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Hybrid-driven-based stabilisation for networked control systems

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Abstract: This study investigates controller design for networked control systems under hybrid driven scheme. A hybrid driven communication scheme is proposed, which can improve the system performance and reduce the network transmission. A Bernoulli distributed stochastic variable is introduced to describe the switch law of the communication scheme. A general system model under hybrid driven scheme is then constructed. Based on this model, sufficient conditions are derived to guarantee the desired system performance. Furthermore, criteria for co-designing both the feedback gain and the trigger parameters are established. Finally, simulation results show the usefulness of the proposed method.

1 Introduction

Due to the advantages of flexible architectures, low installation and maintenance cost, networked control systems (NCSs) have been employed in some fields such as intelligent manufacturing control systems, DC motor systems, smart grid and so on [1–6]. Different from the traditional point-to-point hard wiring connection, the components of the NCSs, such as sensors, controllers and actuators, are connected over a communication network medium. Due to the insertion of the network, some challenges are also met due to the network-induced imperfections. The communication network transmission is not always reliable, resulted by the existing network-induced delays, data packet dropout and noise interference, which may lead to performance degradation or even instability of an NCSs. Fortunately, there has been some effective control strategies proposed to deal with the above problems [7–10].

With respect to the constraints in the NCSs, most available results adopted a time-triggered control method for system modelling and analysis, whose communication interval is designed a priori to reduce the complexity for the easy implementation and analysis. The drawback of this strategy is that it does not consider efficient usage of limited communication resources. In this triggering method, a fixed sampling interval should be selected to ensure a desired performance under worst conditions such as channel bandwidth, capacity, and external disturbances in the network environment. In order to avoid the unnecessary waste of network resources, a discrete event-triggered scheme has been proposed in [11], in which the transmitted signal must satisfy certain conditions. A considerable amount of the network resource occupancy can be saved while maintaining the guaranteed control performance. Recently, event-triggered scheme has motivated lots of interesting research, leading a growing number of significant publications [12–19]. For instance, the problem of event-triggered output-feedback H_∞ control for NCSs with non-uniform sampling is investigated, the stability and synthesis conditions are presented to guarantee the uniform ultimate bounded stability and the desired performance [16]. A systematic approach to the understanding, analysis and design of the event-based filters for time-varying systems with fading channels and multiplicative noise is provided in [17]. An event-based H_∞ filter design for a class of T–S fuzzy systems with stochastic sensor faults is studied, sufficient conditions are established to ensure the filtering error system exponentially stable in the mean square [18]. Compared with the time-triggered communication scheme, the event triggered schemes provides a useful way to improve the communication efficiency. However, most of the above mentioned event triggered schemes are always at the expense of system performance, it should often get a balance between the amount of communication and the system performance. Method for optimising data transmission in the NCSs is still a challenge problem. In practical systems, utilisation of the communication bandwidth is not always high, in a certain period of time, the information transmission can be less. As to this system, neither the method of the event triggered scheme nor the time triggered scheme can ensure the best system performance. How to deal with this situation adequately is still a challenge problem. The aim of this study is to shorten such a gap.

In this note, we aim to provide a hybrid driven communication scheme for NCSs with network-induced delays. The contribution of the paper is as follows: (i) In order to obtain the desired system performance and make full use of the network bandwidth, two separate communication channels can be switched between the sensor and the controller. The event triggered communication channel will be selected when the transmission line bandwidth is over occupied, otherwise, we can switch to the other channel. (ii) A Bernoulli distributed stochastic variable is introduced to describe the switch of the two channels. (iii) Under the hybrid event-triggering scheme, a general system model with communication delay is established. Based on the model, criteria for the stability and controller design are derived in the form of linear matrix inequality.

The paper is organised as follows. Section 2 is the problem formulation. Main results are presented in Section 3. A simulation example is presented in Section 4 to demonstrate the main results obtained. Finally, the paper is concluded in Section 5.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Eculidean space, and the set of $n\times m$ real matrices; the superscript ' T' stands for matrix transposition; I is the identity matrix of appropriate dimension; the notation $X>0$ (respectively, $X\geq 0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). For a matrix B and two symmetric matrices A and C,

$$\begin{bmatrix} A & * \\ B & C \end{bmatrix}$$

denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

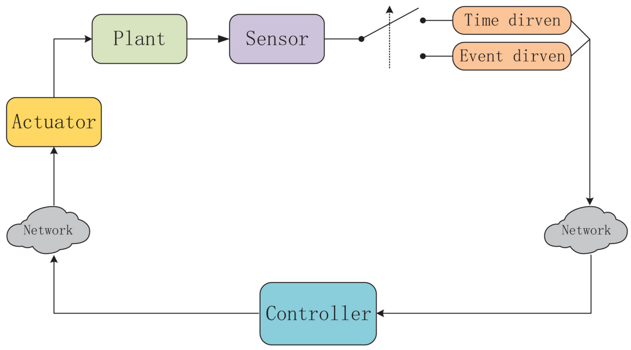

Fig. 1 *Structure of hybrid driven NCS*

2 System description

For the purpose of reducing data communication frequency, designing two independent channels that work in different frequencies is one method while we can control the whole channel with a switch. In this section, we will model and design a networked controller for NCSs. A networked control structure for system (1) is shown in Fig. 1.

Consider an NCS

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are the system state vector and the control input, respectively. A and B are constant matrices with appropriate dimensions. For further development, we give the following assumptions, which are common in the NCS research in the literature [20].

i. We assume that the sensors are time-triggered and all measurement data packets are time stamped. The controller is event driven and is triggered by the arrival of a measurement data packet. The actuator has a logic zero order hold (ZOH). Only if the time stamp of the packet is greater than that of the packet currently stored in the ZOH, can the logic ZOH accept the received control packet.
ii. The sensors, controller and actuators are connected through unreliable network medium. There exists random network-induced delay and packet dropout in both forward and feedback channels.
iii. The communication strategy is assumed to be switched between the time-triggered scheme and event-triggered scheme depending on the needs of the system performance and the capacity of network resources.
iv. We assume the distribution of the random networked delay and packet dropout can be observed. Only the network-induced delay from sensor to controller and controller to actuator are considered,

First, in one of the two channels, we consider the case when the system (1) is in the networked environments with quality-of-service (QoS) constraints, i.e. the selecting switch turn to 'time driven', the controller can be described as [21, 22]

$$u(t) = Kx(t_k h), \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{2}$$

where h is the sampling period, $t_k h$ are the instants when the control signal reaches the ZOH, τ_{t_k} is the communication delay, and $\{t_1, t_2, t_3, \ldots\} \subset \{1, 2, 3, \ldots\}$. It is easy to see that the holding interval of the ZOH are $\Omega = [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$.

