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Event-triggered H_∞ filter design for delayed neural network with quantization

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ABSTRACT

This paper is concerned with H_∞ filter design for a class of neural network systems with event-triggered communication scheme and quantization. Firstly, a new event-triggered communication scheme is introduced to determine whether or not the current sampled sensor data should be broadcasted and transmitted to quantizer, which can save the limited communication resource. Secondly, a logarithmic quantizer is used to quantify the sampled data, which can reduce the data transmission rate in the network. Thirdly, considering the influence of the constrained network resource, we investigate the problem of H_∞ filter design for a class of event-triggered neural network systems with quantization. By using Lyapunov functional and linear matrix inequality (LMI) techniques, some delay-dependent stability conditions for the existence of the desired filter are obtained. Furthermore, the explicit expression is given for the designed filter parameters in terms of LMIs. Finally, a numerical example is given to show the usefulness of the obtained theoretical results.

1. Introduction

Nowadays, neural networks have been more and more prevalent due to their extensive application in image processing, associative memory, and optimization problems. Recently, many important results have been reported on neural networks, see, e.g., Gong, Liang, and Cao (2015), He, Ji, Zhang, and Wu (2016), Luo, Wang, Wei, Alsaadi, and Hayat (2016), Ma, Sun, Liu, and Xing (2016), and Yang, Li, and Huang (2016). The analysis problems of exponential stability for the delayed recurrent neural networks have stirred a great deal of research interests. Furthermore, the filtering problems for neural network systems have been widely investigated by many researchers via various methodologies (Huang, Huang, & Chen, 2013; Mathiyalagan, Anbuvithya, Sakthivel, Park, & Prakash, 2016). So the studies of the stability and filtering of delayed neural networks have significant theoretic meaning and application value. In recent years, several methods have been proposed to solve the H_∞ filter design problem (Cao, Sun, & Lam, 1998; Huang & Feng, 2009; Liu, Fei, Tian, & Gu, 2015; Wang & Ho, 2003; Wang, Shi, & Zhang, 2015). The authors in Liu et al. (2015) investigate the reliable H_∞ filter design for a class of $T-S$ fuzzy systems with stochastic sensor faults under an event triggered scheme. In the literature (Wang & Ho, 2003), the problem of H_∞ filtering of nonlinear stochastic systems is also considered. As the basic problem in the area of network, H_∞ filter problem has received researchers' attention for a long time, but the study on neural network only has a short history, many problems should be studied widely and deeply. Therefore, it is essential to pay attention to filter design in the various aspects of the neural network.

As an alternative of the time-triggered control scheme, event triggered scheme is utilized as an efficient way to reduce the burden of communication networks and improve the transmission efficiency. Compared with the time-triggered control scheme, the advantage of the event triggered scheme is that it can facilitate the efficient usage of the shared communication resources, and whether the current sampled information will be transmitted or not depends on pre-designed conditions, avoiding much of the unnecessary transmission. Up to now, event triggered scheme has received a lot of research interest and some important results have been published (Hu & Yue, 2012a; Li et al., 2016; Liu et al., 2015; Liu & Yue, 2013b; Yue, Tian, & Han, 2013). To name a few results, the authors in Yue et al. (2013) proposed a novel event-triggering scheme and event-triggered H_∞ controller design for

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networked control systems which are investigated. Based on the results of Yue et al. (2013), the authors in Liu and Yue (2013b) propose an event-triggering sampling strategy with probabilistic sensor and actuator fault and investigate the reliable control design for networked control system under the proposed event-triggered scheme. In Hu and Yue (2012a), the authors are concerned with the problem of event-based H_∞ filtering for networked systems with communication delay. In Liu et al. (2015), the authors investigate reliable H_∞ filter design for a class of $T - S$ fuzzy systems with stochastic sensor faults under an event triggered scheme. The authors in Li et al. (2016) consider the event-triggered distributed average-consensus of discrete-time first-order multi agent systems with limited communication data rate and general directed network topology. Motivated by the above references, it is necessary to design an event-triggered communication scheme to save the limited communication resources in the delayed neural network system. This is one of the motivations of this work.

At present, the quantitative processing has been paid attention by more and more researchers. Considering the limited communication capacity in the networks, quantization of measurement and/or input signals is an indispensable step which aims at saving limited bandwidth and energy consumption. It can be considered as the process of encoding, which is realized by the quantizer. Quantization plays an important role in information exchange among agents. In the literature (Hu & Yue, 2012b; Li, Chang, Du, & Yu, 2016; Li, Chen, Liao, & Huang, 2016), a series of quantitative methods are proposed in time-varying quantizer or logarithmic quantizer. In Li, Chang et al. (2016), the authors introduce H_∞ control of discrete-time for uncertain linear systems with quantized feedback. The authors in Hu and Yue (2012b) discuss the event-triggered control design of linear networked systems with quantization. In the literature (Li, Chen et al., 2016), quantized data-based leader-following consensus of general discrete-time multi-agent systems is described. The effect of the quantization on the networked control systems is much larger than the traditional control systems. To the best of our knowledge, event-triggered scheme for a class of neural network systems with quantization has not been well addressed. This situation motivates our current investigation.

Motivated by the observations above, we focus on the event-based H_∞ filter design problem for a class of delayed neural networks with quantization. To reduce the computation load or to reduce the exchange of information, we introduce an event-triggering sampling mechanism. Then, an event-based filter design model for neural network systems is constructed by taking the effect of event-triggered scheme and the quantization into consideration. Besides, sufficient conditions for the existence of the filter are established and the explicit expression is given for the designed filter parameters. Finally, a numerical example is given to show the effectiveness.

The paper is organized as follows. In Section 2, an H_∞ filter design is addressed for the delayed neural network systems with event triggered communication scheme and quantization. Sufficient conditions for the existence of the desired filter are established and a filter design method is provided in Section 3. Moreover, we derive the explicit solution of filter parameters. A numerical example is given in Section 4 to show the effectiveness and applicability of the proposed method. The conclusion is drawn in the final part.

Notation: In this paper, $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$, respectively, denote the n-dimensional Euclidean space and the set of $m \times n$ real matrices. Matrix $X > 0$ (respectively, $X \geq 0$) denotes that X is a real symmetric positive definite (positive semi-definite). In a symmetry matrix $*$ is used to describe the symmetric terms. I is the identity matrix of appropriate dimension. In addition, T stands for the transpose of matrix.

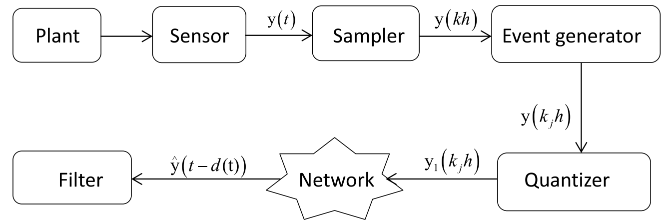

Fig. 1. The structure of event-triggered H_∞ filter design for delayed neural network with quantization.

2. Problem formulation and preliminaries

As is shown in Fig. 1, consider a delayed neural network with n neurons:

$$\begin{cases} \dot{x}(t) = -Ax(t) + W_0 g(x(t)) + W_1 g(x(t-\tau(t))) + A_w \omega(t) \\ y(t) = Cx(t) \\ z(t) = Lx(t) \end{cases} \tag{1}$$

where $x(t) = [x_1, x_2, \ldots, x_n]^T \epsilon \mathbb{R}^n$ is the state vector of the neural network; $A = \text{diag}\{a_1, a_2, \ldots, a_n\}$ is a diagonal matrix with positive entries $a_i > 0$; W_0 and W_1 are the connection weight matrix and the delayed connection weight matrix, respectively; $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$ denotes the neuron activation function; and $\tau(t)$ denotes the time-varying bounded state delay satisfying $\tau(t)\epsilon[\tau_m, \tau_M]$, where τ_m and τ_M are the lower and upper bounds of $\tau(t)$; $\omega(t)\epsilon\mathbb{R}^p$ is the external disturbance and $\omega(t)\epsilon L_2[0, \infty)$; A_w, C, L are the parameter matrices with appropriate dimensions; $y(t) = [y_1, y_2, \ldots, y_r]^T \epsilon \mathbb{R}^r$ is the measurement output; $z(t) = [z_1, z_2, \ldots, z_p]^T \epsilon \mathbb{R}^p$ is the objective vector.

