Neurocomputing 273 (2018) 1–8

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Event-triggered non-fragile state estimation for delayed neural networks with randomly occurring sensor nonlinearity

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ARTICLE INFO

Article history:
Received 29 April 2017
Revised 15 July 2017
Accepted 3 August 2017
Available online 18 August 2017

Communicated by Hongli Dong

Keywords:
Event-triggered scheme
Neural networks
Sensor nonlinearity
State estimation

ABSTRACT

This paper is concerned with event-triggered non-fragile state estimator design for delayed neural networks subject to randomly occurring sensor nonlinearity. Different from the existing event-triggered scheme, a new event-triggered scheme is designed which is dependent on the incomplete measurement. The adopted event-triggered scheme is introduced between the neural networks and state estimator for the purpose of energy saving. Considering the sensor nonlinearity and using the event-triggered scheme, a new estimation error system is modeled. Based on this model, a sufficient condition is derived to guarantee the asymptotical stability of estimation error system. Furthermore, a desired event-triggered non-fragile estimator is designed by solving a set of linear matrix inequalities. Finally, a numerical example is provided to illustrate the usefulness of the proposed method.

1. Introduction

In the past several decades, neural networks have received considerable attention due to the fact that they have widely applications in signal processing, target tracking, pattern recognition and so on. Many outstanding results have been achieved [1–4]. It is well known that understanding the neuron states is an essential step to apply the neural networks and realize the desired performance in practice. However, it is often difficult to acquire the complete information immediately of all the neuron states, and only a series of observations can we obtain. Therefore, much effort has been devoted to the neuron state estimation problem [5–7].

In many practical systems, there exist many unavoidable factors such as drastic variations of flow rates, pressures, and temperatures. Such harsh environments can make the sensor measurement imperfect, which result in nonlinear characteristic of sensors. The method based on linear measurements can not deal with the sensor nonlinearity effectively. Therefore, how to solve sensor nonlinearity on filter and controller design has been paid much attention [8–14]. For example, in [8], the authors are concerned with the problem of asynchronous $l_2 - l_\infty$ filtering for discrete-time stochastic Markov jump systems with sensor nonlinearity. In [10], the issue of finite state estimation is investigated for coupled Markovian neural networks subject to sensor nonlinearities. The filter design is discussed in [9] for Markov jump systems with incomplete transition probabilities subject to sensor nonlinearities. The H_∞ filtering problem is considered for discrete time-delay systems with quantization and stochastic sensor nonlinearity in [14]. However, in most aforementioned literatures, communication constraints between the system plant and the state estimator is not considered. All the sampled sensor measurements will be transmitted to the estimator leading to an unnecessary transmission of sampled data. In some cases, the network bandwidth is limited, it is of great necessity to make high utilization of the precious communication resources in neural networks with sensor nonlinearity, which is the first motivation of this paper.

In recent years, the event-triggered communication strategy dependent on state or output measurements has attracted considerable attention in control community [15–18]. Different from the time-triggered case where the sampled signals are executed periodically, the sampled data under event-triggered scheme can be transmitted only if the predefined triggering condition is violated. The main advantage of the event-triggered scheme is that it can reduce the transmission amount while maintaining the desired stability and performance criteria. Due to the explicit engineering prospects, event-triggered schemes have been active areas of research [19–24]. In [19], the reliable control design is considered for networked control systems against probabilistic actuator and sensor fault with different failure rates. The event-triggered robust fusion estimation problem is discussed in [20] for uncertain multirate sampled-data systems with stochastic nonlinearities and

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http://dx.doi.org/10.1016/j.neucom.2017.08.011
0925-2312/

the colored measurement noises. The authors in [21] construct a discrete event-triggered communication scheme to save the limited network resource while preserving the desired performance. The problem of fault detection is investigated in [22] for nonlinear discrete-time event-triggered networked systems. In [23], the consensus problem under event-triggered communication scheme is considered for discrete-time heterogeneous multi-agent systems on a directed interconnection topology. The event-triggered multi-objective control problems are considered in [24] for time-varying systems with randomly occurring saturations, stochastic nonlinearities and state multiplicative noise. It is noticed that the majority of the existing event-triggered strategies do not take the sensor nonlinearity into consideration, then, the existing performance and stability results in the published work may not valid in the presence of sensor nonlinearity. Therefore, it is challenging to study event-triggered scheme which takes the sensor nonlinearity into account. This is the second motivation of this paper.

Motivated by the above statement, in this paper, the problem of event-triggered non-fragile state estimation is investigated for delayed neural networks with randomly occurring sensor nonlinearity. The sensor nonlinearity is assumed to occur randomly according to a Bernoulli distributed stochastic variable. For the purpose of energy saving, a event triggered scheme is constructed based on the available sensor measurement. Under the event-triggered scheme, a new state estimation error system is constructed. Sufficient conditions are obtained which can ensure the performance and the stability of the augmented system. Furthermore, the desired state estimator is derived by solving certain linear matrix inequalities. Finally, a simulation example is provided to demonstrate the usefulness of the proposed event-triggered non-fragile state estimator.

The rest of this paper is organized as follows. In Section 2, we introduce the state estimation error system considering sensor nonlinearity and the event-triggered scheme. In Section 3, sufficient conditions are provided to ensure the stability of the augmented system. Moreover, the estimator gain is designed. An illustrative example is given in Section 4. Finally, the conclusion is drawn in Section 5.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidian space, and the set of $n\times m$ real matrices; the superscript T stands for matrix transposition; I is the identity matrix of appropriate dimension; the notation $X>0$ (respectively, $X\ge 0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite); $Prob\{X\}$ denotes probability of event X to occur; $\mathcal{E}$ denotes the expectation operator; for a matrix B and two symmetric matrices A and C, $\left[\begin{smallmatrix} A & * \\ B & C\end{smallmatrix}\right]$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. System description

Consider the following continuous-time delayed neural network with n neurons:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bg(x(t)) + Eh(x(t-d(t))) + Dw(t) \\ y(t) = (1-\alpha(t))Cx(t) + \alpha(t)f(Cx(t)) \\ z(t) = Lx(t) \end{cases} \tag{1}$$

where $x(t)=[x_1(t), x_2(t), \ldots, x_n(t)]^T\in\mathbb{R}^n$ is the state vector, and $y(t)\in\mathbb{R}^m$ is the measurable output, $z(t)\in\mathbb{R}^p$ is the signal to be estimated, $w(t)\in\mathbb{R}^q$ is the noise input belonging to $\mathcal{L}_2[0,\infty)$. $d(t)$ is a time-varying delay satisfying $0\le d(t)\le d_M$, where d_M is a constant. $A=diag\{a_1, a_2, \ldots, a_n\}<0$ is a constant real matrix, $B\in\mathbb{R}^{n\times n}$ and $E\in\mathbb{R}^{n\times n}$ are the interconnection matrices representing coefficients of the neurons. D, C and L are known real constant matrices with appropriate dimensions.

Remark 1. Nowadays, due to the simplicity and low cost, smart sensor technology has become increasingly active development and widely used in various fields of aerospace, aviation, defense, science and technology and industrial and agricultural production etc. It should be noticed that smart sensor in Fig. 1 is embedded to sense, sample, exchange information, transmit useful collected information over a network. Smart sensor in this paper is made up of sensor, sampler, event generator and data memory.

