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H_∞ filtering for networked systems with hybrid-triggered communication mechanism and stochastic cyber attacks

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Abstract

This paper concentrates on investigating H_∞ filtering for networked systems with hybrid-triggered communication mechanism and stochastic cyber attacks. Random variables satisfying Bernoulli distribution are introduced to describe the hybrid-triggered scheme and stochastic cyber attacks, respectively. Firstly, a mathematical H_∞ filtering error model with hybrid-triggered communication mechanism is constructed under the stochastic cyber attacks. Secondly, by using Lyapunov stability theory and linear matrix inequality (LMI) techniques, the sufficient conditions which can guarantee the stability of augmented filtering error system are obtained and the parameters of the designed filter can be presented in an explicit form. Finally, numerical examples are given to demonstrate the feasibility of the designed filter.

1. Introduction

Networked control systems (NCSs) are a kind of control systems wherein feedback signals and control signals are exchanged through a network in a form of information package. It's

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characterized by enabling the execution of several tasks far away when it connects cyber space to physical space [1]. Due to the advantages such as high flexibility, low cost and simple installation [2,3], NCSs are applied in different kinds of fields referring to aircrafts, automobiles, vehicles and so on [4–6]. Therefore, NCSs are attracting more and more interest owing to the development and the advantages of the Internet [7]. In [8], the robust H_∞ controller is designed for networked control systems with uncertainties such like network-induced delay and data dropouts. The synchronization problem of feedback control is investigated in [9] with time-varying delay for complex dynamic networks. The authors in [10] study the robust fault tolerant control problem for distributed network control systems.

In the past few years, time-triggered method (periodic sampling) is widely adopted for system modeling and analysis in the NCSs [11], however, periodic sampling will generate lots of redundant signals if all the sampled data is transmitted through the network. To make full use of the limited network resource, lots of researchers propose the event-triggered schemes to overcome the problem caused by periodic sampling, for example, a novel event-triggered scheme is proposed in [12], in which a H_∞ controller is designed for NCSs. The core idea of the novel event-triggered scheme in [12] is that whether the newest sampled data is released or not is dependent on a threshold, and the adoption of event-triggered scheme can largely help alleviate the burden of the network [13]. Consequently, there are large numbers of researchers interested in the investigations about the novel event-triggered scheme proposed in [12]. An event-triggered non-parallel distribution compensation control problem in [14] is addressed for networked Takagi–Sugeno (T–S) fuzzy systems. The authors of [15] consider the event-triggered filtering problem for discrete-time linear system with package dropouts satisfying Bernoulli distribution. In [16], a discrete event-triggered scheme is proposed for fuzzy filter design in a class of nonlinear NCSs. Inspired by the aforementioned event-triggered scheme in [12], the hybrid-triggered scheme which consists of time-triggered scheme and event-triggered scheme is firstly proposed in [17], which investigates the problem of control stabilization for networked control systems under the hybrid-triggered scheme. Based on the hybrid-triggered scheme above, the authors in [18] are concerned with the hybrid-driven-based reliable control design for a class of T–S fuzzy systems with probabilistic actuator faults and nonlinear perturbations. Motivated by the proposed hybrid-triggered scheme in [17], this paper is devoted to the hybrid-triggered H_∞ filtering subject to stochastic cyber attacks on the measurement outputs.

Due to the insertion of the network in the control systems, challenges including packet dropouts, network-induced delay and randomly occurring nonlinearities [19–22] are inevitable. It is nonnegligible that another phenomena named cyber attacks can be more destroyable. Cyber attacks are offensive maneuvers which target networked information systems, infrastructures and networked devices by various means of malicious acts. By hacking into a susceptible system, cyber attacks can be the biggest threat to the security of network. As the description in [23], there are three kinds of common attacks containing denial of service attacks [24,25], relay attacks [26,27] and deception attacks [28,29]. With the rapid development of the network, the influence of cyber attacks can not be neglected any more. Based on the cyber attacks mentioned above, lots of researches are investigated and impressive results are yielded. The authors are concerned with extended Kalman filter design for stochastic nonlinear systems under cyber attacks in [30]. The distributed recursive filtering problem is studied in [31] with quantization and deception attacks for a class of discrete time-delayed systems. In [32], a novel state filtering approach and sensor scheduling co-design with random deception attacks are presented.

This paper addresses the issue about H_∞ filtering for networked systems under stochastic cyber attacks with hybrid-triggered communication mechanism. The main contributions of this paper are as follows. (1) In order to make full use of networked bandwidth and guarantee the desired system performance, the hybrid-triggered scheme which consists of time-triggered scheme and event-triggered scheme is introduced. (2) Due to the insertion of network, the stochastic cyber attacks are considered, and the launching probability of cyber attacks is governed by Bernoulli random variable. (3) By taking the hybrid-triggered communication mechanism and stochastic cyber attacks into consideration, an H_∞ filter is designed for networked systems. Although there are several researches concerned with filter design, to the best of our knowledge, there is no research investigating the H_∞ filter design for networked systems by considering both hybrid-triggered scheme and cyber attacks.

The rest of this paper is organized as follows. In the Section 2, a filtering error system is constructed by introducing the hybrid-triggered communication mechanism and taking the stochastic cyber attacks into account. Section 3 gives the sufficient conditions which can guarantee the augmented filtering system stable by using Lyapunov functional approach and LMI techniques. Moreover, the design algorithm of H_∞ filter is presented and the filtering parameters are obtained in an explicit form. Section 4 gives illustrative examples to demonstrate the usefulness of desired H_∞ filter.

Notation: R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n\times m$ real matrices; the su-perscript T stands for matrix transposition; I is the identity matrix of appropriate dimension; the notation $X>0$ (respectively, $X\geq 0$), for $X\in R^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite). For a matrix B and two symmetric matrices A, C and $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. Problem description and preliminaries

In this paper, the problem of H_∞ filtering for networked system with hybrid-triggered communication mechanism and stochastic cyber attacks is investigated. The framework of hybrid-triggered H_∞ filtering for networked systems under stochastic cyber attacks is shown as Fig. 1. From Fig. 1, one can see that the framework consists of the sensor, the hybrid-triggered scheme, the filter, a zero-order-hold (ZOH), a network channel.

Consider the following continuous-time linear system.

$$\begin{cases} \dot{x}(t) = Ax(t) + Bw(t) \\ y(t) = Cx(t) \\ z(t) = Lx(t) \end{cases} \tag{1}$$

where $x(t)\in R^n$ is the state vector, $y(t)\in R^m$ is the ideal measurement, $z(t)\in R^p$ is the signal to be estimated, $w(t)\in \mathcal{L}_2[0,+\infty)$ represents the disturbance input vector, A, B, C and L are known real matrices with appropriate dimensions.

The purpose of this paper is to design a hybrid-triggered H_∞ filter under stochastic cyber attacks for networked systems. Consider the following filter system.

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f \hat{y}(t) \\ z_f(t) = C_f x_f(t) \end{cases} \tag{2}$$

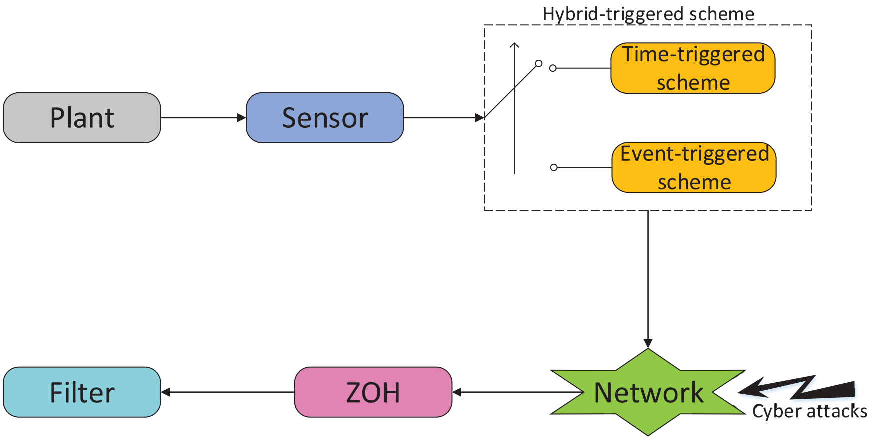

Fig. 1. The framework of H_∞ filtering for networked systems with hybrid-triggered scheme and stochastic cyber attacks.

where $x_f(t)$ is the filter state vector, $\hat{y}(t)$ is the real input of the filter, $z_f(t)$ is the estimation of the $z(t)$, A_f, B_f and C_f are filter parameters to be determined.

Remark 1. Different from the classic filter design, this paper investigates the H_∞ filtering problem with hybrid-triggered scheme and stochastic cyber attacks. As is shown in Fig. 1, the data transmission is assumed to work in a non-ideal network condition, hence some uncertainties such as network-induced delay and packet dropouts along with cyber attacks are taken into consideration.

Considering the limitation of the networked resources, a hybrid-triggered scheme in Fig. 1 is used for data transmission to alleviate the burden of the networked communication, which consists of time-triggered scheme and event-triggered scheme. Both of the two schemes are discussed in detail in the following, respectively.

Time-triggered scheme: Suppose that the sensor is time-triggered and each process signal is sampled at periodic intervals. The ideal measurement $y_1(t)$ can be expressed as follows:

$$y_1(t) = Cx(t_k h), t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) \tag{3}$$

where h represents the sampling period, t_k $(k = \{0, 1, 2, \ldots\})$ are integers and $\{t_1, t_2, t_3, \ldots\} \subset \{1, 2, 3, \ldots\}$, the corresponding network-induced delay is represented by τ_{t_k}. Similar to [8], define $\tau(t) = t - t_k h$, Eq. (3) can be written as follows.

$$y_1(t) = Cx(t - \tau(t)) \tag{4}$$

where $\tau(t) \in [0, \tau_M]$, τ_M is the upper bound of the networked delay.

Event-triggered scheme: To further enhance the bandwidth utilization, by taking [12] as a reference, an event-triggered scheme is applied to determine whether the current measurements should be transmitted or not. We use kh and $t_k h$ to represent the sampling instants and the triggering instants. Once $y(t_k h)$ is transmitted, whether the next triggered instant $y(t_{k+1} h)$ should be transmitted or not is determined by comparing the latest transmitted sampled-data with the error which is shown as follows:

$$t_{k+1} h = t_k h + \inf_{l \geqslant 1} \{lh | e_k^T(t_k h) \Omega e_k(t_k h) \geqslant \sigma y^T(t_k h) \Omega y(t_k h)\} \tag{5}$$

where $\Omega > 0$, $\sigma \in [0, 1)$ and $l = 1, 2, \ldots$. The threshold error $e_k(t_k h) = y(t_k h) - y(t_k h + lh)$.

