Neurocomputing 291 (2018) 35–49

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Quantized state estimation for neural networks with cyber attacks and hybrid triggered communication scheme

Jinliang Liu [a], Jilei Xia [a], Jie Cao [a,*], Engang Tian [b]

[a] College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, China
[b] Institute of Information and Control Engineering Technology, Nanjing Normal University, Nanjing, Jiangsu 210042, China

ARTICLE INFO

Article history:
Received 12 November 2017
Revised 25 December 2017
Accepted 10 February 2018
Available online 21 February 2018

Communicated by Prof. Wei Guoliang

Keywords:
Neural networks
Hybrid triggered scheme
State estimation
Cyber attacks

ABSTRACT

This paper is concerned with the issue of quantized state estimation for neural networks with cyber attacks and hybrid triggered communication scheme. In order to reduce the pressure of the network transmission and save the network resources, the hybrid triggered scheme and quantization are introduced. The hybrid triggered scheme consists of time triggered scheme and event triggered scheme, in which the stochastic switch is described by a variable satisfying Bernoulli distribution. First, by taking the effect of hybrid triggered scheme and quantization into consideration, a mathematical model for estimating the state of neural networks is constructed. Second, by using linear matrix inequality (LMI) techniques and Lyapunov stability theory, the sufficient conditions are given which can ensure the stability of estimating error system under hybrid triggered scheme, and the designing algorithm of desired state estimator is also presented in terms of LMIs. Finally, a numerical example is given to show the usefulness of the proposed approach.

1. Introduction

In recent years, the dynamic behaviors of neural networks have been paid increasing attention due to their potential applications in automation, image recognition, data compression and large volume of high speed data processing. It is well known that understanding the neuron states is an indispensable step to apply the neural networks and realize the desired performance in practical application. A large number of researchers have made great achievements in the neuron state estimation problem [1–4]. In [1], the authors investigate the variance-constrained state estimation problem for a class of networked multi-rate systems with network-induced probabilistic sensor failures and measurement quantization. The authors in [2] concentrate on the problem of stochastic finite-time state estimation for discrete time-delay neural networks with Markovian jumps. In literature [3], the authors consider the problem of state estimation with guaranteed performance for a class of switching fuzzy neural networks.

Over the past few decades, time triggered scheme (periodic sampling) is intensively applied for systems modeling in conventional control systems [5–7]. In [5], the authors consider the robust H_∞ filtering problem for a class of uncertain nonlinear time-delay stochastic systems. In view of the limited network resources, the drawback of time triggered scheme which contains the transmitting of large repetitive signals is prominent day by day. To avoid the shortcoming of time triggered scheme, another triggered scheme named event triggered scheme is proposed, which is used to deal with many issues of data transmission for networked control systems. For example, a novel event triggered scheme, which is used to reduce the communication load of the network, is proposed in [8] to investigate the problem of H_∞ controllers design for networked control systems with network-induced delay. Different from the time triggered scheme, the main idea of event triggered scheme mentioned above is that whether the settled threshold is violated or not can be the key rule of newly sampling data transmission. Based on the proposed event triggered scheme in [8], a large number of scholars concentrate on the investigations of event triggered scheme and have obtained abundant research results [9–12]. For example, an adaptive event-triggered communication scheme is proposed in [9] which is applied for a class of networked Takagi–Sugeno (T–S) fuzzy control systems. In [10], the problem of event-triggered reliable H_∞ filtering for networked systems with multiple sensor distortions is investigated. The authors in [11] are devoted to the design of event-triggered non-fragile state estimator for delayed neural networks subject to randomly occurring sensor nonlinearity. On the basis of the proposed event triggered scheme in [8], another triggered scheme named hybrid triggered scheme is proposed in [13], which is used to

* Corresponding author.
E-mail addresses: liujinliang@vip.163.com (J. Liu), caojie690929@163.com (J. Cao).

https://doi.org/10.1016/j.neucom.2018.02.060
0925-2312/

investigate the design of hybrid driven controller for networked control systems with network-induced delays. Inspired by the work mentioned above, some researchers make a further study about hybrid triggered scheme [14–17]. The authors in [14] are devoted to the reliable controller design for hybrid-driven nonlinear systems via T–S fuzzy model with probabilistic actuator faults and probabilistic nonlinear perturbations. The problem of H_∞ filter design for neural networks is investigated in [15] with hybrid triggered scheme and deception attacks. The hybrid-triggered H_∞ filtering problem is considered for networked systems under stochastic cyber attacks in [16]. In this paper, the hybrid triggered scheme is introduced to investigate the state estimation for neural networks with cyber attacks.