Event generator is introduced between the sensor and the quantizer which is used to determine whether the newly sampled state will be sent out to the quantizer by using the following judgement algorithm, which is the same as Yue et al. (2013)

$$\begin{aligned} &[y((k+j)h) - y(kh)]^T \Phi [y((k+j)h) - y(kh)] \\ &\quad \leq \sigma y^T((k+j)h)\Phi y((k+j)h) \end{aligned} \tag{2}$$

where the Φ is a symmetric positive define matrix, $j = 1, 2, \ldots$, $\sigma\epsilon[0, 1)$, $y((k+j)h)$ is the current sampled sensor measurements, and $y(kh)$ is the latest transmitted sensor measurements. The sampled state $y((k+j)h)$ satisfying the inequality (2) will not be transmitted, only the one that exceeds the threshold in (2) will be sent to the quantizer, which means that, in the sensor side, only some of the sampled states that violate (2) will be sent out to the quantizer side.

Remark 1. From the event-triggered algorithm (2), we can easily see that the sensor measurements are sampled at time kh by sampler with a given period h, the next sensor measurement is at time $(k+1)h$. Suppose that the release times are $k_0h, k_1h, k_2h, \ldots$, it is easily seen that $t_ih = k_{i+1}h - k_ih$ denotes the release period of event generator in (2), t_ih means that the sampling between the two conjoint transmitted instant.

Remark 2. It is easily seen from event-triggered algorithm (2) that the set of the release instants $\{k_0h, k_1h, k_2h, \ldots\} \subseteq \{0, 1, 2, \ldots\}$. The amount of $\{k_0h, k_1h, k_2h, \ldots\}$ depends on the value of σ and the variation of the sensor measurements.

When the new sampled states $y(k_jh)$ arriving at the quantizer, we determine to quantize $y(k_jh)$. Define a function (Qu, Guan, He, & Chi, 2015)

$$q(y) = \text{diag}\{q_1, q_2, \ldots, q_m\} \tag{3}$$

where $q_i(\cdot)$ is symmetric, i.e. $q_i(-y_i) = -q_i(y_i)$, and the logarithmic quantizer can be described by the following sector mode:

$$q_i(y_i) = (1 + \Delta_{q_i}(y_i))y_i. \tag{4}$$

Define

$$\Delta_q = \mathrm{diag}\{\Delta_{q_1}\ \Delta_{q_2}\ \dots\ \Delta_{q_m}\} \tag{5}$$

and we have

$$q(y) = (I + \Delta_q)y. \tag{6}$$

Then

$$q(y(k_jh)) = (I + \Delta_q)y(k_jh). \tag{7}$$

Then the new sampled states $y(k_jh)$ via the quantizer can be described by the following equation:

$$y_1(k_jh) = q(y(k_jh)) = (I + \Delta_q)y(k_jh). \tag{8}$$

If $\Delta_q = 0$, then $q(y) = y$, i.e. no quantization.

In order to transform the system into a time delay system and use the time delay system to deal with the problem of our research, for technical convenience, similar to Liu and Yue (2013a), Liu and Yue (2013b), Yue et al. (2013) and Wang et al. (2015), we consider the following two cases:

Cases A: If $k_jh + h + \bar{d} \geq k_{j+1}h + d_{j+1}$, where $\bar{d} = \max d_j$, we define a function $d(t)$ as:

$$d(t) = t - k_jh, \quad t\epsilon[k_jh + d_j, k_{j+1}h + d_{j+1}). \tag{9}$$

It can easily be obtained that $d_j \leq d(t) \leq (k_{j+1}-k_j)h+d_{j+1} \leq h+\bar{d}$.

Cases B: If $k_jh+h+\bar{d} < k_{j+1}h+d_{j+1}$, we consider the following two intervals:

$$[k_jh + d_j, k_jh + h + \bar{d}), \quad [k_jh + ih + \bar{d}, k_jh + ih + h + \bar{d}).$$

Since $d_j \leq \bar{d}$, it can be shown that there exists a positive integer $m \geq 1$ such that

$$k_jh + mh + \bar{d} < k_{j+1}h + d_{j+1} \leq k_jh + mh + \bar{d}.$$

Besides, $y(k_jh)$ and $k_jh + ih$ with $i = 1, 2, \dots, m$ satisfy (2). Let:

$$\begin{cases} I_1 = [k_jh + d_j, k_jh + h + \bar{d}) \\ I_2 = \bigcup_{i=0}^{m-1}[k_jh + ih + \bar{d}, k_jh + ih + h + \bar{d}) \\ I_3 = [k_jh + mh + \bar{d}, k_{j+1}h + d_{j+1}). \end{cases} \tag{10}$$

Define a function:

$$d(t) = \begin{cases} t - k_jh, & t\epsilon I_1 \\ t - k_jh - ih, & t\epsilon I_2^{(i)} \quad (i = 1, 2, \dots, m-1) \\ t - k_jh - mh, & t\epsilon I_3. \end{cases} \tag{11}$$

In conclusion, by the definition of $d(t)$, we can get:

$$\begin{cases} 0 \leq d_j \leq d(t) < h + \bar{d}, & t\epsilon I_1 \\ 0 \leq d_j \leq \bar{d} \leq d(t) < h + \bar{d}, & t\epsilon I_2^{(i)} \quad (i = 1, 2, \dots, m-1) \\ 0 \leq d_j \leq \bar{d} \leq d(t) < h + \bar{d}, & t\epsilon I_3 \end{cases} \tag{12}$$

where the third row in (10) holds since $k_{j+1}h + d_{j+1} \leq k_j + (d_M + 1)h + \bar{d}$. Obviously,

$$0 \leq d_j \leq d(t) \leq h + \bar{d} \triangleq d_M, \quad t\epsilon[k_jh + d_j, k_{j+1}h + d_{j+1}). \tag{13}$$

In the Case A, for $t\epsilon[k_jh+d_j, k_{j+1}h+d_{j+1})$, we define an error vector $e_k(t) = 0$.

In Case B, define the sensor measurement error between the current sampling instant and the latest transmission instant:

$$e_k(t) = \begin{cases} 0, & t\epsilon I_1 \\ y(k_jh + ih) - y(k_jh), & t\epsilon I_2^{(i)} \quad (i = 1, 2, \dots, m-1) \\ y(k_jh + d_Mh) - y(k_jh), & t\epsilon I_3. \end{cases} \tag{14}$$

Then after quantization, from the definition of $e_k(t)$ and the triggering algorithm (2), it can be easily seen that for $t\epsilon[k_jh + d_j, k_{j+1}h + d_{j+1})$

$$e_k{}^T(t)\Phi e_k(t) \leq \sigma y^T(t - d(t))\Phi y(t - d(t)). \tag{15}$$

Remark 3. Notice that the relation of $k_jh+h+\bar{d} \geq k_{j+1}h+d_{j+1}$ in Case A means the newly sampled sensor measurement $y(k_jh + h)$ will be transmitted and arrive at the quantizer side at the instant $k_jh + h + d_{j+1}$; $k_jh + h + \bar{d} < k_{j+1}h + d_{j+1}$ in Case B means the newly sampled sensor measurement $y(k_jh + h)$ and the latest sensor measurement $y(k_jh)$ variate the judgement algorithm (2), and $y(k_jh + h)$ will not be transmitted to the quantizer side.

