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Brief Papers

Adaptive event-triggered $H\infty$ filtering for T-S fuzzy system with time delay

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ABSTRACT

This paper is concerned with adaptive event-triggered H_∞ filter design for a class of T-S fuzzy systems with time delay. Firstly, an adaptive event-triggered scheme is introduced, which can adaptively adjust the communication threshold to save the limited communication resource. Secondly, a T-S fuzzy model is applied to approximate the nonlinear dynamics of the plant. By using Lyapunov function, sufficient conditions for the existence of the desired filter are established in terms of linear matrix inequalities such that the filtering error dynamics is locally mean square asymptotically stable. Then, the explicit expression is provided for the designed filter parameters. Finally, a simulation example is employed to illustrate the design method.

1. Introduction

Many industrial systems exhibit severe nonlinear characteristics, which usually make the stability analysis and design more difficult [1–3]. On modeling nonlinear behavior, Takagi-Sugeno (T-S) fuzzy models are qualified to represent a class of nonlinear dynamic systems. Much attention has been attracted by T-S fuzzy system, which can be analyzed by many methods of conventional linear systems and described by a family of IF-THEN rules to approximate any continuous functions [4–7]. As is well known, many efforts have been paid to T-S fuzzy system [8–17]. In [15], the paper is concerned with the problem of robust H_∞ control for uncertain T-S fuzzy systems with interval time-varying delay, of which the delay is assumed to be a time-varying function belonging to an interval. In [16], the author investigates the problems of state estimation for nonlinear positive systems based on T-S fuzzy model. The authors in [17] are concerned with event-triggered fuzzy control design for a class of discrete-time nonlinear networked control systems (NCSs) with time-varying communication delays. Specially, the filtering problem have been widely investigated over the past years [18–22]. In [19], reliable H_∞ filter design for a class of T-S fuzzy systems with stochastic sensor faults under an event-triggered scheme has been investigated. In [20], the robust and reliable H_∞ filter design for a class of nonlinear NCSs with random sensor faults via T-S fuzzy model have been investigated. In [22], the authors investigate a combined event-triggered communication scheme and H_∞ fuzzy filter co-design method for a class of nonlinear networked control system.

Compared with the periodic sampling method, the event-triggered scheme could not only reduce the burden of the communication but also preserve the desired properties of the ideal continuous state feedback system, such as stability and convergence. The outstanding application on event-triggered scheme could be found in many literatures [24,23,25–27,29]. For example, in [23], the paper is concerned with the control design problem of event-triggered networked systems with both state and control input quantizations. In [25], authors investigate the reliable control design for networked control system under event-triggered scheme. The author in [26] investigated the event-triggered H_∞ controller design problem for nonlinear networked control systems (NCSs) with time delay and uncertainties. In [27], the problem of event-triggered fuzzy filtering is investigated for a class of NCSs. The authors in [29] proposed a novel event-triggered scheme and constructed a delay system model for the analysis, then they derived the criteria for stability with an H_∞ norm bound and criteria for co-designing both the feedback gain and the trigger parameters. As we all know, the network-induced delays, packet dropouts and disorder are mainly caused by the limited network bandwidth. As communication bandwidth is scarce in a shared communication channel, one obvious problem when considering NCSs is whether there is sufficient communication bandwidth to feedback information to the controller and then

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send the control commands to the actuators and the plants [30,31]. However, the above proposed event-triggered scheme are based on the inequality $e_k^T(t)\Phi e_k(t) > \delta x^T(t_{k+1})\Phi x(t_{k+1})$, where Φ is a symmetric positive definite matrix, δ is a positive constant. Since the trigger parameter is a constant, it cannot adjust the sampling interval dynamically, which can waste the communication resources. Therefore, to mitigate the unnecessary waste of computation and communication resources in the conventional event-triggered scheme while keeping the control performance, the adaptive event-triggered scheme has beenproposed. This is also the motivation of this work.

In this paper, an adaptive event-triggered scheme has been introduced to a T-S fuzzy system with time delay. The adaptive scheme shows an effective way to keep balance for the system control performance and network communication bandwidth burden. An algorithm is presented to find the adaptive threshold instead of a pre-given constant to achieve good performance of the proposed adaptive scheme, which is the challenge of this work. Moreover, the filtering problem for T-S fuzzy system is investigated under the adaptive event-triggered scheme. By using Lyapunov functional approach, a sufficient condition for the existence of the desired filter is established in terms of linear matrix inequalities. Finally, a simulation example is provided to illustrate the effectiveness of the proposed method.

Notation: The superscript "T" represents matrix transposition, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n\times m$ real matrices; $\|\cdot\|$ represents the Euclidean vector norm or the induced matrix 2-norm as appropriate; I is the identity matrix of appropriate dimension.

$$\begin{bmatrix} A & * \\ B & C \end{bmatrix}$$

denote a symmetric matrix, where $*$ denotes the entries implied by symmetry, for a matrix B and two symmetric matrices A and C. The notation $X>0$ (respectively, $X\geq 0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite).

2. Problem statement and preliminaries

Consider the following nonlinear system represented by the following T-S fuzzy system with r plant rules. Plant rule R^i: IF $\theta_1(t)$ is ${W_1}^i$ and $\cdots$ and $\theta_g(t)$ is ${W_g}^i$, THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di}x(t-\tau(t)) + A_{\omega i}\omega(t) \\ y(t) = C_i x(t) \\ z(t) = L_i x(t) \end{cases} \tag{1}$$

where r is the number of IF-THEN rules, $\theta_1(t), \theta_2(t), \ldots, \theta_g(t)$ are the premise variables, $x(t)\in\mathbb{R}^n$, $y(t)\in\mathbb{R}^m$ and $z(t)\in\mathbb{R}^p$ are the state vector, output vector and the signal to be estimated, respectively. A_i, A_{di}, $A_{\omega i}$, C_i, L_i are parameter matrices with appropriate dimensions, $\omega(t)\in L_2[0,\infty)$ denotes the exogenous disturbance signal, $\tau(t)$ is a time-varying delay taking values on the interval $[\tau_m, \tau_M]$, where τ_m and τ_M are positive real numbers.

By using center-average defuzzifier, product interference and singleton fuzzifier, the obtained fuzzy system (1) is inferred as follows:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + A_d(t)x(t-\tau(t)) + A_\omega(t)\omega(t) \\ y(t) = C(t)x(t) \\ z(t) = L(t)x(t) \end{cases} \tag{2}$$

where $A(t) = \sum_{i=1}^{r} h_i A_i$, $A_d(t) = \sum_{i=1}^{r} h_i A_{di}$, $A_\omega(t) = \sum_{i=1}^{r} h_i A_{\omega i}$, $C(t) = \sum_{i=1}^{r} h_i C_i$, $L(t) = \sum_{i=1}^{r} h_i L_i$. h_i is the abbreviation for $h_i(\theta(t))$, $h_i(\theta(t)) = \frac{\mu_i(\theta(t))}{\sum_{i=1}^{r}\mu_i(\theta(t))}$, $\mu_i(\theta(t)) = \prod_{j=1}^{g} W_j^i(\theta_j(t))$, $W_j^i(\theta_j(t))$ is the grade membership value of $\theta_j(t)$ in W_j^i and $h_i(\theta(t))$ satisfies $h_i(\theta(t))\geq 0$, $\sum_{i=1}^{r} h_i(\theta(t)) = 1$. For notational simplicity, we use h_i to represent $h_i(\theta(t))$.

