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H_∞ filter design for a class of T-S fuzzy systems with quantization and event-triggered communication scheme

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Abstract: This paper concerns the H_∞ filter design for T-S fuzzy systems with quantization and event-triggered communication scheme. In general, it is difficult to gain the mathematical model for nonlinear systems. T-S fuzzy systems described by a family of IF-THEN rules, are valid tools to establish model of nonlinear systems. This paper aims to design H_∞ filter, which satisfied closed-loop system is exponentially stable in mean square. Firstly, an event-triggered scheme is introduced to the networked control systems, which decide whether the sensor data was transmitted or not. Secondly, quantization is taken into consideration, which may save the limited bandwidth resources. Thirdly, based on the event-triggered scheme and quantization, we construct filter design model for T-S fuzzy systems. Lastly, the corresponding conditions for the desired filter gains were derived and the explicit expression of filter parameters is given. Two numerical example is provided to demonstrate the design method.

Key Words: H_∞ filting; Event-trigger generator; T-S fuzzy systems; Networked control systems.

1 Introduction

Nowadays, the T-S fuzzy model described by a family of IF-THEN rules has gained widely attention, which provided an valid method to turn a set of nonlinear systems into a class of linear sub-systems [1–3]. The problem of stability and stabilization for T-S fuzzy model has received widely concern. Over the past several decades, there have been significant accomplishments on stability analysis and synthesis for T-S fuzzy systems. More particularly, many researchers have considered the filtering issues for T-S fuzzy systems via various methods [4–6]. In [4], the authors concerned with a class of T-S fuzzy stochastic systems. In [5], the problem of robust and reliable H_∞ filter design was concerned, which contain random sensor faults via T-S fuzzy model. In [6], the authors addressed the problem of H_∞ filter design for nonlinear systems, which contain time delay via T-S fuzzy systems.

Different from the traditional point-to-point wiring, the advantages of communication channels is reduced the production cost of cables. However, there are some drawbacks when the control signals conveyed through communication channels. Several challenging issues needs to be considered including network-induced delays, packet dropouts and so on. Taking these into concerned, some effective methods have been proposed for the analysis and synthesis of NCSs [7–11]. However, the problem of above mentioned results is that we supposed that the precision of all data transmissions is infinite, while we ignore the influences of data quantization. The limited bandwidth resources and some devices in the closed-loop systems determines that the precision of data transmission can not be infinite, which means that data should be quantized before conveyed to the next network node. Motivated by the above discussion, the influence of data quantization on systems should be taken into consideration for the better performance of the considered systems. Unfortunately, the researchers in the field have paid much less attention on the quantization problem than what it deserves. As far as we know, there are only there are only handful of results [12–14] about the quantization problem when considering the network conditions. Therefore, the problem of quantization needs further more attention.

In the past decades, event-triggered communication scheme has attracted increasing attention, which can save communication energy. Compared with traditional time-triggered control, event-triggered control determines whether the current sample-data should be conveyed to the next network code or not, which will save the limited bandwidth resources.

In this paper, an event-triggered communication scheme and quantizer are employed to save the limited network resources while keeping the required performance. Whether or not the sample-data to be conveyed is determined by the event-triggered communication scheme. The current data can be transmitted when they dissatisfied a special condition, otherwise, will be not. This paper aims to research the problem of H_∞ filter design for a class of T-S fuzzy systems with quantization and event-triggered communication scheme.

2 System description

Consider the following T-S fuzzy system

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_d(t)x(t-\tau(t)) + A_\omega(t)\omega(t) \\ y(t) = C(t)x(t) \\ z(t) = L(t)x(t) \end{cases} \tag{1}$$

where $A(t) = \sum_{i=1}^{r} \mu_i A_i$, $A_d(t) = \sum_{i=1}^{r} \mu_i A_{di}$, $A_\omega(t) = \sum_{i=1}^{r} \mu_i A_{\omega i}$, $C(t) = \sum_{i=1}^{r} \mu_i C_i$, $L(t) = \sum_{i=1}^{r} \mu_i L_i$. h_i is the abbreviation for $\mu_i(\theta(t))$, $\mu_i(\theta(t)) = \frac{\alpha_i(\theta(t))}{\sum_{i=1}^{r}\alpha_i(\theta(t))}$, $\alpha_i(\theta(t)) = \prod_{j=1}^{g} W_j^i(\theta_j(t))$, $W_j^i(\theta(t))$ is the grade member-ship value of $\theta_j(t)$ in W_j^i and $\mu_i(\theta(t))$ satisfies $\mu_i(\theta(t)) \geq 0$, $\sum_{i=1}^{r} \mu_i(\theta(t)) = 1$.

It is generally known, the traditional periodic sampling mechanism may produce many useless message through the

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network, which may waste communication bandwidth. For reducing the load of network conveying and saving the communication energy of the network, there is a great need to introduce an event-triggered scheme, which determines whether or not the newly data should be conveyed to the filter. The framework of system is shown in Fig.1, similar to [15], an event generator is employed between the quantizer and the filter to determine whether or not the newly data should be conveyed to the filter, which is the following judgment algorithm:

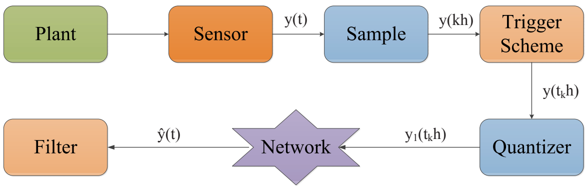

Fig. 1: The structure of filtering error system

$$[y((k+j)h)-y(kh)]^T\Omega[y((k+j)h)-y(kh)] \quad (2)$$
$$\le \sigma y((k+j)h)^T\Omega y((k+j)h)$$

where $\Omega > 0$, $j = 1, 2, \cdots$ and $\sigma \in [0, 1)$.