In the following, we select the filter for the estimation of $z(t)$ as follows:

$$\begin{cases} \dot{x}_f(t) = A_fx_f(t) + B_f\hat{y}(t) \\ z_f(t) = C_fx_f(t) \end{cases} \tag{16}$$

where $x_f(t)\epsilon\mathbb{R}^n$ is the state estimation of the filter; $z_f(t)\epsilon\mathbb{R}^p$ is the output of the filter representing an estimation of $z(t)$; $A_f\epsilon\mathbb{R}^{n\times n}$, $B_f\epsilon\mathbb{R}^{n\times m}$, $C_f\epsilon\mathbb{R}^{p\times n}$ are the filter parameter matrices to be determined; $\hat{y}(t)$ is the input of the filter, and based on the sampling technique, the actual output can be described as

$$\hat{y}(t) = (I + \Delta_q)[Cx(t - d(t)) - e_k(t)]. \tag{17}$$

Then after quantization and network, combining (16) and (17), the H_∞ filter system can be rewritten as:

$$\begin{cases} \dot{x}_f(t) = A_fx_f(t) + B_f(I + \Delta_q)[Cx(t - d(t)) - e_k(t)] \\ z_f(t) = C_fx_f(t). \end{cases} \tag{18}$$

By setting $\bar{x}(t) = [x^T(t)\ x_f^T(t)]^T$, $\bar{z}(t) = z(t) - z_f(t)$, the following augmented system can be obtained from (1) and (18):

$$\begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{W}_0\bar{g}(H\bar{x}(t)) + \bar{W}_1\bar{g}(H\bar{x}(t - \tau(t))) \\ \quad +\bar{A}_w\omega(t) + \bar{B}H\bar{x}(t - d(t)) + \bar{B}_1e_k(t) \\ \bar{z}(t) = \bar{L}\bar{x}(t) \end{cases} \tag{19}$$

where

$$\bar{A} = \begin{bmatrix} -A & 0 \\ 0 & A_f \end{bmatrix}, \quad \bar{W}_0 = \begin{bmatrix} W_0 \\ 0 \end{bmatrix},$$

$$\bar{W}_1 = \begin{bmatrix} W_1 \\ 0 \end{bmatrix}, \quad \bar{A}_w = \begin{bmatrix} A_w \\ 0 \end{bmatrix}, \quad H^T = \begin{bmatrix} I \\ 0 \end{bmatrix},$$

$$\bar{B} = \begin{bmatrix} 0 \\ B_f(I + \Delta_q)C \end{bmatrix}, \quad \bar{B}_1 = \begin{bmatrix} 0 \\ -B_f(I + \Delta_q) \end{bmatrix},$$

$$\bar{L} = \begin{bmatrix} L & -C_f \end{bmatrix}.$$

In the following, we introduce a definition and some lemmas, which will help us in deriving the main results.

Definition 1 (*Mao, 1996*). The system is exponentially stable, if there exist two constants, $u > 0$, $k > 0$, satisfying:

$$\|\bar{x}(t)\|^2 \leq ue^{-kt} \sup_{-r\leq\theta\leq 0} \|\phi(\theta)\|^2 \tag{20}$$

where $\phi(\cdot)$ is initial state of the system, such as $\phi(t) = \bar{x}(t)$, $t \in [-r, 0]$.

Assumption 1 (*Li, Hu, Hu, & Li, 2012*). The neuron activation function satisfies one of the following conditions, and U_1, U_2 are real constant matrices and satisfy $U_2 - U_1 \geq 0$:

$$[g(x) - U_1x]^T[g(x) - U_2x] \leq 0. \tag{21}$$

Lemma 1 (*Gu, Chen, & Kharitonov, 2003*). *For the given instant τ_1 and matrix $R > 0$, the following inequalities are established:*

$$-\tau_1\int_{t-\tau_1}^{t} \dot{x}^T(s)R\dot{x}(s) \leq \begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}. \tag{22}$$

Lemma 2 (*Wang, Xie, & de Souza, 1992*). (1) *For any vector $x, y \in \mathbb{R}^n$ and matrices $Q \in \mathbb{R}^{n\times n}$ with appropriate dimensions, the following*

$$\Gamma = \begin{bmatrix} \Sigma_{11} & * & * & * & * \\ \Sigma_{21} & \Sigma_{22} & * & * & * \\ \Sigma_{31} & \Sigma_{32} & \Sigma_{33} & * & * \\ \Sigma_{41}^{(p)} & 0 & 0 & -R_1 & * \\ \Sigma_{51}^{(q)} & 0 & 0 & 0 & -R_3 \end{bmatrix} < 0 \quad (p = 1, 2; q = 1, 2) \tag{27}$$

where

$$\Sigma_{11} = \begin{bmatrix} \Phi_1 & * & * & * & * & * & * & * \\ R_2 & \Phi_2 & * & * & * & * & * & * \\ 0 & M_3 - M_2^T & \Phi_3 & * & * & * & * & * \\ 0 & 0 & N_4 - N_3^T & \Phi_4 & * & * & * & * \\ H^T\bar{B}^T P + S_5 - S_1^T & 0 & 0 & 0 & \Phi_5 & * & * & * \\ 0 & 0 & 0 & 0 & Z_6 - Z_5^T & \Phi_6 & * & * \\ \bar{W}_0^T P - \alpha \bar{U}_2^T & 0 & 0 & 0 & 0 & 0 & -\alpha I & * \\ \bar{W}_1^T P & 0 & -\beta \bar{U}_2 & 0 & 0 & 0 & 0 & -\beta I \end{bmatrix},$$

$$\Phi_1 = P\bar{A} + \bar{A}^T P + Q_1 + Q_2 + Q_3 - R_2 + S_1 + S_1^T - \alpha \bar{U}_1, \quad \Phi_2 = -Q_1 + M_2 + M_2^T - R_2,$$

$$\Phi_3 = -M_3 - M_3^T + N_3 + N_3^T - \beta \bar{U}_1, \quad \Phi_4 = -Q_2 - N_4 - N_4^T, \quad \Sigma_{22} = \mathrm{diag}\{-\Phi, -\gamma^2 I, -I\},$$

$$\Phi_5 = -S_5 - S_5^T + Z_5 + Z_5^T + \Psi, \quad \Phi_6 = -Q_3 - Z_6 - Z_6^T, \quad \Psi = \begin{bmatrix} \sigma C^T \Phi C & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Sigma_{31} = \begin{bmatrix} c_{21}R_1\bar{A} & 0 & 0 & 0 & c_{21}R_1\bar{B}H & 0 & c_{21}R_1\bar{W}_0 & c_{21}R_1\bar{W}_1 \\ c_1R_2\bar{A} & 0 & 0 & 0 & c_1R_2\bar{B}H & 0 & c_1R_2\bar{W}_0 & c_1R_2\bar{W}_1 \\ d_1R_3\bar{A} & 0 & 0 & 0 & d_1R_3\bar{B}H & 0 & d_1R_3\bar{W}_0 & d_1R_3\bar{W}_1 \end{bmatrix},$$

$$\Sigma_{21} = \begin{bmatrix} \bar{B}^T P & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{A}_w^T P & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Sigma_{32} = \begin{bmatrix} c_{21}R_1\bar{B}_1 & c_{21}R_1\bar{A}_w & 0 \\ c_1R_2\bar{B}_1 & c_1R_2\bar{A}_w & 0 \\ d_1R_3\bar{B}_1 & d_1R_3\bar{A}_w & 0 \end{bmatrix},$$

$$\Sigma_{41}^{(1)} = \begin{bmatrix} 0 & c_{21}M_2^T & c_{21}M_3^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Sigma_{41}^{(2)} = \begin{bmatrix} 0 & 0 & c_{21}N_3^T & c_{21}N_4^T & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Sigma_{51}^{(1)} = \begin{bmatrix} d_1S_1^T & 0 & 0 & 0 & d_1S_5^T & 0 & 0 & 0 \end{bmatrix}, \quad \Sigma_{51}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_1Z_5^T & d_1Z_6^T & 0 & 0 \end{bmatrix},$$

$$\Sigma_{33} = \mathrm{diag}\{-R_1, -R_2, -R_3\}, \quad c_{21} = \sqrt{\tau_M - \tau_m}, \quad c_1 = \tau_m, \quad d_1 = \sqrt{d_M}.$$

Box I.

inequality is established:

$$2x^T y \le x^T Q x + y^T Q^{-1} y \tag{23}$$

(2) D, E, F *are matrices with appropriate dimensions and satisfy* $\|F\| \le 1$, *the following inequality is established:*
For any variable $\varepsilon > 0$,

$$DFE + E^T F^T D^T \le \varepsilon^{-1} DD^T + \varepsilon E^T E. \tag{24}$$

Lemma 3 (Tian, Yue, & Zhang, 2009)**.** *Suppose* $\tau(t)\epsilon[\tau_m, \tau_M]$, $d(t)\epsilon[0, d_M]$, $Q_i (i = 1, 2, 3, 4, 5)$ *are matrices with appropriate dimensions, the inequality* $Q_1 + (\tau_M - \tau(t))Q_2 + (\tau(t) - \tau_m)Q_3 + (d_M - d(t))Q_4 + d(t)Q_5 < 0$ *is established, if and only if the following inequalities are established:*

$$\begin{cases} Q_1 + (\tau_M - \tau_m)Q_2 + d_M Q_4 < 0 \\ Q_1 + (\tau_M - \tau_m)Q_3 + d_M Q_4 < 0 \\ Q_1 + (\tau_M - \tau_m)Q_2 + d_M Q_5 < 0 \\ Q_1 + (\tau_M - \tau_m)Q_3 + d_M Q_5 < 0. \end{cases} \tag{25}$$

Lemma 4 (Xiong & Lam, 2009)**.** *For matrix* $R > 0$, X *and any real number* η, *then we have:*

$$-XR^{-1}X \le \eta^2 R - 2\eta X. \tag{26}$$

3. Main results

In Theorem 1, assuming A, C, W_0, W_1, A_w and filter gain A_f, B_f, C_f are known, and using the method of Lyapunov function and linear matrix inequality technique, considering the event-triggered generator and quantizer, we analysis the stability of the system (19).