The purpose of this paper is to design a H_∞ fuzzy filter for system (2), the following time-varying filter structure is proposed:

$$\begin{cases} \dot{x}_f(t) = A_{fi}x_f(t) + B_{fi}\hat{y}(t) \\ z_f(t) = C_{fi}x_f(t) \end{cases} \tag{3}$$

where $x_f(t)\in R^n$ is the filter state vector, $z_f(t)\in R^p$ the estimation of $z(t)$, $\hat{y}(t)$ is the real input of the filter. The matrices $A_{fi}\in R^{n\times n}$, $B_{fi}\in R^{n\times m}$, $C_{fi}\in R^{p\times n}$ are to be determined.

The defuzzified output of (3) is referred by

$$\begin{cases} \dot{x}_f(t) = A_f(t)x_f(t) + B_f(t)\hat{y}(t) \\ z_f(t) = C_f(t)x_f(t) \end{cases} \tag{4}$$

where $A_f(t) = \sum_{i=1}^{r} h_i A_{fi}$, $B_f(t) = \sum_{i=1}^{r} h_i B_{fi}$, $C_f(t) = \sum_{i=1}^{r} h_i C_{fi}$.

Remark 1. In traditional filtering problem, the effect of the communication network can be neglected, thus $\hat{y}(t) = y(t)$. However, in networked control systems, the existence of network-induced delays should be take into account. We have $\hat{y}(t)\neq y(t)$ in this paper.

The sensor sample the measurement output $y(t)$ with the regular sampling period in Fig. 1, which leads to transmit many unnecessary signals, reduce bandwidth utilization and increase consumption of limited energy of wireless sensor nodes because every sampled-signal must be transmitted to a fuzzy filter through a network channel. In order to mitigate the burden of communication and prolong the lifetime of wireless sensor nodes in Fig. 1, an adaptive event generator is attached to the sensor, which is used to determine whether or not the current sampled measurement $y(t)$ should be transmitted. We use kh and $t_k h$ to represent the sampling instants and the triggered instants, respectively, where h is the sampling period. Specifically, once $t_k h$ is transmitted, the next triggered instant is determined by

$$t_{k+j}h = t_k h + \min\{nh \mid [y(t_{k+j}h) - y(t_k h)]^T \Phi [y(t_{k+j}h) - y(t_k h)] > \delta(t_{k+j}h) y^T(t_{k+j}h)\Phi y(t_{k+j}h)\} \tag{5}$$

where Φ is a symmetric positive definite matrix, $\delta(t)\in[0.1, 0.5]$. nh means the sampling instants between the current transmitted sampling instant t_k and the future transmitted sampling instant t_{k+j}. The trigger parameter $\delta(t)$ in the adaptive event-triggered scheme is not a constant, which presents a differential function

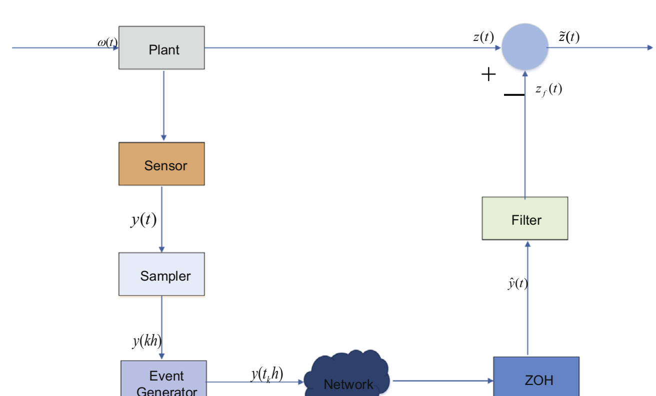

Fig. 1. A typical filtering for networked control system with adaptive event triggered scheme.

satisfying [33]

$$\dot{\delta}(t) = d\delta(t) \tag{6}$$

where $\delta(0) = 0.1$ is the initial condition of parameter $\delta(t)$ and

$$d = \begin{cases} 1, & e_k^T(t)\Phi e_k(t) < \rho \\ 0, & e_k^T(t)\Phi e_k(t) = \rho \quad \text{or} \quad 0 \\ -1, & e_k^T(t)\Phi e_k(t) > \rho \end{cases} \tag{7}$$

where ρ is a non-negative constant number.

Algorithm to find the communication parameter ρ:

Step 1: For given δ_M and $\tau_1 = 0$, set $\tau_2 = \tau_2 + \mu$, where μ is the step increment of τ_2, τ_2 is the upper delay bound of $\tau(t)$.
Step 2: For a given τ_2 in step 1, if there exists a feasible solution satisfying LMIs (36), go to step 3; otherwise go to Step 1.
Step 3: Use the Matlab LMI Toolbox to find the maximum τ_2, and the corresponding Φ based on Theorem 2.
Step 4: Set a simulation time T, and based on an ordinary event-triggered scheme with an initial triggering parameter $\delta(0)$, where $\delta(0) \le \delta_M$, use Simulink to find the state error $e(t)$ at every sampling instant.
Step 5: Based on step 4, calculate the average error function $e_k^T(t)\Phi e_k(t)$, then output the parameter ρ, where ρ is equal to the average state error function.

Remark 2. It is noticed that if the trigger parameter $\delta(t)$ in our proposed adaptive event triggered scheme is not a constant, but a differential function satisfying (6). Combining (6) and (7) and the Algorithm for the communication parameter ρ, we can see that when $e_k^T(t)\Phi e_k(t) > \rho$, $d = -1$ occurs, the value of ρ will decrease, which means the discussed system (19) is unstable in this situation and more sampled sensor measurements should be transmitted; when $e_k^T(t)\Phi e_k(t) = \rho$ or $0, d = 0$ occurs, the adaptive event triggered scheme reduces to a event triggered scheme with a constant trigger parameter; when $e_k^T(t)\Phi e_k(t) < \rho, d = 1$ occurs, the value of ρ will increase, the transmitted sensor measurements will be reduced. We can see that the proposed event triggered scheme can reduce the burden of transmission and it is more adaptive than some existing ones.

The communication delays in the network are assumed to be $d_k, d_k \in [0, \overline{d}]$, where $\overline{d}$ is a positive real number. Under the event-triggered scheme (5), suppose the triggered instants are t_0h, $t_1h, t_2h, \cdots$, the correspondent triggered signals $y(t_0h), y(t_1h), y(t_2h), \ldots, y(t_kh)$ will arrive at the filter at instants $t_0h + d_0, t_1h + d_1$, $t_2h + d_2, \cdots$, respectively.

Remark 3. The sensors in the communication network are time-triggered with a constant sampling period h. The transmission of the sampled data is determined by the adaptive event-triggered communication scheme (5). The sensor samples measurements output $y(t)$ at time $kh, k \in R$, the release instants are $t_k, k \in R$, the transmitted measurement output are represented by $y(t_0h), y(t_1h)$, $y(t_2h), \ldots, y(t_kh), \cdots$ will reach the filter, where $t_0 = 0$ is the initial released instant.