Define the network allowable equivalent delay $\eta(t) = t - t_k h$, (2) can be rewritten as

$$u(t) = Kx(t - \eta(t)), \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{3}$$

where $\eta(t) \in [\eta_0, \eta_1]$.

In order to save the network resources and improve the system performance, an event-triggered scheme is introduced to the other channel, which can decide whether or not the sampled-data should be transmitted. We define event generator function as follows:

$$f(e(t_k h), \sigma) = e^{\mathrm{T}}(t_k h)\Omega e(t_k h) - \sigma x^{\mathrm{T}}(i_k^j h)\Omega x(i_k^j h) \tag{4}$$

where $e(t_k h) = x(t_k h) - x(i_k^j h)$, $i_k^j h = t_k h + jh$, $x(t_k h)$ is latest transmitted sampling data. $i_k^j h$ is the current sampling data. The newly sampled data will be sent to the controller as long as the condition

$$f(e(t_k h), \sigma) \le 0 \tag{5}$$

is violated. Under this circumstance, similar to [14], the holding interval of the ZOH $t \in \Omega$ are divided into sampling-interval-like subsets $\Omega = \cup_{k=1}^{\infty}[i_k^j h + \tau_{t_k}, i_k^j h + h + \tau_{t_{k+1}}), j = 0, \ldots, t_{k+1} - t_k - 1$. Define $d(t) = t - i_k^j h$, It is clear that $0 < d(t) < h + \tau_{t_{k+1}} \triangleq d_M$. Then, the control of (2) is

$$u(t) = K[x(t - d(t)) + e(t_k h)], \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{6}$$

Remark 1: NCSs have widespread applications in the fields of power systems and teleoperation systems and so on, a plenty of important results have been reported in [1–6]. It is to note that continuous communication can lead a waste of energy. To remove the quality of communication and reduce the communication cost, researchers have begun to investigate the systems under event-triggered scheme [3, 10, 11, 13, 14]. On the basis of the existing results, we investigate the hybrid-driven-based stabilisation for NCSs.

According to the analysis of the two channels above, the following discussed system model is constructed: (see (7))

where $\alpha(t)$ is a Bernoulli distributed stochastic variable with $\mathrm{Prob}\{\alpha(t) = 1\} = \bar{\alpha}$ and $\mathrm{Prob}\{\alpha(t) = 0\} = 1 - \bar{\alpha}$. The mathematical variance of $\alpha(t)$ is δ^2. For system (7), we supplement the initial condition of the state $x(t)$ on $[-\bar{d}; 0]$ as $x(t) = \phi(t); t \in [-\bar{d}; 0]$, where $\phi(t)$ is a continuous function on $[-\bar{d}; 0]$, $-\bar{d} = \max\{\eta_1, d_M\}$.

Remark 2: The sojourn probability $\bar{\alpha}$ can be obtained through the following statistical method:

$$\bar{\alpha} = \lim_{n \to \infty} \frac{k_i}{n} \tag{8}$$

where $k_i, n \in Z^+$, k_i is the times of $\alpha = 1$ in the interval $[1, n]$.

Remark 3: From the above analysis, the event triggered scheme (5) can be rewritten as

$$e^{\mathrm{T}}(t_k h)\Omega e(t_k h) - \sigma x^{\mathrm{T}}(t - d(t))\Omega x(t - d(t)) \le 0 \tag{9}$$

which will be useful in the stability analysis of the system (7) later.

Remark 4: In this paper, the sojourn probability in each channel is assumed to be known a prior, i.e. $\Pr\{\alpha(t) = 1\} = \bar{\alpha}$. When $\alpha(t) = 1$, the system (7) becomes $\dot{x}(t) = Ax(t) + BKx(t - \eta(t)$,

$$\begin{aligned}\dot{x}(t) &= Ax(t) + \alpha(t)BKx(t - \eta(t)) \\ &\quad + (1 - \alpha(t))BK[x(t - d(t)) + e(t_k h)], t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})\end{aligned} \tag{7}$$

which means the system is under time driven scheme; when $\alpha(t) = 0$, the system (7) becomes $\dot{x}(t) = Ax(t) + BK[x(t - d(t)) + e(t_k h)]$, which means the system is under event-triggered scheme. In other words, the system (7) is a more general model.

Remark 5: Different from the existing networked control systems, there are two schemes can be selected to guarantee a desired performance in this paper. When the signal transmitted in the closed-loop system can always be obtained consecutively and perfectly, $\alpha(t) = 1$ happen. Otherwise, When networked delay/packet dropout, network congestion and network disturbance happen, which lead to intolerable performance degradation and even instability, $\alpha(t) = 0$ happen.

The objective of this paper is to design the controller for NCSs based on two transmission channels. The following lemmas would be employed in deriving our results.

Definition 1 ([23]): For a given function $V: C^b_{F_0}([-\tau_M, 0], \mathbb{R}^n) \times S$, its infinitesimal operator $\mathcal{L}$ is defined as

$$\mathcal{L}(V\eta(t)) = \lim_{\Delta \to 0^+} \frac{1}{\Delta}[\mathbb{E}(V(\eta_t + \Delta) \mid \eta_t) - V(\eta_t)] \tag{10}$$

Lemma 1 ([24]): For any vectors $x,\ y \in R^n$, and positive definite matrix $Q \in R^{n \times n}$, the following inequality holds:

$$2x^{\mathrm{T}}y \le x^{\mathrm{T}}Qx + y^{\mathrm{T}}Q^{-1}y$$

Lemma 2 ([25]): Suppose $\eta(t) \in [\eta_0, \eta_1]$, $d(t) \in [0, d_M]$, Ξ_1, Ξ_2, Ξ_3, Ξ_4 and Ω are matrices with appropriate dimensions, then (see (11))

if and only if

$$(\eta_1 - \eta_0)\Xi_1 + d_M\Xi_3 + \Omega < 0$$
$$(\eta_1 - \eta_0)\Xi_2 + d_M\Xi_3 + \Omega < 0$$
$$(\eta_1 - \eta_0)\Xi_1 + d_M\Xi_4 + \Omega < 0$$
$$(\eta_1 - \eta_0)\Xi_2 + d_M\Xi_4 + \Omega < 0$$

3 Main results

In this section, we will develop an approach for stability analysis and controller synthesis of system (7).