We design a quantizer with similar form in [16]. Suppose $y(t_k h)$ is transmitted by the event trigger, the quantized output signal can be expressed as

$$y_1(t_k h) = \sum_{i=1}^{r} \mu_i(\theta(t_k h))q(y(t_k h)) \quad (3)$$

Then the output signal from quantizer is $y_1(t_k h)$

$$y_1(t_k h) = \sum_{i=1}^{r} \mu_i(I+\triangle_q)y(t_k h) \quad (4)$$

then $y_1(t_k h)$, which transmitted to filter through the network, become $\hat{y}(t)$

$$\begin{aligned}\hat{y}(t) &= \hat{y}(t_k h + d_k) = \sum_{i=1}^{r} \mu_i y_1(t_k h) \\ &= \sum_{i=1}^{r} \mu_i (I+\triangle_q) y(t_k h) \\ & \quad t \in [t_k h + d_k, t_{k+1}h + d_{k+1})\end{aligned}$$

where d_k is network delay.

Remark 1 *Since the existence of communication network between the sensor and the filter, the premises in the system and the ones in the filter should not be synchronous. That means, at the same instant $t \in [t_k h + d_k, t_{k+1}h + d_{k+1})$, while $\theta_i(t)$ is obtainable in (1), only $\theta_i(t_k h)$ is obtainable in the filter. In this paper, we suppose the mechanical model of the studied system is known a priori, depended on the known mechanical model, when the initial condition is set and the state of the studied system can be counted. Due to $\theta_i(t_k h)$ is obtainable at the filter, $\theta_i(t)$ can be counted for $t \in [t_k h, t_{k+1} h)$. Therefore, the synchronous premise variables $\theta_i(t)$ can be gained in the filter side.*

Suppose the form of H_∞ fuzzy filter is following, where ith rule is shown in the following IF-THEN rule

$R^i : IF \theta_1(t) is W_1^i and \cdots and \theta_g(t) is W_g^i$,

THEN

$$\begin{cases} \dot{x}_f(t) = A_{fi}x_f(t) + B_{fi}\hat{y}(t) \\ z_f(t) = C_{fi}x_f \end{cases} \quad (5)$$

where $r \in S = \{1, 2, \cdots, r\}$. $x_f(t) \in R^n$, $z_f(t) \in R^p$ are the state and output of the filter, respectively. $A_{fi} \in R^{n\times n}$, $B_{fi} \in R^{n\times m}$, $C_{fi} \in R^{p\times n}$ are unknown parameter which to be decided.

The defuzzified output of (5) is referred by

$$\begin{cases} \dot{x}_f(t) = A_f(t)x_f(t) + B_f(t)\hat{y}(t) \\ z_f(t) = C_f(t)x_f(t) \end{cases} \quad (6)$$

where $A_f(t) = \sum_{i=1}^{r} \mu_i A_{fi}$, $B_f(t) = \sum_{i=1}^{r} \mu_i B_{fi}$, $C_f(t) = \sum_{i=1}^{r} \mu_i C_{fi}$.

Similar to [15, 17, 18] for technical convenience, we define an error vector:

$$e_k(t) = \begin{cases} 0, t \in I_0 \\ y(t_k h) - y(t_k h + ih), t \in I_i, \\ i = 1, 2, \cdots, \delta_M - 1 \\ y(t_k h) - y(t_k h + \delta_M h), t \in I_{\delta_M} \end{cases} \quad (7)$$

Through the definition of $e_k(t)$ and the event-triggering scheme (2), it can be obtained that for $t \in [t_k h + d_k, t_{k+1}h + d_{k+1})$

$$e_k^T(t)\Omega e_k(t) \le \sigma y^T(t-d(t))\Omega y(t-d(t)) \quad (8)$$

Based on the above description, combine (4) and (7), the actual output can be written as

$$\hat{y}(t) = \sum_{i=1}^{r} \mu_i (I+\triangle_q)(y(t-d(t)) + e_k(t)) \quad (9)$$

then filter can be rewritten as

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j[A_{fj}x_f(t) \\ +B_{fj}(I+\triangle_q)(C_i x(t-d(t)+e_k(t))] \\ z_f(t) = \sum_{j=1}^{r} \mu_j C_{fj}x_f(t) \end{cases} \quad (10)$$

Define

$$e(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix} \tilde{z(t)} = z(t) - z_f(t)$$

From (5) and (10) the following filtering-error system funded on (1) and (10) can be written as

$$\begin{cases} \dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j[\bar{A}e(t) + \bar{A}_d He(t-\tau(t)) \\ +\bar{B}He(t-d(t)) + \bar{B}_1 e_k(t) + \bar{A}_\omega \omega(t)] \\ \tilde{z}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \mu_i\mu_j \bar{L}e(t) \end{cases} \quad (11)$$

$$\bar{A} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fj} \end{bmatrix}, \bar{A}_d = \begin{bmatrix} A_{di} \\ 0 \end{bmatrix}, H = \begin{bmatrix} I & 0 \end{bmatrix}$$
$$\bar{B} = \begin{bmatrix} 0 \\ B_{fj}(I+\triangle_q)C_i \end{bmatrix}, \bar{B}_1 = \begin{bmatrix} 0 \\ B_{fj}(I+\triangle_q) \end{bmatrix}$$
$$\bar{A}_\omega = \begin{bmatrix} A_\omega \\ 0 \end{bmatrix}, \bar{L} = \begin{bmatrix} L_i & -C_{fj} \end{bmatrix}$$

3 Main results

In this part, we will first derive sufficient conditions to ensure the composite system (11) is exponentially stable in the mean square. Then we will propose the explicit parameters of the designed filter.

