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Research article

Distributed event-triggered H_∞ filtering over sensor networks with sensor saturations and cyber-attacks

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ABSTRACT

This paper investigates the problem of distributed event-triggered H_∞ filtering over sensor networks with sensor saturations and cyber-attacks. By taking the effects of sensor saturations existing in spatially distributed sensors and randomly occurring cyber-attacks into consideration, a distributed event-triggered filtering error system is firstly established. Then, sufficient conditions guaranteeing the system asymptotically stable with H_∞ performance are obtained by means of Lyapunov stability theory. Moreover, the explicit expressions of distributed H_∞ filters and the weighting matrices of distributed event-triggered scheme are achieved by solving a set of linear matrix inequalities (LMIs). Finally, two examples are given to illustrate the usefulness of the designed distributed event-triggered H_∞ filters.

1. Introduction

Sensor networks have attracted considerable attention by researchers in the fields of target detection and tracking [1], disaster monitoring and forecasting [2], since there are a large amount of geographically distributed micro-sensor devices [3] collecting environmental data with various sensing modalities [4]. Due to the advantages of robustness, reliability and low requirement on network bandwidth [5], a distributed manner is better than a centralized method in signal processing among the large-scale sensors. In particular, for the purpose of estimating system state over sensor networks, distributed H_∞ filtering has been a significant problem [6] since the statistic of exogenous disturbance is not prescribed beforehand [7], where each filter estimates the measurements from not only a corresponding sensor node but also its neighboring sensors. For example, the issue of distributed sampled-data asynchronous H_∞ filtering is addressed in Ref. [8] for a continuous-time Markovian jump linear system over a sensor network.

It is worth noting that sensors are usually driven by batteries possessing limited energy storage capacity, which is one of the constraints for sensor networks. The periodic sampling (time-triggered scheme) is a traditional signal processing method, where both sampling and transmitting events are triggered along with the elapse of a fixed time period [9]. However, in some cases, where the system reaches equilibrium and no disturbance vibrates the system, the system states fluctuate slightly over a certain time interval [10]. As a result, the time-triggered scheme may result in the waste of energy resources and limited bandwidth [11] since it periodically samples almost identical data and releases the redundant sampled signals to a shared communication network. In order to improve the situation, many scholars have proposed more efficient sampling or transmitting strategies (see Ref. [12–14] and the references therein). In the field of networked systems, event-triggered scheme, as a digital signal processing strategy, is the most prevalent since it can significantly save communication resources while preserving satisfactory system performance [15]. In Ref. [16], an event-triggered controller is designed for networked control system, where the system state is supervised in discrete instants and the periodically sampled data can be transmitted only when a predefined triggering condition is violated. The event-triggered scheme proposed in Ref. [16] has been further investigated for other systems or been improved for more practical strategies in recent years. For example, by exploiting the event-triggered scheme proposed in Ref. [16], the problem of finite-time state estimation for Markovian jump systems is addressed in Ref. [17] and the issue of decentralized control for neural networks is studied in Ref. [18]. Based on the event-triggered scheme in Ref. [16], an adaptive event-triggered scheme is investigated in Ref. [19] for a nonlinear networked interconnected control system. A novel hybrid triggered scheme is firstly proposed by the authors in Ref. [20], which combines the advantages of both the time-triggered scheme and the event-triggered scheme mentioned in Ref. [16]. Moreover, the innovative hybrid-

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triggered scheme proposed in Ref. [20] has been successfully applied when the authors in Ref. [21] are concerned with the quantized stabilization for Takagi-Sugeno (T-S) fuzzy systems. In the practical engineering, the event-triggered scheme proposed in Ref. [16] has been widely utilized in filtering for sensor networks. For example, the issue of distributed H_∞ consensus filtering over mobile sensor networks is investigated in Ref. [22] where event-triggered scheme is applied on each sensor node. A problem concerning distributed event-triggered filtering over wireless sensor networks is studied in Ref. [23] for a class of discrete time-varying systems.

However, the aforementioned results about sensor networks overlook sensor saturations, which always exist in sensors and may destabilize the sensor networks to some extent. Actually, the sensor saturations have been investigated for several years, and many research results are available. For instance, the issue of designing filter for Markov jump systems is studied in Ref. [25] with sensor saturations. In Ref. [24], H_∞ filtering for nonlinear networked systems is investigated by taking sensor saturations into consideration. The authors in Ref. [26] propose a distributed H_∞ filter design method for discrete-time T-S fuzzy complex networks subject to sensor saturation. It should be pointed out that the sensor saturations concerning with sensor networks have not been fully investigated yet, which is the first motivation of this paper.

Additionally, the communication among sensors is highly important for sensor networks to achieve a desired control goal. Nevertheless, besides the common communication constraints such as limited bandwidth, time-delays and packet dropouts, cyber-attacks have gradually been formidable barriers to smooth communication in sensor networks due to the increasing dependence on more cyber elements and the vulnerability of communication channels as well. Recently, the cyber security has been a hot topic in the networked control area and potential cyber-attacks have been discussed by many scholars, see survey paper [27] and the references therein. There are mainly two kinds of cyber-attacks, namely, denial of service (DoS) attacks [28] and deception attacks [29]. Generally speaking, the cyber-attacks may corrupt data delivery, integrity or confidentiality [30], which means the control and measurement signals may be targeted so that the system performance might be ruined. In order to achieve the attack purpose, adversaries usually launch the cyber-attacks in a consecutive or random manner [31]. For instance, with consideration of the random cyber-attacks in networked control systems, the distributed control problem is addressed in Ref. [32], where the energy consumption is saved and the transmission load of network is alleviated by introducing event-triggered scheme. However, as far as we concerned, few attention has been paid to the cyber-attacks randomly occurring in sensor networks, which is the second motivation of this paper.

Enlighted by the observations above, this paper focuses on distributed event-triggered H_∞ filter design over sensor networks with sensor saturations and cyber-attacks. The main contributions of this paper are summarized as follows. (i) A distributed filtering structure is proposed over the sensor networks by taking sensor saturations, distributed event-triggered scheme and cyber-attakcs into consideration. The key of distributed event-triggered scheme is the event monitor (EM) deployed on each sensor node, which can independently decide whether the periodically sampled data from the saturated sensors should be broadcast over a communication network or not. The cyber-attacks are modeled as nonlinear functions and their randomly occurring features are identically indicated by a Bernoulli variable with certain statistical property. The input of filter i is the aggregated data, which is collected from the sampled output of saturated sensor i and its potentially neighboring sensors. (ii) A new model of distributed H_∞ filtering error system is firstly constructed by considering sensor saturations, distributed event-triggered scheme and cyber-attakcs. (iii) Based on the constructed model, the sufficient conditions guaranteeing the asymptotical stability of distributed H_∞ filtering error system over sensor networks are derived by means of Lyapunov stability theory. Moreover, the explicit expressions of distributed H_∞ filters and the weighting matrices of distributed event-triggered scheme are achieved by solving a set of LMIs.

The remainder of this paper is organized as follows. In Section 2, the problem formulation is stated. Section 3 derives new sufficient conditions which can guarantee the distributed H_∞ filtering error system asymptotically stable. Moreover, the parameters with regard to distributed H_∞ filters and distributed event-triggered scheme are obtained simultaneously. Two examples are presented in Section 4 to show the usefulness of the designed distributed H_∞ filters. The conclusion of this paper is drawn in Section 5.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n\times m$ real matrices. Matrix $X>0\,(X\in\mathbb{R}^{n\times n})$ means that the matrix X is real symmetric positive definite. $\mathbb{E}$ represents a mathematical expectation. $\|\cdot\|$ denotes the Euclidean norm. I and T stand for the identity matrix with appropriate dimension and the transpose of matrix, respectively. $*$ in a symmetric matrix denotes the terms implied by symmetry. $\mathscr{L}_2[0,\infty)$ represents the space of square-integrable vector functions over $[0,\infty)$. In addition, $\mathrm{diag}_N\{\cdot\}$ and $\mathrm{diag}_N^i\{\cdot\}$ denote N-block-diagonal matrix and N-block-diagonal matrix only with i-th block is nonzero, respectively. Similarly, $vec_N\{\cdot\}$ and $\mathrm{vec}_N^i\{\cdot\}$ denote N-rows vector and N-rows vector only with ith row is nonzero; $\mathrm{col}_N\{\cdot\}$ and $\mathrm{col}_N^i\{\cdot\}$ denote N-columns vector and N-columns vector only with ith column is nonzero. $\otimes$ stands for the Kronecker product for matrices.

2. Problem formulation and modeling

2.1. System description

Consider a structure of distributed event-triggered H_∞ filtering over sensor networks in Fig. 1, which can be characterized by a directed graph $\mathscr{G}=\{\mathscr{V},\mathscr{E},\mathscr{W}\}$, where $\mathscr{V}=\{1,2,...,N\}$ denotes a set of sensor nodes; $\mathscr{E}\in\mathscr{V}\times\mathscr{V}$ is a set of directed edges in the directed graph $\mathscr{G}$; The element $(i,j)\in\mathscr{E}$ $(i\in\mathscr{V}, j\in\mathscr{V})$ means that the node i can receive data from node j; $\mathscr{W}=[w_{ij}]\in\mathbb{R}^{N\times N}$ stands for an adjacent matrix with $w_{ij}=1$ if edge $(i,j)\in\mathscr{E}$ or with $w_{ij}=0$ if edge $(i,j)\notin\mathscr{E}$. Suppose the plant is described as following continuous-time linear-invariant system:

$$\begin{cases}\dot{x}(t)=Ax(t)+B\omega(t)\\ z(t)=Lx(t)\\ x(0)=x_0\end{cases}\tag{1}$$

where $x(t)\in\mathbb{R}^n$ is the state vector; $z(t)\in\mathbb{R}^m$ is the output vector to be estimated; $\omega(t)\in\mathbb{R}^p$ is the external noise signal, which belongs to $\mathscr{L}_2[0,\infty)$ and aims to disturb the plant; x_0 denotes the initial state; A, B, L are known matrices with appropriate dimensions.

N sensor nodes are deployed to detect the system state dispersedly and the measurement of ith sensor node is considered as follows:

$$y_i(t)=C_ix(t),\quad i\in\mathscr{V}\tag{2}$$

where $y_i(t)\in\mathbb{R}^q$ $(i\in\mathscr{V})$ are the sensor measurements; C_i $(i\in\mathscr{V})$ are known constant matrices with appropriate dimensions.