Based on the above analysis, considering the behavior of ZOH, the input of the filter $\hat{y}(t)$ can be described as

$$\hat{y}(t) = \sum_{i=1}^{r} h_i y(t_kh), t \in [t_kh + d_k, t_{k+1}h + d_{k+1}) \tag{8}$$

For technical convenience, the following two cases will be considered:

Case 1: If $t_kh + h + \overline{d} \ge t_{k+1}h + d_{k+1}$, where $\overline{d} = \max d_k$, define a function $d(t)$ as

$$d(t) = t - t_kh, t \in [t_kh + d_k, t_{k+1}h + d_{k+1}) \tag{9}$$

Case 2: If $t_kh + h + \overline{d} < t_{k+1}h + d_{k+1}$, consider the following two intervals:

$$[t_kh + d_k, t_kh + h + \overline{d}), \quad [t_kh + ih + \overline{d}, t_kh + ih + h + \overline{d})$$

Since $d_k \le \overline{d}$, it can be easily shown that there exists a positive integer $\delta_M \ge 1$ such that

$$t_kh + \delta_M h + \overline{d} < t_{k+1}h + d_{k+1} \le t_kh + \delta_M h + h + \overline{d} \tag{10}$$

Moreover, $y(t_kh)$ and $t_kh + ih$ with $i = 1, 2, \ldots, \delta_M$ satisfy the event triggered scheme (5). Let

$$\begin{cases} I_0 = [t_kh + d_k, t_kh + h + \overline{d}) \\ I_i = [t_kh + ih + \overline{d}, t_kh + ih + h + \overline{d}) \\ I_{d_M} = [t_kh + \delta_M h + \overline{d}, t_{k+1}h + d_{k+1}) \end{cases} \tag{11}$$

where $i = 1, 2, \ldots, \delta_{M-1}$. It can be easily shown that

$$[t_kh + d_k, t_{k+1}h + d_{k+1}) = \bigcup_{i=0}^{i=\delta_M} I_i \tag{12}$$

Define

$$d(t) = \begin{cases} t - t_kh, t \in I_0 \\ t - t_kh - ih, t \in I_i, i = 1, 2, \cdots, \delta_M - 1 \\ t - t_kh - \delta_M h, t \in I_{\delta_M} \end{cases} \tag{13}$$

From the definition of $d(t)$, we can define

$$e_k(t) = \begin{cases} 0, t \in I_0 \\ y(t_kh) - y(t_kh + ih), t \in I_i, i = 1, 2, \cdots, \delta_M - 1 \\ y(t_kh) - y(t_kh + \delta_M h), t \in I_{\delta_M} \end{cases} \tag{14}$$

Remark 4. From the definition of $e_k(t)$ and the triggering algorithm (5), we can obtain that for $[t_kh + d_k, t_{k+1}h + d_{k+1})$

$$e_k(t) = y(t_kh) - y(t - d(t)). \tag{15}$$

Remark 5. Noticed that $d(t)$ is different from the traditional time-varying delay. $d(t)$ depends not only on the release times, but also on the network induce delay d_k and the sampling period h.

Remark 6. Since there exists a communication network between the sensor and the filter, the premises in the system and the ones at the filter side are asynchronous. That is, when $\theta_i(t)$ is available in (2), only $\theta_i(t_kh)$ is available at the filter side at the same instant $t \in [t_kh + d_k, t_k + lh + d_{k+1})$. In this paper, we assume that the mechanical model of the studied system is known a priori, once the initial condition is given, based on the known mechanical model, the state of the studied system can be calculated. Since $\theta_i(t_kh)$ is available at the filter side, $\theta_i(t)$ can be calculated for $t \in [t_kh, t_{k+1}h)$. Therefore, the synchronous premise variables $\theta_i(t)$ can be derived at the filter side.

From the above description, the adaptive event-triggered scheme can be expressed as

$$t_{k+j}h = t_kh + \min\{nh \mid e_k^T(t)\Phi e_k(t) > \delta(t_{k+j}h) y^T(t - d(t))\Phi y(t - d(t))\} \tag{16}$$

Based on the above description, the actual output $\hat{y}(t)$ in (8) can be described as

$$\hat{y}(t) = \sum_{i=1}^{r} h_i(e_k(t) + C_i x(t - d(t))), t \in [t_k + d_k, t_{k+1} + d_{k+1}) \tag{17}$$

Combine (4) and (17), the defuzzified value of the filter can be rewritten as

$$\begin{cases} \dot{x}_f(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j [A_{fj} x_f(t) + B_{fj}(t) e_k(t) + B_{fj} C_i x(t-d(t))] \\ z_f(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j C_{fj} x_f(t) \end{cases} \tag{18}$$

By introducing a new vector $e(t) = \left[x^T(t) \;\; x_f^T(t)]^T\right]^T$ and letting the filtering error be $\tilde{z}(t) = z(t) - z_f(t)$, based on Eqs. (2) and (18), the following augment system can be obtained:

$$\begin{cases} \dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left\{ \overline{A} e(t) + \overline{A}_d H e(t-\tau(t)) + \overline{B} H e(t-d(t)) + \overline{B}_1 e_k(t) + \overline{A}_w \omega(t) \right\} \\ \tilde{z}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \overline{L} e(t) \end{cases} \tag{19}$$

where

$$\overline{A} = \begin{bmatrix} A_i & 0 \\ 0 & A_{fj} \end{bmatrix}, \quad \overline{A}_d = \begin{bmatrix} A_{di} \\ 0 \end{bmatrix}, \quad \overline{B} = \begin{bmatrix} 0 \\ B_{fj} C_i \end{bmatrix},$$

$$\overline{B_1} = \begin{bmatrix} 0 \\ B_{fj} \end{bmatrix}, \quad \overline{A}_\omega = \begin{bmatrix} A_{\omega i} \\ 0 \end{bmatrix}, \quad \overline{L} = \begin{bmatrix} L_i & -C_{fj} \end{bmatrix},$$

$$\overline{H} = \begin{bmatrix} I & 0 \end{bmatrix}$$

The following lemmas and definitions are needed in the proof of our main results.

Lemma 1 (*see Peng and Tian [16]*). *For any constant matrix $R \in \mathbb{R}, R = R^T > 0$, vector function $\dot{x} : [-\tau_M, 0] \to \mathbb{R}^n$ and constant $\tau_M > 0$, so that the following integration is well defined, it holds that*

$$-\tau_M \int_{t-\tau_M}^{t} \dot{x}^T(s) R \dot{x}(s)\, ds \le \begin{bmatrix} x(t) \\ x(t-\tau_M) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau_M) \end{bmatrix} \tag{20}$$

Lemma 2 (*see Wang and Xie [32]*). *For any vectors $x, y \in R^n$, and positive definite matrix $Q \in R^{n \times n}$, the following inequality holds that:*

$$2x^T y \le x^T Q x + y^T Q^{-1} y \tag{21}$$

Lemma 3 (*see Tian et al. [28]*). *Suppose Ξ_1, Ξ_2 and Ω are constant matrices of appropriate dimensions, $0 \le \tau_m \le \tau(t) \le \tau_M$, then*

$$(\tau(t) - \tau_m)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + \Omega < 0 \tag{22}$$

if and only if

$$(\tau_M - \tau_m)\Xi_1 + \Omega < 0$$

and

$$(\tau_M - \tau_m)\Xi_2 + \Omega < 0$$

hold.

3. Main results

Theorem 1. *For some given constants $0 \le \tau_m \le \tau_M, d_M, \gamma$ and δ_M, the system (19) is exponentially mean-square (EMSS) with a prescribed H_∞ performance γ under the adaptive event-triggered scheme, if there exist matrices $Q_1 > 0, Q_2 > 0, Q_3 > 0, R_1 > 0, R_2 > 0, R_3 > 0, \Phi > 0, M_{ij} > 0, N_{ij} > 0, T_{ij} > 0, S_{ij} > 0$ with appropriate dimensions so that the following matrix inequalities hold:*

$$\Pi^{ij}(s) + \Pi^{ji}(s) < 0, \quad i \le j \in R \tag{23}$$

where

$$\Pi^{ij}(s) = \begin{bmatrix} \Pi_{11}^{ij} & * & * & * \\ \Pi_{21}^{ij} & -I & * & * \\ \Pi_{31}^{ij} & 0 & \Pi_{33}^{ij} & * \\ \Pi_{41}^{ij}(s) & 0 & 0 & \Pi_{44}^{ij} \end{bmatrix} < 0, \quad s = 1,2,3,4.$$