Theorem 1 *For given parameters γ, τ_m, τ_M, d_M and σ, under the event trigger scheme (2) and quantization, system (11) is exponentially stable in the mean square with an H_∞ disturbance attenuation level γ, if there exist matrices $P > 0$, $Q_k > 0$, $R_k > 0 (k = 1, 2, 3)$, $\Omega > 0$ and M_{ij}, N_{ij}, T_{ij}, S_{ij} with appropriate dimensions satisfying*

$$\Xi^{ij} + \Xi^{ji} < 0, i \leq j \in S \tag{12}$$

where

$$\begin{aligned}
\Xi^{ij} &= \begin{bmatrix} \Omega_{11}^{ij} & * & * \\ \Omega_{21}^{ij} & \Omega_{22}^{ij} & * \\ \Omega_{31}^{ij}(s) & 0 & \Omega_{33}^{ij} \end{bmatrix}, (s = 1, 2, 3, 4) \\
\Omega_{11}^{ij} &= \begin{bmatrix} \Omega_{1111}^{ij} & \Omega_{1112}^{ij} \\ \Omega_{1121}^{ij} & \Omega_{1122}^{ij} \end{bmatrix} \\
\Omega_{1111}^{ij} &= \begin{bmatrix} \Gamma_{ij1} & * & * & * \\ R_2 & \Gamma_{ij2} & * & * \\ H^T\bar{A}_d{}^T P & \bar{M} & \Gamma_{ij3} & * \\ 0 & 0 & \bar{N} & \Gamma_{ij4} \end{bmatrix} \\
\bar{M} &= M_{ij3} - M_{ij2}^T, \bar{N} = N_{ij4} - N_{ij3}^T \\
\Omega_{1112}^{ij} &= \begin{bmatrix} * & * & * & * \\ g1 & * & * & * \\ g2 & * & * & * \\ g3 & * & * & * \end{bmatrix} \\
g1 &= g2 = g3 = * \\
\Omega_{1121}^{ij} &= \begin{bmatrix} H^T\bar{B}^T P + T_{ij5} - T_{ij1}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{B}_1^T P & 0 & 0 & 0 \\ \bar{A}_\omega^T P & 0 & 0 & 0 \end{bmatrix} \\
\Omega_{1122}^{ij} &= \begin{bmatrix} \Gamma_{ij4} & * & * & * \\ \Gamma_{ij5} & * & * & * \\ 0 & 0 & -\Omega & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix} \\
\Gamma_{ij1} &= P\bar{A} + \bar{A}^T P + Q_1 + Q_2 + Q_3 \\
&- R_1 + T_{ij1} + T_{ij1}^T \\
\Gamma_{ij2} &= -Q_1 - R_2 + M_{ij2} + M_{ij2}^T \\
\Gamma_{ij3} &= -M_{ij3} - M_{ij3}^T + N_{ij3} + N_{ij3}^T \\
\Gamma_{ij4} &= -Q_2 - N_{ij4} - N_{ij4}^T \\
\Gamma_{ij6} &= -Q_3 - S_{ij6} - S_{ij6}^T \\
\Gamma_{ij5} &= \sigma H^T C_i^T \Omega C_i H - T_{ij5} - T_{ij5}^T + S_{ij5} + S_{ij5}^T \\
\Omega_{21}^{ij} &= \begin{bmatrix} \Omega_{2111}^{ij} & \Omega_{2112}^{ij} \end{bmatrix} \\
\Omega_{2111}^{ij} &= \begin{bmatrix} \bar{L} & 0 & 0 & 0 \\ \sqrt{\tau_{21}}P\bar{A} & 0 & \sqrt{\tau_{21}}P\bar{A}_d H & 0 \\ \tau_m P\bar{A} & 0 & \tau_m P\bar{A}_d H & 0 \\ \sqrt{d_M}P\bar{A} & 0 & \sqrt{d_M}P\bar{A}_d H & 0 \end{bmatrix} \\
\Omega_{2112}^{ij} &= \begin{bmatrix} 0 & 0 & 0 & 0 \\ \sqrt{\tau_{21}}P\bar{B}H & 0 & \sqrt{\tau_{21}}P\bar{B}_1 & \sqrt{\tau_{21}}P\bar{A}_\omega \\ \tau_m P\bar{B}H & 0 & \tau_m P\bar{B}_1 & \tau_m P\bar{A}_\omega \\ \sqrt{d_M}P\bar{B} & 0 & \sqrt{d_M}P\bar{B}_1 H & \sqrt{d_M}P\bar{A}_\omega \end{bmatrix} \\
\Omega_{22}^{ij} &= diag\{-I, -PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\} \\
\sqrt{\tau_{21}} &= \sqrt{\tau_M - \tau_m}, \Omega_{33}^{ij} = diag\{-R_1, -R_2\} \\
\Omega_{31}^{ij}(1) &= \begin{bmatrix} \sqrt{\tau_{21}}M_{ij}^T \\ \sqrt{d_M}T_{ij}^T \end{bmatrix}, \Omega_{31}^{ij}(2) = \begin{bmatrix} \sqrt{\tau_{21}}M_{ij}^T \\ \sqrt{d_M}S_{ij}^T \end{bmatrix} \\
\Omega_{31}^{ij}(3) &= \begin{bmatrix} \sqrt{\tau_{21}}N_{ij}^T \\ \sqrt{d_M}S_{ij}^T \end{bmatrix}, \Omega_{31}^{ij}(4) = \begin{bmatrix} \sqrt{\tau_{21}}N_{ij}^T \\ \sqrt{d_M}T_{ij}^T \end{bmatrix} \\
M_{ij}^T &= \begin{bmatrix} 0 & M_{ij2}^T & M_{ij3}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
N_{ij}^T &= \begin{bmatrix} 0 & 0 & N_{ij3}^T & N_{ij4}^T & 0 & 0 & 0 & 0 \end{bmatrix} \\
T_{ij}^T &= \begin{bmatrix} T_{ij1}^T & 0 & 0 & 0 & T_{ij5}^T & 0 & 0 & 0 \end{bmatrix} \\
S_{ij}^T &= \begin{bmatrix} 0 & 0 & 0 & 0 & S_{ij5}^T & S_{ij6}^T & 0 & 0 \end{bmatrix}
\end{aligned}$$