2.2. Distributed H_∞ filters

By considering the effects of sensor saturations, distributed event-triggered scheme and cyber-attacks step by step, the sensor measurements will be received by distributed H_∞ filters eventually. As shown in Fig. 1, N corresponding zero-order-holders (ZOHs) are set according to N sensor nodes. Inspired by Ref. [22], the function of ith ZOH is to collect the sampled data from ith sensor as well as some of other $N-1$ sensors, which constitute the aggregated data $\sum_{j=1}^{N}w_{ij}\hat{y}_j(t)$ in accordance with the sensing topology $\mathscr{G}$. In addition, the ZOHs are assumed to be event-driven, which means that only when a new aggregated data is collected can the output of ZOH i be updated to actuate

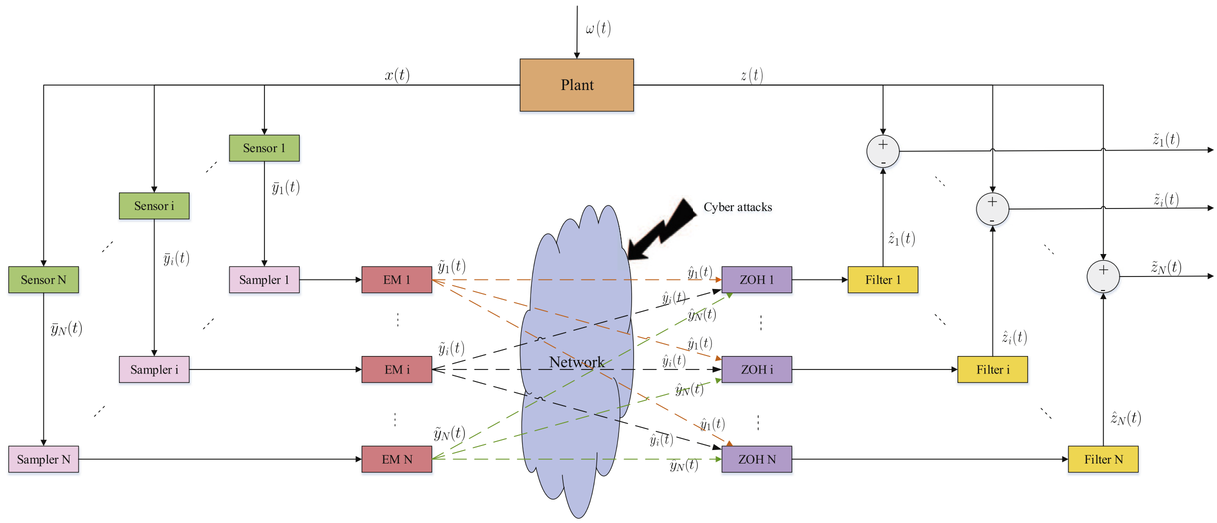

Fig. 1. The structure of distributed event-triggered H_∞ filtering with sensor saturations and cyber-attacks.

the filter i. In other words, the ZOH i will keep the latest input of filter i until a new aggregated data is collected.

Based on the description above, the distributed H_∞ filters to be designed in this paper are proposed as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = D_i\hat{x}_i(t) + K_i\sum_{j=1}^{N} w_{ij}\hat{y}_j(t) \\ \hat{z}_i(t) = E_i\hat{x}_i(t) + G_i\sum_{j=1}^{N} w_{ij}\hat{y}_j(t) \end{cases} \tag{3}$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ is the state of filter i; $\hat{z}_i(t) \in \mathbb{R}^m$ is the estimation of $z(t)$, which is also the output of filter i ($i \in \mathscr{V}$); the aggregated data $\sum_{j=1}^{N} w_{ij}\hat{y}_j(t)$ collected by ZOH i is the input of filter i and $\hat{y}_j(t)$ in $\sum_{j=1}^{N} w_{ij}\hat{y}_j(t)$ represents the broadcast signal of sensor node j, which arrives at the ZOH i as shown in Fig. 1. D_i, K_i, E_i, G_i ($i \in \mathscr{V}$) are the filter parameter matrices to be designed later.

Remark 1. *The model of distributed H_∞ filters (3) proposed in this paper is similar to the model of distributed full order linear dynamic filters in Ref. [34], where the input of filters are all aggregated data. However, the collection of the aggregated data received by filter i is different between this paper and Ref. [34]. To be specific, the aggregated data in this paper is assumed to be collected by ZOH i, which is inspired by Ref. [22], while the aggregated data in Ref. [34] is collected by sensor i itself.*

2.3. Sensor saturations

In this paper, sensor saturation is characterized as a saturation function $sat(v) = [sat(v_1), sat(v_2), \ldots, sat(v_m)]^T \in \mathbb{R}^s$, where each element is defined as follows [33]:

$$sat(v_i) = \begin{cases} \rho_i, & v_i > \rho_i \\ v_i, & -\rho_i \le v_i \le \rho_i, \quad i = 1, 2, \ldots, s. \\ -\rho_i, & v_i < -\rho_i \end{cases} \tag{4}$$

where ρ_i is the threshold value of ith sensor saturation.

Further more, the saturation function can be separated into two parts, that is $sat(v) = v - \varphi(v)$, where $\varphi(v)$ is a nonlinear function satisfying the following constraint.

There exists a positive number $\varepsilon_i \in (0, 1)$ such that

$$\varepsilon_i v_i^T v_i \ge \varphi^T(v_i)\varphi(v_i). \tag{5}$$

It is worth mentioning that the restraint coefficient ε_i can be further confined according to the Proof in Ref. [33], namely,

$$\varepsilon_i \ge (1 - (\rho_i/|v_i|_{\max}))^2 . \tag{6}$$

Therefore, the real output of sensor node i with sensor saturation is

$$\bar{y}_i(t) = sat(y_i(t)) = y_i(t) - \varphi(y_i(t)) = C_i x(t) - \varphi(y_i(t)), \quad i \in \mathscr{V} \tag{7}$$

subject to the constraint that for $\varepsilon_i \in (0, 1)$,

$$\varepsilon_i x^T(t) C_i^T C_i x(t) \ge \varphi^T(y_i(t))\varphi(y_i(t)). \tag{8}$$

2.4. Distributed event-triggered scheme

In order to alleviate the burden of network bandwidth, the distributed event-triggered scheme is introduced for data transmission. For convenience of development, Assumption 1 is given in the following, which is partially followed from Ref. [34].

Assumption 1. *The samplers are time-driven and synchronized, that is, all of them dispersedly sample the corresponding sensor output in the same sampling period h, where $h > 0$ is a constant. Hence, the sampling time sequence can be described as $\mathbb{L} = \{lh | l \in \mathbb{N} = \{0, 1, 2, \ldots\}\}$. The broadcasting time sequence of ith sensor node is assumed as $\mathbb{T}_i = \{t_l^i h | t_l^i \in \mathbb{N}\}$. Between two consecutive broadcasting instants, the sampling time sequence of ith sensor node is assumed to be $\mathbb{S}_i^{t_l^i} = \{s_l^i h | s_l^i = t_l^i + m_i,\ m_i = 0, 1, \ldots, M_i,\ M_i = t_{l+1}^i - t_l^i - 1\}$.*

From Assumption 1, the latest broadcast data and the newly sampled data can be represented by $\bar{y}_i(t_l^i h)$ and $\bar{y}_i(s_l^i h)$, respectively. The idea of distributed event-triggered scheme is realized by event monitors (EMs) separately deployed on every sensor node, each of them is composed of an event generator and a storer. The adoption of event generator on node i is to produce a series of events according to the following predefined event-triggered condition [10], which determines whether the periodically sampled data will be broadcast or not.

$$(\bar{y}_i(t_l^i h) - \bar{y}_i(s_l^i h))^T \Omega_i (\bar{y}_i(t_l^i h) - \bar{y}_i(s_l^i h)) < \sigma_i (\bar{y}_i(t_l^i h))^T \Omega_i (\bar{y}_i(t_l^i h)) \tag{9}$$

where $\sigma_i \in (0, 1)$ is the trigger threshold parameter to be given in advance, and $\Omega_i > 0$ is the weighting matrix to be designed later. The event-triggered condition (9) is checked by event generator in discrete instants, which depends on the latest broadcast data $\bar{y}_i(t_l^i h)$ and the error, i.e. defining $e_{\bar{y}}^i(t) = \bar{y}_i(t_l^i h) - \bar{y}_i(s_l^i h)$, between the latest broadcast data and the current sampled sensor measurements. In order to fulfill the comparison in the current sampling instant, the storer is adopted to reserve the latest broadcast data.

Remark 2. *When the phenomena of sensor saturations are overlooked on each sensor node, an alternative event-triggered condition can be defined as , which recently has been adopted to address the event-triggered H_∞ filtering* *in Refs.* [7] *and* [10]. *However, the event-triggered condition* (9) *on each node is related to the sampled measurements from corresponding saturated sensor.*

To be specific, only when the event-triggered condition (9) is violated, will the newly sampled data $\bar{y}_i(s_l^i h)$ be broadcast and will the storer be updated with the newest broadcast data. Otherwise, the newly sampled data is discarded directly and the storer keeps the latest broadcast data. Therefore, the next broadcasting instant of sensor node i can be expressed as follows

$$t_{l+1}^i h = t_l^i h + \min_{m_i \geq 0}\{(m_i+1)h|(e_{\bar{y}}^i(t))^T \Omega_i (e_{\bar{y}}^i(t)) \geq \sigma_i (\bar{y}_i(t_l^i h))^T \Omega_i (\bar{y}_i(t_l^i h))\} \tag{10}$$

Remark 3. *From* (10)*, it can be seen that not all the periodically sampled data are broadcast to the filters, which means the broadcasting time sequence is a subset of the sampling time sequence, i.e.* $\mathbb{T}_i \subseteq \mathbb{L}$*. Consequently, the occupancy of network bandwidth can be reduced and the burden of network communication can be alleviated as well. Notice that when* $\sigma_i = 0$*, the event-triggered condition* (9) *is violated all the time. Then the* $\mathbb{T}_i \subseteq \mathbb{L}$ *becomes* $\mathbb{T}_i = \mathbb{L}$*, which means the event-triggered scheme on node i reduces to the time-triggered scheme.*

Remark 4. *In this paper, the EM on each sensor node works independently, which means the trigger threshold parameters* σ_i $(i \in \mathscr{V})$ *in* (10) *are not required to be equal. In other words, the broadcasting time sequence on each node is not equal to each other, i.e.* $\mathbb{T}_i \neq \mathbb{T}_j$ $(i \neq j)$*, since the event-triggered conditions at all the nodes are different. As a result, the aggregated data received by filters may be collected from sensors asynchronously.*

It is worth noting that due to the introduction of network, the time-delay of the released measurement from the sensor node i at instant $t_l^i h$ is assumed to be $\tau_{t_l^i}^i$. The instant when the broadcast data from sensor node i arrives at ZOHs can be expressed by $t_l^i h + \tau_{t_l^i}^i$. Then, the broadcast data $\bar{y}_i(t_l^i h)$ are maintained by ZOHs in the holding interval $\left[t_l^i h + \tau_{t_l^i}^i, t_{l+1}^i h + \tau_{t_{l+1}^i}^i\right)$, which can be further denoted by a union of a series of smaller intervals [35], i.e. $\left[t_l^i h + \tau_{t_l^i}^i, t_{l+1}^i h + \tau_{t_{l+1}^i}^i\right) = \bigcup_{m_i=0}^{M_i} \mathscr{I}_l^{m_i}$, where $\mathscr{I}_l^{m_i} = \left[s_l^i h + \tau_{s_l^i}^i, s_l^i h + h + \tau_{s_{l+1}^i}^i\right)$. Without loss of generality, the time-delays $\tau_{t_l^i}^i$ $(i \in \mathscr{V})$, which are induced in the sampled data transmission from sensor node i to ZOHs, are assumed to be upper-bounded, namely $0 < \tau_{t_l^i}^i \leq \bar{\tau}_i$ $(i \in \mathscr{V})$.

