

State estimation for complex network systems with quantization and event-triggered communication scheme

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Abstract: The problem of state estimation is investigated for complex network systems with quantization and event-triggered communication scheme. A quantization method is introduced to quantize the output signal, which can reduce the burden of data transmission; An event-triggered communication scheme is cited, which is useful to reduce the communication burden of transmission channel. The main purpose of this paper is to analyze and design a reliable estimator. Firstly, using event-triggered scheme to determine whether the newly sampled signal will be sent out. Secondly, signals will be transmitted to estimator by quantizer. A Lyapunov functional approach and linear matrix inequality techniques are employed to construct a system model with time-varying delay. Criteria for globally asymptotically stability in the mean square sense and criteria for the existence of state estimator are derived. Finally, simulation results have shown the effectiveness of the proposed method.

Key Words: Event-triggered scheme; Quantization; Complex network system; State estimation; Linear matrix inequality.

1 Introduction

Human beings live in the world with a variety of networks. With the rapid development of modern science and technology, people have found that in the real world, there is a certain internal relation among these networks. Thus the complex networks theory is introduced to describe the common features among these networks. In recent years, complex networks have become a research hotspot issues^[1-2].

In recent years, scholars have put great interests in complex network systems and gained many research results^[3]. Complex network systems have complex connection among network nodes structure and dynamical complexity in time and space. So, using network bandwidth and reduce network load effectively, arises great attention of scholars. At present, commonly used methods are time-triggering^[4] and event-triggering mechanism^[5]. In real life, periodic sampling are easier to accept, but from the aspect of network resource utilization, event-triggering mechanism is better, for it can greatly improve the efficiency of communication, only when a particular event occurs can an execution perform, thus saving network bandwidth resources.

So far, the research and application based on event-triggered mechanism have many research results^[2,5]. In literature [2], discusses the H_∞ performance and controller design problem of network control system based on triggering mechanism; In literature [5], for the quantization of state and control input, studies the stability problem of the network control systems based on event-trigger mechanism. Now the state estimation of complex systems has been attracting the attention of people, however, based on the event-trigger mechanism, the problem hasn't attracted much attention^[6,7]. In the actual network system, there exists the phenomenon of data loss and data approximation, and the capacity of trans-

mission is limited. To avoid it, data should be quantified before transmission. Quantization can be thought of the coding process by using quantizer. So far, there have been a lot of research results^[5,8].

Based on the above analysis, this paper studies the state estimation problem of complex network systems with event-triggered communication scheme and quantization. By introducing a random variable submitted to Bernulli distribution to represent the node random changes; By using Lyapunov stability theory and stochastic system theory, criteria for the existence of the state estimator and criteria for the global mean square asymptotically stability of complex network systems are obtained.

2 System description

Considering the following stochastic complex network system consisting of N coupled nodes with time-varying delayed, every node is a n -dimensional semi-system. The system can be described as:^[9]

$$\begin{aligned} \dot{x}_i(t) = & \delta(t)A f_1(x_i(t)) + (1 - \delta(t))B f_2(x_i(t)) \\ & + \sum_{j=1}^N g_{ij} \Gamma_1 x_j(t) + \sum_{j=1}^N g_{ij} \Gamma_2 x_j(t - \tau(t)) \end{aligned} \quad (1)$$

where the state vector of the i -th node is $\hat{x}_i(t) = (x_{i1}(t), x_{i2}(t), \dots, x_{in}(t))^T \in R^n$, $f_1(x_i(t))$ and $f_2(x_i(t))$ are nonlinear vector valued functions, A and B are constant matrices with appropriate dimensions, Γ_1 and Γ_2 are the inner coupling matrices of the network. $G = (g_{ij}) \in R^{N \times N}$ is the configuration matrix of the topological structure of the complex network. g_{ij} can be defined as: if the i -th node has connection with the j -th, ($i \neq j$), then $g_{ij} = g_{ji} = 1$; otherwise, $g_{ij} = g_{ji} = 0$. The elements of G can be defined as: $g_{ii} = - \sum_{j=1, j \neq i}^N g_{ij}$ ($i=1,2,3,\dots,N$). $\tau(t)$ is the time-varying delay in the network system, satisfying that: $\tau_1 \leq \tau(t) \leq \tau_2, 0 \leq \tau_1 \leq \tau_2$. $\delta(t)$ submits to Bernoulli distribu-

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tion, defined as:

$$\delta(t) = \begin{cases} 1, & f_1(\cdot) \text{ happens} \\ 0, & f_2(\cdot) \text{ happens} \end{cases} \quad (2)$$

Suppose $\delta(t)$ satisfies that:

$$Prob\{\delta(t) = 1\} = \delta_0, Prob\{\delta(t) = 0\} = 1 - \delta_0, \quad (3)$$

where δ_0 is known constant.

Remark 1 The model (1) takes the random variation of the complex network system into consideration, using $\delta(t)$ to stand for the random variable. Moreover, the model (1) considers the time-varying delay in the system.