Recently, the quantitative processing has drawn much attention which is an effective way to save the network resources besides the event triggered scheme mentioned above. In networked control systems, the quantitative processing which contains the compression and decompression of sampling data aims at reducing the pressure of transmission and save network resources. Motivated by the merit of quantization mentioned above, many researchers have taken the effect of quantization into consideration for networked systems [18–20]. The problem of H_∞ output feedback control for event-triggered Markovian jump system is investigated in [18] with measured output quantizations. In [19], an event-driven networked control systems has been studied subject to quantization and packet losses. The authors in [20] investigate the dynamic output-feedback control problem for a class of discrete-time nonlinear stochastic systems with uniform quantization effects. In this paper, due to the quantitative processing which can reduce the pressure of network transmission in networked systems, the quantization is introduced in the investigation of state estimation for neural networks. To the best of authors' knowledge, the state estimation for neural networks with hybrid triggered scheme and quantization has not been investigated yet, which is the motivation of this paper.

The insertion of network has posed many conveniences in data transmission, but it also brings a lot of challenges such as packet losses, nonlinear disturbance and network-induced delay. In addition, another offensive behavior named cyber attacks aims at destroying data transmission system, real-time sampling data, communication infrastructures and networked devices. According to descriptions in [21,22], cyber attacks, which include an inaccurate sampling data and misidentification of the receiving devices, have generated greater threats to the safety of network. Given the importance of network security, enough attention should be paid to the risks of the cyber attacks. In recent years, large numbers of researchers are devoted to the investigations of cyber attacks and have obtained abundant research results [23–25]. For example, a novel approach of state filtering scheme and the sensor scheduling co-design for cyber–physical systems subject to random deception attacks is investigated in [23]. The authors in [24] investigate online deception attack strategy against remote state estimation with sensor-to-estimator communication rate constraint. A novel distributed state estimator with an event-triggered scheme is proposed in [25] to defend against false data injection attack. Drawing lessons from the existing achievements about cyber attacks, this paper is concerned with the design of state estimator for neural networks under hybrid triggered scheme subject to cyber attacks.

Motivated by the observations above, this paper is concerned with state estimation for neural networks with hybrid triggered scheme and quantization subject to cyber attacks. The hybrid triggered scheme and quantization are introduced to reduce the pressure of network transmission. Cyber attacks are also considered to make the state estimation for neural networks closer to the working condition of the actual system. Finally, a numerical example is given to show the usefulness of the proposed method.

This paper is organized as follows. In Section 2, a mathematical model of the estimating error system is constructed by taking the effects of hybrid triggered scheme and cyber attacks into consideration. Sufficient conditions which can guarantee stability of the estimating error system are established and a state estimator design method is provided in Section 3. A numerical example is given in Section 4 to show the usefulness of the proposed method. The conclusion is given in the final part.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n\times m$ real matrices; Matrix $X>0$, for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite. $\mathbb{E}(X)$ represents the mathematical expectation of X. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry. I is the identity matrix of appropriate dimension. In addition, T stands for the transpose of matrix.

2. System description

This paper is concerned with state estimation for neural networks with hybrid triggered scheme and quantization subject to cyber attacks. As is shown in Fig. 1, the hybrid triggered scheme and quantization are introduced to reduce the pressure of network transmission. A random variable which satisfies Bernoulli distribution is employed to describe the stochastic switch between the time triggered scheme and event triggered scheme in the hybrid triggered scheme. Consider the following neural networks with time-varying delays and n neurons [4,26,27]:

$$\begin{cases}\dot{x}(t)=-Ax(t)+W_0g(x(t))+W_1g(x(t-\phi(t)))\\ y(t)=Cx(t)\end{cases}\tag{1}$$

where $x(t)=[x_1(t),x_2(t),\cdots,x_n(t)]^T\in\mathbb{R}^n$ is the state vector of neural networks and $y(t)=[y_1(t),y_2(t),\ \cdots,y_r(t)]^T\in\mathbb{R}^r$ is the measurement output; $A=diag\{a_1,a_2,\cdots,a_n\}$ is a diagonal matrix with positive entries $a_i>0$; W_0 and W_1 are the connection weight matrix and the delayed connection weight matrix, respectively; C is a given constant matrix of appropriate dimensions. $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\cdots,g_n(x_n(t))]^T$ denotes the neuron activation function, and $\phi(t)$ denotes the time-varying bounded state delay satisfying $\phi(t)\in[\phi_m,\phi_M]$, where ϕ_m and ϕ_M are the lower and upper bounds of $\phi(t)$.

In order to estimate the state of neural network system (1), the following state estimation system [28,29] is introduced:

$$\begin{cases}\dot{\hat{x}}(t)=-A\hat{x}(t)+K(\tilde{y}(t)-\hat{y}(t))\\ \hat{y}(t)=C\hat{x}(t)\end{cases}\tag{2}$$

where $\hat{x}(t)\in\mathbb{R}^n$ is the estimated state vector and K is the estimator gain matrix to be determined. $\tilde{y}(t)$ is the actual input of the estimator and $\hat{y}(t)$ is the estimated measurement output.

In this paper, the state estimation for neural networks is investigated with hybrid triggered scheme and quantization subject to stochastic cyber attacks. The structure of the state estimator is shown in Fig. 1. The sensor measurements are sampled at time kh with a period h.