Theorem 1. *For given parameters* τ_m, τ_M, d_M *and* σ, *the system* (19) *is exponentially stable with an* H_∞ *disturbance attenuation level* γ *under the event trigger scheme* (2) *and the action of quantizer* (3), *if there exist matrices* $P > 0$, $Q_i > 0$, $R_i > 0 (i = 1, 2, 3)$, $\Phi > 0$, $M_j (j = 2, 3)$, $N_k (k = 3, 4)$, $S_l (l = 1, 5)$, $Z_s (s = 5, 6)$ *with appropriate dimensions and parameters* $\alpha > 0$, $\beta > 0$, *satisfying Eq.* (27) *given in* Box I.

Proof. Choose the following Lyapunov functional as:

$$V(x_t) = V_1(x_t) + V_2(x_t) + V_3(x_t) \tag{28}$$

where

$$V_1(x_t) = \bar{x}^T(t) P \bar{x}(t)$$

$$V_2(x_t) = \int_{t-\tau_m}^{t} \bar{x}^T(s) Q_1 \bar{x}(s) ds + \int_{t-\tau_M}^{t} \bar{x}^T(s) Q_2 \bar{x}(s) ds + \int_{t-d_M}^{t} \bar{x}^T(s) Q_3 \bar{x}(s) ds$$

$$V_3(x_t) = \int_{t-\tau_M}^{t-\tau_m} \int_{s}^{t} \dot{\bar{x}}^T(v) R_1 \dot{\bar{x}}(v) dv ds + \tau_m \int_{t-\tau_m}^{t} \int_{s}^{t} \dot{\bar{x}}^T(v) R_2 \dot{\bar{x}}(v) dv ds + \int_{t-d_M}^{t} \int_{s}^{t} \dot{\bar{x}}^T(v) R_3 \dot{\bar{x}}(v) dv ds.$$

Taking the time derivative of $V_1(x_t), V_2(x_t), V_3(x_t)$ with respect to t, we can obtain:

$$\begin{aligned}\dot{V}(x_t) &= \dot{V}_1(x_t) + \dot{V}_2(x_t) + \dot{V}_3(x_t) \\ &= 2\bar{x}^T(t)P\dot{\bar{x}}(t) + \bar{x}^T(t)(Q_1+Q_2+Q_3)\bar{x}(t) \\ &\quad - \bar{x}^T(t-\tau_m)Q_1\bar{x}(t-\tau_m) - \bar{x}^T(t-\tau_M)Q_2\bar{x}(t-\tau_M) \\ &\quad - \bar{x}^T(t-d_M)Q_3\bar{x}(t-d_M) + (\tau_M-\tau_m)\dot{\bar{x}}^T(t)R_1\dot{\bar{x}}(t) \\ &\quad - \int_{t-\tau_M}^{t-\tau_m}\dot{\bar{x}}^T(s)R_1\dot{\bar{x}}(s)ds + \tau_m^2\dot{\bar{x}}^T(t)R_2\dot{\bar{x}}(t) \\ &\quad - \tau_m\int_{t-\tau_m}^{t}\dot{\bar{x}}^T(s)R_2\dot{\bar{x}}(s)ds + d_M\dot{\bar{x}}^T(t)R_3\dot{\bar{x}}(t) \\ &\quad - \int_{t-d_m}^{t}\dot{\bar{x}}^T(s)R_3\dot{\bar{x}}(s)ds. \end{aligned} \quad (29)$$

Applying the free-weighting matrices method, it is easily derived that:

$$\begin{cases} 2\xi^T(t)M\left[\bar{x}(t-\tau_m) - \bar{x}(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_m}\dot{\bar{x}}(s)ds\right] = 0 \\ 2\xi^T(t)N\left[\bar{x}(t-\tau(t)) - \bar{x}(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot{\bar{x}}(s)ds\right] = 0 \\ 2\xi^T(t)S\left[\bar{x}(t) - \bar{x}(t-d(t)) - \int_{t-d(t)}^{t}\dot{\bar{x}}(s)ds\right] = 0 \\ 2\xi^T(t)Z\left[\bar{x}(t-d(t)) - \bar{x}(t-d_M) - \int_{t-d_M}^{t-d(t)}\dot{\bar{x}}(s)ds\right] = 0 \end{cases} \quad (30)$$

where M, N, S and T are matrices with appropriate dimensions, and

$$\xi^T(t) = [\bar{x}^T(t), \bar{x}^T(t-\tau_m), \bar{x}^T(t-\tau(t)), \bar{x}^T(t-\tau_M), \bar{x}^T(t-d(t)), \bar{x}^T(t-d_M), \bar{g}(H(\bar{x}(t))), \bar{g}(H(\bar{x}(t-\tau(t))))]^T.$$

By using Lemma 2, we have:

$$\begin{cases} -2\xi^T(t)M\int_{t-\tau(t)}^{t-\tau_m}\dot{\bar{x}}(s)ds \le (\tau(t)-\tau_m)\xi^T(t) \\ \quad MR_1^{-1}MR_1^{-1}M^T\xi(t) + \int_{t-\tau(t)}^{t-\tau_m}\dot{\bar{x}}^TR_1\dot{\bar{x}}(s)ds \\ -2\xi^T(t)N\int_{t-\tau_M}^{t-\tau(t)}\dot{\bar{x}}(s)ds \le (\tau_M-\tau(t))\xi^T(t) \\ \quad NR_1^{-1}N^T\xi(t) + \int_{t-\tau_M}^{t-\tau(t)}\dot{\bar{x}}^TR_1\dot{\bar{x}}(s)ds \\ -2\xi^T(t)S\int_{t-d(t)}^{t}\dot{\bar{x}}(s)ds \le d(t)\xi^T(t)SR_3^{-1}S^T\xi(t) \\ \quad + \int_{t-d(t)}^{t}\dot{\bar{x}}^TR_3\dot{\bar{x}}(s)ds \\ -2\xi^T(t)Z\int_{t-d_M}^{t-d(t)}\dot{\bar{x}}(s)ds \le (d_M-d(t))\xi^T(t) \\ \quad ZR_3^{-1}Z^T\xi(t) + \int_{t-d_M}^{t-d(t)}\dot{\bar{x}}^TR_3\dot{\bar{x}}(s)ds. \end{cases} \quad (31)$$

By using Lemma 1, notice that:

$$-\tau_m\int_{t-\tau_m}^{t}\dot{\bar{x}}^T(s)R_2\dot{\bar{x}}(s)ds \le \begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-\tau_m)\end{bmatrix}^T\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix} \times \begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-\tau_m)\end{bmatrix}. \quad (32)$$

By Assumption 1, we obtain:

$$\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t)))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & \bar{U}_2\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t)))\end{bmatrix} \le 0, \quad (33)$$

where $\bar{U}_1 = H^T\hat{U}_1H,\ \bar{U}_2 = -H^T\hat{U}_2,\ \hat{U}_1 = \frac{U_1^TU_2+U_2^TU_1}{2},\ \hat{U}_2 = \frac{U_1^T+U_2^T}{2}$. So for the parameters $\alpha > 0, \beta > 0$, it is easy to get:

$$-\alpha\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t)))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & \bar{U}_2\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t)))\end{bmatrix} \ge 0, \quad (34)$$

$$-\beta\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t-\tau(t))))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & \bar{U}_2\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t-\tau(t))))\end{bmatrix} \ge 0. \quad (35)$$

