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Reliable Control for Nonlinear Systems with Stochastic Actuators Fault and Random Delays Through a T-S Fuzzy Model Approach

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Abstract This paper considers the reliable control design for T-S fuzzy systems with probabilistic actuators faults and random time-varying delays. The faults of each actuator occurs randomly and its failure rates are governed by a set of unrelated random variables satisfying certain probabilistic distribution. In terms of the probabilistic failures of each actuator and time-varying random delays, new fault model is proposed. Based on the new fuzzy model, reliable controller is designed and sufficient conditions for the exponentially mean square stability (EMSS) of T-S fuzzy systems are derived by using Lyapunov functional method and linear matrix inequality (LMI) technique. It should be noted that the obtained criteria depend on not only the size of the delay, but also the probability distribution of it. Finally, a numerical example is given to show the effectiveness of the proposed method.

Keywords reliable control; fuzzy system; probabilistic failures; random delays
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1 Introduction

It is well known that T-S fuzzy systems have been widely studied and have many practical applications during the past decades. Much effort has been devoted to both theoretical research and implementation techniques for fuzzy control (see [6,10, 14] and the references therein). Actually, large number of studies show that fuzzy control is a practical control approach for complex nonlinear systems. In many model-based fuzzy control methods, the well-known Takagi-Sugeno (T-S) fuzzy model, which had been first proposed in [10], has been widely used in the study of system analysis and synthesis of a class of nonlinear systems through membership functions (see [1,6,14]) for some representative works. However, in the real world, nonlinear dynamic systems with time delay exist extensively in many industrial and engineering systems, such as rolling mill systems, chemical processes, communication networks, and it is well known that the T-S fuzzy model is qualified to represent a certain class of nonlinear dynamic systems (of course including time delay systems), and thus, it is natural to investigate nonlinear systems with time delay via the corresponding T-S fuzzy model. Some nice results on T-S fuzzy model have

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been reported in the literature and there are two kinds of results, namely delay-independent T-S fuzzy systems[1,2] and delay-dependent T-S fuzzy systems [3,18]. The delay-dependent results are usually less conservative than the delay-independent ones, especially when the delay is small.

Furthermore, recent studies on the delay-dependent stability and stabilization for the T-S fuzzy systems are also fruitful, and many valuable results have been reported in the open literature, for constant delay (see [3,18]) and for time-varying delay (see [8,9]). However, it should be noted that most of the aforementioned results are obtained under a full reliability assumption that all sensors, control components and actuators of the systems are in good working condition. As is known to all, due to the sensors or actuators aging, zero shift, electromagnetic interference, network disturbance *etal.*, sensor failures, actuator failures and data distortion are unavoidable, which may lead to intolerable system performance[19]. Recently, there have been a deep look into the reliable control design (see [4,11,12,20] and the references there in). In the aforementioned papers, the fault model is described in a static way and most of them only considered actuator failures [11,20], few of them consider the reliable control with sensor failures [4,12]. In [11], the reliable control design was considered for networked control systems (NCSs) against probabilistic actuator fault with different failure rates, measurements distortion, random network-induced delay and packet dropout, and a new distribution-based fault model is proposed, which contains the probability distribution information of the random delay and packet dropout. In [12], a comprehensive model is developed in this paper to discuss a class of T-S fuzzy model based nonlinear systems with probabilistic sensor and actuator failure.

Up to now, to the best of the authors' knowledge, the reliable control has not yet been addressed for T-S fuzzy systems with both probabilistic actuators failures and random delays, which still remains as a challenging problem. In this paper, the reliable control for T-S fuzzy systems is investigated. A new distribution-based fault model is proposed, which includes both probabilistic actuators fault and random time-varying delays. The actuators can be in different characters and their failures happen in a random way, which are governed by a set of random variables satisfying certain probabilistic distribution on the interval $[0, \theta_l](\theta_l \geq 1,\ l = 1, 2)$. By using Lyapunov functional approach, new criteria for designing the fuzzy reliable controller is obtained in term of linear matrix inequality.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n \times m$ real matrices, the superscript "T" stands for matrix transposition, I is the identity matrix of appropriate dimension. $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. The notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). When x is a stochastic variable, $\mathcal{E}\{\S\}$ stands for the expectation of x. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denote a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2 Systems Description and Preliminaries

In this paper, we consider the following T-S fuzzy systems:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(k)\big[A_i x(t) + B_i u(t)\big], \tag{1}$$

where A_i, B_i are matrices with appropriate dimensions, $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$ are the state and control respectively. $i \in \{1, 2, \cdots, r\} \triangleq \mathbb{S}$, where r is the member of IF-THEN rules, and

$$h_i(\theta(t)) = \frac{\omega_i(\theta(t))}{\sum_{i=1}^{r} \omega_i(\theta(t))}, \qquad \omega_i(\theta(t)) = \prod_{j=1}^{g} W_j^i(\theta_j(t)),$$

$W_j^i(\theta_j(t))$ is the grade membership value of $\theta_j(t)$ in W_j^i and $h_i(\theta(t))$ satisfies

$$h_i(\theta(t)) \geq 0, \qquad \sum_{i=1}^{r} h_i(\theta(t)) = 1.$$

For notational simplicity, we use h_i to represent $h_i(\theta(t))$ in the following description.