Define time-varying delay function as $\tau^i(t) = t - s_l^i h$, $t \in \mathscr{I}_l^{m_i}$ $(i \in \mathscr{V})$. Then it can be derived from $\mathscr{I}_l^{m_i}$ that $0 < \tau_{s_l^i}^i \leq \tau^i(t) < h + \bar{\tau} \triangleq \tau_M$ where $\bar{\tau} \triangleq \max_{i \in \mathscr{V}}\{\bar{\tau}_i\}$. Now, the real transmitted data of sensor node i can be expressed as follows:

$$\tilde{y}_i(t) = \bar{y}_i(t_l^i h) = \bar{y}_i(t_l^i h) - \bar{y}_i(s_l^i h) + \bar{y}_i(s_l^i h) = e_{\bar{y}}^i(t) + \bar{y}_i(t - \tau^i(t)) \tag{11}$$

2.5. Cyber-attacks

In this paper, cyber-attacks are assumed to occur occasionally in the data transmission process and the randomly occurring features can be indicated by a series of Bernoulli random variables $\alpha^i(t) \in \{0, 1\}$ $(i \in \mathscr{V})$ according to the ideas in Ref. [31,36,37]. To be more concrete, on the one hand, when the cyber-attacks are launched by malicious adversaries, $\alpha^i(t) = 1$. On the other hand, the sampled data is successfully broadcast to the filters and then $\alpha^i(t) = 0$. In this sense, the occurring frequency can be estimated by a certain random probability. It is noted that due to a communication network is shared in broadcasting sampled data, the cyber-attacks are assumed to take place identically, which can be denoted by a Bernoulli random variable $\alpha(t)$. In addition, the statistical properties of $\alpha(t)$ are identically supposed, i.e. $\mathbb{E}\{\alpha(t)\} = \bar{\alpha}$, $\mathbb{E}\{(\alpha(t) - \bar{\alpha})^2\} = \delta^2$ for all the transmission between sensors and filters. By referring to the definition in Refs. [38] and [39], the cyber-attacks are modeled as nonlinear functions $f(y_i(t))$ in this paper. In the sequel, the real broadcast signals of node i, which arrive at the ZOHs, can be represented by

$$\begin{aligned} \hat{y}_i(t) &= \alpha(t) f(y_i(t - d^i(t))) + (1 - \alpha(t))\tilde{y}_i(t) \\ &= \alpha(t) f(y_i(t - d^i(t))) + (1 - \alpha(t)) \\ &\quad [C_i x(t - \tau^i(t)) - \varphi(y_i(t - \tau^i(t))) + e_{\bar{y}}^i(t)] \end{aligned} \tag{12}$$

where $d^i(t) \in (0, d_M^i]$ is assumed to be the corresponding time-delay of cyber-attacks signal that might be transmitted on the ith communication channel. Define $d_M = \max_{i \in \mathscr{V}}\{d_M^i\}$ to represent the upper-bounded delay of cyber-attacks signals.

Remark 5. *An attacker may cheat the systems by corrupted sensor output or designed input* [40]. *Hence, a cyber attacker may have access to the initial sensor measurements and then pretend the attack signals highly similar to the real sensor measurements to achieve a perfect attack goal. This paper takes cyber-attacks into consideration shown in* (12)*, where the nonlinear functions associated with initial sensor measurements are supposed to represent the signals of cyber-attacks, which are limited in* Assumption 2.

Remark 6. *From the definition of* $\alpha(t)$ *in* (12)*, the cyber attacks are called stochastic deception attacks in a strict sense since they may substitute the normal broadcast data with their vicious information in the transmission process. Note that the cyber-attacks might take place at any time during the sampled data transmission, and once they successfully replace the sampled data, they will continue the transmission over a communication network. Therefore, the transmission for the cyber-attacks signals on each transmission channel will induce the time-delays, which are assumed as* $d^i(t)$ *in* (12) *and bounded by* d_M.

2.6. The filtering error system

Define $\tilde{z}_i(t) = z(t) - \hat{z}_i(t)$ as a filtering error vector for filter i, and $\bar{A} = \mathrm{diag}_N\{A\}$, $\bar{B} = \mathrm{col}_N\{B\}$, $\bar{C}_i = \mathrm{diag}_N^i\{C_i\}$, $\tilde{C} = \mathrm{vec}_N^i\{\mathrm{col}_N\{C_i\}\}$, $\bar{D} = \mathrm{diag}_N\{D_i\}$, $\bar{E} = \mathrm{diag}_N\{E_i\}$, $\bar{F}_i = \mathrm{diag}_N^i\{F_i\}$, $\bar{G}_i = \mathrm{diag}_N^i\{G_i\}$, $\bar{K}_i = \mathrm{diag}_N^i\{K_i\}$, $\bar{L} = \mathrm{diag}_N\{L\}$, $\bar{x}(t) = \mathrm{col}_N\{x(t)\}$, $\bar{\hat{x}}(t) = \mathrm{col}_N\{\hat{x}_i(t)\}$, $\bar{e}_{\bar{y}}^i(t) = col_N^i\{e_{\bar{y}}^i(t)\}$, $\bar{f}(y_i(t - d^i(t))) = col_N^i\{f(y_i(t - d^i(t)))\}$, $\bar{\varphi}(y_i(t - \tau^i(t))) = col_N^i\{\varphi(y_i(t - \tau^i(t)))\}$.

Based on the above discussions of sensor saturations, distributed event-triggered scheme and cyber-attacks, further let $\xi(t) = \mathrm{col}_2\{\bar{x}(t), \bar{\hat{x}}(t)\}$ and $\tilde{z}(t) = \mathrm{col}_N\{\tilde{z}_i(t)\}$ and then by combining (1), (3) and (12), a filtering error system can be expressed as

$$\begin{cases} \dot{\xi}(t) = \tilde{A}\xi(t) + \tilde{B}\omega(t) + (1 - \alpha(t)) \sum_{i=1}^N \check{C}_i \xi(t - \tau^i(t)) \\ \quad + (1 - \alpha(t)) \sum_{i=1}^N \check{D}_i \bar{e}_{\bar{y}}^i(t) \\ \quad -(1 - \alpha(t)) \sum_{i=1}^N \check{D}_i \bar{\varphi}(y_i(t - \tau^i(t))) \\ \quad + \alpha(t) \sum_{i=1}^N \check{D}_i \bar{f}(y_i(t - d^i(t))) \\ \tilde{z}(t) = \tilde{E}\xi(t) - (1 - \alpha(t)) \sum_{i=1}^N \check{F}_i \xi(t - \tau^i(t)) \\ \quad - (1 - \alpha(t)) \sum_{i=1}^N \check{G}_i \bar{e}_{\bar{y}}^i(t) \\ \quad +(1 - \alpha(t)) \sum_{i=1}^N \check{G}_i \bar{\varphi}(y_i(t - \tau^i(t))) \\ \quad - \alpha(t) \sum_{i=1}^N \check{G}_i \bar{f}(y_i(t - d^i(t))) \end{cases} \tag{13}$$

for all $t \in \mathscr{I}_l^{m_i}$, where

$$\tilde{A} = \begin{bmatrix} \overline{A} & 0 \\ 0 & \overline{D} \end{bmatrix}, \ \widetilde{B} = \begin{bmatrix} \overline{B} \\ 0 \end{bmatrix},$$
$$\check{C}_i = \begin{bmatrix} 0 & 0 \\ \overline{K}_i(\mathscr{W} \otimes I)\widetilde{C} & 0 \end{bmatrix},$$
$$\check{D}_i = \begin{bmatrix} 0 \\ \overline{K}_i(\mathscr{W} \otimes I) \end{bmatrix},$$
$$\widetilde{E} = [\overline{L} \ \ -\overline{E}], \ \check{F}_i = [\overline{G}_i(\mathscr{W} \otimes I)\widetilde{C} \ \ 0], \ \check{G}_i = [\overline{G}_i(\mathscr{W} \otimes I)].$$

To facilitate the acquisition of the main results, the following assumption, definition and lemma are presented.

Assumption 2. [38,39] *The signals of cyber-attacks are modeled as nonlinear functions* $f(y_i(t - d^i(t)))$ $(i \in \mathscr{V})$*, which are subject to the following condition:*

$$\|f(y_i(t - d^i(t)))\|^2 \le \|F_i y_i(t - d^i(t))\|^2 \tag{14}$$

where F_i $(i \in \mathscr{V})$ *are constant matrices denoting the upper bound.*

Definition 1. [34] *The filtering error system* (13) *is asymptotically stable satisfying a prescribed* H_∞ *performance if the following conditions hold:*

(1) *The filtering error system* (13) *subject to* Eqs. (8), (9) and (14) *is asymptotically stable when* $\omega(t) = 0$.
(2) *Under zero initial condition, the estimation error* $\widetilde{z}_i(t)$ *satisfies*

$$\frac{1}{N}\sum_{i=1}^{N} \mathbb{E}\left\{\int_0^\infty \widetilde{z}_i^T(t)\widetilde{z}_i(t)dt\right\} < \gamma^2 \mathbb{E}\left\{\int_0^\infty \omega^T(t)\omega(t)dt\right\} \tag{15}$$

for any nonzero $\omega(t) \in \mathscr{L}[0, \infty)$*, where* $\gamma > 0$ *is a prescribed value.*

Lemma 1. [34] *Suppose a time-varying function* $\tau(t)$ *is subject to the interval* $(0, \tau_M]$ *and a vector-valued function* $\dot{\xi}(t)$: $(0, \tau_M] \to \mathbb{R}^n$*, there exist matrices* $R = R^T \in \mathbb{R}^{n\times n}$ *and* $S \in \mathbb{R}^{n\times n}$ *such that* $\begin{bmatrix} R & * \\ S^T & R \end{bmatrix} \ge 0$. *Then the following integral inequality holds:*

$$-\tau_M \int_{t-\tau_M}^{t} \dot{\xi}^T(s)R\dot{\xi}(s)ds \le \begin{bmatrix} \xi(t) \\ \xi(t-\tau_M) \\ \xi(t-\tau(t)) \end{bmatrix}^T \begin{bmatrix} -R & * & * \\ -S^T & -R & * \\ R+S^T & R+S & -2R-S-S^T \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t-\tau_M) \\ \xi(t-\tau(t)) \end{bmatrix} \tag{16}$$

3. Main results

In this section, by using Lyapunov-Krasovskii functional approach and linear matrix inequality technique, considering the sensor saturations, the distributed event-triggered scheme and stochastic cyber-attacks, the sufficient conditions will be obtained which guarantee the filtering error system (13) asymptotically stable with the constraint of H_∞ performance (15).

Theorem 1. *For given the upper bound of time-delays* τ_M, d_M*, expectation* $\overline{\alpha}$*, scalar* γ*, the restraint coefficients of sensor saturations* ε_i*, trigger threshold parameters* σ_i*, matrices* F_i *and filter parameters* D_i, K_i, E_i, G_i, $(i \in \mathscr{V})$*, the filtering error system* (13) *is asymptotically stable, if there exist matrices* $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_{1i} > 0$, $R_{2i} > 0$, $\Omega_i > 0$, S_i, W_i *and* X *with appropriate dimensions satisfying: for* $i \in \mathscr{V}$,

$$\Psi = \begin{bmatrix} \Phi & * \\ \Xi & \Lambda \end{bmatrix} < 0, \begin{bmatrix} R_{1i} & * \\ S_i^T & R_{1i} \end{bmatrix} \ge 0, \begin{bmatrix} R_{2i} & * \\ W_i^T & R_{2i} \end{bmatrix} \ge 0 \tag{17}$$

where

$$\Phi = \begin{bmatrix} \Phi_1 & * \\ \Phi_2 & \Phi_3 \end{bmatrix}, \ \Lambda = \begin{bmatrix} -NI & * \\ 0 & -NI \end{bmatrix},$$

$$\Xi = \begin{bmatrix} \widetilde{E} & 0 & 0 & vec_N\{-\overline{\alpha}_1\check{F}_i\} & 0 & 0 & vec_N\{-\overline{\alpha}_1\check{G}_i\} & vec_N\{-\overline{\alpha}\check{G}_i\} & vec_N\{\overline{\alpha}_1\check{G}_i\} & 0 \\ 0 & 0 & 0 & vec_N\{\delta\check{F}_i\} & 0 & 0 & vec_N\{\delta\check{G}_i\} & vec_N\{-\delta\check{G}_i\} & vec_N\{-\delta\check{G}_i\} & 0 \end{bmatrix}$$