Assumption 1 [6] For any $u, v \in R^n$, Nonlinear vector valued functions $f_1(\cdot)$ and $f_2(\cdot)$ are continuous, satisfying the following condition:

$$\begin{aligned} [f_1(u) - f_1(v) - \Xi_1(u - v)]^T [f_1(u) - f_1(v) - \Xi_2(u - v)] &\leq 0, \\ [f_2(u) - f_2(v) - \Xi_3(u - v)]^T [f_2(u) - f_2(v) - \Xi_4(u - v)] &\leq 0. \end{aligned}$$

Remark 2 On the basis of Assumption 1, we can get:

$$\begin{aligned} \begin{bmatrix} x_i(t) \\ f_1(x_i(t)) \end{bmatrix}^T \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & I_n \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_1(x_i(t)) \end{bmatrix} &\geq 0, \\ \begin{bmatrix} x_i(t) \\ f_2(x_i(t)) \end{bmatrix}^T \begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} \\ \bar{\Omega}_{21} & I_n \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_2(x_i(t)) \end{bmatrix} &\geq 0, \end{aligned}$$

where

$$\begin{aligned} \Omega_{11} &= \frac{\Xi_1^T \Xi_2 + \Xi_2^T \Xi_1}{2}, \quad \Omega_{21}^T = \Omega_{12} = -\frac{\Xi_1^T + \Xi_2^T}{2}, \\ \bar{\Omega}_{11} &= \frac{\Xi_3^T \Xi_4 + \Xi_4^T \Xi_3}{2}, \quad \bar{\Omega}_{21} = \bar{\Omega}_{12} = -\frac{\Xi_3^T + \Xi_4^T}{2}. \end{aligned}$$

The system structure is shown in Figure 1. Next, we should design a state estimator for the complex network system.

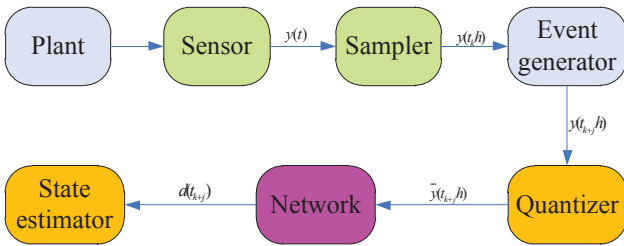


Figure 1. The structure of an event-triggered complex networked system.

By using the matrix Kronecker product, we can rewrite the networks (1) in the following compact form:

$$\begin{aligned} \dot{x}(t) &= \delta(t)I_A F_1(x(t)) + (1 - \delta(t))I_B F_2(x(t)) \\ &\quad + (G \otimes \Gamma_1)x(t) + (G \otimes \Gamma_2)x(t - \tau(t)) \end{aligned} \quad (4)$$

where $I_A = I_N \otimes A$, $I_B = I_N \otimes B$, $G_1 = G \otimes \Gamma_1$, $G_2 = G \otimes \Gamma_2$, $x^T(t) = [x_1^T(t), x_2^T(t), \dots, x_N^T(t)]$, $F_1^T(x(t)) = [f_1^T(x_1(t)), f_1^T(x_2(t)), \dots, f_1^T(x_N(t))]$,

$$F_2^T(x(t)) = [f_2^T(x_1(t)), f_2^T(x_2(t)), \dots, f_2^T(x_N(t))].$$

In this article, suppose that the output $y(t)$ of the complex network(5) is:

$$y(t) = Cx(t). \quad (5)$$

The event-triggered controllers and quantizers are constructed between sensors and state estimators, and suppose sensors and samplers are time-triggered, the sampling period is h , and the sampling time is $kh(k = 0, 1, 2, \dots)$. However, whether the newly sampled state $y((k + j)h)$ will be sent out or not is determined by the following judgement algorithm:

$$\begin{aligned} [y((k + j)h) - y(kh)]^T \Omega [y((k + j)h) - y(kh)] &\leq \\ \sigma y^T((k + j)h) \Omega y((k + j)h), & \end{aligned} \quad (6)$$

where Ω is a symmetric positive definite matrix with appropriate dimension, $\sigma \in [0, 1)$, the newly sampled state $y((k + j)h)$ satisfying the above inequality (6) will not be transmitted.

Based on the above analysis, the real measurement output will be:

$$\begin{aligned} \tilde{y}(t) &= y(t_k h) = Cx(t_k h), \\ t &\in [t_k h + d_k, t_{k+1} h + d_{k+1}). \end{aligned} \quad (7)$$

In the following article, the sampled signal $y(t_k h)$ can be described as $\bar{y}(t_k h)$, that is:

$$\begin{aligned} \bar{y}(t_k h) &= g(\tilde{y}(t)) = g(y(t_k h)) = g(Cx(t_k h)), \\ t &\in [t_k h + d_k, t_{k+1} h + d_{k+1}), \end{aligned} \quad (8)$$

where the quantizer $g(\cdot)$ can be defined as:

$$g(y) = \text{diag}\{g_1(y_1), g_2(y_2), \dots, g_n(y_n)\}, \quad (9)$$

where $g_j(\cdot)$ ($j=1,2,\dots,n$) is symmetric, that is: $g_j(-y_j) = -g_j(y_j)$, and the logarithmic quantizer $g_j(\cdot)$ ($j=1,2,\dots,n$) can be defined as:

