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Co-design of event generator and state estimator for complex network systems with quantization

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Abstract

This paper studies the problem of event-triggered state estimation for complex network systems with quantization. The event-triggered communication scheme and quantization are employed to reduce the burden of network transmission. By utilizing Lyapunov stability theory and linear matrix inequality techniques, sufficient conditions are established which can ensure the augmented estimation error system to be asymptotically stable. Furthermore, the explicit expressions of the desired state estimators are derived in terms of linear matrix inequalities. Finally, an example is provided to illustrate the usefulness of the proposed method.

1. Introduction

Complex networks have gained a lot of attention in various fields of science and humanity worldwide, such as gas transportation network, the Internet, our country highway network, etc. With the rapid development of modern science and technology, people have found that in the real world, although network nodes have different meanings in different situations, the complex network

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structure constrains the network dynamical behaviors. Thus the complex network theory is introduced to describe the common features among different kinds of network nodes. In recent years, scholars have paid great attention to complex network systems and they have gained many research results, including network control, reliability analysis and state estimation, and so on [1–3].

With the rapid expansion of network information, it needs a high data traffic rates on the network. However, network bandwidth and network resource are limited [4–6]. Effective methods should be adopted to overcome this problem. Luckily, different methods have been proposed on how to make full use of network bandwidth and how to reduce the network load effectively. Generally speaking, the existing methods can be classified into time-triggered mechanism and event-triggered mechanism [7–10]. Time triggered scheme (periodic sampling method) is easier to accept from the aspect of system analysis; but in terms of network resource utilization, time triggered scheme sometimes cannot show its advantage. Especially, when the state is close to equilibrium, there is little new information. If we still use the time triggered scheme, the computing resources will be wasted and the cost of computing will be increased. Fortunately, event-triggered mechanism has been proposed to overcome this problem, for it can greatly improve utilization of network resource. The key idea of event-triggered mechanism is that only the current sampled data satisfies a predesigned condition, can it be transmitted. The event-triggered scheme can save network bandwidth to some extent.

So far, we have acquired a lot of research results based on event-triggered mechanism [7–12]. The authors in [7] propose a novel event-triggering scheme and address event-triggered H_∞ control for networked systems. Based on the work of [7], the authors in [8] investigate the reliable control design for networked control system under event-triggered scheme by taking probabilistic sensor and actuator fault into consideration. The event-based fault detection problem is studied in [9] for networked systems with communication delay, unknown input and nonlinear perturbation. The problem of event-triggered output-feedback H_∞ control for networked control systems with non-uniform sampling is addressed in [10]. In [11], the design problems of the observer-based event-driven controllers are investigated for the state-dependent nonlinear systems. The authors in [12] address an event-driven observer-based fault-tolerant controller design for a state-dependent system with external disturbance and fault.

In the actual networked control system, due to the limited communication capacity, measurement output should be quantified before transmission. An important aspect is that utilizing quantization schemes can not only have sufficient precision, but also require low communication rate. Quantization can be considered to a coding process by using quantizer. So far, there have been a lot of relevant research results [13–16]. For example, the authors in [13] address the observer-based output feedback control for networked control systems with two quantizers. In [14], the authors are concerned with the control design problem of event-triggered networked systems with both state and control input quantizations. The networked H_∞ stabilization of linear time-invariant systems under quantized state feedback control has been investigated in [15]. The authors in [16] are concerned with the variance-constrained state estimation problem for a class of networked multi-rate systems with network-induced probabilistic sensor failures and measurement quantization. To the best of the authors knowledge, the state estimation problem for complex network systems with event-triggered mechanism and quantization has not been investigated yet, which is a starting point of this article.

Inspired by the above observations, this paper studies the state estimation problem for complex network systems with event-triggered communication scheme and quantization. Firstly, taking into consideration of event-triggered scheme and quantization, we construct a state error system model for complex network systems. Based on this constructed model, by using

Lyapunov stability theory and linear matrix inequality techniques, a sufficient condition for the asymptotically stability of complex network systems is obtained. Furthermore, the relevant parameters of the state estimator are derived in terms of linear matrix inequalities. Finally, numerical simulation verifies the usefulness of the proposed method. The main contributions of this paper lie in the following aspects: (1) The event-triggered communication mechanism is introduced to reduce the pressure of data transmission. (2) Quantization is employed to reduce network load and save network bandwidth. (3) Suitable state estimators are designed for complex network systems with quantization and event-triggered communication scheme.

The rest of this paper is organized as follows. In Section 2, problem formulation and preliminaries are briefly outlined. In Section 3, a flexible approach to the desired state estimator design is established. In Section 4, a numerical example is given to demonstrate the usefulness of the designed state estimators. Finally, conclusion is given in Section 5.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n\times m$ real matrices, respectively. $\|\cdot\|$ stands for the Euclidean vector norm or the induced matrix 2-norm as appropriate. The superscript T stands for matrix transposition. I is the identity matrix of appropriate dimensions. The notation $X>0$ (respectively, $X\geq 0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is a real symmetric positive definite (respectively, positive semi-definite). For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C\end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. Model and preliminaries

Consider the following stochastic complex network systems consisting of N coupled nodes with time-varying delay, and every node is a n-dimensional dynamical subsystem. The complex network systems can be described as [17]:

$$\dot{x}_i(t) = \delta(t)Af_1(x_i(t)) + (1-\delta(t))Bf_2(x_i(t)) + \sum_{j=1}^{N} g_{ij}\Gamma_1 x_j(t) + \sum_{j=1}^{N} g_{ij}\Gamma_2 x_j(t-\tau(t)), \tag{1}$$

where the state vector of the ith node is $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in\mathbb{R}^n$, $f_1(x_i(t))$ and $f_2(x_i(t))$ are nonlinear vector functions, Γ_1 and Γ_2 are the inner coupling matrices of the network, A and B are constant matrix with appropriate dimensions. $G=(g_{ij})\in\mathbb{R}^{N\times N}$ is the outer-coupling matrix of the networks representing the coupling strength and topological structure of complex networks. g_{ij} can be defined as: if the ith node has connection with the jth, $(i\neq j)$, then $g_{ij}=g_{ji}$; if there is no connection between the ith node and the jth node, then $g_{ij}\neq g_{ji}$. $\tau(t)$ is the time-varying delay in the network system, satisfying that: $\tau_m\leq\tau(t)\leq\tau_M$, $0\leq\tau_m\leq\tau_M$. $\delta(t)$ is subject to Bernoulli distribution, defined as

$$\delta(t) = \begin{cases} 1, & f_1(.) \text{ happens,} \\ 0, & f_2(.) \text{ happens,} \end{cases} \tag{2}$$

Suppose $\delta(t)$ satisfies that:

$$\text{Probability } \{\delta(t)=1\}=\delta_0, \quad \text{Probability } \{\delta(t)=0\}=1-\delta_0, \tag{3}$$

where δ_0 is a known constant.

Remark 1. In the model (1), we use $\delta(t)$ to stand for the random switching between $f_1(x_i(t))$ and $f_2(x_i(t))$. When $\delta(t)=1$, it means that nonlinear vector function $f_1(x_i(t))$ occurs. When $\delta(t)=0$, it means that nonlinear character is described as $f_2(x_i(t))$.

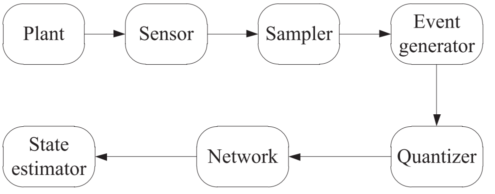

Fig. 1. The structure of an event-triggered complex network systems.