In this paper, the sensors are time-driven, the controller and actuators are event-driven. A proposed framework of System (1) is shown in figure **??**. The role of the networked synchronizer is to evaluate the premises employed in (1). Due to the network impact, the premise variables for the control rules in the controller side are $\theta(t-\tau(t))$, where $0 \leq \tau(t) \leq \tau_2$ is the network-induced delay. To design the PDC control rules, if we use $\theta(t - \tau(t))$ as the premise variables, wherein the resulting time scale will be different from that in (1), which may lead to some problems, such ad different membership function expression in the derivation of the main result. To avoid this problems, in the following, we propose a model-based method for the re-construction of the premises in the control rule design.

From the above assumption on the System (1), when the initial condition is given, the state of the state of the system can be calculated based on the system Model (1). Since d_k is known by the networked synchronizer, based on the system Model (1), we can compute $x(t - \tau(t))$. Based on the above analysis, the premise synchronizer included in Fig. 1 can derive the premises $\theta(t - \tau(t))$ in controller side.

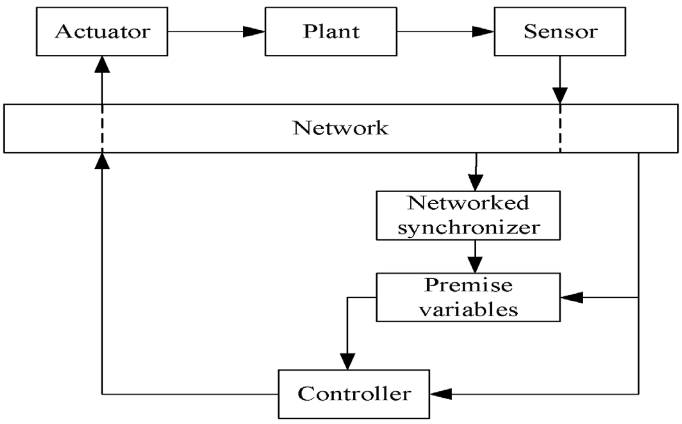

Fig. 1. A General Framework of System

If we do not consider the unreliable communication channels, that is, only the network-induced delay from sensor to controller and controller to actuator are considered, from the above assumption, the i^{kh} controller rule can be naturally designed as

$$\begin{aligned} R^i: \quad & \text{If } \theta_1(t-\tau(t)) \text{ is } W_1^i \text{ and}, \cdots, \text{if } \theta_g(t-\tau(t)) \text{ is } W_g^i, \\ & \text{then } u(t) = K_j x(t-\tau(t)) \end{aligned} \tag{2}$$

Based on the above description, the defuzzified output of controller rule (2) is designed as

$$u(t)=\sum_{j=1}^{r}h_j(\theta(t))K_jx(t-\tau(t)). \tag{3}$$

Consider the unreliable channel from controller to actuator, (3) can be described as

$$u(t)=\sum_{j=1}^{r}h_j\Xi K_jx(k-\tau(t)), \tag{4}$$

where $\Xi=\mathrm{diag}\{\Xi_1,\Xi_2,\cdots,\Xi_m\}$ with Ξ_i $(i=1,2,\cdots,m)$ being m unrelated random variables taking values on the interval $[0,\rho_1]$, where $\rho_1\geq 1$, the mathematical expectation and variance of Ξ_i $(i=1,2,\cdots,m)$ are α_i and δ_i^2.

Remark 1. In [5, 15–17], the random variables γ_k taking values in$\{0,1\}$ represent the meaning of completely failure or not. In [5, 16], the authors assume that the random variables γ_k taking values in the interval [0, 1], when $0\leq\gamma_k\leq 1$, it means partial failure. In this paper, we assume that the variables Ξ_i belongs to the interval $[0,\ \rho_1]$ with $\rho_1\geq 1$. For $\Xi_j=0$, it means failure of the j-th actuator or packet loss during the transmission from controller to actuator, for $\Xi_j=1$, it means that the j-th actuator is in good working condition. Different from [5,15–17], we also consider data distortion of the sensors and actuators. As is known, when the sensors or actuators have faults, the output signal maybe larger or smaller than what it should be. Considering this, we use $0<\Xi_i<1$ and $\Xi_i>1$, to represent the condition of data distortion.

Assumption 1. *The distribution of the time-varying delay $\tau(t)$ can be obtained, and for a constant $\tau_1\in[0,\tau_2]$, the probability of $\tau(t)\in[0,\tau_1)$ and $\tau(t)\in[\tau_1,\tau_2]$ can be known.*

Define two functions

$$\tau_1(t)=\begin{cases}\tau(t), & \text{for } \tau(t)\in[0,\tau_1),\\ 0, & \text{for } \tau(t)\in[\tau_1,\tau_2],\end{cases} \tag{5}$$

$$\tau_2(t)=\begin{cases}\tau(t), & \text{for } \tau(t)\in[\tau_1,\tau_2],\\ \tau_1, & \text{for } \tau(t)\in[0,\tau_1).\end{cases} \tag{6}$$

In order to employ the information of the probabilistic information of the stochastic delay, a stochastic variable $\beta(t)$ is defined as

$$\beta(t)=\begin{cases}1, & \tau(t)\in[0,\tau_1),\\ 0, & \tau(t)\in[\tau_1,\tau_2].\end{cases} \tag{7}$$

Assumption 2. *$\beta(t)$ is a Bernoulli distributed sequence with*

$$\mathrm{Prob}\{\beta(t)=1\}=\mathcal{E}\{\beta(t)\}=\beta_0,\qquad \mathrm{Prob}\{\beta(t)=0\}=1-\mathcal{E}\{\beta(t)\}=1-\beta_0, \tag{8}$$

where $0\leq\beta_0\leq 1$ is a constant, and $\beta(t)$ is unrelated with Ξ_i $(i=1,2,\cdots,m)$.