$$g_j(y_j) = \begin{cases} u_l^j, & \frac{1}{1+\delta_{g_j}} u_l^j < y_j \leq \frac{1}{1-\delta_{g_j}} u_l^j, y_j > 0 \\ 0, & y_j = 0 \\ -g_j(-y_j), & y_j < 0 \end{cases} \quad (10)$$

where $\delta_{g_j} = \frac{1-\rho_{g_j}}{1+\rho_{g_j}}$ ($0 < \rho_{g_j} < 1$), and ρ_{g_j} represents the density of quantitative of g_j . For the sake of simplicity, suppose $\delta_g = \delta_{g_j}$, where δ_g is a constant. By the above discussion, we can get: $\rho_{g_j} = \rho_g = \frac{1-\delta_g}{1+\delta_g}$. Furthermore, similar to the methods in [10], we define quantitative series set as:

$$\begin{aligned} U_j &= \{\pm u_l^{(j)}, u_l^{(j)} = \rho_{g_j}^l \cdot u_0^{(j)}, l = \pm 1, \pm 2, \dots\} \\ &\cup \{\pm u_0^{(j)}\} \cup \{0\}, u_0^{(j)} > 0, \end{aligned} \quad (11)$$

define:

$$\begin{aligned} \Delta_g &= \text{diag}\{\Delta_{g_1}, \Delta_{g_2}, \dots, \Delta_{g_n}\}, \text{ where } \Delta_{g_j} \in \\ &[-\delta_{g_j}, \delta_{g_j}], j = 1, 2, \dots, n, \end{aligned} \quad (12)$$

The logarithmic quantizer $g_j(\cdot)$ can be described by using the following sector bound approach:

$$g_j(y_j) = (1 + \Delta_{g_j}(y_j))y_j, \quad (13)$$

then $g(\cdot)$ can be represented as:

$$g(y) = (I + \Delta_g)y. \quad (14)$$

Combine (8) and (14), by quantization, $\bar{y}(t)$ can be described as:

$$\begin{aligned} \bar{y}(t) &= g(y(t_k h)) = (I + \Delta_g)y(t_k h), \\ t &\in [t_k h + d_k, t_{k+1} h + d_{k+1}). \end{aligned} \quad (15)$$

Based on the real measurement output, the state estimation system can be given as:

$$\begin{cases} \dot{\hat{x}}(t) = G_1 \hat{x}(t) + G_2 \hat{x}(t - \tau(t)) + K(\bar{y}(t) - \hat{y}(t)) \\ \hat{y}(t) = C \hat{x}(t) \end{cases} \quad (16)$$

where $\hat{x}(t)$ is the estimation state vector, $\hat{y}(t)$ is estimation measurement output, K is the feedback gain matrix.

In order to shift the system to time-delay system, similar to the methods in [2], now consider the following two cases:

Case 1. If $t_k h + h + \bar{d} \geq t_{k+1} h + d_{k+1}$, where $\bar{d} = \max\{d_k\}$, define $d(t)$ as:

$$d(t) = t - t_k h, t \in [t_k h + d_k, t_{k+1} h + d_{k+1}), \quad (17)$$

obviously,

$$d_k \leq d(t) \leq (t_{k+1} - t_k)h + d_{k+1} < h + \bar{d}. \quad (18)$$

Case 2. If $t_k h + h + \bar{d} < t_{k+1} h + d_{k+1}$, consider the following intervals:

$$\begin{aligned} &[t_k h + d_k, t_k h + h + \bar{d}), \\ &[t_k h + ih + \bar{d}, t_k h + ih + h + \bar{d}), \end{aligned} \quad (19)$$

Since $d_k \leq \bar{d}$, it can be easily shown that there exists d_M , such that $t_k h + d_M h + \bar{d} < t_{k+1} h + d_{k+1} \leq t_k h + d_M h + h + \bar{d}$.

Let

$$\begin{cases} I_0 = [t_k h + d_k, t_k h + h + \bar{d}) \\ I_i = [t_k h + ih + \bar{d}, t_k h + ih + h + \bar{d}), \\ I_{d_M} = [t_k h + d_M h + \bar{d}, t_{k+1} h + d_{k+1}) \end{cases} \quad (20)$$

where $i = 1, 2, \dots, d_M - 1$, obviously, we can get

$$\bigcup_{i=0}^{d_M} I_i = [t_k h + d_k, t_{k+1} h + d_{k+1}). \quad (21)$$

Define a function

$$d(t) = \begin{cases} t - t_k h, t \in I_0, \\ t - t_k h - ih, t \in I_i, i = 1, 2, \dots, d_M - 1, \\ t - t_k h - d_M h, t \in I_{d_M}, \end{cases} \quad (22)$$

then, we can conclude that:

$$\begin{cases} d_k \leq d(t) \leq h + \bar{d}, t \in I_0, \\ d_k \leq \bar{d} \leq d(t) \leq h + \bar{d}, t \in I_i, \\ d_k \leq \bar{d} \leq d(t) \leq h + \bar{d}, t \in I_{d_M}, \end{cases} \quad (23)$$

where $i = 1, 2, \dots, d_M - 1$. In Case 1, for $t \in [t_k h + d_k, t_{k+1} h + d_{k+1})$, define $e_k(t) = 0$; In Case 2, define:

$$e_k(t) = \begin{cases} 0, t \in I_0, \\ y(t_k h) - y(t_k h + ih), t \in I_i, \\ y(t_k h) - y(t_k h + d_M h), t \in I_{d_M}, \end{cases} \quad (24)$$

where $i = 1, 2, \dots, d_M - 1$, from the definition of $e_k(t)$ and the event-triggered scheme, (6) can be rewritten as:

$$\begin{aligned} e_k^T(t) \Omega e_k(t) &\leq \sigma x^T(t - d(t)) C^T \Omega C x(t - d(t)), \\ t &\in [t_k h + d_k, t_{k+1} h + d_{k+1}) \end{aligned} \quad (25)$$