Theorem 1: For given scalars η_0, η_1, d_M, δ, σ and $\bar{\alpha}$, system (7) is asymptotically stable if there exist matrices $P > 0$, $Q_i > 0$, $R_i > 0 (i = 1, 2, 3)$, $\Omega > 0$, and M, N, T, S with appropriate dimensions such that for $l = 1, 2, 3, 4$

$$\Pi(l) = \begin{bmatrix} \Omega_{11} + \Gamma + \Gamma^{\mathrm{T}} & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * \\ \Omega_{31} & 0 & \Omega_{33} & * \\ \Omega_{41}(l) & 0 & 0 & \Omega_{44} \end{bmatrix} < 0 \tag{12}$$

where (see equation below)

Proof: Construct a Lyapunov–Krasovskii functional for system (7)

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{13}$$

where

$$\begin{aligned} V_1(t) &= x^{\mathrm{T}}(t)Px(t) \\ V_2(t) &= \int_{t-\eta_0}^{t} x^{\mathrm{T}}(s)Q_1x(s)\,\mathrm{d}s \\ &\quad + \int_{t-\eta_1}^{t} x^{\mathrm{T}}(s)Q_2x(s) \\ &\quad + \int_{t-d_M}^{t} x^{\mathrm{T}}(s)Q_3x(s)\,\mathrm{d}s \\ V_3(t) &= \int_{t-\eta_1}^{t-\eta_0}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)R_1\dot{x}(v)\,\mathrm{d}v\,\mathrm{d}s \\ &\quad + \eta_0\int_{t-\eta_0}^{t}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)R_2\dot{x}(v)\,\mathrm{d}v\,\mathrm{d}s \\ &\quad + \int_{t-d_M}^{t}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)R_3\dot{x}(v)\,\mathrm{d}v\,\mathrm{d}s \end{aligned}$$

Taking the derivative of $V(t)$ along the trajectory of the system (7), taking its mathematical expectation and applying free weighing matrix method [26, 27], we have (see (14))

where

$$\mathcal{A} = Ax(t) + \bar{\alpha}BKx(t - \eta(t)) + (1 - \bar{\alpha})BK(x(t - d(t)) + e_k(t))$$

$$\Gamma_1 = 2\zeta^{\mathrm{T}}(t)M\left[x(t - \eta_0) - x(t - \eta(t)) - \int_{t-\eta(t)}^{t-\eta_0} \dot{x}(s)\,\mathrm{d}s\right]$$

$$\Gamma_2 = 2\zeta^{\mathrm{T}}(t)N\left[x(t - \eta(t)) - x(t - \eta_1) - \int_{t-\eta_1}^{t-\eta(t)} \dot{x}(s)\,\mathrm{d}s\right]$$

$$\Gamma_3 = 2\zeta^{\mathrm{T}}(t)T\left[x(t) - x(t - d(t)) - \int_{t-d(t)}^{t} \dot{x}(s)\,\mathrm{d}s\right]$$

$$\Gamma_4 = 2\zeta^{\mathrm{T}}(t)S\left[x(t - d(t)) - x(t - d_M) - \int_{t-d_M}^{t-d(t)} \dot{x}(s)\,\mathrm{d}s\right]$$

in which $\zeta^{\mathrm{T}}(t) = [x^{\mathrm{T}}(t)x^{\mathrm{T}}(t - \eta_0)x^{\mathrm{T}}(t - \eta(t))x^{\mathrm{T}}(t - \eta_1)x^{\mathrm{T}}(t - d(t))x^{\mathrm{T}}(t - d_M)e^{\mathrm{T}}(t_k h)]^{\mathrm{T}}$, $\tilde{R} = (\eta_1 - \eta_0)R_1 + \eta_0^2R_2 + d_MR_3$

Note that

$$\begin{aligned} &-\eta_0\int_{t-\eta_0}^{t} \dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\,\mathrm{d}s \\ &\quad \le \begin{bmatrix} x(t) \\ x(t - \eta_0) \end{bmatrix}^{\mathrm{T}}\begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix}\begin{bmatrix} x(t) \\ x(t - \eta_0) \end{bmatrix} \end{aligned} \tag{15}$$

$$\begin{aligned} -2\zeta^{\mathrm{T}}(t)M\int_{t-\eta(t)}^{t-\eta_0} \dot{x}(s)\,\mathrm{d}s &\le (\eta(t) - \eta_0)\zeta^{\mathrm{T}}(t)MR_1^{-1}M^{\mathrm{T}}\zeta(t) \\ &\quad + \int_{t-\eta(t)}^{t-\eta_0} \dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\,\mathrm{d}s \end{aligned} \tag{16}$$

$$\begin{aligned} -2\zeta^{\mathrm{T}}(t)N\int_{t-\eta_1}^{t-\eta(t)} \dot{x}(s)\,\mathrm{d}s &\le (\eta_1 - \eta(t))\zeta^{\mathrm{T}}(t)NR_1^{-1}N^{\mathrm{T}}\zeta(t) \\ &\quad + \int_{t-\eta_1}^{t-\eta(t)} \dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\,\mathrm{d}s \end{aligned} \tag{17}$$

$$\begin{aligned} -2\zeta^{\mathrm{T}}(t)T\int_{t-d(t)}^{t} \dot{x}(s)\,\mathrm{d}s &\le d(t)\zeta^{\mathrm{T}}(t)TR_1^{-1}T^{\mathrm{T}}\zeta(t) \\ &\quad + \int_{t-d(t)}^{t} \dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\,\mathrm{d}s \end{aligned} \tag{18}$$