Assumption 1 ([17]). For any $u, v \in \mathbb{R}^n$, nonlinear vector functions $f_1(\cdot)$ and $f_2(\cdot)$ satisfy the following sector-bounded conditions:

$$[f_1(u)-f_1(v)-\Xi_1(u-v)]^T[f_1(u)-f_1(v)-\Xi_2(u-v)] \le 0,$$
$$[f_2(u)-f_2(v)-\Xi_3(u-v)]^T[f_2(u)-f_2(v)-\Xi_4(u-v)] \le 0. \tag{4}$$

Remark 2. From Assumption 1, we can easily get that:

$$\begin{bmatrix} x_i(t) \\ f_1(x_i(t)) \end{bmatrix}^T \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & I_n \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_1(x_i(t)) \end{bmatrix} \ge 0, \quad \begin{bmatrix} x_i(t) \\ f_2(x_i(t)) \end{bmatrix}^T \begin{bmatrix} \overline{\Omega}_{11} & \overline{\Omega}_{12} \\ \overline{\Omega}_{21} & I_n \end{bmatrix} \begin{bmatrix} x_i(t) \\ f_2(x_i(t)) \end{bmatrix} \ge 0, \tag{5}$$

where

$$\Omega_{11} = \frac{\Xi_1^T\Xi_2 + \Xi_2^T\Xi_1}{2}, \quad \Omega_{21}^T = \Omega_{12} = -\frac{\Xi_1^T + \Xi_2^T}{2},$$
$$\overline{\Omega}_{11} = \frac{\Xi_3^T\Xi_4 + \Xi_4^T\Xi_3}{2}, \quad \overline{\Omega}_{21}^T = \overline{\Omega}_{12} = -\frac{\Xi_3^T + \Xi_4^T}{2}.$$

The system structure is shown in Fig. 1. The main purpose of this paper is to design a suitable state estimator for complex network systems with quantization and event-triggered scheme.

With the matrix Kronecker product, the system (1) can be rewritten in the following compact form:

$$\dot{x}(t) = \delta(t)I_A F_1(x(t)) + (1-\delta(t))I_B F_2(x(t)) + (G \otimes \Gamma_1)x(t) + (G \otimes \Gamma_2)x(t-\tau(t)) \tag{6}$$

where $I_A = I_N \otimes A$, $I_B = I_N \otimes B$, $x^T(t) = [x_1^T(t), x_2^T(t), \ldots, x_N^T(t)]$, $G_1 = G \otimes \Gamma_1$, $G_2 = G \otimes \Gamma_2$,

$$F_1^T(x(t)) = [f_1^T(x_1(t)), f_1^T(x_2(t)), \ldots, f_1^T(x_N(t))], \quad F_2^T(x(t)) = [f_2^T(x_1(t)), f_2^T(x_2(t)), \ldots, f_2^T(x_N(t))].$$

In this article, the measurement output $y(t)$ in Eq. (6) is:

$$y(t) = Cx(t). \tag{7}$$

The event generator and quantizer are constructed between sensors and state estimators, and we suppose sensors and samplers are time-triggered, the sampling period is h, the sampling time is kh $(k = 0, 1, 2, \ldots)$. Also, at the time kh, the current measurement output is $y(kh)$, however, whether the newly measurement output $y((k+j)h)$ will be sent out or not is determined by the following judgement algorithm [7]:

$$[y((k+j)h) - y(kh)]^T \Omega [y((k+j)h) - y(kh)] \le \sigma y^T((k+j)h)\Omega y((k+j)h), \tag{8}$$

where Ω is a symmetric positive definite matrix with appropriate dimension, $\sigma \in [0,1)$, the newly measurement output $y((k+j)h)$ satisfying the above inequality (8) will not be transmitted.

Based on the above analysis, the real measurement outputs will be:

$$\tilde{y}(t) = y(t_k h) = Cx(t_k h), \quad t \in [t_k h + d_k, t_{k+1} h + d_{k+1}). \tag{9}$$

By the quantizer, the measurement outputs signal $y(t_k h)$ can be described as $\overline{y}(t_k h)$, that is:

$$\overline{y}(t_k h) = g(\tilde{y}(t)) = g(y(t_k h)) = g(Cx(t_k h)), \quad t \in [t_k h + d_k, t_{k+1} h + d_{k+1}), \tag{10}$$

where $g(y) = \mathrm{diag}\{g_1(y_1), g_2(y_2), \ldots, g_n(y_n)\}$, $g_j(\cdot)$, $(j = 1, 2, \ldots, n)$ is symmetric, that is: $g_j(-y_j) = -g_j(y_j)$, the logarithmic quantizer $g_j(\cdot)$ $(j = 1, 2, \ldots, n)$ can be defined as:

$$g_j(y_j) = \begin{cases} u_l^{(j)}, & \text{if } \dfrac{1}{1+\delta_{g_j}} u_l^{(j)} < y_j \le \dfrac{1}{1-\delta_{g_j}} u_l^{(j)}, \quad y_j > 0, \\ 0, & \text{if } y_j = 0, \\ -g_j(-y_j), & \text{if } y_j < 0, \end{cases} \tag{11}$$

where $\delta_{g_j} = \frac{1-\rho_{g_j}}{1+\rho_{g_j}} \left(0 < \rho_{g_j} < 1\right)$, ρ_{g_j} is the quantitative density of g_j, and it is a constant. For the sake of simplicity, we assume $\delta_g = \delta_{g_j}$, where δ_g is a constant. By the above discussion, we can get: $\rho_{g_j} = \rho_g = \frac{1-\delta_g}{1+\delta_g}$. Furthermore, similar to the methods in references [18], we define quantitative series set as:

$$U_j = \{\pm u_l^{(j)}, u_l^{(j)} = \rho_{g_j}^l \cdot u_0^{(j)}, l = \pm 1, \pm 2, \ldots\} \cup \{\pm u_0^{(j)}\} \cup \{0\}, \quad u_0^{(j)} > 0. \tag{12}$$

Define:

$$\Delta_g = \mathrm{diag}\{\Delta_{g_1}, \Delta_{g_2}, \ldots, \Delta_{g_n}\}, \quad \text{where } \Delta_{g_j} \in [-\delta_{g_j}, \delta_{g_j}],\ j = 1, 2, \ldots, n, \tag{13}$$

the logarithmic quantizer $g_j(\cdot)$ can be described by using the following sector bound approach:

$$g_j(y_j) = (1 + \Delta_{g_j}(y_j)) y_j, \tag{14}$$

then $g(\cdot)$ can be represented as:

$$g(y) = (I + \Delta_g) y. \tag{15}$$

Combine (10) and (15), $\overline{y}(t)$ can be described as:

$$\overline{y}(t) = g(y(t_k h)) = (I + \Delta_g) y(t_k h), \quad t \in [t_k h + d_k, t_{k+1} h + d_{k+1}). \tag{16}$$

In order to facilitate the theoretical development, we consider the following two cases, similar to [7,19]:

Case 1: If $t_k h + h + \overline{d} \ge t_{k+1} h + d_{k+1}$, where $\overline{d} = \max\{d_k\}$, define $d(t)$ as:

$$d(t) = t - t_k h, \quad t \in [t_k h + d_k, t_{k+1} h + d_{k+1}), \tag{17}$$

obviously,

$$d_k \le d(t) \le (t_{k+1} - t_k) h + d_{k+1} < h + \overline{d}. \tag{18}$$

Case 2: If $t_k h + h + \overline{d} < t_{k+1} h + d_{k+1}$, consider the following two intervals

$$[t_k h + d_k, t_k h + h + \overline{d}), \quad [t_k h + ih + \overline{d}, t_k h + ih + h + \overline{d}), \tag{19}$$

since $d_k \le \overline{d}$, it can be easily shown that there exists d_M, such that $t_k h + d_M h + \overline{d} < t_{k+1} h + d_{k+1} \le t_k h + d_M h + h + \overline{d}$.