Using $\beta(t)$, System (4) can be rewritten as

$$u(t)=\sum_{j=1}^{r}h_j\beta(t)\Xi K_jx(k-\tau_1(t))+\sum_{j=1}^{r}h_j(1-\beta(t))\Xi K_jx(k-\tau_2(t)). \tag{9}$$

Substituting (9) into (1), we obtain

$$\begin{aligned}\dot{x}(t) =& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \big[A_i x(t) + \beta(t) B_i \Xi K_j x(k-\tau_1(t)) + (1-\beta(t)) B_i \Xi K_j x(k-\tau_2(t))\big] \\ =& \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \big[A_i x(t) + \beta(t) B_i \overline{\Xi} K_j x(k-\tau_1(t)) + (1-\beta(t)) B_i \overline{\Xi} K_j x(k-\tau_2(t)) \\ &+ \beta(t) B_i (\Xi - \overline{\Xi}) K_j x(k-\tau_1(t)) + (1-\beta(t)) B_i (\Xi - \overline{\Xi}) K_j x(k-\tau_2(t))\big], \end{aligned} \tag{10}$$

where

$$\overline{\Xi} = \mathrm{diag}\{\alpha_1, \cdots, \alpha_m\} = \sum_{i=1}^{m} \alpha_i C_i, C_i = \mathrm{diag}\{\underbrace{0, \cdots, 0}_{i-1}, 1, \underbrace{0, \cdots, 0}_{m-i}.$$

From above discussion, we can obtain $\mathcal{E}\{(\Xi_1 - \overline{\Xi}_1)^2\} = \mathrm{diag}\{\delta_1^2, \cdots, \delta_m^2\}$.

The following Lemmas and Definitions are needed in the proof of our main results.

Definition 1[7]. *For a given function $V : C_{F_0}^b([-d_2, 0], \mathbb{R}^n) \times \mathbb{S}$, its infinitesimal operator $\mathcal{L}$ is defined as*

$$\mathcal{L}V(\eta(k)) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}\big[\mathcal{E}(V(\eta_{k+\Delta})|\eta_k) - v(\eta_k)\big].$$

Definition 2. *The System (10) is said to be exponentially mean-square stable (EMSS) if, for any initial condition $\phi(i) \in \mathbb{R}^n$, $i = -d_2, -d_2+1, \cdots, 0$, there exist constants $\alpha > 0$, $\beta > 0$ such that the following condition holds,*

$$\mathcal{E}\Big\{\sum_{k=0}^{\infty} X^T(k) X(k)\Big\} \le \alpha e^{-\beta t} \sup_{-d_2 \le i \le 0} \mathcal{E}\{\phi^T(i)\phi(i)\}.$$

Lemma 1[13]. *Suppose Ω_1,Ω_2, and Ω are constant matrices of appropriate dimensions. Then*

$$\tau_1(t)\Omega_1 + (\tau_1 - \tau_1(t))\Omega_2 + (\tau_2(t) - \tau_1)\Omega_3 + (\tau_2 - \tau_2(t))\Omega_4 + \Omega < 0 \tag{11}$$

is true for any $\tau_1(t) \in [0, \tau_1)$, $\tau_2(t) \in [\tau_1, \tau_2]$, if and only if

$$\tau_1\Omega_2 + (\tau_2 - \tau_1)\Omega_4 + \Omega < 0, \tag{12}$$

$$\tau_1\Omega_2 + (\tau_2 - \tau_1)\Omega_3 + \Omega < 0, \tag{13}$$

$$\tau_1\Omega_1 + (\tau_2 - \tau_1)\Omega_4 + \Omega < 0, \tag{14}$$

$$\tau_1\Omega_1 + (\tau_2 - \tau_1)\Omega_3 + \Omega < 0. \tag{15}$$

3 Main Result

The following theorem can be obtained for the System (10) with the fuzzy reliable controller (9).

Theorem 1. *For given scalars τ_1, τ_2 and matrix K_j $(j \in \mathbb{S})$, the System (10) is EMSS if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, N_{ij}, M_{ij}, V_{ij} and S_{ij} $(i, j \in \mathbb{S})$ with appropriate dimensions such that the following matrix inequalities hold for $i, j = 1, 2, \cdots, r$ and $1 \le i \le j \le r$.*

$$\Psi_s(i,j) + \Psi_s(i,j) < 0, \qquad s = 1, 2, 3, 4, \tag{16}$$

where

$$\Psi_s(i,j) = \begin{bmatrix} \Omega_{11}(i,j)+\Gamma(i,j)+\Gamma^T(i,j) & * & * & * \\ \Omega_{21}(i,j) & \Omega_{22} & * & * \\ \Omega_{31}(i,j) & 0 & \Omega_{33} & * \\ \Omega_{41}^s(i,j) & 0 & 0 & \Omega_{44} \end{bmatrix}$$