$$\Pi_{11}^{ij} = \begin{bmatrix} \Sigma_{ij1} & * & * & * & * & * & * & * \\ R_2 & \Sigma_{ij2} & * & * & * & * & * & * \\ H^T \overline{A_d}^T P & M_{ij3} - M_{ij2}^T & \Sigma_{ij3} & * & * & * & * & * \\ 0 & 0 & N_{ij4} - N_{ij3}^T & \Sigma_{ij4} & * & * & * & * \\ H^T \overline{B}^T P + S_{ij5} - S_{ij1}^T & 0 & 0 & 0 & \Sigma_{ij5} & * & * & * \\ 0 & 0 & 0 & 0 & T_{ij6} - T_{ij5}^T & \Sigma_{ij6} & * & * \\ \overline{B_1}^T P & 0 & 0 & 0 & 0 & 0 & -\Phi & * \\ \overline{A_w}^T P & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\Sigma_{ij1} = \overline{A}^T P + P\overline{A} + S_{ij1} + S_{ij1}^T + Q_1 + Q_2 + Q_3 - R_2,$$
$$\Sigma_{ij2} = -Q_1 - R_2 + M_{ij2} + M_{ij2}^T$$
$$\Sigma_{ij3} = -M_{ij3} - M_{ij3}^T + N_{ij3} + N_{ij3}^T,$$
$$\Sigma_{ij4} = -N_{ij4} - N_{ij4}^T - Q_2$$
$$\Sigma_{ij5} = -S_{ij5} - S_{ij5}^T + T_{ij5} + T_{ij5}^T + \delta_M H^T C_i^T \Phi C_i H,$$
$$\Sigma_{ij6} = -Q_3 - T_{ij6} - T_{ij6}^T$$
$$\Pi_{21}^{ij} = \begin{bmatrix} \overline{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Pi_{33}^{ij} = \mathrm{diag}\{-PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\}$$
$$\Pi_{31}^{ij} = \begin{bmatrix} \sqrt{\tau_{21}}P\overline{A} & 0 & \sqrt{\tau_{21}}P\overline{A}_d H & 0 & \sqrt{\tau_{21}}P\overline{B}H & 0 & \sqrt{\tau_{21}}P\overline{B}_1 & \sqrt{\tau_{21}}P\overline{A}_\omega \\ \tau_m P\overline{A} & 0 & \tau_m P\overline{A}_d H & 0 & \tau_m P\overline{B}H & 0 & \tau_m P\overline{B}_1 & \tau_m P\overline{A}_\omega \\ \sqrt{d_M}P\overline{A} & 0 & \sqrt{d_M}P\overline{A}_d H & 0 & \sqrt{d_M}P\overline{B}H & 0 & \sqrt{d_M}P\overline{B}_1 & \sqrt{d_M}P\overline{A}_\omega \end{bmatrix},$$

$$\delta_M = \max \delta(t)$$

$$\Pi_{41}^{ij}(1) = \begin{bmatrix} \sqrt{\tau_{21}} M_{ij}^T \\ \sqrt{d_M} S_{ij}^T \end{bmatrix}, \quad \Pi_{41}^{ij}(2) = \begin{bmatrix} \sqrt{\tau_{21}} M_{ij}^T \\ \sqrt{d_M} T_{ij}^T \end{bmatrix}$$

$$\Pi_{41}^{ij}(3) = \begin{bmatrix} \sqrt{\tau_{21}} N_{ij}^T \\ \sqrt{d_M} T_{ij}^T \end{bmatrix}, \quad \Pi_{41}^{ij}(4) = \begin{bmatrix} \sqrt{\tau_{21}} N_{ij}^T \\ \sqrt{d_M} S_{ij}^T \end{bmatrix}, \quad \tau_{21} = \tau_M - \tau_m$$

$$M_{ij}^T = \begin{bmatrix} 0 & M_{ij2}^T & M_{ij3}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$N_{ij}^T = \begin{bmatrix} 0 & 0 & N_{ij3}^T & N_{ij4}^T & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$S_{ij}^T = \begin{bmatrix} S_{ij1}^T & 0 & 0 & 0 & S_{ij5}^T & 0 & 0 & 0 \end{bmatrix},$$
$$T_{ij}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & T_{ij5}^T & T_{ij6}^T & 0 & 0 \end{bmatrix}$$

Proof. Consider the following Lyapunov functional candidate for system (19)

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{24}$$

where

$$V_1(t) = e^T(t) P e(t)$$

$$V_2(t) = \int_{t-\tau_m}^{t} e^T(s) Q_1 e(s)\, ds + \int_{t-\tau_M}^{t} e^T(s) Q_2 e(s)\, ds + \int_{t-d_M}^{t} e^T(s) Q_3 e(s)\, ds$$

$$V_3(t) = \int_{t-\tau_M}^{t-\tau_m} \int_{s}^{t} \dot{e}^T(v) R_1 \dot{e}(v)\, dv\, ds + \tau_m \int_{t-\tau_m}^{t} \int_{s}^{t} \dot{e}^T(v) R_2 \dot{e}(v)\, dv\, ds + \int_{t-d_M}^{t} \int_{s}^{t} \dot{e}^T(v) R_3 \dot{e}(v)\, dv\, ds$$

and $P > 0$, $Q_i > 0, R_i > 0\ (i = 1,2,3)$ are all symmetric positive definite matrices with appropriate dimensions. Taking the derivation of $V(t)$, we have

$$\dot{V}_1(t)=2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\dot{e}^T(t)Pe(t) \tag{25}$$

$$\dot{V}_2(t)=e^T(t)(Q_1+Q_2+Q_3)e(t)-x^T(t-\tau_m)Q_1e(t-\tau_m) \tag{26}$$

$$-e^T(t-\tau_M)Q_2e(t-\tau_M)-e^T(t-d_M)Q_3e(t-d_M)$$

$$\dot{V}_3(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\dot{e}^T(t)[(\tau_M-\tau_m)R_1+\tau_m^2R_2+d_MR_3]\dot{e}(t)$$
$$-\int_{t-\tau_M}^{t-\tau_m}\dot{e}^T(s)R_1\dot{x}(s)\,ds-\tau_m\int_{t-\tau_m}^{t}\dot{e}^T(s)R_2\dot{e}(s)\,ds$$
$$-\int_{t-d_M}^{t}\dot{e}^T(s)R_3\dot{e}(s)\,ds \tag{27}$$

Using and employing the free-weighting matrix method, the following is obtained:

$$\dot{V}(t)+\tilde{z}^T(t)\tilde{z}(t)-\gamma^2\omega^T(t)\omega(t)\le 2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\dot{e}^T(t)Pe(t)$$
$$+e^T(t)(Q_1+Q_2+Q_3)e(t)-e^T(t-\tau_m)Q_1e(t-\tau_m)$$
$$-e^T(t-\tau_M)Q_2e(t-\tau_M)-e^T(t-d_M)Q_3e(t-d_M)$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\dot{e}^T(t)[\tau_{21}R_1+\tau_m^2R_2+d_MR_3]\dot{e}(t)$$
$$-\int_{t-\tau_M}^{t-\tau_m}\dot{e}^T(s)R_1\dot{e}(s)\,ds-\tau_m\int_{t-\tau_m}^{t}\dot{x}^T(s)R_2\dot{x}(s)\,ds$$
$$-\int_{t-d_M}^{t}\dot{e}^T(s)R_3\dot{e}(s)\,ds$$
$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)M_{ij}\left[e(t-\tau_m)-e(t-\tau(t))-\int_{t-\tau(t)}^{t-\tau_m}\dot{e}(s)\,ds\right]$$
$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)N_{ij}\left[e(t-\tau(t))-e(t-\tau_M)-\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)\,ds\right]$$
$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)S_{ij}\left[e(t)-e(t-d(t))-\int_{t-d(t)}^{t}\dot{e}(s)\,ds\right] \tag{28}$$

