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Hybrid-driven-based H_∞ filter design for neural networks subject to deception attacks

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ABSTRACT

This paper investigates the problem of H_∞ filter design for neural networks with hybrid triggered scheme and deception attacks. In order to make full use of the limited network resources, a hybrid triggered scheme is introduced, in which the switching between the time triggered scheme and the event triggered scheme obeys Bernoulli distribution. By considering the effect of hybrid triggered scheme and deception attacks, a mathematical model of H_∞ filtering error system is constructed. The sufficient conditions that can ensure the stability of filtering error system are given by using Lyapunov stability theory and linear matrix inequality (LMI) techniques. Moreover, the explicit expressions are provided for the designed filter parameters that is in terms of LMIs. Finally, a numerical example is employed to illustrate the design method.

1. Introduction

Nowadays, neural networks have been triumphantly applied in many areas such as image processing, associative storage, signal optimization, large volume of high speed data processing. Over the last decades, rapidly increasing attention has been paid to the research of neural networks, and a great number of achievements have been obtained by researchers [1–5]. Due to the certain theory meaning and application value of filter design for neural networks, the research on filter design for delayed neural networks has attracted extensive attention in recent years [6–9]. In [7], the authors investigate the resilient finite-time filtering problem for discrete-time uncertain Markov jump neural networks with packet dropouts. The authors in [8] investigate the robust H_∞ filtering problem for neural delay differential systems with parametric uncertainties. In literature [9], the authors are concerned with H_∞ filter design for a class of neural network systems with event-triggered communication scheme and quantization.

During the past few decades, time triggered scheme (periodic sampling) that is applied for the sensor sampling in the analysis and design of control systems has been widely studied by lots of researchers [10,11]. For example, a model of networked control systems is constructed by taking the effects of network-induced delay and data dropout into consideration in [11]. Based on the constructed model of networked control systems above, the design of robust H_∞ controllers for uncertain networked control systems is investigated in [11]. However, if all sampling data is delivered via the network, it will result in plenty of waste for the limited network resources. To avoid the drawbacks of periodic sampling, many researchers propose

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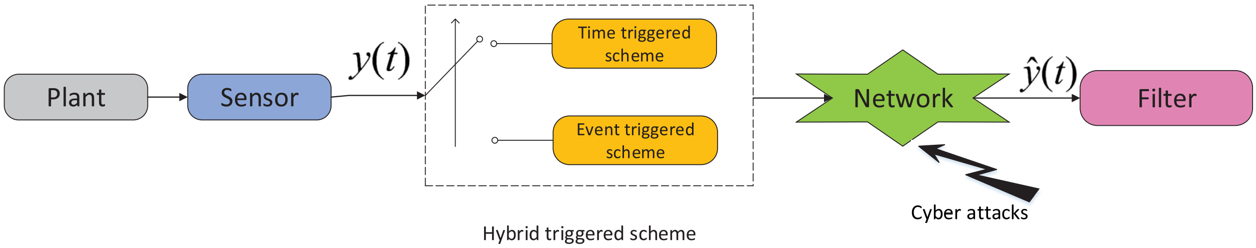

Fig. 1. The structure of hybrid-driven-based H_∞ filter for neural networks subject to deception attacks.

event triggered schemes to reduce the burden of network and enhance the efficiency of transmission. For example, a new event triggered scheme is proposed in [12] to investigate the problem of event-triggered controller design for networked control systems. The basic idea of event triggered scheme in [12] is that whether the pre-designed condition is violated or not can be the key rule of transmission for newly sampling data, in other words, it only be delivered when newly sampling data violates the pre-designed conditions. Based on the research achievement in [12], large numbers of researchers have arisen interest in the investigation of event triggered scheme and obtained fruitful achievements [13–16]. For instance, the authors in [13] investigate the problem of event-triggered state estimation for complex network systems with quantization. A decentralized event triggering communication scheme for large-scale systems is investigated in [14] under network environments. The authors of [15] are devoted to the design of H_∞ event-triggered filter for a class of T-S fuzzy systems. Drawing lessons from existing researches on the event-triggered scheme in [12], the authors in [17] propose a new hybrid triggered scheme, in which the stochastic switching between the time triggered scheme and event triggered scheme is described by a variable obeying Bernoulli distribution. On the basis of hybrid triggered scheme in [17], the problem of reliable control for hybrid-driven T-S fuzzy systems with actuator faults and probabilistic nonlinear perturbations is investigated in [18]. In this paper, inspired by the proposed hybrid triggered scheme in [17], an H_∞ filter design is investigated with hybrid triggered scheme and deception attacks.

Cyber attacks are aggressive behaviors aiming at destroying communication systems, real sampling data, networked infrastructures and devices. As is described in [19], Denial of Service attacks, replay attacks and deception attacks are three categories of cyber attacks. One of the most important attack modes on the safety of network is the deception attacks that include an incorrect sensor measurement or a mistaken identity of the receiving equipment. In view of the significance of network security, the risks of deception attacks should not be ignored any more with the fast development of network. On the basis of the deception attacks mentioned above, more and more researchers are devoting to the exploration of deception attacks [19–21]. For example, the attack scheduling problem for a class of stochastic linear systems with χ_2 detectors is studied in [19]. The authors investigate the distributed recursive filtering problem for a class of discrete time-delayed stochastic systems subject to both uniform quantization and deception attack effects on the measurement outputs in [20]. A novel event based distributed estimator is proposed in [21] to defend against the false injection attack. Based on the deception attacks proposed in [19], this paper is concerned with hybrid-driven-based H_∞ filter design for neural networks subject to deception attacks.

Motivated by the observations above, this paper addresses the issue of H_∞ filter design for neural networks with hybrid-triggered scheme and deception attacks. The hybrid triggered scheme that consists of time triggered scheme and event triggered scheme is introduced to reduce the pressure of network bandwidth. A mathematical model of H_∞ filtering error system is constructed by taking the effect of hybrid triggered scheme and deception attacks into consideration. Sufficient conditions that can guarantee the stability of filtering error system for neural networks are obtained by using Lyapunov stability theory and LMI techniques. Moreover, the parameters of the filter are obtained in explicit expression. A simulation example is provided to show the usefulness of the proposed method.

This paper is organized as follows. In Section 2, a filtering error system is considered for the neural networks with hybrid triggered scheme and deception attacks. Section 3 gives sufficient conditions which can guarantee the stability of filtering error system for neural networks. A numerical example is given in Section 4 to show the usefulness of the proposed method. The conclusion is drawn in the final part.

Notation: R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n\times m$ real matrices; Matrix $X>0$, for $X\in R^{n\times n}$ means that the matrix X is real symmetric positive definite. I is the identity matrix of appropriate dimension. In addition, T stands for the transpose of matrix. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System description

In this paper, a hybrid-driven-based H_∞ filter design for neural networks subject to deception attacks is investigated. As is shown in Fig. 1, supposing that the sampling data is transmitted in a non-ideal networked environment, the hybrid triggered scheme is introduced to reduce the pressure of network bandwidth. The neural networks with n neurons is given as follows:

$$\begin{cases} \dot{x}(t) = -Ax(t) + W_0 g(x(t)) + W_1 g(x(t-\phi(t))) + A_\omega \omega(t) \\ y(t) = Cx(t) \\ z(t) = Lx(t) \end{cases} \tag{1}$$

where $x(t) = [x_1, x_2, \ldots, x_n]^T \in R^n$ is the state vector of neural networks. $A = diag\{a_1, a_2, \ldots, a_n\}$ is a diagonal matrix with positive entries $a_i > 0$. W_0 and W_1 are the connection weight matrix and the delayed connection weight matrix, respectively. $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$ denotes the neuron activation function, and $\phi(t)$ denotes the time-varying bounded state delay taking values on the interval $[\phi_m, \phi_M]$, where ϕ_m and ϕ_M are the lower and upper bounds of $\phi(t)$. $\omega(t) \in L_2[0, \infty)$ is the external disturbance. A_ω, C, and L are the parameter matrices with appropriate dimension. $y(t) = [y_1, y_2, \ldots, y_r]^T \in R^r$ is the measurement output. $z(t) = [z_1, z_2, \ldots, z_p]^T \in R^p$ are the output vector.

The purpose of this paper is to design the following filter system (2):

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f \hat{y}(t) \\ z_f(t) = C_f x_f(t) \end{cases} \tag{2}$$

where $x_f(t) \in R^n$ is the state estimation of $x(t)$. $z_f(t) \in R^n$ is the output of the filter representing a estimation of $z(t)$; $A_f \in R^{n \times n}$, $B_f \in R^{n \times m}$, $C_f \in R^{p \times n}$ are the filter parameter matrices to be determined. $\hat{y}(t)$ is the real input of the filter.

In this paper, the problem of hybrid-driven-based H_∞ filter design for system (1) is investigated. As is shown in Fig. 1, the hybrid triggered scheme which consists of time triggered scheme and event triggered scheme is firstly adopted in neural networks. The detailed discussion about the two triggered scheme is given in **Case A** and **Case B**, respectively.

Case A: As is shown in Fig. 1, when the selecting switch in the hybrid triggered scheme turns to "time triggered scheme", the H_∞ filter design for neural networks becomes the classical networked H_∞ filter design for neural networks. Then, utilizing methods in [17,22], the input signal $y_1(t)$ of the filter transmitted through the network channel can be described as:

$$y_1(t) = Cx(t_k h), t \in [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) \tag{3}$$

where h is the sampling period, $t_k h$ is the instants when the sampling data arrives at the filter, $\tau_{t_k}$is the network-induced delay. By using the method in [23,24], define the network allowable equivalent delay $\tau(t) = t - t_k h$, Eq. (3) can be rewritten as follows:

$$y_1(t) = Cx(t - \tau(t)) \tag{4}$$

where $\tau(t) \in [0, \tau_M]$, τ_M is the upper bound of $\tau(t)$.

As is well known, the time triggered scheme is widely used along with the transmitting of large repetitive signals, which leads to the waste of network resources. In order to reduce the waste of network resources and improve the system performance, the event triggered scheme is introduced to decide whether the newly sampling data should be transmitted or not.

Case B: As is shown in Fig. 1, when the selecting switch in the hybrid triggered scheme turns to "event triggered scheme", the H_∞ filter design for neural networks becomes the event triggered H_∞ filter design for neural networks. The event generator function can be defined as the following judgement algorithm, which is the same as [25]:

$$[y(((k+j)h)) - y(kh)]^T \Phi [y(((k+j)h) - y(kh)] \leq \sigma y^T((k+j)h) \Phi y((k+j)h) \tag{5}$$

where Φ is a symmetric positive define matrix, $j = 1, 2, \ldots,$ $\sigma \in [0, 1)$, $y((k+j)h)$ is the current sampling data and $y(kh)$ is the latest one. The sampling data $y((k+j)h)$ that satisfies the pre-design condition (5) will not be delivered, only the one that violates the pre-design condition in (5) will be transmitted.

Remark 1. According to the event triggered algorithm (5), it is evident that the sampling data is sampled at instant kh by sampler with a given period h, and the next sampling data is sampled at instant $(k+1)h$. Suppose that the release times are $t_0 h$, $t_1 h$, $t_2 h$, $\cdots$, it is easily deduced that $\nu_i h = t_{i+1} h - t_i h$ denotes the released period, where $\nu_i h$ represents the sampling interval between the two conjoint transmitted instants. In accordance with the event triggered algorithm (5), the set of the release instants $\{t_0 h, t_1 h, t_2 h, \ldots\} \subseteq \{0, 1, 2, \ldots\}$. The amount of $\{t_0 h, t_1 h, t_2 h, \ldots\}$ depends on the value of σ and the variation of sampling data.