For analyzing more easily, the interval $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$ can be divided into several subintervals. Suppose that there exists a constant g which satisfies $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) = \bigcup_{l=1}^{g} \Lambda_l$, where $\Lambda_l = [t_k h + lh + \eta_{t+l}, t_k h + lh + h + \eta_{t+l+1}]$, $l = \{1, 2, \ldots, g\}$, $g = t_{k+1} - t_k - 1$. Define $\eta(t) = t - t_k h - lh$, $0 \leq \tau_{t_k} \leq \eta(t) \leq h + \eta_{t_{k+l+1}} \triangleq \eta_M$. Let $e_k(t) = x(t_k h) - x(t_k h + lh)$, the measurement $y_2(t)$ can be written as

$$y_2(t) = Cx(t - \eta(t)) + Ce_k(t) \tag{6}$$

Remark 2. From the event-triggered judgement algorithm above, for a given period h, the sampler samples the data can be found at time kh, and the next sensor measurement is at time $(k+1)h$. Suppose that $t_0 h, t_1 h, t_2 h, \cdots$ are the release time, then it is easily obtained that $s_i h = t_{i+1} h - t_i h$ denote the release period of event generator in (5), where $s_i h$ mean that the sampling instants between the two conjoint releasing instants.

Remark 3. According to the event-triggered algorithm (5), a set of the releasing instants $\{t_1, t_2, t_3, \ldots\} \subseteq \{0, 1, 2, \cdots\}$. Learning from the research [8], it is not required that $t_{k+1} > t_k$. The packet dropouts will not occur only when $\{t_1, t_2, t_3, \ldots\} = \{1, 2, 3, \ldots\}$. If $t_{k+1} = t_k + 1$, then $h + \tau_{t_{k+1}} > \tau_{t_k}$, two special cases about $\tau_{t_k} = \bar{\tau}$ and $\tau_{t_k} < h$ imply the packet dropouts and the network-induced delay, where $\bar{\tau}$ is a constant. Thus, the frequency of releasing instants depends on the value of σ and the variations of the sensor measurements.

Combine Eq. (4) in time-triggered scheme and Eq. (6) in event-triggered scheme, similar to [17], the measurement $\bar{y}(t)$ via hybrid-triggered scheme can be expressed as follows:

$$\begin{aligned}\bar{y}(t) &= \alpha(t) y_1(t) + (1 - \alpha(t)) y_2(t) \\ &= \alpha(t) Cx(t - \tau(t)) + (1 - \alpha(t))[Cx(t - \eta(t)) + Ce_k(t)]\end{aligned} \tag{7}$$

where $\alpha(t) \in [0, 1]$, $\bar{\alpha}$ is utilized to represent the expectation of $\alpha(t)$, and ρ_1^2 represents the mathematical variance of $\alpha(t)$.

Remark 4. To describe the stochastic switching rule between the time-triggered scheme and the event-triggered scheme, the random variable $\alpha(t)$ which satisfies the Bernoulli distribution is introduced. In (7), when $\alpha(t) = 1$, $\bar{y}(t) = Cx(t - \tau(t))$, it is observed that the data is transmitted via time-triggered scheme; When $\alpha(t) = 0$, $\bar{y}(t) = Cx(t - \eta(t)) + Ce_k(t)$, then, the event-triggered scheme is activated in data transmission.

The cyber attacks in this paper belong to deception attacks which aim to destroy the stability and performance of networked system. A nonlinear function $f(x(t))$ is utilized to describe the deception attacks which is assumed to satisfy the following assumption:

Assumption 1. Suppose that deception attacks $f(x(t))$ satisfy the following condition:

$$||f(x(t))||_2 \leq ||Gx(t)||_2 \tag{8}$$

where G is a constant matrix representing the upper bound of the nonlinearity.

Remark 5. In order to describe the restrictive condition of nonlinear perturbation, the information of upper bound is introduced in [36,37]. Similarly, we use matrix G to represent the upper bound of stochastic cyber attacks in Assumption 1, and its value depends on the actual situation of networked attacks.

In the transmitting process, normal signals are subject to cyber attacks randomly in the networked channel, then we use variable $d(t)$ to represent the time-varying delay of the aggressive signals which are delivered to the filter.

By using the similar methods in [23,33], the Bernoulli variable $\theta(t)$ is introduced to govern the stochastic cyber attacks, then the real input $\hat{y}(t)$ of filter can be written as

$$\begin{aligned}\hat{y}(t) &= \theta(t)Cf(x(t-d(t))) + (1-\theta(t))\bar{y}(t) \\ &= \theta(t)Cf(x(t-d(t)))+(1-\theta(t))\{\alpha(t)Cx(t-\tau(t))+(1-\alpha(t))[Cx(t-\eta(t))+Ce_k(t)]\}\end{aligned} \tag{9}$$

where $d(t) \in [0, d_M]$, $\theta(t) \in [0, 1]$. $\bar{\theta}$ is utilized to represent the expectation of $\theta(t)$, ρ_2^2 is utilized to represent the mathematical variance of $\theta(t)$.

Remark 6. Bernoulli variables are used to describe the stochastic characteristic in the control systems. In [34], the stochastic delay is described by random Bernoulli variable for NCSs. In [35], the occurring probability of the two different sampling periods are described by Bernoulli variable. In this paper, the random variables $\alpha(t)$ and $\theta(t)$ which satisfy Bernoulli distribution are used to describe the stochastic switching changes between the two different schemes and the stochastic cyber attacks, respectively. It is noted that the Bernoulli variables $\alpha(t)$ and $\theta(t)$ are mutually independent.

According to (9), we can obtain the real input $\hat{y}(t)$ of the filter. Substitute $\hat{y}(t)$ into Eq. (2), then the filter can be written as follows:

$$\begin{cases}\dot{x}_f(t) = A_f x_f(t) + B_f\{\theta(t)Cf(x(t-d(t))) + (1-\theta(t))\{\alpha(t)Cx(t-\tau(t)) \\ \qquad + (1-\alpha(t))[Cx(t-\eta(t)) + Ce_k(t)]\}\} \\ z_f(t) = C_f x_f(t)\end{cases} \tag{10}$$

Define

$$e(t) = \begin{bmatrix} x(t) \\ x_f(t)\end{bmatrix}, \tilde{z}(t) = z(t) - z_f(t)$$

Based on Eqs. (1) and (10), the filtering error system can be described as

$$\begin{cases}\dot{e}(t) = \bar{A}_f e(t)+(1-\theta(t))\alpha(t)\bar{B}_f He(t-\tau(t))+\bar{B}w(t)+(1-\theta(t))(1-\alpha(t))\bar{B}_f He(t-\eta(t)) \\ \qquad + (1-\theta(t))(1-\alpha(t))\bar{B}_f e_k(t) + \theta(t)\bar{B}_f f(x(t-d(t))) \\ \tilde{z}(t) = \bar{C}_f e(t)\end{cases} \tag{11}$$

where

$$\bar{A}_f = \begin{bmatrix} A & 0 \\ 0 & A_f\end{bmatrix}, \bar{B}_f = \begin{bmatrix} 0 \\ B_f C\end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0\end{bmatrix}, H = \begin{bmatrix} I & 0\end{bmatrix}, \bar{C}_f = \begin{bmatrix} L & -C_f\end{bmatrix}$$

Some important lemmas are introduced in the following:

Lemma 1 ([38]). *For any vectors $x, y \in R^n$, and positive definite matrix $Q \in R^{n\times n}$, the following inequality holds:*

$$2x^T y \le x^T Qx + y^T Q^{-1} y \tag{12}$$

Lemma 2 ([39]). *Suppose* $\tau(t)\in[0,\ \tau_M]$, $d(t)\in[0,\ d_M]$, $\eta(t)\in[0,\ \eta_M]$, Ξ_1, Ξ_2, Ξ_3, Ξ_4, Ξ_5, Ξ_6 *and* Ω *are matrices with appropriate dimensions, then*

$$\tau(t)\Xi_1+(\tau_M-\tau(t))\Xi_2+d(t)\Xi_3+(d_M-d(t))\Xi_4+\eta(t)\Xi_5+(\eta_M-\eta(t))\Xi_6+\Omega<0 \tag{13}$$

if and only if

$$\begin{cases}\tau_M\Xi_1+d_M\Xi_3+\eta_M\Xi_5+\Omega<0, & \tau_M\Xi_2+d_M\Xi_3+\eta_M\Xi_5+\Omega<0\\ \tau_M\Xi_1+d_M\Xi_4+\eta_M\Xi_5+\Omega<0, & \tau_M\Xi_2+d_M\Xi_4+\eta_M\Xi_5+\Omega<0\\ \tau_M\Xi_1+d_M\Xi_3+\eta_M\Xi_6+\Omega<0, & \tau_M\Xi_2+d_M\Xi_3+\eta_M\Xi_6+\Omega<0\\ \tau_M\Xi_1+d_M\Xi_4+\eta_M\Xi_6+\Omega<0, & \tau_M\Xi_2+d_M\Xi_4+\eta_M\Xi_6+\Omega<0\end{cases} \tag{14}$$

3. Main results

In this section, by using Lyapunov functional approach, the main results will be summarized and the sufficient conditions which can guarantee the stability of networked system will be obtained.