with

$$\Phi_1 = \begin{bmatrix} \Phi_{11} & * & * \\ P - X^T + X\tilde{A} & \Phi_{12} & * \\ -\sum_{i=1}^N S_i^T & 0 & -Q_1 - \sum_{i=1}^N R_{1i} \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} \mathrm{col}_N\{R_{1i} + S_i^T + \overline{\alpha}_1\check{C}_i^T X^T\} & \mathrm{col}_N\{\overline{\alpha}_1\check{C}_i^T X^T\} & \mathrm{col}_N\{R_{1i} + S_i\} \\ -\sum_{i=1}^N W_i^T & 0 & 0 \\ \mathrm{col}_N\{R_{2i} + W_i^T\} & 0 & 0 \\ \mathrm{col}_N\{\overline{\alpha}_1\check{D}_i^T X^T\} & \mathrm{col}_N\{\overline{\alpha}_1\check{D}_i^T X^T\} & 0 \\ \mathrm{col}_N\{\overline{\alpha}\check{D}_i^T X^T\} & \mathrm{col}_N\{\overline{\alpha}\check{D}_i^T X^T\} & 0 \\ \mathrm{col}_N\{-\overline{\alpha}_1\check{D}_i^T X^T\} & \mathrm{col}_N\{-\overline{\alpha}_1\check{D}_i^T X^T\} & 0 \\ \widetilde{B}^T X^T & \widetilde{B}^T X^T & 0 \end{bmatrix}$$

$$\Phi_3 = \begin{bmatrix} \Phi_{31} & * & * & * & * & * & * \\ 0 & -Q_2 - \sum_{i=1}^N R_{2i} & * & * & * & * & * \\ 0 & \mathrm{col}_N\{R_{2i} + W_i\} & \Phi_{34} & * & * & * & * \\ \Phi_{32} & 0 & 0 & \mathrm{diag}_N\{(\sigma_i - 1)\overline{\Omega}\} & * & * & * \\ 0 & 0 & 0 & 0 & \mathrm{diag}_N\{-\overline{\alpha}I\} & * & * \\ \Phi_{33} & 0 & 0 & \mathrm{diag}_N\{-\sigma_i\overline{\Omega}\} & 0 & \mathrm{diag}_N\{\sigma_i\overline{\Omega} - I\} & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\begin{aligned}
\Phi_{11} &= Q_1 + Q_2 - \textstyle\sum_{i=1}^N R_{1i} - \sum_{i=1}^N R_{2i} + X\tilde{A} + \tilde{A}^T X^T, \\
\Phi_{12} &= \tau_M^2 \textstyle\sum_{i=1}^N R_{1i} + d_M^2 \sum_{i=1}^N R_{2i} - X - X^T, \\
\Phi_{31} &= \mathrm{diag}_N\{-2R_{1i} - S_i - S_i^T + \varepsilon_i\hat{C}_i^T\hat{C}_i + \sigma_i\hat{C}_i^T\overline{\Omega}\hat{C}_i\}, \\
\Phi_{32} &= \mathrm{diag}_N\{\sigma_i\overline{\Omega}\hat{C}_i\}, \ \hat{C}_i = [\overline{C}_i \ \ 0], \\
\Phi_{33} &= \mathrm{diag}_N\{-\sigma_i\overline{\Omega}\hat{C}_i\}, \\
\Phi_{34} &= \mathrm{diag}_N\{-2R_{2i} - W_i - W_i^T + \overline{\alpha}\hat{C}_i^T\overline{F}_i^T\overline{F}_i\hat{C}_i\}, \ \overline{\Omega} = \mathrm{diag}_N\{\Omega_i\}, \\
\overline{\alpha}_1 &= 1 - \overline{\alpha}, \ \delta = \sqrt{\overline{\alpha}\,\overline{\alpha}_1}.
\end{aligned}$$

Proof. Choose the following Lyapunov-Krasovskii functional candidate as

$$\begin{aligned}
V(t) = {} & \xi^T(t)P\xi(t) + \int_{t-\tau_M}^{t} \xi^T(s)Q_1\xi(s)ds + \int_{t-d_M}^{t} \xi^T(s)Q_2\xi(s)ds \\
& + \tau_M \int_{t-\tau_M}^{t} \int_s^t \dot{\xi}^T(\theta) \textstyle\sum_{i=1}^N R_{1i}\dot{\xi}(\theta)d\theta ds \\
& + d_M \int_{t-d_M}^{t} \int_s^t \dot{\xi}^T(\theta) \textstyle\sum_{i=1}^N R_{2i}\dot{\xi}(\theta)d\theta ds
\end{aligned} \tag{18}$$

for all $t \in \mathscr{I}_l^{m_i}$. Two cases without and with system disturbance will be discussed in the following to prove that the filtering error system (13) is asymptotically stable and satisfies the constraint of H_∞ performance (15), respectively.

On the one hand, the analysis is without disturbance, namely, $\omega(t) = 0$. By taking the derivative on $V(t)$ along the trajectory of the filtering error system (13), it yields that:

$$\begin{aligned}
\dot{V}(t) = {} & 2\xi^T(t)P\dot{\xi}(t) + \xi^T(t)(Q_1 + Q_2)\xi(t) - \xi^T(t-\tau_M)Q_1\xi(t-\tau_M) \\
& - \xi^T(t-d_M)Q_2\xi(t-d_M) + \tau_M^2\dot{\xi}^T(t)\textstyle\sum_{i=1}^N R_{1i}\dot{\xi}(t) \\
& + d_M^2\dot{\xi}^T(t)\textstyle\sum_{i=1}^N R_{2i}\dot{\xi}(t) - \tau_M\int_{t-\tau_M}^{t}\dot{\xi}^T(s)\sum_{i=1}^N R_{1i}\dot{\xi}(s)ds \\
& - d_M\int_{t-d_M}^{t}\dot{\xi}^T(s)\textstyle\sum_{i=1}^N R_{2i}\dot{\xi}(s)ds
\end{aligned} \tag{19}$$

for all $t \in \mathscr{I}_l^{m_i}$.

By applying Lemma 1, it can be obtained that

$$-\tau_M \int_{t-\tau_M}^{t} \dot{\xi}^T(s) \sum_{i=1}^{N} R_{1i}\dot{\xi}(s)ds \le [\phi_\tau^i(t)]^T \Pi_{1i}[\phi_\tau^i(t)] \tag{20}$$

$$-d_M \int_{t-d_M}^{t} \dot{\xi}^T(s) \sum_{i=1}^{N} R_{2i}\dot{\xi}(s)ds \le [\phi_d^i(t)]^T \Pi_{2i}[\phi_d^i(t)] \tag{21}$$

where

$$\Pi_{1i} = \begin{bmatrix} -R_{1i} & * & * \\ -S_i^T & -R_{1i} & * \\ R_{1i}+S_i^T & R_{1i}+S_i & -2R_{1i}-S_i-S_i^T \end{bmatrix},$$
$$\phi_\tau^i(t) = \begin{bmatrix} \xi(t) \\ \xi(t-\tau_M) \\ \xi(t-\tau^i(t)) \end{bmatrix}, \quad \tau^i(t) \in (0, \tau_M],$$
$$\Pi_{2i} = \begin{bmatrix} -R_{2i} & * & * \\ -W_i^T & -R_{2i} & * \\ R_{2i}+W_i^T & R_{2i}+W_i & -2R_{2i}-W_i-W_i^T \end{bmatrix},$$
$$\phi_d^i(t) = \begin{bmatrix} \xi(t) \\ \xi(t-d_M) \\ \xi(t-d^i(t)) \end{bmatrix}, \quad d^i(t) \in (0, d_M],$$

and

$$\begin{bmatrix} R_{1i} & * \\ S_i^T & R_{1i} \end{bmatrix} \ge 0, \begin{bmatrix} R_{2i} & * \\ W_i^T & R_{2i} \end{bmatrix} \ge 0 \tag{22}$$

In addition, the following equation holds for any constant matrix X from (13) that

$$\begin{aligned} &\mathbb{E}\{2(\xi^T(t)X + \dot{\xi}^T(t)X)(\check{A}\xi(t) + (1-\alpha(t))\textstyle\sum_{i=1}^{N}\check{C}_i\xi(t-\tau^i(t)) \\ &+ (1-\alpha(t))\textstyle\sum_{i=1}^{N}\check{D}_i\bar{e}_{\bar{y}}^i(t) - (1-\alpha(t))\sum_{i=1}^{N}\check{D}_i\bar{\varphi}(y_i(t-\tau^i(t))) \\ &\quad + \alpha(t)\textstyle\sum_{i=1}^{N}\check{D}_i\bar{f}(y_i(t-d^i(t))) - \dot{\xi}(t))\} = 0 \end{aligned} \tag{23}$$

The constraint of nonlinear function in (8) can be rewritten as

$$\varepsilon_i\xi^T(t-\tau^i(t))\hat{C}_i^T\hat{C}_i\xi(t-\tau^i(t)) \ge \bar{\varphi}^T(y_i(t-\tau^i(t)))\bar{\varphi}(y_i(t-\tau^i(t))) \tag{24}$$

where $t \in \mathscr{I}_l^{m_i}$, $\hat{C}_i = [\bar{C}_i \ \ 0]$, $i \in \mathscr{V}$.

It can be obtained from the event-triggered conditions in (9) that for all $t \in \mathscr{I}_l^{m_i}$,

$$\begin{aligned} &\sigma_i[\hat{C}_i\xi(t-\tau^i(t)) - \bar{\varphi}(y_i(t-\tau^i(t))) + \bar{e}_{\bar{y}}^i(t)]^T\bar{\Omega} \\ &\times [\hat{C}_i\xi(t-\tau^i(t)) - \bar{\varphi}(y_i(t-\tau^i(t))) + \bar{e}_{\bar{y}}^i(t)] > (\bar{e}_{\bar{y}}^i(t))^T\bar{\Omega}\bar{e}_{\bar{y}}^i(t) \end{aligned} \tag{25}$$

where $\bar{\Omega} = \text{diag}_N\{\Omega_i\}$, $i \in \mathscr{V}$.

From the Assumption 2, the cyber-attacks signals satisfy the following inequality

$$\begin{aligned} &\bar{\alpha}\xi^T(t-d^i(t))\hat{C}_i^T\bar{F}_i^T\bar{F}_i\hat{C}_i\xi(t-d^i(t)) \\ &- \bar{\alpha}\bar{f}^T(y_i(t-d^i(t)))\bar{f}(y_i(t-d^i(t))) \ge 0 \end{aligned} \tag{26}$$

Considering the constraints of sensor saturations (24), distributed event-triggered scheme (25) and cyber-attacks signals (26) simultaneously, we can get the following inequality:

$$[\psi^i(t)]^T \begin{bmatrix} \varepsilon_i\hat{C}_i^T\hat{C}_i + \sigma_i\hat{C}_i^T\bar{\Omega}\hat{C}_i & * & * & * & \\ 0 & \bar{\alpha}\hat{C}_i^T\bar{F}_i^T\bar{F}_i\hat{C}_i & * & * & \\ \sigma_i\bar{\Omega}\hat{C}_i & 0 & (\sigma_i-1)\bar{\Omega} & * & * \\ 0 & 0 & 0 & -\bar{\alpha}I & * \\ -\sigma_i\bar{\Omega}\hat{C}_i & 0 & -\sigma_i\bar{\Omega} & 0 & \sigma_i\bar{\Omega}-I \end{bmatrix} [\psi^i(t)] > 0 \tag{27}$$

where $\psi^i(t) = \left[\xi^T(t-\tau^i(t)) \ \ \xi^T(t-d^i(t)) \ \ (\bar{e}_{\bar{y}}^i(t))^T \ \ \bar{f}^T(y_i(t-d^i(t))) \ \ \bar{\varphi}^T(y_i(t-\tau^i(t)))\right]^T$.

By combining (18)–(21), (23) and (27), we can get $\dot{V}(t) \le \eta^T(t)\widetilde{\Phi}\eta(t)$, where

$$\eta(t) = \text{col}_{5N+4}\{\xi(t), \dot{\xi}(t), \ \xi(t-\tau_M), \ \text{col}_N\{\xi(t-\tau^i(t))\}, \ \xi(t-d_M), \text{col}_N\{\xi(t-d^i(t))\}, \ \text{col}_N\{\bar{e}_{\bar{y}}^i(t)\}, \text{col}_N\{\bar{f}(y_i(t-\tau^i(t)))\}, \text{col}_N\{\bar{\varphi}(y_i(t - \tau^i(t)))\}\}$$

and $\widetilde{\Phi}$ is made up by Φ defined in (17) except its last row and column. Then, it is clear that $\widetilde{\Phi} < 0$ and $\dot{V}(t) \le -\kappa\xi^T(t)\xi(t) < 0$ for $\xi(t) \neq 0$, where $\kappa = \lambda_{\min}(-\widetilde{\Phi})$. Therefore, the filtering error system (13) is asymptotically stable.