Similar to [26], the interval $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$ is divided into several subintervals like subsets $\Lambda = \bigcup_{j=1}^{\infty} [t_k h + jh + \eta_{t+j}, t_k h + jh + h + \eta_{t+j+1})$, $j = \{1, 2, \ldots, t_{k+1} - t_k - 1\}$. Define $\eta(t) = t - t_k h - jh$, $0 \leq \tau_{t_k} \leq \eta(t) \leq h + \eta_{t_{k+j+1}} \triangleq \eta_M$, $e_k(t) = x(t_k h) - x(t_k h + jh)$. With the event triggered scheme, the input signal of the filter $y_2(t)$ can be written as:

$$y_2(t) = Cx(t - \eta(t)) + Ce_k(t) \tag{6}$$

Combine **Case A** and **Case B**, utilizing the methods in [17], the hybrid triggered scheme is introduced to reduce the pressure of network bandwidth. The stochastic switching between the time triggered scheme and the event triggered scheme is described by a random variable which obeys the Bernoulli distribution. The input signal $\tilde{y}(t)$ of the filter via the network channel can be expressed as follows:

$$\begin{aligned}\tilde{y}(t) &= \alpha(t)y_1(t) + (1-\alpha(t))y_2(t)\\ &= \alpha(t)Cx(t-\tau(t)) + (1-\alpha(t))[Cx(t-\eta(t)) + Ce_k(t)]\end{aligned} \tag{7}$$

where $\alpha(t) \in [0, 1]$, $\alpha(t)$ has the following statistical properties.

$$\mathbf{E}\{\alpha(t)\} = \bar{\alpha}, \mathbf{E}\{(\alpha(t)-\bar{\alpha})^2\} = \bar{\alpha}(1-\bar{\alpha}) = \gamma_1^2$$

$\bar{\alpha}$ represents the expectation of $\alpha(t)$, γ_1^2 is utilized to represent the mathematical variance of $\alpha(t)$.

Remark 2. Different from the communication strategies in the existing publications, the hybrid triggered scheme in this paper is based on a switching between the time triggered scheme and the event triggered scheme. $\alpha(t) = 1$ means the time triggered scheme is activated. $\alpha(t) = 0$ means the event triggered scheme is activated.

As is shown in Fig. 1, the transmitting data is susceptible to aggressive signals. In this paper, the H_∞ filter design with hybrid triggered scheme is investigated by taking the effect of deception attacks into consideration. The aggressive signals combined with real sampling data are transmitted to the filter after the deception attacks launched. It is assumed that the time-varying delay of aggressive signals can be described by variable $d(t)$ and deception attacks $f(x(t-d(t)))$ meet the following criterion:

$$||f(x(t-d(t)))||_2 \le ||Gx(t-d(t))||_2 \tag{8}$$

where G is a constant matrix representing the upper bound of the nonlinearity.

Remark 3. The upper bounds information of the nonlinearity is introduced in [27,28] to describe the inhibiting condition of nonlinear disturbance. Analogously, in this paper, matrix G is used to represent the upper bound of deception attacks mentioned above, and its value depends on the practical state of deception attacks.

Taking the hybrid triggered scheme (7) and the deception attack (8) into consideration, the real input $\hat{y}(t)$ of filter can be written as:

$$\begin{aligned}\hat{y}(t) &= (1-\theta(t))\tilde{y}(t) + \theta(t)Cf(x(t-d(t)))\\ &= (1-\theta(t))\alpha(t)Cx(t-\tau(t)) + (1-\theta(t))(1-\alpha(t))[Cx(t-\eta(t)) + Ce_k(t)] + \theta(t)Cf(x(t-d(t)))\end{aligned} \tag{9}$$

where $d(t) \in [0, d_M]$, $\theta(t) \in [0, 1]$, $\theta(t)$ is used to describe the occurring probability of deception attacks.

It is assumed that Bernoulli variables $\alpha(t)$ and $\theta(t)$ are independent of each other. $\theta(t)$ have the following statistical properties.

$$\mathbf{E}\{\theta(t)\} = \bar{\theta}, \mathbf{E}\{(\theta(t)-\bar{\theta})^2\} = \bar{\theta}(1-\bar{\theta}) = \gamma_2^2$$

where $\bar{\theta}$ represents the expectation of $\theta(t)$, γ_2^2 is utilized to represent the mathematical variance of $\theta(t)$.

Remark 4. In this paper, $\theta(t)$ obeying Bernoulli distribution is utilized to depict the randomly occurring cyber attacks. When $\theta(t) = 1$, the real input of the filter in (9) is $\hat{y}(t) = Cf(x(t-d(t)))$, which means that the real signal is replaced by the deception attacks. When $\theta(t) = 0$, the real input of the filter can be expressed as $\hat{y}(t) = \alpha(t)Cx(t-\tau(t)) + (1-\alpha(t))[Cx(t-\eta(t)) + Ce_k(t)]$ according to (9), which means that the data is delivered in the network without deception attacks.

Remark 5. In [29,30], the missing measurements are described by a binary switching sequence satisfying Bernoulli distribution. In this paper, the stochastic switching between the two different triggered schemes is described by a variable satisfying Bernoulli distribution. The occurring probability of the stochastic cyber-attacks is also described by Bernoulli variables.

Combine (9) and (2), the filter system can be written as follows:

$$\begin{cases}\dot{x}_f(t) = A_f x_f(t) + B_f\theta(t)Cf(x(t-d(t))) + B_f(1-\theta(t))\alpha(t)Cx(t-\tau(t))\\ \qquad + B_f(1-\theta(t))(1-\alpha(t))Cx(t-\eta(t)) + B_f(1-\theta(t))(1-\alpha(t))Ce_k(t)\\ z_f(t) = C_f x_f(t)\end{cases} \tag{10}$$

By setting $e(t) = [x^T(t) \quad x_f^T(t)]^T$, $\tilde{z}(t) = z(t) - z_f(t)$, the following augmented system can be obtained from (1) and (10):

$$\begin{cases}\dot{e}(t) = \bar{A}_f e(t) + \bar{W}_0 g(x(t)) + \bar{W}_1 g(x(t-\phi(t))) + (1-\theta(t))\alpha(t)\bar{B}_f He(t-\tau(t)) + \bar{A}_\omega \omega(t)\\ \qquad + (1-\theta(t))(1-\alpha(t))\bar{B}_f He(t-\eta(t)) + (1-\theta(t))(1-\alpha(t))\bar{B}_f e_k(t) + \theta(t)\bar{B}_f f(x(t-d(t)))\\ \tilde{z}(t) = \bar{C}_f e(t)\end{cases} \tag{11}$$

where

$$\bar{A}_f = \begin{bmatrix}-A & 0\\ 0 & A_f\end{bmatrix}, \quad \bar{B}_f = \begin{bmatrix}0\\ B_f C\end{bmatrix}, \quad \bar{W}_0 = \begin{bmatrix}W_0\\ 0\end{bmatrix}, \quad \bar{W}_1 = \begin{bmatrix}W_1\\ 0\end{bmatrix}, \quad \bar{A}_\omega = \begin{bmatrix}A_\omega\\ 0\end{bmatrix}, \quad H^T = \begin{bmatrix}I\\ 0\end{bmatrix}, \quad \bar{C}_f^T = \begin{bmatrix}L\\ -C_f\end{bmatrix}$$

Definition 1 [31]**.** For given function $V:C^b_{F_0}([-\tau_M, 0], R^n)\times S$, its infinitesimal operator $\mathcal{L}$ is defined as:

$$\mathcal{L}(V_{\eta(t)}) = \lim_{\triangle\to 0+}\frac{1}{\triangle}[\mathbb{E}(V(\eta_t+\triangle)|\eta_t) - V(\eta_t)] \tag{12}$$

Assumption 1 [32]**.** The neural activation function satisfies one of the following conditions, and U_1, U_2 are real constant matrices and satisfy $U_2 - U_1 \geq 0$:

$$[g(x) - U_1x]^T[g(x) - U_2x] \leq 0 \tag{13}$$

Lemma 1 [33,34]**.** *For the given instant τ_1 and matrix $R>0$, the following inequalities are established:*

$$-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R\dot{x}(s) \leq \begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix}^T \begin{bmatrix} -R & R \\ R & -R \end{bmatrix}\begin{bmatrix} x(t) \\ x(t-\tau_1) \end{bmatrix} \tag{14}$$

Lemma 2 [35]**.** *For any vector x, $y\in R^n$ and matrix $Q\in R^{n\times n}$ with appropriate dimensions, the following inequality is established:*

$$2x^Ty \leq x^TQx + y^TQ^{-1}y \tag{15}$$

Lemma 3 [36]**.** *Supposed $\tau(t)\in[0, \tau_M]$, $\phi(t)\in[\phi_m, \phi_M]$, $\eta(t)\in[0, \eta_M]$, $d(t)\in[0, d_M]$, $\Psi_i(i=1,\dots,8)$ are matrices with appropriate dimensions, the inequality $(\phi(t)-\phi_m)\Psi_1 + (\phi_M-\phi(t))\Psi_2 + \tau(t)\Psi_3 + (\tau_M-\tau(t))\Psi_4 + \eta(t)\Psi_5 + (\eta_M-\eta(t))\Psi_6 + d(t)\Psi_7 + (d_M-d(t))\Psi_8 + \Xi < 0$ is established, if and only if the following inequalities are established:*

$$\begin{cases}
(\phi_M-\phi_m)\Psi_2+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_2+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\
(\phi_M-\phi_m)\Psi_2+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_2+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_7+\Xi<0\\
(\phi_M-\phi_m)\Psi_2+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_2+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\
(\phi_M-\phi_m)\Psi_2+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_2+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_7+\Xi<0\\
(\phi_M-\phi_m)\Psi_1+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_1+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\
(\phi_M-\phi_m)\Psi_1+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_1+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_7+\Xi<0\\
(\phi_M-\phi_m)\Psi_1+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_1+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\
(\phi_M-\phi_m)\Psi_1+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & (\phi_M-\phi_m)\Psi_1+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_7+\Xi<0
\end{cases} \tag{16}$$

3. Main results

In this section, by using Lyapunov stability theory and LMI techniques, the sufficient conditions will be derived which guarantee the stability of the filtering error system.