$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)T_{ij}\left[e(t-d(t))-e(t-d_M)-\int_{t-d_M}^{t-d(t)}\dot{e}(s)\,ds\right]$$
$$+\sigma_Me^T(t-d(t))H^TC_i^T\Phi C_iHe(t-d(t))$$
$$-e_k^T(t)\Phi e_k(t)+\tilde{z}^T(t)\tilde{z}(t)-\gamma^2\omega^T(t)\omega(t)$$

where N_{ij}, M_{ij}, T_{ij} and S_{ij} are matrices with appropriate dimensions, and

$$\zeta^T(t)=\begin{bmatrix}e^T(t) & e^T(t-\tau_m) & e^T(t-\tau(t)) & e^T(t-\tau_M) & e^T(t-d(t)) & e^T(t-d_M) & e_k^T(t) & \omega^T(t)\end{bmatrix}$$
$$M_{ij}^T=\begin{bmatrix}0 & M_{ij2}^T & M_{ij3}^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$
$$N_{ij}^T=\begin{bmatrix}0 & 0 & N_{ij3}^T & N_{ij4}^T & 0 & 0 & 0 & 0\end{bmatrix}$$
$$S_{ij}^T=\begin{bmatrix}S_{ij1}^T & 0 & 0 & 0 & S_{ij5}^T & 0 & 0 & 0\end{bmatrix}$$
$$T_{ij}^T=\begin{bmatrix}0 & 0 & 0 & 0 & T_{ij5}^T & T_{ij6}^T & 0 & 0\end{bmatrix}$$

Applying Lemma 1, we have that

$$-\tau_m\int_{t-\tau_m}^{t}\dot{e}^T(s)R_2\dot{e}(s)\,ds\le\begin{bmatrix}e(t)\\ e(t-\tau_m)\end{bmatrix}^T\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}e(t)\\ e(t-\tau_m)\end{bmatrix} \tag{29}$$

By using Lemma 2, we have that

$$-2\zeta^T(t)M_{ij}\int_{t-\tau(t)}^{t-\tau_m}\dot{e}(s)\,ds$$
$$\le\int_{t-\tau(t)}^{t-\tau_m}\dot{e}^T(s)R_1\dot{e}(s)\,ds+(\tau(t)-\tau_m)\zeta^T(t)M_{ij}R_1^{-1}M_{ij}^T\zeta(t) \tag{30}$$

$$-2\zeta^T(t)N_{ij}\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)\,ds$$
$$\le\int_{t-\tau_M}^{t-\tau(T)}\dot{e}^T(s)R_1\dot{e}(s)\,ds+(\tau_M-\tau(t))\zeta^T(t)N_{ij}R_1^{-1}N_{ij}^T\zeta(t) \tag{31}$$

$$-2\zeta^T(t)S_{ij}\int_{t-d(t)}^{t}\dot{e}(s)\,ds$$
$$\le\int_{t-d(t)}^{t}\dot{e}^T(s)R_3\dot{e}(s)\,ds+d(t)\zeta^T(t)S_{ij}R_3^{-1}S_{ij}^T\zeta(t) \tag{32}$$

$$-2\zeta^T(t)T_{ij}\int_{t-d_M}^{t-d(t)}\dot{e}(s)\,ds$$
$$\le\int_{t-d_M}^{t-d(t)}\dot{e}^T(s)R_3\dot{e}(s)\,ds+(d_M-d(t))\zeta^T(t)T_{ij}R_3^{-1}T_{ij}^T\zeta(t) \tag{33}$$

Combining the event triggered scheme (5) and (33)–(35), one can obtain

$$\dot{V}(t)+\tilde{z}^T(t)\tilde{z}(t)-\gamma^2\omega^T(t)\omega(t)$$
$$\le 2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\dot{e}^T(t)Pe(t)+e^T(t)(Q_1+Q_2+Q_3)e(t)$$
$$-e^T(t-\tau_m)Q_1e(t-\tau_m)$$
$$-e^T(t-\tau_M)Q_2e(t-\tau_M)-e^T(t-d_M)Q_3e(t-d_M)$$
$$+\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\dot{e}^T(t)[\tau_{21}R_1+\tau_m^2R_2+d_MR_3]\dot{e}(t)$$
$$+\begin{bmatrix}e(t)\\ e(t-\tau_m)\end{bmatrix}^T\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix}\begin{bmatrix}e(t)\\ e(t-\tau_m)\end{bmatrix}$$
$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)M_{ij}[e(t-\tau_m)-e(t-\tau(t))]$$
$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)N_{ij}[e(t-\tau(t))-e(t-\tau_M)]$$
$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)S_{ij}[e(t)-e(t-d(t))]$$
$$+2\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\zeta^T(t)T_{ij}[e(t-d(t))-e(t-d_M)]$$
$$+\sigma_Me^T(t-d(t))H^TC_i^T\Phi C_iHe(t-d(t))$$
$$-e_k^T(t)\Phi e_k(t)+\tilde{z}^T(t)\tilde{z}(t)-\gamma^2\omega^T(t)\omega(t)$$
$$+(\tau(t)-\tau_m)\zeta^T(t)M_{ij}R_1^{-1}M_{ij}^T\zeta(t)$$
$$+(\tau_M-\tau(t))\zeta^T(t)N_{ij}R_1^{-1}N_{ij}^T\zeta(t)+d(t)\zeta^T(t)S_{ij}R_3^{-1}S_{ij}^T\zeta(t)$$
$$+(d_M-d(t))\zeta^T(t)T_{ij}R_3^{-1}T_{ij}^T\zeta(t)$$
$$\le\sum_{i=1}^{r}\sum_{j=1}^{r}h_ih_j\Big\{\zeta^T(t)\Pi_{11}^{ij}\zeta(t)+\dot{e}^T(t)\left[\tau_{21}R_1+\tau_m^2R_2+d_MR_3\right]\dot{e}(t)$$
$$+\tilde{z}^T(t)\tilde{z}(t)+(\tau(t)-\tau_m)\zeta^T(t)M_{ij}R_1^{-1}M_{ij}^T\zeta(t)$$
$$+(\tau_M-\tau(t))\zeta^T(t)N_{ij}R_1^{-1}N_{ij}^T\zeta(t)+d(t)\zeta^T(t)S_{ij}R_3^{-1}S_{ij}^T\zeta(t)$$
$$+(d_M-d(t))\zeta^T(t)T_{ij}R_3^{-1}T_{ij}^T\zeta(t)\Big\} \tag{34}$$

By using well-known Schur complement and Lemma 3, from (23), one can easily see that

$$\dot{V}(t)\le\gamma^2\omega^T(t)\omega(t)-\tilde{z}^T(t)\tilde{z}(t) \tag{35}$$

The remaining part of the proof is similar to those in [19] and so omitted here for simplicity. The proof is complete.□

In the following, based on analysis results in Theorem 1, we give a method design the desired filter. The explicit expression of the designed filter parameters are given in the following theorem.