Theorem 1. *For given positive parameters* $\bar{\theta}$, $\bar{\alpha}$, τ_M, η_M, d_M, σ, ρ_i $(i=1,2)$ *and matrix* G, *with hybrid-triggered communication mechanism and cyber attacks, system (11) is asymptotically stable with an* H_∞ *disturbance attenuation level* γ, *if there exist matrices* $P>0$, $Q_k>0$, $R_k>0$ $(k=1,2,3)$, $\Omega>0$ *and* M, N, U, S, W, V *with appropriate dimensions satisfying*

$$\Omega(s)=\begin{bmatrix}\Omega_{11}+\Gamma+\Gamma^T & * & * & * & * & * & * & * & *\\ \Omega_{21} & -I & * & * & * & * & * & * & *\\ \Omega_{31} & 0 & -I & * & * & * & * & * & *\\ \Omega_{41}(s) & 0 & 0 & \Omega_{44} & * & * & * & * & *\\ \Omega_{51} & 0 & 0 & 0 & \Omega_{55} & * & * & * & *\\ \Omega_{61} & 0 & 0 & 0 & 0 & \Omega_{66} & * & * & *\\ \Omega_{71} & 0 & 0 & 0 & 0 & 0 & \Omega_{77} & * & *\\ \Omega_{81} & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{88} & *\\ \Omega_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Omega_{99}\end{bmatrix}<0(s=1,\cdots,8) \tag{15}$$

where

$$\Omega_{11}=\begin{bmatrix}\Phi_{11} & * & * & * & * & * & * & * & * & *\\ \Phi_{21} & 0 & * & * & * & * & * & * & * & *\\ 0 & 0 & -Q_1 & * & * & * & * & * & * & *\\ \Phi_{31} & 0 & 0 & \Phi_{33} & * & * & * & * & * & *\\ 0 & 0 & 0 & 0 & -Q_2 & * & * & * & * & *\\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & *\\ 0 & 0 & 0 & 0 & 0 & 0 & -Q_3 & * & * & *\\ \Phi_{41} & 0 & 0 & 0 & 0 & 0 & 0 & -C^T\Omega C & * & *\\ \bar{B}^TP & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2I & *\\ \bar{\theta}\bar{B}_f^TP & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\theta}I\end{bmatrix}$$

$\Phi_{11}=P\bar{A}_f+\bar{A}_f^TP+Q_1+Q_2+Q_3$, $\Phi_{21}=\bar{\theta}_1\bar{\alpha}H^T\bar{B}_f^TP$, $\Phi_{31}=\bar{\theta}_1\bar{\alpha}_1H^T\bar{B}_f^TP$

$\Phi_{33}=\sigma C^TH^T\Omega HC$, $\Phi_{41}=\bar{\theta}_1\bar{\alpha}_1\bar{B}_f^TP$, $\Upsilon_1=\bar{\theta}_1\bar{\alpha}P\bar{B}_fH$, $\Upsilon_2=\bar{\theta}_1\bar{\alpha}_1P\bar{B}_fH$

$$\Gamma=\begin{bmatrix}M+U+W & N-M & -N & -U+S & -S & -W+V & -V & 0 & 0 & 0\end{bmatrix}$$

$$\Omega_{21} = \begin{bmatrix} \bar{C}_f & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Omega_{31} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \bar{\theta}_2 GH & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Omega_{41}(1) = \begin{bmatrix} \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}U^T \\ \sqrt{d_M}W^T \end{bmatrix}, \Omega_{41}(2) = \begin{bmatrix} \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}U^T \\ \sqrt{d_M}V^T \end{bmatrix}, \Omega_{41}(3) = \begin{bmatrix} \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}W^T \end{bmatrix}, \Omega_{41}(4) = \begin{bmatrix} \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}V^T \end{bmatrix}$$

$$\Omega_{41}(5) = \begin{bmatrix} \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}U^T \\ \sqrt{d_M}W^T \end{bmatrix}, \Omega_{41}(6) = \begin{bmatrix} \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}U^T \\ \sqrt{d_M}V^T \end{bmatrix}, \Omega_{41}(7) = \begin{bmatrix} \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}W^T \end{bmatrix}, \Omega_{41}(8) = \begin{bmatrix} \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}V^T \end{bmatrix}$$

$$\Omega_{51} = \begin{bmatrix} \sqrt{\tau_M}P\bar{A}_f & \sqrt{\tau_M}\Upsilon_1 & 0 & \sqrt{\tau_M}\Upsilon_2 & 0 & 0 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}P\bar{B}_f & \sqrt{\tau_M}P\bar{B} & \bar{\theta}\sqrt{\tau_M}P\bar{B}_f \\ \sqrt{\eta_M}P\bar{A}_f & \sqrt{\eta_M}\Upsilon_1 & 0 & \sqrt{\eta_M}\Upsilon_2 & 0 & 0 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}P\bar{B}_f & \sqrt{\eta_M}P\bar{B} & \bar{\theta}\sqrt{\eta_M}P\bar{B}_f \\ \sqrt{d_M}P\bar{A}_f & \sqrt{d_M}\Upsilon_1 & 0 & \sqrt{d_M}\Upsilon_2 & 0 & 0 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}P\bar{B}_f & \sqrt{d_M}P\bar{B} & \bar{\theta}\sqrt{d_M}P\bar{B}_f \end{bmatrix}$$

$$\Omega_{61} = \begin{bmatrix} 0 & \bar{\theta}_1\rho_1\sqrt{\tau_M}P\bar{B}_fH & 0 & -\bar{\theta}_1\rho_1\sqrt{\tau_M}P\bar{B}_fH & 0 & 0 & 0 & -\bar{\theta}_1\rho_1\sqrt{\tau_M}P\bar{B}_f & 0 & 0 \\ 0 & \bar{\theta}_1\rho_1\sqrt{\eta_M}P\bar{B}_fH & 0 & -\bar{\theta}_1\rho_1\sqrt{\eta_M}P\bar{B}_fH & 0 & 0 & 0 & -\bar{\theta}_1\rho_1\sqrt{\eta_M}P\bar{B}_f & 0 & 0 \\ 0 & \bar{\theta}_1\rho_1\sqrt{d_M}P\bar{B}_fH & 0 & -\bar{\theta}_1\rho_1\sqrt{d_M}P\bar{B}_fH & 0 & 0 & 0 & -\bar{\theta}_1\rho_1\sqrt{d_M}P\bar{B}_f & 0 & 0 \end{bmatrix}$$

$$\Omega_{71} = \begin{bmatrix} 0 & \bar{\alpha}\rho_2\sqrt{\tau_M}P\bar{B}_fH & 0 & \bar{\alpha}_1\rho_2\sqrt{\tau_M}P\bar{B}_fH & 0 & 0 & 0 & \bar{\alpha}_1\rho_2\sqrt{\tau_M}P\bar{B}_f & 0 & 0 \\ 0 & \bar{\alpha}\rho_2\sqrt{\eta_M}P\bar{B}_fH & 0 & \bar{\alpha}_1\rho_2\sqrt{\eta_M}P\bar{B}_fH & 0 & 0 & 0 & \bar{\alpha}_1\rho_2\sqrt{\eta_M}P\bar{B}_f & 0 & 0 \\ 0 & \bar{\alpha}\rho_2\sqrt{d_M}P\bar{B}_fH & 0 & \bar{\alpha}_1\rho_2\sqrt{d_M}P\bar{B}_fH & 0 & 0 & 0 & \bar{\alpha}_1\rho_2\sqrt{d_M}P\bar{B}_f & 0 & 0 \end{bmatrix}$$

$$\Omega_{81} = \begin{bmatrix} 0 & \rho_1\rho_2\sqrt{\tau_M}P\bar{B}_fH & 0 & -\rho_1\rho_2\sqrt{\tau_M}P\bar{B}_fH & 0 & 0 & 0 & -\rho_1\rho_2\sqrt{\tau_M}P\bar{B}_f & 0 & 0 \\ 0 & \rho_1\rho_2\sqrt{\eta_M}P\bar{B}_fH & 0 & -\rho_1\rho_2\sqrt{\eta_M}P\bar{B}_fH & 0 & 0 & 0 & -\rho_1\rho_2\sqrt{\eta_M}P\bar{B}_f & 0 & 0 \\ 0 & \rho_1\rho_2\sqrt{d_M}P\bar{B}_fH & 0 & -\rho_1\rho_2\sqrt{d_M}P\bar{B}_fH & 0 & 0 & 0 & -\rho_1\rho_2\sqrt{d_M}P\bar{B}_f & 0 & 0 \end{bmatrix}$$

$$\Omega_{91} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2\sqrt{\tau_M}P\bar{B}_f \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2\sqrt{\eta_M}P\bar{B}_f \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2\sqrt{d_M}P\bar{B}_f \end{bmatrix}$$

$$\bar{\theta}_1 = 1 - \bar{\theta}, \bar{\alpha}_1 = 1 - \bar{\alpha}, \rho_1^2 = \bar{\alpha}\bar{\alpha}_1, \rho_2^2 = \bar{\theta}\bar{\theta}_1, \bar{\theta}_2 = \sqrt{\bar{\theta}}, \Omega_{44} = diag\{-R_1, -R_2, -R_3\}$$

$$\Omega_{55} = \Omega_{66} = \Omega_{77} = \Omega_{88} = \Omega_{99} = \{-PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\}$$

$$M^T = \begin{bmatrix} M_1^T & M_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, N^T = \begin{bmatrix} 0 & N_2^T & N_3^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$U^T = \begin{bmatrix} U_1^T & 0 & 0 & U_4^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, S^T = \begin{bmatrix} 0 & 0 & 0 & S_4^T & S_5^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$W^T = \begin{bmatrix} W_1^T & 0 & 0 & 0 & 0 & W_6^T & 0 & 0 & 0 & 0 \end{bmatrix}, V^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & V_6^T & V_7^T & 0 & 0 & 0 \end{bmatrix}$$

Proof. Choose the following Lyapunov functional candidate as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{16}$$

where

$$V_1(t) = e^T(t)Pe(t)$$

$$V_2(t) = \int_{t-\tau_M}^{t} e^T(s)Q_1e(s)ds + \int_{t-\eta_M}^{t} e^T(s)Q_2e(s)ds + \int_{t-d_M}^{t} e^T(s)Q_3e(s)ds$$

$$V_3(t) = \int_{t-\tau_M}^{t}\int_{s}^{t} \dot{e}^T(v)R_1\dot{e}(v)dvds + \int_{t-\eta_M}^{t}\int_{s}^{t} \dot{e}^T(v)R_2\dot{e}(v)dvds + \int_{t-d_M}^{t}\int_{s}^{t} \dot{e}^T(v)R_3\dot{e}(v)dvds$$

and $P > 0$, $Q_k > 0$, $R_k > 0$ $(k = 1, 2, 3)$.