Theorem 1. *For given parameters ϕ_m, ϕ_M, τ_M, η_M, d_M, σ, $\bar{\alpha}$, $\bar{\theta}$ and matrix G, by considering hybrid triggered scheme (7) and deception attacks (8), the augmented system (11) is asymptotically stable with an H_∞ disturbance attention level γ, if there exist matrices $P>0$, $Q>0$, $Q_k>0$, $R_k>0$ $(k=1,2,3,4,5)$, $\Omega>0$, K, Y, M, N, S, Z, W, V, with appropriate dimension and parameters $\alpha>0$, $\beta>0$ satisfying:*

$$\Omega(s) = \begin{bmatrix} \Omega_{11}+\Gamma+\Gamma^T & * & * & * & * \\ \Omega_{21} & \Omega_{22} & * & * & * \\ \Omega_{31} & 0 & \Omega_{33} & * & * \\ \Omega_{41} & 0 & 0 & \Omega_{44} & * \\ \Omega_{51}(s) & 0 & 0 & 0 & \Omega_{55} \end{bmatrix} < 0, \quad s = 1,2,3,\dots,16 \tag{17}$$

where

$$\Omega_{11} = \begin{bmatrix} \Gamma_1 & * & * \\ \Gamma_2 & \Gamma_3 & * \\ \Gamma_4 & 0 & \Gamma_5 \end{bmatrix}$$

$$\Gamma = \begin{bmatrix} M+Z+W & K & -K+Y & -Y & -M+N & -N & -Z+S & -S & -W+V & -V & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_1 = \begin{bmatrix} \Lambda_1 & * & * & * & * \\ R_2 & -Q_1-R_2 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & -Q_2 & * \\ \Lambda_2 & 0 & 0 & 0 & -\beta\bar{U}_1 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \Lambda_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Gamma_4 = \begin{bmatrix} \Lambda_4 & 0 & 0 & 0 & 0 \\ \bar{W}_1^TP & 0 & 0 & 0 & -\beta\bar{U}_2^T \\ \Lambda_5 & 0 & 0 & 0 & 0 \\ \bar{A}_\omega^TP & 0 & 0 & 0 & 0 \\ \bar{\theta}\bar{B}_f^TP & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_3 = diag\{-Q_3, \sigma C^TH^T\Omega HC, -Q_4, 0, -Q_5\}, \quad \Gamma_5 = diag\{-\alpha I, -\beta I, -C^T\Omega C, -\gamma^2 I, -\bar{\theta}Q\}$$

$$\Lambda_1 = P\bar{A}_f + \bar{A}_f^TP + Q_1 + Q_2 + Q_3 + Q_4 + Q_5 - R_2 - \alpha\bar{U}_1, \ \Lambda_2 = (1-\bar{\theta})\bar{\alpha}H^T\bar{B}_f^TP$$

$$\Lambda_3 = (1-\bar{\theta})(1-\bar{\alpha})H^T\bar{B}_f^TP, \quad \Lambda_4 = \bar{W}_0^TP - \alpha\bar{U}_2^T, \quad \Lambda_5 = (1-\bar{\theta})(1-\bar{\alpha})\bar{B}_f^TP$$

$$\Omega_{21}=\begin{bmatrix}\bar{C}_f & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\bar{\theta}}GQH & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},\quad \Omega_{22}=diag\{-I,-Q\}$$

$$\Omega_{31}=\begin{bmatrix}\Gamma_6 & \Gamma_7 & \Gamma_8 & \Gamma_9\\ 0 & \Gamma_{10} & 0 & \Gamma_{11}\\ 0 & \Gamma_{12} & 0 & \Gamma_{13}\end{bmatrix},\quad \Omega_{41}=\begin{bmatrix}0 & \Gamma_{14} & 0 & \Gamma_{15}\\ 0 & 0 & 0 & \Gamma_{16}\end{bmatrix}$$

$$\Gamma_6=\begin{bmatrix}\kappa P\bar{A}_f & 0 & 0 & 0\\ \phi_m P\bar{A}_f & 0 & 0 & 0\\ \sqrt{\tau_M}P\bar{A}_f & 0 & 0 & 0\\ \sqrt{\eta_M}P\bar{A}_f & 0 & 0 & 0\\ \sqrt{d_M}P\bar{A}_f & 0 & 0 & 0\end{bmatrix},\quad \Gamma_7=\begin{bmatrix}\bar{\theta}_1\bar{\alpha}\kappa P\bar{B}_fH & 0 & \bar{\theta}_1\bar{\alpha}_1\kappa P\bar{B}_fH & 0\\ \bar{\theta}_1\bar{\alpha}\phi_m P\bar{B}_fH & 0 & \bar{\theta}_1\bar{\alpha}_1\phi_m P\bar{B}_fH & 0\\ \bar{\theta}_1\bar{\alpha}\sqrt{\tau_M}P\bar{B}_fH & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}P\bar{B}_fH & 0\\ \bar{\theta}_1\bar{\alpha}\sqrt{\eta_M}P\bar{B}_fH & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}P\bar{B}_fH & 0\\ \bar{\theta}_1\bar{\alpha}\sqrt{d_M}P\bar{B}_fH & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}P\bar{B}_fH & 0\end{bmatrix}$$

$$\Gamma_8=\begin{bmatrix}0 & 0 & \kappa\bar{W}_0P & \kappa\bar{W}_1P\\ 0 & 0 & \phi_m\bar{W}_0P & \phi_m\bar{W}_1P\\ 0 & 0 & \sqrt{\tau_M}\bar{W}_0P & \sqrt{\tau_M}\bar{W}_1P\\ 0 & 0 & \sqrt{\eta_M}\bar{W}_0P & \sqrt{\eta_M}\bar{W}_1P\\ 0 & 0 & \sqrt{d_M}\bar{W}_0P & \sqrt{d_M}\bar{W}_1P\end{bmatrix},\quad \Gamma_9=\begin{bmatrix}\bar{\theta}_1\bar{\alpha}_1\kappa P\bar{B}_f & \kappa\bar{A}_\omega P & \bar{\theta}\kappa\bar{B}_fP\\ \bar{\theta}_1\bar{\alpha}_1\phi_m P\bar{B}_f & \phi_m\bar{A}_\omega P & \bar{\theta}\phi_m\bar{B}_fP\\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}P\bar{B}_f & \sqrt{\tau_M}\bar{A}_\omega P & \bar{\theta}\sqrt{\tau_M}\bar{B}_fP\\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}P\bar{B}_f & \sqrt{\eta_M}\bar{A}_\omega P & \bar{\theta}\sqrt{\eta_M}\bar{B}_fP\\ \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}P\bar{B}_f & \sqrt{d_M}\bar{A}_\omega P & \bar{\theta}\sqrt{d_M}\bar{B}_fP\end{bmatrix}$$

$$\Gamma_{10}=\begin{bmatrix}\bar{\theta}_1\gamma_1\kappa P\bar{B}_fH & 0 & -\bar{\theta}_1\gamma_1\kappa P\bar{B}_fH & 0\\ \bar{\theta}_1\gamma_1\phi_m P\bar{B}_fH & 0 & -\bar{\theta}_1\gamma_1\phi_m P\bar{B}_fH & 0\\ \bar{\theta}_1\gamma_1\sqrt{\tau_M}P\bar{B}_fH & 0 & -\bar{\theta}_1\gamma_1\sqrt{\tau_M}P\bar{B}_fH & 0\\ \bar{\theta}_1\gamma_1\sqrt{\eta_M}P\bar{B}_fH & 0 & -\bar{\theta}_1\gamma_1\sqrt{\eta_M}P\bar{B}_fH & 0\\ \bar{\theta}_1\gamma_1\sqrt{d_M}P\bar{B}_fH & 0 & -\bar{\theta}_1\gamma_1\sqrt{d_M}P\bar{B}_fH & 0\end{bmatrix},\quad \Gamma_{11}=\begin{bmatrix}-\bar{\theta}_1\gamma_1\kappa P\bar{B}_f & 0 & 0\\ -\bar{\theta}_1\gamma_1\phi_m P\bar{B}_f & 0 & 0\\ -\bar{\theta}_1\gamma_1\sqrt{\tau_M}P\bar{B}_f & 0 & 0\\ -\bar{\theta}_1\gamma_1\sqrt{\eta_M}P\bar{B}_f & 0 & 0\\ -\bar{\theta}_1\gamma_1\sqrt{d_M}P\bar{B}_f & 0 & 0\end{bmatrix}$$

$$\Gamma_{12}=\begin{bmatrix}\bar{\alpha}\gamma_2\kappa P\bar{B}_fH & 0 & \bar{\alpha}_1\gamma_2\kappa P\bar{B}_fH & 0\\ \bar{\alpha}\gamma_2\phi_m P\bar{B}_fH & 0 & \bar{\alpha}_1\gamma_2\phi_m P\bar{B}_fH & 0\\ \bar{\alpha}\gamma_2\sqrt{\tau_M}P\bar{B}_fH & 0 & \bar{\alpha}_1\gamma_2\sqrt{\tau_M}P\bar{B}_fH & 0\\ \bar{\alpha}\gamma_2\sqrt{\eta_M}P\bar{B}_fH & 0 & \bar{\alpha}_1\gamma_2\sqrt{\eta_M}P\bar{B}_fH & 0\\ \bar{\alpha}\gamma_2\sqrt{d_M}P\bar{B}_fH & 0 & \bar{\alpha}_1\gamma_2\sqrt{d_M}P\bar{B}_fH & 0\end{bmatrix},\quad \Gamma_{13}=\begin{bmatrix}\bar{\alpha}_1\gamma_2\kappa P\bar{B}_f & 0 & 0\\ \bar{\alpha}_1\gamma_2\phi_m P\bar{B}_f & 0 & 0\\ \bar{\alpha}_1\gamma_2\sqrt{\tau_M}P\bar{B}_f & 0 & 0\\ \bar{\alpha}_1\gamma_2\sqrt{\eta_M}P\bar{B}_f & 0 & 0\\ \bar{\alpha}_1\gamma_2\sqrt{d_M}P\bar{B}_f & 0 & 0\end{bmatrix}$$

$$\Gamma_{14}=\begin{bmatrix}\kappa\Lambda_6H & 0 & -\kappa\Lambda_6H & 0\\ \phi_m\Lambda_6H & 0 & -\phi_m\Lambda_6H & 0\\ \sqrt{\tau_M}\Lambda_6H & 0 & -\sqrt{\tau_M}\Lambda_6H & 0\\ \sqrt{\eta_M}\Lambda_6H & 0 & -\sqrt{\eta_M}\Lambda_6H & 0\\ \sqrt{d_M}\Lambda_6H & 0 & -\sqrt{d_M}\Lambda_6H & 0\end{bmatrix},\quad \Gamma_{15}=\begin{bmatrix}-\kappa\Lambda_6 & 0 & 0\\ -\phi_m\Lambda_6 & 0 & 0\\ -\sqrt{\tau_M}\Lambda_6 & 0 & 0\\ -\sqrt{\eta_M}\Lambda_6 & 0 & 0\\ -\sqrt{d_M}\Lambda_6 & 0 & 0\end{bmatrix},\quad \Gamma_{16}=\begin{bmatrix}0 & 0 & \kappa\gamma_2P\bar{B}_f\\ 0 & 0 & \phi_m\gamma_2P\bar{B}_f\\ 0 & 0 & \sqrt{\tau_M}\gamma_2P\bar{B}_f\\ 0 & 0 & \sqrt{\eta_M}\gamma_2P\bar{B}_f\\ 0 & 0 & \sqrt{d_M}\gamma_2P\bar{B}_f\end{bmatrix}$$

$$\Lambda_6=\gamma_1\gamma_2P\bar{B}_f,\quad \kappa=\sqrt{\phi_M-\phi_m},\quad \bar{\alpha}_1=1-\bar{\alpha},\quad \bar{\theta}_1=1-\bar{\theta},\quad \gamma_1=\sqrt{\bar{\alpha}(1-\bar{\alpha})},\quad \gamma_2=\sqrt{\bar{\theta}(1-\bar{\theta})}$$

$$\Omega_{33}=diag\{\Upsilon,\Upsilon,\Upsilon\},\quad \Omega_{44}=diag\{\Upsilon,\Upsilon\},\quad \Upsilon=diag\{-PR_1^{-1}P,-PR_2^{-1}P,-PR_3^{-1}P,-PR_4^{-1}P,-PR_5^{-1}P\}$$

$$\Omega_{51}(1)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{51}(2)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}W^T\end{bmatrix},\Omega_{51}(3)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{51}(4)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}W^T\end{bmatrix}$$

$$\Omega_{51}(5)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{51}(6)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}W^T\end{bmatrix},\quad \Omega_{51}(7)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{51}(8)=\begin{bmatrix}\kappa Y^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}W^T\end{bmatrix}$$

$$\Omega_{51}(9)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{51}(10)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}W^T\end{bmatrix},\Omega_{51}(11)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{51}(12)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}N^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}W^T\end{bmatrix}$$

$$\Omega_{51}(13)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{51}(14)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}S^T\\ \sqrt{d_M}W^T\end{bmatrix},\Omega_{51}(15)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}V^T\end{bmatrix},\Omega_{51}(16)=\begin{bmatrix}\kappa K^T\\ \sqrt{\tau_M}M^T\\ \sqrt{\eta_M}Z^T\\ \sqrt{d_M}W^T\end{bmatrix}$$

$$\begin{aligned}
\Omega_{55} &= diag\{-R_1,-R_3,-R_4,-R_5\}\\
Y^T &= \begin{bmatrix}0 & 0 & Y_3^T & Y_4^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
K^T &= \begin{bmatrix}0 & K_2^T & K_3^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
N^T &= \begin{bmatrix}0 & 0 & 0 & 0 & N_5^T & N_6^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
M^T &= \begin{bmatrix}M_1^T & 0 & 0 & 0 & M_5^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
S^T &= \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & S_7^T & S_8^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
Z^T &= \begin{bmatrix}Z_1^T & 0 & 0 & 0 & 0 & 0 & Z_7^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
V^T &= \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & V_9^T & V_{10}^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
W^T &= \begin{bmatrix}W_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & W_9^T & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}
\end{aligned}$$