As is shown in Fig. 1, when "time triggered scheme" is chosen, by using the similar arguments in [30], the actual input of the estimator can be written as follows without considering the effect of quantization.

$$y_1(t)=Cx(t_kh),\quad t\in[t_kh+\tau_{t_k},t_{k+1}h+\tau_{t_{k+1}})\tag{3}$$

where h is the sampling period, t_kh is the instant that the sampled data is transmitting, τ_{t_k} is the corresponding network-induced delay. Similar to [31], define the network allowable equivalent delay $\tau(t)=t-t_kh$, Eq. (3) can be written as follows

$$y_1(t)=Cx(t-\tau(t))\tag{4}$$

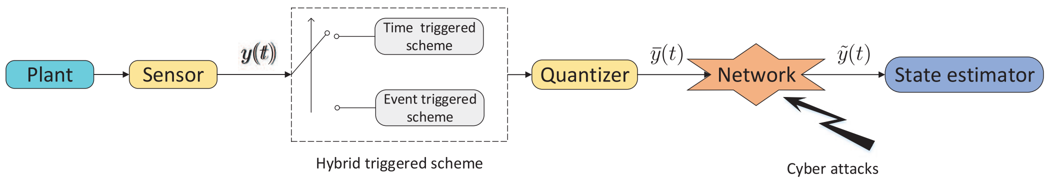


Fig. 1. The structure of hybrid-driven-based state estimator for neural networks with quantization subject to cyber attacks.

where $\tau(t)\in[0,\ \tau_M]$, τ_M is the upper bound of time-delay.

As is shown in Fig. 1, when "event triggered scheme" is activated in the hybrid triggered scheme, the event generator function can be defined as the following judgment algorithm, which is similar in [8,32]:

$$[y(((k+j)h))-y(kh)]^T\Phi[y((k+j)h)-y(kh)] \leq \sigma y^T((k+j)h)\Phi y((k+j)h) \tag{5}$$

where $\Phi>0$ is a symmetric positive define matrix, $j=1,2,\cdots$, $\sigma\in[0,\ 1)$, $y((k+j)h)$ is the newly sampling data, and $y(kh)$ is the latest delivered data. The sampled signal $y((k+j)h)$ which satisfies the inequality (5) will not be transmitted, only the one that exceeds the threshold in (5) will be delivered.

As a matter of convenience, the interval $[t_kh+\tau_{t_k}, t_{k+1}h+\tau_{t_{k+1}})$ can be divided into several subintervals. Suppose that there exists a constant ϱ which satisfies $[t_kh+\tau_{t_k}, t_{k+1}h+\tau_{t_{k+1}})=\bigcup_{j=1}^{\varrho}\Lambda_j$, where $\Lambda_j=[t_kh+jh+\eta_{t_k+j}, t_kh+jh+h+\eta_{t_k+j+1}]$, $j=\{0,1,2,\cdots,\varrho\}$ $\varrho=t_{k+1}-t_k-1$. Define $\eta(t)=t-t_kh-jh$, $0\leq\tau_{t_k}\leq\eta(t)\leq h+\eta_{t_{k+j+1}}\triangleq\eta_M$. Let $e_k(t)=y(t_kh)-y(t_kh+jh)$. Under the event triggered scheme, the input signal of the filter $y_2(t)$ can be written as follows:

$$y_2(t)=Cx(t-\eta(t))+e_k(t) \tag{6}$$

Remark 1. An effective triggered scheme named hybrid triggered scheme is proposed in [13], in which the stochastic switch between the time triggered scheme and the event triggered scheme is described by a random variable satisfying Bernoulli distribution. In this paper, by utilizing the methods in [13], the hybrid triggered scheme is introduced to alleviate the burden of network and increase utilization of network resources. Based on the hybrid triggered scheme, the state estimation for neural networks is investigated in this paper with quantization and cyber attacks.

By applying the modeling approach of hybrid triggered scheme in [13], the real input of the state estimator $\bar{y}(t)$ under the hybrid triggered scheme can be written as follows:

$$\begin{aligned}\bar{y}(t)&=\alpha(t)y_1(t)+(1-\alpha(t))y_2(t)\\&=\alpha(t)Cx(t-\tau(t))+(1-\alpha(t))[Cx(t-\eta(t))+e_k(t)]\end{aligned} \tag{7}$$

where $\alpha(t)\ \in\ [0,\ 1]$, $\alpha(t)$ has the following statistical properties.

$$\mathbb{E}\{\alpha(t)\}=\bar{\alpha}, \mathbb{E}\{(\alpha(t)-\bar{\alpha})^2\}=\bar{\alpha}(1-\bar{\alpha})=\rho_1^2$$

$\bar{\alpha}$ represents the expectation of $\alpha(t)$, ρ_1^2 is utilized to represent the mathematical variance of $\alpha(t)$.

Inspired by the effect of quantization in [33,34], the quantitative process is introduced to save the bandwidth of the network further in the design of state estimator. The following function is introduced:

$$q(y)=diag\{q_1,q_2,\ldots,q_m\}, \tag{8}$$

where $q(\cdot)$ is symmetric, that is $q_i(-y_i)=-q_i(y_i)$.