and

$$\Omega_{11}(i,j) = \begin{bmatrix} PA_i + A_i^T P + Q_1 + Q_2 & * & * & * & * \\ \beta_0 K_j^T \overline{\Xi}^T B_i^T P & 0 & * & * & * \\ 0 & 0 & -Q_1 & * & * \\ 0 & 0 & 0 & 0 & * \\ (1-\beta_0) K_j^T \overline{\Xi}^T B_i^T P & 0 & 0 & 0 & -Q_2 \end{bmatrix},$$

$$\Omega_{21}(i,j) = \begin{bmatrix} \sqrt{\beta_0\tau_1} R_1 A_i & \sqrt{\beta_0\tau_1} R_1 B_i \overline{\Xi} K_j & 0 & 0 & 0 \\ \sqrt{(1-\beta_0)\tau_1} R_1 A_i & 0 & 0 & \sqrt{(1-\beta_0)\tau_1} R_1 B_i \overline{\Xi} K_j & 0 \\ \sqrt{\beta_0\tau_{21}} R_2 A_i & \sqrt{\beta_0\tau_{21}} R_2 B_i \overline{\Xi} K_j & 0 & 0 & 0 \\ \sqrt{(1-\beta_0)\tau_{21}} R_2 A_i & 0 & 0 & \sqrt{(1-\beta_0)\tau_{21}} R_2 B_i \overline{\Xi} K_j & 0 \end{bmatrix},$$

$$\Omega_{31}(i,j) = \begin{bmatrix} 0 & \sqrt{\beta_0}\Upsilon_1 & 0 & 0 & 0 \\ 0 & \sqrt{\beta_0}\Upsilon_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1-\beta_0}\Upsilon_1 & 0 \\ 0 & 0 & 0 & \sqrt{1-\beta_0}\Upsilon_2 & 0 \end{bmatrix},$$

$$\Upsilon_1 = \begin{bmatrix} \sqrt{\tau_1\delta_1} R_1 B_i C_1 K_j \\ \sqrt{\tau_1\delta_2} R_1 B_i C_2 K_j \\ \vdots \\ \sqrt{\tau_1\delta_m} R_1 B_i C_m K_j \end{bmatrix}, \qquad \Upsilon_2 = \begin{bmatrix} \sqrt{\tau_{21}\delta_1} R_2 B_i C_1 K_j \\ \sqrt{\tau_{21}\delta_2} R_2 B_i C_2 K_j \\ \vdots \\ \sqrt{\tau_{21}\delta_m} R_2 B_i C_m K_j \end{bmatrix},$$

$$\Omega_{41}^1(i,j) = \begin{bmatrix} \sqrt{\tau_1} N_{ij}^T \\ \sqrt{\tau_{21}} V_{ij}^T \end{bmatrix}, \qquad \Omega_{41}^2(i,j) = \begin{bmatrix} \sqrt{\tau_1} N_{ij}^T \\ \sqrt{\tau_{21}} S_{ij}^T \end{bmatrix},$$

$$\Omega_{41}^3(i,j) = \begin{bmatrix} \sqrt{\tau_1} M_{ij}^T \\ \sqrt{\tau_{21}} V_{ij}^T \end{bmatrix}, \qquad \Omega_{41}^4(i,j) = \begin{bmatrix} \sqrt{\tau_1} M_{ij}^T \\ \sqrt{\tau_{21}} S_{ij}^T \end{bmatrix},$$

$$\Gamma(i,j) = [\, N_{ij} \quad -N_{ij}+M_{ij} \quad -M_{ij}+V_{ij} \quad -V_{ij}+S_{ij} \quad -S_{ij} \,],$$

$$\Omega_{22} = daig\{-R_1, -R_1, -R_2, -R_2\}, \Omega_{44} = daig\{-R_1, -R_2\},$$

$$\Omega_{33} = daig\{\underbrace{-R_1,\cdots,-R_1}_{m}, \underbrace{-R_2,\cdots,-R_2}_{m}, \underbrace{-R_1,\cdots,-R_1}_{m}, \underbrace{-R_2,\cdots,-R_2}_{m}\}, \qquad \tau_{21} = \tau_2 - \tau_1$$

Proof.

Construct a Lyapunov functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{17}$$

where

$$\begin{aligned} V_1(t) &= x^T(t) P x(t), \\ V_2(t) &= \int_{t-\tau_1}^{t} x^T(s) Q_1 x(s) ds + \int_{t-\tau_2}^{t} x^T(s) Q_2 x(s) ds, \\ V_3(t) &= \int_{t-\tau_1}^{t}\int_{s}^{t} \dot{x}^T(v) R_1 \dot{x}(v) dv ds + \int_{t-\tau_2}^{t-\tau_1}\int_{s}^{t} \dot{x}^T(v) R_2 \dot{x}(v) dv ds. \end{aligned}$$

Using the infinitesimal operator and taking expectation for the Lyapunov functional, we obtain

$$\mathcal{E}\{\mathcal{L}V(t)\}$$

$$
\begin{aligned}
=&\mathcal{E}\Big\{\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j2x^T(t)P\big[A_ix(t)+\beta_0B_i\Xi K_jx(t-\tau_1(t))+(1-\beta_0)B_i\overline{\Xi}K_jx(t-\tau_2(t))\big]\\
&+x^T(t)(Q_1+Q_2)x(t)-x^T(t-\tau_1)Q_1x(t-\tau_1)-x^T(t-\tau_2)Q_2x(t-\tau_2)\\
&+\dot{x}^T(t)(\tau_1R_1+\tau_{21}R_2)\dot{x}(t)-\int_{t-\tau_1}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds-\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds\Big\}. \qquad (18)
\end{aligned}
$$