Proof. Choose the following Lyapunov functional candidate as:

$$V(t)=V_1(t)+V_2(t)+V_3(t) \tag{18}$$

where

$$\begin{aligned}
V_1(t) &= e^T(t)Pe(t)\\
V_2(t) &= \int_{t-\phi_m}^{t} e^T(s)Q_1e(s)ds+\int_{t-\phi_M}^{t} e^T(s)Q_2e(s)ds+\int_{t-\tau_M}^{t} e^T(s)Q_3e(s)ds+\int_{t-\eta_M}^{t} e^T(s)Q_4e(s)ds+\int_{t-d_M}^{t} e^T(s)Q_5e(s)ds\\
V_3(t) &= \int_{t-\phi_M}^{t-\phi_m}\int_s^t \dot{e}^T(v)R_1\dot{e}(v)dvds+\phi_m\int_{t-\phi_m}^{t}\int_s^t \dot{e}^T(v)R_2\dot{e}(v)dvds+\int_{t-\tau_M}^{t}\int_s^t \dot{e}^T(v)R_3\dot{e}(v)dvds\\
&\quad+\int_{t-\eta_M}^{t}\int_s^t \dot{e}^T(v)R_4\dot{e}(v)dvds+\int_{t-d_M}^{t}\int_s^t \dot{e}^T(v)R_5\dot{e}(v)dvds
\end{aligned}$$

Applying the infinitesimal operator (Definition 1) for $V_k(t)$ $(k=1,2,3)$ and taking expectation on it, we can obtain:

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_1(t)\} &= 2e^T(t)P[\bar{A}_fe(t)+\bar{W}_0\bar{g}(x(t))+\bar{W}_1\bar{g}(x(t-\phi(t)))+\bar{A_\omega}w(t)+\bar{\theta}_1\bar{\alpha}\bar{B}_fHe(t-\tau(t))+\bar{\theta}_1\bar{\alpha}_1\bar{B}_fHe(t-\eta(t))\\
&\quad+\bar{\theta}_1\bar{\alpha}_1\bar{B}_fe_k(t)+\bar{\theta}\bar{B}_ff(x(t-d(t)))]
\end{aligned} \tag{19}$$

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_2(t)\} &= e^T(t)(Q_1+Q_2+Q_3+Q_4+Q_5)e(t)-e^T(t-\phi_m)Q_1e(t-\phi_m)-e^T(t-\phi_M)Q_2e(t-\phi_M)\\
&\quad-e^T(t-\tau_M)Q_3e(t-\tau_M)-e^T(t-\eta_M)Q_4e(t-\eta_M)-e^T(t-d_M)Q_5e(t-d_M)
\end{aligned} \tag{20}$$

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_3(t)\} &= \mathbb{E}\left\{\dot{e}^T(t)\tilde{R}\dot{e}(t)\right\}-\int_{t-\phi_M}^{t-\phi_m}\dot{e}^T(s)R_1\dot{e}(s)ds-\phi_m\int_{t-\phi_m}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds-\int_{t-\tau_M}^{t}\dot{e}^T(s)R_3\dot{e}(s)ds\\
&\quad-\int_{t-\eta_M}^{t}\dot{e}^T(s)R_4\dot{e}(s)ds-\int_{t-d_M}^{t}\dot{e}^T(s)R_5\dot{e}(s)ds
\end{aligned} \tag{21}$$

in which

$$\mathbb{E}(\dot{e}^T(t)\tilde{R}\dot{e}(t))=\mathcal{A}^T\tilde{R}\mathcal{A}+\bar{\theta}_1^2\gamma_1^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1+\gamma_2^2\mathcal{B}_2^T\tilde{R}\mathcal{B}_2+\gamma_1^2\gamma_2^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1+\gamma_2^2F_c^T\bar{B}_f^T\tilde{R}\bar{B}_fF_c \tag{22}$$

where

$$\begin{aligned}
\mathcal{A} &= \bar{A}_fe(t)+\bar{W}_0\bar{g}(e(t))+\bar{W}_1\bar{g}(e(t-\phi(t)))+\bar{A_\omega}w(t)+\bar{\theta}_1\bar{\alpha}\bar{B}_fHe(t-\tau(t))\\
&\quad+\bar{\theta}_1\bar{\alpha}_1\bar{B}_fHe(t-\eta(t))+\bar{\theta}_1\bar{\alpha}_1\bar{B}_fe_k(t)+\bar{\theta}\bar{B}_ff(x(t-d(t)))
\end{aligned}$$

$$\mathcal{B}_1 = \bar{B}_f[He(t-\tau(t)) - He(t-\eta(t)) - e_k(t)], \mathcal{B}_2 = \bar{B}_f[\bar{\alpha}He(t-\tau(t)) + \bar{\alpha}_1 He(t-\eta(t)) + \bar{\alpha}_1 e_k(t)]$$
$$\tilde{R} = (\phi_M - \phi_m)R_1 + \phi_m^2 R_2 + \tau_M R_3 + \eta_M R_4 + d_M R_5, F_c = f(x(t-d(t)))$$

From the definition of $f(t, x(t))$ in (8) which is the limited condition of deception attacks, there exists a symmetric matrix Q satisfying the following inequality:

$$\bar{\theta}e^T(t-d(t))H^T G^T QGHe(t-d(t)) - \bar{\theta}f^T(x(t-d(t))Qf(x(t-d(t))) \geq 0 \tag{23}$$

Applying the free-weighting matrices method [11,37] , it can be obtained that:

$$2\xi^T(t)K\left[e(t-\phi_m) - e(t-\phi(t)) - \int_{t-\phi(t)}^{t-\phi_m} \dot{e}(s)d_s\right] = 0 \tag{24}$$

$$2\xi^T(t)Y\left[e(t-\phi(t)) - e(t-\phi_M) - \int_{t-\phi_M}^{t-\phi(t)} \dot{e}(s)d_s\right] = 0 \tag{25}$$

$$2\xi^T(t)M\left[e(t) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t} \dot{e}(s)d_s\right] = 0 \tag{26}$$

$$2\xi^T(t)N\left[e(t-\tau(t)) - e(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)} \dot{e}(s)d_s\right] = 0 \tag{27}$$

$$2\xi^T(t)Z\left[e(t) - e(t-\eta(t)) - \int_{t-\eta(t)}^{t} \dot{e}(s)d_s\right] = 0 \tag{28}$$

$$2\xi^T(t)S\left[e(t-\eta(t)) - e(t-\eta_M) - \int_{t-\eta_M}^{t-\eta(t)} \dot{e}(s)d_s\right] = 0 \tag{29}$$

$$2\xi^T(t)W\left[e(t) - e(t-d(t)) - \int_{t-d(t)}^{t} \dot{e}(s)d_s\right] = 0 \tag{30}$$

$$2\xi^T(t)V\left[e(t-d(t)) - e(t-d_M) - \int_{t-d_M}^{t-d(t)} \dot{e}(s)d_s\right] = 0 \tag{31}$$

where K, Y, N, M, Z, S, W, V are matrices with appropriate dimensions, and $\xi^T(t)$ is defined as follows:

$$\xi^T(t) = \begin{bmatrix}\xi_1^T(t) & \xi_2^T(t)\end{bmatrix}$$
$$\xi_1^T(t) = \begin{bmatrix}e^T(t) & e^T(t-\phi_m) & e^T(t-\phi(t)) & e^T(t-\phi_M) & e^T(t-\tau(t)) & e^T(t-\tau_M) & e^T(t-\eta(t) & e^T(t-\eta_M)\end{bmatrix}$$
$$\xi_2^T(t) = \begin{bmatrix}e^T(t-d(t) & e^T(t-d_M) & \bar{g}^T(He(t)) & \bar{g}^T(He(t-\phi(t))) & e_k^T(t) & w^T(t) & f^T(He(t-d(t)))\end{bmatrix}$$

By using Lemma 2, we have

$$-2\xi^T(t)K\int_{t-\phi(t)}^{t-\phi_m} \dot{e}(s)ds \leq (\phi(t)-\phi_m)\xi^T(t)KR_1^{-1}K^T\xi(t) + \int_{t-\phi(t)}^{t-\phi_m} \dot{e}^T(s)R_1\dot{e}(s)ds \tag{32}$$

$$-2\xi^T(t)Y\int_{t-\phi_M}^{t-\phi(t)} \dot{e}(s)ds \leq (\phi_M-\phi(t))\xi^T(t)YR_1^{-1}Y^T\xi(t) + \int_{t-\phi_M}^{t-\phi(t)} \dot{e}^T(s)R_1\dot{e}(s)ds \tag{33}$$

$$-2\xi^T(t)M\int_{t-\tau(t)}^{t} \dot{e}(s)ds \leq \tau(t)\xi^T(t)MR_3^{-1}M^T\xi(t) + \int_{t-\tau(t)}^{t} \dot{e}^T(s)R_3\dot{e}(s)ds \tag{34}$$

$$-2\xi^T(t)N\int_{t-\tau_M}^{t-\tau(t)} \dot{e}(s)ds \leq (\tau_M-\tau(t))\xi^T(t)NR_3^{-1}N^T\xi(t) + \int_{t-\eta_M}^{t-\eta(t)} \dot{e}^T(s)R_3\dot{e}(s)ds \tag{35}$$

$$-2\xi^T(t)Z\int_{t-\eta(t)}^{t} \dot{e}(s)ds \leq \eta(t)\xi^T(t)ZR_4^{-1}Z^T\xi(t) + \int_{t-\eta(t)}^{t} \dot{e}^T(s)R_4\dot{e}(s)ds \tag{36}$$

$$-2\xi^T(t)S\int_{t-\eta_M}^{t-\eta(t)} \dot{e}(s)ds \leq (\eta_M-\eta(t))\xi^T(t)SR_4^{-1}S^T\xi(t) + \int_{t-\eta_M}^{t-\eta(t)} \dot{e}^T(s)R_4\dot{e}(s)ds \tag{37}$$

$$-2\xi^T(t)W\int_{t-d(t)}^{t} \dot{e}(s)ds \leq d(t)\xi^T(t)WR_5^{-1}W^T\xi(t) + \int_{t-d(t)}^{t} \dot{e}^T(s)R_5\dot{e}(s)ds \tag{38}$$

$$-2\xi^T(t)V\int_{t-d_M}^{t-d(t)} \dot{e}(s)ds \leq (d_M-d(t))\xi^T(t)VR_5^{-1}V^T\xi(t) + \int_{t-d_M}^{t-d(t)} \dot{e}^T(s)R_5\dot{e}(s)ds \tag{39}$$