Assumption 1. [25] The neuron activation function $g(x(t)) = [g_1(x_1(t), g_2(x_2(t), \ldots, g_n(x_n(t))]^T$ and $h(x(t-d(t))) = [h_1(x_1(t-d(t), h_2(x_2(t-d(t)), \ldots, h_n(x_n(t-d(t)))]^T$ satisfy for $l=1,2,\ldots,n$, $\forall s_1\neq s_2$

$$\phi_{gl}^- \le \frac{g_l(s_1)-g_l(s_2)}{s_1-s_2} \le \phi_{gl}^+ \tag{2}$$

$$\phi_{hl}^- \le \frac{h_l(s_1)-h_l(s_2)}{s_1-s_2} \le \phi_{hl}^+ \tag{3}$$

where ϕ_{gl}^-, ϕ_{gl}^+, ϕ_{hl}^-, ϕ_{hl}^+ are known constants.

$f(Cx(t))$ is assumed to be continuous with $f(0)=0$, and satisfies the following condition [10]:

$$\|f(C\varsigma_1(t)) - f(C\varsigma_2(t))\|^2 \le \|\mathcal{F}C(\varsigma_1(t)-\varsigma_2(t))\|^2 \tag{4}$$

for all $\varsigma_1(t), \varsigma_2(t)\in\mathbb{R}^n$, in which $\mathcal{F}$ is a constant matrix. The stochastic variable $\alpha(t)\in\{0,1\}$ is a Bernoulli distributed white sequence which accounts for the phenomena of randomly occurring sensor nonlinearity and $Prob\{\alpha(t)=0\}=\bar{\alpha}$.

Remark 2. In this paper, the output measurement model in (1) is introduced to reflect the phenomenon of sensor nonlinearity, which is often encountered due to the harsh environment. A Bernoulli distributed stochastic variable $\alpha(t)$ is used to describe the randomly occurring sensor nonlinearity.

Remark 3. As is known to all, time delays are unavoidable due to the finite switching speed of the amplifiers and the finite signal propagation time. In practice, time delays always result in poor performances or even instability of the system. In view of this, it makes sense to deal with the effect of time delays on the system stability. Many important results have been achieved in the past few years [26,27].

In this paper, we are interested in designing the non-fragile state estimator as follows:

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + Bg(\hat{x}(t)) + Eh(\hat{x}(t-d(t))) + (K+\Delta K)(\bar{y}(t) \\ \qquad - \hat{y}(t)) \\ \hat{y}(t) = C\hat{x}(t) \\ \hat{z}(t) = L\hat{x}(t) \end{cases} \tag{5}$$

where $\hat{x}(t)$, $\hat{y}(t)$ and $\hat{z}(t)$ are the estimations of $x(t)$, $y(t)$ and $z(t)$, respectively. K is the estimator gain to be designed, ΔK denote the structure $HF(t)G$, in which H and G are the given constant matrices, $F(t)$ is an unknown time-varying matrix satisfying $F^T(t)F(t)\le I$.

Remark 4. Considering the complex and changeable environment, the state estimator gains may undergo drafts and fluctuations, then, the realized parameter of the state estimator may be inaccurate [28]. Hence, it is necessary to consider the non-fragile state estimator problem. In this paper, we assume ΔK is the corresponding small uncertainties on state estimator implementation.

In order to achieve high utilization of the communication bandwidth, as is shown in Fig. 1, between the neural networks and the state estimator, we introduce an event-triggered scheme which depends on the available output measurements. The event-triggered condition is predesigned as follows [18]:

$$e_k^T(t)W_1e_k(t) \le \sigma y^T(t_kh+jh)W_2y(t_kh+jh) \tag{6}$$

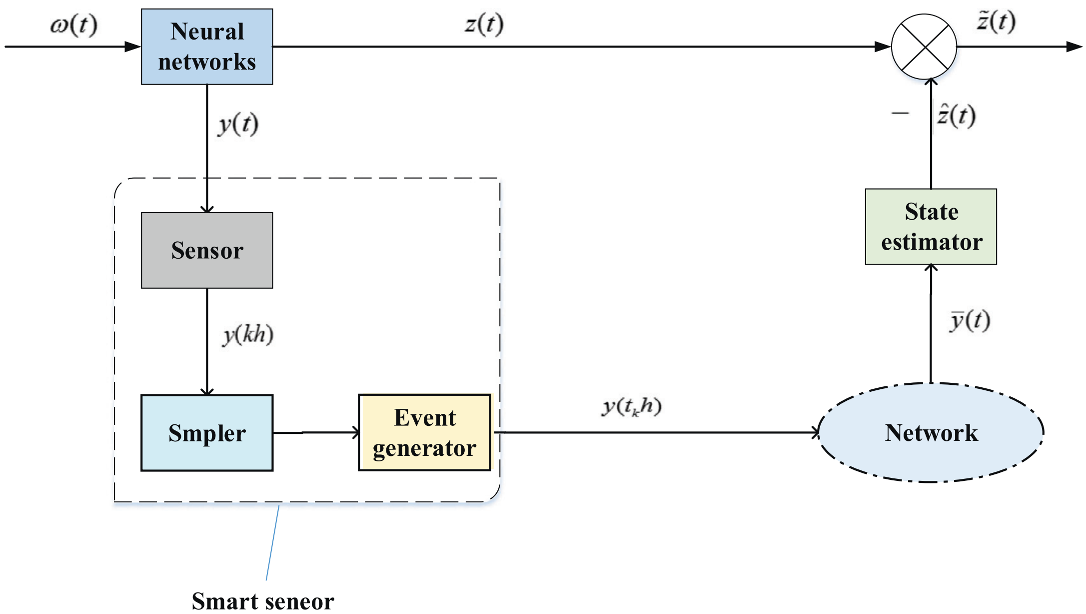

Fig. 1. The structure of event-triggered neural networks.

where $e_k(t) = y(t_k h + jh) - y(t_k h)$, σ is a given positive scalar, $W_1 > 0$ and $W_2 > 0$ are the matrices with appropriate dimension, kh is the sampling instants, $t_k h$ is the sequence of the event-triggering instants. The latest sampled signal $y(t_k h + jh)$ can be triggered as long as the predesigned condition (6) is violated.

Remark 5. Different from the continuous case of the event-triggered scheme, in which one will always face a problem called "Zeno behavior", in this paper, the sensor measures are only sampled at discrete instants $kh(k = 1, 2, \ldots)$, which means the inter-event times are at least h. Therefore, the event-triggered scheme in this paper can avoid this Zeno behavior.

Remark 6. It can be observed that the amount of transmission in the communication channel can be adjusted by setting different triggering parameters of σ in (6). The smaller value of σ, the more data can be sent to the state estimator. When $\sigma = 0$, the event-triggered scheme reduces to time-triggered scheme.