Proof. Choose the following Lyapunov functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{13}$$

where

$$\begin{aligned}
V_1(t) &= e^T(t)Pe(t) \\
V_2(t) &= \int_{t-\tau_m}^{t} e^T(s)Q_1 e(s)ds + \int_{t-\tau_M}^{t} e^T(s)Q_2 e(s)ds \\
&+ \int_{t-d_M}^{t} e^T(s)Q_3 e(s)ds \\
V_3(t) &= \int_{t-\tau_M}^{t-\tau_m}\int_{s}^{t} \dot{e}^T(v)R_1\dot{e}(v)dvds \\
&+ \tau_m \int_{t-\tau_m}^{t}\int_{s}^{t} \dot{e}^T(v)R_2\dot{e}(v)dvds \\
&+ \int_{t-d_M}^{t}\int_{s}^{t} \dot{e}^T(v)R_3\dot{e}(v)dvds
\end{aligned}$$

$P > 0, Q_k > 0, R_k > 0 (k = 1, 2, 3)$ Through applying a similar method to the proof in [20], we can get that the system (11) is exponentially stable with an H_∞ norm bound γ if (12) is satisfied. Due to the limited page, we omit the proof.

Remark 2 *Through Theorem 1 we can get that system (11) is exponentially stable in the mean square. Based on the Theorem 1, we design a filter in form of (6). Due to the exists of nonlinear term in the Theorem 1, we separate I and $\triangle_q$, which can transformed nonlinear term into LMIs. The algorithm of the designed filter are gained in the following theorem.*

Theorem 2 *Considering filter error System (11) with quantizer. For given scalar γ, τ_m, τ_M, d_M, ε_1, ε_2, ε_3, σ and* *δ, under the event trigger scheme* (2), *system* (11) *is exponentially stable in the mean square if there exist variable* $m_1 > 0$ *and* $P_1 > 0$, $\bar{P}_3 > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{Q}_3 > 0$, $\bar{R}_1 > 0$, $\bar{R}_2 > 0$, $\bar{R}_3 > 0$, $\Omega > 0$, $\bar{A}_{fj}$, $\bar{B}_{fj}$, $\bar{C}_{fj}$, $\bar{M}_{ij}$, $\bar{N}_{ij}$, $\bar{T}_{ij}$, $\bar{S}_{ij}$ *with appropriate dimensions, such that*

$$\bar{\Sigma}^{ij} + \bar{\Sigma}^{ji} < 0, i \leq j \in S \quad (14)$$

$$P_1 - \bar{P}_3 > 0 \quad (15)$$

where

$$\bar{\Sigma}^{ij} = \begin{bmatrix} \bar{\Phi}_{11}^{ij} & * & * & * & * \\ \bar{\Phi}_{21}^{ij} & \Phi_{22}^{ij} & * & * & * \\ \Phi_{31}^{ij}(s) & 0 & \Phi_{33}^{ij} & * & * \\ \Phi_{41}^{T} & \Phi_{42}^{T} & 0 & -m_1 I & * \\ \Phi_{51}^{T} & 0 & 0 & 0 & -m_1 I \end{bmatrix}$$

$$\bar{\Phi}_{11}^{ij} = \begin{bmatrix} \bar{\Phi}_{1111}^{ij} & \bar{\Phi}_{1112}^{ij} \\ \bar{\Phi}_{1121}^{ij} & \bar{\Phi}_{1122}^{ij} \end{bmatrix}$$

$$\bar{\Phi}_{1111}^{ij} = \begin{bmatrix} \bar{\Gamma}_{ij1} & * & * & * \\ \bar{R}_2 & \bar{\Gamma}_{ij2} & * & * \\ \tilde{\Upsilon}_{ij31} & \bar{M}_{ij3} - \bar{M}_{ij2}^T & \bar{\Gamma}_{ij3} & * \\ 0 & 0 & \bar{N}_{ij4} - \bar{N}_{ij3}^T & \bar{\Gamma}_{ij4} \end{bmatrix}$$