Considering the condition of event-triggered scheme (5), we can obtain that:

$$\sigma e^T(t-\eta(t))H^TC^T\Omega CHe(t-\eta(t)) - e_k^T(t)C^T\Omega Ce_k(t) \geq 0 \tag{40}$$

By using Lemma 1, notice that:

$$-\phi_m\int_{t-\phi_m}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds \leq \begin{bmatrix} e(t) \\ e(t-\phi_m)\end{bmatrix}^T\begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\phi_m)\end{bmatrix} \tag{41}$$

By employing Assumption 1, we obtain that:

$$\begin{bmatrix} e(t) \\ \bar{g}(H(e(t)))\end{bmatrix}^T\begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix} e(t) \\ \bar{g}(H(e(t)))\end{bmatrix} \leq 0, \tag{42}$$

where $\bar{U}_1 = H^T\hat{U}_1H$, $\bar{U}_2 = -H^T\hat{U}_2$, $\hat{U}_1 = \frac{U_1^TU_2+U_2^TU_1}{2}$, $\hat{U}_2 = \frac{U_1^T+U_2^T}{2}$.

There exists the parameters $\alpha > 0$, $\beta > 0$, it is easy to get:

$$-\alpha\begin{bmatrix} e(t) \\ \bar{g}(H(e(t)))\end{bmatrix}^T\begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix} e(t) \\ \bar{g}(H(e(t)))\end{bmatrix} \geq 0, \tag{43}$$

$$-\beta\begin{bmatrix} e(t) \\ \bar{g}(H(e(t-\phi(t))))\end{bmatrix}^T\begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix} e(t) \\ \bar{g}(H(e(t-\phi(t))))\end{bmatrix} \geq 0, \tag{44}$$

Combine $(18)-(44)$, we can obtain that:

$$\begin{aligned}
&\mathbb{E}\{\mathcal{L}V(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2w^T(t)w(t)\} \\
&\leq 2e^T(t)P\mathcal{A} + e^T(t)(Q_1+Q_2+Q_3+Q_4+Q_5)e(t) - e^T(t-\phi_m)Q_1e(t-\phi_m) - e^T(t-\phi_M)Q_2e(t-\phi_M) \\
&\quad - e^T(t-\tau_M)Q_3e(t-\tau_M) - e^T(t-\eta_M)Q_4e(t-\eta_M) - e^T(t-d_M)Q_5x(t-d_M) \\
&\quad + \begin{bmatrix} e(t) \\ e(t-\phi_m)\end{bmatrix}^T\begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\phi_m)\end{bmatrix} + (\phi(t)-\phi_m)\xi^T(t)KR_1^{-1}K^T\xi(t) + 2\xi^T(t)K\int_{t-\phi(t)}^{t-\phi_m}\dot{e}(s)ds \\
&\quad + (\phi_M-\phi(t))\xi^T(t)YR_1^{-1}Y^T\xi(t) + 2\xi^T(t)Y\int_{t-\phi_M}^{t-\phi(t)}\dot{e}(s)ds + \tau(t)\xi^T(t)MR_3^{-1}M^T\xi(t) + 2\xi^T(t)M\int_{t-\tau(t)}^{t}\dot{e}(s)ds \\
&\quad + (\tau_M-\tau(t))\xi^T(t)NR_3^{-1}N^T\xi(t) + 2\xi^T(t)N\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)ds + \eta(t)\xi^T(t)ZR_4^{-1}Z^T\xi(t) + 2\xi^T(t)Z\int_{t-\eta(t)}^{t}\dot{e}(s)ds \\
&\quad + (\eta_M-\eta(t))\xi^T(t)SR_4^{-1}S^T\xi(t) + 2\xi^T(t)S\int_{t-\eta_M}^{t-\eta(t)}\dot{e}(s)ds + d(t)\xi^T(t)WR_5^{-1}W^T\xi(t) + 2\xi^T(t)W\int_{t-d(t)}^{t}\dot{e}(s)ds \\
&\quad + (d_M-d(t))\xi^T(t)VR_5^{-1}V^T\xi(t) + 2\xi^T(t)V\int_{t-d_M}^{t-d(t)}\dot{e}(s)ds + \mathcal{A}^T\tilde{R}\mathcal{A} + \bar{\theta}_1\gamma_1^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 \\
&\quad - \alpha\begin{bmatrix} e(t) \\ \bar{g}(H(e(t)))\end{bmatrix}^T\begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix} e(t) \\ \bar{g}(H(e(t)))\end{bmatrix} - \beta\begin{bmatrix} e(t) \\ \bar{g}(H(e(t-\phi(t))))\end{bmatrix}^T\begin{bmatrix} \bar{U}_1 & \bar{U}_2 \\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix} e(t) \\ \bar{g}(H(e(t-\phi(t))))\end{bmatrix} \\
&\quad + \gamma_1^2\gamma_2^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 + \gamma_2^2f^T(x(t-d(t)))\bar{B}_f^T\tilde{R}\bar{B}_ff(x(t-d(t))) + \sigma e^T(t-\eta(t))H^TC^T\Omega CHe(t-\eta(t)) \\
&\quad + \gamma_2^2\mathcal{B}_2^T\tilde{R}\mathcal{B}_2 - e_k^T(t)C^T\Omega Ce_k(t) - \gamma^2w^T(t)w(t) + e^T(t)\bar{C}_f^T\bar{C}_fe(t) + \bar{\theta}e^T(t-d(t))H^TG^TQGHe(t-d(t)) \\
&\quad - \bar{\theta}f^T(x(t-d(t))Qf(x(t-d(t))) \\
&\leq \xi^T(t)(\Omega_{11}+\Gamma+\Gamma^T)\xi(t) + (\phi(t)-\phi_m)\xi^T(t)KR_1^{-1}K^T\xi(t) + (\phi_M-\phi(t))\xi^T(t)YR_1^{-1}Y^T\xi(t) \\
&\quad + \tau(t)\xi^T(t)MR_3^{-1}M^T\xi(t) + (\tau_M-\tau(t))\xi^T(t)NR_3^{-1}N^T\xi(t) + \eta(t)\xi^T(t)ZR_4^{-1}Z^T\xi(t) \\
&\quad + (\eta_M-\eta(t))\xi^T(t)SR_4^{-1}S^T\xi(t) + d(t)\xi^T(t)WR_5^{-1}W^T\xi(t) + (d_M-d(t))\xi^T(t)VR_5^{-1}V^T\xi(t) \\
&\quad + \mathcal{A}^T\tilde{R}\mathcal{A} + \bar{\theta}_1^2\gamma_1^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 + \gamma_2^2\mathcal{B}_2^T\tilde{R}\mathcal{B}_2 + \gamma_1^2\gamma_2^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 + \gamma_2^2f^T(x(t-d(t)))\bar{B}_f^T\tilde{R}\bar{B}_ff(x(t-d(t))) \\
&\quad + \bar{\theta}e^T(t-d(t))H^TG^TQGHe(t-d(t)) - \bar{\theta}f^T(x(t-d(t))Qf(x(t-d(t)))
\end{aligned} \tag{45}$$

By using Lemma 3 and Schur complement, combining Eqs. (17) and (45), we can obtain that (11) is sufficient to guarantee $\mathbb{E}\{\mathcal{L}(V(t)) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2w^T(t)w(t)\} < 0$. The proof is completed.

Based on Theorem 1, the parameters of desired filter are given in the following theorem.

Theorem 2. *For given positive parameters $\bar{\theta}$, $\bar{\alpha}$, γ, ϕ_m, ϕ_M, τ_M, η_M, d_M, σ, ϵ_k $(k = 1, 2, 3, 4, 5)$ and matrix G, if there exist matrices $P_1 > 0$, $\tilde{P}_3 > 0$, $Q > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$ $(k = 1, 2, 3, 4, 5)$, $\Omega > 0$, $\tilde{K}$, $\tilde{Y}$, $\tilde{M}$, $\tilde{N}$, $\tilde{S}$, $\tilde{Z}$, $\tilde{W}$, $\tilde{V}$, $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$ with appropriate* *dimensions and parameters $\alpha > 0$, $\beta > 0$, the filtering error system (11) with hybrid triggered scheme*(7) *and deception attacks* (8) *is asymptotically stable if the following LMIs hold:*

$$\tilde{\Omega}(s) = \begin{bmatrix} \tilde{\Omega}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * & * \\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * & * & * \\ \tilde{\Omega}_{31} & 0 & \tilde{\Omega}_{33} & * & * \\ \tilde{\Omega}_{41} & 0 & 0 & \tilde{\Omega}_{44} & * \\ \tilde{\Omega}_{51}(s) & 0 & 0 & 0 & \tilde{\Omega}_{55} \end{bmatrix} < 0, \quad s = 1, 2, 3, \cdots, 16 \tag{46}$$

$$P_1 - \bar{P}_3 > 0 \tag{47}$$

where

$$\tilde{\Omega}_{11} = \begin{bmatrix} \tilde{\Gamma}_1 & * & * \\ \tilde{\Gamma}_2 & \tilde{\Gamma}_3 & * \\ \tilde{\Gamma}_4 & 0 & \tilde{\Gamma}_5 \end{bmatrix}$$

$$\tilde{\Gamma} = \begin{bmatrix} \tilde{M} + \tilde{Z} + \tilde{W} & \tilde{K} & -\tilde{K} + \tilde{Y} & -\tilde{Y} & -\tilde{M} + \tilde{N} & -\tilde{N} & -\tilde{Z} + \tilde{S} & -\tilde{S} & -\tilde{W} + \tilde{V} & -\tilde{V} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_1 = \begin{bmatrix} \tilde{\Lambda}_1 & * & * & * & * \\ R_2 & -\tilde{Q}_1 - \tilde{R}_2 & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & -\tilde{Q}_2 & * \\ \tilde{\Lambda}_2 & 0 & 0 & 0 & \tilde{\Lambda}_3 \end{bmatrix}, \tilde{\Gamma}_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tilde{\Gamma}_4 = \begin{bmatrix} \tilde{\Lambda}_5 & 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_6 & 0 & 0 & 0 & \tilde{\Lambda}_7 \\ \tilde{\Lambda}_8 & 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_9 & 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_{10} & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_3 = diag\{-\tilde{Q}_3, \Pi_3, -\tilde{Q}_4, 0, -\tilde{Q}_5\}, \quad \tilde{\Gamma}_5 = diag\{-\alpha I, -\beta I, -C^T\Omega C, -\gamma^2 I, -\bar{\theta}Q\}$$