On the other hand, introduce an integral term $J \triangleq \int_0^\infty \mathscr{J}(t)dt$ with $\mathscr{J}(t) = \frac{1}{N}\mathbb{E}\{\tilde{z}^T(t)\tilde{z}(t)\} - \gamma^2\mathbb{E}\{\omega^T(t)\omega(t)\}$ in the case of nonzero system disturbance, i.e. $\omega(t) \in \mathscr{L}[0, \infty)$. Then, $\dot{V}(t) + \mathscr{J}(t) < \zeta^T(t)\Psi\zeta(t)$ can be obtained by applying Schur complement, where $\zeta(t) = col_{5N+5}\{\eta(t), \omega(t)\}$. Therefore, (17) is a sufficient condition for

$$\dot{V}(t) + \mathscr{J}(t) < 0, \quad t \in \mathscr{I}_l^{m_i} \tag{28}$$

Similar to the Proof in Ref. [34], by integrating both sides of (28) from 0 to t, it can be derived that

$$V(t) - V(0) + \int_0^t \mathscr{J}(s)ds < 0 \tag{29}$$

Letting $t \to \infty$ and under zero-initial condition, it can be obtained from (29) that

$$\int_0^\infty \mathscr{J}(t)dt < -V(t)\Big|_{t=\infty} \tag{30}$$

which implies that $J = \frac{1}{N}\mathbb{E}\{\int_0^\infty \tilde{z}^T(t)\tilde{z}(t)dt\} - \gamma^2\mathbb{E}\{\int_0^\infty \omega^T(t)\omega(t)dt\} < 0$ can be guaranteed under zero-initial condition, i.e.

$$\frac{1}{N}\sum_{i=1}^{N}\mathbb{E}\left\{\int_0^\infty \tilde{z}_i^T(t)\tilde{z}_i(t)dt\right\} < \gamma^2\mathbb{E}\left\{\int_0^\infty \omega^T(t)\omega(t)dt\right\} \tag{31}$$

Therefore, the predefined constraint of H_∞ performance (15) is satisfied.

This completes the Proof. □

Theorem 1 presents sufficient conditions for the asymptotically stability of filtering error system (13). Based on Theorem 1, Theorem 2 is dedicated to designing filters in the form of (3).

Theorem 2. *For given the upper bound of time-delays τ_M, d_M, expectation $\bar{\alpha}$, scalar γ, the restraint coefficients of sensor saturations ε_i, trigger threshold parameters σ_i, matrices F_i, the filtering error system (13) subject to (8), (9) and (14) is asymptotically stable with H_∞ performance defined in (15), if there exist real matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_{1i} > 0$, $R_{2i} > 0$, $\Omega_i > 0$, S_i, W_i, X_1, X_3 and diagonal matrices X_2, $\hat{D}$, $\hat{K}_i$, $\bar{E}$, $\bar{G}_i$ with appropriate dimensions such that for $i \in \mathscr{V}$,* the following LMIs hold:

$$\hat{\Psi} = \begin{bmatrix} \hat{\Phi} & * \\ \Xi & \Lambda \end{bmatrix} < 0, \begin{bmatrix} R_{1i} & * \\ S_i^T & R_{1i} \end{bmatrix} \ge 0, \begin{bmatrix} R_{2i} & * \\ W_i^T & R_{2i} \end{bmatrix} \ge 0 \tag{32}$$

where

$$\hat{\Phi} = \begin{bmatrix} \hat{\Phi}_1 & * \\ \hat{\Phi}_2 & \Phi_3 \end{bmatrix},$$

with

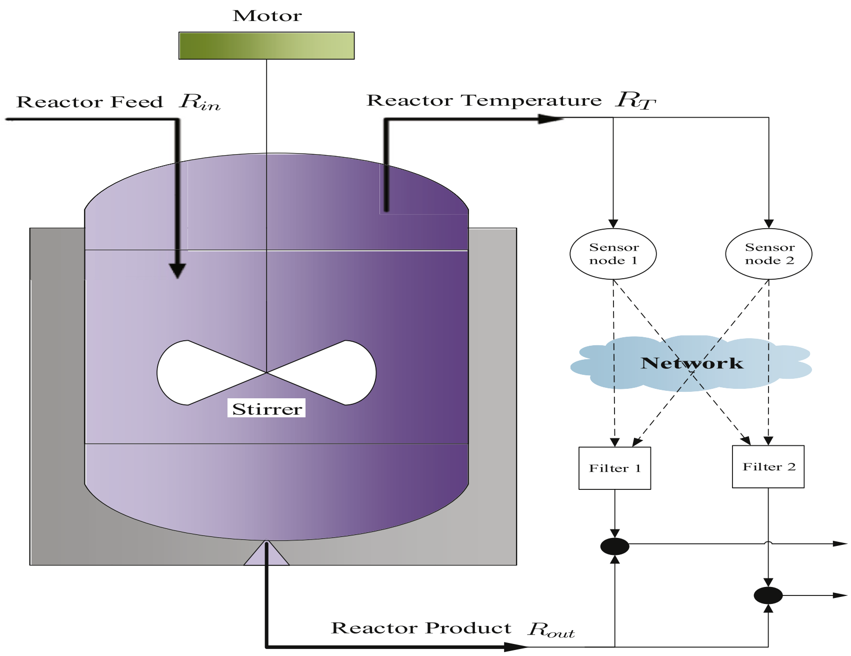

Fig. 2. A continuous stirred tank reactor with two sensor nodes.

$$\begin{aligned}
\hat{\Phi}_1 &= \begin{bmatrix} \hat{\Phi}_{11} & * & * \\ P - X^T + \Upsilon_A & \hat{\Phi}_{12} & * \\ -\sum_{i=1}^{N} S_i^T & 0 & -Q_1 - \sum_{i=1}^{N} R_{1i} \end{bmatrix}, \\
\hat{\Phi}_{11} &= Q_1 + Q_2 - \sum_{i=1}^{N} R_{1i} - \sum_{i=1}^{N} R_{2i} - \Upsilon_A - \Upsilon_A^T, \\
\hat{\Phi}_{12} &= \tau_M^2 \sum_{i=1}^{N} R_{1i} + d_M^2 \sum_{i=1}^{N} R_{2i} - X - X^T,
\end{aligned}$$

$$\hat{\Phi}_2 = \begin{bmatrix} \mathrm{col}_N\{R_{1i} + S_i^T + \bar{\alpha}_1 \Upsilon_{C_i}^T\} & \mathrm{col}_N\{\bar{\alpha}_1 \Upsilon_{C_i}^T\} & \mathrm{col}_N\{R_{1i} + S_i\} \\ -\sum_{i=1}^{N} W_i^T & 0 & 0 \\ \mathrm{col}_N\{R_{2i} + W_i^T\} & 0 & 0 \\ \mathrm{col}_N\{\bar{\alpha}_1 \Upsilon_{D_i}^T\} & \mathrm{col}_N\{\bar{\alpha}_1 \Upsilon_{D_i}^T\} & 0 \\ \mathrm{col}_N\{\bar{\alpha} \Upsilon_{D_i}^T\} & \mathrm{col}_N\{\bar{\alpha} \Upsilon_{D_i}^T\} & 0 \\ \mathrm{col}_N\{-\bar{\alpha}_1 \Upsilon_{D_i}^T\} & \mathrm{col}_N\{-\bar{\alpha}_1 \Upsilon_{D_i}^T\} & 0 \\ \Upsilon_B^T & \Upsilon_B^T & 0 \end{bmatrix},$$

$$\begin{aligned}
X &= \begin{bmatrix} X_1 & X_2 \\ X_3 & X_2 \end{bmatrix}, \quad \Upsilon_A = \begin{bmatrix} X_1 A & \hat{D} \\ X_3 A & \hat{D} \end{bmatrix}, \quad \Upsilon_B = \begin{bmatrix} X_1 B \\ X_3 B \end{bmatrix}, \\
\Upsilon_{C_i} &= \begin{bmatrix} \hat{K}_i(\mathscr{W} \otimes I)\tilde{C} & 0 \\ \hat{K}_i(\mathscr{W} \otimes I)\tilde{C} & 0 \end{bmatrix}, \quad \Upsilon_{D_i} = \begin{bmatrix} \hat{K}_i(\mathscr{W} \otimes I) \\ \hat{K}_i(\mathscr{W} \otimes I) \end{bmatrix}.
\end{aligned}$$

Other symbols have been defined in Theorem 1.

Moreover, the filter parameters in (3) *are given as follows*:

$$\bar{D} = X_2^{-1}\hat{D}, \quad \bar{K}_i = X_2^{-1}\hat{K}_i, \quad \bar{E} \quad and \quad \bar{G}_i, \quad i \in \mathscr{V} \tag{33}$$

where $\bar{D} = \mathrm{diag}_N\{D_i\}$, $\bar{K}_i = \mathrm{diag}_N^i\{K_i\}$, $\bar{E} = \mathrm{diag}_N\{E_i\}$, $\bar{G}_i = \mathrm{diag}_N^i\{G_i\}$.

Proof. It can be derived from $\Psi < 0$ in (17) that

$$\Phi_{12} = \tau_M^2 \sum_{i=1}^{N} R_{1i} + d_M^2 \sum_{i=1}^{N} R_{2i} - X - X^T < 0 \tag{34}$$

Then, by combining $R_{1i} > 0$, $R_{2i} > 0$ and (34), $X > 0$ is obtained.

Furthermore, define $X = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_2 \end{bmatrix}$ with nonsingular matrix X_2 and apply the expression into all the X in Theorem 1. In addition, replace the $\tilde{A}$, $\tilde{B}$, $\check{C}_i$, $\check{D}_i$, $\tilde{E}$, $\tilde{F}_i$, $\check{G}_i$ in Theorem 1 with the definitions in (13). Let the definitions of $\hat{C}_i$ in (24) and $\bar{\Omega}$ in (25) substitute into the $\hat{C}_i$ and $\bar{\Omega}$ in Theorem 1, respectively. As a consequence, Theorem 2 is directly yielded from Theorem 1 by substituting $X_2\bar{D}$ and $X_2\bar{K}_i$ with $\hat{D}$ and $\hat{K}_i$, respectively.

This completes the Proof. □

4. Simulation examples

In this section, in order to confirm the usefulness of the distributed H_∞ filters (3) proposed in this paper, two examples will be given in the following.

Example 1. Consider a continuous stirred tank reactor [8] diagrammed in Fig. 2, where the stirrer driven by a motor can stir the reactor feed R_{in} and then the reactor product R_{out} can be obtained at the bottom of reactor. The concentration of reactor product plays an important role in getting the desired product, which should be monitored all the time. Although the traditional chemical approaches to measuring the concentration of the desired product can achieve the monitoring objective, it will increase the cost by using such direct measurements. Signal processing approaches such as the distributed event-triggered H_∞ filter design method proposed in this paper can be chosen as a substitution, where the reactor temperature R_T is measured by two sensors to estimate the concentration of reactor product as shown in Fig. 2.