$$(\eta_1(t) - \eta_0)\Xi_1 + (\eta_1 - \eta(t))\Xi_2 + d(t)\Xi_3 + (d_M - d(t))\Xi_4 + \Omega < 0 \tag{11}$$

$$-2\zeta^{\mathrm{T}}(t)S\int_{t-d_M}^{t-d(t)}\dot{x}(s)\,\mathrm{d}s \le (d_M - d(t))\zeta^{\mathrm{T}}(t)SR_1^{-1}S^{\mathrm{T}}\zeta(t) + \int_{t-d_M}^{t-d(t)}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)\,\mathrm{d}s \tag{19}$$

$$\dot{x}^{\mathrm{T}}(t)\tilde{R}\dot{x}(t) = \mathcal{A}^{\mathrm{T}}\tilde{R}\mathcal{A} + \delta^2\mathcal{B}^{\mathrm{T}}\tilde{R}\mathcal{B} \tag{20}$$

where $\mathcal{B} = BKx(t-\eta(t)) - BKx(t-d(t)) - BKe(t_k h)$
Combining (14)–(20) and (5), we have (see (21))
Applying Schur complements, it can be concluded that (12) guarantees $\mathbb{E}\{\mathcal{L}V(t)\} < 0$ in (21). □

Based on Theorem 1, we are in a position to design the state feedback controller for system (7).

Theorem 2: For given constants η_0, η_1, d_M δ, $\bar{\alpha}$, and $\varepsilon_r, r = 1,2,3$, system (7) is asymptotically stable with state feedback gain $K = YX^{-1}$, if there exist real matrices $X > 0$, $\tilde{Q}_i > 0$, $\tilde{R}_i > 0 (i = 1,2,3)$, $\tilde{\Omega} > 0$, and $\tilde{M}$, $\tilde{N}$, $\tilde{T}$, $\tilde{S}$ with appropriate dimensions such that for $l = 1,2,3,4$

$$\tilde{\Pi}(l) = \begin{bmatrix} \tilde{\Omega}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^{\mathrm{T}} & * & * & * \\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * & * \\ \tilde{\Omega}_{31} & 0 & \tilde{\Omega}_{33} & * \\ \tilde{\Omega}_{41}(l) & 0 & 0 & \tilde{\Omega}_{44} \end{bmatrix} < 0 \tag{22}$$

where (see equation below)

Proof: Due to

$$(R_i - \varepsilon_i^{-1}P)R_i^{-1}(R_i - \varepsilon_i^{-1}P) \ge 0,$$

we can get

$$-PR_i^{-1}P \le -2\varepsilon_i P + \varepsilon_i^2 R_i$$

Substituting $-PR_i^{-1}P$ with $-2\varepsilon_i P + \varepsilon_i^2 R$ $(i = 1,2,3)$ into (12), one can have

$$\Omega_{11} = \begin{bmatrix} \Lambda_1 & * & * & * & * & * & * \\ R_2 & -Q_1 - R_2 & * & * & * & * & * \\ \bar{\alpha}K^{\mathrm{T}}B^{\mathrm{T}}P & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & -Q_2 & * & * & * \\ (1-\bar{\alpha})K^{\mathrm{T}}B^{\mathrm{T}}P & 0 & 0 & 0 & \sigma\Omega & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q_3 & * \\ (1-\bar{\alpha})K^{T}B^{T}P & 0 & 0 & 0 & 0 & 0 & -\Omega \end{bmatrix}$$

$$\Lambda_1 = PA + A^{\mathrm{T}}P + Q_1 + Q_2 + Q_3 - R_2,\ \ \Gamma = [T \quad M \quad -M+N \quad -N \quad -T+S \quad -S \quad 0]$$

$$\Omega_{21} = \begin{bmatrix} \sqrt{\eta_1-\eta_0}PA & 0 & \bar{\alpha}\sqrt{\eta_1-\eta_0}PBK & 0 & (1-\bar{\alpha})\sqrt{\eta_1-\eta_0}PBK & 0 & (1-\bar{\alpha})\sqrt{\eta_1-\eta_0}PBK \\ \eta_0 PA & 0 & \bar{\alpha}\eta_0 PBK & 0 & (1-\bar{\alpha})\eta_0 PBK & 0 & (1-\bar{\alpha})\eta_0 PBK \\ \sqrt{d_M}PA & 0 & \bar{\alpha}\sqrt{d_M}PBK & 0 & (1-\bar{\alpha})\sqrt{d_M}PBK & 0 & (1-\bar{\alpha})\sqrt{d_M}PBK \end{bmatrix}$$

$$\Omega_{31} = \begin{bmatrix} 0 & 0 & \delta\sqrt{\eta_1-\eta_0}PBK & 0 & -\delta\sqrt{\eta_1-\eta_0}PBK & 0 & -\delta\sqrt{\eta_1-\eta_0}PBK \\ 0 & 0 & \delta\eta_0 PBK & 0 & -\delta\eta_0 PBK & 0 & -\delta\eta_0 PBK \\ 0 & 0 & \delta\sqrt{d_M}PBK & 0 & -\delta\sqrt{d_M}PBK & 0 & -\delta\sqrt{d_M}PBK \end{bmatrix}$$

$$\Omega_{22} = \Omega_{33} = \mathrm{diag}\{-PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\},\quad \Omega_{44} = \mathrm{diag}\{-R_1, -R_3\}$$

$$\Omega_{41}(1) = \begin{bmatrix} \sqrt{\eta_1-\eta_0}M^{\mathrm{T}} \\ \sqrt{d_M}T^{\mathrm{T}} \end{bmatrix},\quad \Omega_{41}(2) = \begin{bmatrix} \sqrt{\eta_1-\eta_0}M^{\mathrm{T}} \\ \sqrt{d_M}S^{\mathrm{T}} \end{bmatrix},\quad \Omega_{41}(3) = \begin{bmatrix} \sqrt{\eta_1-\eta_0}N^{\mathrm{T}} \\ \sqrt{d_M}T^{\mathrm{T}} \end{bmatrix},$$

$$\Omega_{41}(4) = \begin{bmatrix} \sqrt{\eta_1-\eta_0}N^{\mathrm{T}} \\ \sqrt{d_M}S^{\mathrm{T}} \end{bmatrix}$$