Let

$$\begin{cases} I_0 = [t_k h + d_k, t_k h + h + \overline{d}), \\ I_i = [t_k h + ih + \overline{d}, t_k h + ih + h + \overline{d}), \quad i = 1, 2, \ldots, d_M - 1, \\ I_{d_M} = [t_k h + d_M h + \overline{d}, t_{k+1} h + d_{k+1}), \end{cases} \tag{20}$$

obviously, we can get

$$\bigcup_{i=0}^{d_M} I_i = [t_k h + d_k, t_{k+1} h + d_{k+1}). \tag{21}$$

Define a function

$$d(t) = \begin{cases} t - t_k h, \quad t \in I_0, \\ t - t_k h - ih, \quad t \in I_i, \ i = 1, 2, \ldots, d_M - 1, \\ t - t_k h - d_M h, \quad t \in I_{d_M}, \end{cases} \tag{22}$$

then, we can conclude that:

$$\begin{cases} d_k \le d(t) \le h + \overline{d}, \quad t \in I_0, \\ d_k \le \overline{d} \le d(t) \le h + \overline{d}, \quad t \in I_i, \ i = 1, 2, \ldots, d_M - 1, \\ d_k \le \overline{d} \le d(t) \le h + \overline{d}, \quad t \in I_{d_M}, \end{cases} \tag{23}$$

due to $t_{k+1} h + d_{k+1} \le t_k h + (d_M + 1)h + \overline{d}$, the third row in Eq. (23) holds. In Case 1, for $t \in [t_k h + d_k, t_{k+1} h + d_{k+1})$, define $e_k(t) = 0$; In Case 2, define:

$$e_k(t) = \begin{cases} 0, \quad t \in I_0, \\ y(t_k h) - y(t_k h + ih), \quad t \in I_i, \ i = 1, 2, \ldots, d_M - 1, \\ y(t_k h) - y(t_k h + d_M h), \quad t \in I_{d_M}, \end{cases} \tag{24}$$

from the definition of $e_k(t)$ and the event-triggered communication scheme, Eq. (8) can be rewritten as:

$$e_k^T(t) \Omega e_k(t) \le \sigma x^T(t - d(t)) C^T \Omega C x(t - d(t)), \quad t \in [t_k h + d_k, t_{k+1} h + d_{k+1}) \tag{25}$$

Based on the real measurement output, the main purpose of this article is to construct the following state estimation system:

$$\begin{cases} \dot{\hat{x}}(t) = G_1 \hat{x}(t) + G_2 \hat{x}(t - \tau(t)) + K(\overline{y}(t) - \hat{y}(t)), \\ \hat{y}(t) = C \hat{x}(t), \end{cases} \tag{26}$$

where $\hat{x}(t)$ is the estimator state vector, $\hat{y}(t)$ is estimator output, K is the feedback gain matrix, which is also the state estimator we should design next.

Define $e(t) = x(t) - \hat{x}(t)$, combining Eqs. (6), (16), (24) and (26), we can get:

$$\begin{aligned} \dot{e}(t) = {} & \delta(t) \cdot (I_N \otimes A) F_1(x(t)) + (1 - \delta(t)) \cdot (I_N \otimes B) F_2(x(t)) + (G \otimes \Gamma_1 - KC) e(t) \\ & + (G \otimes \Gamma_2) e(t - \tau(t)) + KCx(t) - K(I + \Delta_g) C x(t - d(t)) - K(I + \Delta_g) e_k(t). \end{aligned} \tag{27}$$

Define $\overline{x}(t) = [x^T(t), e^T(t)]^T$, combining Eqs. (6) and (27), we can get the following augmented system:

$$\dot{\overline{x}}(t) = \delta(t) I_{A_1} F_1(H \overline{x}(t)) + (1 - \delta(t)) I_{B_1} F_2(H \overline{x}(t)) + A_1 \overline{x}(t)$$

$$+B_1\overline{x}(t-\tau(t))+C_1\overline{x}(t-d(t))+D_1e_k(t), \tag{28}$$

where

$$A_1=\begin{bmatrix} G\otimes\Gamma_1 & 0 \\ KC & G\otimes\Gamma_1-KC \end{bmatrix},\quad B_1=\begin{bmatrix} G\otimes\Gamma_2 & 0 \\ 0 & G\otimes\Gamma_2 \end{bmatrix},\quad C_1=\begin{bmatrix} 0 & 0 \\ -K(I+\Delta_g)C & 0 \end{bmatrix},$$

$$D_1=\begin{bmatrix} 0 \\ -K(I+\Delta_g) \end{bmatrix},\quad I_{A_1}=\begin{bmatrix} I_A \\ I_A \end{bmatrix},\quad I_{B_1}=\begin{bmatrix} I_B \\ I_B \end{bmatrix},\quad H^T=\begin{bmatrix} I \\ 0 \end{bmatrix}.$$

Define a new vector:

$$\xi^T(t)=[F_1^T(H\overline{x}(t)),F_2^T(H\overline{x}(t)),\overline{x}^T(t),\overline{x}^T(t-\tau_m),\overline{x}^T(t-\tau(t)),$$
$$\overline{x}^T(t-\tau_M),\overline{x}^T(t-d_M),\overline{x}^T(t-d(t)),e_k^T(t)],$$

and define

$$\varphi_1=[I_{A_1},0,A_1,0,B_1,0,0,C_1,D_1],\quad \varphi_2=[0,I_{B_1},A_1,0,B_1,0,0,C_1,D_1],$$

the system (28) can be rewritten as follows:

$$\dot{\overline{x}}(t)=\delta(t)\varphi_1\xi(t)+(1-\delta(t))\varphi_2\xi(t). \tag{29}$$

Before giving the main results, the following lemmas are introduced as follows:

Lemma 1 (*[20]*). *For any vectors* $x, y\in\mathbb{R}^n$ *and positive definite matrix* $Q\in\mathbb{R}^{n\times n}$, *the following inequality holds*:

$$2x^Ty\le x^TQx+y^TQ^{-1}y. \tag{30}$$

Lemma 2 (*[21]*). *If* $\tau_1\le\tau(t)$, $x(t)\in\mathbb{R}^n$, *for any positive definite matrix* R, *we have*:

$$-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R\dot{x}(s)ds\le\begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}^T\cdot\begin{bmatrix} -R & * \\ R & -R \end{bmatrix}\cdot\begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}. \tag{31}$$

Lemma 3 (*[22]*). *For any positive definite matrix* $R\in\mathbb{R}^{n\times n}$, *if* $0<\tau_1\le\tau(t)\le\tau_2$ *and vector function* $\dot{x}(t):[-\tau_2,-\tau_1]\to\mathbb{R}^n$, *the following inequality holds*:

$$-(\tau_2-\tau_1)\int_{t-\tau_2}^{t-\tau_1}\dot{x}^T(s)R\dot{x}(s)ds\le\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}^T\cdot\begin{bmatrix} -R & * & * \\ R & -2R & * \\ 0 & R & -R \end{bmatrix}\cdot\begin{bmatrix} x(t-\tau_1) \\ x(t-\tau(t)) \\ x(t-\tau_2) \end{bmatrix}. \tag{32}$$

Lemma 4 (*[23]*). Ω_1, Ω_2 *and* Ω *are matrices with appropriate dimensions, then for* $\tau(t)\in[\tau_1,\tau_2]$,

$$(\tau(t)-\tau_1)\Omega_1+(\tau_2-\tau(t))\Omega_2+\Omega<0 \tag{33}$$

holds, if and only if the following inequalities hold:

$$\begin{cases} (\tau_2-\tau_1)\Omega_1+\Omega<0, \\ (\tau_2-\tau_1)\Omega_2+\Omega<0. \end{cases} \tag{34}$$

Lemma 5 ([24]). *A, D, E and F are matrices with appropriate dimensions, and satisfying $\|F\| \le 1$, then the following inequalities hold*:

(a) *For any variable $\varepsilon>0$, we have*:

$$DFE+E^TF^TD^T \le \varepsilon^{-1}DD^T+\varepsilon E^TE; \tag{35}$$

(b) *For any positive definite matrix $P>0$ and variable $\varepsilon>0$ such that $\varepsilon I-EPE^T>0$, then we have*:

$$(A+DFE)P(A+DFE)^T \le APA^T+APE^T(\varepsilon I-EPE^T)^{-1}EPA^T+\varepsilon DD^T. \tag{36}$$

3. Main results

In this section, to make the augmented estimation error system (29) be asymptotically stable, sufficient conditions are proposed in Theorem 1 firstly. Then, based on the obtained conditions, the designed method of the desired state estimator is given subsequently.

