.9 & 0 \\ -0.2 & -1.1\end{bmatrix},$$

$$A_{d1} = \begin{bmatrix}-1.1 & 0.1 \\ -0.8 & -0.9\end{bmatrix}, A_{d2} = \begin{bmatrix}-0.9 & 0 \\ -1.1 & -1.2\end{bmatrix},$$

$$A_{w1} = \begin{bmatrix}1 \\ -0.2\end{bmatrix} A_{w2} = \begin{bmatrix}0.3 \\ 0.1\end{bmatrix}, C_1 = \begin{bmatrix}1 & 0\end{bmatrix}$$

$$C_2 = \begin{bmatrix}0.5 & -0.6\end{bmatrix}, L_1 = \begin{bmatrix}1 & -0.5\end{bmatrix},$$

$$L_2 = \begin{bmatrix}-0.2 & 0.3\end{bmatrix}, h_1(\theta(t)) = \sin^2 t, h_2(\theta(t)) = \cos^2 t$$

$$w(t) = \begin{cases} 1, & 5 \le t \le 10 \\ -1 & 15 \le t \le 20 \\ 0 & \text{else}\end{cases}$$

when the system is under the event-triggered scheme, let the corresponding trigger parameter $\rho = 0.9$ and $\bar{\Xi} = 0.8$, $\delta = 0.05$, $\tau_m = 0.1$, $\tau_M = 0.3$, $d_M = 0.6$, $\gamma = 1.2$, by applying Theorem 2, we can obtain the corresponding trigger matrix $\Omega = 0.1926$ and by applying Theorem 2, the filter parameters are derived as follows:

$$\begin{aligned} A_{f1} &= \begin{bmatrix} -1.9589 & -0.0081 \\ 1.2934 & -1.2695 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.0440 \\ -0.0695 \end{bmatrix}, \\ A_{f2} &= \begin{bmatrix} -1.0594 & 0.3548 \\ -0.5006 & -1.2287 \end{bmatrix}, B_{f2} = \begin{bmatrix} -0.0319 \\ 0.0503 \end{bmatrix}, \\ C_{f1} &= \begin{bmatrix} -0.8457 & 0.4302 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.3089 & -0.1238 \end{bmatrix} \end{aligned}$$

For the initial condition $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$, $x_f(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ and the sampling period $h = 0.1$, the event-triggering release instants and intervals are shown in Fig.1. the simulation result for the responses of $e(t)$ are shown in Fig.2, which demonstrate that the designed filter can satisfy the system performance.

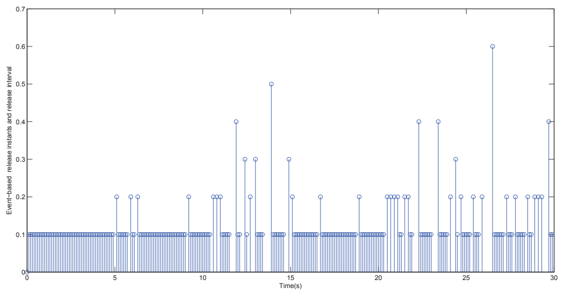

Fig. 1: the release instants and the release interval

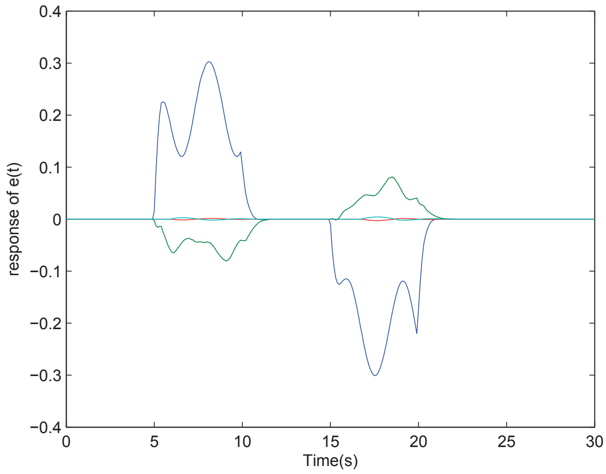

Fig. 2: Responses of $e(t)$

5 Conclusion

The reliable H_∞ filter design for a event-based network control systems via T-S fuzzy model has been investigated. In particular, the event-triggered scheme inserted in the network has the advantages of reducing the communication load in the network and gearing up its efficiency. Moreover, employing the networked T-S fuzzy model with probabilistic sensor faults and the event triggered scheme, the fundamental stability criteria are obtained, and a filter design method is developed. Then the explicit expression of the desired filter parameters has been derived. Lastly, A numerical example has been provided to show the usefulness and effectiveness of the proposed method.

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