$$M^{\mathrm{T}} = \begin{bmatrix} M_1^{\mathrm{T}} & M_2^{\mathrm{T}} & M_3^{\mathrm{T}} & M_4^{\mathrm{T}} & M_5^{\mathrm{T}} & M_6^{\mathrm{T}} & M_7^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
$$N^{\mathrm{T}} = \begin{bmatrix} N_1^{\mathrm{T}} & N_2^{\mathrm{T}} & N_3^{\mathrm{T}} & N_4^{\mathrm{T}} & N_5^{\mathrm{T}} & N_6^{\mathrm{T}} & N_7^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$
$$T^{\mathrm{T}} = \begin{bmatrix} T_1^{\mathrm{T}} & T_2^{\mathrm{T}} & T_3^{\mathrm{T}} & T_4^{\mathrm{T}} & T_5^{\mathrm{T}} & T_6^{\mathrm{T}} & T_7^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}},$$
$$S^{\mathrm{T}} = \begin{bmatrix} S_1^{\mathrm{T}} & S_2^{\mathrm{T}} & S_3^{\mathrm{T}} & S_4^{\mathrm{T}} & S_5^{\mathrm{T}} & S_6^{\mathrm{T}} & S_7^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}}$$

$$\begin{aligned} \mathbb{E}\{\mathcal{L}V(t)\} = {} & 2\dot{x}^{\mathrm{T}}(t)P\mathcal{A} + x^{\mathrm{T}}(t)Q_1x(t) - x^{\mathrm{T}}(t-\eta_0)Q_1x(t-\eta_0) \\ & + x^{\mathrm{T}}(t)Q_2x(t) - x^{\mathrm{T}}(t-\eta_1)Q_2x(t-\eta_1) + x^{\mathrm{T}}(t)Q_3x(t) \\ & - x^{\mathrm{T}}(t-d_M)Q_3x(t-d_M) + \dot{x}^{\mathrm{T}}(t)\tilde{R}\dot{x}(t) \\ & - \int_{t-\eta_1}^{t-\eta_0}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)\,\mathrm{d}s - \eta_0\int_{t-\eta_0}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)\,\mathrm{d}s \\ & - \int_{t-d_M}^{t}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s) + \Gamma_1 + \Gamma_2 + \Gamma_3 + \Gamma_4 \end{aligned} \tag{14}$$

$$\Pi(l) = \begin{bmatrix} \Omega_{11} + \Gamma + \Gamma^{\mathrm{T}} & * & * & * \\ \Omega_{21} & \bar{\Omega}_{22} & * & * \\ \Omega_{31} & 0 & \bar{\Omega}_{33} & * \\ \Omega_{41}(l) & 0 & 0 & \Omega_{44} \end{bmatrix} < 0 \tag{23}$$

where $\bar{\Omega}_{22} = \bar{\Omega}_{33} = \mathrm{diag}\{-2\varepsilon_1 X + \varepsilon_1^2 R_1, -2\varepsilon_2 X + \varepsilon_2^2 R_2, -2\varepsilon_3 X + \varepsilon_3^2 R_3\}$.

Define $Y = KX$, $X = P^{-1}$, $XQ_iX = \tilde{Q}_i (i = 1, 2, 3)$, $XR_iX = \tilde{R}_i (i = 1, 2, 3)$, $XMX = \tilde{M}$, $XNX = \tilde{N}$, $XTX = \tilde{T}$, $XSX = \tilde{S}$, pre- and post-multiplying both sides of (23) with $\mathrm{diag}\{\underbrace{X, \ldots, X}_{15}\}$, we obtain (22). This completes the proof. □

4 Simulation examples

Consider a special system of (7), the settings of the system are as follows:

$$A = \begin{bmatrix} -2 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.05 \\ 0.02 \end{bmatrix} \tag{24}$$

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V(t)\} \le\ & 2\dot{x}^{\mathrm{T}}(t)P\mathcal{A} + x^{\mathrm{T}}(t)(Q_1 + Q_2 + Q_3)x(t) \\
& -x^{\mathrm{T}}(t-\eta_0)Q_1x(t-\eta_0) - x^{\mathrm{T}}(t-\eta_1)Q_2x(t-\eta_1) \\
& -x^{\mathrm{T}}(t-d_M)Q_3x(t-d_M) + \mathcal{A}^{\mathrm{T}}\tilde{R}\mathcal{A} + \delta^2\mathcal{B}^{\mathrm{T}}\tilde{R}\mathcal{B} \\
& +\begin{bmatrix} x(t) \\ x(t-\eta_0) \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\eta_0) \end{bmatrix} \\
& +2\zeta^{\mathrm{T}}(t)M[x(t-\eta_0) - x(t-\eta(t))] \\
& +2\zeta^{\mathrm{T}}(t)N[x(t-\eta(t)) - x(t-\eta_1)] \\
& +2\zeta^{\mathrm{T}}(t)T[x(t) - x(t-d(t))] \\
& +2\zeta^{\mathrm{T}}(t)S[x(t-d(t)) - x(t-d_M)] \\
& +(\eta(t)-\eta_0)\zeta^{\mathrm{T}}(t)MR_1^{-1}M^{\mathrm{T}}\zeta(t) \\
& +(\eta_1-\eta(t))\zeta^{\mathrm{T}}(t)NR_1^{-1}N^{\mathrm{T}}\zeta(t) + d(t)\zeta^{\mathrm{T}}(t)TR_1^{-1}T^{\mathrm{T}}\zeta(t) \\
& +(d_M - d(t))\zeta^{\mathrm{T}}(t)SR_1^{-1}S^{\mathrm{T}}\zeta(t) \\
& +e^{\mathrm{T}}(t_kh)\Omega e(t_kh) - \sigma x^{\mathrm{T}}(t-d(t))\Omega x(t-d(t)) \\
=\ & \zeta^{\mathrm{T}}(t)(\Omega_{11} + \Gamma + \Gamma^{\mathrm{T}})\zeta(t) + \mathcal{A}^{\mathrm{T}}\tilde{R}\mathcal{A} + \delta^2\mathcal{B}^{\mathrm{T}}\tilde{R}\mathcal{B} \\
& +(\eta(t)-\eta_0)\zeta^{\mathrm{T}}(t)MR_1^{-1}M^{\mathrm{T}}\zeta(t) \\
& +(\eta_1-\eta(t))\zeta^{\mathrm{T}}(t)NR_1^{-1}N^{\mathrm{T}}\zeta(t) + d(t)\zeta^{\mathrm{T}}(t)TR_1^{-1}T^{\mathrm{T}}\zeta(t) \\
& +(d_M - d(t))\zeta^{\mathrm{T}}(t)SR_1^{-1}S^{\mathrm{T}}\zeta(t)
\end{aligned} \tag{21}$$