For convenience of analysis, let $d = t_{k+1} - t_k - 1$, τ_{t_k} and $\tau_{t_{k+1}}$ denote the time delay in the network communication channel at instant $t_k h$ and $t_{k+1} h$, respectively. The holding interval $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$ under the influence of the logic ZOH can be expressed as $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) = \bigcup_{l=1}^{d} \mathcal{U}_l$, where $\mathcal{U}_l = [t_k h + lh + \tau_{t_{k+l}}, t_k h + lh + h + \tau_{t_{k+l+1}}]$. Denote $\tau(t) = t - t_k h - lh$, $0 \le \tau_{t_k} \le \tau(t) \le h + \tau_{t_{k+l+1}} \triangleq \tau_M$. Then, the real input of the state estimator can be rewritten as

$$\bar{y}(t) = (1 - \alpha(t - \tau(t)))Cx(t - \tau(t)) + \alpha(t - \tau(t))f(Cx(t - \tau(t))) + e_k(t) \tag{7}$$

By defining the estimation error $e(t) = x(t) - \hat{x}(t)$, $g(e(t)) = g(x(t)) - g(\hat{x}(t))$, $h(e(t - d(t))) = h(x(t - d(t))) - h(\hat{x}(t - d(t)))$, we obtain the estimation error dynamical system as follows:

$$\begin{aligned}\dot{e}(t) &= Ae(t) + Bg(e(t)) + Eh(e(t - d(t))) + Dw(t) \\ &\quad - (K + \Delta K)e_k(t) - (1 - \bar{\alpha})(K + \Delta K)Cx(t - \tau(t)) \\ &\quad - \bar{\alpha}(K + \Delta K)f(Cx(t - \tau(t))) - [\bar{\alpha} - \alpha(t - \tau(t))](K + \Delta K) \\ &\quad \times Cx(t - \tau(t)) - [\alpha(t - \tau(t)) \\ &\quad - \bar{\alpha}](K + \Delta K)f(Cx(t - \tau(t))) + (K + \Delta K)C(x(t) - e(t))\end{aligned} \tag{8}$$

Set $\xi(t) = [e^T(t)\ x^T(t)]^T$, $\tilde{z}(t) = z(t) - \hat{z}(t)$. Then, the estimation error system (8) and the neural network (1) can be expressed as follows:

$$\begin{cases}\dot{\xi}(t) = \bar{A}\xi(t) + \bar{B}g(\xi(t)) + \bar{E}h(\xi(t - d(t))) + \bar{D}w(t) \\ -H_1(K + \Delta K)e_k(t) \\ -H_1(1 - \overline{\alpha})(K + \Delta K)CH_2\xi(t - \tau(t)) - \overline{\alpha}H_1(K + \Delta K) \\ f(CH_2\xi(t - \tau(t))) \\ -[\overline{\alpha} - \alpha(t - \tau(t))]H_1(K + \Delta K)CH_2\xi(t - \tau(t)) \\ -[\alpha(t - \tau(t)) - \overline{\alpha}]H_1(K + \Delta K)f(CH_2\xi(t - \tau(t))) \\ +H_1\Delta KCH_3\xi(t) \\ \tilde{z}(t) = \bar{L}\xi(t)\end{cases} \tag{9}$$

where

$$\bar{A} = \begin{bmatrix} A - KC & KC \\ 0 & A \end{bmatrix}, \quad B = diag\{B, B\}, \quad E = diag\{E, E\},$$

$$\bar{D} = \begin{bmatrix} D \\ D \end{bmatrix}, \quad H_1 = \begin{bmatrix} I \\ 0 \end{bmatrix}, \quad H_2 = \begin{bmatrix} 0 & I \end{bmatrix}$$

$$H_3 = \begin{bmatrix} -I & I \end{bmatrix}, \quad \bar{L} = LH_1^T, g(\xi(t)) = \begin{bmatrix} g(e(t)) \\ g(x(t)) \end{bmatrix},$$

$$h(\xi(t - d(t))) = \begin{bmatrix} g(e(t - d(t)) \\ g(x(t - d(t)) \end{bmatrix}$$

Remark 7. In this paper, in order to reduce the number of transmitted sampled data, we introduce an event-triggered scheme between the neural networks and state estimator. The triggering condition is dependent on the error between the latest transmitted data and newly sampled one. Up to now, there are no publications taking both of the event-triggered scheme and the sensor nonlinearity into consideration. This is an obvious difference between the event-triggered scheme in this paper and the ones in the existing results. We will show the advantage of the adopted event-triggered scheme in saving precious network resources in the simulation part.

Lemma 1. [29] *Consider the augmented system with $\tau(t)$ that satisfies $0 < \tau(t) \le \bar{\tau}$. For any matrices $X \in R^{n \times n}$ and $U \in R^{n \times n}$ that satisfy $\begin{bmatrix} X & U \\ U^T & X \end{bmatrix} \ge 0$, the following inequality holds:*

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t}\dot{\xi}^T(s)X\dot{\xi}(s) \le \begin{bmatrix}\xi(t)\\ \xi(t-\tau(t))\\ \xi(t-\bar{\tau})\end{bmatrix}^T \times \begin{bmatrix}-X & * & *\\ X^T-U^T & -2X+U+U^T & *\\ U^T & X^T-U^T & -X\end{bmatrix}\begin{bmatrix}\xi(t)\\ \xi(t-\tau(t))\\ \xi(t-\bar{\tau})\end{bmatrix} \tag{10}$$

Lemma 2. *[30] Given matrices $F_1=F_1^T$, F_2 and F_3 of appropriate dimensions, we have $F_1+F_3\Delta(t)F_2+F_2^T\Delta^T(t)F_3^T<0$ for all $\Delta(t)$ satisfying $\Delta^T(t)\Delta(t)\le I$, if and only if there exists a positive scalar $\varepsilon<0$, such that $F_1+\varepsilon^{-1}F_3F_3^T+\varepsilon F_2^TF_2<0$*

3. Main results

For presentation convenience, we denote

$$\Phi_g^- = diag\{\phi_{g1}^-\phi_{g1}^+, \phi_{g2}^-\phi_{g2}^+, \dots, \phi_{gn}^-\phi_{gn}^+\},$$
$$\Phi_g^+ = diag\left\{\frac{\phi_{g1}^-+\phi_{g1}^+}{2}, \frac{\phi_{g2}^-+\phi_{g2}^+}{2}, \dots, \frac{\phi_{gn}^-+\phi_{gn}^+}{2}\right\}$$
$$\Phi_h^- = diag\{\phi_{h1}^-\phi_{h1}^+, \phi_{h2}^-\phi_{h2}^+, \dots, \phi_{hn}^-\phi_{hn}^+\},$$
$$\Phi_h^+ = diag\left\{\frac{\phi_{h1}^-+\phi_{h1}^+}{2}, \frac{\phi_{h2}^-+\phi_{h2}^+}{2}, \dots, \frac{\phi_{hn}^-+\phi_{hn}^+}{2}\right\}$$
$$\Phi_1^- = diag\{\Phi_g^-, \Phi_g^-\},\ \Phi_1^+ = diag\{\Phi_g^+, \Phi_g^+\},$$
$$\Phi_2^- = diag\{\Phi_h^-, \Phi_h^-\},\ \Phi_2^+ = diag\{\Phi_h^+, \Phi_h^+\}$$
$$\zeta^T(t) = [r_1^T(t)\ \xi^T(t-\tau(t))\ \xi^T(t-\tau_M)\ e_k(t)\ g^T(\xi(t))\ g^T(\xi(t-d(t)))\ f^T(CH_2\xi(t))\ w^T(t)]$$
$$r_1^T(t) = [\xi^T(t)\ \xi^T(t-d(t))\ \xi^T(t-d_M)],$$
$$r_2^T(t) = [\xi^T(t)\ \xi^T(t-\tau(t))\ \xi^T(t-\tau_M)]$$

The following Theorem will give sufficient asymptotical stability conditions for the estimation error system (9) by using Lyapunov–Krasovskii functional approach.