$$\tilde{\Lambda}_1 = \Pi_1 + \Pi_1^T + +\tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 + \tilde{Q}_4 + \tilde{Q}_5 - \tilde{R}_2 - \Pi_2, \quad \tilde{\Lambda}_2 = \begin{bmatrix} (1-\bar{\theta})\bar{\alpha}C^T\hat{B}_f^T & (1-\bar{\theta})\bar{\alpha}C^T\hat{B}_f^T \\ 0 & 0 \end{bmatrix}$$

$$\tilde{\Lambda}_3 = \begin{bmatrix} -\beta\frac{U_1^TU_2+U_2^TU_1}{2} & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{\Lambda}_4 = \begin{bmatrix} (1-\bar{\theta})(1-\bar{\alpha})C^T\hat{B}_f^T & (1-\bar{\theta})(1-\bar{\alpha})C^T\hat{B}_f^T \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Lambda}_5 = \begin{bmatrix} W_0^TP_1 + \alpha\frac{U_1+U_2}{2} & W_0^T\bar{P}_3 \end{bmatrix}$$

$$\tilde{\Lambda}_6 = \begin{bmatrix} W_0^TP_1 & W_0^T\bar{P}_3 \end{bmatrix}, \quad \tilde{\Lambda}_7 = \beta\frac{U_1^T+U_2^T}{2}, \quad \tilde{\Lambda}_8 = \begin{bmatrix} \bar{\theta}_1\bar{\alpha}_1C^T\hat{B}_f^T & \bar{\theta}_1\bar{\alpha}_1C^T\hat{B}_f^T \end{bmatrix}, \quad \tilde{\Lambda}_9 = \begin{bmatrix} A_\omega^TP_1 & A_\omega^T\bar{P}_3 \end{bmatrix}$$

$$\tilde{\Lambda}_{10} = \begin{bmatrix} \bar{\theta}C^T\hat{B}_f^T & \bar{\theta}C^T\hat{B}_f^T \end{bmatrix}, \quad \Pi_1 = \begin{bmatrix} -P_1A & \hat{A}_f \\ -\bar{P}_3A & \hat{A}_f \end{bmatrix}, \quad \Pi_2 = \begin{bmatrix} \alpha\frac{U_1^TU_2+U_2^TU_1}{2} & 0 \\ 0 & 0 \end{bmatrix}, \quad \Pi_3 = \begin{bmatrix} \sigma C^T\Omega C & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{\Omega}_{21} = \begin{bmatrix} \Pi_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{\bar{\theta}}GQ & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \Pi_4 = \begin{bmatrix} L & -\hat{C}_f \end{bmatrix}, \quad \tilde{\Omega}_{22} = diag\{-I, -Q\}$$

$$\tilde{\Omega}_{31} = \begin{bmatrix} \tilde{\Gamma}_6 & \tilde{\Gamma}_7 & \tilde{\Gamma}_8 & \tilde{\Gamma}_9 \\ 0 & \tilde{\Gamma}_{10} & 0 & \tilde{\Gamma}_{11} \\ 0 & \tilde{\Gamma}_{12} & 0 & \tilde{\Gamma}_{13} \end{bmatrix}, \quad \tilde{\Omega}_{41} = \begin{bmatrix} 0 & \tilde{\Gamma}_{14} & 0 & \tilde{\Gamma}_{15} \\ 0 & 0 & 0 & \tilde{\Gamma}_{16} \end{bmatrix}, \quad \tilde{\Omega}_{33} = diag\{\tilde{\Upsilon}, \tilde{\Upsilon}, \tilde{\Upsilon}\}, \tilde{\Omega}_{44} = diag\{\tilde{\Upsilon}, \tilde{\Upsilon}\}$$

$$\tilde{\Upsilon} = diag\{-2\epsilon_1\tilde{P} + \epsilon_1^2\tilde{R}_1, -2\epsilon_2\tilde{P} + \epsilon_2^2\tilde{R}_2, -2\epsilon_3\tilde{P} + \epsilon_3^2\tilde{R}_3, -2\epsilon_4\tilde{P} + \epsilon_4^2\tilde{R}_4, -2\epsilon_5\tilde{P} + \epsilon_5^2\tilde{R}_5\}$$

$$\tilde{\Gamma}_6 = \begin{bmatrix} \kappa\Pi_5 & 0 & 0 & 0 \\ \phi_m\Pi_5 & 0 & 0 & 0 \\ \sqrt{\tau_M}\Pi_5 & 0 & 0 & 0 \\ \sqrt{\eta_M}\Pi_5 & 0 & 0 & 0 \\ \sqrt{d_M}\Pi_5 & 0 & 0 & 0 \end{bmatrix}, \quad \tilde{\Gamma}_7 = \begin{bmatrix} \bar{\theta}_1\bar{\alpha}\kappa\Pi_6^T & 0 & \bar{\theta}_1\bar{\alpha}_1\kappa\Pi_6^T & 0 \\ \bar{\theta}_1\bar{\alpha}\phi_m\Pi_6^T & 0 & \bar{\theta}_1\bar{\alpha}_1\phi_m\Pi_6^T & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{\tau_M}\Pi_6^T & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\Pi_6^T & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{\eta_M}\Pi_6^T & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\Pi_6^T & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{d_M}\Pi_6^T & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\Pi_6^T & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_8 = \begin{bmatrix} 0 & 0 & \kappa\Pi_7^T & \kappa\Pi_8^T \\ 0 & 0 & \phi_m\Pi_7^T & \phi_m\Pi_8^T \\ 0 & 0 & \sqrt{\tau_M}\Pi_7^T & \sqrt{\tau_M}\Pi_8^T \\ 0 & 0 & \sqrt{\eta_M}\Pi_7^T & \sqrt{\eta_M}\Pi_8^T \\ 0 & 0 & \sqrt{d_M}\Pi_7^T & \sqrt{d_M}\Pi_8^T \end{bmatrix}$$

$$\tilde{\Gamma}_9 = \begin{bmatrix} \bar{\theta}_1\bar{\alpha}_1\kappa\Pi_9^T & \kappa\psi_{10}^T & \bar{\theta}\kappa\Pi_9^T \\ \bar{\theta}_1\bar{\alpha}_1\phi_m\Pi_9^T & \phi_m\psi_{10}^T & \bar{\theta}\phi_m\Pi_9^T \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\Pi_9^T & \sqrt{\tau_M}\psi_{10}^T & \bar{\theta}\sqrt{\tau_M}\Pi_9^T \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\Pi_9^T & \sqrt{\eta_M}\psi_{10}^T & \bar{\theta}\sqrt{\eta_M}\Pi_9^T \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\Pi_9^T & \sqrt{d_M}\psi_{10}^T & \bar{\theta}\sqrt{d_M}\Pi_9^T \end{bmatrix}, \quad \tilde{\Gamma}_{10} = \begin{bmatrix} \bar{\theta}_1\gamma_1\kappa P\Pi_6^T & 0 & -\bar{\theta}_1\gamma_1\kappa\Pi_6^T & 0 \\ \bar{\theta}_1\gamma_1\phi_m P\Pi_6^T & 0 & -\bar{\theta}_1\gamma_1\phi_m\Pi_6 & {}^T0 \\ \bar{\theta}_1\gamma_1\sqrt{\tau_M}P\Pi_6^T & 0 & -\bar{\theta}_1\gamma_1\sqrt{\tau_M}\Pi_6^T & 0 \\ \bar{\theta}_1\gamma_1\sqrt{\eta_M}P\Pi_6^T & 0 & -\bar{\theta}_1\gamma_1\sqrt{\eta_M}\Pi_6^T & 0 \\ \bar{\theta}_1\gamma_1\sqrt{d_M}P\Pi_6^T & 0 & -\bar{\theta}_1\gamma_1\sqrt{d_M}\Pi_6^T & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_{11} = \begin{bmatrix} -\bar{\theta}_1\gamma_1\kappa\Pi_9^T & 0 & 0 \\ -\bar{\theta}_1\gamma_1\phi_m\Pi_9^T & 0 & 0 \\ -\bar{\theta}_1\gamma_1\sqrt{\tau_M}\Pi_9^T & 0 & 0 \\ -\bar{\theta}_1\gamma_1\sqrt{\eta_M}\Pi_9^T & 0 & 0 \\ -\bar{\theta}_1\gamma_1\sqrt{d_M}\Pi_9^T & 0 & 0 \end{bmatrix}, \quad \tilde{\Gamma}_{12} = \begin{bmatrix} \bar{\alpha}\gamma_2\kappa\Pi_6^T & 0 & \bar{\alpha}_1\gamma_2\kappa\Pi_6^T & 0 \\ \bar{\alpha}\gamma_2\phi_m\Pi_6^T & 0 & \bar{\alpha}_1\gamma_2\phi_m P\Pi_6^T & 0 \\ \bar{\alpha}\gamma_2\sqrt{\tau_M}\Pi_6^T & 0 & \bar{\alpha}_1\gamma_2\sqrt{\tau_M}\Pi_6^T & 0 \\ \bar{\alpha}\gamma_2\sqrt{\eta_M}\Pi_6^T & 0 & \bar{\alpha}_1\gamma_2\sqrt{\eta_M}\Pi_6^T & 0 \\ \bar{\alpha}\gamma_2\sqrt{d_M}\Pi_6^T & 0 & \bar{\alpha}_1\gamma_2\sqrt{d_M}\Pi_6^T & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_{13} = \begin{bmatrix} \bar{\alpha}_1\gamma_2\kappa\Pi_9^T & 0 & 0 \\ \bar{\alpha}_1\gamma_2\phi_m\Pi_9^T & 0 & 0 \\ \bar{\alpha}_1\gamma_2\sqrt{\tau_M}\Pi_9^T & 0 & 0 \\ \bar{\alpha}_1\gamma_2\sqrt{\eta_M}\Pi_9^T & 0 & 0 \\ \bar{\alpha}_1\gamma_2\sqrt{d_M}\Pi_9^T & 0 & 0 \end{bmatrix} \quad \tilde{\Gamma}_{14} = \begin{bmatrix} \kappa\gamma_1\gamma_2\Pi_6^T & 0 & -\kappa\gamma_1\gamma_2\Pi_9^T & 0 \\ \phi_m\gamma_1\gamma_2\Pi_6^T & 0 & -\phi_m\gamma_1\gamma_2\Pi_9^T & 0 \\ \sqrt{\tau_M}\gamma_1\gamma_2\Pi_6^T & 0 & -\sqrt{\tau_M}\gamma_1\gamma_2\Pi_9^T & 0 \\ \sqrt{\eta_M}\gamma_1\gamma_2\Pi_6^T & 0 & -\sqrt{\eta_M}\gamma_1\gamma_2\Pi_9^T & 0 \\ \sqrt{d_M}\gamma_1\gamma_2\Pi_6^T & 0 & -\sqrt{d_M}\gamma_1\gamma_2\Pi_9^T & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_{15} = \begin{bmatrix} -\kappa\gamma_1\gamma_2\Pi_9^T & 0 & 0 \\ -\phi_m\gamma_1\gamma_2\Pi_9^T & 0 & 0 \\ -\sqrt{\tau_M}\gamma_1\gamma_2\Pi_9^T & 0 & 0 \\ -\sqrt{\eta_M}\gamma_1\gamma_2\Pi_9^T & 0 & 0 \\ -\sqrt{d_M}\gamma_1\gamma_2\Pi_9^T & 0 & 0 \end{bmatrix}, \quad \tilde{\Gamma}_{16} = \begin{bmatrix} 0 & 0 & \kappa\gamma_2\Pi_9^T \\ 0 & 0 & \phi_m\gamma_2\Pi_9^T \\ 0 & 0 & \sqrt{\tau_M}\gamma_2\Pi_9^T \\ 0 & 0 & \sqrt{\eta_M}\gamma_2\Pi_9^T \\ 0 & 0 & \sqrt{d_M}\gamma_2\Pi_9^T \end{bmatrix}$$