At time instant t, assume $x_1(t)$ and $x_2(t)$ denote the concentration of the educt from R_{in} and the concentration of product R_{out}, respectively. $x_3(t)$ stands for the reactor temperature R_T [8]. Then, near the operating point, the linearized state-space model of the continuous stirred tank reactor can be represented as the form (1) with

$$A = \begin{bmatrix} -0.9388 & 0 & 0.0459 \\ 0.6250 & -0.9388 & -0.0125 \\ -0.9355 & 2.4449 & -0.8894 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad L = [0 \;\; 1 \;\; 0] \tag{35}$$

and $x(t) = [x_1(t) \;\; x_2(t) \;\; x_3(t)]^T$, $x_0 = [0 \;\; -1 \;\; 0.8]^T$, $\omega(t) = \begin{cases} 0.3\sin(t)e^{-0.01t}, & 0 < t < 3; \\ 0, & \mathit{otherwise}. \end{cases}$

The adjacency matrix of the two sensors is given as $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. The

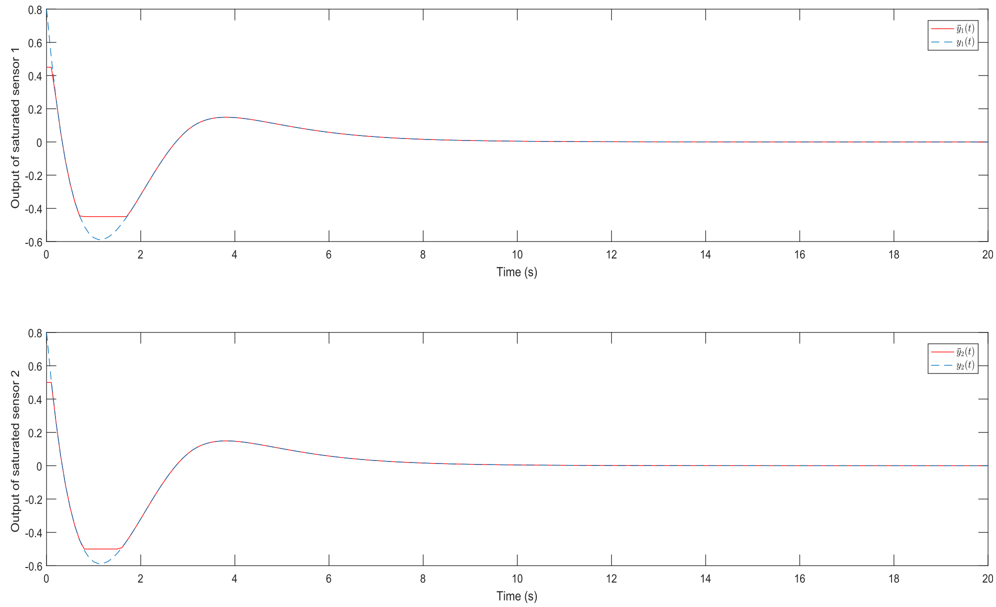

Fig. 3. Output of saturated sensors in Example 1.

parameter matrices of (2) are chosen as $C_1 = C_2 = [0 \;\; 0 \;\; 1]$, take the nonlinear signals of cyber-attacks as $f(y_1(t)) = -\tanh(0.15y_1(t))$, $f(y_2(t)) = -\tanh(0.08y_2(t))$. Then from Assumption 2, the constraint condition (14) can be satisfied when choosing the upper bounds $F_1 = 0.15$ and $F_2 = 0.08$.

Further more, given upper bound of time-delays $\tau_M = 0.6$s, $d_M = 0.4$s, threshold values $\rho_1 = 0.45$, $\rho_2 = 0.5$, trigger threshold parameters $\sigma_1 = 0.4$, $\sigma_2 = 0.5$ and scalar $\gamma = 2$. According to the in equation (6), the restraint coefficients of sensor saturations are chosen as $\varepsilon_1 = \varepsilon_2 = 0.2$. Specify the expectation of occurring cyber-attacks as $\bar{\alpha} = 0.03$. Then, under sampling period $h = 0.1$s, the following parameters of distributed H_∞ filters and the weighting matrices of distributed event-trigger scheme are derived from Theorem 2.

$$D_1 = \begin{bmatrix} -1.0690 & -0.0018 & 0.0225 \\ 0.2193 & -0.7705 & -0.0963 \\ -0.1894 & 0.7697 & -0.9262 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -1.0830 & -0.0261 & 0.0413 \\ 0.6267 & -0.3288 & -0.1510 \\ -0.2988 & 0.8101 & -0.6358 \end{bmatrix}$$

$$K_1 = \begin{bmatrix} -0.0019 \\ -0.0022 \\ -0.0423 \end{bmatrix}, \; K_2 = \begin{bmatrix} -0.0016 \\ 0.0024 \\ -0.0184 \end{bmatrix},$$

$$E_1 = [-0.0213 \;\; -1.2144 \;\; 0.1770]$$

$$E_2 = [0.4763 \;\; -1.0342 \;\; 0.0662], \; G_1 = 0.0793,$$

$$G_2 = 0.0416, \; \Omega_1 = 0.0679, \; \Omega_2 = 0.0375$$

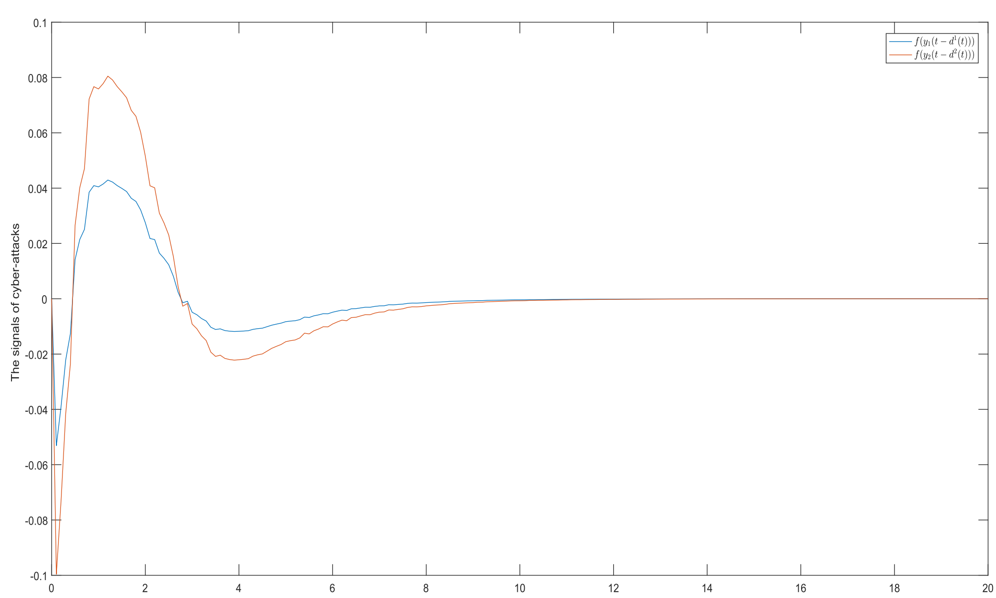

Fig. 4. The cyber-attacks signals in Example 1.

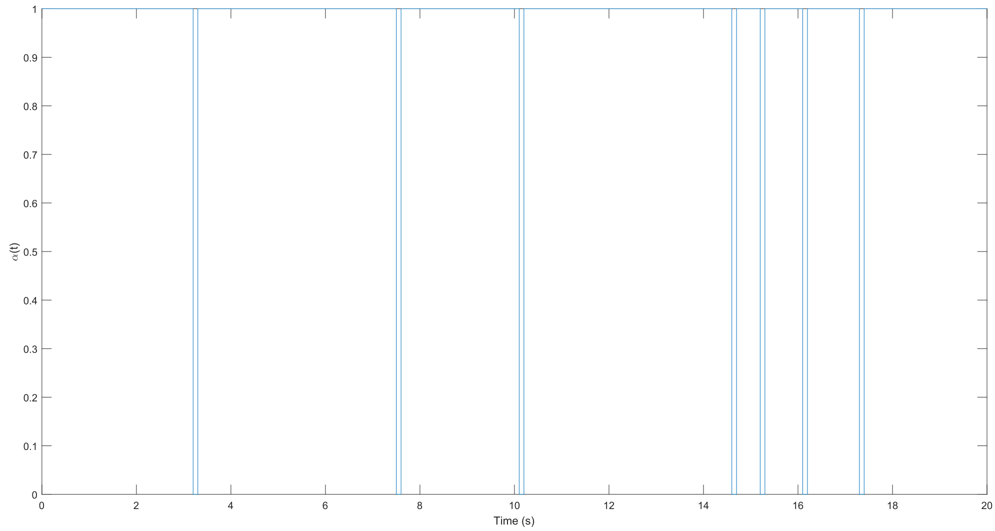

Fig. 5. Bernoulli distribution variable $\alpha(t)$ in Example 1.

In addition, the following simulation results from Figs. 3–7 can be obtained at the same time. Fig. 3 shows the output of saturated sensors. The signals of cyber-attacks and the Bernoulli distribution variable $\alpha(t)$ are shown in Figs. 4 and 5, respectively. The release instants and intervals of sensors are presented in Fig. 6. It is clearly that the transmitted data packets are reduced under the distributed event-triggered scheme, which relieves the load of network bandwidth effectively. The filtering errors of filters are depicted in Fig. 7, which shows that the concentration of the desired reactor product R_{out} is well estimated by the distributed H_∞ filters designed in this paper.

Example 2. Consider a continuous linear system as

$$\begin{cases} \dot{x}(t) = \begin{bmatrix} -0.1 & 0.4 \\ 0 & -0.5 \end{bmatrix} x(t) + \begin{bmatrix} 0.2 \\ 0.5 \end{bmatrix} \omega(t) \\ \quad z(t) = [0.1 \quad 0.1] x(t) \end{cases} \tag{36}$$

where $x_0 = \begin{bmatrix} -1.5 \\ 0.8 \end{bmatrix}$ and $\omega(t) = \begin{cases} 1.2\sin(2t), & 0 < t < 10; \\ 0, & otherwise. \end{cases}$ Given a sensor network topology with three sensor nodes shown in Fig. 8, where each sensor node has default self-loop. The constant matrices of sensors are $C_1 = [3 \quad 0]$, $C_2 = [1 \quad -2]$, $C_3 = [-1 \quad 1.2]$. The cyber-attacks are modeled as nonlinear functions, namely, $f(y_1(t)) = -\tanh(0.12y_1(t))$, $f(y_2(t)) = -\tanh(0.25y_2(t))$, $f(y_3(t)) = -\tanh(0.15y_3(t))$, which are subject to Assumption 2 with upper bounds $F_1 = 0.12$, $F_2 = 0.25$ and $F_3 = 0.15$.

In light of Theorem 2, where the parameters choose $\tau_M = 0.5$, $d_M = 0.3$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0.2$, $\sigma_1 = 0.05$, $\sigma_2 = 0.1$, $\sigma_3 = 0.25$, $\gamma = 2$, $\bar{\alpha} = 0.08$, the parameters of distributed H_∞ filters and distributed event-triggered scheme can be obtained under sampling period $h = 0.01$s as follows:

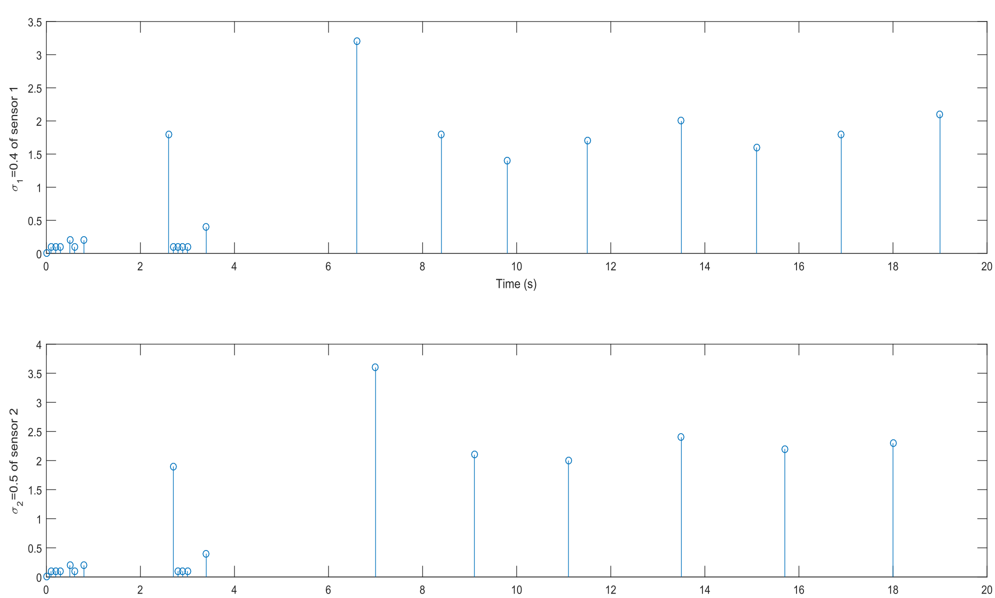

Fig. 6. Release instants and intervals of all the sensors in Example 1.