Combine (15) and (28)–(35) we can obtain that:

$$\begin{aligned}&\dot{V}(t) - \gamma^2w^T(t)w(t) + \bar{z}^T(t)\bar{z}(t) \\ &\le 2\bar{x}^T(t)P\dot{\bar{x}}(t) + \bar{x}^T(Q_1+Q_2+Q_3)\bar{x}(t) \\ &\quad - \bar{x}^T(t-\tau_m)Q_1\bar{x}(t-\tau_m) \\ &\quad - \bar{x}^T(t-\tau_M)Q_2\bar{x}(t-\tau_M) - \bar{x}^T(t-d_M)Q_3\bar{x}(t-d_M) \\ &\quad + \begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-\tau_m)\end{bmatrix}^T\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-\tau_m)\end{bmatrix} \\ &\quad + 2\xi^T(t)M[\bar{x}(t-\tau_m) - \bar{x}(t-\tau(t))] \\ &\quad + 2\xi^T(t)N[\bar{x}(t-\tau(t)) - \bar{x}(t-\tau_M)] \\ &\quad + 2\xi^T(t)S[\bar{x}(t) - \bar{x}(t-d(t))] \\ &\quad + 2\xi^T(t)Z[\bar{x}(t-d(t)) - \bar{x}(t-d_M)] \\ &\quad + (\tau(t)-\tau_m)\xi^T(t)MR_1^{-1}M^T\xi(t) \\ &\quad + (\tau_M-\tau(t))\xi^T(t)NR_1^{-1}N^T\xi(t) \\ &\quad + d(t)\xi^T(t)SR_3^{-1}S^T\xi(t) + (d_M-d(t))\xi^T(t)ZR_3^{-1}Z^T\xi(t) \\ &\quad - \alpha\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t)))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & \bar{U}_2\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t)))\end{bmatrix} \\ &\quad - \beta\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t-\tau(t))))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & \bar{U}_2\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H(\bar{x}(t-\tau(t))))\end{bmatrix} \\ &\quad + \sigma\bar{x}^T(t-d(t))\Psi\bar{x}(t-d(t)) - e_k^T(t)\Phi e_k(t) \\ &\quad + (\tau_M-\tau_m)\dot{\bar{x}}^T(t)R_1\dot{\bar{x}}(t) + \tau_m^2\dot{\bar{x}}^T(t)R_2\dot{\bar{x}}(t) \\ &\quad + d_M\dot{\bar{x}}^T(t)R_3\dot{\bar{x}}(t) - \gamma^2w^T(t)w(t) + \bar{z}^T(t)\bar{z}(t). \end{aligned} \quad (36)$$

By Schur complement, from Eq. (36) we can obtain:

$$\begin{aligned}&\dot{V}(t) - \gamma^2w^T(t)w(t) + \bar{z}^T(t)\bar{z}(t) \\ &\le \xi^T(t)\Sigma_{11}\xi(t) + (\tau(t)-\tau_m)\xi^T(t)MR_1^{-1}M^T\xi(t) \\ &\quad + (\tau_M-\tau(t))\xi^T(t)NR_1^{-1}N^T\xi(t) \\ &\quad + d(t)\xi^T(t)SR_3^{-1}S^T\xi(t) + (d_M-d(t))\xi^T(t)ZR_3^{-1}Z^T\xi(t) \\ &\quad + (\tau_M-\tau_m)\dot{\bar{x}}^T(t)R_1\dot{\bar{x}}(t) + \tau_m^2\dot{\bar{x}}^T(t)R_2\dot{\bar{x}}(t) \\ &\quad + d_M\dot{\bar{x}}^T(t)R_3\dot{\bar{x}}(t) + \bar{z}^T(t)\bar{z}(t). \end{aligned} \quad (37)$$

By using Lemma 3 and Schur complement, from Eq. (37), it is easy to see that Eq. (27) with $p = 1, 2, q = 1, 2$ can lead to:

$$\dot{V}(t) \le \gamma^2w^T(t)w(t) - \bar{z}^T(t)\bar{z}(t). \quad (38)$$

Then integrating both sides of Eq. (38) from 0 to t and letting $t \to +\infty$, we can get $\|z(t)\|_2 \le \gamma\,\|w(t)\|_2$.

Suppose $w(t) = 0$, $\Xi(t) = e^{2kt}V(t)$, and taking the time derivative of $\Xi(t)$ yields, repeated above prove and using the similar method (Mao, 1996), there exist $l_0 > 0, k > 0$ such that $V(t) \le l_0e^{-2kt}\sup_{-\delta\le\theta\le0}\|\Phi\|^2$.

$$\Pi=\begin{bmatrix}\Phi_{11} & * & * & * & * & * & * \\ \Phi_{21} & \Sigma_{22} & * & * & * & * & * \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & * & * & * & * \\ \Sigma_{41}^{(p)} & 0 & 0 & R_1 & * & * & * \\ \Sigma_{51}^{(q)} & 0 & 0 & 0 & R_3 & * & * \\ \Phi_{61} & 0 & \Phi_{63} & 0 & 0 & -\frac{\varepsilon_4}{\delta^2}I & * \\ \Phi_{71} & 0 & \Phi_{73} & 0 & 0 & 0 & -\varepsilon_4 I\end{bmatrix}<0 \quad (p=1,2;\ q=1,2) \tag{40}$$

where

$$\Phi_{11}=\begin{bmatrix}\Phi_1 & * & * & * & * & * & * & * \\ R_2 & \Phi_2 & * & * & * & * & * & * \\ 0 & M_3-M_2^T & \Phi_3 & * & * & * & * & * \\ 0 & 0 & N_4-N_3^T & \Phi_4 & * & * & * & * \\ H^T\tilde{B}^TP+S_5-S_1^T & 0 & 0 & 0 & \Phi_5 & * & * & * \\ 0 & 0 & 0 & 0 & Z_6-Z_5^T & \Phi_6 & * & * \\ \bar{W}_0^TP-\alpha\bar{U}_2^T & 0 & 0 & 0 & 0 & 0 & -\alpha I & * \\ \bar{W}_1^TP & 0 & -\beta\bar{U}_2 & 0 & 0 & 0 & 0 & -\beta I\end{bmatrix},$$

$$\Phi_{21}=\begin{bmatrix}\tilde{B}^TP & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{A}_w^TP & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \bar{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \qquad \Phi_{32}=\begin{bmatrix}c_{21}R_1\tilde{B}_1 & c_{21}R_1\bar{A}_w & 0 \\ c_1R_2\tilde{B}_1 & c_1R_2\bar{A}_w & 0 \\ d_1R_3\tilde{B}_1 & d_1R_3\bar{A}_w & 0\end{bmatrix},$$

$$\Phi_{31}=\begin{bmatrix}c_{21}R_1\bar{A} & 0 & 0 & 0 & c_{21}R_1\tilde{B}H & 0 & c_{21}R_1\bar{W}_0 & c_{21}R_1\bar{W}_1 \\ c_1R_2\bar{A} & 0 & 0 & 0 & c_1R_2\tilde{B}H & 0 & c_1R_2\bar{W}_0 & c_1R_2\bar{W}_1 \\ d_1R_3\bar{A} & 0 & 0 & 0 & d_1R_3\tilde{B}H & 0 & d_1R_3\bar{W}_0 & d_1R_3\bar{W}_1\end{bmatrix}, \quad \tilde{B}=\begin{bmatrix}0 \\ B_fC\end{bmatrix}, \quad \tilde{B}_1=\begin{bmatrix}0 \\ -B_f\end{bmatrix},$$

$$\Phi_{33}=\mathrm{diag}\{-2\varepsilon_1P_1+\varepsilon_1^2R_1,-2\varepsilon_2P_1+\varepsilon_2^2R_2,-2\varepsilon_3P_1+\varepsilon_3^2R_3\},$$

$$\Phi_{61}=\begin{bmatrix}\tilde{B}_f^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \quad \Phi_{71}=\begin{bmatrix}0 & 0 & 0 & 0 & \varepsilon_4C & 0 & 0 & 0\end{bmatrix},$$

$$\Phi_{63}=\begin{bmatrix}c_{21}\tilde{B}_f^T & c_1\tilde{B}_f^T & d_1\tilde{B}_f^T\end{bmatrix}, \quad \Phi_{73}=\begin{bmatrix}-\varepsilon_4I & 0 & 0\end{bmatrix}, \quad \tilde{B}_f=\begin{bmatrix}B_f^TP_2 & B_f^TP_3 \\ 0 & 0\end{bmatrix}.$$

Box II.