Theorem 1. *For given positive scalars $\bar{\alpha}$, τ_M, d_M, γ, σ, $\varepsilon_i(i=1,2,3,4,5)$ and matrix K, H, G, the augmented system (9) is asymptotically stable if there exist positive matrices $P>0$, $Q_1>0$, $Q_2>0$, $R_1>0$, $R_2>0$, $W_1>0$, $W_2>0$, appropriately dimensioned M, N, and appropriately dimensioned diagonal matrices U, V satisfying the following inequalities*

$$\Xi = \begin{bmatrix}\Xi_{11} & * & * & *\\ \Xi_{21} & \Xi_{22} & * & *\\ \Xi_{31} & 0 & \Xi_{33} & *\\ LH_1^T & 0 & 0 & -I\end{bmatrix} < 0 \tag{11}$$

$$\begin{bmatrix}R_1 & *\\ M & R_1\end{bmatrix} > 0 \tag{12}$$

$$\begin{bmatrix}R_2 & *\\ N & R_2\end{bmatrix} > 0 \tag{13}$$

where

$$\Xi_{11} = \begin{bmatrix}\Xi_{111} & *\\ \Xi_{112} & \Xi_{113}\end{bmatrix},$$

$$\Xi_{111} = \begin{bmatrix}\Gamma_1 & * & *\\ R_1-M & -2R_1+M+M^T-V\Phi_2^- & *\\ M & R_1-M & -Q_1-R_1\end{bmatrix}$$

$$\Xi_{112} = \begin{bmatrix}-(1-\bar{\alpha})H_2^TC^T(K+\Delta K)^TH_1^TP+R_2-N & 0 & 0\\ N & 0 & 0\\ -(K+\Delta K)^TH_1^TP & 0 & 0\\ \bar{B}^TP+\Phi_1^+U & 0 & 0\\ \bar{E}^TP & \Phi_2^+V & 0\\ -\bar{\alpha}(K+\Delta K)^TH_1^TP & 0 & 0\\ \bar{D}^TP & 0 & 0\end{bmatrix},$$

$$\Xi_{33} = diag\{\Xi_{22}, \Xi_{22}\}$$

$$\Xi_{113}=\begin{bmatrix}\Gamma_2 & * & * & * & * & * & *\\ R_2-N & -Q_2-R_2 & * & * & * & * & *\\ 0 & 0 & -W_1 & * & * & * & *\\ 0 & 0 & 0 & -UI & * & * & *\\ 0 & 0 & 0 & 0 & -VI & * & *\\ 0 & 0 & 0 & 0 & 0 & -\lambda I+2\sigma\bar{\alpha}^2W_2 & *\\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2I\end{bmatrix}$$

$$\Gamma_1 = P\bar{A}+\bar{A}^TP+PH_1\Delta KCH_3+H_3^TC^T(\Delta K)^TH_1^TP+Q_1 + Q_2-R_1-R_2-U\Phi_1^-$$
$$\Gamma_2 = -2R_2+N+N^T+\lambda H_2^TC^T\mathcal{F}^T\mathcal{F}CH_2+2\sigma(1-\bar{\alpha})^2H_2^TC^TW_2CH_2,$$
$$\delta = \sqrt{\bar{\alpha}(1-\bar{\alpha})}$$
$$\Xi_{21} = \begin{bmatrix}d_M\Xi_{211} & d_M\Xi_{212}\\ \tau_M\Xi_{211} & \tau_M\Xi_{212}\end{bmatrix},\ \Xi_{22} = diag\{-PR_1^{-1}P, -PR_2^{-1}P\},$$
$$U = diag\{U_1, U_2\}, V = diag\{V_1, V_2\}$$
$$\Xi_{31} = \begin{bmatrix}0_{1\times3} & \sqrt{2}\delta d_MPH_1(K+\Delta K)CH_2 & 0_{1\times4} & 0 & 0\\ 0_{1\times3} & \sqrt{2}\delta\tau_MPH_1(K+\Delta K)CH_2 & 0_{1\times4} & 0 & 0\\ 0_{1\times3} & 0 & 0_{1\times4} & \sqrt{2}\delta d_MPH_1(K+\Delta K) & 0\\ 0_{1\times3} & 0 & 0_{1\times4} & \sqrt{2}\delta\tau_MPH_1(K+\Delta K) & 0\end{bmatrix}$$
$$\Xi_{211} = [P\bar{A}+PH_1\Delta KCH_3\ 0\ 0\ -(1-\bar{\alpha})PH_1(K+\Delta K)CH_2\ 0\ -PH_1(K+\Delta K)]$$
$$\Xi_{212} = \left[P\bar{B}\ P\bar{E}\ -\bar{\alpha}PH_1(K+\Delta K)\ P\bar{D}\right]$$

Proof. Lyapunov–Krasovskii functional candidate for system (9) is chosen as follows:

$$V(t) = V_1(t)+V_2(t)+V_3(t) \tag{14}$$

where

$$V_1(t) = \xi^T(t)P\xi(t)$$
$$V_2(t) = \int_{t-d_M}^{t}\xi^T(s)Q_1\xi(s)ds+\int_{t-\tau_M}^{t}\xi^T(s)Q_2\xi(s)ds$$
$$V_3(t) = d_M\int_{t-d_M}^{t}\int_{s}^{t}\dot{\xi}^T(v)R_1\dot{\xi}(v)dvds + \tau_M\int_{t-\tau_M}^{t}\int_{s}^{t}\dot{\xi}^T(v)R_2\dot{\xi}(v)dvds$$

Taking derivation on $V_i(t)$ and taking expectation on them, we obtain

$$\mathcal{E}\{\dot{V}_1(t)\} = \mathcal{E}\{2\xi^T(t)P\dot{\xi}(t)\} \tag{15}$$

$$\mathcal{E}\{\dot{V}_2(t)\} = \xi^T(t)(Q_1+Q_2)\xi(t)-\xi^T(t-\tau_M)Q_2\xi(t-\tau_M) - x^T(t-d_M)Q_1x(t-d_M) \tag{16}$$

$$\mathcal{E}\{\dot{V}_3(t)\} = \dot{\xi}^T(t)\mathcal{R}\dot{\xi}(t)-d_M\int_{t-d_M}^{t}\dot{\xi}^T(s)R_1\dot{\xi}(s)ds - \tau_M\int_{t-\tau_M}^{t}\dot{\xi}^T(s)R_2\dot{\xi}(s)ds \tag{17}$$

where $\mathcal{R}=d_MR_1+\tau_MR_2$.

Notice that

$$\mathcal{E}\{2\xi^T(t)P\dot{x}(t)\} = 2\xi^T(t)P\mathcal{A} \tag{18}$$

$$\mathcal{E}\{\dot{\xi}^T(t)\bar{R}\dot{\xi}(t)\} = \mathcal{A}^T\mathcal{R}\mathcal{A}+2\delta^2[H_1(K+\Delta K)CH_2\xi(t-\tau(t))]^T \times\mathcal{R}[H_1(K+\Delta K)CH_2\xi(t-\tau(t))]$$

$$
\begin{aligned}
&+2\delta^2[H_1(K+\Delta K)f(CH_2\xi(t-\tau(t)))]^T \\
&\times \mathcal{R}[H_1(K+\Delta K)f(CH_2\xi(t-\tau(t)))]
\end{aligned} \tag{19}
$$

where

$$
\begin{aligned}
\mathcal{A} = {} & \bar{A}\xi(t) + \bar{B}g(\xi(t)) + \bar{E}h(\xi(t-d(t))) \\
& + \bar{D}w(t) - H_1(K+\Delta K)e_k(t) + H_1\Delta KCH_3\xi(t) \\
& - (1-\bar{\alpha})H_1(K+\Delta K)CH_2\xi(t-\tau(t)) \\
& - \bar{\alpha}H_1(K+\Delta K)f(CH_2\xi(t-\tau(t)))
\end{aligned}
$$