Define $e(t) = x(t) - \hat{x}(t)$, then by (4), (14), (15) and (24), we can get:

$$\begin{aligned} \dot{e}(t) &= \delta(t) \cdot (I_N \otimes A) F_1(x(t)) \\ &\quad + (1 - \delta(t)) \cdot (I_N \otimes B) F_2(x(t)) \\ &\quad + (G \otimes \Gamma_1 - KC) e(t) + (G \otimes \Gamma_2) e(t - \tau(t)) \\ &\quad + KC x(t) - K(I + \Delta_g) C x(t - d(t)) \\ &\quad - K(I + \Delta_g) e_k(t). \end{aligned} \quad (26)$$

Define $\bar{x}(t) = [x^T(t), e^T(t)]^T$, combine (4) and (26), we can get the following augmented system:

$$\begin{aligned} \dot{\bar{x}}(t) &= \delta(t) I_{A_1} F_1(H \bar{x}(t)) + (1 - \delta(t)) I_{B_1} F_2(H \bar{x}(t)) \\ &\quad + A_1 \bar{x}(t) + B_1 \bar{x}(t - \tau(t)) + C_1 \bar{x}(t - d(t)) \\ &\quad + D_1 e_k(t), \end{aligned} \quad (27)$$

where

$$\begin{aligned} A_1 &= \begin{bmatrix} G \otimes \Gamma_1 & 0 \\ KC & G \otimes \Gamma_1 - KC \end{bmatrix}, \\ B_1 &= \begin{bmatrix} G \otimes \Gamma_2 & 0 \\ 0 & G \otimes \Gamma_2 \end{bmatrix}, C_1 = \begin{bmatrix} 0 & 0 \\ K_1 C & 0 \end{bmatrix}, \\ D_1 &= \begin{bmatrix} 0 \\ K_1 \end{bmatrix}, K_1 = -K(I + \Delta_g), \\ I_{A_1} &= \begin{bmatrix} I_A \\ I_A \end{bmatrix}, I_{B_1} = \begin{bmatrix} I_B \\ I_B \end{bmatrix}, H^T = \begin{bmatrix} I \\ 0 \end{bmatrix}. \end{aligned}$$

Introduce a new vector:

$$\begin{aligned} \xi^T(t) &= [F_1^T(H \bar{x}(t)), F_2^T(H \bar{x}(t)), \bar{x}^T(t), \bar{x}^T(t - \tau_m), \\ &\quad \bar{x}^T(t - \tau(t)), \bar{x}^T(t - \tau_M), \bar{x}^T(t - d_M), \\ &\quad \bar{x}^T(t - d(t)), e_k^T(t)], \end{aligned}$$

and let $\varphi_1 = [I_{A_1}, 0, A_1, 0, B_1, 0, 0, C_1, D_1]$, $\varphi_2 = [0, I_{B_1}, A_1, 0, B_1, 0, 0, C_1, D_1]$, then (27) can be rewritten as the following form:

$$\dot{\bar{x}}(t) = \delta(t) \varphi_1 \xi(t) + (1 - \delta(t)) \varphi_2 \xi(t). \quad (28)$$

3 Main results

In this section, we will present a criteria for the existence of desirable estimator of complex network system (28), the state feedback gain matrix K will be given by Theorem 2.

Theorem 1 For given scalars $0 \leq \tau_m \leq \tau_M$, event-trigger parameter σ and the estimator gain matrix K , the complex network system (28) is globally asymptotically stable in the mean square sense, if there exist positive definite matrices $P > 0$, $Q_i > 0 (i = 1, 2, 3)$, $R_i > 0 (i = 1, 2, 3)$, and M, N, Z_1, Z_2 with appropriate dimensions, such that the following matrix inequalities hold:

$$\Sigma(s) = \begin{bmatrix} \Phi_{11} + \Gamma + \Gamma^T & * & * \\ \Phi_{21} & \Phi_{22} & * \\ \Phi_{31}(s) & 0 & -R_2 \end{bmatrix} < 0, \quad (29)$$