Notice that

$$
\begin{aligned}
\mathcal{E}\{\dot{x}^T(t)\widetilde{R}\dot{x}(t)\}\leq&\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big\{\beta_0[A_ix(t)+B_i\Xi K_jx(t-\tau_1(t))]^T\widetilde{R}[A_ix(t)+B_i\Xi K_jx(t-\tau_1(t))]\\
&+(1-\beta_0)\big[A_ix(t)+B_i\overline{\Xi}K_jx(t-\tau_2(t))\big]^T\widetilde{R}\big[A_ix(t)+B_i\overline{\Xi}K_jx(t-\tau_2(t))\big]\\
&+\sum_{l=1}^{m}\delta_l^2\beta_0x^T(t-\tau_1(t))(B_iC_lK_j)^T\widetilde{R}(B_iC_lK_j)x(t-\tau_1(t))\\
&+\sum_{l=1}^{m}\delta_l^2(1-\beta_0)x^T(t-\tau_2(t))(B_iC_lK_j)^T\widetilde{R}(B_iC_lK_j)\,x(t-\tau_2(t))\Big\}, \qquad (19)
\end{aligned}
$$

where $\widetilde{R}=\tau_1R_1+\tau_{21}R_2$.

Employing the free-weighting matrices $M_{ij},N_{ij},V_{ij},S_{ij}\ (i,j\in\mathbb{S})$, we can obtain

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big\{2\xi^T(t)N_{ij}\Big[x(t)-x(t-\tau_1(t))-\int_{t-\tau_1(t)}^{t}\dot{x}(s)ds\Big]\Big\}=0, \qquad (20)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big\{2\xi^T(t)M_{ij}\Big[x(t-\tau_1(t))-x(t-\tau_1)-\int_{t-\tau_1}^{t-\tau_1(t)}\dot{x}(s)ds\Big]\Big\}=0, \qquad (21)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big\{2\xi^T(t)V_{ij}\Big[x(t-\tau_1)-x(t-\tau_2(t))-\int_{t-\tau_2(t)}^{t-\tau_1}\dot{x}(s)ds\Big]\Big\}=0, \qquad (22)$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big\{2\xi^T(t)S_{ij}\Big[x(t-\tau_2(t))-x(t-\tau_2)-\int_{t-\tau_2}^{t-\tau_2(t)}\dot{x}(s)ds\Big]\Big\}=0, \qquad (23)$$

where $\xi^T(k)=[\,x^T(t)\quad x^T(t-\tau_1(t))\quad x^T(t-\tau_1)\quad x^T(t-\tau_2(t))\quad x^T(t-\tau_2)\,]$ and N_{ij},M_{ij}, V_{ij},S_{ij} are matrices with appropriate dimensions and

$$-2\xi^T(t)N_{ij}\int_{t-\tau_1(t)}^{t}\dot{x}(s)ds\leq\tau_1(t)\xi^T(t)N_{ij}R_1^{-1}N_{ij}^T\xi(t)+\int_{t-\tau_1(t)}^{t}\dot{x}^T(s)R_1\dot{x}(s)ds, \qquad (24)$$

$$-2\xi^T(t)M_{ij}\int_{t-\tau_1}^{t-\tau_1(t)}\dot{x}(s)ds\leq(\tau_1-\tau_1(t))\xi^T(t)M_{ij}R_1^{-1}M_{ij}^T\xi(t)+\int_{t-\tau_1}^{t-\tau_1(t)}\dot{x}^T(s)R_1\dot{x}(s)ds, \qquad (25)$$

$$-2\xi^T(t)V_{ij}\int_{t-\tau_2(t)}^{t-\tau_1}\dot{x}(s)ds\leq(\tau_2(t)-\tau_1)\xi^T(t)V_{ij}R_2^{-1}V_{ij}^T\xi(t)+\int_{t-\tau_2(t)}^{t-\tau_1}\dot{x}^T(s)R_2\dot{x}(s)ds, \qquad (26)$$

$$-2\xi^T(t)S_{ij}\int_{t-\tau_2}^{t-\tau_2(t)}\dot{x}(s)ds\leq(\tau_2-\tau_2(t))\xi^T(t)S_{ij}R_2^{-1}S_{ij}^T\xi(t)+\int_{t-\tau_2}^{t-\tau_2(t)}\dot{x}^T(s)R_2\dot{x}(s)ds. \qquad (27)$$

Combining (19)–(27) and using Schur complement, from (18), we obtain

$$\mathcal{E}\{\mathcal{L}V(t)\}\leq\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big\{\xi^T(t)\big[\Omega_{11}(i,j)+\Gamma(i,j)+\Gamma^T(i,j)\big]\xi(t)$$