By Lemma 1, for matrices M and N satisfying (12) and (13), we derive

$$
-d_M\int_{t-d_M}^{t}\dot{\xi}^T(s)R_1\dot{\xi}(s)ds \le r_1^T(t)S_1r_1(t) \tag{20}
$$

$$
-\tau_M\int_{t-\tau_M}^{t}\dot{\xi}^T(s)R_2\dot{\xi}(s)ds \le r_2^T(t)S_2r_2(t) \tag{21}
$$

where

$$
S_1 = \begin{bmatrix} -R_1 & R_1 - M^T & M^T \\ R_1 - M & -2R_1 + M + M^T & R_1 - M^T \\ M & R_1 - M & -R_1 \end{bmatrix},
$$

$$
S_2 = \begin{bmatrix} -R_2 & R_2 - N^T & N^T \\ R_2 - N & -2R_2 + N + N^T & R_2 - N^T \\ N & R_2 - N & -R_2 \end{bmatrix}
$$

It is not difficult to get from (2) that

$$
\begin{bmatrix} e(t) \\ g(e(t)) \end{bmatrix}^T \begin{bmatrix} -U_1\Phi_g^- & U_1\Phi_g^+ \\ \Phi_g^+U_1 & -U_1 \end{bmatrix} \begin{bmatrix} e(t) \\ g(e(t)) \end{bmatrix} \ge 0 \tag{22}
$$

$$
\begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix}^T \begin{bmatrix} -U_2\Phi_g^- & U_2\Phi_g^+ \\ \Phi_g^+U_2 & -U_2 \end{bmatrix} \begin{bmatrix} x(t) \\ g(x(t)) \end{bmatrix} \ge 0 \tag{23}
$$

Remind that $\xi(t) = [e^T(t) \quad x^T(t)]^T$, $g(\xi(t)) = [g^T(e(t)) \quad g^T(x(t))]^T$, from (22) and (23), one can get

$$
\begin{bmatrix} \xi(t) \\ g(\xi(t)) \end{bmatrix}^T \begin{bmatrix} -U\Phi_g^- & U\Phi_g^+ \\ \Phi_g^+U & -U \end{bmatrix} \begin{bmatrix} \xi(t) \\ g(\xi(t)) \end{bmatrix} \ge 0 \tag{24}
$$

Similarly, from (3), one can have

$$
\begin{bmatrix} e(t-d(t)) \\ h(e(t-d(t))) \end{bmatrix}^T \begin{bmatrix} -V_1\Phi_h^- & V_1\Phi_h^+ \\ \Phi_h^+V_1 & -V_1 \end{bmatrix} \begin{bmatrix} e(t-d(t)) \\ h(e(t-d(t))) \end{bmatrix} \ge 0 \tag{25}
$$

$$
\begin{bmatrix} x(t-d(t)) \\ h(x(t-d(t))) \end{bmatrix}^T \begin{bmatrix} -V_2\Phi_h^- & V_2\Phi_h^+ \\ \Phi_h^+V_2 & -V_2 \end{bmatrix} \begin{bmatrix} x(t-d(t)) \\ h(x(t-d(t))) \end{bmatrix} \ge 0 \tag{26}
$$

From the definitions $\xi(t-d(t)) = [e^T(t-d(t)) \quad x^T(t-d(t))]^T$, $g(\xi(t-d(t))) = [g^T(e(t-d(t))) \quad g^T(x(t-d(t)))]^T$, from (25) and (26), we can obtain that

$$
\begin{bmatrix} \xi(t-d(t)) \\ h(\xi(t-d(t))) \end{bmatrix}^T \begin{bmatrix} -V\Phi_h^- & V\Phi_h^+ \\ \Phi_h^+V & -V \end{bmatrix} \begin{bmatrix} \xi(t-d(t)) \\ h(\xi(t-d(t))) \end{bmatrix} \ge 0 \tag{27}
$$

where U_1, U_2 V_1 and V_2 are appropriately dimensioned diagonal matrices, $U = diag\{U_1, U_2\}$, $V = diag\{V_1, V_2\}$.

From the definition in (4), for any appropriately dimensioned diagonal matrix Λ, it can be seen that

$$
\begin{aligned}
&\Lambda\xi^T(t-\tau(t))H_2^TC^T\mathcal{F}^T\mathcal{F}CH_2\xi(t-\tau(t)) \\
&-\Lambda f^T(CH_2\xi(t-\tau(t)))f(CH_2\xi(t-\tau(t))) \ge 0
\end{aligned} \tag{28}
$$

Recalling the definition of $\tau(t)$, it follows from (6) that

$$
\begin{aligned}
&\bar{\mathcal{C}}^TW_2\bar{\mathcal{C}} + 2\sigma\delta^2[Cx(t-\tau(t))]W_2[Cx(t-\tau(t))] \\
&+2\sigma\delta^2f^T(Cx(t-\tau(t)))W_2f(Cx(t-\tau(t))) \\
&-e_k^T(t)\Omega e_k(t) \ge 0
\end{aligned} \tag{29}
$$

in which $\bar{\mathcal{C}} = (1-\bar{\alpha})Cx(t-\tau(t)) + \bar{\alpha}f(Cx(t))$

Note that (29) can be rewritten as

$$
\begin{aligned}
&\hat{\mathcal{C}}^TW_2\hat{\mathcal{C}} + 2\sigma\delta^2[CH_2\xi(t-\tau(t))]W_2[CH_2\xi(t-\tau(t))] \\
&+2\sigma\delta^2f^T(CH_2\xi(t-\tau(t)))W_2f(CH_2\xi(t-\tau(t))) \\
&-e_k^T(t)W_1e_k(t) \ge 0
\end{aligned} \tag{30}
$$

where $\bar{\mathcal{C}} = (1-\bar{\alpha})CH_2\xi(t-\tau(t)) + \bar{\alpha}f(CH_2\xi(t))$

Combining (15)–(30), we can obtain that

$$
\begin{aligned}
&\mathcal{E}\{\dot{V}(t)\} + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2w^T(t)w(t) \\
&\le \zeta^T(t)\Xi_{11}\zeta(t) + \mathcal{A}^T\mathcal{R}\mathcal{A} \\
&+2\delta^2[H_1(K+\Delta K)CH_2\xi(t-\tau(t))]^T \\
&\times\mathcal{R}[H_1(K+\Delta K)CH_2\xi(t-\tau(t))] \\
&+2\delta^2[H_1(K+\Delta K)f(CH_2\xi(t-\tau(t)))]^T \\
&\times\mathcal{R}[H_1(K+\Delta K)f(CH_2\xi(t-\tau(t)))] + \xi^T(t)H_1L^TLH_1^T\xi(t)
\end{aligned} \tag{31}
$$

It can be concluded that $\mathcal{E}\{\dot{V}(t)\} < 0$ can be easily obtained from (11). Therefore, the asymptotical stability of (9) can be ensured by (11)–(13).

This completes the proof. □

Based on the obtained result in Theorem 1, we are now in position to design the gain matrix of the estimator.