Theorem 2. *For some given constants* $0 \le \tau_m \le \tau_M, d_M, \gamma, \varepsilon_1, \varepsilon_2, \varepsilon_3$ *and* δ_M, *systems (19) is exponentially mean-square stable (EMSS) with a prescribed* H_∞ *performance* γ *under the adaptive event-triggered scheme, if there exist* $P_1>0, \overline{P}_3>0, \overline{Q}_1>0, \overline{Q}_2>0, \overline{Q}_3>0, \overline{R}_1>0, \overline{R}_2>0, \overline{R}_3>0, \Phi>0, \overline{A}_{fj}, \overline{B}_{fj}, \overline{C}_{fj}$, *and* $\overline{M}_{ij}>0, \overline{N}_{ij}>0, \overline{S}_{ij}>0, \overline{T}_{ij}>0$ *with appropriate dimensions so that the following LMIs hold:*

$$\Omega^{ij}(s)+\Omega^{ji}(s)<0, \quad i \le j \in R \tag{36}$$

where

$$\Omega^{ij}(s)=\begin{bmatrix} \Omega_{11}^{ij} & * & * & * \\ \Omega_{21}^{ij} & -I & * & * \\ \Omega_{31}^{ij} & 0 & \Omega_{33}^{ij} & * \\ \Omega_{41}^{ij}(s) & 0 & 0 & \Omega_{44}^{ij} \end{bmatrix}<0, \quad s=1,2,3,4.$$

$$\Omega_{11}^{ij}=\begin{bmatrix} \overline{\Sigma}_{ij1} & * & * & * & * & * & * & * \\ \overline{R}_2 & \overline{\Sigma}_{ij2} & * & * & * & * & * & * \\ \Upsilon_{ij31} & \overline{M}_{ij3}-\overline{M}_{ij2}^T & \overline{\Sigma}_{ij3} & * & * & * & * & * \\ 0 & 0 & \overline{N}_{ij4}-\overline{N}_{ij3}^T & \overline{\Sigma}_{ij4} & * & * & * & * \\ \Upsilon_{ij51}+\overline{S}_{ij5}-\overline{S}_{ij1}^T & 0 & 0 & 0 & \overline{\Sigma}_{ij5} & * & * & * \\ 0 & 0 & 0 & 0 & \overline{T}_{ij6}-\overline{T}_{ij5}^T & \overline{\Sigma}_{ij6} & * & * \\ \Upsilon_{ij71} & 0 & 0 & 0 & 0 & 0 & -\overline{\Phi} & * \\ \Upsilon_{ij81} & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\overline{\Sigma}_{ij1}=\Upsilon_{ij11}+\Upsilon_{ij11}^T+\overline{S}_{ij1}+\overline{S}_{ij1}^T+\overline{Q}_1+\overline{Q}_2+\overline{Q}_3-\overline{R}_2, \quad \Upsilon_{ij11}=\begin{bmatrix} P_1A_i & \overline{A}_{fj} \\ P_3A_i & \overline{A}_{fj} \end{bmatrix}$$

$$\overline{\Sigma}_{ij2}=-\overline{Q}_1-\overline{R}_2+\overline{M}_{ij2}+\overline{M}_{ij2}^T, \quad \overline{\Sigma}_{ij3}=-\overline{M}_{ij3}-\overline{M}_{ij3}^T+\overline{N}_{ij3}+\overline{N}_{ij3}^T,$$

$$\overline{\Sigma}_{ij4}=-\overline{N}_{ij4}-\overline{N}_{ij4}^T-\overline{Q}_2$$

$$\overline{\Sigma}_{ij5}=-\overline{S}_{ij5}-\overline{S}_{ij5}^T+\overline{T}_{ij5}+\overline{T}_{ij5}^T+\Upsilon_{ij55},$$

$$\overline{\Sigma}_{ij6}=-\overline{Q}_3-\overline{T}_{ij6}-\overline{T}_{ij6}^T, \quad \Upsilon_{ij55}=\begin{bmatrix} \delta_M C_i^T \Phi C_i & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Upsilon_{ij31}=\begin{bmatrix} A_{di}^T P_1 & A_{di}^T \overline{P}_3 \\ 0 & 0 \end{bmatrix}, \quad \Upsilon_{ij51}=\begin{bmatrix} C_i^T \overline{B}_{fj}^T & C_i^T \overline{B}_{fj}^T \\ 0 & 0 \end{bmatrix},$$

$$\Upsilon_{ij71}=\begin{bmatrix} \overline{B}_{fj}^T & \overline{B}_{fj}^T \end{bmatrix}, \quad \Upsilon_{ij81}=\begin{bmatrix} A_{\omega i}^T P_1 & A_{\omega i}^T \overline{P}_3 \end{bmatrix}$$

$$\Omega_{21}^{ij}=\begin{bmatrix} \Upsilon_{ij91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Upsilon_{ij91}=\begin{bmatrix} L_i & -\overline{C}_{fj} \end{bmatrix}$$

$$\Omega_{31}^{ij}=\begin{bmatrix} \sqrt{\tau_{21}}\Upsilon_{ij11} & 0 & \sqrt{\tau_{21}}\Upsilon_{ij31}^T & 0 & \sqrt{\tau_{21}}\Upsilon_{ij51}^T & 0 & \sqrt{\tau_{21}}\Upsilon_{ij71}^T & \sqrt{\tau_{21}}\Upsilon_{ij81}^T \\ \tau_m\Upsilon_{ij11} & 0 & \tau_m\Upsilon_{ij31}^T & 0 & \tau_m\Upsilon_{ij51}^T & 0 & \tau_m\Upsilon_{ij71}^T & \tau_m\Upsilon_{ij81}^T \\ \sqrt{d_M}\Upsilon_{ij11} & 0 & \sqrt{d_M}\Upsilon_{ij31}^T & 0 & \sqrt{d_M}\Upsilon_{ij51}^T & 0 & \sqrt{d_M}\Upsilon_{ij71}^T & \sqrt{d_M}\Upsilon_{ij81}^T \end{bmatrix},$$

$$\delta_M=\max\delta(t)$$

$$\Omega_{33}^{ij}=\mathrm{diag}\{-2\varepsilon_1\overline{P}+\varepsilon_1^2\overline{R}_1, -2\varepsilon_1\overline{P}+\varepsilon_1^2\overline{R}_2, -2\varepsilon_1\overline{P}+\varepsilon_1^2\overline{R}_3\},$$

$$\overline{P}=\begin{bmatrix} P_1 & \overline{P}_3 \\ \overline{P}_3 & \overline{P}_3 \end{bmatrix}, \quad \Omega_{44}^{ij}=\mathrm{diag}\{-\overline{R}_1, -\overline{R}_3\}$$

$$\Pi_{41}^{ij}(1)=\begin{bmatrix} \sqrt{\tau_{21}}\overline{M}_{ij}^T \\ \sqrt{d_M}\overline{S}_{ij}^T \end{bmatrix}, \quad \Pi_{41}^{ij}(2)=\begin{bmatrix} \sqrt{\tau_{21}}\overline{M}_{ij}^T \\ \sqrt{d_M}\overline{T}_{ij}^T \end{bmatrix},$$

$$\Pi_{41}^{ij}(3)=\begin{bmatrix} \sqrt{\tau_{21}}\overline{N}_{ij}^T \\ \sqrt{d_M}\overline{T}_{ij}^T \end{bmatrix}, \quad \Pi_{41}^{ij}(4)=\begin{bmatrix} \sqrt{\tau_{21}}\overline{N}_{ij}^T \\ \sqrt{d_M}\overline{S}_{ij}^T \end{bmatrix}, \tau_{21}=\tau_M-\tau_m$$

$$\overline{M}_{ij}^T=\begin{bmatrix} 0 & \overline{M}_{ij2}^T & \overline{M}_{ij3}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{N}_{ij}^T=\begin{bmatrix} 0 & 0 & \overline{N}_{ij3}^T & \overline{N}_{ij4}^T & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\overline{S}_{ij}^T=\begin{bmatrix} \overline{S}_{ij1}^T & 0 & 0 & 0 & \overline{S}_{ij5}^T & 0 & 0 & 0 \end{bmatrix},$$

$$\overline{T}_{ij}^T=\begin{bmatrix} 0 & 0 & 0 & 0 & \overline{T}_{ij5}^T & \overline{T}_{ij6}^T & 0 & 0 \end{bmatrix}$$

Furthermore, if the above conditions are feasible, the parameter matrices of the filter can be obtained by

$$\begin{cases} A_{fj}=\overline{A}_{fj}\overline{P}_3^{-1} \\ B_{fj}=\overline{B}_{fj} \\ C_{fj}=\overline{C}_{fj}\overline{P}_3^{-1} \end{cases} \tag{37}$$