The logarithmic quantizer can be defined as follows:

$$q_i(y_i)=\begin{cases}u_l^{(i)}, & if\ \dfrac{1}{1+\delta_{q_i}}u_l^{(i)}<y_i<\dfrac{1}{1-\delta_{q_i}}u_l^{(i)},\ y_i>0,\\ 0, & if\ y_i=0,\\ -q_i(-y_i), & if\ y_i<0\end{cases} \tag{9}$$

where $\delta_{q_i}=\frac{1-\rho_{q_i}}{1+\rho_{q_i}}$ $(0<\rho_{q_i}<1)$, ρ_{q_i} is quantization resolution. For simplicity, we assume that $\delta_{q_i}=\delta_q$, δ_q is a constant, $q(\cdot)$ $(i=1,2,\cdots,m)$ can be described as follows:

$$q(y)=(I+\Delta_q)y \tag{10}$$

in which $\Delta_q=diag\{\Delta_{q_1},\Delta_{q_2},\cdots,\Delta_{q_m}\}$.

Combine (7) and (10), the real input of the state estimator can be written as:

$$\begin{aligned}\bar{y}(t)&=(I+\Delta_q)\alpha(t)Cx(t-\tau(t))+(I+\Delta_q)(1-\alpha(t))\\&\quad\times[Cx(t-\eta(t))+e_k(t)]\end{aligned} \tag{11}$$

Remark 2. The quantitative process which aims at saving limited bandwidth and network resources further has drawn much attention of many researchers in recent years [18,19,33]. In this paper, the quantization is considered for the state estimation of neural networks with hybrid triggered scheme subject to cyber attacks.

As is shown in Fig. 1, the sampling data is vulnerable to the malicious signals. In this paper, when the sampling data is delivered in the network, the effect of cyber attacks is considered. After the cyber attacks lunched, the real sampling data combined with the aggressive signals is delivered to the state estimator. The real input of the state estimator can be described as follows:

$$\begin{aligned}\tilde{y}(t)&=(1-\theta(t))\bar{y}(t)+\theta(t)Cf(x(t-d(t)))\\&=(1-\theta(t))(I+\Delta_q)\alpha(t)Cx(t-\tau(t))+(1-\theta(t))(I+\Delta_q)\\&\quad\times(1-\alpha(t))[Cx(t-\eta(t))+e_k(t)]+\theta(t)Cf(x(t-d(t)))\end{aligned} \tag{12}$$

where $f(x(t))$ represents cyber attacks, $d(t)$ is time-varying delay of cyber attacks, $d(t)\in[0,\ d_M]$, d_M is the upper bound of $d(t)$. $\theta(t)$ is used to describe the occurring probability of the stochastic cyber attacks, $\theta(t)\ \in\ [0,\ 1]$.

Remark 3. As is described in [21,22], cyber attacks are essentially a kind of methods, processes, or means which are utilized to maliciously reduce network reliability. In this paper, in order to be closer to the actual situation of neural network, cyber attacks are considered for the state estimation of neural networks with hybrid triggered scheme and quantization.

It should be noted that Bernoulli variables $\alpha(t)$ and $\theta(t)$ are independent of each other. $\theta(t)$ has the following statistical properties.

$$\mathbb{E}\{\theta(t)\}=\bar{\theta},\quad \mathbb{E}\{(\theta(t)-\bar{\theta})^2\}=\bar{\theta}(1-\bar{\theta})=\rho_2^2$$

where $\bar{\theta}$ represents the expectation of $\theta(t)$, ρ_2^2 is utilized to represent the mathematical variance of $\theta(t)$.

Remark 4. In (12), $\theta(t)$ which satisfies Bernoulli distribution is employed to depict the randomly occurring probability of cyber attacks. When $\theta(t)=1$, the aggressive signals are delivered instead of real sensor measurement, the real input in (12) is $\tilde{y}(t)=Cf(x(t-d(t)))$. $\theta(t)=0$ means the network environment is safe and the effect of cyber attacks does not be taken into consideration, the real input can be expressed as $\tilde{y}(t)=\alpha(t)(I+\triangle_q)Cx(t-\tau(t))+(1-\alpha(t))(I+\triangle_q)[Cx(t-\eta(t))+Ce_k(t)]$.

Remark 5. In [35–38], random variables which satisfy Bernoulli distribution are employed to described the stochastic disturbance and missing measurements. In this paper, $\alpha(t)$ obeying Bernoulli distribution describes the stochastic switch between the time triggered scheme and the event triggered scheme. $\theta(t)$ which obeys Bernoulli distribution is employed to describe randomly occurring probability of cyber attacks. It should be noted that $\alpha(t)$ and $\theta(t)$ are independent of each other.