$$\kappa = \sqrt{\phi_M - \phi_m}, \quad \bar{\alpha}_1 = 1 - \bar{\alpha}, \quad \bar{\theta}_1 = 1 - \bar{\theta}, \quad \gamma_1 = \sqrt{\bar{\alpha}(1-\bar{\alpha})}, \quad \gamma_2 = \sqrt{\bar{\theta}(1-\bar{\theta})}, \quad \Pi_5 = \begin{bmatrix} -P_1 A & \hat{A}_f \\ -\bar{P}_3 A & \hat{A}_f \end{bmatrix}$$

$$\Pi_6 = \begin{bmatrix} C^T\hat{B}_f^T & C^T\hat{B}_f^T \\ 0 & 0 \end{bmatrix}, \quad \Pi_7 = \begin{bmatrix} W_0^T P_1 & W_0^T\bar{P}_3 \end{bmatrix}, \quad \Pi_8 = \begin{bmatrix} W_1^T P_1 & W_1^T\bar{P}_3 \end{bmatrix}, \quad \Pi_9 = \begin{bmatrix} C^T\hat{B}_f^T & C^T\hat{B}_f^T \end{bmatrix}$$

$$\Pi_{10} = \begin{bmatrix} \bar{A}_\omega^T P_1 & \bar{A}_\omega^T\bar{P}_3 \end{bmatrix}$$

$$\tilde{\Omega}_{51}(1) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(2) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}, \tilde{\Omega}_{51}(3) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(4) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\tilde{\Omega}_{51}(5) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(6) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(7) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(8) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\tilde{\Omega}_{51}(9) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(10) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}, \tilde{\Omega}_{51}(11) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(12) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\tilde{\Omega}_{51}(13) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(14) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(15) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{51}(16) = \begin{bmatrix} \kappa\tilde{K}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\begin{aligned}
\tilde{\Omega}_{55} &= diag\{-\tilde{R}_1, -\tilde{R}_3, -\tilde{R}_4, -\tilde{R}_5\} \\
\tilde{Y}^T &= \begin{bmatrix} 0 & 0 & \tilde{Y}_3^T & \tilde{Y}_4^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{K}^T &= \begin{bmatrix} 0 & \tilde{K}_2^T & \tilde{K}_3^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{N}^T &= \begin{bmatrix} 0 & 0 & 0 & 0 & \tilde{N}_5^T & \tilde{N}_6^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{M}^T &= \begin{bmatrix} \tilde{M}_1^T & 0 & 0 & 0 & \tilde{M}_5^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{S}^T &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \tilde{S}_7^T & \tilde{S}_8^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{Z}^T &= \begin{bmatrix} \tilde{Z}_1^T & 0 & 0 & 0 & 0 & 0 & \tilde{Z}_7^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{V}^T &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{V}_9^T & \tilde{V}_{10}^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{W}^T &= \begin{bmatrix} \tilde{W}_1^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{W}_9^T & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\end{aligned}$$

Moreover, in condition of feasible conditions above, the parameter matrices of the filter are given by

$$\begin{cases} A_f = \hat{A}_f \bar{P}_3^{-1} \\ B_f = \hat{B}_f \\ C_f = \hat{C}_f \bar{P}_3^{-1} \end{cases} \tag{48}$$

Proof. Due to $(R_i - \epsilon_i^{-1}P)R_i^{-1}(R_i - \epsilon_i^{-1}P) \geq 0, (i = 1, 2, 3, 4, 5)$, we have $-PR_i^{-1}P \leq -2\epsilon_i P + \epsilon_i^2 R_i$. Substitute $-PR_i^{-1}P$ with $-2\epsilon_i P + \epsilon_i^2 R_i$ into (17), we obtain

$$\hat{\Omega}(s) = \begin{bmatrix} \tilde{\Omega}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * & * \\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * & * & * \\ \tilde{\Omega}_{31} & 0 & \tilde{\Omega}_{33} & * & * \\ \tilde{\Omega}_{41} & 0 & 0 & \tilde{\Omega}_{44} & * \\ \tilde{\Omega}_{51}(s) & 0 & 0 & 0 & \tilde{\Omega}_{55} \end{bmatrix} < 0, \quad s = 1, 2, 3, \ldots, 16 \tag{49}$$

$$P_1 - \bar{P}_3 > 0$$

where

$$\begin{aligned}
\tilde{\Omega}_{33} &= diag\{\tilde{\Upsilon}, \tilde{\Upsilon}, \tilde{\Upsilon}\}, \tilde{\Omega}_{44} = diag\{\tilde{\Upsilon}, \tilde{\Upsilon}\} \\
\tilde{\Upsilon} &= diag\{-2\epsilon_1\tilde{P} + \epsilon_1^2\tilde{R}_1, -2\epsilon_2\tilde{P} + \epsilon_2^2\tilde{R}_2, -2\epsilon_3\tilde{P} + \epsilon_3^2\tilde{R}_3, -2\epsilon_4\tilde{P} + \epsilon_4^2\tilde{R}_4, -2\epsilon_5\tilde{P} + \epsilon_5^2\tilde{R}_5\}
\end{aligned}$$

Since $\bar{P}_3 > 0$, there exist P_2 and $P_3 > 0$ satisfying $\bar{P}_3 = P_2^T P_3^{-1} P_2$.
Define

$$P = \begin{bmatrix} P_1 & P_2^T \\ P_2 & P_3 \end{bmatrix}, \quad J = \begin{bmatrix} I & 0 \\ 0 & P_2^T P_3^{-1} \end{bmatrix}$$

$$\Lambda = diag\left\{ \underbrace{J, \ldots, J}_{10}, \underbrace{I, \ldots, I}_{7}, \underbrace{J, \ldots, J}_{29} \right\}$$

Multiplying Λ and Λ^T on both sides of (17), respectively, and define

$$\begin{aligned}
&\tilde{Q}_i = JQ_iJ^T, \quad \tilde{R}_i = JR_iJ^T (i = 1, 2, 3, 4, 5), \quad \tilde{K}_i = JK_iJ^T (i = 2, 3) \\
&\tilde{Y}_i = JY_iJ^T (i = 3, 4), \quad \tilde{M}_i = JM_iJ^T (i = 1, 5), \quad \tilde{N}_i = JN_iJ^T (i = 5, 6), \quad \tilde{S}_i = JS_iJ^T (i = 7, 8) \\
&\tilde{Z}_i = JZ_iJ^T (i = 1, 7), \quad \tilde{W}_i = JW_iJ^T (i = 1, 9), \quad \tilde{V}_i = JV_iJ^T (i = 9, 10)
\end{aligned}$$

Define variables

$$\begin{cases} \hat{A}_f = \tilde{A}_f \bar{P}_3, \quad \tilde{A}_f = P_2^T A_f P_2^{-T} \\ \hat{B}_f = P_2^T B_f \\ \hat{C}_f = \tilde{C}_f \bar{P}_3, \quad \tilde{C}_f = C_f P_2^{-T} \end{cases} \tag{50}$$

On the basis of the descriptions above, utilizing the methods in [25], the filter parameters (A_f, B_f, C_f) can be expressed by $(P_2^{-T}\tilde{A}_f P_2, P_2^{-T}\hat{B}_f, \tilde{C}_f P_2)$, the filter system (2) can be written as

$$\begin{cases} \dot{x}_f(t) = P_2^{-T}\tilde{A}_f P_2 x_f(t) + P_2^{-T}\hat{B}_f \hat{y}(t) \\ z_f(t) = \tilde{C}_f P_2 x_f(t) \end{cases} \tag{51}$$

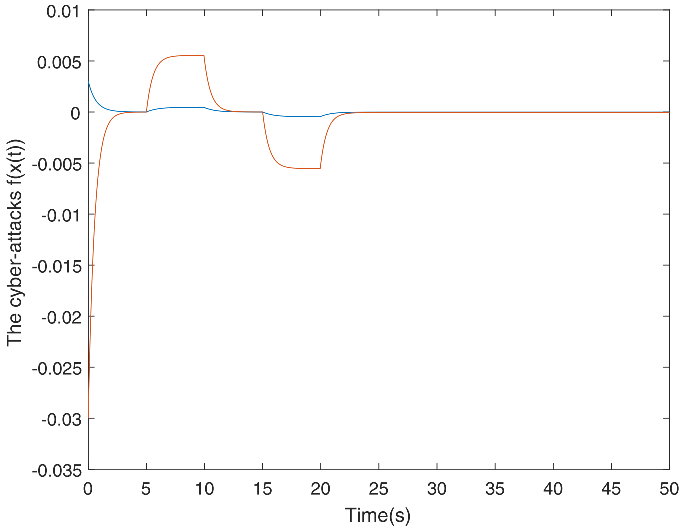

Fig. 2. The cyber-attacks$f(x(t))$.

Similar to the analysis of [25], define $\hat{x}(t) = P_2^T x_f(t)$, from (51), we can obtain

$$\begin{cases} \dot{\hat{x}}_f(t) = \hat{A}_f \hat{x}(t) + \hat{B}_f \hat{y}(t) \\ z_f(t) = \hat{C}_f \hat{x}(t) \end{cases} \tag{52}$$

From (50) and (52), we can obtain the parameters of the designed filter given by (48). This completes the proof. □

4. Simulation examples

In this section, a numerical example is given to demonstrate the effectiveness of the designed H_∞ filter for neural networks.

The parameters of system (1) are given as follows:

$$A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad A_\omega = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad C = \begin{bmatrix} 0.9 & 0.8 \end{bmatrix}, \quad L = \begin{bmatrix} 0.2 & -0.5 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.3 & -0.4 \\ -0.4 & 0.3 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix},$$

$$U_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}, \quad \omega(t) = \begin{cases} 1, & 5 \le t \le 10, \\ -1, & 15 \le t \le 20, \\ 0, & else \end{cases}$$

The neuron activation function $g(x(t))$ in (11) is defined as

$$g(x(t)) = \begin{bmatrix} 0.5x_1(t) - tanh(0.2x_1(t)) + 0.2x_2(t) \\ 0.95x_2(t) - tanh(0.75x_1(t)) \end{bmatrix}.$$

In (8), supposing the deception attack $f(x(t)) = [\begin{smallmatrix} -tanh(0.2x_2(t)) \\ -tanh(0.02x_1(t)) \end{smallmatrix}]$, there exists a matrix $G = diag\{0.02, 0.2\}$ which satisfies the inequation $||f(x(t))||_2 \le ||Gx(t)||_2$.

Suppose the initial condition $x(0) = [1 \quad -1]^T$, $x_f(0) = [0.8 \quad -0.8]^T$, sampling period $h = 0.1$, $\phi_m = 0.1, \phi_M = 0.2, \tau_M = 0.2$, $\eta_M = 0.2$, $d_M = 0.15$, $\gamma = 7$ and $\epsilon_k = 1$ $(k = 1, 2, 3, 4, 5)$.

In the following, three cases will be given to illustrate the effectiveness of designed H_∞ filter subject to deception attacks.