Proof. Due to

$$(R_k-\varepsilon^{-1}P)R_k^{-1}(R_k-\varepsilon^{-1}P)\ge 0$$

we have

$$-PR_k^{-1}P \le -2\varepsilon P+\varepsilon^2 R_k$$

Substituting $-PR_k^{-1}P$ with $-2\varepsilon P+\varepsilon^2 R_k$ into (23), we obtain

$$\overline{\Pi}^{ij}(s)+\overline{\Pi}^{ji}(s)<0, \quad i<j \in R \tag{38}$$

$$\overline{\Pi}^{ij}(s)=\begin{bmatrix} \Pi_{11}^{ij} & * & * & * \\ \Pi_{21}^{ij} & -I & * & * \\ \Pi_{31}^{ij} & 0 & \overline{\Pi}_{33}^{ij} & * \\ \Pi_{41}^{ij}(s) & 0 & 0 & \Pi_{44}^{ij} \end{bmatrix}<0, \quad s=1,2,3,4. \tag{39}$$

where

$$\overline{\Pi}_{33}^{ij}=\mathrm{diag}\{-2\varepsilon_1 P+\varepsilon_1^2 R_1, -2\varepsilon_1 P+\varepsilon_1^2 R_2, -2\varepsilon_1 P+\varepsilon_1^2 R_3\}$$

Since $\overline{P}_3>0$, there exist P_2 and $P_3>0$ satisfying $\overline{P}_3=P_2^T P_3^{-1} P_2$. Let the matrix P be partitioned as

$$P=\begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix},$$

where $P_1>0$ and $P_3>0$. Define the following invertible matrix:

$$J=\begin{bmatrix} I & 0 \\ 0 & P_2^T P_3^{-1} \end{bmatrix}$$

and $\Lambda=\mathrm{diag}\{\underbrace{J,\ldots,J}_{6}, I, I, I, \underbrace{J,\ldots,J}_{5}\}$, then, multiply (38) by Λ from the left side and its transpose from the right side, respectively. Define variables

$$\begin{cases} \overline{A}_{fj}=\hat{A}_{fj}\overline{P}_3, \hat{A}_{fj}=P_2^T A_{fj} P_2^{-T} \\ \overline{B}_{fj}=P_2^T B_{fj} \\ \overline{C}_{fj}=\hat{C}_{fj}\overline{P}_3, \hat{C}_{fj}=C_{fj}P_2^{-T} \end{cases} \tag{40}$$

Defining

$$\overline{P}=JPJ^T=\begin{bmatrix} P_1 & \overline{P}_3 \\ \overline{P}_3 & \overline{P}_3 \end{bmatrix},$$

$\overline{Q}_k=JQ_kJ^T, \overline{R}_k=JR_kJ^T, (k=1,2,3)\overline{M}_{ijv_1}=JM_{ijv_1}J^T,$
$\overline{N}_{ijv_2}=JN_{ijv_2}J^T, \overline{S}_{ijv_3}=JS_{ijv_3}J^T, \overline{T}_{ijv_4}=JT_{ijv_4}$
$J^T, (v_1=2,3.\ v_2=3,4.\ v_3=1,5. v_4=5,6).$

Then, Eq. (23) is equivalent to Eq. (36) for $s=1,2,3,4$, respectively.

Replacing the filter parameters (A_{fj}, B_{fj}, C_{fj}) by $(P_2^{-T}\hat{A}_{fj}P_2^T, P_2^{-T}\overline{B}_{fj}, \hat{C}_{fj}P_2^T)$ in (4), then, the filter (4) can be written as

$$\begin{cases} \dot{x}_f(t)=P_2^{-T}\hat{A}_{fj}P_2^T x_f(t)+P_2^{-T}\overline{B}_{fj}\hat{y}(t) \\ z_f(t)=\hat{C}_{fj}P_2^T x_f(t) \end{cases}, \tag{41}$$

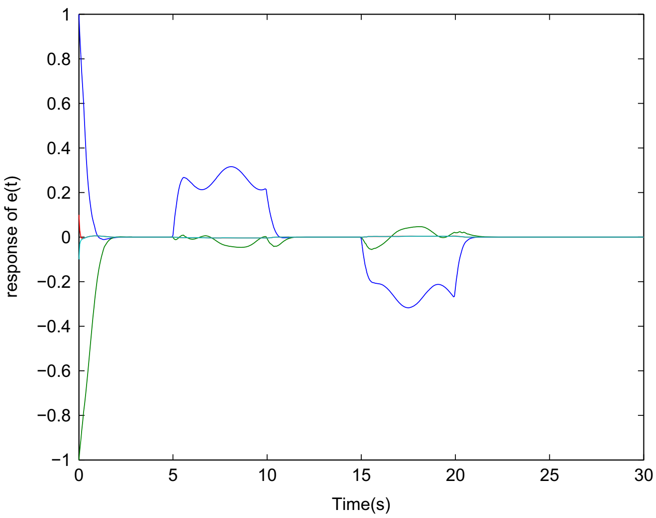

Fig. 2. Response of $e(t)$.

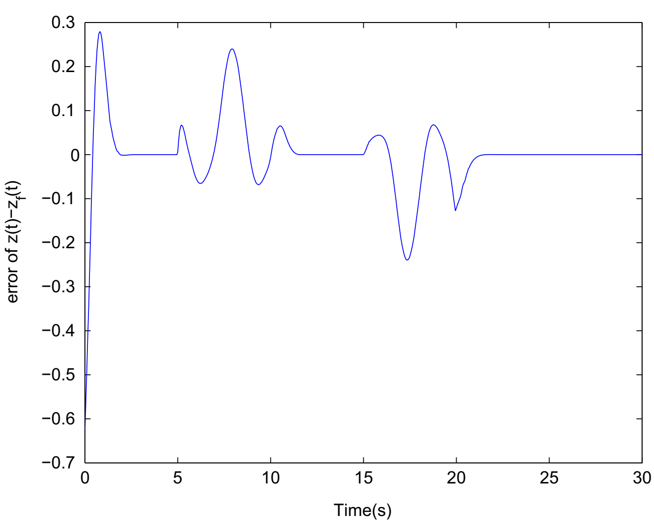

Fig. 3. Response of $\tilde{z}(t)$.

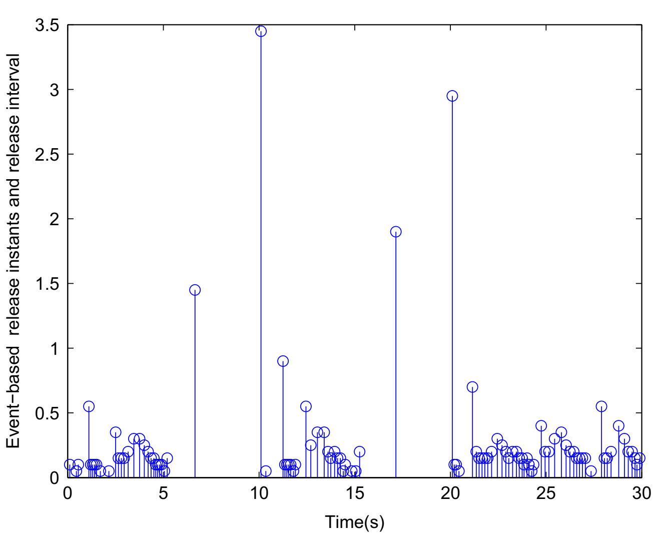

Fig. 4. Event-based release instants and release interval.