By applying the infinitesimal operator (15) for $V_k(t)$ $(k = 1, 2, 3)$ and taking expectation on it, we can obtain

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V_1(t)\} = {} & 2e^T(t)P[\bar{A}_f e(t) + \bar{\theta}_1\bar{\alpha}\bar{B}_f He(t-\tau(t)) + \bar{\theta}_1\bar{\alpha}_1\bar{B}_f He(t-\eta(t)) + \bar{\theta}_1\bar{\alpha}_1\bar{B}_f e_k(t) \\ & + \bar{B}w(t) + \bar{\theta}\bar{B}_f f(x(t-d(t)))]\end{aligned} \tag{17}$$

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V_2(t)\} = {} & e^T(t)(Q_1+Q_2+Q_3)e(t) - e^T(t-\tau_M)Q_1e(t-\tau_M) - e^T(t-\eta_M)Q_2e(t-\eta_M) \\ & - e^T(t-d_M)Q_3e(t-d_M)\end{aligned} \tag{18}$$

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V_3(t)\} = {} & \mathbb{E}\big\{\dot{e}^T(t)(\tau_M R_1 + \eta_M R_2 + d_M R_3)\dot{e}(t)\big\} - \int_{t-\tau_M}^{t} \dot{e}^T(s)R_1\dot{e}(s)ds - \int_{t-\eta_M}^{t} \dot{e}^T(s)R_2\dot{e}(s)ds \\ & - \int_{t-d_M}^{t} \dot{e}^T(s)R_3\dot{e}(s)ds\end{aligned} \tag{19}$$

Notice that

$$\begin{aligned}\mathbb{E}\big\{\dot{e}^T(t)(\tau_M R_1 + \eta_M R_2 + d_M R_3)\dot{e}(t)\big\} = {} & \mathcal{A}^T\tilde{R}\mathcal{A} + \bar{\theta}_1^2\rho_1^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 + \rho_2^2\mathcal{B}_2^T\tilde{R}\mathcal{B}_2 + \rho_1^2\rho_2^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 \\ & + \rho_2^2 f^T(x(t-d(t)))\bar{B}_f^T\tilde{R}\bar{B}_f f(x(t-d(t)))\end{aligned} \tag{20}$$

where

$$\begin{aligned}\mathcal{A} &= \bar{A}_f e(t) + \bar{\theta}_1\bar{\alpha}\bar{B}_f He(t-\tau(t)) + \bar{\theta}_1\bar{\alpha}_1\bar{B}_f He(t-\eta(t)) + \bar{\theta}_1\bar{\alpha}_1\bar{B}_f e_k(t) + \bar{B}w(t) \\ &\quad + \bar{\theta}\bar{B}_f f(x(t-d(t))) \\ \mathcal{B}_1 &= \bar{B}_f[He(t-\tau(t)) - He(t-\eta(t)) - e_k(t)] \\ \mathcal{B}_2 &= \bar{B}_f[\bar{\alpha}He(t-\tau(t)) + \bar{\alpha}_1 He(t-\eta(t)) + \bar{\alpha}_1 e_k(t)] \\ \tilde{R} &= \tau_M R_1 + \eta_M R_2 + d_M R_3\end{aligned}$$

From the assumption in Eq.(8), we can obtain

$$\bar{\theta}e^T(t-d(t))H^TG^TGHe(t-d(t)) - \bar{\theta}f^T(x(t-d(t))f(x(t-d(t))) \geq 0 \tag{21}$$

Applying the free-weighting matrices method [8,40], it can be obtained that

$$2\xi^T(t)M\left[e(t) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{e}(s)d_s\right] = 0 \tag{22}$$

$$2\xi^T(t)N\left[e(t-\tau(t)) - e(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{e}(s)d_s\right] = 0 \tag{23}$$

$$2\xi^T(t)U\left[e(t) - e(t-\eta(t)) - \int_{t-\eta(t)}^{t} \dot{e}(s)d_s\right] = 0 \tag{24}$$

$$2\xi^T(t)S\left[e(t-\eta(t))-e(t-\eta_M)-\int_{t-\eta_M}^{t-\eta(t)}\dot{e}(s)d_s\right]=0 \tag{25}$$

$$2\xi^T(t)W\left[e(t)-e(t-d(t))-\int_{t-d(t)}^{t}\dot{e}(s)d_s\right]=0 \tag{26}$$

$$2\xi^T(t)V\left[e(t-d(t))-e(t-d_M)-\int_{t-d_M}^{t-d(t)}\dot{e}(s)d_s\right]=0 \tag{27}$$

where N, M, T, S, W, V are matrices with appropriate dimensions, and

$$\xi^T(t)=\begin{bmatrix}\xi_1^T(t) & \xi_2^T(t)\end{bmatrix}$$
$$\xi_1^T(t)=\begin{bmatrix}e^T(t) & e^T(t-\tau(t)) & e^T(t-\tau_M) & e^T(t-\eta(t) & e^T(t-\eta_M)\end{bmatrix}$$
$$\xi_2^T(t)=\begin{bmatrix}e^T(t-d(t) & e^T(t-d_M) & e_k^T(t) & w^T(t) & f^T(x(t-d(t)))\end{bmatrix}$$

By Lemma 1, we have

$$-2\xi^T(t)M\int_{t-\tau(t)}^{t}\dot{e}(s)ds\le\tau(t)\xi^T(t)MR_1^{-1}M^T\xi(t)+\int_{t-\tau(t)}^{t}\dot{e}^T(s)R_1\dot{e}(s)ds \tag{28}$$

$$-2\xi^T(t)N\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)ds\le(\tau_M-\tau(t))\xi^T(t)NR_1^{-1}N^T\xi(t)+\int_{t-\tau_M}^{t-\tau(t)}\dot{e}^T(s)R_1\dot{e}(s)ds \tag{29}$$

$$-2\xi^T(t)U\int_{t-\eta(t)}^{t}\dot{e}(s)ds\le\eta(t)\xi^T(t)UR_2^{-1}U^T\xi(t)+\int_{t-\eta(t)}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds \tag{30}$$

$$-2\xi^T(t)S\int_{t-\eta_M}^{t-\eta(t)}\dot{e}(s)ds\le(\eta_M-\eta(t))\xi^T(t)SR_2^{-1}S^T\xi(t)+\int_{t-\eta_M}^{t-\eta(t)}\dot{e}^T(s)R_2\dot{e}(s)ds \tag{31}$$

$$-2\xi^T(t)W\int_{t-d(t)}^{t}\dot{e}(s)ds\le d(t)\xi^T(t)WR_3^{-1}W^T\xi(t)+\int_{t-d(t)}^{t}\dot{e}^T(s)R_3\dot{e}(s)ds \tag{32}$$

$$-2\xi^T(t)V\int_{t-d_M}^{t-d(t)}\dot{e}(s)ds\le(d_M-d(t))\xi^T(t)VR_3^{-1}V^T\xi(t)+\int_{t-d_M}^{t-d(t)}\dot{e}^T(s)R_3\dot{e}(s)ds \tag{33}$$

Considering the condition of event-triggered scheme (5), we can obtain that

$$\sigma e^T(t-\eta(t))C^T H^T \Omega H C e(t-\eta(t)) - e_k^T(t) C^T \Omega C e_k(t) \geq 0 \tag{34}$$

By applying free-weighting matrices method and reciprocally convex approach, combining Eqs. (16)–(34), we can obtain that

$$\begin{aligned}
&\mathbb{E}\{\mathcal{L}V(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 w^T(t) w(t)\} \\
&\leq \xi^T(t)[\Omega_{11} + \Gamma + \Gamma^T - \Omega_{21}^T\Omega_{21} - \Omega_{31}^T P \Omega_{31} + \Omega_{51}^T\Omega_{55}\Omega_{51} + \Omega_{61}^T\Omega_{66}\Omega_{61} + \Omega_{71}^T\Omega_{77}\Omega_{71} \\
&+\Omega_{81}^T\Omega_{88}\Omega_{81} + \Omega_{91}^T\Omega_{99}\Omega_{91} + \tau(t) M R_1^{-1} M^T + (\tau_M - \tau(t)) N R_1^{-1} N^T + \eta(t) U R_2^{-1} U^T \\
&+(\eta_M - \eta(t)) S R_2^{-1} S^T + d(t) W R_3^{-1} W^T + (d_M - d(t)) V R_3^{-1} V^T]\xi(t)
\end{aligned} \tag{35}$$

Similar to [13,41], by using Schur complement and Lemma 2, we can obtain that (15) can guarantee $\mathbb{E}\{\mathcal{L}(V(t)) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 w^T(t) w(t)\} < 0$. The proof is completed. □

Based on Theorem 1, the sufficient conditions have been obtained which can guarantee the stability of system. In order to solve the nonlinear terms in Theorem 1, LMI techniques are used to compute the parameters of the desired filter in the following:

Theorem 2. *For given positive parameters γ, $\bar{\theta}$, $\bar{\alpha}$, τ_M, η_M, d_M, σ, ρ_i $(i = 1, 2)$ and ϵ_k, matrix G, system (11) is asymptotically stable with hybrid-triggered scheme and stochastic cyber attacks, if there exist matrices $P_1 > 0$, $\bar{P}_3 > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$ $(k = 1, 2, 3)$, $\Omega > 0$, $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$, $\tilde{M}$, $\tilde{N}$, $\tilde{U}$, $\tilde{S}$, $\tilde{W}$, $\tilde{V}$ are matrices with appropriate dimensions, such that the following LMIs hold:*

$$\tilde{\Omega}(s) = \begin{bmatrix}
\tilde{\Omega}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * & * & * & * & * & * \\
\tilde{\Omega}_{21} & -I & * & * & * & * & * & * & * \\
\tilde{\Omega}_{31} & 0 & -I & * & * & * & * & * & * \\
\tilde{\Omega}_{41}(s) & 0 & 0 & \tilde{\Omega}_{44} & * & * & * & * & * \\
\tilde{\Omega}_{51} & 0 & 0 & 0 & \tilde{\Omega}_{55} & * & * & * & * \\
\tilde{\Omega}_{61} & 0 & 0 & 0 & 0 & \tilde{\Omega}_{66} & * & * & * \\
\tilde{\Omega}_{71} & 0 & 0 & 0 & 0 & 0 & \tilde{\Omega}_{77} & * & * \\
\tilde{\Omega}_{81} & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\Omega}_{88} & * \\
\tilde{\Omega}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\Omega}_{99}
\end{bmatrix} < 0 (s = 1, \cdots, 8) \tag{36}$$