where $s = 1, 2$, and

$$\begin{aligned} \Phi_{11} &= \begin{bmatrix} \Gamma_3 & \Gamma_4 \\ Z_1 \otimes I_n & * & * & * \\ 0 & Z_2 \otimes I_n & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ 0 & 0 & R_1 & \bar{R}_1 \\ \Gamma_3 &= & 0 & 0 & B_1^T P & 0 \\ & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & 0 & 0 & \bar{C}_1 & 0 \\ & & 0 & 0 & D_1^T P & 0 \end{bmatrix} \\ \bar{C}_1 &= R_3 + C_1^T P, \bar{R}_1 = -R_1 - Q_1 \\ \Gamma_4 &= \begin{bmatrix} 0 & * & * & * & * \\ 0 & * & * & * & * \\ PB_1 & * & * & * & * \\ 0 & * & * & * & * \\ \Gamma_4 &= & 0 & * & * & * \\ & & 0 & -Q_2 & * & * \\ & & 0 & 0 & \bar{R}_3 & * \\ & & 0 & 0 & R_3 & \bar{W} \\ & & 0 & 0 & 0 & -W_1 \end{bmatrix} \\ \bar{R}_3 &= -R_3 - Q_3, \bar{W} = \sigma W - 2R_3 \\ \Phi_{21} &= [\Gamma_5 \quad \Gamma_6], \tau_{21} = \tau_M - \tau_m, \delta_{10} = 1 - \delta_0 \\ \Gamma_5 &= \begin{bmatrix} \theta_1 R_1 I_{A_1} & 0 & \theta_1 R_1 A_1 & 0 \\ \theta_2 R_2 I_{A_1} & 0 & \theta_2 R_2 A_1 & 0 \\ \theta_3 R_3 I_{A_1} & 0 & \theta_3 R_3 A_1 & 0 \\ 0 & \theta_{10} R_1 I_{B_1} & \theta_{10} R_1 A_1 & 0 \\ 0 & \theta_{20} R_2 I_{B_1} & \theta_{20} R_2 A_1 & 0 \\ 0 & \theta_{30} R_3 I_{B_1} & \theta_{30} R_3 A_1 & 0 \end{bmatrix} \\ \Gamma_6 &= \begin{bmatrix} \theta_1 R_1 B_1 & 0 & 0 & \theta_1 R_1 C_1 & \theta_1 R_1 D_1 \\ \theta_2 R_2 B_1 & 0 & 0 & \theta_2 R_2 C_1 & \theta_2 R_2 D_1 \\ \theta_3 R_3 B_1 & 0 & 0 & \theta_3 R_3 C_1 & \theta_3 R_3 D_1 \\ \theta_{10} R_1 B_1 & 0 & 0 & \theta_{10} R_1 C_1 & \theta_{10} R_1 D_1 \\ \theta_{20} R_2 B_1 & 0 & 0 & \theta_{20} R_2 C_1 & \theta_{20} R_2 D_1 \\ \theta_{30} R_3 B_1 & 0 & 0 & \theta_{30} R_3 C_1 & \theta_{30} R_3 D_1 \end{bmatrix} \\ \Phi_{22} &= \text{diag}\{-R_1, -R_2, -R_3, -R_1, -R_2, -R_3\} \\ \Pi_{31} &= H^T(Z_1 \otimes \Omega_{21})^T + \delta_0 P I_{A_1} \\ \Pi_{32} &= H^T(Z_2 \otimes \bar{\Omega}_{21})^T + \delta_{10} P I_{B_1} \\ \Pi_{33} &= -R_3 - R_1 + Q_1 + Q_2 + Q_3 + A_1^T P + P A_1 \\ &\quad + H^T(Z_1 \otimes \Omega_{11})H + H^T(Z_2 \otimes \bar{\Omega}_{11})H \\ \Phi_{31}(1) &= \sqrt{\delta} N^T, \Phi_{31}(2) = \sqrt{\delta} M^T \\ W &= \begin{bmatrix} C^T \Omega C & 0 \\ 0 & 0 \end{bmatrix}, W_1 = \begin{bmatrix} \Omega & 0 \\ 0 & 0 \end{bmatrix} \\ \Gamma &= [\Gamma_1 \quad \Gamma_2], \Gamma_1 = [0 \quad 0 \quad 0 \quad N] \\ \Gamma_2 &= [-N + M \quad -M \quad 0 \quad 0 \quad 0] \\ M^T &= [\Gamma_7 \quad \Gamma_8], N^T = [\Gamma_9 \quad \Gamma_{10}] \\ \Gamma_7 &= [M_1^T \quad M_2^T \quad M_3^T \quad M_4^T] \\ \Gamma_8 &= [M_5^T \quad M_6^T \quad M_7^T \quad M_8^T \quad M_9^T] \\ \Gamma_9 &= [N_1^T \quad N_2^T \quad N_3^T \quad N_4^T] \\ \Gamma_{10} &= [N_5^T \quad N_6^T \quad N_7^T \quad N_8^T \quad N_9^T] \\ \theta_1 &= \tau_m \sqrt{\delta_0}, \theta_2 = \sqrt{\tau_{21} \delta_0}, \theta_3 = d_M \sqrt{\delta_0} \\ \theta_{10} &= \tau_m \sqrt{\delta_{10}}, \theta_{20} = \sqrt{\tau_{21} \delta_{10}}, \theta_{30} = d_M \sqrt{\delta_{10}} \end{aligned}$$

$$R = \tau_m^2 R_1 + (\tau_M - \tau_m) R_2 + d_M^2 R_3$$

Proof: Construct the following Lyapunov functional candidate:

$$V(t, \bar{x}(t)) = V_1(t, \bar{x}(t)) + V_2(t, \bar{x}(t)) + V_3(t, \bar{x}(t)), \quad (30)$$

where

$$\begin{aligned} V_1(t, \bar{x}(t)) &= \bar{x}^T(t) P \bar{x}(t), \\ V_2(t, \bar{x}(t)) &= \int_{t-\tau_m}^t \bar{x}^T(s) Q_1 \bar{x}(s) ds \\ &\quad + \int_{t-\tau_M}^t \bar{x}^T(s) Q_2 \bar{x}(s) ds \\ &\quad + \int_{t-d_M}^t \bar{x}^T(s) Q_3 \bar{x}(s) ds, \\ V_3(t, \bar{x}(t)) &= \tau_m \int_{t-\tau_m}^t \int_s^t \dot{\bar{x}}^T(v) R_1 \dot{\bar{x}}(v) dv ds \\ &\quad + \int_{t-\tau_M}^{t-\tau_m} \int_s^t \dot{\bar{x}}^T(v) R_2 \dot{\bar{x}}(v) dv ds \\ &\quad + d_M \int_{t-d_M}^t \int_s^t \dot{\bar{x}}^T(v) R_3 \dot{\bar{x}}(v) dv ds. \end{aligned}$$

Then the remaining part of the proof is similar to those in [9]. Due to page limitation, we omit the details in the proof.

Theorem 2 For given constants $0 \leq \tau_m \leq \tau_M$, event-trigger parameter σ , where $g(\cdot)$ is the form of (9), and the density of quantitative is ρ_g . When Assumption 1 holds, the complex networks systems (28) is globally asymptotically stable in the mean square sense, if there exists positive definite matrices $P_1 > 0, P_2 > 0, Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, R_2 > 0, R_3 > 0$ and $Y, M_k, N_k (k = 1, 2, \dots, 9)$, Z_1 and Z_2 with appropriate dimensions, such that for given $\varepsilon_i > 0 (i = 1, 2, 3, 4)$, the following linear matrix inequalities hold:

$$\tilde{\Sigma}(s) = \begin{bmatrix} \tilde{\Phi}_{11} + \Gamma + \Gamma^T & * & * & * \\ \tilde{\Phi}_{21} & \tilde{\Phi}_{22} & * & * \\ \Phi_{31}(s) & 0 & -R_2 & * \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix} < 0, \quad (31)$$

where $s = 1, 2$, and

$$\begin{aligned} \tilde{\Phi}_{11} &= [\Gamma_{11} \quad \Gamma_{12}] \\ \Gamma_{11} &= \begin{bmatrix} Z_1 \otimes I_n & * & * & * \\ 0 & Z_2 \otimes I_n & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * \\ 0 & 0 & R_1 & \bar{R}_1 \\ 0 & 0 & B_1^T P & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \bar{C}_{11} & 0 \\ 0 & 0 & \bar{D}^T P & 0 \end{bmatrix} \\ \bar{C}_{11} &= R_3 + \bar{C}^T P \end{aligned}$$

$$\Gamma_{12} = \begin{bmatrix} 0 & * & * & * & * \\ 0 & * & * & * & * \\ PB_1 & * & * & * & * \\ 0 & * & * & * & * \\ 0 & * & * & * & * \\ 0 & -Q_2 & * & * & * \\ 0 & 0 & \bar{R}_3 & * & * \\ 0 & 0 & R_3 & \bar{W} & * \\ 0 & 0 & 0 & 0 & -W_1 \end{bmatrix}$$

$$\bar{C} = \begin{bmatrix} 0 & 0 \\ -KC & 0 \end{bmatrix}, \bar{D} = \begin{bmatrix} 0 \\ -K \end{bmatrix}$$

$$\tilde{\Phi}_{21} = [\Gamma_{13} \quad \Gamma_{14}]$$

$$\Gamma_{13} = \begin{bmatrix} \theta_1 P I_{A_1} & 0 & \theta_1 P A_1 & 0 \\ \theta_2 P I_{A_1} & 0 & \theta_2 P A_1 & 0 \\ \theta_3 P I_{A_1} & 0 & \theta_3 P A_1 & 0 \\ 0 & \theta_{10} P I_{B_1} & \theta_{10} P A_1 & 0 \\ 0 & \theta_{20} P I_{B_1} & \theta_{20} P A_1 & 0 \\ 0 & \theta_{30} P I_{B_1} & \theta_{30} P A_1 & 0 \end{bmatrix}$$

$$\Gamma_{14} = \begin{bmatrix} \theta_1 P B_1 & 0 & 0 & \theta_1 P \bar{C} & \theta_1 P \bar{D} \\ \theta_2 P B_1 & 0 & 0 & \theta_2 P \bar{C} & \theta_2 P \bar{D} \\ \theta_3 P B_1 & 0 & 0 & \theta_3 P \bar{C} & \theta_3 P \bar{D} \\ \theta_{10} P B_1 & 0 & 0 & \theta_{10} P \bar{C} & \theta_{10} P \bar{D} \\ \theta_{20} P B_1 & 0 & 0 & \theta_{20} P \bar{C} & \theta_{20} P \bar{D} \\ \theta_{30} P B_1 & 0 & 0 & \theta_{30} P \bar{C} & \theta_{30} P \bar{D} \end{bmatrix}$$