Define $e(t)=x(t)-\hat{x}(t)$, combine (1), (2), (10) and (12), one can get

$$\begin{aligned}\dot{e}(t)=&-(A+KC)e(t)+KCx(t)-(1-\theta(t))\alpha(t)\\&\times KC(I+\Delta_q)x(t-\tau(t))\\&-(1-\theta(t))(1-\alpha(t))KC(I+\Delta_q)x(t-\eta(t))\\&-(1-\theta(t))(1-\alpha(t))K(I+\Delta_q)e_k(t)\\&-\theta(t)KCf(x(t-d(t)))+W_0g(x(t))+W_1g(x(t-(\phi(t))))\end{aligned} \tag{13}$$

Let $\bar{x}(t)=[x^T(t)\quad e^T(t)]^T$, combine (1) and (13), the following augmented system can be obtained:

$$\begin{aligned}\dot{\bar{x}}(t)=&\bar{A}\bar{x}(t)+(1-\theta(t))\alpha(t)\bar{B}\bar{x}(t-\tau(t))\\&+(1-\theta(t))(1-\alpha(t))\bar{B}\bar{x}(t-\eta(t))\\&+(1-\theta(t))(1-\alpha(t))\bar{C}e_k(t)+\theta(t)\bar{D}f(H\bar{x}(t-d(t)))\\&+\bar{W}_0g(H\bar{x}(t))+\bar{W}_1g(H\bar{x}(t-\phi(t)))\end{aligned} \tag{14}$$

where

$$\bar{A}=\begin{bmatrix}-A & 0\\ KC & -(A+KC)\end{bmatrix},\quad \bar{B}=\begin{bmatrix}0 & 0\\ -KC(I+\Delta_q) & 0\end{bmatrix},$$
$$\bar{C}=\begin{bmatrix}0\\ -K(I+\Delta_q)\end{bmatrix},\quad \bar{D}=\begin{bmatrix}0\\ -KC\end{bmatrix}$$
$$\bar{W}_0=\begin{bmatrix}W_0\\ 0\end{bmatrix},\quad \bar{W}_1=\begin{bmatrix}W_1\\ 0\end{bmatrix},\quad H^T=\begin{bmatrix}I\\ 0\end{bmatrix}$$

In the following, a definition, some lemmas and assumptions will be introduced, which will help us in deriving the main results.

Definition 1 [39]. For given function $V{:}C^b_{F_0}([-\tau_M,0],R^n)\times S$, its infinitesimal operator $\mathcal{L}$ is defined as

$$\mathcal{L}(V_{\eta(t)})=\lim_{\triangle\to 0+}\frac{1}{\triangle}[\mathbb{E}(V(\eta_t+\triangle)|\eta_t)-V(\eta_t)] \tag{15}$$

Assumption 1 [15,40]. Cyber attacks $f(x(t))$ satisfy the following condition:

$$||f(x(t))||_2\le ||Gx(t)||_2 \tag{16}$$

where G is a constant matrix representing the upper bound of the nonlinearity.

Assumption 2 [41]. The neural activation function satisfies the following condition, and U_1, U_2 are real constant matrices and satisfy $U_2-U_1\ge 0$:

$$[g(x)-U_1x]^T[g(x)-U_2x]\le 0 \tag{17}$$

Lemma 1 [42]. *For the given instant τ_1 and matrix $R>0$, the following inequality is established:*

$$-\tau_1\int_{t-\tau_1}^{t}\dot{x}^T(s)R\dot{x}(s)\le\begin{bmatrix}x(t)\\ x(t-\tau_1)\end{bmatrix}^T\begin{bmatrix}-R & R\\ R & -R\end{bmatrix}\begin{bmatrix}x(t)\\ x(t-\tau_1)\end{bmatrix} \tag{18}$$

Lemma 2 [43]. *For any vector x, $y\in R^n$ and matrix $Q\in R^{n\times n}$ with appropriate dimensions, the following inequality is established:*

$$2x^Ty\le x^TQx+y^TQ^{-1}y \tag{19}$$

Lemma 3 [44]. *Given matrices M, N and P of appropriate dimensions, supposing P is symmetrical,*

$$P+MF(t)N+N^TF^T(t)M^T<0 \tag{20}$$

for any $F(t)$ satisfying $F(t)^TF(t)\le I$, if and only if there exists a parameter $\epsilon>0$ such that

$$P+\epsilon^{-1}MM^T+\epsilon NN^T<0 \tag{21}$$

Lemma 4 [45]. *Supposed $\tau(t)\in[0,\ \tau_M]$, $\phi(t)\in[\phi_m,\ \phi_M]$, $\eta(t)\in[0,\ \eta_M]$, $d(t)\in[0,\ d_M]$, $\Psi_i(i=1,\cdots,8)$ are matrices with appropriate dimensions, the inequality $(\phi(t)-\phi_m)\Psi_1+(\phi_M-\phi(t))\Psi_2+\tau(t)\Psi_3+(\tau_M-\tau(t))\Psi_4+\eta(t)\Psi_5+(\eta_M-\eta(t))\Psi_6+d(t)\Psi_7+(d_M-d(t))\Psi_8+\Xi<0$ is established, if and only if the following inequalities are established:*