For $V(t)\geq V_1(t)\geq\lambda_{\min}(P)\|\bar{x}(t)\|^2$, we get $\lambda_{\min}(P)\|x(t)\|^2\leq l_0e^{-2kt}\sup_{-\delta\leq\theta\leq0}\|\phi\|^2$, i.e.

$$\|\bar{x}(t)\|^2\leq\frac{l_0}{\lambda_{\min}(P)}e^{-2kt}\sup_{-\delta\leq\theta\leq0}\|\phi\|^2. \tag{39}$$

Squaring both sides we get $\|\bar{x}(t)\|\leq\sqrt{\frac{l_0}{\lambda_{\min}(P)}}e^{-2kt}\sup_{-\delta\leq\theta\leq0}\|\phi\|$.

Suppose $l=\sqrt{\frac{l_0}{\lambda_{\min}(P)}}$ and $\|\bar{x}(t)\|^2\leq le^{-2kt}\sup_{-\delta\leq\theta\leq0}\|\phi\|^2$. By Definition 1, the system (19) is exponentially stable. This complete the proof.

Remark 4. In Theorem 1, a sufficient condition is given which can guarantee the exponential stability of the augmented system (19). Note that there exist nonlinear terms Δ_q in Theorem 1. Similar to Qu et al. (2015), applying a well-known bounding inequality, we eliminate Δ_q and an equivalent expression of (27) is obtained in the following Theorem 2.

Theorem 2. *For given positive parameters $\gamma,\tau_m,\tau_M,d_M,\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4$, and σ, the system (19) is exponentially stable under the event trigger scheme (2) and the action of quantizer (3), if there exist matrices $P>0,Q_i>0,R_i>0(i=1,2,3),\Phi>0,M_j(j=2,3),N_k(k=3,4),S_l(l=1,5),Z_s(s=5,6)$ with appropriate dimensions and parameters $\alpha>0,\beta>0$ such that the inequality given in Box II holds.*

Other parameters are the same as Theorem 1.

Proof. Pre-and post-multiplying (27) with $\mathrm{diag}=\{\underbrace{I,\ldots,I}_{11},PR_1^{-1},PR_2^{-1},PR_3^{-1},I,I\}$ and its transpose, one has:

$$\tilde{\Gamma}=\begin{bmatrix}\Sigma_{11} & * & * & * & * \\ \Sigma_{21} & \tilde{\Sigma}_{22} & * & * & * \\ \tilde{\Sigma}_{31} & \tilde{\Sigma}_{32} & \tilde{\Sigma}_{33} & * & * \\ \Sigma_{41}^{(p)} & 0 & 0 & -R_1 & * \\ \Sigma_{51}^{(q)} & 0 & 0 & 0 & -R_3\end{bmatrix} \tag{41}$$

where

$$\tilde{\Sigma}_{31}=\begin{bmatrix}c_{21}P\bar{A} & 0 & 0 & 0 & c_{21}P\bar{B}H & 0 & c_{21}P\bar{W}_0 & c_{21}P\bar{W}_1 \\ c_1P\bar{A} & 0 & 0 & 0 & c_1P\bar{B}H & 0 & c_1P\bar{W}_0 & c_1P\bar{W}_1 \\ d_1P\bar{A} & 0 & 0 & 0 & d_1P\bar{B}H & 0 & d_1P\bar{W}_0 & d_1P\bar{W}_1\end{bmatrix},$$

$$\tilde{\Sigma}_{32}=\begin{bmatrix}c_{21}P\bar{B}_1 & c_{21}P\bar{A}_w & 0 \\ c_1P\bar{B}_1 & c_1P\bar{A}_w & 0 \\ d_1P\bar{B}_1 & d_1P\bar{A}_w & 0\end{bmatrix},$$

$$\tilde{\Sigma}_{33}=\mathrm{diag}\{-PR_1P,-PR_2P,-PR_3P\}.$$

By using Lemma 4, we can obtain the following inequality:

$$\begin{cases}-PR_1^{-1}P\leq-2\varepsilon_1P+\varepsilon_1^2R_1\\ -PR_2^{-1}P\leq-2\varepsilon_2P+\varepsilon_2^2R_2\\ -PR_3^{-1}P\leq-2\varepsilon_3P+\varepsilon_3^2R_3.\end{cases} \tag{42}$$

Define $P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}$,

since $\bar{P}_3 > 0$, there exist nonsingular matrices P_2 and $P_3 > 0$ satisfying $\bar{P}_3 = P_2^T P_3^{-1} P_2$. Applying Schur complement, $P > 0$ is equal to $P_1 - \bar{P}_3 > 0$.

The matrix (27) can be rewritten as the following form:

$$\Pi = \Pi_{11} + J_B \Delta_q J_C + J_C^T \Delta_q J_B^T \tag{43}$$

where

$$J_B^T = \begin{bmatrix} \tilde{B}_f^T & \underbrace{0, \dots, 0}_{10} & c_{21}\tilde{B}_f^T & c_1 \tilde{B}_f^T & d_1 \tilde{B}_f^T & 0 & 0 \end{bmatrix},$$

$$J_C = \begin{bmatrix} 0 & 0 & 0 & 0 & C & 0 & 0 & 0 & -I & \underbrace{0, \dots, 0}_{7} \end{bmatrix},$$

$$\tilde{B}_f = \begin{bmatrix} B_f^T P_2 & B_f^T P_3 \\ 0 & 0 \end{bmatrix},$$

$$\Pi_{11} = \begin{bmatrix} \Phi_{11} & * & * & * & * \\ \Phi_{21} & \Sigma_{22} & * & * & * \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & * & * \\ \Sigma_{41}^{(p)} & 0 & 0 & R_1 & * \\ \Sigma_{51}^{(q)} & 0 & 0 & 0 & R_3 \end{bmatrix}.$$

Using Lemma 2, there exists $\varepsilon_4 > 0$,

$$\Pi \leq \Pi_{11} + \varepsilon_4^{-1} J_B \Delta_q^2 J_B^T + \varepsilon_4 J_C^T J_C \tag{44}$$

and

$$\Delta_q^2 \leq \delta^2 I. \tag{45}$$

By Schur complement, we can get (40) according to (27).

This completes the proof.

Remark 5. From Theorem 2, the solution of inequality (40) is not only dependent on the upper and lower bounds of the network delay, but also affected by the quantization parameter δ of the quantizer.

Based on the event trigger scheme (2) and the action of quantizer (3), the stability condition is conducted in Theorem 2. The explicit expression of the parameters of the designed filter are given in the following theorem in terms of LMIs.

Theorem 3. *For given positive parameters $\gamma, \tau_m, \tau_M, d_M, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4$ and σ, the augmented system (19) is exponentially stable under the event trigger scheme (2) and the action of quantizer (3), if there exist matrices $P_1 > 0, \bar{P}_3 > 0, \bar{Q}_i > 0, \bar{R}_i > 0 (i = 1, 2, 3), \Phi > 0, \bar{A}_f, \bar{B}_f, \bar{C}_f, \bar{M}_j (j = 2, 3), \bar{N}_k (k = 3, 4), \bar{S}_l (l = 1, 5), \bar{Z}_s (s = 5, 6)$ with appropriate dimensions and parameters $\alpha > 0, \beta > 0$ such that the LMIs given in Box III hold, and other parameters are the same as Theorem 2.*

Moreover, if the above conditions are feasible, the parameter matrices of the filter are given by:

$$\begin{cases} A_f = \bar{A}_f \bar{P}_3^{-1} \\ B_f = \bar{B}_f \\ C_f = \bar{C}_f \bar{P}_3^{-1}. \end{cases}$$

Proof. Let $J = \begin{bmatrix} I & 0 \\ 0 & P_2^T P_3^{-1} \end{bmatrix}$, $\Lambda = \mathrm{diag}\{\underbrace{J, \dots, J}_{6}, \underbrace{I, \dots, I}_{5}, \underbrace{J, \dots, J}_{5}, I, I\}$.