$$\tilde{\Phi}_{22} = \text{diag}\{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_1, \tilde{R}_2, \tilde{R}_3\}$$

$$\tilde{R}_i = \varepsilon_i^2 R_i - 2\varepsilon_i P, i = 1, 2, 3,$$

$$P I_{A_1} = \begin{bmatrix} \mathbf{P}_1 I_A \\ \mathbf{P}_2 I_A \end{bmatrix}, P I_{B_1} = \begin{bmatrix} \mathbf{P}_1 I_B \\ \mathbf{P}_2 I_B \end{bmatrix}$$

$$P A_1 = \begin{bmatrix} \mathbf{P}_1(G \otimes \Gamma_1) & 0 \\ YC & \mathbf{P}_2(G \otimes \Gamma_1) - YC \end{bmatrix}$$

$$P B_1 = \begin{bmatrix} \mathbf{P}_1(G \otimes \Gamma_2) & 0 \\ 0 & \mathbf{P}_2(G \otimes \Gamma_2) \end{bmatrix}$$

$$P \bar{C} = \begin{bmatrix} 0 & 0 \\ -YC & 0 \end{bmatrix}, P \bar{D} = \begin{bmatrix} 0 \\ -Y \end{bmatrix}$$

$$\Phi_{41} = [\Gamma_{15} \quad \Gamma_{16}], \Phi_{42} = [\Gamma_{17} \quad \Gamma_{18}]$$

$$\Gamma_{15} = \begin{bmatrix} 0 & 0 & 0 & -I & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_{16} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -\varepsilon_4 YC & 0 & -\varepsilon_4 Y \end{bmatrix}$$

$$\Gamma_{17} = \begin{bmatrix} 0 & -\theta_1 I & 0 & -\theta_2 I & 0 & -\theta_3 I \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_{18} = \begin{bmatrix} 0 & -\theta_{10} I & 0 & -\theta_{20} I & 0 & -\theta_{30} I \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_{43} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \Phi_{44} = \text{diag}\{-\varepsilon_4 I, -\frac{\varepsilon_4}{\delta_g} I\}$$

Proof: For the convenience of derivation and the conservation of the solution, define: $P = \text{diag}\{\mathbf{P}_1, \mathbf{P}_2\} > 0$, $Q_1 = \text{diag}\{\mathbf{Q}_1, \mathbf{Q}_1\} > 0$, $Q_2 = \text{diag}\{\mathbf{Q}_2, \mathbf{Q}_2\} > 0$, $Q_3 = \text{diag}\{\mathbf{Q}_3, \mathbf{Q}_3\} > 0$, $R_1 = \text{diag}\{\mathbf{R}_1, \mathbf{R}_1\} > 0$, $R_2 = \text{diag}\{\mathbf{R}_2, \mathbf{R}_2\} > 0$, $R_3 = \text{diag}\{\mathbf{R}_3, \mathbf{R}_3\} > 0$, $M_k = \text{diag}\{\mathbf{M}_k, \mathbf{M}_k\}$, $N_k = \text{diag}\{\mathbf{N}_k, \mathbf{N}_k\}$ ($k = 1, 2, \dots, 9$). Similar to the proof methods in [9], perform a congruence transformation of diagonal matrix to (29) $\text{diag}\{I, PR_1^{-1}, PR_2^{-1}, PR_3^{-1}, PR_1^{-1}, PR_2^{-1}, PR_3^{-1}, I\}$,

and combine the following inequality: $-PR_i^{-1}P \leq \varepsilon_i^2 R_i - 2\varepsilon_i P$, $i = 1, 2, 3$, by using Lyapunov functional methods and linear matrix inequality techniques, we can obtain (31). For simplicity, we omit the details in the proof.

4 Simulation examples

Considering the following continuous complex network systems consisting of 5 coupled nodes, where the dynamical function of every node can be described as:

$$\dot{x}_i(t) = \delta(t)A f_1(x_i(t)) + (1 - \delta(t))B f_2(x_i(t)) + \sum_{j=1}^N g_{ij} \Gamma_1 x_j(t) + \sum_{j=1}^N g_{ij} \Gamma_2 x_j(t - \tau(t)), \quad (32)$$

where

$$x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix}, A = \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.3 \end{bmatrix},$$

$$B = \begin{bmatrix} -0.2 & 0.25 \\ 0.25 & -0.3 \end{bmatrix}.$$

The external coupling configuration matrix G and the inner-coupling matrix Γ_1, Γ_2 are the following form:

$$G = \begin{bmatrix} -15 & 12.01 & 0 & 0 & 0.01 \\ 12.01 & -15 & 0 & 0 & 0 \\ 0.01 & 0.02 & -16 & 0 & 0.02 \\ 0.02 & 0.01 & 0 & -16 & 0.01 \\ 0 & 0 & 0.01 & 0.01 & -14 \end{bmatrix},$$

$$\Gamma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Gamma_2 = 0.1\Gamma_1.$$