$$P_1 - \bar{P}_3 > 0 \tag{37}$$

where

$$\tilde{\Phi}_{11} = \psi_{11} + \psi_{11}^T + \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3, \psi_{31} = \begin{bmatrix} C^T\hat{B}_f^T & C^T\hat{B}_f^T \end{bmatrix}, \psi_{41} = \begin{bmatrix} B^T P_1 & B^T \bar{P}_3 \end{bmatrix}$$

$$\psi_{11} = \begin{bmatrix} P_1 A & \hat{A}_f \\ \bar{P}_3 A & \hat{A}_f \end{bmatrix}, \psi_{21} = \begin{bmatrix} C^T\hat{B}_f^T & C^T\hat{B}_f^T \\ 0 & 0 \end{bmatrix}, \psi_{44} = \begin{bmatrix} \sigma C^T \Omega C & 0 \\ 0 & 0 \end{bmatrix}, \tilde{P} = \begin{bmatrix} P_1 & \bar{p}_3 \\ \bar{P}_3 & \bar{p}_3 \end{bmatrix}$$

$$\tilde{\Gamma} = \begin{bmatrix} \tilde{M} + \tilde{U} + \tilde{W} & \tilde{N} - \tilde{M} & -\tilde{N} & -\tilde{U} + \tilde{S} & -\tilde{S} & -\tilde{W} + \tilde{V} & -\tilde{V} & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Omega}_{21} = \begin{bmatrix} \psi_{51} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \psi_{51} = \begin{bmatrix} L & -\hat{C}_f \end{bmatrix}$$

$$\tilde{\Omega}_{11}=\begin{bmatrix} \tilde{\Phi}_{11} & * & * & * & * & * & * & * & * & * \\ \bar{\theta}_1\bar{\alpha}\psi_{21} & 0 & * & * & * & * & * & * & * & * \\ 0 & 0 & -\tilde{Q}_1 & * & * & * & * & * & * & * \\ \bar{\theta}_1\bar{\alpha}_1\psi_{21} & 0 & 0 & \psi_{44} & * & * & * & * & * & * \\ 0 & 0 & 0 & 0 & -\tilde{Q}_2 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\tilde{Q}_3 & * & * & * \\ \bar{\theta}_1\bar{\alpha}_1\psi_{31} & 0 & 0 & 0 & 0 & 0 & 0 & -C^T\Omega C & * & * \\ \psi_{41} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \bar{\theta}\psi_{31} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\theta}I \end{bmatrix}$$

$$\tilde{\Omega}_{31}=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \bar{\theta}_2 G & 0 & 0 & 0 & 0\end{bmatrix}, \tilde{\Omega}_{44}=diag\{-\tilde{R}_1,-\tilde{R}_2,-\tilde{R}_3\}$$

$$\tilde{\Omega}_{41}(1)=\begin{bmatrix}\sqrt{\tau_M}\tilde{M}^T\\ \sqrt{\eta_M}\tilde{U}^T\\ \sqrt{d_M}\tilde{W}^T\end{bmatrix}, \tilde{\Omega}_{41}(2)=\begin{bmatrix}\sqrt{\tau_M}\tilde{M}^T\\ \sqrt{\eta_M}\tilde{U}^T\\ \sqrt{d_M}\tilde{V}^T\end{bmatrix}, \tilde{\Omega}_{41}(3)=\begin{bmatrix}\sqrt{\tau_M}\tilde{M}^T\\ \sqrt{\eta_M}\tilde{S}^T\\ \sqrt{d_M}\tilde{W}^T\end{bmatrix}, \tilde{\Omega}_{41}(4)=\begin{bmatrix}\sqrt{\tau_M}\tilde{M}^T\\ \sqrt{\eta_M}\tilde{S}^T\\ \sqrt{d_M}\tilde{V}^T\end{bmatrix}$$

$$\tilde{\Omega}_{41}(5)=\begin{bmatrix}\sqrt{\tau_M}\tilde{N}^T\\ \sqrt{\eta_M}\tilde{U}^T\\ \sqrt{d_M}\tilde{W}^T\end{bmatrix}, \tilde{\Omega}_{41}(6)=\begin{bmatrix}\sqrt{\tau_M}\tilde{N}^T\\ \sqrt{\eta_M}\tilde{U}^T\\ \sqrt{d_M}\tilde{V}^T\end{bmatrix}, \tilde{\Omega}_{41}(7)=\begin{bmatrix}\sqrt{\tau_M}\tilde{N}^T\\ \sqrt{\eta_M}\tilde{S}^T\\ \sqrt{d_M}\tilde{W}^T\end{bmatrix}, \tilde{\Omega}_{41}(8)=\begin{bmatrix}\sqrt{\tau_M}\tilde{N}^T\\ \sqrt{\eta_M}\tilde{S}^T\\ \sqrt{d_M}\tilde{V}^T\end{bmatrix}$$

$$\tilde{\Omega}_{55}=\tilde{\Omega}_{66}=\tilde{\Omega}_{77}=\tilde{\Omega}_{88}=\tilde{\Omega}_{99}=\left\{-2\epsilon_1\tilde{P}+\epsilon_1^2\tilde{R}_1, -2\epsilon_2\tilde{P}+\epsilon_2^2\tilde{R}_2, -2\epsilon_3\tilde{P}+\epsilon_3^2\tilde{R}_3\right\}$$

$$\tilde{\Omega}_{51}=\begin{bmatrix} \sqrt{\tau_M}\psi_{11} & \bar{\theta}_1\bar{\alpha}\sqrt{\tau_M}\psi_{21}^T & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\psi_{21}^T & 0 & 0 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\psi_{41}^T & \sqrt{\tau_M}\psi_{31}^T & \bar{\theta}\sqrt{\tau_M}\psi_{41}^T \\ \sqrt{\eta_M}\psi_{11} & \bar{\theta}_1\bar{\alpha}\sqrt{\eta_M}\psi_{21}^T & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\psi_{21}^T & 0 & 0 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\psi_{41}^T & \sqrt{\eta_M}\psi_{31}^T & \bar{\theta}\sqrt{\eta_M}\psi_{41}^T \\ \sqrt{d_M}\psi_{11} & \bar{\theta}_1\bar{\alpha}\sqrt{d_M}\psi_{21}^T & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\psi_{21}^T & 0 & 0 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\psi_{41}^T & \sqrt{d_M}\psi_{31}^T & \bar{\theta}\sqrt{d_M}\psi_{41}^T \end{bmatrix}$$

$$\tilde{\Omega}_{61}=\begin{bmatrix} 0 & \bar{\theta}_1\rho_1\sqrt{\tau_M}\psi_{21}^T & 0 & -\bar{\theta}_1\rho_1\sqrt{\tau_M}\psi_{21}^T & 0 & 0 & 0 & -\bar{\theta}_1\rho_1\sqrt{\tau_M}\psi_{41}^T & 0 & 0 \\ 0 & \bar{\theta}_1\rho_1\sqrt{\eta_M}\psi_{21}^T & 0 & -\bar{\theta}_1\rho_1\sqrt{\eta_M}\psi_{21}^T & 0 & 0 & 0 & -\bar{\theta}_1\rho_1\sqrt{\eta_M}\psi_{41}^T & 0 & 0 \\ 0 & \bar{\theta}_1\rho_1\sqrt{d_M}\psi_{21}^T & 0 & -\bar{\theta}_1\rho_1\sqrt{d_M}\psi_{21}^T & 0 & 0 & 0 & -\bar{\theta}_1\rho_1\sqrt{d_M}\psi_{41}^T & 0 & 0 \end{bmatrix}$$

$$\tilde{\Omega}_{71}=\begin{bmatrix} 0 & \bar{\alpha}\rho_2\sqrt{\tau_M}\psi_{21}^T & 0 & \bar{\alpha}_1\rho_2\sqrt{\tau_M}\psi_{21}^T & 0 & 0 & 0 & \bar{\alpha}_1\rho_2\sqrt{\tau_M}\psi_{41}^T & 0 & 0 \\ 0 & \bar{\alpha}\rho_2\sqrt{\eta_M}\psi_{21}^T & 0 & \bar{\alpha}_1\rho_2\sqrt{\eta_M}\psi_{21}^T & 0 & 0 & 0 & \bar{\alpha}_1\rho_2\sqrt{\eta_M}P\psi_{41}^T & 0 & 0 \\ 0 & \bar{\alpha}\rho_2\sqrt{d_M}\psi_{21}^T & 0 & \bar{\alpha}_1\rho_2\sqrt{d_M}\psi_{21}^T & 0 & 0 & 0 & \bar{\alpha}_1\rho_2\sqrt{d_M}\psi_{41}^T & 0 & 0 \end{bmatrix}$$

$$\tilde{\Omega}_{81}=\begin{bmatrix} 0 & \rho_1\rho_2\sqrt{\tau_M}\psi_{21}^T & 0 & -\rho_1\rho_2\sqrt{\tau_M}\psi_{21}^T & 0 & 0 & 0 & -\rho_1\rho_2\sqrt{\tau_M}\psi_{41}^T & 0 & 0 \\ 0 & \rho_1\rho_2\sqrt{\eta_M}\psi_{21}^T & 0 & -\rho_1\rho_2\sqrt{\eta_M}\psi_{21}^T & 0 & 0 & 0 & -\rho_1\rho_2\sqrt{\eta_M}\psi_{41}^T & 0 & 0 \\ 0 & \rho_1\rho_2\sqrt{d_M}\psi_{21}^T & 0 & -\rho_1\rho_2\sqrt{d_M}\psi_{21}^T & 0 & 0 & 0 & -\rho_1\rho_2\sqrt{d_M}\psi_{41}^T & 0 & 0 \end{bmatrix}$$

$$\tilde{\Omega}_{91}=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2\sqrt{\tau_M}\psi_{41}^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2\sqrt{\eta_M}\psi_{41}^T \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_2\sqrt{d_M}\psi_{41}^T \end{bmatrix}$$

$$\tilde{M}^T=\begin{bmatrix}\tilde{M}_1^T & M_2^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \tilde{N}^T=\begin{bmatrix}0 & \tilde{N}_2^T & \tilde{N}_3^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\tilde{U}^T=\begin{bmatrix}\tilde{U}_1^T & 0 & 0 & \tilde{U}_4^T & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \tilde{S}^T=\begin{bmatrix}0 & 0 & 0 & \tilde{S}_4^T & \tilde{S}_5^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\tilde{W}^T=\begin{bmatrix}\tilde{W}_1^T & 0 & 0 & 0 & 0 & \tilde{W}_6^T & 0 & 0 & 0 & 0\end{bmatrix}, \tilde{V}^T=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \tilde{V}_6^T & \tilde{V}_7^T & 0 & 0 & 0\end{bmatrix}$$