Theorem 2. *For given positive scalars $\bar{\alpha}$, τ_M, d_M, γ, σ, $\varepsilon_i(i = 1, 2, 3, 4, 5)$ and matrices K, H, G, the augmented system (9) is asymptotically stable if there exist positive matrices $P = diag\{P_1, P_2\}$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $W_1 > 0$, $W_2 > 0$, and appropriate dimensioned M, N, and diagonal matrices U_1, U_2 V_1, V_2 with appropriate dimensions satisfying the following inequalities*

$$
\Xi = \begin{bmatrix} \bar{\Xi}_{11} & * & * & * & * \\ \bar{\Xi}_{21} & \bar{\Xi}_{22} & * & * & * \\ \bar{\Xi}_{31} & 0 & \bar{\Xi}_{33} & * & * \\ \bar{L} & 0 & 0 & -I & * \\ \bar{\Xi}_{51} & \bar{\Xi}_{52} & \bar{\Xi}_{53} & 0 & \bar{\Xi}_{55} \end{bmatrix} < 0 \tag{32}
$$

$$
\begin{bmatrix} R_1 & * \\ M & R_1 \end{bmatrix} > 0 \tag{33}
$$

$$
\begin{bmatrix} R_2 & * \\ N & R_2 \end{bmatrix} > 0 \tag{34}
$$

where

$$
\bar{\Xi}_{11} = \begin{bmatrix} \bar{\Xi}_{111} & * \\ \bar{\Xi}_{112} & \bar{\Xi}_{113} \end{bmatrix},
$$

$$
\bar{\Xi}_{111} = \begin{bmatrix} \bar{\Gamma}_1 & * & * \\ R_1 - M & -2R_1 + M + M^T - V\Phi_2^- & * \\ M & R_1 - M & -Q_1 - R_1 \end{bmatrix}
$$

$$
\bar{\Xi}_{112} = \begin{bmatrix} -(1-\bar{\alpha})\Pi_1^T + R_2 - N & 0 & 0 \\ N & 0 & 0 \\ \Pi_2^T & 0 & 0 \\ \Pi_3 + \Phi_1^+U & 0 & 0 \\ \Pi_4 & \Phi_2^+V & 0 \\ \bar{\alpha}\Pi_2^T & 0 & 0 \\ \Pi_5 & 0 & 0 \end{bmatrix},
$$

$$
\Pi_1 = \begin{bmatrix} 0 & -YC \\ 0 & 0 \end{bmatrix}, \Pi_2 = \begin{bmatrix} -Y^T & 0 \end{bmatrix}
$$

$$
\Pi_3 = \begin{bmatrix} B^TP_1 & 0 \\ 0 & B^TP_2 \end{bmatrix}, \Pi_4 = \begin{bmatrix} E^TP_1 & 0 \\ 0 & E^TP_2 \end{bmatrix}, \Pi_5 = \begin{bmatrix} D^TP_1 & D^TP_2 \end{bmatrix}
$$

$$
\bar{\Xi}_{113} = \begin{bmatrix} \bar{\Gamma}_2 & * & * & * & * & * & * \\ R_2 - N & -Q_2 - R_2 & * & * & * & * & * \\ 0 & 0 & -W_1 & * & * & * & * \\ 0 & 0 & 0 & -U & * & * & * \\ 0 & 0 & 0 & 0 & -V & * & * \\ 0 & 0 & 0 & 0 & 0 & -\lambda I + 2\sigma\bar{\alpha}^2W_2 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2I \end{bmatrix}
$$

$$\bar{\Gamma}_1 = \bar{\Gamma}_{11} + \bar{\Gamma}_{11}^T + Q_1 + Q_2 - R_1 - R_2 - U\Phi_1^-, \bar{\Gamma}_{11} = \begin{bmatrix} P_1A - YC & YC \\ 0 & P_2A \end{bmatrix}$$

$$\bar{\Gamma}_2 = -2R_2 + N + N^T + \lambda \begin{bmatrix} 0 & 0 \\ 0 & C^T\mathcal{F}^T\mathcal{F}C \end{bmatrix} + 2\sigma(1-\bar{\alpha})^2 \begin{bmatrix} 0 & 0 \\ 0 & C^TW_2C \end{bmatrix}$$

$$\bar{\Xi}_{21} = \begin{bmatrix} d_M\bar{\Xi}_{211} & d_M\bar{\Xi}_{212} \\ \tau_M\bar{\Xi}_{211} & \tau_M\bar{\Xi}_{212} \end{bmatrix}, \bar{\Xi}_{22} = diag\{-2\varepsilon_1P + \varepsilon_1^2R_1, -2\varepsilon_2P + \varepsilon_2^2R_2\}$$

$$\bar{\Xi}_{31} = \begin{bmatrix} 0_{1\times3} & -\sqrt{2}\delta d_M\Pi_1 & 0_{1\times4} & 0 & 0 \\ 0_{1\times3} & -\sqrt{2}\delta\tau_M\Pi_1 & 0_{1\times4} & 0 & 0 \\ 0_{1\times3} & 0 & 0_{1\times4} & -\sqrt{2}\delta d_M\Pi_2 & 0 \\ 0_{1\times3} & 0 & 0_{1\times4} & -\sqrt{2}\delta\tau_M\Pi_2 & 0 \end{bmatrix},$$

$$\bar{\Xi}_{33} = diag\{\bar{\Xi}_{22}, \bar{\Xi}_{22}\}$$

$$\bar{\Xi}_{211} = \begin{bmatrix} \bar{\Gamma}_{11} & 0 & 0 & (1-\bar{\alpha})\Pi_1 & 0 & \Pi_2^T \end{bmatrix},$$

$$\bar{\Xi}_{212} = \begin{bmatrix} \Pi_3^T & \Pi_4^T & \bar{\alpha}\Pi_2^T & \Pi_5^T \end{bmatrix}, \bar{L} = \begin{bmatrix} L & 0 \end{bmatrix}$$

$$\bar{\Xi}_{51} = \begin{bmatrix} \varepsilon_3\Theta_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \Theta_2 & 0 & 0 & -(1-\bar{\alpha})\Theta_3 & 0 & -G & 0 & 0 & -\bar{\theta}G & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Theta_3 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & G & 0 \end{bmatrix}$$

$$\bar{\Xi}_{55} = diag\{-\varepsilon_3I, -\varepsilon_3I, -\varepsilon_4I, -\varepsilon_4I, -\varepsilon_5I, -\varepsilon_5I\}$$

$$\Theta_1 = \begin{bmatrix} H^TP_1 & 0 \end{bmatrix}, \Theta_2 = \begin{bmatrix} -GC & GC \end{bmatrix}, \Theta_3 = \begin{bmatrix} 0 & GC \end{bmatrix}$$

$$\bar{\Xi}_{52} = \begin{bmatrix} \varepsilon_3d_M\Theta_1 & \varepsilon_3\tau_M\Theta_1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Xi}_{53} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \varepsilon_4\sqrt{2}d_M\delta\Theta_1 & \varepsilon_4\sqrt{2}\tau_M\delta\Theta_1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \varepsilon_5\sqrt{2}d_M\delta\Theta_1 & \varepsilon_5\sqrt{2}\tau_M\delta\Theta_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Moreover, the desired state estimator gain matrix is given by

$$K = P_1^{-1}Y \tag{35}$$

Proof. It can be seen that (11) in Theorem 1 can be written as

$$\Xi = \Sigma + \Upsilon_1^TF(t)\Upsilon_2 + \Upsilon_3^TF(t)\Upsilon_4 + \Upsilon_5^TF(t)\Upsilon_6 < 0 \tag{36}$$