The activation function of networks nodes is described as:

$$f_1(x_i(t)) = \begin{bmatrix} f_{11}(x_i(t)) \\ f_{12}(x_i(t)) \end{bmatrix}, f_2(x_i(t)) = \begin{bmatrix} f_{21}(x_i(t)) \\ f_{22}(x_i(t)) \end{bmatrix}$$

$$f_{11}(x_i(t)) = 0.4x_{i1}(t) - \tanh(0.3x_{i2}(t)) + 0.2x_{i2}(t - \tau(t))$$

$$f_{12}(x_i(t)) = 0.9x_{i2}(t) - \tanh(0.7x_{i2}(t))$$

$$f_{21}(x_i(t)) = 0.3x_{i1}(t) - \tanh(0.2x_{i1}(t)) + 0.1x_{i2}(t - \tau(t))$$

$$f_{22}(x_i(t)) = 0.8x_{i2}(t) - \tanh(0.6x_{i2}(t)),$$

suppose the measurement output matrix C is: $C = [0.2 \quad -0.5 \quad 0.2 \quad 0 \quad 0.2 \quad -0.6 \quad 0.2 \quad 0 \quad -0.7 \quad 0.2]$, the initial condition of the system is: $x_0 = [0.4 \quad -0.3 \quad 0.5 \quad -0.1 \quad 0.2 \quad -0.2 \quad 0.1 \quad -0.5 \quad 0.3 \quad -0.4]^T$.

Suppose the random switching probability of networks nodes is $\delta_0 = 0.6$, the lower bound of time-varying delays is $\tau_m = 0$, the upper bound is $\tau_M = 0.2$, the event-trigger parameter is $\sigma = 0.2$, sampling period is $h = 0.05$. In fact, for $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$, by applying Theorem 2, using LMI tool box to solve the inequality(31), we can get: the estimator matrix is:

$$K = [-0.0114, 0.0286, -0.0040, -0.0001, -0.0073, 0.0215, -0.0074, -0.0000, 0.0083, -0.0025]^T,$$

the event-trigger matrix is:

$$\Omega = 51.6877.$$

Simulation results are shown in Figures 2-5 by using the state estimator K . From Figure 4-5, we can see the quantitative comparison by using quantizer or not. Clearly, we can see that there is few difference before and after quantification, but using quantification can reduce data transmission, save networks bandwidth. This indicates that in the process of data transmission, considering the quantitative method is reasonable.

5 Conclusion

In order to save networks bandwidth, this paper introduces event-triggered mechanism; Considering to avoid excessive amount of data transmission, by using quantitative methods, this paper establishes the complex network systems model with random nodes structure, and studies the state estimation problem. By using Lyapunov stability theory and linear matrix inequality tools, criteria for globally asymptotically stability in the mean square and criteria for the existence of state estimator are derived. Finally, simulation results verifies the effectiveness of the proposed method.

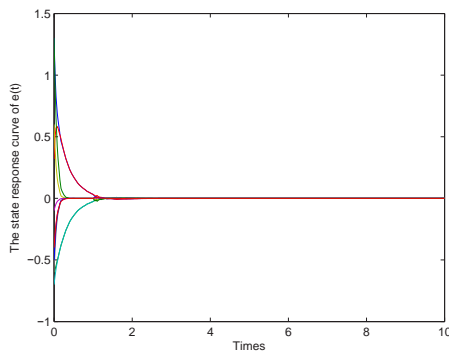


Figure 2. The state response curve of $e(t)$.

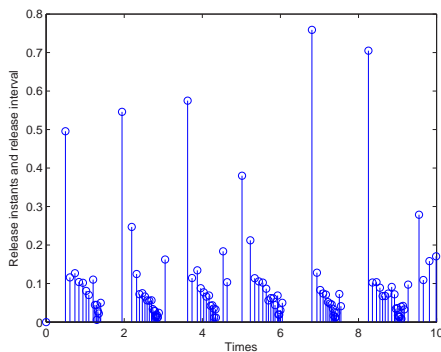


Figure 3. The release instants and release intervals.

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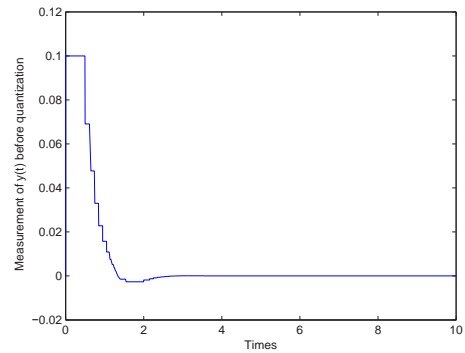


Figure 4. The output of $y(t)$ before quantization.

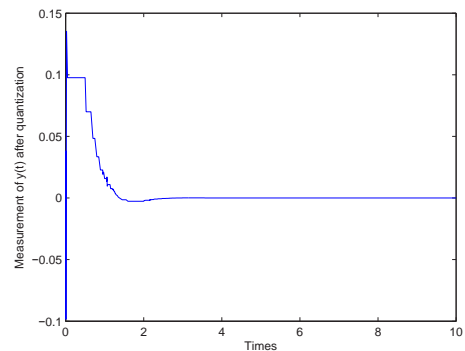


Figure 5. The output of $y(t)$ after quantization.

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