$$\begin{cases}\phi_1\Psi_2+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & \phi_1\Psi_2+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\ \phi_1\Psi_2+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & \phi_1\Psi_2+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_7+\Xi<0\\ \phi_1\Psi_2+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & \phi_1\Psi_2+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\ \phi_1\Psi_2+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & \phi_1\Psi_2+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_7+\Xi<0\\ \phi_1\Psi_1+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & \phi_1\Psi_1+\tau_M\Psi_4+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\ \phi_1\Psi_1+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & \phi_1\Psi_1+\tau_M\Psi_4+\eta_M\Psi_5+d_M\Psi_7+\Xi<0\\ \phi_1\Psi_1+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_8+\Xi<0, & \phi_1\Psi_1+\tau_M\Psi_3+\eta_M\Psi_6+d_M\Psi_7+\Xi<0\\ \phi_1\Psi_1+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_8+\Xi<0, & \phi_1\Psi_1+\tau_M\Psi_3+\eta_M\Psi_5+d_M\Psi_7+\Xi<0\end{cases} \tag{22}$$

where $\phi_1=\phi_M-\phi_m$

3. Main results

In this section, by applying Lyapunov functional approach and linear matrix inequality technique, considering the hybrid triggered scheme and cyber attacks, the sufficient conditions will be derived which guarantee the stability of the estimation error system (14).

Theorem 1. *For given parameters: the bounds of time-delay ϕ_m, ϕ_M, τ_M, η_M and d_M, the event triggered parameter σ, the expectations of Bernoulli variables $\bar{\alpha}$ and $\bar{\theta}$, the constant matrix G, the augmented system (14) is asymptotically stable, if there exist matrices $P>0$, $Q_k>0$, $R_k>0$ $(k=1,2,3,4,5)$, $\Omega>0$, L, Y, M, N, W, V, Z, S with appropriate dimension and parameters $\alpha>0$, $\beta>0$ satisfying:*

$$\Omega(s)=\begin{bmatrix}\Omega_{11}+\Gamma+\Gamma^T & * & * & *\\ \Omega_{21} & \Omega_{22} & * & *\\ \Omega_{31} & 0 & \Omega_{33} & *\\ \Omega_{41}(s) & 0 & 0 & \Omega_{44}\end{bmatrix}<0,$$
$$s=1,2,3,\cdots,16 \tag{23}$$

where

$$\Omega_{11}=\begin{bmatrix}\Gamma_1 & * & * \\ \Gamma_2 & \Gamma_3 & * \\ \Gamma_4 & 0 & \Gamma_5\end{bmatrix},\quad \Gamma=\begin{bmatrix}\Phi_1 & \Phi_2\end{bmatrix},$$

$$\Phi_1=\begin{bmatrix}M+Z+W & L & -L+Y & -Y & -M+N\end{bmatrix}$$

$$\Phi_2=\begin{bmatrix}-N & -Z+S & -S & -W+V & -V & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Gamma_1=\begin{bmatrix}\Lambda_1 & * & * & * & * \\ R_2 & -Q_1-R_2 & * & * & * \\ 0 & 0 & -\beta\bar{U}_1 & * & * \\ 0 & 0 & 0 & -Q_2 & * \\ \Lambda_2 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Gamma_2=\begin{bmatrix}0 & 0 & 0 & 0 & 0 \\ \Lambda_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\Gamma_4=\begin{bmatrix}\Lambda_4 & 0 & 0 & 0 & 0 \\ \bar{W}_1^TP & 0 & -\beta\bar{U}_2^T & 0 & 0 \\ \Lambda_5 & 0 & 0 & 0 & 0 \\ \bar{\theta}\bar{D}^TP & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Gamma_3=diag\{-Q_3,\sigma C^TH^T\Omega HC,-Q_4,\bar{\theta}H^TGGH,-Q_5\}$$

$$\Gamma_5=diag\{-\alpha I,-\beta I,-\Omega,-\bar{\theta}I\},\quad \Lambda_1=P\bar{A}+\bar{A}^TP+Q_1+Q_2+Q_3+Q_4+Q_5-R_2-\alpha\bar{U}_1$$

$$\Lambda_2=(1-\bar{\theta})\bar{\alpha}\bar{B}^TP,\quad \Lambda_3=(1-\bar{\theta})(1-\bar{\alpha})\bar{B}^TP,$$

$$\Lambda_4=\bar{W}_0^TP-\alpha\bar{U}_2^T,\quad \Lambda_5=(1-\bar{\theta})(1-\bar{\alpha})\bar{C}^TP$$

$$\bar{\alpha}_1=1-\bar{\alpha},\quad \bar{\theta}_1=1-\bar{\theta},\quad \rho_1=\sqrt{\bar{\alpha}(1-\bar{\alpha})},$$

$$\rho_2=\sqrt{\bar{\theta}(1-\bar{\theta})}$$

$$\Omega_{21}=\begin{bmatrix}\Gamma_6 & \Gamma_7 & \Gamma_8 & \Gamma_9 \\ 0 & \Gamma_{10} & 0 & \Gamma_{11} \\ 0 & \Gamma_{12} & 0 & \Gamma_{13}\end{bmatrix},\quad \Omega_{31}=\begin{bmatrix}0 & \Gamma_{14} & 0 & \Gamma_{15} \\ 0 & 0 & 0 & \Gamma_{16}\end{bmatrix},$$