where

$$\Sigma = \begin{bmatrix} \widehat{\Xi}_{11} & * & * & * \\ \widehat{\Xi}_{21} & \Xi_{22} & * & * \\ \widehat{\Xi}_{31} & 0 & \Xi_{33} & * \\ LH_1^T & 0 & 0 & -I \end{bmatrix} < 0, \widehat{\Xi}_{11} = \begin{bmatrix} \widehat{\Xi}_{111} & * \\ \widehat{\Xi}_{112} & \Xi_{113} \end{bmatrix},$$

$$\widehat{\Xi}_{21} = \begin{bmatrix} d_M\widehat{\Xi}_{211} & d_M\widehat{\Xi}_{212} \\ \tau_M\widehat{\Xi}_{211} & \tau_M\widehat{\Xi}_{212} \end{bmatrix}$$

$$\widehat{\Xi}_{111} = \begin{bmatrix} \overline{\Gamma}_1 & * & * \\ R_1 - M & -2R_1 + M + M^T - V\Phi_2^- & * \\ M & R_1 - M & -Q_1 - R_1 \end{bmatrix}$$

$$\widehat{\Xi}_{112} = \begin{bmatrix} -(1-\overline{\alpha})H_2^TC^TK^TH_1^TP & 0 & 0 \\ N & 0 & 0 \\ -K^TH_1^TP & 0 & 0 \\ \overline{B}^TP + \Phi_2^+U & 0 & 0 \\ \overline{E}^TP & \Phi_2^+V & 0 \\ -\overline{\alpha}K^TH_1^TP & 0 & 0 \\ \overline{D}^TP & 0 & 0 \end{bmatrix}$$

$$\widehat{\Xi}_{31} = \begin{bmatrix} 0_{1\times3} & \sqrt{2}\delta d_MPH_1KCH_2 & 0_{1\times4} & 0 & 0 \\ 0_{1\times3} & \sqrt{2}\delta\tau_MPH_1KCH_2 & 0_{1\times4} & 0 & 0 \\ 0_{1\times3} & 0 & 0_{1\times4} & \sqrt{2}\delta d_MPH_1K & 0 \\ 0_{1\times3} & 0 & 0_{1\times4} & \sqrt{2}\delta\tau_MPH_1K & 0 \end{bmatrix}$$

$$\overline{\Gamma}_1 = P\overline{A} + \overline{A}^TP + Q_1 + Q_2 - R_1 - R_2 - U\Phi_2^-$$

$$\widehat{\Xi}_{211} = \begin{bmatrix} P\overline{A} & 0 & 0 & -(1-\overline{\alpha})PH_1KCH_2 & 0 & -PH_1K \end{bmatrix}$$

$$\widehat{\Xi}_{212} = \begin{bmatrix} P\overline{B} & P\overline{E} & -\overline{\alpha}PH_1K & P\overline{D} \end{bmatrix}$$

$$\Upsilon_1 = \begin{bmatrix} H^TH_1^TP & 0_{1\times9} & d_MH^TH_1^TP & \tau_MH^TH_1^TP & 0 \end{bmatrix}$$

$$\Upsilon_2 = \begin{bmatrix} GCH_3 \; 0 \; 0 \; -(1-\overline{\alpha})GCH_2 \; 0 \; -G \; 0 \; 0 \; \overline{G} \; 0_{1\times4} \end{bmatrix}$$

$$\Upsilon_3 = \begin{bmatrix} 0_{1\times12} & \sqrt{2}d_M\delta H^TH_1^TP & \sqrt{2}\tau_M\delta H^TH_1^TP & 0_{1\times3} \end{bmatrix},$$

$$\Upsilon_4 = \begin{bmatrix} 0_{1\times3} & GCH_2 & 0_{1\times13} \end{bmatrix}$$

$$\Upsilon_5 = \begin{bmatrix} 0_{1\times14} & \sqrt{2}d_M\delta H^TH_1^TP & \sqrt{2}\tau_M\delta H^TH_1^TP & 0 \end{bmatrix},$$

$$\Upsilon_6 = \begin{bmatrix} 0_{1\times8} & G & 0_{1\times8} \end{bmatrix}$$

By using Lemma 2, there exist positive scalar ε_3, ε_4 and ε_5, such that

$$\Xi \le \Sigma + \varepsilon_3\Upsilon_1^T\Upsilon_1 + \varepsilon_3^{-1}\Upsilon_2^T\Upsilon_2 + \varepsilon_4\Upsilon_3^T\Upsilon_3 + \varepsilon_4^{-1}\Upsilon_4^T\Upsilon_4 + \varepsilon_5\Upsilon_5^T\Upsilon_5 + \varepsilon_5^{-1}\Upsilon_6^T\Upsilon_6 < 0 \tag{37}$$

By using Schur complement, we can obtain

$$\Sigma = \begin{bmatrix} \widehat{\Xi}_{11} & * & * & * & * \\ \widehat{\Xi}_{21} & \Xi_{22} & * & * & * \\ \widehat{\Xi}_{31} & 0 & \Xi_{33} & * & * \\ LH_1^T & 0 & 0 & -I & * \\ \bar{\Xi}_{51} & \bar{\Xi}_{52} & \bar{\Xi}_{53} & 0 & \bar{\Xi}_{54} \end{bmatrix} < 0 \tag{38}$$

From

$$(R_k - \varepsilon_kP)R_k^{-1}(R_k - \varepsilon_kP) \ge 0 \tag{39}$$

we can get

$$-PR_k^{-1}P \le -2\varepsilon_kP + \varepsilon_k^2R_k \tag{40}$$

Denote $Y = P_1K$ and replace $-PR_k^{-1}P$ with $-2\varepsilon_kP + \varepsilon_k^2R_k$, we can get (32). It is easy to see that (32) holds implies (11) holds and the augmented system is asymptotically stable. This completes the proof. □

Remark 8. In this paper, the event-triggered non-fragile state estimation problem is investigated for delayed neural works in presence of sensor nonlinearity. According to Theorem 2, the estimator gain can be designed by solving equalities (32)–(34). The following simulation example validates the proposed design method.

Remark 9. With the development of the computer, we can find the feasible solution to the LMIs in Theorem 2 by using LMI SOLVER FEASP in MATLAB LMI tool box even when the size of the state vector is big.

4. Simulation examples

In this section, a simulation example is provided to illustrate the effectiveness of the obtained event-triggered state estimation scheme for neural networks with randomly occurring sensor nonlinearities.

Consider the discussed neural networks (1) with parameters

$$A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & -2 \\ -2 & 1 \end{bmatrix}, \quad E = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix},$$

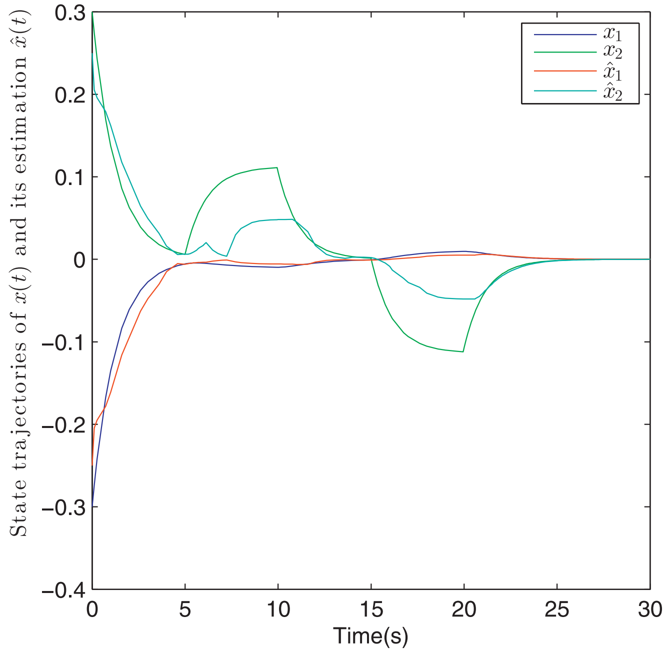

Fig. 2. Evaluation of state $x(t)$ and its estimation $\hat{x}(t)$.