$$\bar{\Phi}_{1112}^{ij} = \begin{bmatrix} * & * & * & * \\ k1 & * & * & * \\ k2 & * & * & * \\ k3 & * & * & * \end{bmatrix}$$

$$k1 = k2 = k3 = *$$

$$\bar{\Phi}_{1121}^{ij} = \begin{bmatrix} \tilde{\Lambda}_{ij51} + T_{ij5} - T_{ij1}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_{ij71} & 0 & 0 & 0 \\ \tilde{\Upsilon}_{ij81} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Phi}_{1122}^{ij} = \begin{bmatrix} \bar{\Gamma}_{ij5} & * & * & * \\ \bar{S}_{ij6} - \bar{S}_{ij5} & \bar{\Gamma}_{ij6} & * & * \\ 0 & 0 & -\Omega & * \\ 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\bar{\Gamma}_{ij1} = \tilde{\Upsilon}_{ij1} + \tilde{\Upsilon}_{ij1}^T + \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 - \bar{R}_2 + \bar{T}_{ij1} + \bar{T}_{ij1}^T$$

$$\tilde{\Upsilon}_{ij1} = \begin{bmatrix} P_1 A_i & \bar{A}_{fj} \\ \bar{P}_3 A_i & \bar{A}_{fj} \end{bmatrix}$$

$$\tilde{\Upsilon}_{ij31} = \begin{bmatrix} A_{di}^T P_1 & A_{di}^T \bar{P}_3 \\ 0 & 0 \end{bmatrix}$$

$$\bar{\Gamma}_{ij2} = -\bar{Q}_1 - \bar{R}_2 + \bar{M}_{ij2} + \bar{M}_{ij2}^T$$

$$\bar{\Gamma}_{ij3} = -\bar{M}_{ij3} - \bar{M}_{ij3}^T + \bar{N}_{ij3} + \bar{N}_{ij3}^T$$

$$\bar{\Gamma}_{ij4} = -\bar{Q}_2 - \bar{N}_{ij4} - \bar{N}_{ij4}^T$$

$$\bar{\Gamma}_{ij5} = \tilde{\Upsilon}_{ij5} - \bar{T}_{ij5} - \bar{T}_{ij5}^T + \bar{S}_{ij5} + \bar{S}_{ij5}^T$$

$$\bar{\Gamma}_{ij6} = -\bar{Q}_3 - \bar{S}_{ij6} - \bar{S}_{ij6}^T$$

$$\tilde{\Lambda}_{ij51} = \begin{bmatrix} C_i^T I^T \bar{B}_{fj}^T & C_i^T I^T \bar{B}_{fj}^T \\ 0 & 0 \end{bmatrix}$$

$$\tilde{\Upsilon}_{ij5} = \begin{bmatrix} \rho C_i^T \Omega C_i & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{\Lambda}_{ij71} = \begin{bmatrix} I^T \bar{B}_{fj}^T & I^T \bar{B}_{fj}^T \end{bmatrix}$$

$$\tilde{\Upsilon}_{ij81} = \begin{bmatrix} A_{\omega i}^T P_1 & A_{\omega i}^T \bar{P}_3 \end{bmatrix}$$

$$\bar{\Phi}_{21}^{ij} = \begin{bmatrix} \bar{\Phi}_{211}^{ij} & \bar{\Phi}_{212}^{ij} \end{bmatrix}$$

$$\bar{\Phi}_{211}^{ij} = \begin{bmatrix} \tilde{L} & 0_{p\times 2n} & 0_{p\times 2n} & 0_{p\times 2n} \\ \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij1} & 0_{2n\times 2n} & \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij31}^T & 0_{2n\times 2n} \\ \tau_m \tilde{\Upsilon}_{ij1} & 0_{2n\times 2n} & \tau_m \tilde{\Upsilon}_{ij31}^T & 0_{2n\times 2n} \\ \sqrt{d_M}\tilde{\Upsilon}_{ij1} & 0_{2n\times 2n} & \sqrt{d_M}\tilde{\Upsilon}_{ij31}^T & 0_{2n\times 2n} \end{bmatrix}$$

$$\bar{\Phi}_{212}^{ij} = \begin{bmatrix} 0_{p\times 2n} & 0_{p\times 2n} & 0_{p\times m} & 0_{p\times m} \\ \sqrt{\tau_{21}}\tilde{\Lambda}_{ij51}^T & 0_{2n\times 2n} & \sqrt{\tau_{21}}\tilde{\Lambda}_{ij71}^T & \sqrt{\tau_{21}}\tilde{\Upsilon}_{ij81}^T \\ \tau_m \tilde{\Lambda}_{ij51}^T & 0_{2n\times 2n} & \tau_m \tilde{\Lambda}_{ij71}^T & \tau_m \tilde{\Upsilon}_{ij81}^T \\ \sqrt{d_M}\tilde{\Lambda}_{ij51}^T & 0_{2n\times 2n} & \sqrt{d_M}\tilde{\Lambda}_{ij71}^T & \sqrt{d_M}\tilde{\Upsilon}_{ij81}^T \end{bmatrix}$$

$$\Phi_{22}^{ij} = diag\{-I, -2\varepsilon_1 \bar{P} + \varepsilon_1^2 \bar{R}_1, -2\varepsilon_2 \bar{P} + \varepsilon_2^2 \bar{R}_2, -2\varepsilon_3 \bar{P} + \varepsilon_3^2 \bar{R}_3\}$$