Moreover, if the above conditions are feasible, the parameter matrices of the filter are given by

$$\begin{cases} A_f = \hat{A}_f \bar{P}_3^{-1} \\ B_f = \hat{B}_f \\ C_f = \hat{C}_f \bar{P}_3^{-1} \end{cases} \tag{38}$$

Proof. Due to

$(R_k - \epsilon_k^{-1}P)R_k^{-1}(R_k - \epsilon_k^{-1}P) \geq 0, (k = 1, 2, 3)$

we have

$-PR_k^{-1}P \leq -2\epsilon_k P + \epsilon_k^2 R_k$

Substitute $-PR_k^{-1}P$ with $-2\epsilon_k P + \epsilon_k^2 R_k$ into Eq. (15), we obtain Eq. (39) above.

$$\hat{\Omega}(s) = \begin{bmatrix} \tilde{\Omega}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * & * & * & * & * & * \\ \tilde{\Omega}_{21} & -I & * & * & * & * & * & * & * \\ \tilde{\Omega}_{31} & 0 & -I & * & * & * & * & * & * \\ \tilde{\Omega}_{41}(s) & 0 & 0 & \tilde{\Omega}_{44} & * & * & * & * & * \\ \tilde{\Omega}_{51} & 0 & 0 & 0 & \hat{\Omega}_{55} & * & * & * & * \\ \tilde{\Omega}_{61} & 0 & 0 & 0 & 0 & \hat{\Omega}_{66} & * & * & * \\ \tilde{\Omega}_{71} & 0 & 0 & 0 & 0 & 0 & \hat{\Omega}_{77} & * & * \\ \tilde{\Omega}_{81} & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\Omega}_{88} & * \\ \tilde{\Omega}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\Omega}_{99} \end{bmatrix} < 0 \quad (s = 1, \cdots, 8)$$

$$\hat{\Omega}_{55} = \hat{\Omega}_{66} = \hat{\Omega}_{77} = \hat{\Omega}_{88} = \hat{\Omega}_{99} = \{-2\epsilon_1 P + \epsilon_1^2 R_1, -2\epsilon_2 P + \epsilon_2^2 R_2, -2\epsilon_3 P + \epsilon_3^2 R_3\} \tag{39}$$

Since $\bar{P}_3 > 0$, there exist P_2 and $P_3 > 0$ satisfying $\bar{P}_3 = P_2^T P_3^{-1} P_2$.

Define

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}, J = \begin{bmatrix} I & 0 \\ 0 & P_2^T P_3^{-1} \end{bmatrix}, \chi = diag\{\underbrace{J, \cdots, J}_{7}, \underbrace{I, \cdots, I}_{5}, \underbrace{J, \cdots, J}_{18}\}$$

By Schur complement, $P > 0$ is equivalent to P_1 - $\bar{P}_3 > 0$. Multiplying Eq. (39) by χ from the left side and its transpose from the right side, and defining $\tilde{P} = JPJ^T = \begin{bmatrix} P_1 & \bar{P}_3 \\ \bar{P}_3 & \bar{P}_3 \end{bmatrix}$, $\tilde{Q}_k = JQ_kJ^T$, $\tilde{R}_k = JR_kJ^T (k = 1, 2, 3)$, $\tilde{M}_{v1} = JM_{v1}J^T$, $\tilde{N}_{v2} = JN_{v2}J^T$, $\tilde{U}_{v3} = JU_{v3}J^T$, $\tilde{S}_{v4} = JS_{v4}J^T$, $\tilde{W}_{v5} = JW_{v5}J^T$, $\tilde{V}_{v6} = JV_{v6}J^T$, $(v_1 = 1, 2; v_2 = 2, 3; v_3 = 1, 4; v_4 = 4, 5; v_5 = 1, 6; v_6 = 6, 7)$, then, we can derive Eqs. (36) and (37). Define variables

$$\begin{cases} \hat{A}_f = \tilde{A}_f \bar{P}_3, \tilde{A}_f = P_2^T A_f P_2^{-T} \\ \hat{B}_f = P_2^T B_f \\ \hat{C}_f = \tilde{C}_f \bar{P}_3, \tilde{C}_f = C_f P_2^{-T} \end{cases} \tag{40}$$

Based on the descriptions above, similar to the analysis of [42], the filter parameters (A_f, B_f, C_f) can be expressed by $(P_2^{-T}\tilde{A}_f P_2, P_2^{-T}\hat{B}_f, \tilde{C}_f P_2)$, the filter model (2) can be written as

$$\begin{cases} \dot{x}_f(t) = P_2^{-T}\tilde{A}_f P_2 x_f(t) + P_2^{-T}\hat{B}_f \hat{y}(t) \\ z_f(t) = \tilde{C}_f P_2 x_f(t) \end{cases} \tag{41}$$

Define $\hat{x}(t) = P_2^T x_f(t)$, Eq.(41) can be written as follows.

$$\begin{cases} \hat{x}_f(t) = \tilde{A}_f \hat{x}(t) + \hat{B}_f \hat{y}(t) \\ z_f(t) = \tilde{C}_f \hat{x}(t) \end{cases} \tag{42}$$

That is why $(\tilde{A}_f, \hat{B}_f, \tilde{C}_f)$ can be chosen as the filter parameters. This completes the proof. □

According to Theorem 2, the H_∞ filtering parameters are obtained in an explicit form when the designed filter is under the hybrid-triggered scheme. To make the main results more abundant and more substantial, two corollaries are given to describe the time-triggered filtering error system and event-triggered filtering error system, respectively. Thus, the filtering parameters are expressed precisely.

Corollary 1. *For given positive parameters γ, $\bar{\theta}$, τ_M, d_M, σ, ρ_2 and ϵ_k, matrix G, system is asymptotically stable with time-triggered scheme and stochastic cyber attacks, if there exist matrices $P_1 > 0$, $\bar{P}_3 > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$ $(k = 1, 2)$, $\Omega > 0$, $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$, $\tilde{M}$, $\tilde{N}$, $\tilde{W}$, $\tilde{V}$ are matrices with appropriate dimensions, such that the following LMIs hold:*

$$\begin{bmatrix} \Xi_{11} + \Theta + \Theta^T & * & * & * & * \\ \Xi_{21} & -I & * & * & * \\ \Xi_{31} & 0 & -I & * & * \\ \Xi_{41}(s) & 0 & 0 & \Xi_{44} & * \\ \Xi_{51} & 0 & 0 & 0 & \Xi_{55} \end{bmatrix} < 0, (s = 1, 2, 3, 4) \tag{43}$$

$$P_1 - \bar{P}_3 > 0 \tag{44}$$

where

$$\Xi_{11} = \begin{bmatrix} \psi_{11} + \psi_{11}^T + \tilde{Q}_1 + \tilde{Q}_2 & * & * & * & * & * & * \\ \bar{\theta}_1 \psi_{21} & 0 & * & * & * & * & \\ 0 & 0 & -\tilde{Q}_1 & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * \\ 0 & 0 & 0 & 0 & -\tilde{Q}_2 & * & * \\ \psi_{41} & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \bar{\theta}\psi_{41} & 0 & 0 & 0 & 0 & 0 & -\bar{\theta} I \end{bmatrix}$$

$$\Theta = \begin{bmatrix} \tilde{M} + \tilde{W} & \tilde{N} - \tilde{M} & -\tilde{N} & -\tilde{W} + \tilde{V} & -\tilde{V} & 0 & 0 \end{bmatrix}$$

$$\Xi_{21} = \begin{bmatrix} \psi_{51} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \Xi_{31} = \begin{bmatrix} 0 & 0 & 0 & \bar{\theta}_2 G & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{41}(1) = \begin{bmatrix} \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}, \Xi_{41}(2) = \begin{bmatrix} \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \Xi_{41}(3) = \begin{bmatrix} \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}, \Xi_{41}(4) = \begin{bmatrix} \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}$$

$$\Xi_{51} = \begin{bmatrix} \sqrt{\tau_M}\psi_{11} & \bar{\theta}_1\sqrt{\tau_M}\psi_{21}^T & 0 & 0 & 0 & \sqrt{\tau_M}\psi_{41}^T & \bar{\theta}\sqrt{\tau_M}\psi_{31}^T \\ \sqrt{d_M}\psi_{11} & \bar{\theta}_1\sqrt{d_M}\psi_{21}^T & 0 & 0 & 0 & \sqrt{d_M}\psi_{41}^T & \bar{\theta}\sqrt{d_M}\psi_{31}^T \\ 0 & \rho_2\sqrt{\tau_M}\psi_{21}^T & 0 & 0 & 0 & 0 & \rho_2\sqrt{\tau_M}\psi_{31}^T \\ 0 & \rho_2\sqrt{d_M}\psi_{21}^T & 0 & 0 & 0 & 0 & \rho_2\sqrt{d_M}\psi_{31}^T \end{bmatrix}$$

$$\Xi_{44} = diag\{-\tilde{R}_1, -\tilde{R}_2\}, \Xi_{55} = \{-2\epsilon_1\tilde{P} + \epsilon_1^2\tilde{R}_1, -2\epsilon_2\tilde{P} + \epsilon_2^2\tilde{R}_2, -2\epsilon_1\tilde{P} + \epsilon_1^2\tilde{R}_1, -2\epsilon_2\tilde{P} + \epsilon_2^2\tilde{R}_2\}$$

Besides, the parameters matrices of the filter under the time-triggered scheme are obtained if the conditions above are hold.

$$\begin{cases} A_f = \hat{A}_f \bar{P}_3^{-1} \\ B_f = \hat{B}_f \\ C_f = \hat{C}_f \bar{P}_3^{-1} \end{cases} \tag{45}$$

Corollary 1 has discussed the filtering error system which is under the time-triggered scheme. In Corollary 2, the case of event-triggered scheme is described meticulously and the filtering parameters are expressed accurately.