$$C=\begin{bmatrix}1&0\\0&1\end{bmatrix},\ D=\begin{bmatrix}0\\0.1\end{bmatrix},\ L=\begin{bmatrix}1&0\\0&1\end{bmatrix}\ H=\begin{bmatrix}0.3&0\\0&0.3\end{bmatrix},$$

$$G=\begin{bmatrix}0.1&0\\0&0.1\end{bmatrix},\ w(t)=\begin{cases}c, & 5\le t\le 10\\ -1 & 15\le t\le 20\\ 0 & \text{else}\end{cases},F(t)=sintI$$

The active neuron active functions are chosen as

$$g(x(t))=\begin{bmatrix}tanh(0.08x(t))\\tanh(0.06x(t))\end{bmatrix},$$

$$h(x(t-d(t)))=\begin{bmatrix}tanh(0.08x(t-d(t)))\\tanh(0.08x(t-d(t)))\end{bmatrix}$$

which means $\Phi_g^-=0$ and $\Phi_g^+=diag\{0.04,0.03\}$, $\Phi_h^-=0$ and $\Phi_h^+=diag\{0.04,0.04\}$

The sensor nonlinearities are assumed to be

$$f(x(t))=\begin{bmatrix}tanh(0.01x(t))\\tanh(0.03x(t))\end{bmatrix}$$

which satisfy (4) with $\mathcal{F}=diag\{0.01,0.03\}$

The occurring probability of sensor nonlinearity is $\bar{\alpha}=0.8$. Setting $d_M=0.8$, $\tau_M=0.7$, $\gamma=1$, $\varepsilon_i(i=1,\ldots,5)$, the parameter of the triggering condition is given as $\sigma=0.1$. With the above parameters, by application of LMI Toolbox in MATLAB, from Theorem 2, we can obtain a feasible solution as follows:

$$P_1=\begin{bmatrix}2.7079&0.1849\\0.1849&2.5436\end{bmatrix},Y=\begin{bmatrix}2.5347&0.1393\\0.1196&2.2980\end{bmatrix}$$

$$W1=\begin{bmatrix}29.0350&0.1019\\0.1019&28.4225\end{bmatrix},W_2=\begin{bmatrix}16.1224&0.1958\\0.1958&16.0293\end{bmatrix}$$

According to (35), we obtain the corresponding non-fragile estimator gain matrix as

$$K=\begin{bmatrix}0.9375&-0.0103\\-0.0211&0.9042\end{bmatrix}$$

From the structure of ΔK, we have $H=0.3I$, $G=0.1I$ and $F(t)=sin(t)I$. The initial conditions of system (1) and the estimator are chosen as

$$x_0=\begin{bmatrix}-0.3&0.3\end{bmatrix}^T,\hat{x}_0=\begin{bmatrix}-0.25&0.25\end{bmatrix}^T$$

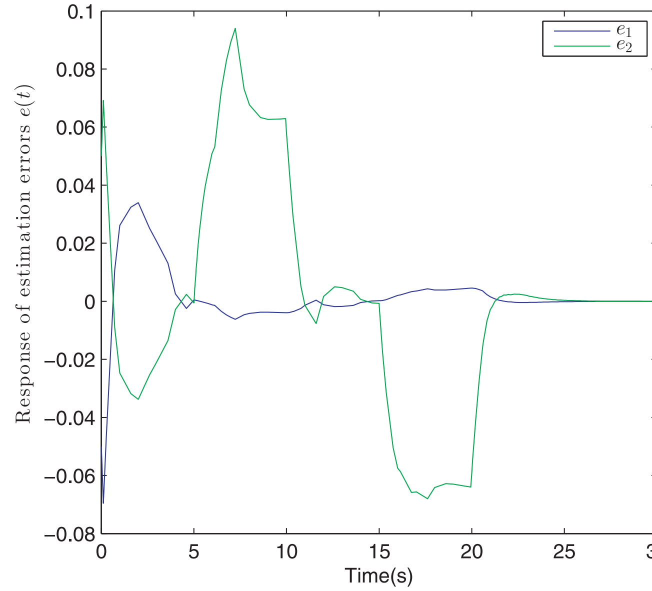

Fig. 3. Responses of estimation error $e(t)$.

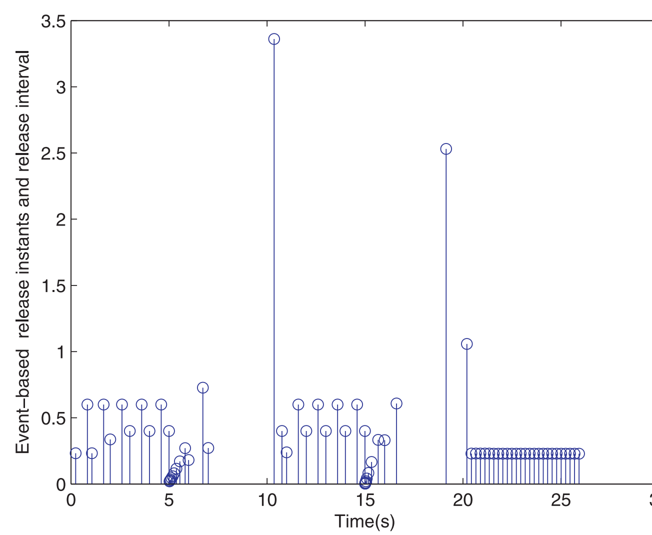

Fig. 4. Release instants and intervals.

We can get the following simulation results from Figs. 2–4. Fig. 2 shows the response of the state and estimation of the neural networks. Fig. 3 shows the estimation error dynamics. From Fig. 3, we can see the estimation error converges to zero asymptotically. The event-triggered release instants and the corresponding release intervals are shown in Fig. 4, where the average transmission rate is 37% . Therefore, the amount of transmission is reduced by using the event-triggered scheme. These simulation results demonstrate that the designed non-fragile state estimator performs well.

5. Conclusion

In this paper, the problem of event-triggered non-fragile state estimator design has been investigated for delayed neural networks with randomly occurring sensor nonlinearity. We introduce a Bernoulli distributed random variable to describe the randomly occurring sensor nonlinearity. In order to reduce data transmission in bandwidth limited network, a novel event-triggered communication scheme has been introduced between the neural networks and its estimator. Under the proposed event-triggered scheme, the non-fragile estimation error system has been modeled. Based on this model, a criteria for asymptotical stability of the estimation error system is obtained. In addition, the designed gain matrix of the estimator is presented by solving certain matrix inequalities. The usefulness of the proposed method has been demonstrated by the simulation results of a numerical example.

Acknowledgments

This work is supported by the National Natural Science Foundation of China (Nos. 61403185, 61640313, 61473156), Six Talent Peaks Project in Jiangsu Province (No. 2015-DZXX-021), the Natural Science Foundation of Jiangsu Province of China (No. BK20171481), and Major project supported by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (Grant No. 15KJA120001). Jiangsu Agricultural Science and Technology Independent Innovation Fund Project (No. CX(15)1051).

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