Theorem 1. *For given scalars $0 \le \tau_m \le \tau_M, d_M$, event-triggered parameter σ and the estimator gain matrix K, complex network systems (29) is asymptotically stable, if there exist matrices $P>0$, $Q_i>0$ $(i=1,2,3)$, $R_i>0$ $(i=1,2,3)$, and M, N, Z_1, Z_2 with appropriate dimensions, such that the following matrix inequalities hold*:

$$\Sigma(s)=\begin{bmatrix} \Phi_{11}+\Gamma+\Gamma^T & * & * \\ \Phi_{21} & \Phi_{22} & * \\ \Phi_{31}(s) & 0 & -R_2 \end{bmatrix}<0, \quad s=1,2, \tag{37}$$

where

$$\Phi_{11}=\begin{bmatrix} Z_1\otimes I_n & * & * & * & * & * & * & * & * \\ 0 & Z_2\otimes I_n & * & * & * & * & * & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * & * & * & * & * & * \\ 0 & 0 & R_1 & -R_1-Q_1 & * & * & * & * & * \\ 0 & 0 & B_1^TP & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q_2 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -R_3-Q_3 & * & * \\ 0 & 0 & R_3+C_1^TP & 0 & 0 & 0 & R_3 & \sigma W-2R_3 & * \\ 0 & 0 & D_1^TP & 0 & 0 & 0 & 0 & 0 & -W_1 \end{bmatrix},$$

$$\Phi_{21} = \begin{bmatrix} \theta_1 R_1 I_{A_1} & 0 & \theta_1 R_1 A_1 & 0 & \theta_1 R_1 B_1 & 0 & 0 & \theta_1 R_1 C_1 & \theta_1 R_1 D_1 \\ \theta_2 R_2 I_{A_1} & 0 & \theta_2 R_2 A_1 & 0 & \theta_2 R_2 B_1 & 0 & 0 & \theta_2 R_2 C_1 & \theta_2 R_2 D_1 \\ \theta_3 R_3 I_{A_1} & 0 & \theta_3 R_3 A_1 & 0 & \theta_3 R_3 B_1 & 0 & 0 & \theta_3 R_3 C_1 & \theta_3 R_3 D_1 \\ 0 & \theta_{10} R_1 I_{B_1} & \theta_{10} R_1 A_1 & 0 & \theta_{10} R_1 B_1 & 0 & 0 & \theta_{10} R_1 C_1 & \theta_{10} R_1 D_1 \\ 0 & \theta_{20} R_2 I_{B_1} & \theta_{20} R_2 A_1 & 0 & \theta_{20} R_2 B_1 & 0 & 0 & \theta_{20} R_2 C_1 & \theta_{20} R_2 D_1 \\ 0 & \theta_{30} R_3 I_{B_1} & \theta_{30} R_3 A_1 & 0 & \theta_{30} R_3 B_1 & 0 & 0 & \theta_{30} R_3 C_1 & \theta_{30} R_3 D_1 \end{bmatrix},$$

$$\Phi_{22} = \mathrm{diag}\{-R_1, -R_2, -R_3, -R_1, -R_2, -R_3\},$$
$$\Pi_{31} = H^T(Z_1 \otimes \Omega_{21})^T + \delta_0 P I_{A_1}, \quad \Pi_{32} = H^T(Z_2 \otimes \overline{\Omega}_{21})^T + \delta_{10} P I_{B_1},$$
$$\Pi_{33} = -R_3 - R_1 + Q_1 + Q_2 + Q_3 + A_1^T P + P A_1 + H^T(Z_1 \otimes \Omega_{11})H + H^T(Z_2 \otimes \overline{\Omega}_{11})H,$$

$$\Phi_{31}(1) = \sqrt{\delta} N^T, \quad \Phi_{31}(2) = \sqrt{\delta} M^T, \quad \tau_{21} = \tau_M - \tau_m, \quad \delta_{10} = 1 - \delta_0,$$
$$W = \begin{bmatrix} C^T \Omega C & 0 \\ 0 & 0 \end{bmatrix}, \quad W_1 = \begin{bmatrix} \Omega & 0 \\ 0 & 0 \end{bmatrix}, \quad \Gamma = [0\ \ 0\ \ 0\ \ N\ \ -N+M\ \ -M\ \ 0\ \ 0\ \ 0],$$
$$M^T = \begin{bmatrix} M_1^T & M_2^T & M_3^T & M_4^T & M_5^T & M_6^T & M_7^T & M_8^T & M_9^T \end{bmatrix},$$
$$N^T = \begin{bmatrix} N_1^T & N_2^T & N_3^T & N_4^T & N_5^T & N_6^T & N_7^T & N_8^T & N_9^T \end{bmatrix},$$
$$\theta_1 = \tau_m \sqrt{\delta_0}, \quad \theta_2 = \sqrt{\tau_{21}\delta_0}, \quad \theta_3 = d_M \sqrt{\delta_0},$$
$$\theta_{10} = \tau_m \sqrt{\delta_{10}}, \quad \theta_{20} = \sqrt{\tau_{21}\delta_{10}}, \quad \theta_{30} = d_M \sqrt{\delta_{10}},$$
$$R = \tau_m^2 R_1 + \tau_{21} R_2 + d_M^2 R_3.$$

Proof. Construct the following Lyapunov functional candidate:

$$V(t, \overline{x}(t)) = V_1(t, \overline{x}(t)) + V_2(t, \overline{x}(t)) + V_3(t, \overline{x}(t)), \tag{38}$$

where

$$V_1(t, \overline{x}(t)) = \overline{x}^T(t) P \overline{x}(t),$$
$$V_2(t, \overline{x}(t)) = \int_{t-\tau_m}^{t} \overline{x}^T(s) Q_1 \overline{x}(s) ds + \int_{t-\tau_M}^{t} \overline{x}^T(s) Q_2 \overline{x}(s) ds + \int_{t-d_M}^{t} \overline{x}^T(s) Q_3 \overline{x}(s) ds,$$
$$V_3(t, \overline{x}(t)) = \tau_m \int_{t-\tau_m}^{t} \int_{s}^{t} \dot{\overline{x}}^T(v) R_1 \dot{\overline{x}}(v) dv ds + \int_{t-\tau_M}^{t-\tau_m} \int_{s}^{t} \dot{\overline{x}}^T(v) R_2 \dot{\overline{x}}(v) dv ds$$
$$+ d_M \int_{t-d_M}^{t} \int_{s}^{t} \dot{\overline{x}}^T(v) R_3 \dot{\overline{x}}(v) dv ds.$$