Corollary 2. *For given positive parameters γ, $\bar{\theta}$, η_M, d_M, σ, ρ_2 and ϵ_k, matrix G, system is asymptotically stable with event-triggered scheme and stochastic cyber attacks, if there exist matrices $P_1 > 0$, $\bar{P}_3 > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$ $(k = 2, 3)$, $\Omega > 0$, $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$, $\tilde{U}$, $\tilde{S}$, $\tilde{W}$, $\tilde{V}$ are matrices with appropriate dimensions, such that the following LMIs hold:*

$$\begin{bmatrix} \bar{\Xi}_{11} + \bar{\Theta} + \bar{\Theta}^T & * & * & * & * \\ \bar{\Xi}_{21} & -I & * & * & * \\ \bar{\Xi}_{31} & 0 & -I & * & * \\ \bar{\Xi}_{41}(s) & 0 & 0 & \bar{\Xi}_{44} & * \\ \bar{\Xi}_{51} & 0 & 0 & 0 & \bar{\Xi}_{55} \end{bmatrix} < 0, \ (s = 1, 2, 3, 4) \tag{46}$$

$$P_1 - \bar{P}_3 > 0 \tag{47}$$

where

$$\bar{\Xi}_{11} = \begin{bmatrix} \psi_{11} + \psi_{11}^T + \tilde{Q}_2 + \tilde{Q}_3 & * & * & * & * & * & * & * \\ \bar{\theta}_1 \psi_{21} & \psi_{44} & * & * & * & * & * & * \\ 0 & 0 & -\tilde{Q}_2 & * & * & * & * & * \\ 0 & 0 & 0 & 0 & * & * & * & * \\ 0 & 0 & 0 & 0 & -\tilde{Q}_3 & * & * & * \\ \bar{\theta}_1 \psi_{31} & 0 & 0 & 0 & 0 & -C^T \Omega C & * & * \\ \psi_{41} & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I & * \\ \bar{\theta} \psi_{31} & 0 & 0 & 0 & 0 & 0 & 0 & -\bar{\theta} I \end{bmatrix}$$

$$\bar{\Theta} = \begin{bmatrix} \tilde{U} + \tilde{W} & -\tilde{U} + \tilde{S} & -\tilde{S} & -\tilde{W} + \tilde{V} & -\tilde{V} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Xi}_{21} = \begin{bmatrix} \psi_{51} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \ \bar{\Xi}_{31} = \begin{bmatrix} 0 & 0 & 0 & \bar{\theta}_2 G & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Xi}_{41}(1) = \begin{bmatrix} \sqrt{\eta_M} \tilde{U}^T \\ \sqrt{d_M} \tilde{W}^T \end{bmatrix}, \ \bar{\Xi}_{41}(2) = \begin{bmatrix} \sqrt{\eta_M} \tilde{U}^T \\ \sqrt{d_M} \tilde{V}^T \end{bmatrix}, \ \bar{\Xi}_{41}(3) = \begin{bmatrix} \sqrt{\eta_M} \tilde{S}^T \\ \sqrt{d_M} \tilde{W}^T \end{bmatrix}, \ \bar{\Xi}_{41}(4) = \begin{bmatrix} \sqrt{\eta_M} \tilde{S}^T \\ \sqrt{d_M} \tilde{V}^T \end{bmatrix}$$

$$\bar{\Xi}_{51} = \begin{bmatrix} \sqrt{\eta_M} \psi_{11} & \bar{\theta}_1 \sqrt{\eta_M} \psi_{21}^T & 0 & 0 & 0 & \bar{\theta}_1 \sqrt{\eta_M} \psi_{31}^T & \sqrt{\eta_M} \psi_{41}^T & \bar{\theta} \sqrt{\eta_M} \psi_{31}^T \\ \sqrt{d_M} \psi_{11} & \bar{\theta}_1 \sqrt{d_M} \psi_{21}^T & 0 & 0 & 0 & \bar{\theta}_1 \sqrt{d_M} \psi_{31}^T & \sqrt{d_M} \psi_{41}^T & \bar{\theta} \sqrt{d_M} \psi_{31}^T \\ 0 & \rho_2 \sqrt{\tau_M} \psi_{21}^T & 0 & 0 & 0 & \rho_2 \sqrt{\tau_M} \psi_{41}^T & 0 & \rho_2 \sqrt{\eta_M} \psi_{31}^T \\ 0 & \rho_2 \sqrt{d_M} \psi_{21}^T & 0 & 0 & 0 & \rho_2 \sqrt{d_M} \psi_{41}^T & 0 & \rho_2 \sqrt{d_M} \psi_{31}^T \end{bmatrix}$$

$$\bar{\Xi}_{44} = diag\{-\tilde{R}_2, -\tilde{R}_3\}, \ \bar{\Xi}_{55} = \{-2\epsilon_2 \tilde{P} + \epsilon_2^2 \tilde{R}_2, -2\epsilon_3 \tilde{P} + \epsilon_3^2 \tilde{R}_3, -2\epsilon_2 \tilde{P} + \epsilon_2^2 \tilde{R}_2, -2\epsilon_3 \tilde{P} + \epsilon_3^2 \tilde{R}_3\}$$

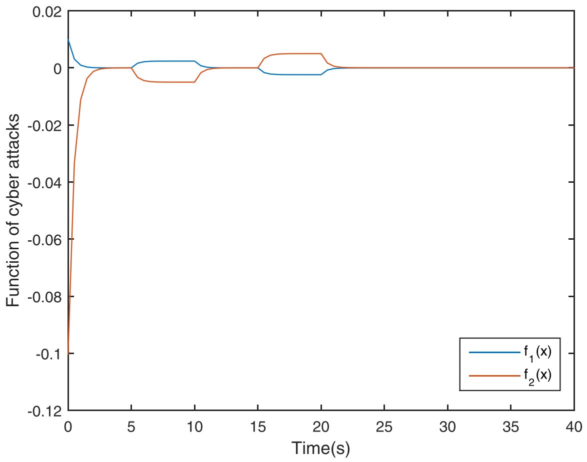

Fig. 2. The cyber attacks $f(x(t))$ in Example 1 and Example 3.

The parameter matrices of the filter under the event-triggered scheme are given as follows:

$$\begin{cases} A_f = \hat{A}_f \bar{P}_3^{-1} \\ B_f = \hat{B}_f \\ C_f = \hat{C}_f \bar{P}_3^{-1} \end{cases} \tag{48}$$

Remark 7. In this section, two corollaries are added to make a comparison with Theorem 2 which presents the accurate expressions of filtering parameters under the case of hybrid-triggered scheme. By comparing with the reference [34] which investigates the problem of H_∞ filtering for systems with time-varying delay, Corollary 1 describes that the H_∞ filter design with stochastic cyber attacks is feasible when the system is under the time-triggered scheme. In [43], the event-based H_∞ filtering for networked systems with communication delay is investigated. Compared with the reference [43], an extension is made in Corollary 2 which shows the algorithm of designed event-triggered H_∞ filter with stochastic cyber attacks. Moreover, in the simulation section, three examples are given to demonstrate the effectiveness of the designed H_∞ filter with stochastic cyber attacks under hybrid-triggered scheme, time-triggered scheme and event-triggered scheme, respectively.

4. Simulation examples

In this section, three examples are given to demonstrate the effectiveness of the designed filter.

Example 1. Consider the system (1) with the following matrix parameters.

$$A = \begin{bmatrix} -2.1 & 0 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, C = \begin{bmatrix} 0.8 & 0.6 \end{bmatrix}, L = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}, \omega(t) = \begin{cases} 1, & 5 \le t \le 10 \\ -1, & 15 \le t \le 20 \\ 0, & else \end{cases}$$

The function of cyber attacks $f(x(t))$ which is shown in Fig. 2 is supposed as $f(x(t)) = \begin{bmatrix} -tanh(0.1x_2(t)) \\ -tanh(0.01x_1(t)) \end{bmatrix}$. According to Assumption 1, the upper bound $G = diag\{0.01, 0.1\}$ can

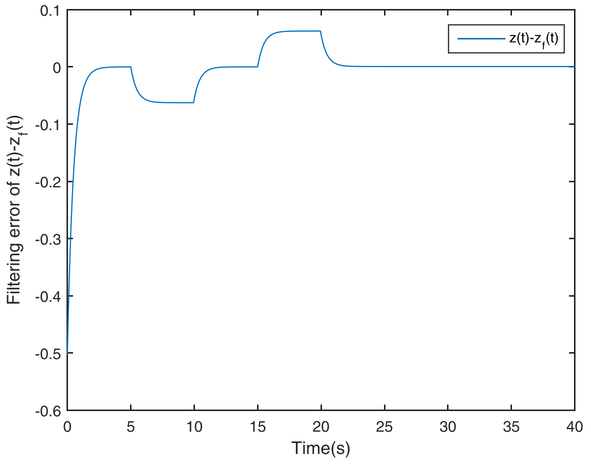

Fig. 3. Response of $\tilde{z}(t)$ in Example 1.

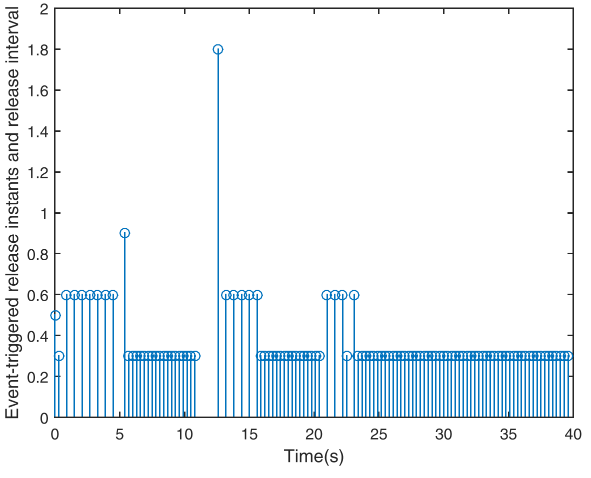

Fig. 4. The event-triggered instants and intervals in Example 1.

be derived, it can be verified easily that the upper bound confirm to the restrictive condition of nonlinear stochastic cyber attacks (8).