$$\tilde{\Omega}_{11} = \begin{bmatrix} \tilde{\Lambda}_1 & * & * & * & * & * & * \\ \tilde{R}_2 & -\tilde{Q}_1 - \tilde{R}_2 & * & * & * & * & * \\ \bar{\alpha}Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & -\tilde{Q}_2 & * & * & * \\ (1-\bar{\alpha})Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & 0 & \sigma\tilde{\Omega} & * & * \\ 0 & 0 & 0 & 0 & 0 & -\tilde{Q}_3 & * \\ (1-\bar{\alpha})Y^TB^T & 0 & 0 & 0 & 0 & 0 & -\tilde{\Omega} \end{bmatrix}$$

$$\tilde{\Lambda}_1 = AX^{\mathrm{T}} + XA^{\mathrm{T}} + \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 - \tilde{R}_2, \quad \tilde{\Gamma} = \begin{bmatrix} \tilde{T} & \tilde{M} & -\tilde{M} + \tilde{N} & -\tilde{N} & -\tilde{T} + \tilde{S} & -\tilde{S} & 0 \end{bmatrix}$$

$$\tilde{\Omega}_{21} = \begin{bmatrix} \sqrt{\eta_1 - \eta_0}AX & 0 & \bar{\alpha}\sqrt{\eta_1 - \eta_0}BY & 0 & (1-\bar{\alpha})\sqrt{\eta_1 - \eta_0}BY & 0 & (1-\bar{\alpha})\sqrt{\eta_1 - \eta_0}BY \\ \eta_0 AX & 0 & \bar{\alpha}\eta_0 BY & 0 & (1-\bar{\alpha})\eta_0 BY & 0 & (1-\bar{\alpha})\eta_0 BY \\ \sqrt{d_M}AX & 0 & \bar{\alpha}\sqrt{d_M}BY & 0 & (1-\bar{\alpha})\sqrt{d_M}BY & 0 & (1-\bar{\alpha})\sqrt{d_M}BY \end{bmatrix}$$

$$\tilde{\Omega}_{31} = \begin{bmatrix} 0 & 0 & \delta\sqrt{\eta_1 - \eta_0}BY & 0 & -\delta\sqrt{\eta_1 - \eta_0}BY & 0 & -\delta\sqrt{\eta_1 - \eta_0}BY \\ 0 & 0 & \delta\eta_0 BY & 0 & -\delta\eta_0 BY & 0 & -\delta\eta_0 BY \\ 0 & 0 & \delta\sqrt{d_M}BY & 0 & -\delta\sqrt{d_M}BY & 0 & -\delta\sqrt{d_M}BY \end{bmatrix}$$

$$\tilde{\Omega}_{22} = \tilde{\Omega}_{33} = \mathrm{diag}\{-2\varepsilon_1 X + \varepsilon_1^2\tilde{R}_1, -2\varepsilon_2 X + \varepsilon_2^2\tilde{R}_2, -2\varepsilon_3 X + \varepsilon_3^2\tilde{R}_3\},$$

$$\tilde{\Omega}_{44} = \mathrm{diag}\{-\tilde{R}_1, -\tilde{R}_3\}$$

$$\tilde{\Omega}_{41}(1) = \begin{bmatrix} \sqrt{\eta_1 - \eta_0}\tilde{M}^{\mathrm{T}} \\ \sqrt{d_M}\tilde{T}^{\mathrm{T}} \end{bmatrix}, \quad \tilde{\Omega}_{41}(2) = \begin{bmatrix} \sqrt{\eta_1 - \eta_0}\tilde{M}^{\mathrm{T}} \\ \sqrt{d_M}\tilde{S}^{\mathrm{T}} \end{bmatrix},$$

$$\tilde{\Omega}_{41}(3) = \begin{bmatrix} \sqrt{\eta_1 - \eta_0}\tilde{N}^{\mathrm{T}} \\ \sqrt{d_M}\tilde{T}^{\mathrm{T}} \end{bmatrix}, \tilde{\Omega}_{41}(4) = \begin{bmatrix} \sqrt{\eta_1 - \eta_0}\tilde{N}^{\mathrm{T}} \\ \sqrt{d_M}\tilde{S}^{\mathrm{T}} \end{bmatrix}$$