Take the derivative of $V_i(t, \overline{x}(t))$ along the trajectory of system (29) and take expectation on $\mathcal{L}V_i(t, \overline{x}(t))$, we can get:

$$\begin{aligned} \mathbb{E}\{\mathcal{L}v_1(t, \overline{x}(t))\} &= 2\overline{x}^T(t) P \dot{\overline{x}}(t) \\ &= 2\overline{x}^T(t) P [\delta_0 \varphi_1 \xi(t) + (1-\delta_0)\varphi_2 \xi(t))], \end{aligned} \tag{39}$$

$$\begin{aligned} \mathbb{E}\{\mathcal{L}v_2(t, \overline{x}(t))\} &= \overline{x}^T(t)(Q_1 + Q_2 + Q_3)\overline{x}(t) - \overline{x}^T(t-\tau_m) Q_1 \overline{x}(t-\tau_m) - \overline{x}^T(t-\tau_M) Q_2 \overline{x}(t-\tau_M) \\ &\quad - \overline{x}^T(t-d_M) Q_3 \overline{x}(t-d_M), \end{aligned} \tag{40}$$

$$\mathbb{E}\{\mathcal{L}v_3(t, \overline{x}(t))\} = \delta_0 \xi^T(t) \varphi_1^T R \varphi_1 \xi(t) + (1-\delta_0) \xi^T(t) \varphi_2^T R \varphi_2 \xi(t) - \tau_m \int_{t-\tau_m}^{t} \dot{\overline{x}}^T(v) R_1 \dot{\overline{x}}(v) dv$$

$$-d_M\int_{t-d_M}^{t}\dot{\overline{x}}^T(v)R_3\dot{\overline{x}}(v)dv-\int_{t-\tau_M}^{t-\tau_m}\dot{\overline{x}}^T(v)R_2\dot{\overline{x}}(v)dv. \tag{41}$$

Combining Eqs. (39), (40) and (41) and employing the free matrix method [25], then we can get:

$$\begin{aligned}\mathbb{E}\{\mathcal{L}v(t,\overline{x}(t))\} &= 2\overline{x}^T(t)P[\delta_0\varphi_1\xi(t)+(1-\delta_0)\varphi_2\xi(t))]+\overline{x}^T(t)(Q_1+Q_2+Q_3)\overline{x}(t)\\ &\quad +\delta_0\xi^T(t)\varphi_1^TR\varphi_1\xi(t)+(1-\delta_0)\xi^T(t)\varphi_2^TR\varphi_2\xi(t)-\tau_m\int_{t-\tau_m}^{t}\dot{\overline{x}}^T(v)R_1\dot{\overline{x}}(v)dv\\ &\quad -d_M\int_{t-d_M}^{t}\dot{\overline{x}}^T(v)R_3\dot{\overline{x}}(v)dv\\ &\quad -\int_{t-\tau_M}^{t-\tau_m}\dot{\overline{x}}^T(v)R_2\dot{\overline{x}}(v)dv-\overline{x}^T(t-\tau_m)Q_1\overline{x}(t-\tau_m)-\overline{x}^T(t-\tau_M)Q_2\overline{x}(t-\tau_M)\\ &\quad -\overline{x}^T(t-d_M)Q_3\overline{x}(t-d_M)+\gamma_1+\gamma_2,\end{aligned} \tag{42}$$

where M, N are free matrices, and

$$\gamma_1=2\xi^T(t)N\left[\overline{x}(t-\tau_m)-\overline{x}(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_m}\dot{\overline{x}}(s)ds\right]=0, \tag{43}$$

$$\gamma_2=2\xi^T(t)M\left[\overline{x}(t-\tau(t))-\overline{x}(t-\tau_M)-\int_{t-\tau_M}^{t-\tau(t)}\dot{\overline{x}}(s)ds\right]=0. \tag{44}$$

By lemma 1, the following inequalities hold:

$$-2\xi^T(t)N\int_{t-\tau(t)}^{t-\tau_m}\dot{\overline{x}}(s)ds\le\int_{t-\tau(t)}^{t-\tau_m}\dot{\overline{x}}^T(s)R_2\dot{\overline{x}}(s)ds+(\tau(t)-\tau_m)\xi^T(t)NR_2^{-1}N^T\xi(t), \tag{45}$$

$$-2\xi^T(t)M\int_{t-\tau_M}^{t-\tau(t)}\dot{\overline{x}}(s)ds\le\int_{t-\tau_M}^{t-\tau(t)}\dot{\overline{x}}^T(s)R_2\dot{\overline{x}}(s)ds+(\tau_M-\tau(t))\xi^T(t)MR_2^{-1}M^T\xi(t). \tag{46}$$

By lemma 2, we can get:

$$-\tau_m\int_{t-\tau_m}^{t}\dot{\overline{x}}^T(v)R_1\dot{\overline{x}}(v)dv\le\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-\tau_m)\end{bmatrix}^T\begin{bmatrix}-R_1 & *\\ R_m & -R_1\end{bmatrix}\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-\tau_m)\end{bmatrix}. \tag{47}$$

Similarly, by Lemma 3, we have:

$$-d_M\int_{t-d_M}^{t}\dot{\overline{x}}^T(v)R_3\dot{\overline{x}}(v)dv\le\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-d(t))\\ \overline{x}(t-d_M)\end{bmatrix}^T\begin{bmatrix}-R_3 & * & *\\ R_3 & -2R_3 & *\\ 0 & R_3 & -R_3\end{bmatrix}\begin{bmatrix}\overline{x}(t)\\ \overline{x}(t-d(t))\\ \overline{x}(t-d_M)\end{bmatrix}. \tag{48}$$

Recalling Assumption 1 and Remark 2, the following inequalities hold:

$$\begin{bmatrix}\overline{x}(t)\\ F_1(H\overline{x}(t))\end{bmatrix}^T\begin{bmatrix}H^T(Z_1\otimes\Omega_{11})H & *\\ (Z_1\otimes\Omega_{21})H & Z_1\otimes I_n\end{bmatrix}\begin{bmatrix}\overline{x}(t)\\ F_1(H\overline{x}(t))\end{bmatrix}\ge 0, \tag{49}$$

$$\begin{bmatrix} \bar{x}(t) \\ F_2(H\bar{x}(t)) \end{bmatrix}^T \begin{bmatrix} H^T(Z_2 \otimes \overline{\Omega}_{11})H & * \\ (Z_2 \otimes \overline{\Omega}_{21})H & Z_2 \otimes I_n \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ F_2(H\bar{x}(t)) \end{bmatrix} \geq 0. \tag{50}$$

From Eq. (25), we can obtain:

$$\sigma x^T(t-d(t))C^T\Omega Cx(t-d(t)) - e_k^T(t)\Omega e_k(t) \geq 0, \quad t \in [t_kh + d_k, t_{k+1}h + d_{k+1}). \tag{51}$$

Combining Eqs. (43)–(51), the following inequality holds:

$$\begin{aligned}
&\mathbb{E}\{\mathcal{L}v(t,\bar{x}(t))\} \\
&\quad \leq 2\bar{x}^T(t)P\left[\delta_0\varphi_1\xi(t) + (1-\delta_0)\varphi_2\xi(t)\right] + \bar{x}^T(t)(Q_1+Q_2+Q_3)\bar{x}(t) \\
&\qquad + \delta_0\xi^T(t)\varphi_1^T R\varphi_1\xi(t) + (1-\delta_0)\xi^T(t)\varphi_2^T R\varphi_2\xi(t) \\
&\qquad + \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t-\tau_m) \end{bmatrix}^T \begin{bmatrix} -R_1 & * \\ R_1 & -R_1 \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t-\tau_m) \end{bmatrix} + \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t-d(t)) \\ \bar{x}(t-d_M) \end{bmatrix}^T \\
&\qquad \begin{bmatrix} -R_3 & * & * \\ R_3 & -2R_3 & * \\ 0 & R_3 & -R_3 \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ \bar{x}(t-d(t)) \\ \bar{x}(t-d_M) \end{bmatrix} \\
&\qquad -\bar{x}^T(t-\tau_m)Q_1\bar{x}(t-\tau_m) - \bar{x}^T(t-\tau_M)Q_2\bar{x}(t-\tau_M) - \bar{x}^T(t-d_M)Q_3\bar{x}(t-d_M) \\
&\qquad + \begin{bmatrix} \bar{x}(t) \\ F_1(H\bar{x}(t)) \end{bmatrix}^T \begin{bmatrix} H^T(Z_1 \otimes \Omega_{11})H & * \\ (Z_1 \otimes \Omega_{21})H & Z_1 \otimes I_n \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ F_1(H\bar{x}(t)) \end{bmatrix} \\
&\qquad + \begin{bmatrix} \bar{x}(t) \\ F_2(H\bar{x}(t)) \end{bmatrix}^T \begin{bmatrix} H^T(Z_2 \otimes \overline{\Omega}_{11})H & * \\ (Z_2 \otimes \overline{\Omega}_{21})H & Z_2 \otimes I_n \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ F_2(H\bar{x}(t)) \end{bmatrix} \\
&\qquad + 2\xi^T(t)N[\bar{x}(t-\tau_m) - \bar{x}(t-\tau(t))] + 2\xi^T(t)M[\bar{x}(t-\tau(t)) - \bar{x}(t-\tau_M)] \\
&\qquad + (\tau(t)-\tau_m)\xi^T(t)NR_2^{-1}N^T\xi(t) + (\tau_M-\tau(t))\xi^T(t)MR_2^{-1}M^T\xi(t) \\
&\qquad + \sigma\bar{x}^T(t-d(t))W\bar{x}(t-d(t)) - e_k^T(t)\Omega e_k(t).
\end{aligned} \tag{52}$$