$$\kappa=\sqrt{\phi_M-\phi_m}$$

$$\Gamma_6=\begin{bmatrix}\kappa R_1\bar{A} & 0 & 0 & 0 \\ \phi_mR_2\bar{A} & 0 & 0 & 0 \\ \sqrt{\tau_M}R_3\bar{A} & 0 & 0 & 0 \\ \sqrt{\eta_M}R_4\bar{A} & 0 & 0 & 0 \\ \sqrt{d_M}R_5\bar{A} & 0 & 0 & 0\end{bmatrix},$$

$$\Gamma_7=\begin{bmatrix}\bar{\theta}_1\bar{\alpha}\kappa R_1\bar{B} & 0 & \bar{\theta}_1\bar{\alpha}_1\kappa R_1\bar{B} & 0 \\ \bar{\theta}_1\bar{\alpha}\phi_mR_2\bar{B} & 0 & \bar{\theta}_1\bar{\alpha}_1\phi_mR_2\bar{B} & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{\tau_M}R_3\bar{B} & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}R_3\bar{B} & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{\eta_M}R_4\bar{B} & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}R_4\bar{B} & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{d_M}R_5\bar{B} & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}R_5\bar{B} & 0\end{bmatrix}$$

$$\Gamma_8=\begin{bmatrix}0 & 0 & \kappa R_1\bar{W}_0 & \kappa R_1\bar{W}_1 \\ 0 & 0 & \phi_mR_2\bar{W}_0 & \phi_mR_2\bar{W}_1 \\ 0 & 0 & \sqrt{\tau_M}R_3\bar{W}_0 & \sqrt{\tau_M}R_3\bar{W}_1 \\ 0 & 0 & \sqrt{\eta_M}R_4\bar{W}_0 & \sqrt{\eta_M}R_4\bar{W}_1 \\ 0 & 0 & \sqrt{d_M}R_5\bar{W}_0 & \sqrt{d_M}R_5\bar{W}_1\end{bmatrix},$$

$$\Gamma_9=\begin{bmatrix}\bar{\theta}_1\bar{\alpha}_1\kappa R_1\bar{C} & \bar{\theta}\kappa R_1\bar{D} \\ \bar{\theta}_1\bar{\alpha}_1\phi_mR_2\bar{C} & \bar{\theta}\phi_mR_2\bar{D} \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}R_3\bar{C} & \bar{\theta}\sqrt{\tau_M}R_3\bar{D} \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}R_4\bar{C} & \bar{\theta}\sqrt{\eta_M}R_4\bar{D} \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}R_5\bar{C} & \bar{\theta}\sqrt{d_M}R_5\bar{D}\end{bmatrix}$$

$$\Gamma_{10}=\begin{bmatrix}\bar{\theta}_1\rho_1\kappa R_1\bar{B} & 0 & -\bar{\theta}_1\rho_1\kappa R_1\bar{B} & 0 \\ \bar{\theta}_1\rho_1\phi_mR_2\bar{B} & 0 & -\bar{\theta}_1\rho_1\phi_mR_2\bar{B} & 0 \\ \bar{\theta}_1\rho_1\sqrt{\tau_M}R_3\bar{B} & 0 & -\bar{\theta}_1\rho_1\sqrt{\tau_M}R_3\bar{B} & 0 \\ \bar{\theta}_1\rho_1\sqrt{\eta_M}R_4\bar{B} & 0 & -\bar{\theta}_1\rho_1\sqrt{\eta_M}R_4\bar{B} & 0 \\ \bar{\theta}_1\rho_1\sqrt{d_M}R_5\bar{B} & 0 & -\bar{\theta}_1\rho_1\sqrt{d_M}R_5\bar{B} & 0\end{bmatrix},$$

$$\Gamma_{11}=\begin{bmatrix}-\bar{\theta}_1\rho_1\kappa R_1\bar{C} & 0 \\ -\bar{\theta}_1\rho_1\phi_mR_2\bar{C} & 0 \\ -\bar{\theta}_1\rho_1\sqrt{\tau_M}R_3\bar{C} & 0 \\ -\bar{\theta}_1\rho_1\sqrt{\eta_M}R_4\bar{C} & 0 \\ -\bar{\theta}_1\rho_1\sqrt{d_M}R_5\bar{C} & 0\end{bmatrix}$$

$$\Gamma_{12}=\begin{bmatrix}\bar{\alpha}\rho_2\kappa R_1\bar{B} & 0 & \bar{\alpha}_1\rho_2\kappa R_1\bar{B} & 0 \\ \bar{\alpha}\rho_2\phi_mR_2\bar{B} & 0 & \bar{\alpha}_1\rho_2\phi_mR_2\bar{B} & 0 \\ \bar{\alpha}\rho_2\sqrt{\tau_M}R_3\bar{B} & 0 & \bar{\alpha}_1\rho_2\sqrt{\tau_M}R_3\bar{B} & 0 \\ \bar{\alpha}\rho_2\sqrt{\eta_M}R_4\bar{B} & 0 & \bar{\alpha}_1\rho_2\sqrt{\eta_M}R_4\bar{B} & 0 \\ \bar{\alpha}\rho_2\sqrt{d_M}R_5\bar{B} & 0 & \bar{\alpha}_1\rho_2\sqrt{d_M}R_5\bar{B} & 0\end{bmatrix},$$