$$\bar{P} = \begin{bmatrix} P_1 & \bar{P}_3 \\ \bar{P}_3 & \bar{P}_3 \end{bmatrix}, \tilde{L}_{ij} = \begin{bmatrix} L_i & -\bar{C}_{fj} \end{bmatrix}$$

$$\Phi_{33}^{ij} = diag\{-\bar{R}_1, -\bar{R}_3\}$$

$$\Phi_{31}^{ij}(1) = \begin{bmatrix} \sqrt{\tau_{21}}\bar{M}_{ij}^T \\ \sqrt{d_M}\bar{T}_{ij}^T \end{bmatrix}, \Phi_{31}^{ij}(2) = \begin{bmatrix} \sqrt{\tau_{21}}\bar{M}_{ij}^T \\ \sqrt{d_M}\bar{S}_{ij}^T \end{bmatrix}$$

$$\Phi_{31}^{ij}(3) = \begin{bmatrix} \sqrt{\tau_{21}}\bar{N}_{ij}^T \\ \sqrt{d_M}\bar{S}_{ij}^T \end{bmatrix}, \Phi_{31}^{ij}(4) = \begin{bmatrix} \sqrt{\tau_{21}}\bar{N}_{ij}^T \\ \sqrt{d_M}\bar{T}_{ij}^T \end{bmatrix}$$

$$\bar{M}_{ij}^T = \begin{bmatrix} \bar{M}11_{ij}^T & \bar{M}12_{ij}^T \end{bmatrix}$$

$$\bar{N}_{ij}^T = \begin{bmatrix} \bar{N}11_{ij}^T & \bar{N}12_{ij}^T \end{bmatrix}$$

$$\bar{T}_{ij}^T = \begin{bmatrix} \bar{T}11_{ij}^T & \bar{T}12_{ij}^T \end{bmatrix}$$

$$\bar{S}_{ij}^T = \begin{bmatrix} \bar{S}11_{ij}^T & \bar{S}12_{ij}^T \end{bmatrix}$$

$$\bar{M}11_{ij}^T = \begin{bmatrix} 0_{2n\times 2n} & \bar{M}_{ij2}^T & \bar{M}_{ij3}^T & 0_{2n\times 2n} \end{bmatrix}$$

$$\bar{M}12_{ij}^T = \begin{bmatrix} 0_{2n\times 2n} & 0_{2n\times 2n} & 0_{2n\times m} & 0_{2n\times m} \end{bmatrix}$$

$$\bar{N}11_{ij}^T = \begin{bmatrix} 0_{2n\times 2n} & 0_{2n\times 2n} & \bar{N}_{ij3}^T & \bar{N}_{ij4}^T \end{bmatrix}$$

$$\bar{N}12_{ij}^T = \begin{bmatrix} 0_{2n\times 2n} & 0_{2n\times 2n} & 0_{2n\times m} & 0_{2n\times m} \end{bmatrix}$$

$$\bar{T}11_{ij}^T = \begin{bmatrix} \bar{T}_{ij1}^T & 0_{2n\times 2n} & 0_{2n\times 2n} & 0_{2n\times 2n} \end{bmatrix}$$

$$\bar{T}12_{ij}^T = \begin{bmatrix} \bar{T}_{ij5}^T & 0_{2n\times 2n} & 0_{2n\times m} & 0_{2n\times m} \end{bmatrix}$$

$$\bar{S}11_{ij}^T = \begin{bmatrix} 0_{2n\times 2n} & 0_{2n\times 2n} & 0_{2n\times 2n} & 0_{2n\times 2n} \end{bmatrix}$$

$$\bar{S}12_{ij}^T = \begin{bmatrix} \bar{S}_{ij5}^T & \bar{S}_{ij6}^T & 0_{2n\times m} & 0_{2n\times m} \end{bmatrix}$$

$$\Phi_{41}^T = \begin{bmatrix} \bar{\mathcal{M}}_{1111}^T & \bar{\mathcal{M}}_{1112}^T \end{bmatrix}$$

$$\Phi_{42}^T = \begin{bmatrix} 0_{2n\times m} & \Psi_{ij2}^T & \Psi_{ij3}^T & \Psi_{ij4}^T \end{bmatrix}$$

$$\bar{\mathcal{M}}_{1111}^T = \begin{bmatrix} \Psi_{ij1}^T & 0_{2n\times 2n} & 0_{2n\times 2n} & 0_{2n\times 2n} \end{bmatrix}$$

$$\bar{\mathcal{M}}_{1112}^T = \begin{bmatrix} 0_{2n\times 2n} & 0_{2n\times 2n} & 0_{2n\times m} & 0_{2n\times m} \end{bmatrix}$$

$$\Psi_{ij1}^T = \begin{bmatrix} \delta \bar{B}_{fj}^T & \bar{B}_{fj}^T \end{bmatrix}$$

$$\Psi_{ij2}^T = \begin{bmatrix} \delta\sqrt{\tau_{21}}\bar{B}_{fj}^T & \delta\sqrt{\tau_{21}}\bar{B}_{fj}^T \end{bmatrix}$$

$$\Psi_{ij3}^T = \begin{bmatrix} \delta\tau_m \bar{B}_{fj}^T & \delta\tau_m \bar{B}_{fj}^T \end{bmatrix}$$