Set the sampling period $h = 0.3$, networked-induced delays $\tau_M = 0.2$, $d_M = 0.2$, $\eta_M = 0.1$ and $\gamma = 6.7$. Let $\bar{\theta} = 0.1$, $\bar{\alpha} = 0.5$ and $\sigma = 0.8$, then, the H_∞ filter is under the hybrid-triggered scheme and stochastic cyber attacks. With the initial condition $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$, $x_f(0) = \begin{bmatrix} 0.8 & -0.8 \end{bmatrix}^T$, by using Eq. (38) in Theorem 2, we can derive the filtering parameters as follows:

$$A_f = \begin{bmatrix} -2.1992 & 0.1169 \\ -0.5097 & -1.5204 \end{bmatrix}, B_f = \begin{bmatrix} -1.4374 \\ 0.6644 \end{bmatrix}, C_f = \begin{bmatrix} 0.0024 & -0.0005 \end{bmatrix}$$

Fig. 3 shows the response of $\tilde{z}(t)$ when the designed filter is under the hybrid-triggered scheme (9). The diagram of event-triggered release instants and intervals is shown in Fig. 4,

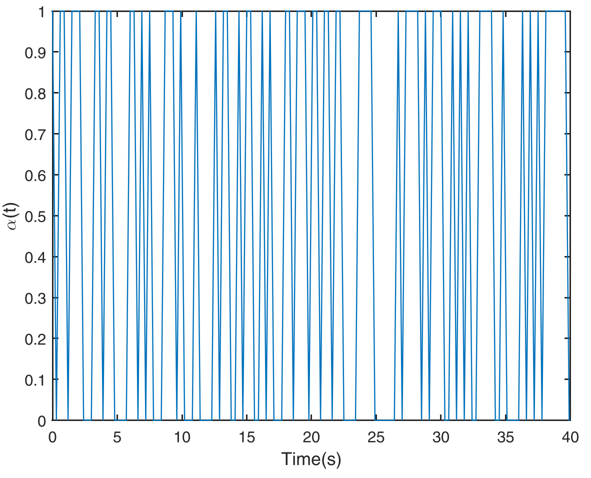

Fig. 5. Bernoulli distribution of $\alpha(t)$ in Example 1.

and Fig. 5 presents the Bernoulli distribution of variable $\alpha(t)$. According to the figures above, it is easy to know that the designed H_∞ filter for networked systems with event-triggered communication mechanism and stochastic cyber attacks is feasible.

Example 2. Consider a mechanical example borrowed from [34], x_1 and x_2 are the positions of masses m_1 and m_2, respectively, and k_1 and k_2 are the spring constants. c denotes viscous friction coefficient between the massed and the horizontal surface. The parameters of this system (1) are given as follows.

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{k_1+k_2}{m_1} & \frac{k_2}{m_1} & -\frac{c}{m_1} & 0 \\ \frac{k_2}{m_2} & -\frac{k_2}{m_2} & 0 & -\frac{c}{m_2} \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_1} \\ 0 \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}, L = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

where $m_1 = 1,\ m_2 = 0.5,\ k_1 = k_2 = 1,\ c = 0.5$.

This example discusses the case when the designed H_∞ filtering system is under the time-triggered scheme and cyber attacks. Suppose that $f^T(x(t)) = [-tanh^T(x_2(t)) \quad -tanh^T(0.05x_1(t)) \quad -tanh^T(x_4(t)) \quad -tanh^T(0.1x_2(t) + 0.1x_3(t) + 0.1x_4(t))]^T$, then, by applying Assumption 1, the upper bound $G = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0.05 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0.1 & 0.1 & 0.1 \end{bmatrix}$ which satisfies the inequality (8) can be obtained. As is shown in Fig. 6, it indicates the diagram of cyber attacks.

Choose $h = 0.01$ as sampling period and $\omega(t)$ is assumed to be the same in Example 1. Set the probability of cyber attacks $\bar{\theta} = 0.1$, disturbance attenuation level $\gamma = 1$, time delay $t_M = 0.1,\ d_M = 0.1$. Set the initial condition $x(0) = [1; 1; -1; -1]$, $x_f(0) = [1; 1; -1; -1]$.

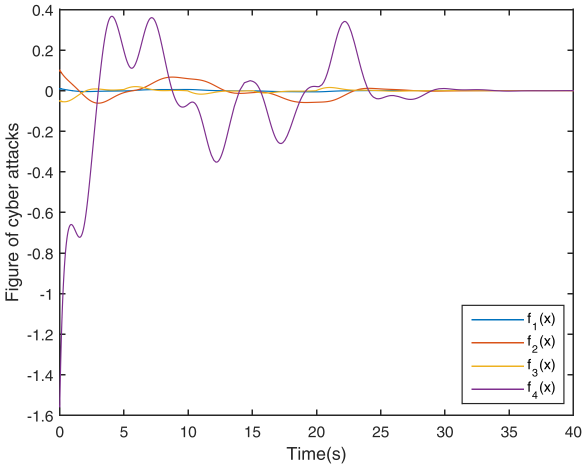

Fig. 6. The cyber attacks $f(x(t))$ in Example 2.

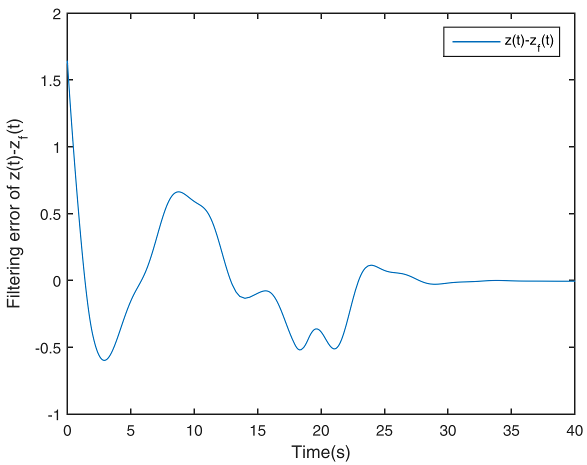

Fig. 7. Response of $\tilde{z}(t)$ in Example 2.

Based on the Eq. (45) in Corollary 1, we can derive the filtering parameters as follows:

$$A_f = \begin{bmatrix} -0.3029 & 0.1825 & 2.5359 & -1.6726 \\ 0.2703 & -0.7889 & -1.5524 & 2.2522 \\ -0.7796 & -0.0019 & -1.3047 & 0.1534 \\ 0.1257 & -0.9524 & -0.6249 & 0.4390 \end{bmatrix}, B_f = \begin{bmatrix} -1.7394 \\ 0.0979 \\ 0.2549 \\ -0.4362 \end{bmatrix}$$

$$C_f = \begin{bmatrix} -0.0443 & -0.3874 & 0.0819 & 0.1254 \end{bmatrix}$$

Fig. 7 represents the response of $\tilde{z}(t)$ when the designed filter is under the time-triggered scheme, which illustrates the effectiveness of the designed filter with stochastic cyber attacks.

Example 3. Consider that the designed filter is under the event-triggered scheme, the system parameters and the function of cyber attacks are supposed to be the same in Example 1. Set sampling period $h = 0.5$, $\bar{\theta} = 0.1$, $\gamma = 5$ and the triggered factor $\sigma = 0.2$, time delay

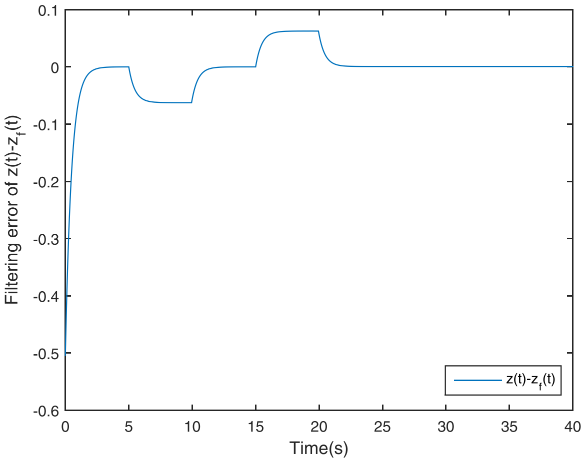

Fig. 8. Response of $\tilde{z}(t)$ in Example 3.

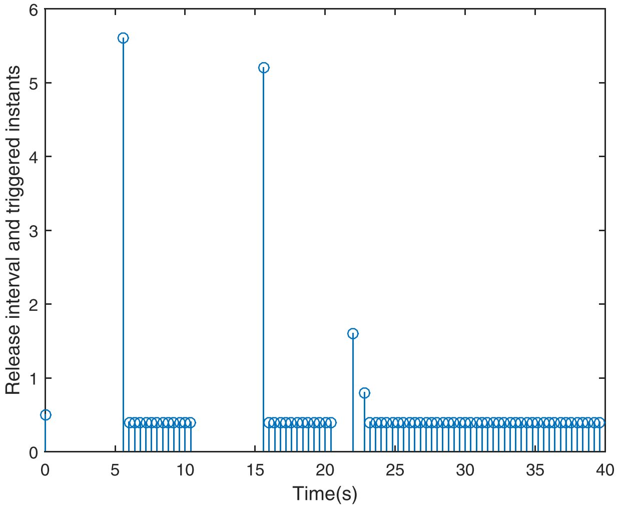

Fig. 9. The event-triggered instants and intervals in Example 3.

$\eta_M = 0.1,\ d_M = 0.1$. By applying Eq. (48) in Corollary 2, the parameters of the designed filter are obtained as follows:

$$A_f = \begin{bmatrix} -6.0963 & 0.8104 \\ -16.1440 & 1.1743 \end{bmatrix}, B_f = \begin{bmatrix} -0.8326 \\ 8.4596 \end{bmatrix}, C_f = \begin{bmatrix} 0.0034 & -0.0007 \end{bmatrix}$$

Under the event-triggered scheme, Fig. 8 depicts the response of $\tilde{z}(t)$ in Eq.(11) and Fig. 9 represents the event-triggered released instants and intervals. According to Fig. 8, one can see that the designed H_∞ filtering with event-triggered scheme and stochastic cyber attacks is useful.

5. Conclusion

This paper is devoted to the investigation of the hybrid-triggered H_∞ filtering problem for networked systems under stochastic cyber attacks. In order to alleviate the burden of the network, a hybrid-triggered scheme is introduced, in which the switching rule between the time-triggered scheme and the event-triggered scheme is described by Bernoulli variable. By taking the hybrid-triggered scheme and the effects of stochastic cyber attacks into consideration, a mathematical H_∞ filtering model has been constructed for networked systems. By applying Lyapunov stability theory and the LMI techniques, sufficient conditions for the stability of filtering error system have been developed. In addition, the parameters of designed H_∞ filter are obtained in an explicit form. Illustrative examples are given to demonstrate the usefulness of desired filter under hybrid-triggered scheme and cyber attacks.

Acknowledgments

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