Recalling Eqs. (37), (52), and using Lemma 4 and Schur supplement, we can conclude that:

$$\mathbb{E}\{\mathcal{L}v(t,\bar{x}(t))\} < 0, \tag{53}$$

then by Lyapunov stability theory, we can easily see that: the system (29) is asymptotically stable. This completes the proof. □

The following theorem is derived to design the parameters of the desired estimator defined in Eq. (26) by using the sufficient conditions established in Theorem 1.

Theorem 2. *For given constants $0 \leq \tau_m \leq \tau_M, d_M$, event-trigger parameter σ, and the quantitative density ρ_g, complex network systems (29) is asymptotically stable, if there exist matrices $\mathbf{P}_1 > 0, \mathbf{P}_2 > 0$, $\mathbf{Q}_1 > 0$, $\mathbf{Q}_2 > 0$, $\mathbf{Q}_3 > 0$, $\mathbf{R}_1 > 0$, $\mathbf{R}_2 > 0$, $\mathbf{R}_3 > 0$ and Y, $\mathbf{M}_k$, $\mathbf{N}_k$ $(k = 1, 2, \ldots, 9)$, Z_1, Z_2 with appropriate dimensions, such that for given $\varepsilon_i > 0$ $(i = 1, 2, 3, 4)$, the following linear matrix inequalities hold:*

$$\tilde{\Sigma}(s) = \begin{bmatrix} \tilde{\Phi}_{11} + \Gamma + \Gamma^T & * & * & * \\ \tilde{\Phi}_{21} & \tilde{\Phi}_{22} & * & * \\ \Phi_{31}(s) & 0 & -R_2 & * \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix} < 0, \quad s = 1, 2. \tag{54}$$

where:

$$\tilde{\Phi}_{11} = \begin{bmatrix} Z_1 \otimes I_n & * & * & * & * & * & * & * & * \\ 0 & Z_2 \otimes I_n & * & * & * & * & * & * & * \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & * & * & * & * & * & * \\ 0 & 0 & R_1 & -R_1 - Q_1 & * & * & * & * & * \\ 0 & 0 & B_1^T P & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & -Q_2 & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -R_3 - Q_3 & * & * \\ 0 & 0 & R_3 + \overline{C}^T P & 0 & 0 & 0 & R_3 & \sigma W - 2R_3 & * \\ 0 & 0 & \overline{D}^T P & 0 & 0 & 0 & 0 & 0 & -W_1 \end{bmatrix},$$

$$\overline{C} = \begin{bmatrix} 0 & 0 \\ -KC & 0 \end{bmatrix}, \quad \overline{D} = \begin{bmatrix} 0 \\ -K \end{bmatrix},$$

$$\tilde{\Phi}_{21} = \begin{bmatrix} \theta_1 PI_{A_1} & 0 & \theta_1 PA_1 & 0 & \theta_1 PB_1 & 0 & 0 & \theta_1 P\overline{C} & \theta_1 P\overline{D} \\ \theta_2 PI_{A_1} & 0 & \theta_2 PA_1 & 0 & \theta_2 PB_1 & 0 & 0 & \theta_2 P\overline{C} & \theta_2 P\overline{D} \\ \theta_3 PI_{A_1} & 0 & \theta_3 PA_1 & 0 & \theta_3 PB_1 & 0 & 0 & \theta_3 P\overline{C} & \theta_3 P\overline{D} \\ 0 & \theta_{10} PI_{B_1} & \theta_{10} PA_1 & 0 & \theta_{10} PB_1 & 0 & 0 & \theta_{10} P\overline{C} & \theta_{10} P\overline{D} \\ 0 & \theta_{20} PI_{B_1} & \theta_{20} PA_1 & 0 & \theta_{20} PB_1 & 0 & 0 & \theta_{20} P\overline{C} & \theta_{20} P\overline{D} \\ 0 & \theta_{30} PI_{B_1} & \theta_{30} PA_1 & 0 & \theta_{30} PB_1 & 0 & 0 & \theta_{30} P\overline{C} & \theta_{30} P\overline{D} \end{bmatrix},$$

$$\tilde{\Phi}_{22} = \text{diag}\{\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_1, \tilde{R}_2, \tilde{R}_3\}, \quad \tilde{R}_i = \varepsilon_i^2 R_i - 2\varepsilon_i P, i = 1, 2, 3,$$

$$PI_{A_1} = \begin{bmatrix} \mathbf{P}_1 I_A \\ \mathbf{P}_2 I_A \end{bmatrix}, \quad PI_{B_1} = \begin{bmatrix} \mathbf{P}_1 I_B \\ \mathbf{P}_2 I_B \end{bmatrix}, \quad PA_1 = \begin{bmatrix} \mathbf{P}_1(G \otimes \Gamma_1) & 0 \\ YC & \mathbf{P}_2(G \otimes \Gamma_1) - YC \end{bmatrix},$$

$$PB_1 = \begin{bmatrix} \mathbf{P}_1(G \otimes \Gamma_2) & 0 \\ 0 & \mathbf{P}_2(G \otimes \Gamma_2) \end{bmatrix}, \quad P\overline{C} = \begin{bmatrix} 0 & 0 \\ -YC & 0 \end{bmatrix}, \quad P\overline{D} = \begin{bmatrix} 0 \\ -Y \end{bmatrix},$$

$$\Phi_{41} = \begin{bmatrix} 0_{1\times3} & -I & 0_{1\times8} & 0 & 0 & 0 \\ 0_{1\times3} & 0 & 0_{1\times8} & -\varepsilon_4 YC & 0 & -\varepsilon_4 Y \end{bmatrix}, \quad \Phi_{42} = [\Gamma_1 \;\; \Gamma_2 \;\; \Gamma_3 \;\; \Gamma_{10} \;\; \Gamma_{20} \;\; \Gamma_{30}],$$

$$\Gamma_i = \begin{bmatrix} 0 & -\theta_i I \\ 0 & 0 \end{bmatrix}, \quad i = 1, 2, 3, \quad \Gamma_{i0} = \begin{bmatrix} 0 & -\theta_{i0} I \\ 0 & 0 \end{bmatrix}, \quad i = 1, 2, 3,$$

$$\Phi_{43} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \Phi_{44} = \text{diag}\left\{ -\varepsilon_4 I, -\frac{\varepsilon_4}{\delta_g^2} I \right\}.$$

Moreover, the parameter of the desired state estimator is given as: $K = \mathbf{P}_2^{-1} Y$.