Defining $\hat{x}(t)=P_2^T x_f(t)$, similar to the analysis of [19], combing (41), we can get

$$\begin{cases} \dot{\hat{x}}_f(t)=\hat{A}_{fj}\hat{x}(t)+\overline{B}_{fj}\hat{y}(t) \\ z_f(t)=\hat{C}_{fj}\hat{x}(t) \end{cases} \tag{42}$$

Obviously, $(\hat{A}_{fj},\hat{B}_{fj},\hat{C}_{fj})$ can be chosen as the filter parameters and guarantee the filtering error system (19) to be asymptotically stable with an H_∞ disturbance attenuation level γ.

Then, from (40) and (42), we can obtain that the parameter matrices of the filter are given by (37). This completes the proof.□

4. Numerical example

Consider the discussed T-S fuzzy system (19) with the following parameters:

$$A_1=\begin{bmatrix}-2.1 & 0.1\\ 1 & -1\end{bmatrix},\quad A_2=\begin{bmatrix}-2 & 0\\ -0.2 & -1.1\end{bmatrix},$$

$$A_{d1}=\begin{bmatrix}-1 & 0.1\\ -0.8 & -0.9\end{bmatrix},\quad A_{d2}=\begin{bmatrix}-0.9 & 0\\ -1 & -0.8\end{bmatrix}$$

$$A_{\omega1}=\begin{bmatrix}1\\ -0.2\end{bmatrix},\quad A_{\omega2}=\begin{bmatrix}0.6\\ 0.3\end{bmatrix},\quad C_1=\begin{bmatrix}1 & 2\end{bmatrix},\quad C_2=\begin{bmatrix}1 & 0.5\end{bmatrix}$$

$$L_1=\begin{bmatrix}0.5 & -2\end{bmatrix},\quad L_2=\begin{bmatrix}-0.3 & 0.3\end{bmatrix}$$

$$w(t)=\begin{cases}1, & 5\le t\le 10\\ -1, & 15\le t\le 20\\ 0, & \text{others}\end{cases}$$

$$h_1(\theta(t))=\sin^2 t,\quad h_2(\theta(t))=\cos^2 t$$

In the following, we will show the effectiveness of the adaptive event-triggered scheme and co-design an H_∞ filter in the form of (4) and the adaptive event-triggered communication (5).

Set $\tau_m=0.1$, $\tau_M=0.4$, $d_M=0.5$, $\gamma=1.2$, $\rho=0.0092$, $\delta_M=0.5$, by applying Theorem 2, we can obtain the corresponding trigger matrix $\Phi=3.0442$, and the following desired filter parameters:

$$A_{f1}=\begin{bmatrix}-30.8276 & -4.6034\\ -2.7563 & -27.4202\end{bmatrix},$$

$$B_{f1}=\begin{bmatrix}-0.6320\\ -0.4958\end{bmatrix},\quad C_{f1}=\begin{bmatrix}0.8001 & 0.6263\end{bmatrix}$$

$$A_{f2}=\begin{bmatrix}-30.9176 & -4.5640\\ -2.6688 & -27.5706\end{bmatrix},$$

$$B_{f2}=\begin{bmatrix}-0.6113\\ -0.5294\end{bmatrix},\quad C_{f2}=\begin{bmatrix}0.6879 & 0.4471\end{bmatrix},$$

For the initial conditions $x(0)=[1-1], x_f(0)=[0.1;-0.1]$ and the sampling period $h=0.05$, the simulation result for the response of $e(t)$ is shown in Fig. 2, the response of filtering error $\tilde{z}(t)$ is depicted in Fig. 3 and the adaptive event triggering release instants and intervals are shown in Fig. 4. The images of $\delta(t)$ and d are shown in Figs. 5 and 6. From the above simulation results, we confirm that the adaptive event-triggered filtering scheme is effective to reduce the communication load in the network and the designed filter can also satisfy the system performance.

5. Conclusions

In this paper, we discuss an adaptive event-triggered H_∞ filter problem for T-S fuzzy system with time delay. Considering unreliable communication networks and limited network resources, an adaptive event triggered scheme is introduced to reduce the

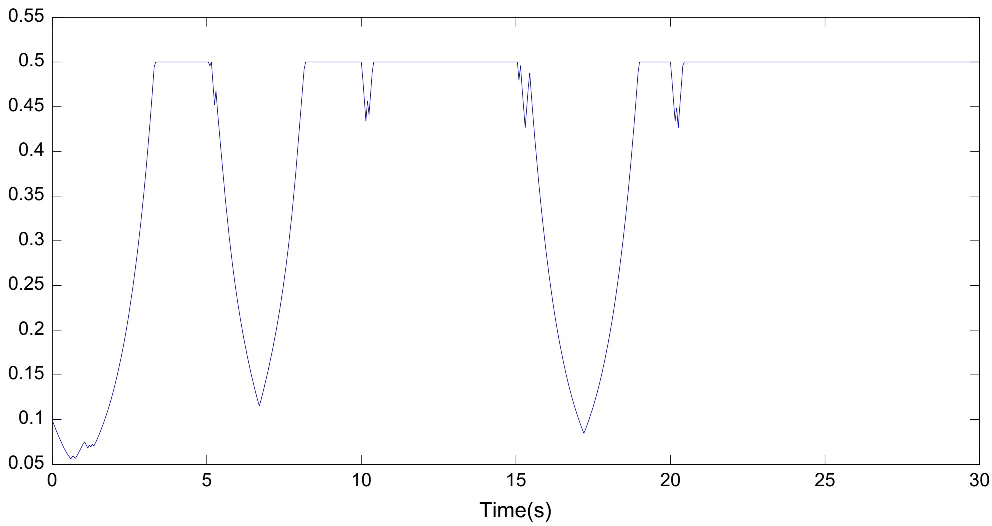

Fig. 5. Parameter δ.

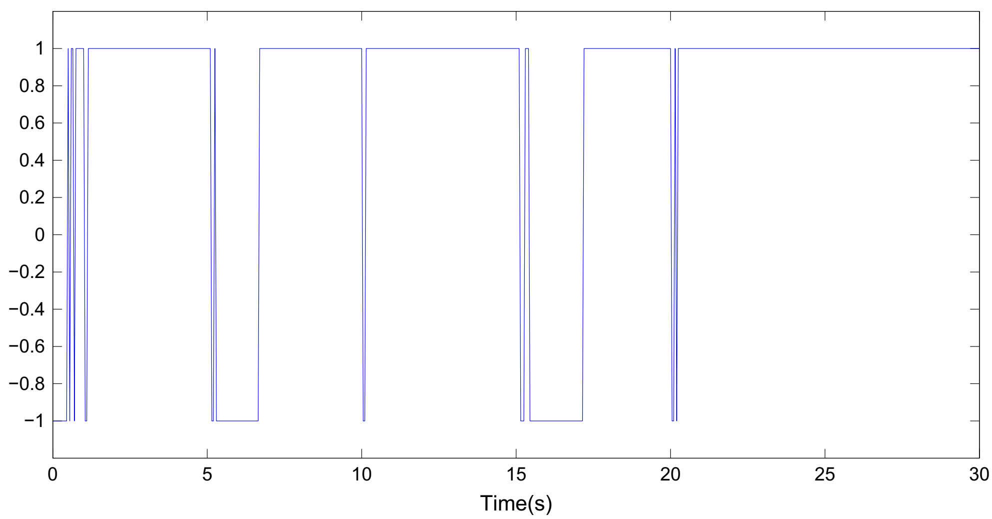

Fig. 6. Parameter d.

utilization of network bandwidth and gearing up its efficiency. Moreover, employing the networked*T-S* fuzzy model under the adaptive event triggered scheme, the fundamental stability criteria are obtained and a filter design method is developed. Finally, numerical examples have been carried out to show the effectiveness of the proposed method.

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