Case 1: Set $\bar{\alpha} = 1$, as is shown in Fig. 1, the selecting switch turns to "time triggered scheme". When the probability of deception attacks $\bar{\theta} = 0.1$, the following matrices can be obtained by applying Theorem 2:

$$\bar{P}_3 = \begin{bmatrix} 1835.3 & -630.2 \\ -630.2 & 792.3 \end{bmatrix}, \quad \hat{A}_f = \begin{bmatrix} -3157.1 & 522.1 \\ 698.9 & -664.7 \end{bmatrix}, \quad \hat{B}_f = \begin{bmatrix} 49.927 \\ -660.973 \end{bmatrix}, \quad \hat{C}_f = \begin{bmatrix} -0.167 & 0.4908 \end{bmatrix}$$

By using Eq. (48) in Theorem 2, the corresponding parameters of designed filter can be obtained as follows:

$$A_f = \begin{bmatrix} -2.0552 & -0.9758 \\ 0.1276 & -0.7374 \end{bmatrix}, \quad B_f = \begin{bmatrix} 49.927 \\ -660.973 \end{bmatrix}, \quad C_f = \begin{bmatrix} 0.0002 & 0.0008 \end{bmatrix}$$

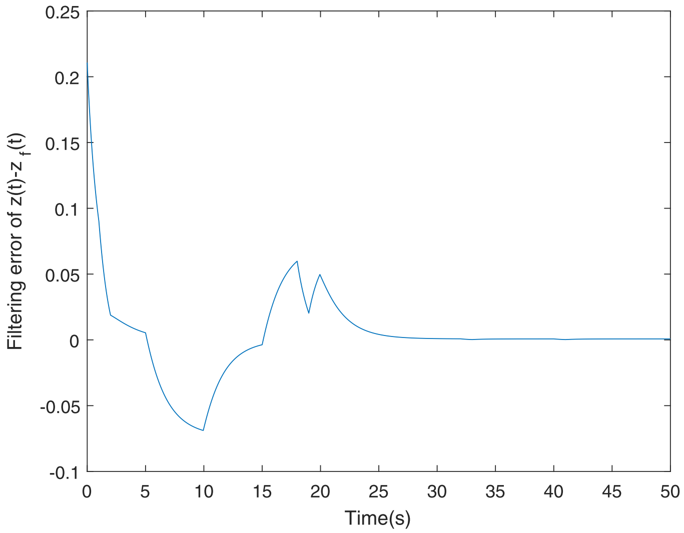

Fig. 3. The filtering error $z(t) - z_f(t)$ in case 1.

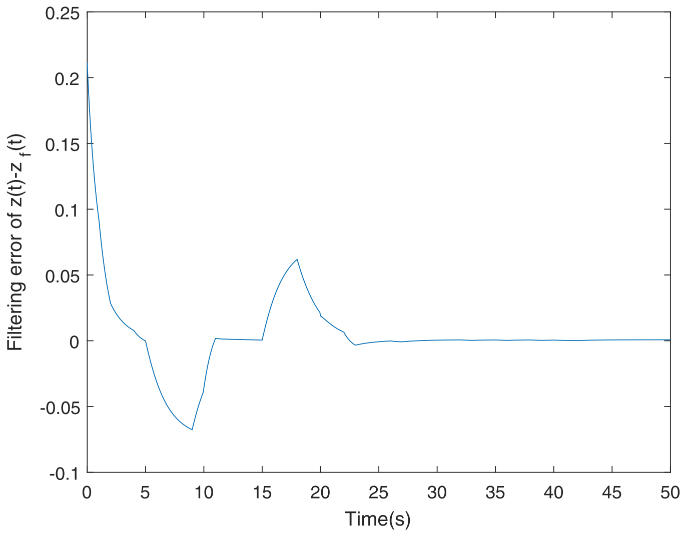

Fig. 4. The filtering error $z(t) - z_f(t)$ in case 2.

The function of deception attacks $f(x(t))$ is shown in Fig. 2. Fig. 3 shows the response of filtering error $z(t) - z_f(t)$ when the "time triggered scheme" is selected in the hybrid triggered scheme. It can be found that the proposed method is useful and the designed filter can satisfy the system performance.

Case 2: Set $\bar{\alpha} = 0$, the triggered parameter $\sigma = 0.2$, as is shown in Fig. 1, the selecting switch turns to "event triggered scheme". Let $\bar{\theta} = 0.2$, the following matrices can be derived as follows by applying Theorem 2:

$$\bar{P}_3 = \begin{bmatrix} 1466.6 & -407.7 \\ -407.7 & 460.6 \end{bmatrix}, \quad \hat{A}_f = \begin{bmatrix} -2549.9 & 375.2 \\ 526.9 & -290.9 \end{bmatrix}, \quad \hat{B}_f = \begin{bmatrix} 24.7349 \\ -241.5662 \end{bmatrix}, \quad \hat{C}_f = \begin{bmatrix} -0.1438 & 0.483 \end{bmatrix}$$

Applying Eq. (48) in Theorem 2, the corresponding filter parameters are shown as follows:

$$A_f = \begin{bmatrix} -2.0059 & -0.9609 \\ 0.2437 & -0.4159 \end{bmatrix}, \quad B_f = \begin{bmatrix} 24.7349 \\ -241.5662 \end{bmatrix}, \quad C_f = \begin{bmatrix} 0.0003 & 0.0013 \end{bmatrix}$$

Fig. 4 depicts the response of the estimation error when the "event triggered scheme" is selected in the hybrid triggered scheme. The release instants and intervals are shown in Fig. 5. From Fig. 4, it demonstrates the usefulness of designed filter with event triggered scheme subject to deception attacks.

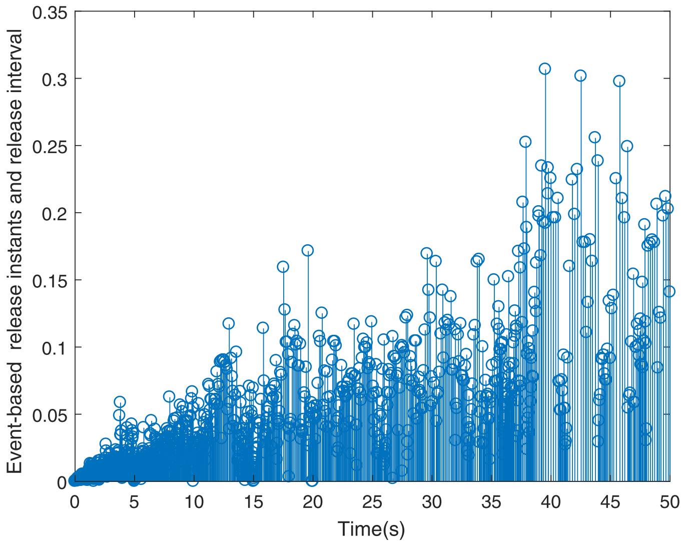

Fig. 5. Release instances and release interval in case 2.

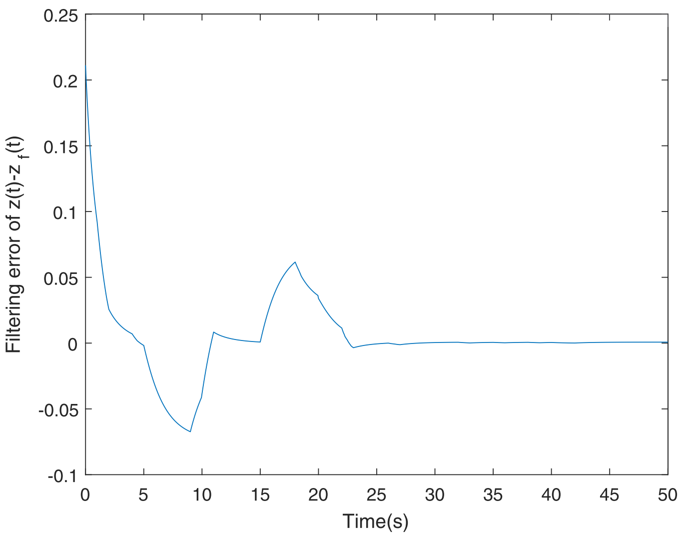

Fig. 6. The filtering error $z(t) - z_f(t)$ in case 3.

Case 3: Let $\bar{\alpha} = 0.5$, the triggered parameter $\sigma = 0.2$, as is shown in Fig. 1, the designed filter is under the hybrid triggered scheme. Set the probability of deception attacks $\bar{\theta} = 0.2$, the following matrices can be obtained by Theorem 2:

$$\bar{P}_3 = \begin{bmatrix} 1263.1 & -342.2 \\ -342.2 & 469.8 \end{bmatrix}, \quad \hat{A}_f = \begin{bmatrix} -2149.5 & 279.6 \\ 412.6 & -294.1 \end{bmatrix}, \quad \hat{B}_f = \begin{bmatrix} 3.2576 \\ -274.9055 \end{bmatrix}, \quad \hat{C}_f = \begin{bmatrix} -0.1033 & 0.4735 \end{bmatrix}$$

According to Eq. (48) in Theorem 2, the parameters of designed filter are obtained as follows:

$$A_f = \begin{bmatrix} -1.9192 & -0.8029 \\ 0.1956 & -0.4837 \end{bmatrix}, \quad B_f = \begin{bmatrix} 3.2576 \\ -274.9055 \end{bmatrix}, \quad C_f = \begin{bmatrix} 0.0002 & 0.0012 \end{bmatrix}$$

Fig. 7 describes the stochastic switch between time triggered scheme and event triggered scheme. The response of estimation error is shown in Fig. 6. From Fig. 6, it can be easily seen that the H_∞ filter design for neural networks subject to deception attacks is feasible.

5. Conclusion

This paper is concerned with H_∞ filter design for neural networks with hybrid triggered scheme and deception attacks. The hybrid triggered scheme which consists of time triggered scheme and event triggered scheme is introduced to reduce

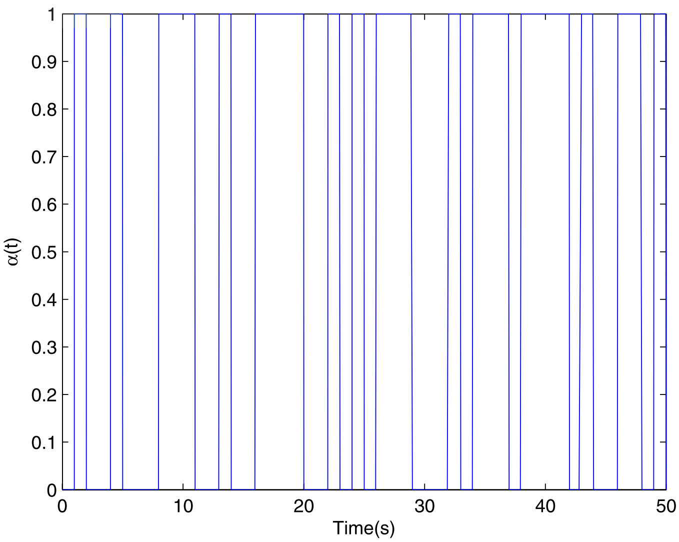

Fig. 7. Bernoulli distribution $\alpha(t)$ in case 3.

the pressure of network bandwidth. By taking the hybrid triggered scheme and the deception attacks into consideration for neural networks, the mathematical model of filtering error system has been constructed. Particularly, the switching rule between the two triggered schemes and the occurring probability of deception attacks are described by variables obeying Bernoulli distribution. Sufficient conditions that can guarantee the stability of filtering error system for neural networks have been developed with the application of Lyapunov stability theory and LMI techniques. In addition, the parameters of designed H_∞ filter are derived in an explicit form. An illustrative example is given to demonstrate the usefulness of desired filter with the hybrid triggered scheme and deception attacks.

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