$$\Psi_{ij4}^T = \begin{bmatrix} \delta\sqrt{d_M}\bar{B}_{fj}^T & \delta\sqrt{d_M}\bar{B}_{fj}^T \end{bmatrix}$$

$$\Phi_{51}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & \tilde{C}_i & 0 & I & 0 \end{bmatrix}$$

$$\tilde{C}_i = \begin{bmatrix} C_i & 0 \end{bmatrix}$$

Moreover, if the above conditions are feasible, the parameter matrices of the filter are given by

$$\begin{cases} A_{fj} = \bar{A}_{fj}\bar{P}_3^{-1} \\ B_{fj} = P_2^{-T}\bar{B}_{fj} \\ C_{fj} = \bar{C}_{fj}\bar{P}_3^{-1} \end{cases}, j \in S \quad (16)$$

Proof. Through utilizing a similar technique to the proof in [16] and [21], there exist P_2 and $P_3 > 0$ satisfying $\bar{P}_3 =$

$P_2^T P_3^{-1} P_2$, and define variables

$$\left\{\begin{array}{l} \bar{A}_{fj} = \hat{A}_{fj}, \hat{A}_{fj} = P_2^T A_{fj} P_2^{-1} \\ \bar{B}_f j = P_2^T B_{fj} \\ \bar{C}_{fj} = \hat{C}_{fj} \bar{P}_3, \hat{C}_{fj} = C_{fj} P_2^{-1} \end{array}\right. \quad (17)$$

substituting the filter parameters (A_{fj}, B_{fj}, C_{fj}) by $(P_2^{-T}\hat{A}_{fj}P_2^T, P_2^{-T}\bar{B}_{fj}, \hat{C}_{fj}P_2^T)$ then, the filter (1) can be rewritten as

$$\left\{\begin{array}{l} \dot{x}_f(t) = P_2^{-T}\hat{A}_{fj}P_2^T x_f(t) + P_2^{-T}\bar{B}_{fj}\hat{y}(t) \\ z_f(t) = \hat{C}_{fj}P_2^T x_f(t) \end{array}\right. \quad (18)$$

Defining $\hat{x}(t) = P_2^T x_f(t)$, (18) becomes

$$\left\{\begin{array}{l} \hat{x}_f(t) = \hat{A}_{fj}\hat{x}(t) + \bar{B}_{fj}\hat{y}(t) \\ z_f(t) = \hat{C}_{fj}\hat{x}(t) \end{array}\right. \quad (19)$$

That is $(\hat{A}_{fj}, \hat{B}_{fj}, \hat{C}_{fj})$ can be selected as the filter parameters. we can obtain that the system (11) is exponentially stable with an H_∞ norm bound γ if satisfy (14) and (15). Due to limited page, we omit the proof.

4 Simulation examples

Consider the T-S fuzzy system in (1) with the following parameters, similar to [21]:

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}$$

$$A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, A_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}$$

$$A_{\omega 1} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, A_{\omega 2} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix}, L_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}$$

$$\mu_1(\theta(t)) = sin^2 t, \mu_2(\theta(t)) = cos^2 t$$

$$\omega(t) = \left\{\begin{array}{ll} 1, & 5 \le t \le 10 \\ -1, & 15 \le t \le 20 \\ 0, & else \end{array}\right.$$

In the following, two possible cases will be considered for showing the influences of the the event-triggered scheme and the effect of the quantizer on the system performance.

Case 1: we set $\tau_m = 0.1, \tau_M = 0.3, d_M = 0.6, \gamma = 1.2$ and the trigger parameter $\sigma = 0.9, \delta = 0.818$ through utilizing the LMI toolbox of Matlab, we can get the trigger matrix $\Omega = 0.1469, m_1 = 2.9985, m_2 = 3.1480$ and the following matrices:

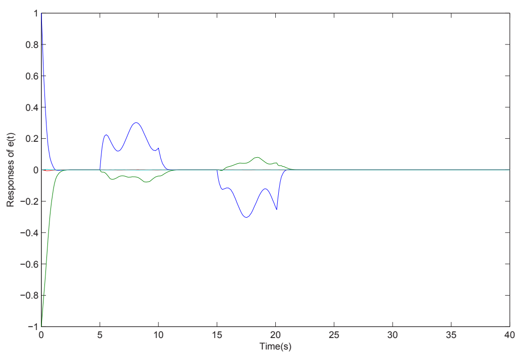

Fig. 2: Responses of $e(t)$ in Case1.

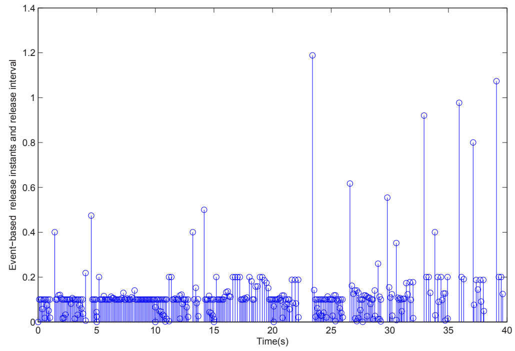

Fig. 3: The event-triggering release instants and intervals in Case1.