Multiplying Λ and Λ^T on both sides of (40), respectively, and defining $\bar{P} = JPJ^T = \begin{bmatrix} P_1 & \bar{P}_3 \\ \bar{P}_3 & \bar{P}_3 \end{bmatrix}$, $\bar{Q}_i = JQ_iJ^T$, $\bar{R}_i = JR_iJ^T (i = 1, 2, 3)$, $\bar{M}_j = JM_jJ^T (j = 2, 3)$, $\bar{N}_k = JN_kJ^T (k = 3, 4)$, $\bar{S}_l = JS_lJ^T (l = 1, 5)$, $\bar{Z}_s = JZ_sJ^T (s = 5, 6)$.

Define variables:

$$\begin{cases} \bar{A}_f = \hat{A}_f \bar{P}_3, & \hat{A}_f = P_2^T A_f P_2^{-T} \\ \bar{B}_f = P_2^T B_f \\ \bar{C}_f = \hat{C}_f \bar{P}_3, & \hat{C}_f = C_f P_2^{-T}. \end{cases} \tag{47}$$

Then, by the above equivalent linear transformation, we can obtain (46) is equivalent to (40).

Next, we will solve the filter parameters.

By observing (46), we cannot solve P_2 and P_3 directly. The continuous transfer function from $\hat{y}(t)$ and $z_f(t)$ can be expressed as:

$$\begin{aligned} T_{z_f \hat{y}} &= C_f (sI - A_f)^{-1} B_f \\ &= \bar{C}_f P_2^{-T} P_3 (sI - P_2^{-1} \bar{A}_f P_2^{-T} P_3)^{-1} P_2^{-1} \bar{B}_f \\ &= \bar{C}_f (s\bar{P}_3 - \bar{A}_f)^{-1} \bar{B}_f \\ &= \bar{C}_f \bar{P}_3^{-1} (sI - \bar{A}_f \bar{P}_3^{-1})^{-1} \bar{B}_f. \end{aligned} \tag{48}$$

So the filter parameters can be described as follows:

$$\begin{cases} A_f = \bar{A}_f \bar{P}_3^{-1} \\ B_f = \bar{B}_f \\ C_f = \bar{C}_f \bar{P}_3^{-1}. \end{cases} \tag{49}$$

This complete the proof.

4. Simulation examples

In this section, an illustrative example is presented to demonstrate the effectiveness of the proposed filter design approach for the neural network (1).

Consider a neural network (1) with the following parameters:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.3 & -0.4 \\ -0.4 & 0.3 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad C = \begin{bmatrix} 0.9 & 0.8 \end{bmatrix},$$

$$A_w = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix},$$

$$w(t) = \begin{cases} 0.5, & t \epsilon [5, 10) \\ -0.5, & t \epsilon [15, 20) \\ 0, & \text{else} \end{cases},$$

$$g(t) = \begin{bmatrix} 0.5x_1(t) - \tanh(0.2x_1(t)) + 0.2x_2(t) \\ 0.95x_2(t) - \tanh(0.75x_1(t)) \end{bmatrix}.$$

We assume $\alpha = 0.64604$, $\beta = 0.57723$, $\gamma = 1.5$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$, time delay $\tau_m = 0.1$, $\tau_M = 0.2$, $d_M = 0.1$, the corresponding trigger parameter $\sigma = 0.05$ and quantization parameters $\rho = 0.8$, where $\delta = \frac{1-\rho}{1+\rho}$, by using the LMI toolbox of Matlab, it is easy to obtain the following matrices:

$$P_1 = \begin{bmatrix} 4.0462 & 0.3022 \\ 0.3022 & 2.6851 \end{bmatrix}, \quad \bar{P}_3 = \begin{bmatrix} 1.7982 & 0.1632 \\ 0.1632 & 1.2316 \end{bmatrix},$$

$$A_f = \begin{bmatrix} -5.8282 & -1.2179 \\ -1.7277 & -3.7041 \end{bmatrix}, \quad B_f = \begin{bmatrix} -2.7993 \\ -3.2925 \end{bmatrix},$$

$$C_f = \begin{bmatrix} -0.4944 & 0.2872 \end{bmatrix},$$

$$\bar{A}_f = \begin{bmatrix} -3.1898 & -0.5663 \\ -0.6963 & -2.9152 \end{bmatrix}, \quad \bar{B}_f = \begin{bmatrix} -2.7993 \\ -3.2925 \end{bmatrix},$$

$$\bar{C}_f = \begin{bmatrix} -0.2997 & 0.2729 \end{bmatrix}, \quad \Phi = 14.3288.$$

$$\Upsilon = \begin{bmatrix} \Upsilon_{11} & * & * & * & * & * & * \\ \Upsilon_{21} & \Sigma_{22} & * & * & * & * & * \\ \Upsilon_{31} & \Upsilon_{32} & \Upsilon_{33} & * & * & * & * \\ \Upsilon_{41}^{(p)} & 0 & 0 & \bar{R}_1 & * & * & * \\ \Upsilon_{51}^{(q)} & 0 & 0 & 0 & \bar{R}_3 & * & * \\ \Upsilon_{61} & 0 & \Upsilon_{63} & 0 & 0 & -\frac{\varepsilon_4}{\delta^2} I & * \\ \Upsilon_{71} & 0 & \Upsilon_{73} & 0 & 0 & 0 & -\varepsilon_4 I \end{bmatrix} < 0 \quad (p = 1, 2; q = 1, 2) \tag{46}$$

where

$$\Upsilon_{11} = \begin{bmatrix} \Lambda_1 & * & * & * & * & * & * & * \\ \bar{R}_2 & \Lambda_2 & * & * & * & * & * & * \\ 0 & \bar{M}_3 - \bar{M}_2^T & \Lambda_3 & * & * & * & * & * \\ 0 & 0 & \bar{N}_4 - \bar{N}_3^T & \Lambda_4 & * & * & * & * \\ \Lambda_5 & 0 & 0 & 0 & \Lambda_6 & * & * & * \\ 0 & 0 & 0 & 0 & \bar{T}_6 - \bar{T}_5^T & \Lambda_7 & * & * \\ \Lambda_8 + \Gamma_6 & 0 & 0 & 0 & 0 & 0 & -\alpha I & * \\ \Lambda_9 & 0 & \Gamma_7 & 0 & 0 & 0 & 0 & -\beta I \end{bmatrix},$$

$$\Lambda_1 = \Gamma_1 + \Gamma_1^T + \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 - \bar{R}_2 + \bar{S}_1 + \bar{S}_1^T - \Gamma_2,$$

$$\Gamma_1 = \begin{bmatrix} -P_1 A & \bar{A}_f \\ -\bar{P}_3 A & \bar{A}_f \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} \alpha \frac{U_1^T U_2 + U_2^T U_1}{2} & 0 \\ 0 & 0 \end{bmatrix}, \quad \Gamma_3 = \begin{bmatrix} \beta \frac{U_1^T U_2 + U_2^T U_1}{2} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Lambda_2 = -\bar{Q}_1 - \bar{R}_2 + \bar{M}_2 + \bar{M}_2^T, \quad \Lambda_3 = \bar{M}_3 - \bar{M}_3^T + \bar{N}_3 + \bar{N}_3^T - \Gamma_3,$$

$$\Lambda_4 = -\bar{Q}_2 - \bar{N}_4 - \bar{N}_4^T, \quad \Lambda_5 = \bar{S}_5 - \bar{S}_1^T + \Gamma_4,$$

$$\Gamma_4 = \begin{bmatrix} C^T \bar{B}_f^T & C^T \bar{B}_f^T \\ 0 & 0 \end{bmatrix}, \quad \Gamma_5 = \begin{bmatrix} \sigma C^T \Phi C & 0 \\ 0 & 0 \end{bmatrix},$$

$$\Lambda_6 = -\bar{S}_5 - \bar{S}_5^T + \bar{Z}_5 + \bar{Z}_5^T + \Gamma_5, \quad \Lambda_7 = -\bar{Q}_3 - \bar{Z}_6 - \bar{Z}_6^T, \quad \tilde{L} = \begin{bmatrix} L & -\bar{C}_f \end{bmatrix},$$