Proof. For the convenience of derivation, define: $P = \text{diag}\{\mathbf{P}_1, \mathbf{P}_2\} > 0$, $Q_1 = \text{diag}\{\mathbf{Q}_1, \mathbf{Q}_1\} > 0$, $Q_2 = \text{diag}\{\mathbf{Q}_2, \mathbf{Q}_2\} > 0$, $Q_3 = \text{diag}\{\mathbf{Q}_3, \mathbf{Q}_3\} > 0$, $R_1 = \text{diag}\{\mathbf{R}_1, \mathbf{R}_1\} > 0$, $R_2 = \text{diag}\{\mathbf{R}_2, \mathbf{R}_2\} > 0$, $R_3 = \text{diag}\{\mathbf{R}_3, \mathbf{R}_3\} > 0$, $M_k = \text{diag}\{\mathbf{M}_k, \mathbf{M}_k\}$, $N_k = \text{diag}\{\mathbf{N}_k, \mathbf{N}_k\}$ $(k = 1, 2\ldots, 9)$. Similar to the methods in [23], pre- and post-multiplying both sides of Eq. (37) with $\mathrm{diag}\{I, PR_1^{-1}, PR_2^{-1}, PR_3^{-1}, PR_1^{-1}, PR_2^{-1}, PR_3^{-1}, I\}$ and its transpose, respectively, and combining the following inequality: $-PR_i^{-1}P \le \varepsilon_i^2 R_i - 2\varepsilon_i P,\ i=1,2,3$, we can obtain that:

$$\Sigma = \begin{bmatrix} \Phi_{11} + \Gamma + \Gamma^T & * & * \\ \hat{\Phi}_{21} & \tilde{\Phi}_{22} & * \\ \Phi_{31}(s) & 0 & -R_2 \end{bmatrix} < 0, \quad s=1,2, \tag{55}$$

where

$$\hat{\Phi}_{21} = \begin{bmatrix} \theta_1 PI_{A_1} & 0 & \theta_1 PA_1 & 0 & \theta_1 PB_1 & 0 & 0 & \theta_1 PC_1 & \theta_1 PD_1 \\ \theta_2 PI_{A_1} & 0 & \theta_2 PA_1 & 0 & \theta_2 PB_1 & 0 & 0 & \theta_2 PC_1 & \theta_2 PD_1 \\ \theta_3 PI_{A_1} & 0 & \theta_3 PA_1 & 0 & \theta_3 PB_1 & 0 & 0 & \theta_3 PC_1 & \theta_3 PD_1 \\ 0 & \theta_{10} PI_{B_1} & \theta_{10} PA_1 & 0 & \theta_{10} PB_1 & 0 & 0 & \theta_{10} PC_1 & \theta_{10} PD_1 \\ 0 & \theta_{20} PI_{B_1} & \theta_{20} PA_1 & 0 & \theta_{20} PB_1 & 0 & 0 & \theta_{20} PC_1 & \theta_{20} PD_1 \\ 0 & \theta_{30} PI_{B_1} & \theta_{30} PA_1 & 0 & \theta_{30} PB_1 & 0 & 0 & \theta_{30} PC_1 & \theta_{30} PD_1 \end{bmatrix},$$

$$PC_1 = \begin{bmatrix} 0 & 0 \\ -\mathbf{P}_2 K(I+\Delta_g)C & 0 \end{bmatrix}, \quad PD_1 = \begin{bmatrix} 0 \\ -\mathbf{P}_2 K(I+\Delta_g) \end{bmatrix},$$

and Eq. (55) can be rewritten as following:

$$\Sigma = \Sigma_{11} + \mathcal{L}_I \mathbf{P}_2 K \mathcal{L}_g + \mathcal{L}_g^T K^T \mathbf{P}_2 \mathcal{L}_I^T, \tag{56}$$

where

$$\Sigma_{11} = \begin{bmatrix} \tilde{\Phi}_{11} + \Gamma + \Gamma^T & * & * \\ \tilde{\Phi}_{21} & \tilde{\Phi}_{22} & * \\ \Phi_{31}(s) & 0 & -R_2 \end{bmatrix}, \quad s=1,2,$$

$$\mathcal{L}_g = [0_{1\times 12}\ \ \Delta_g C\ \ 0\ \ \Delta_g\ \ 0_{1\times 14}], \quad \mathcal{L}_Y = [0_{1\times 12}\ \ -YC\ \ 0\ \ -Y\ \ 0_{1\times 14}],$$
$$\mathcal{L}_I = [0_{1\times 3}\ \ -I\ \ 0_{1\times 12}\ \ -\theta_1 I\ \ 0\ \ -\theta_2 I\ \ 0\ \ -\theta_3 I\ \ 0\ \ -\theta_{10} I\ \ 0\ \ -\theta_{20} I\ \ 0\ \ -\theta_{30} I\ \ 0_{1\times 2}]^T.$$

Applying Lemma 5, there exists $\varepsilon_4 > 0$, such that:

$$\Sigma \le \Sigma_{11} + \varepsilon_4^{-1} \mathcal{L}_I \mathcal{L}_I^T + \varepsilon_4 \mathcal{L}_Y^T \Delta_g^2 \mathcal{L}_Y, \tag{57}$$

notice that:

$$\Delta_g^2 \le \delta_g^2 I. \tag{58}$$

By using Schur supplement, according to Eqs. (37) and (57), we can obtain Eq. (54). Defining $Y = \mathbf{P}_2 K$, thus the parameter of the desired state estimator is given as $K = \mathbf{P}_2^{-1} Y$. This completes the proof. □

Remark 3. From Theorem 2, for given σ and $\varepsilon_i, i=1,2,3$, by solving the linear matrix inequality (54), we can obtain the state estimator K and the event-triggered matrix Ω. Meanwhile, from Theorem 2, it should be noted that the state estimator K is not only related to the event-triggered matrix Ω, but also related to the density of quantization ρ_g.

4. Numerical results

Consider the following continuous complex network systems which consist of 5 coupled nodes, and the dynamical function of every node can be described by the following model:

$$\dot{x}_i(t) = \delta(t)Af_1(x_i(t)) + (1-\delta(t))Bf_2(x_i(t)) + \sum_{j=1}^{N} g_{ij}\Gamma_1 x_j(t) + \sum_{j=1}^{N} g_{ij}\Gamma_2 x_j(t-\tau(t)), \quad (i=1,2,3,4,5), \tag{59}$$

where

$$x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix}, \quad A = \begin{bmatrix} -0.1 & 0.2 \\ 0.1 & -0.3 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2 & 0.25 \\ 0.25 & -0.3 \end{bmatrix}.$$

The external coupling configuration matrix G and the inner-coupling matrix Γ_1, Γ_2 are given by:

$$G = \begin{bmatrix} -17 & 0.01 & 0 & 0 & 0.01 \\ 0.01 & -15 & 0 & 0 & 0 \\ 0.01 & 0.02 & -16 & 0 & 0.02 \\ 0.02 & 0.01 & 0 & -16 & 0.01 \\ 0 & 0 & 0.01 & 0.01 & -14 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Gamma_2 = 0.1\Gamma_1.$$

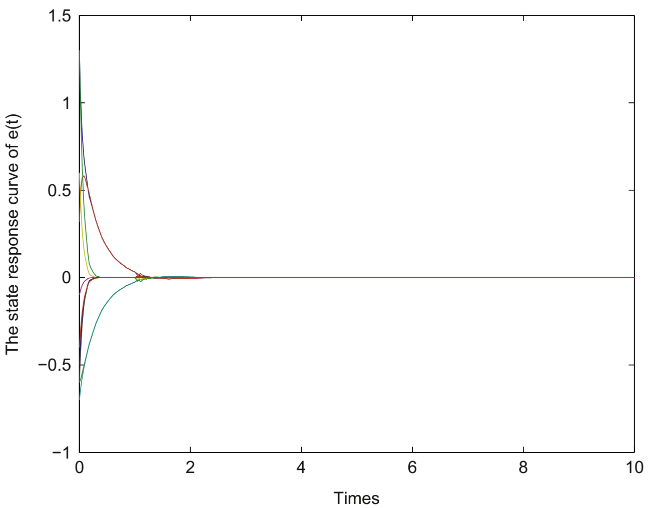

Fig. 2. The state response curve of e(t).