$$
\begin{aligned}
&+ \beta_0\big[A_i x(t) + B_i\overline{\Xi}K_j x(t-\tau_1(t))\big]^T\widetilde{R}\big[A_i x(t) + B_i\overline{\Xi}K_j x(t-\tau_1(t))\big] \\
&+ (1-\beta_0)\big[A_i x(t) + B_i\overline{\Xi}K_j x(t-\tau_2(t))\big]^T\widetilde{R}\big[A_i x(t) + B_i\overline{\Xi}K_j x(t-\tau_2(t))\big] \\
&+ \sum_{l=1}^{m}\delta_l^2\beta_0 x^T(t-\tau_1(t))(B_iC_lK_j)^T\widetilde{R}(B_iC_lK_j)x(t-\tau_1(t)) \\
&+ \sum_{l=1}^{m}\delta_l^2(1-\beta_0) x^T(t-\tau_2(t))(B_iC_lK_j)^T\widetilde{R}(B_iC_lK_j)x(t-\tau_2(t)) \\
&+ \tau_1(t)\xi^T(t)N_{ij}R_1^{-1}N_{ij}^T\xi(t) + (\tau_1-\tau_1(t))\xi^T(t)M_{ij}R_1^{-1}M_{ij}^T\xi(t) \\
&+ (\tau_2(t)-\tau_1)\xi^T(t)V_{ij}R_2^{-1}V_{ij}^T\xi(t) + (\tau_2-\tau_2(t))\xi^T(t)S_{ij}R_2^{-1}S_{ij}^T\xi(t)\Big\}. \qquad (28)
\end{aligned}
$$

By using Schur complement and Lemma 1, we can conclude that (16) is the sufficient condition to guarantee

$$\mathcal{E}\{\mathcal{L}V(t)\} \le 0. \qquad (29)$$

Then, using the Lyapunov stability theory, we can obtain that the System (10) under the fuzzy reliable controller (9) is EMSS. □

In the following, we will focus on designing the fuzzy reliable controller (9) to stabilize System (10) with stochastic actuators fault and random delays.

Theorem 2. *For given scalars $\tau_1, \tau_2, \varepsilon_1$ and ε_2, System (10) under fuzzy reliable controller (9) is EMSS if there exist matrices $X > 0$, $\widehat{Q}_1 > 0$, $\widehat{Q}_2 > 0$, $\widehat{N}_{ij}$, $\widehat{M}_{ij}$, V_{ij} and S_{ij} $(i,j \in \mathbb{S})$ with appropriate dimensions such that the following linear matrix inequalities hold for $i,j = 1,2,\cdots,r$ and $1 \le i \le j \le r$,*

$$\Phi_s(i,j) + \Phi_s(i,j) < 0, \qquad s = 1,2,3,4, \qquad (30)$$

where

$$\Phi_s(i,j) = \begin{bmatrix} \overline{\Omega}_{11}(i,j) + \overline{\Gamma}(i,j) + \overline{\Gamma}^T(i,j) & * & * & * \\ \overline{\Omega}_{21}(i,j) & \overline{\Omega}_{22} & * & * \\ \overline{\Omega}_{31}(i,j) & 0 & \overline{\Omega}_{33} & * \\ \overline{\Omega}_{41}^s(i,j) & 0 & 0 & \overline{\Omega}_{44} \end{bmatrix}$$

and

$$\overline{\Omega}_{11}(i,j) = \begin{bmatrix} A_iX + XA_i^T + \widehat{Q}_1 + \widehat{Q}_2 & * & * & * & * \\ \beta_0 Y_j^T\overline{\Xi}^T B_i^T & 0 & * & * & * \\ 0 & 0 & -\widehat{Q}_1 & * & * \\ 0 & 0 & 0 & 0 & * \\ (1-\beta_0)Y_j^T\overline{\Xi}^T B_i^T & 0 & 0 & 0 & -\widehat{Q}_2 \end{bmatrix},$$

$$\overline{\Gamma}(i,j) = \begin{bmatrix} \widehat{N}_{ij} & -\widehat{N}_{ij} + \widehat{M}_{ij} & -\widehat{M}_{ij} + \widehat{V}_{ij} & -\widehat{V}_{ij} + \widehat{S}_{ij} & -\widehat{S}_{ij} \end{bmatrix},$$

$$\overline{\Omega}_{21}(i,j) = \begin{bmatrix} \sqrt{\beta_0\tau_1}A_iX & \sqrt{\beta_0\tau_1}B_i\overline{\Xi}Y_j & 0 & 0 & 0 \\ \sqrt{(1-\beta_0)\tau_1}A_iX & 0 & 0 & \sqrt{(1-\beta_0)\tau_1}B_i\overline{\Xi}Y_j & 0 \\ \sqrt{\beta_0\tau_{21}}A_iX & \sqrt{\beta_0\tau_{21}}B_i\overline{\Xi}Y_j & 0 & 0 & 0 \\ \sqrt{(1-\beta_0)\tau_{21}}A_iX & 0 & 0 & \sqrt{(1-\beta_0)\tau_{21}}B_i\overline{\Xi}Y_j & 0 \end{bmatrix},$$

$$\overline{\Omega}_{22}(i,j) = diag\{-2\varepsilon_1X + \varepsilon_1^2\widehat{R}_1, -2\varepsilon_1X + \varepsilon_1^2\widehat{R}_1, -2\varepsilon_2X + \varepsilon_1^2\widehat{R}_2, -2\varepsilon_2X + \varepsilon_1^2\widehat{R}_2\},$$

$$\overline{\Omega}_{31}(i,j) = \begin{bmatrix} 0 & \sqrt{\beta_0\tau_1}\widehat{\Upsilon} & 0 & 0 & 0 \\ 0 & \sqrt{\beta_0\tau_{21}}\widehat{\Upsilon} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{(1-\beta_0)\tau_1}\widehat{\Upsilon} & 0 \\ 0 & 0 & 0 & \sqrt{(1-\beta_0)\tau_{21}}\widehat{\Upsilon} & 0 \end{bmatrix}, \qquad \widehat{\Upsilon} = \begin{bmatrix} \delta_1B_iC_1Y_j \\ \delta_2B_iC_2Y_j \\ \vdots \\ \delta_mB_iC_mY_j \end{bmatrix},$$