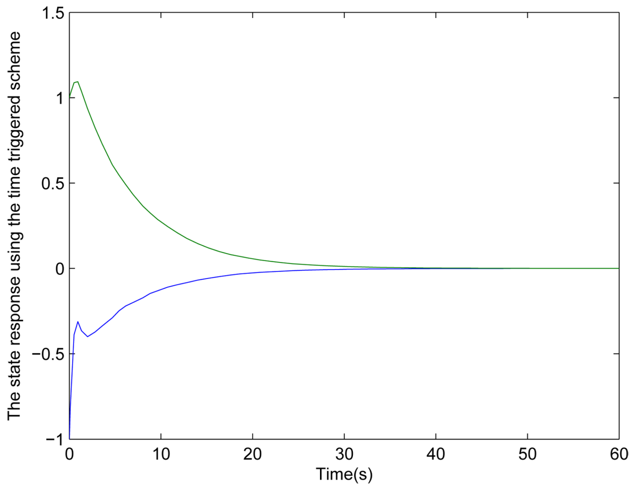

Fig. 2 *State responses under feedback gain (25) for (7)*

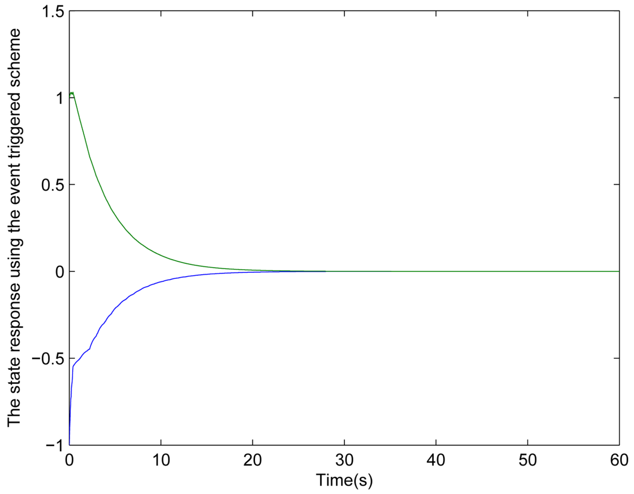

Fig. 3 *State responses under feedback gain (26) for (7)*

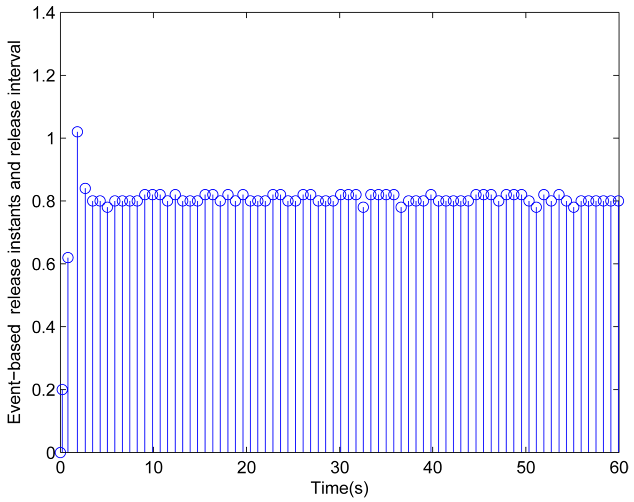

Fig. 4 *Release instants and release interval with feedback gain (26) into (7)*

We can easily see that the system is unstable without a controller. Choose the initial condition $x_0 = [-1 \quad 1]^{\mathrm{T}}$.

In the following, our purpose is to demonstrate the design process of the feedback gain and the proposed hybrid driven scheme.

Case 1: When the selecting switch turn to 'time driven' in Fig. 1, the system (7) is under time-triggered schemes. We assume $\eta_0 = 0.5$, $\eta_1 = 1$, $\sigma_2 = 0.01$ and $\varepsilon_1 = \varepsilon_2 = 1$, based on Matlab/LMIs toolbox and applying Theorem 2, we can get the controller feedback gain

$$K = [0.5864 \quad -13.1863] \tag{25}$$

With the feedback gain (25), the state trajectories of (7) are shown in Fig. 2.

Case 2: When the selecting switch turn to 'event driven' in Fig. 1, i.e. the system (7) is under event-triggered scheme, for given $\sigma = 0.12$, $d_M = 1$, and $\varepsilon_3 = 1$, we can obtain

$$K = [1.7151 \quad -16.2432] \tag{26}$$

the corresponding trigger matrix

$$\Omega = \begin{bmatrix} 16.9797 & -2.7202 \\ -2.7202 & 10.6299 \end{bmatrix},$$

respectively. the state response of (7) and communication instants and communication intervals are shown in Figs. 3 and 4, respectively.

Case 3: When the system (7) is a hybrid driven system, letting $\bar{\alpha} = 0.1$, $\sigma = 0.01$, and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, we can obtain

$$K = [5.2158 \quad -8.9618] \tag{27}$$

and the corresponding trigger matrix

$$\Omega = \begin{bmatrix} 1.3559 & -0.2178 \\ -0.2178 & 1.4307 \end{bmatrix},$$

respectively. The simulation result of the discussed closed-loop system dynamics are shown in Fig. 5. A random switching sequence for the switched triggered schemes is shown in Fig. 6. From the simulation results, it can be found that the designed feedback gain can stabilise the system even when the hybrid driven occurs.

Compared with the results above, from Figs. 2, 3 and 5, we can see that the studied system can be stabilised in above three cases. The amount of the network transmission can be reduced by using the event-triggered transmission strategy or the hybrid driven strategy which is illustrated in Fig. 4. The hybrid driven scheme in this paper gives an alternative method of event-triggered scheme.

5 Conclusion

The hybrid driven controller design for NCSs with network induced delays has been investigated. New hybrid driven communication schemes are proposed to reduce the network bandwidth utilisation and improve the desired system performance. A delay system model has been employed to describe the prosperities of the hybrid driven scheme and the effect of the transmission delay on the system. Sufficient conditions are established to ensure the stability of the discussed system and the existence of the feedback gain matrices of the controller and triggering parameters. An illustrative example has highlighted the usefulness of the proposed method. The problems of hybrid driven state estimation, output feedback hybrid driven H_∞ control and hybrid driven H_∞ filtering will be discussed in our future work.

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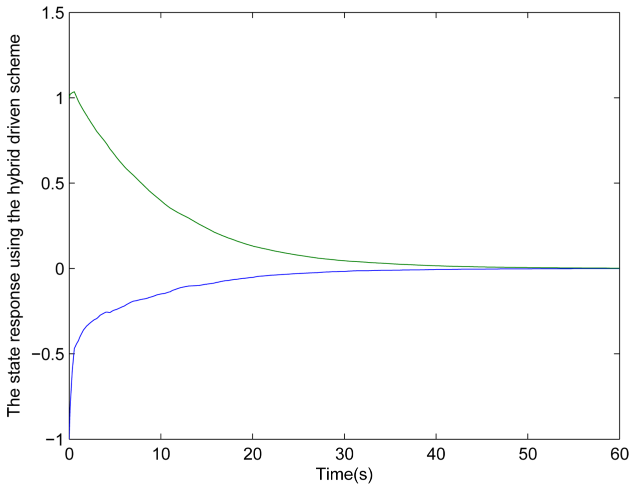

Fig. 5 *State responses under feedback gain (27) for (7)*

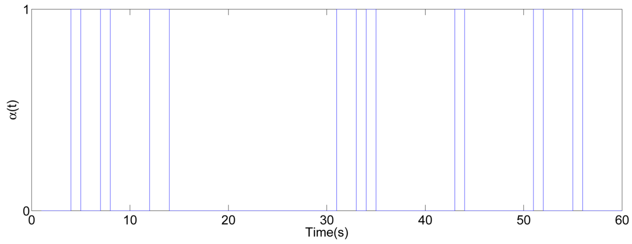

Fig. 6 *Bernoulli stochastic variable* $\alpha(t)$ *with* $\bar{\alpha} = 0.2$ *in case 3*