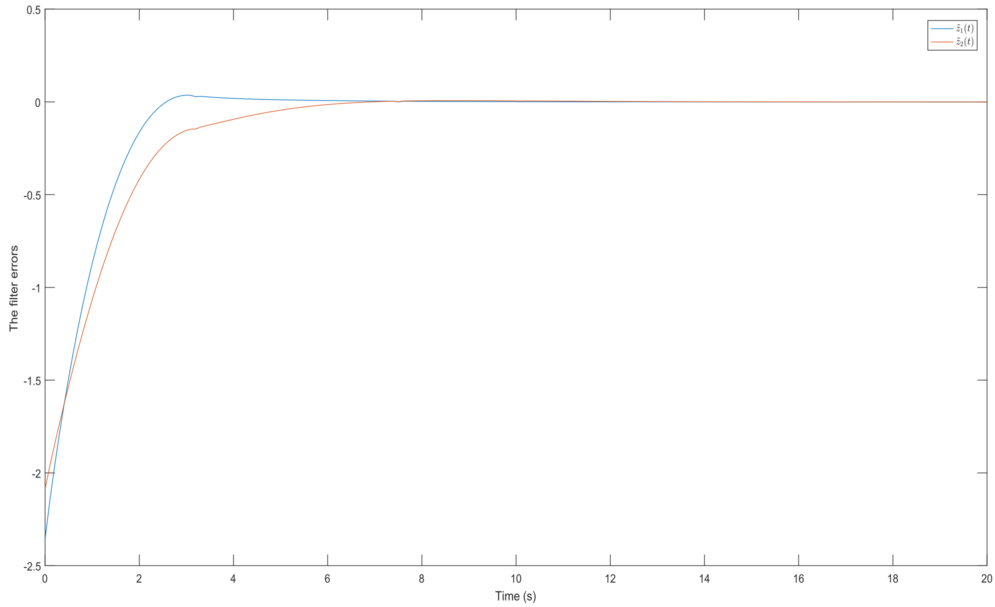

Fig. 7. Filtering errors in Example 1.

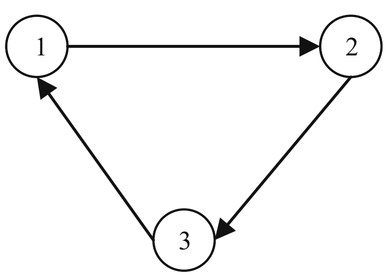

Fig. 8. The topology of the sensor networks in Example 2.

$$D_1 = \begin{bmatrix} -0.9251 & 0.1017 \\ -0.2249 & -0.8484 \end{bmatrix},\ D_2 = \begin{bmatrix} -0.8956 & 0.0291 \\ -0.0839 & -0.9637 \end{bmatrix},$$
$$D_3 = \begin{bmatrix} -0.9000 & 0.1065 \\ -0.0058 & -0.8637 \end{bmatrix}$$
$$K_1 = \begin{bmatrix} -0.0284 \\ -0.0109 \end{bmatrix},\ K_2 = \begin{bmatrix} -0.0171 \\ 0.0109 \end{bmatrix},\ K_3 = \begin{bmatrix} 0.0047 \\ -0.0038 \end{bmatrix}$$
$$E_1 = [-0.0763 \quad -0.0850],\ E_2 = [-0.1025 \quad -0.0866],$$
$$E_3 = [-0.1010 \quad -0.0761]$$
$$G_1 = 0.0096,\ G_2 = 0.0027,\ G_3 = -0.0007,$$
$$\Omega_1 = 2.0923,\ \Omega_2 = 2.0908,\ \Omega_3 = 1.8196$$

Fig. 9 depicts the output of sensors, where the phenomena of sensor saturations can be observed. The signals of cyber-attacks and the Bernoulli distribution variable $\alpha(t)$ are shown in Figs. 10 and 11,

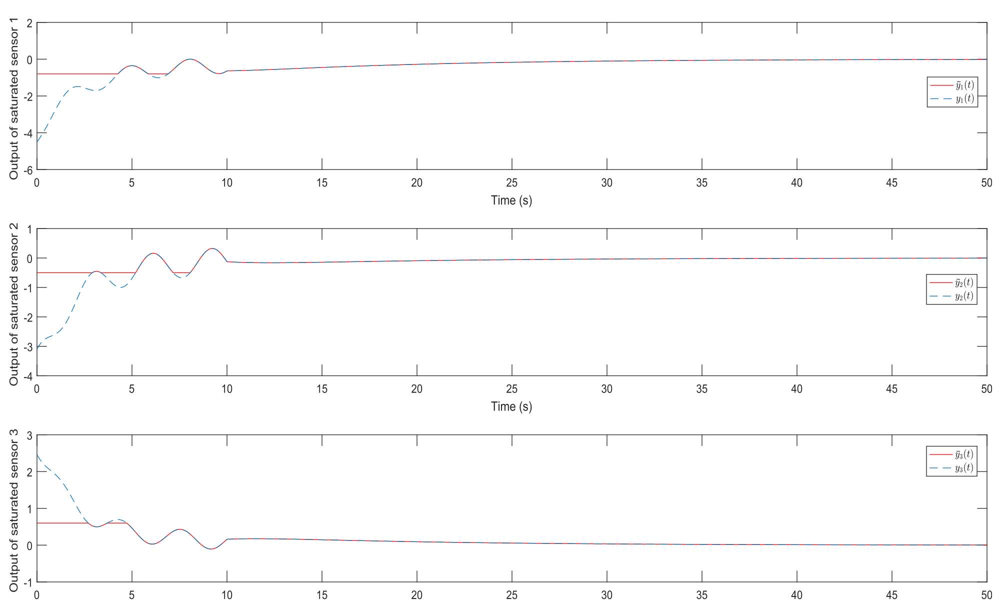

Fig. 9. Output of saturated sensors in Example 2.

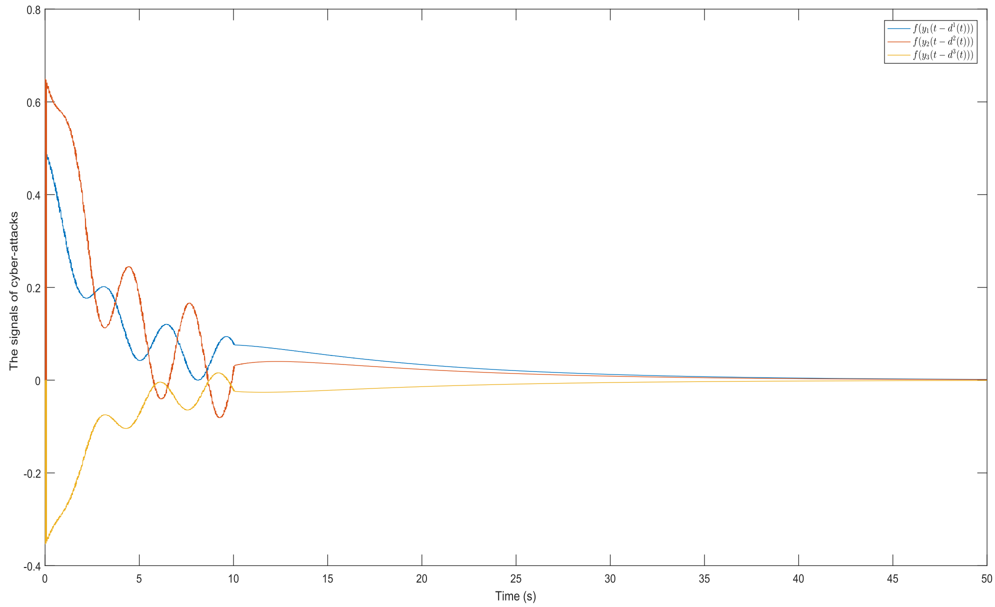

Fig. 10. The cyber-attacks signals in Example 2.

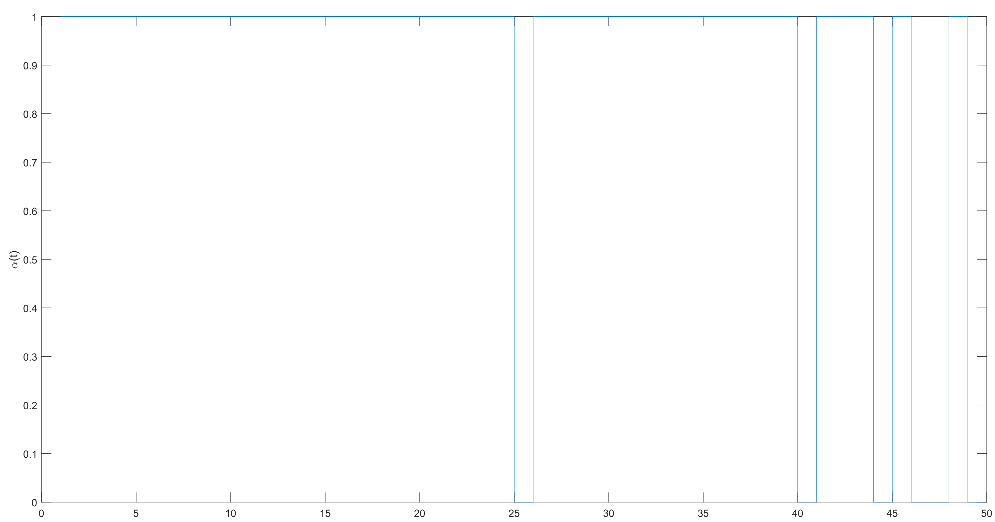

Fig. 11. Bernoulli distribution variable $\alpha(t)$ in Example 2.

respectively. The release instants and intervals of three sensors are presented in Fig. 12. Moreover, the comparisons of trigger times for data transmission under different schemes are shown in Table 1, where the simulation is performed in time interval [0, 50s] with sampling period 0.01s. From Table 1, it can be seen that the trigger times of each node under time-triggered scheme are all 5000, while under distributed event-triggered scheme the trigger times of *node* 1, *node* 2, *node* 3 are 147, 171 and 85, respectively. Therefore, the number of broadcast data can be significantly reduced by adopting the distributed event-triggered scheme. Fig. 13 shows the filtering errors for sensor networks. Obviously, the simulation results above verify the usefulness of the designed distributed H_∞ filters when considering sensor saturations, distributed event-triggered scheme and cyber-attacks.