$$P_1 = \begin{bmatrix} 1.9347 & -0.0286 \\ -0.0286 & 1.9571 \end{bmatrix}$$

$$\bar{P}_3 = \begin{bmatrix} 0.4255 & -0.0616 \\ -0.0616 & 0.6132 \end{bmatrix}$$

$$\bar{A}_{f1} = \begin{bmatrix} -1.0474 & 0.2313 \\ 0.4443 & -1.2756 \end{bmatrix}$$

$$\bar{B}_{f1} = \begin{bmatrix} 0.0339 \\ -0.0329 \end{bmatrix}, \bar{C}_{f1} = \begin{bmatrix} -0.5786 & 0.2790 \end{bmatrix}$$

$$\bar{A}_{f2} = \begin{bmatrix} -0.7317 & -0.0421 \\ -0.3278 & -0.8903 \end{bmatrix}, \bar{B}_{f2} = \begin{bmatrix} -0.0299 \\ 0.0133 \end{bmatrix}$$

$$\bar{C}_{f2} = \begin{bmatrix} 0.1446 & -0.1493 \end{bmatrix}$$

Then, by using Theorem2, we can get the filter parameters as follows:

$$A_{f1} = \begin{bmatrix} -2.4425 & 0.1316 \\ 0.7539 & -2.0045 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.0339 \\ -0.0329 \end{bmatrix}$$

$$C_{f1} = \begin{bmatrix} -1.3131 & 0.3230 \end{bmatrix}, A_{f2} = \begin{bmatrix} -1.7550 & -0.2450 \\ -0.9951 & -1.5519 \end{bmatrix}$$

$$B_{f2} = \begin{bmatrix} -0.0299 \\ 0.0133 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.3091 & -0.2124 \end{bmatrix}$$

For the initial condition $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$, $x_f(0) = \begin{bmatrix} 0.8 & -0.8 \end{bmatrix}^T$ and the sampling period $h = 0.1$, the responses of $e(t)$ is shown in Fig.2, the event-triggering release instants and intervals is shown in Fig.3

Case 2: under the event-triggered scheme, set trigger parameter $\sigma = 0.9$ and $\tau_m = 0.1, \tau_M = 0.3, d_M = 0.6, \gamma = 1.2, \delta = 0.818$ by applying Theorem 2, we can get the trigger matrix$\Omega = 16.1228, m_1 = 5.6251, m_2 = 15.9445$ and the following matrices:

$$P_1 = \begin{bmatrix} 1.9785 & -0.0107 \\ -0.0107 & 2.1648 \end{bmatrix}, \bar{P}_3 = \begin{bmatrix} 0.4228 & -0.0781 \\ -0.0781 & 0.6976 \end{bmatrix}$$

$$\bar{C}_{f1} = \begin{bmatrix} -0.5872 & 0.2928 \end{bmatrix}, \bar{A}_{f2} = \begin{bmatrix} -0.7284 & -0.0328 \\ -0.3583 & -0.9759 \end{bmatrix}$$

$$\bar{A}_{f1} = \begin{bmatrix} -1.0333 & 0.2889 \\ 0.4981 & -1.4951 \end{bmatrix}, \bar{B}_{f1} = \begin{bmatrix} 0.0613 \\ -0.0650 \end{bmatrix}$$

$$\bar{B}_{f2} = \begin{bmatrix} -0.0484 \\ 0.0637 \end{bmatrix}, \bar{C}_{f2} = \begin{bmatrix} 0.1441 & -0.1658 \end{bmatrix}$$

Then, by applying Theorem 2, the filter parameters are derived as follows:

$$C_{f1} = \begin{bmatrix} -1.3391 & 0.2698 \end{bmatrix}$$
$$A_{f2} = \begin{bmatrix} -1.7682 & -0.2450 \\ -1.1293 & -1.5255 \end{bmatrix}$$
$$A_{f1} = \begin{bmatrix} -2.4176 & 0.1434 \\ 0.7985 & -2.0539 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.0613 \\ -0.0650 \end{bmatrix}$$
$$B_{f2} = \begin{bmatrix} -0.0484 \\ 0.0637 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.3032 & -0.2037 \end{bmatrix}$$

For the initial condition $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$, $x_f(0) = \begin{bmatrix} 0.8 & -0.8 \end{bmatrix}^T$ and the sampling period $h = 0.1$, $\hat{y}(t)$ before quantized and $\hat{y}(t)$ after quantized are shown in Fig.4. Where $y_1(t_k h)$ is sample after quantized and $y(t_k h)$ is sample before quantized.

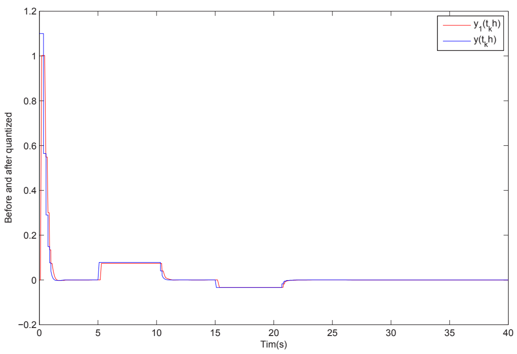

Fig. 4: Responses of $\hat{y}(t)$ before and after quantized in Case 2.

5 Conclusion

The issues of H_∞ filter design with quantization and event-triggered communication scheme network control systems via T-S fuzzy model is proposed in this paper. Sufficient conditions are obtained to guarantee the exponential stability of the proposed system. By applying the method of LMIs inequality, the algorithm of designed filter has been gained. Furthermore, a numerical example has been provided to show the usefulness and effectiveness of the proposed method.

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