$$\Upsilon_{21} = \begin{bmatrix} \Lambda_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Lambda_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \tilde{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Lambda_8 = \begin{bmatrix} W_0^T P_1 & W_0^T \bar{P}_3 \end{bmatrix}, \quad \Gamma_6 = \begin{bmatrix} \alpha \frac{U_1 + U_2}{2} & 0 \end{bmatrix},$$

$$\Gamma_7 = \begin{bmatrix} \beta \frac{U_1^T + U_2^T}{2} & 0 \end{bmatrix}, \quad \Lambda_9 = \begin{bmatrix} W_1^T P_1 & W_1^T \bar{P}_3 \end{bmatrix}, \quad \Lambda_{10} = \begin{bmatrix} -\bar{B}_f^T & -\bar{B}_f^T \end{bmatrix}, \quad \Lambda_{11} = \begin{bmatrix} A_w^T P_1 & A_w^T \bar{P}_3 \end{bmatrix},$$

$$\Upsilon_{31} = \begin{bmatrix} c_{21}\Gamma_1 & 0 & 0 & 0 & c_{21}\Gamma_4^T & 0 & c_{21}\Lambda_8^T & c_{21}\Lambda_9^T \\ c_1\Gamma_1 & 0 & 0 & 0 & c_1\Gamma_4^T & 0 & c_1\Lambda_8^T & c_1\Lambda_9^T \\ d_1\Gamma_1 & 0 & 0 & 0 & d_1\Gamma_4^T & 0 & d_1\Lambda_8^T & d_1\Lambda_9^T \end{bmatrix}, \quad \Upsilon_{32} = \begin{bmatrix} c_{21}\Lambda_{10}^T & c_{21}\Lambda_{11}^T & 0 \\ c_1\Lambda_{10}^T & c_1\Lambda_{11}^T & 0 \\ d_1\Lambda_{10}^T & d_1\Lambda_{11}^T & 0 \end{bmatrix},$$

$$\Upsilon_{33} = \mathrm{diag}\{-2\varepsilon_1\bar{P}_1 + \varepsilon_1^2\bar{R}_1, -2\varepsilon_2\bar{P}_1 + \varepsilon_2^2\bar{R}_2, -2\varepsilon_3\bar{P}_1 + \varepsilon_3^2\bar{R}_3\},$$

$$\Upsilon_{41}^{(1)} = \begin{bmatrix} 0 & c_{21}\bar{M}_2^T & c_{21}\bar{M}_3^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Upsilon_{41}^{(2)} = \begin{bmatrix} 0 & 0 & c_{21}\bar{N}_3^T & c_{21}\bar{N}_4^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{51}^{(1)} = \begin{bmatrix} d_1\bar{S}_1^T & 0 & 0 & 0 & d_1\bar{S}_5^T & 0 & 0 & 0 \end{bmatrix}, \quad \Upsilon_{51}^{(2)} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_1\bar{Z}_5^T & d_1\bar{Z}_6^T & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{61} = \begin{bmatrix} \hat{B}_f^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Upsilon_{71} = \begin{bmatrix} 0 & 0 & 0 & 0 & \varepsilon_4 C & 0 & 0 & 0 \end{bmatrix},$$

$$\Upsilon_{63} = \begin{bmatrix} c_{21}\hat{B}_f^T & c_1\hat{B}_f^T & d_1\hat{B}_f^T \end{bmatrix}, \quad \Upsilon_{73} = \begin{bmatrix} -\varepsilon_4 I & 0 & 0 \end{bmatrix}, \quad \hat{B}_f = \begin{bmatrix} \bar{B}_f & \bar{B}_f \end{bmatrix}.$$

Box III.

Given the initial condition $x(0) = \begin{bmatrix} -0.5 & 0.1 \end{bmatrix}^T$, $x_f(0) = \begin{bmatrix} -0.3 & 0.5 \end{bmatrix}^T$ and the sampling period $h = 0.05$, the event-triggered release instants and intervals are shown in Fig. 2. In Fig. 3, we draw a comparison chart of the state $y(k_j h)$ before the quantization and the quantized state $y_1(k_j h)$, which can better show the changes before and after quantization. The response of the filter error $\bar{z}(t)$ is depicted in Fig. 4, which demonstrates that the designed filter can satisfy the system performance.

5. Conclusion

This paper investigates the event-triggered H_∞ filter design for a class of neural network systems with quantization. In particular, the event-triggered generator and quantizer inserted in network have the advantages of reducing the communication load in the neural network and gearing up its efficiency. Moreover, by employing the neural network model with the event triggered scheme and quantization, the fundamental stability criteria are

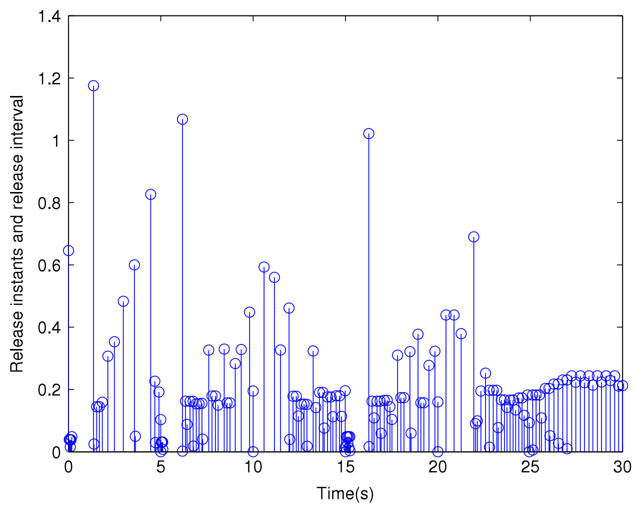

Fig. 2. Release instances and release interval.

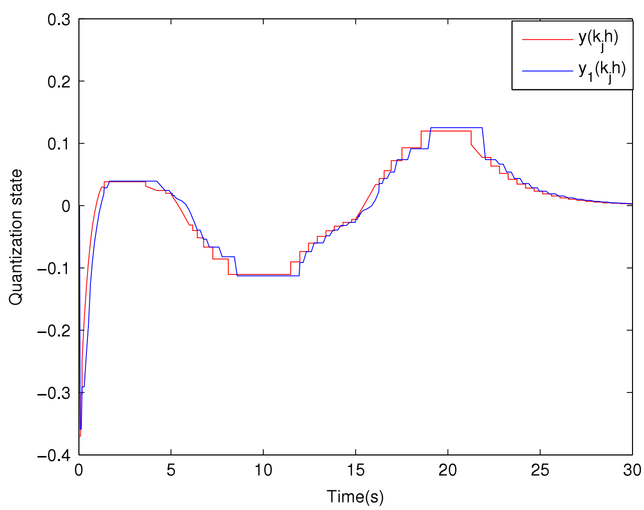

Fig. 3. Before and after quantization.

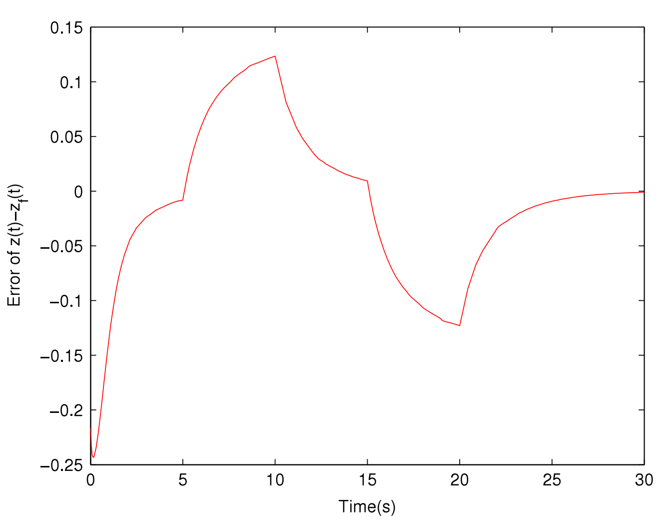

Fig. 4. Filter error $\bar{z}(t)$.

obtained. Furthermore, a filter design method is developed and the explicit expression of the desired filter parameters can be derived. Lastly, a numerical example has been provided to show the usefulness of the proposed method. Further work will study the effects of the distributed event-triggered scheme in the neural networks. In addition, the cases with respect to limited data transmission rate, and encoding–decoding algorithms are also interesting, which is our future research.

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