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7 References

[1] Gupta, R.A., Chow, M.-Y.: 'Networked control system: overview and research trends', *IEEE Trans. Ind. Electron.*, 2010, **57**, (7), pp. 2527–2535

[2] Zhang, L., Gao, H., Kaynak, O.: 'Network-induced constraints in networked control systems: a survey', *IEEE Trans. Ind. Inf.*, 2013, **9**, (1), pp. 403–416

[3] Li, F., Fu, J., Du, D.: 'An improved event-triggered communication mechanism and H-infinity control co-design for network control systems', *Inf. Sci.*, http://dx.doi.org/10.1016/j.ins.2016.01.073

[4] Li, H., Wang, J., Lam, H.-K., *et al.*: 'Adaptive sliding mode control for interval type-2 fuzzy systems', *IEEE Trans. Syst. Man Cybern., Syst.*, DOI: 10.1109/TSMC.2016.2531676

[5] Li, H., Wang, J., Shi, P.: 'Output-feedback based sliding mode control for fuzzy systems with actuator saturation', *IEEE Trans. Fuzzy Syst.*, DOI: 10.1109/TFUZZ.2015.2513085

[6] Li, H., Chen, Z., Wu, L., *et al.*: 'Event-triggered fault detection of nonlinear networked systems', *IEEE Trans. Cybern.*, DOI:10.1109/TCYB.2016.2536750

[7] Sun, S., Ma, J.: 'Linear estimation for networked control systems with random transmission delays and packet dropouts', *Inf. Sci.*, 2014, **269**, pp. 349–365

[8] Peng, C., Yue, D., Fei, M.-R.: 'Relaxed stability and stabilization conditions of networked fuzzy control systems subject to asynchronous grades of membership', *IEEE Trans. Fuzzy Syst.*, 2014, **22**, (5), pp. 1101–1112

[9] Liu, X., Yu, X., Ma, G., *et al.*: 'On sliding mode control for networked control systems with semi-Markovian switching and random sensor delays', *Inf. Sci.*, 2016, **337**, pp. 44–58

[10] Liu, J., Yue, D.: 'Event-triggering in networked systems with probabilistic sensor and actuator faults', *Inf. Sci.*, 2013, **240**, pp. 145–160

[11] Yue, D., Tian, E., Han, Q.-L.: 'A delay system method for designing event-triggered controllers of networked control systems', *IEEE Trans. Autom. Control*, 2013, **58**, (2), pp. 475–481

[12] Du, D., Qi, B., Fei, M., *et al.*: 'Multiple event-triggered H-2/H-infinity filtering for hybrid wired-wireless networked systems with random network-induced delays', *Inf. Sci.*, 2015, **325**, pp. 393–408

[13] Li, S., Sauter, D., Xu, B.: 'Co-design of event-triggered H-infinity control for discrete-time linear parameter-varying systems with network-induced delays', *J. Franklin Inst.*, 2015, **352**, (5), pp. 1867–1892

[14] Peng, C., Yang, T.C.: 'Event-triggered communication and H-infinity control co-design for networked control systems', *Automatica*, 2013, **49**, (5), pp. 1326–1332

[15] Peng, C., Han, Q.-L.: 'On designing a novel self-triggered sampling scheme for networked control systems with data losses and communication delays', *IEEE Trans. Ind. Electron.*, 2016, **63**, (2), pp. 1239–1248

[16] Peng, C., Zhang, J.: 'Event-triggered output-feedback H-infinity control for networked control systems with time-varying sampling', *IET Control Theory Appl.*, 2015, **9**, (9), pp. 1384–1391

[17] Dong, H., Wang, Z., Ding, S.X., *et al.*: 'Event-based filter design for a class of nonlinear time-varying systems with fading channels and multiplicative noises', *IEEE Trans. Signal Process.*, 2015, **63**, (13), pp. 3387–3395

[18] Liu, J., Fei, S., Tian, E., *et al.*: 'Co-design of event generator and filtering for a class of T–S fuzzy systems with stochastic sensor faults', *Fuzzy Sets Syst.*, 2015, **273**, pp. 124–140

[19] Peng, C., Fei, M.-R., Tian, E., *et al.*: 'On hold or drop out-of-order packets in networked control systems', *Inf. Sci.*, 2014, **268**, pp. 436–446

[20] Peng, C., Tian, Y.-C., Yue, D.: 'Output feedback control of discrete-time systems in networked environments', *IEEE Trans. Syst. Man Cybern. A, Syst. Humans*, 2011, **41**, (1), pp. 185–190

[21] Yue, D., Han, Q.-L., Peng, C.: 'State feedback controller design of networked control systems', *IEEE Trans. Circuits Syst. II, Express Briefs*, 2004, **51**, (11), pp. 640–644

[22] Tian, E., Yue, D., Peng, C.: 'Reliable control for networked control systems with probabilistic actuator fault and random delays', *J. Franklin Inst.*, 2010, **347**, (10), pp. 1907–1926

[23] Mao, X.: 'Exponential stability of stochastic delay interval systems with Markovian switching', *IEEE Trans. Autom. Control*, 2002, **47**, (10), pp. 1604–1612

[24] Wang, Y., Xie, L., de Souza, C.E.: 'Robust control of a class of uncertain nonlinear systems', *Syst. Control Lett.*, 1992, **19**, (2), pp. 139–149

[25] Tian, E., Yue, D., Zhang, Y.: 'Delay-dependent robust H-infinity control for t-s fuzzy system with interval time-varying delay', *Fuzzy Sets Syst.*, 2009, **160**, (12), pp. 1708–1719

[26] He, Y., Wu, M., She, J.-H., *et al.*: ': 'Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties', *IEEE Trans. Autom. Control*, 2004, **49**, (5), pp. 828–832

[27] Yue, D., Han, Q.-L., Lam, J.: 'Network-based robust H-infinity control of systems with uncertainty', *Automatica*, 2005, **41**, (6), pp. 999–1007