5. Conclusions

This paper investigates the issue of distributed event-triggered H_∞ filtering over sensor networks with sensor saturations and cyber-attacks. The distributed event-triggered scheme is applied in geographically distributed sensor nodes to reduce the occupancy of limited network bandwidth and then load of network is alleviated. In view of the effects of both sensor saturations and cyber-attacks, a novel model of distributed event-triggered H_∞ filtering system is constructed. On basis of the new model, sufficient conditions guaranteeing the system asymptotically stable are obtained by means of Lyapunov stability theory. Moreover, the explicit expressions of distributed H_∞ filters and the weighting matrices of distributed event-triggered scheme are achieved by solving a set of LMIs. Finally, two examples are provided to

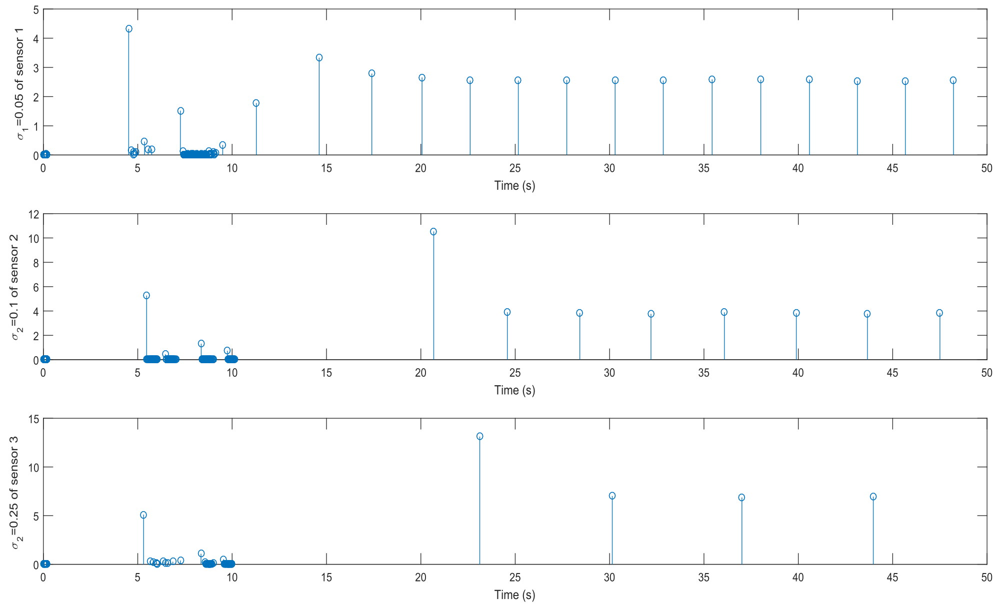

Fig. 12. Release instants and intervals of all the sensors in Example 2.

Table 1
Comparisons of trigger times for data transmission under different schemes.

	node 1	node 2	node 3
time − triggered scheme	5000	5000	5000
distributed event − triggered scheme	147	171	85

demonstrate the usefulness of the distributed H_∞ filters designed in this paper. In our future work, the hybrid triggered scheme will be investigated in the analysis and synthesis of distributed systems with sensor saturations and cyber-attacks.

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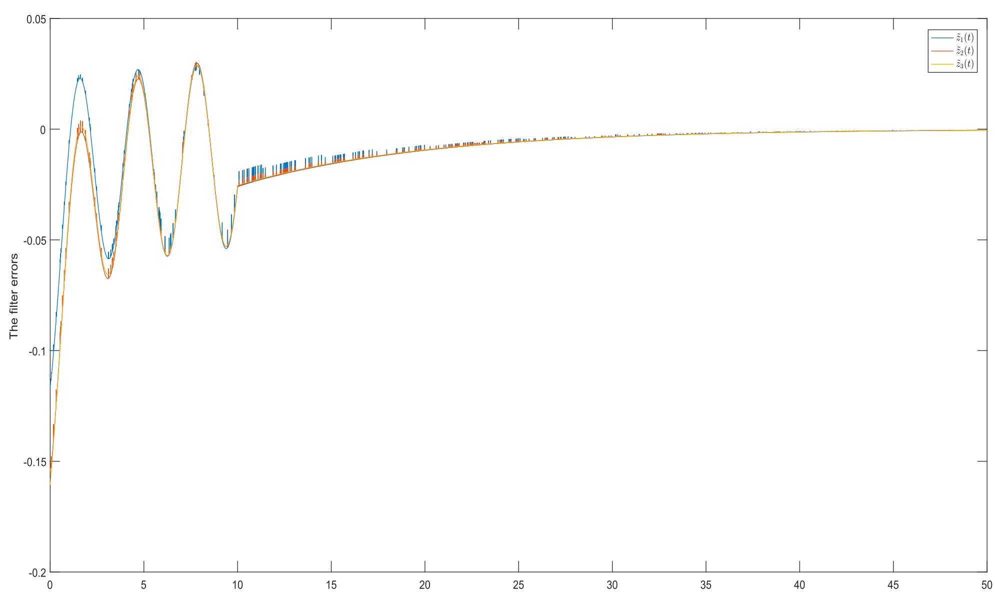

Fig. 13. Filtering errors in Example 2.

References

[1] Su H, Li Z, Ye Y. Event-triggered kalman-consensus filter for two-target tracking sensor networks. ISA Trans 2017;71:103–11.
[2] Zhao Y, Guo G, Ding L. Guaranteed cost control of mobile sensor networks with markov switching topologies. ISA Trans 2015;58:206–13.
[3] Yuan J, Zhang J, Ding S, Dong X. Cooperative localization for disconnected sensor networks and a mobile robot in friendly environments. Inf Fusion 2017;37:22–36.
[4] Shi K, Chen H, Lin Y. Probabilistic coverage based sensor scheduling for target tracking sensor networks. Inf Sci 2015;292:95–110.
[5] Zhang W, Shi L. Sequential fusion estimation for clustered sensor networks. Automatica 2018;89:358–63.
[6] Liang J, Wang Z, Hayat T, Alsaedi A. Distributed H_∞ state estimation for stochastic delayed 2-D systems with randomly varying nonlinearities over saturated sensor networks. Inf Sci 2016;370–371:708–24.
[7] Shen M, Ye D, Wang QG. Event-triggered H_∞ filtering of Markov jump systems with general transition probabilities. Inf Sci 2017;418–419:635–51.
[8] Ge X, Han QL. Distributed sampled-data asynchronous H_∞ filtering of markovian jump linear systems over sensor networks. Signal Process 2016;127:86–99.
[9] Gu Z, Huan Z, Yue D, Yang F. Event-triggered dynamic output feedback control for networked control systems with probabilistic nonlinearities. Inf Sci 2018;457–458:99–112.
[10] Zhang X, Han QL. Event-based H_∞ filtering for sampled-data systems. Automatica 2015;51:55–69.
[11] Zhang X, Han QL, Zhang B. An overview and deep investigation on sampled-data-based event-triggered control and filtering for networked systems. IEEE Trans Ind Inform 2017;13(1):4–16.
[12] Zhang D, Shi P, Wang QG, Yu L. Analysis and synthesis of networked control systems: a survey of recent advances and challenges. ISA Trans 2017;66:376–92.
[13] Zhang D, Xu Z, Wang QG, Zhao YB. Leader-follower H_∞ consensus of linear multi-agent systems with aperiodic sampling and switching connected topologies. ISA Trans 2017;68:150–9.
[14] Ge X, Han QL, Ding D, Zhang XM, Ning B. A survey on recent advances in distributed sampled-data cooperative control of multi-agent systems. Neurocomputing 2018;275:1684–701.
[15] Hu S, Yue D. Event-triggered control design of linear networked systems with quantizations. ISA Trans 2012;51:153–62.
[16] Yue D, Tian E, Han QL. A delay system method for designing event-triggered controllers of networked control systems. IEEE Trans Automat Contr 2013;58(2):475–81.
[17] Zha L, Fang J, Liu J, Tian E. Event-based finite-time state estimation for markovian jump systems with quantizations and randomly occurring nonlinear perturbations. ISA Trans 2017;66:77–85.
[18] Zha L, Tian E, Xie X, Gu Z, Cao J. Decentralized event-triggered H_∞ control for neural networks subject to cyber-attacks. Inf Sci 2018;457–458:141–55.
[19] Gu Z, Yue D, Tian E. On designing of an adaptive event-triggered communication scheme for nonlinear networked interconnected control systems. Inf Sci 2018;422:257–70.
[20] Liu J, Zha L, Cao J, Fei S. Hybrid-driven-based stabilisation for networked control systems. IET Contr Theory & Appl 2016;10(17):2279–85.
[21] Liu J, Wei L, Xie X, Tian E, Fei S. Quantized stabilization for T-S fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks. IEEE Trans Fuzzy Syst 2018. https://doi.org/10.1109/TFUZZ.2018.2849702.
[22] Ding L, Guo G. Distributed event-triggered H_∞ consensus filtering in sensor networks. Signal Process 2015;108:365–75.
[23] Liu Q, Wang Z, He X, Zhou DH. Event-Based recursive distributed filtering over wireless sensor networks. IEEE Trans Automat Contr 2015;60(9):2470–5.
[24] Yan H, Qian F, Yang F, Shi H. H_∞ filtering for nonlinear networked systems with randomly occurring distributed delays, missing measurements and sensor saturation. Inf Sci 2016;370–371:772–82.
[25] Shen M, Ye D, Wang QG. Mode-dependent filter design for markov jump systems with sensor nonlinearities in finite frequency domain. Signal Process 2017;134:1–8.
[26] Sheng S, Zhang X. H_∞ filtering for T-S fuzzy complex networks subject to sensor saturation via delayed information. IET Contr Theory & Appl 2017;11(14):2370–82.
[27] Ding D, Han QL, Xiang Y, Ge X, Zhang X. A survey on security control and attack detection for industrial cyber-physical systems. Neurocomputing 2018;275:1674–83.
[28] Zhang H, Cheng P, Shi L, Chen J. Optimal denial-of-service attack scheduling with energy constraint. IEEE Trans Automat Contr 2015;60(11):3023–8.
[29] Wang Z, Wang D, Shen B, Alsaadi FE. Centralized security-guaranteed filtering in multirate-sensor fusion under deception attacks. J Franklin Inst 2018;355:406–20.
[30] Farraj A, Hammad E, Kundur D. On the impact of cyber attacks on sata integrity in storage-based transient stability control. IEEE Trans Ind Inform 2017;13(6):3322–33.
[31] Ding D, Wei G, Zhang S, Liu Y, Alsaadi FE. On scheduling of deception attacks for discrete-time networked systems equipped with attack detectors. Neurocomputing 2017;219:99–106.
[32] Liu J, Tian E, Xie X, Lin H. Distributed event-triggered control for networked control systems with stochastic cyber-attacks. J Franklin Inst 2018. https://doi.org/10.1016/j.jfranklin.2018.01.048.
[33] Sun L, Wang Y, Feng G. Control design for a class of affine nonlinear descriptor systems with actuator saturation. IEEE Trans Automat Contr 2015;60(8):2195–200.
[34] Ge X, Han QL. Distributed event-triggered H_∞ filtering over sensor networks with communication delays. Inf Sci 2015;291:128–42.
[35] Liu J, Zha L, Xie X, Tian E. Resilient observer-based control for networked nonlinear T-S fuzzy systems with hybrid-triggered scheme. Nonlinear Dynam 2018;91(3):2049–61.
[36] Ding D, Wang Z, Wei G, Alsaadi FE. Event-based security control for discrete-time stochastic systems. IET Contr Theory & Appl 2016;10(15):1808–15.
[37] Ding D, Wang Z, Ho DWC, Wei G. Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks. Automatica 2017;78:231–40.
[38] Liu J, Xia J, Tian E, Fei S. Hybrid-driven-based H_∞ filter design for neural networks subject to deception attacks. Appl Math Comput 2018;320:158–74.
[39] Liu J, Wei L, Tian E, Fei S, Cao J. H_∞ filtering for networked systems with hybrid-triggered communication mechanism and stochastic cyber attacks. J Franklin Inst 2017;354:8490–512.
[40] Ao W, Song Y, Wen C. Adaptive cyber-physical system attack detection and reconstruction with application to power systems. IET Contr Theory & Appl 2016;10(12):1458–68.