$$\overline{\Omega}_{33}(i,j) = diag\{\Upsilon_3, \Upsilon_4, \Upsilon_3, \Upsilon_4\}, \qquad \overline{\Omega}_{44} = diag\{-\widehat{R}_1, -\widehat{R}_2\},$$

$$\Upsilon_3 = \underbrace{-2\varepsilon_1 X + \varepsilon_1^2 \widehat{R}_1, \cdots, -2\varepsilon_1 X + \varepsilon_1^2 \widehat{R}_1}_{m}, \qquad \Upsilon_4 = \underbrace{-2\varepsilon_2 X + \varepsilon_1^2 \widehat{R}_2, \cdots - 2\varepsilon_2 X + \varepsilon_1^2 \widehat{R}_2}_{m},$$

$$\overline{\Omega}_{41}^s(i,j) = \begin{bmatrix} \sqrt{\tau_1}\widehat{N}_{ij}^T \\ \sqrt{\tau_{21}}\widehat{V}_{ij}^T \end{bmatrix}, \qquad \overline{\Omega}_{41}^2(i,j) = \begin{bmatrix} \sqrt{\tau_1}\widehat{N}_{ij}^T \\ \sqrt{\tau_{21}}\widehat{S}_{ij}^T \end{bmatrix},$$

$$\overline{\Omega}_{41}^3(i,j) = \begin{bmatrix} \sqrt{\tau_1}\widehat{M}_{ij}^T \\ \sqrt{\tau_{21}}\widehat{V}_{ij}^T \end{bmatrix}, \qquad \overline{\Omega}_{41}^4(i,j) = \begin{bmatrix} \sqrt{\tau_1}\widehat{M}_{ij}^T \\ \sqrt{\tau_{21}}\widehat{S}_{ij}^T \end{bmatrix},$$

then, the System (10) can be stabilized by control law (9), and the controller gain is $K_j = Y_j X^{-1}$.

Proof. Defining $X = P^{-1}$, and $XQ_1X = \widehat{Q}_1$, $XQ_2X = \widehat{Q}_2$, $XR_1X = \widehat{R}_1$, $XR_2X = \widehat{R}_2$, $\mathrm{diag}\{X,X,X,X\}N_{ij}\mathrm{diag}\{X,X,X,X\} = \widehat{N}_{ij}$, $\mathrm{diag}\{X,X,X,X\}M_{ij}\mathrm{diag}\{X,X,X,X\} = \widehat{M}_{ij}$, $K_jX = Y_j$.

By Schur complement, Equation (16) is equivalent to

$$\widetilde{\Psi}_s(i,j) + \widetilde{\Psi}_s(i,j) < 0, \qquad s = 1,2,3,4, \tag{31}$$

where

$$\Psi_s(i,j) = \begin{bmatrix} \Omega_{11}(i,j) + \Gamma(i,j) + \Gamma^T(i,j) & * & * & * \\ \widetilde{\Omega}_{21}(i,j) & \widetilde{\Omega}_{22} & * & * \\ \widetilde{\Omega}_{31}(i,j) & 0 & \widetilde{\Omega}_{33} & * \\ \Omega_{41}^s(i,j) & 0 & 0 & \Omega_{44} \end{bmatrix}$$

and

$$\widetilde{\Omega}_{21}(i,j) = \begin{bmatrix} \sqrt{\beta_0\tau_1}PA_i & \sqrt{\beta_0\tau_1}PB_i\overline{\Xi}K_j & 0 & 0 & 0 \\ \sqrt{(1-\beta_0)\tau_1}PA_i & 0 & 0 & \sqrt{(1-\beta_0)\tau_1}PB_i\overline{\Xi}K_j & 0 \\ \sqrt{\beta_0\tau_{21}}PA_i & \sqrt{\beta_0\tau_{21}}PB_i\overline{\Xi}K_j & 0 & 0 & 0 \\ \sqrt{(1-\beta_0)\tau_{21}}PA_i & 0 & 0 & \sqrt{(1-\beta_0)\tau_{21}}PB_i\overline{\Xi}K_j & 0 \end{bmatrix},$$

$$\widetilde{\Omega}_{31}(i,j) = \begin{bmatrix} 0 & \sqrt{\beta_0}\widetilde{\Upsilon}_1 & 0 & 0 & 0 \\ 0 & \sqrt{\beta_0}\widetilde{\Upsilon}_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{1-\beta_0}\widetilde{\Upsilon}_1 & 0 \\ 0 & 0 & 0 & \sqrt{1-\beta_0}\widetilde{\Upsilon}_2 & 0 \end{bmatrix},$$

$$\widetilde{\Upsilon}_1 = \begin{bmatrix} \sqrt{\tau_1}PB_iC_1K_j \\ \sqrt{\tau_1}PB_iC_2K_j \\ \vdots \\ \sqrt{\tau_1}PB_iC_mK_j \end{bmatrix}, \qquad \widetilde{\Upsilon}_2 = \begin{bmatrix} \sqrt{\tau_{21}}PB_iC_1K_j \\ \sqrt{\tau_{21}}PB_iC_2K_j \\ \vdots \\ \sqrt{\tau_{21}}PB_iC_mK_j \end{bmatrix}$$