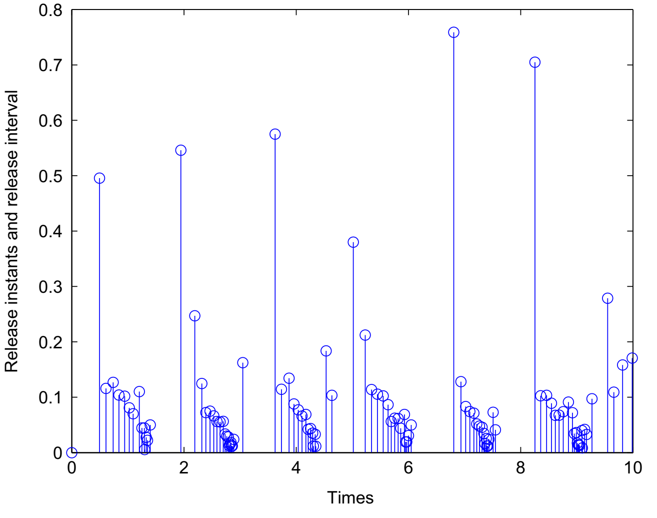

Fig. 3. The release instants and release intervals.

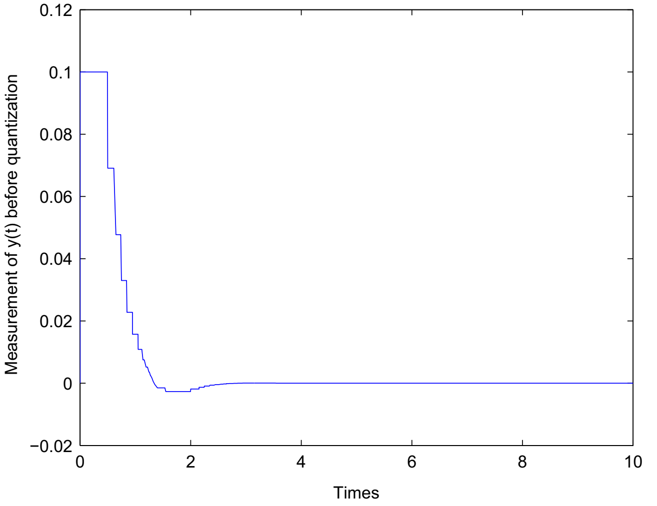

Fig. 4. Measurement of $y(t)$ before quantization.

The dynamical behavior of networks nodes can be described as:

$$f_1(x_i(t)) = \begin{bmatrix} 0.4x_{i1}(t) - \tanh(0.3x_{i2}(t)) + 0.2x_{i2}(t-\tau(t)) \\ 0.9x_{i2}(t) - \tanh(0.7x_{i2}(t)) \end{bmatrix},$$

$$f_2(x_i(t)) = \begin{bmatrix} 0.3x_{i1}(t) - \tanh(0.2x_{i1}(t)) + 0.1x_{i2}(t-\tau(t)) \\ 0.8x_{i2}(t) - \tanh(0.6x_{i2}(t)) \end{bmatrix},$$

suppose the measurement output matrix C is: $C = [0.2\ -0.5\ 0.2\ 0\ 0.2\ -0.6\ 0.2\ 0\ -0.7\ 0.2]$,

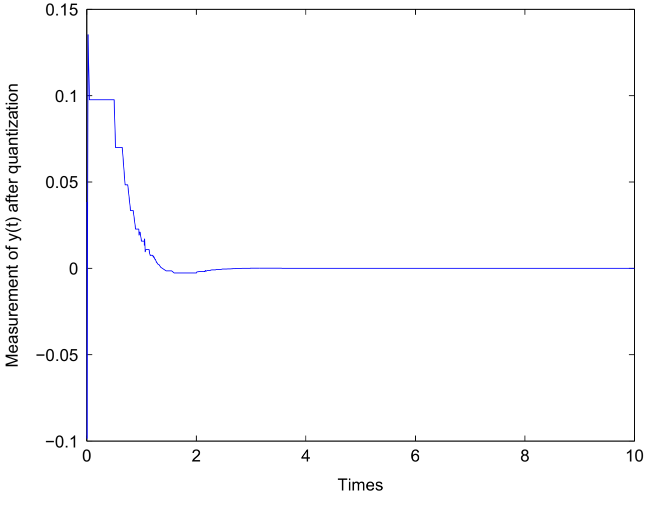

Fig. 5. Measurement of $y(t)$ after quantization.

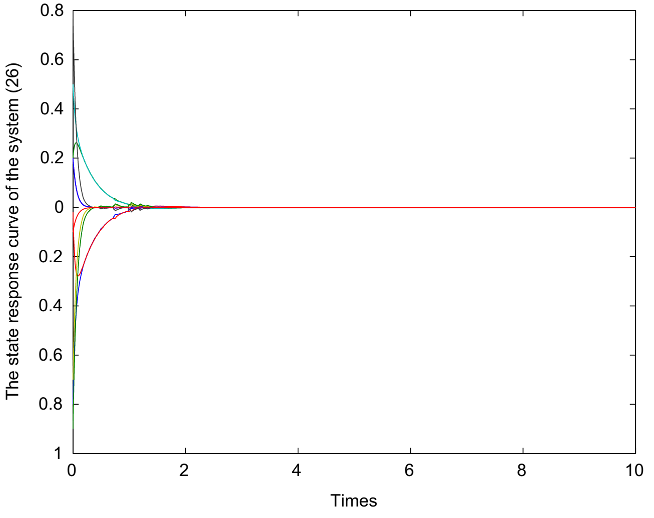

Fig. 6. The state response curve of the system (26).

the initial condition of the system is: $x_0 = [0.4, -0.3, 0.5, -0.1, 0.2, -0.2, 0.1, -0.5, 0.3, -0.4]^T$,

Suppose the random switching probability of networks nodes is $\delta_0 = 0.6$, the lower bound of time-varying delays is $\tau_m = 0$, the upper bound is $\tau_M = 0.2$, the constant d_M is d_M=0.03, the event-triggered parameter is $\sigma = 0.2$, sampling period is h=0.05. In fact, for $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$, by applying Theorem 2, using LMI tool box to solve the inequality (54), we can get the event-triggered matrix is $\Omega = 51.6877$, the estimator matrix is:

$$K = [-0.0114, 0.0286, -0.0040, -0.0001, -0.0073, 0.0215, -0.0074, -0.0000, 0.0083, -0.0025]^T,$$

$$Y = [-0.8494, 2.1313, -0.3194, -0.0103, -0.5630, 1.6663, -0.5737, -0.0002, 0.6938, -0.2128]^T.$$

Simulation results are shown in Figs. 2–6. From Fig. 2 we can see that the error system can achieve asymptotically stability. Fig. 3 shows the release instants and release intervals. From Fig. 3, we can see that the event-triggered communication scheme can save network bandwidth and reduce the energy consumption of state estimator. By comparing Fig. 4 with Fig. 5, we can see the effect of quantization for complex network systems. The advantage of quantization is that it requires low communication rate, and can reduce the risk of data loss for complex network systems. Fig. 6 is the state response curve of the system (26).

5. Conclusion

This paper investigates the event-triggered state estimation problem for a class of complex network systems with quantization. In order to save network bandwidth, reduce the pressure of data transmission and the communication load, this paper introduces the event generator and logarithmic quantizer in the process of state estimator design for complex network systems. The novel asymptotic stability conditions are derived for complex network systems by using Lyapunov stability theory, linear matrix inequality techniques and free-weighting matrix method. Furthermore, based on the stability conditions, the flexible approach of the desired state estimator is derived. Finally, a numerical example verifies the usefulness of the proposed theoretical results.

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