$$\widetilde{\Omega}_{22} = \mathrm{daig}\{-PR_1^{-1}P, -PR_1^{-1}P, -PR_2^{-1}P, -PR_2^{-1}P\},$$

$$\widetilde{\Omega}_{33} = \mathrm{daig}\{\underbrace{-PR_1^{-1}P, \cdots, -PR_1^{-1}P}_{m}, \underbrace{-PR_2^{-1}P, \cdots, -PR_2^{-1}P}_{m}, \underbrace{-PR_1^{-1}P, \cdots, -PR_1^{-1}P}_{m}, \underbrace{-PR_2^{-1}P, \cdots, -PR_2^{-1}P}_{m}\}.$$

Owing to

$$-PR_i^{-1}P \leq -2\varepsilon_i P + \varepsilon_i^2 R_i, \qquad i = 1,2, \tag{32}$$

we have (31) holds if the following equation holds:

$$\check{\Psi}_s(i,j) + \check{\Psi}_s(i,j) < 0, \qquad s = 1,2,3,4, \tag{33}$$

where $\check{\Psi}_s(i,j)$ is obtained by replacing $-PR_i^{-1}P$ with $-2\varepsilon_i P + \varepsilon_i^2 R_i$.

Then, pre and post multiplying (33) with

$$\mathrm{diag}\{\underbrace{X,\cdots,X}_{11+4m}\}$$

then, (30) can be obtained. □

4 Example

To illustrate the effectiveness of the proposed method, consider the T-S fuzzy systems (1) are described as

$$A_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 0.05 & 0 \\ 0 & -0.1 \end{bmatrix}, \qquad B_1 = B_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}.$$

To illustrate the effectiveness of the proposed method, the following two cases are considered.

Case 1. Without considering the stochastic actuators fault that is, $\overline{\Xi} = 1$, $\delta_1 = 0$, and setting $\varepsilon_1 = \varepsilon_2 = 1$, $\beta_0 = 0.8$, $\tau_1 = 0.1$, $\tau_2 = 0.5$, based on the Theorem 2, we can obtain the following controller gains to guarantee the System (1) is EMSS.

$$\begin{cases} K_1 = [0.5252 \ -0.2375], \\ K_2 = [0.4884 \ -0.2475]. \end{cases} \tag{34}$$

With the initial $x_0 = [-0.5\ 0.5]^T$, the time responses of the system states are shown in Fig. 2.

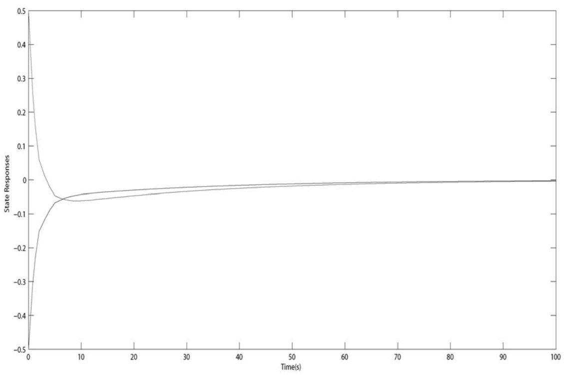

Fig. 2. The State Responses Under Controller Feedback Gain (34)

Case 2. Considering the stochastic actuators fault, set $\overline{\Xi} = 0.7$, $\delta_1 = 0.1$, and $\varepsilon_1 = \varepsilon_2 = 1$, $\beta_0 = 0.8$, $\tau_1 = 0.1$, $\tau_2 = 0.5$, based on the Theorem 2, we can obtain the following controller gains to guarantee the System (1) is EMSS.

$$\begin{cases} K_1 = [0.7440 \ -0.3356], \\ K_2 = [0.6915 \ -0.3498] \end{cases} \tag{35}$$

With the initial $x_0 = [-0.5\ 0.5]^T$, the time responses of the system states are shown in Fig.3.

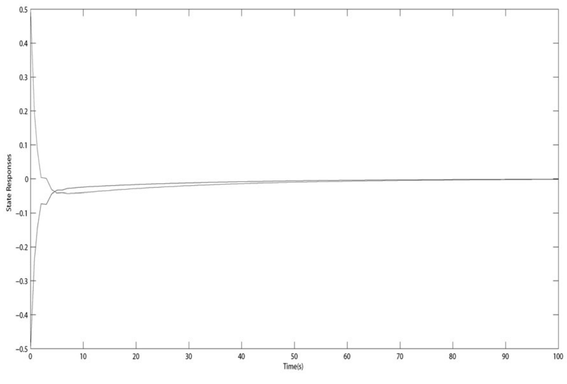

Fig.3. The State Responses Under Controller Feedback Gain (35)

5 Conclusion

This paper considers reliable control design for T-S fuzzy systems with probabilistic actuator failures and random delays. The fault of each actuator occurs in a random way and their failure rates are governed by a set of unrelated random variables satisfying certain probabilistic distribution. In terms of the probabilistic failures of every actuator and random time-varying delays, new model is proposed. By using Lyapunov functional method and linear matrix inequality technique, sufficient conditions for the EMSS of T-S fuzzy systems are obtained. An example with simulation results has been carried out to demonstrate the effectiveness of the proposed method.

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