$$\Gamma_{13}=\begin{bmatrix}\bar{\alpha}_1\rho_2\kappa R_1\bar{C} & 0 \\ \bar{\alpha}_1\rho_2\phi_mR_2\bar{C} & 0 \\ \bar{\alpha}_1\rho_2\sqrt{\tau_M}R_3\bar{C} & 0 \\ \bar{\alpha}_1\rho_2\sqrt{\eta_M}R_4\bar{C} & 0 \\ \bar{\alpha}_1\rho_2\sqrt{d_M}R_5\bar{C} & 0\end{bmatrix}$$

$$\Gamma_{14}=\begin{bmatrix}\kappa\rho_1\rho_2R_1\bar{B} & 0 & -\kappa\rho_1\rho_2R_1\bar{B} & 0 \\ \phi_m\rho_1\rho_2R_2\bar{B} & 0 & -\phi_m\rho_1\rho_2R_2\bar{B} & 0 \\ \sqrt{\tau_M}\rho_1\rho_2R_3\bar{B} & 0 & -\sqrt{\tau_M}\rho_1\rho_2R_3\bar{B} & 0 \\ \sqrt{\eta_M}\rho_1\rho_2R_4\bar{B} & 0 & -\sqrt{\eta_M}\rho_1\rho_2R_4\bar{B} & 0 \\ \sqrt{d_M}\rho_1\rho_2R_5\bar{B} & 0 & -\sqrt{d_M}\rho_1\rho_2R_5\bar{B} & 0\end{bmatrix},$$

$$\Gamma_{15}=\begin{bmatrix}-\kappa\rho_1\rho_2R_1\bar{C} & 0 \\ -\phi_m\rho_1\rho_2R_2\bar{C} & 0 \\ -\sqrt{\tau_M}\rho_1\rho_2R_3\bar{C} & 0 \\ -\sqrt{\eta_M}\rho_1\rho_2R_4\bar{C} & 0 \\ -\sqrt{d_M}\rho_1\rho_2R_5\bar{C} & 0\end{bmatrix},\quad \Gamma_{16}=\begin{bmatrix}0 & \kappa\rho_2R_1\bar{D} \\ 0 & \phi_m\rho_2R_2\bar{D} \\ 0 & \sqrt{\tau_M}\rho_2R_3\bar{D} \\ 0 & \sqrt{\eta_M}\rho_2R_4\bar{D} \\ 0 & \sqrt{d_M}\rho_2R_5\bar{D}\end{bmatrix}$$

$$\Lambda_6=\rho_1\rho_2P\bar{B},\quad \Omega_{22}=diag\{\Upsilon,\Upsilon,\Upsilon\},\quad \Omega_{33}=diag\{\Upsilon,\Upsilon\},$$

$$\Upsilon=diag\{-R_1,-R_2,-R_3,-R_4,-R_5\}$$

$$\Omega_{41}(1)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{41}(2)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}W^T\end{bmatrix},$$

$$\Omega_{41}(3)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{41}(4)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}W^T\end{bmatrix}$$

$$\Omega_{41}(5)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{41}(6)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}W^T\end{bmatrix},$$

$$\Omega_{41}(7)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{41}(8)=\begin{bmatrix}\kappa Y^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}W^T\end{bmatrix}$$

$$\Omega_{41}(9)=\begin{bmatrix}\kappa L^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{41}(10)=\begin{bmatrix}\kappa L^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}W^T\end{bmatrix},$$

$$\Omega_{41}(11)=\begin{bmatrix}\kappa L^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}V^T\end{bmatrix},\quad \Omega_{41}(12)=\begin{bmatrix}\kappa L^T \\ \sqrt{\tau_M}N^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}W^T\end{bmatrix}$$

$$\Omega_{41}(13) = \begin{bmatrix} \kappa L^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}V^T \end{bmatrix}, \quad \Omega_{41}(14) = \begin{bmatrix} \kappa L^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}S^T \\ \sqrt{d_M}W^T \end{bmatrix},$$

$$\Omega_{41}(15) = \begin{bmatrix} \kappa L^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}V^T \end{bmatrix}, \quad \Omega_{41}(16) = \begin{bmatrix} \kappa L^T \\ \sqrt{\tau_M}M^T \\ \sqrt{\eta_M}Z^T \\ \sqrt{d_M}W^T \end{bmatrix}$$

$$\begin{aligned}
\Omega_{44} &= diag\{-R_1, -R_3, -R_4, -R_5\}, \\
Y^T &= \begin{bmatrix} 0_{1\times 2} & Y_3^T & Y_4^T & 0_{1\times 10} \end{bmatrix}, \\
L^T &= \begin{bmatrix} 0 & L_2^T & L_3^T & 0_{1\times 11} \end{bmatrix} \\
N^T &= \begin{bmatrix} 0_{1\times 4} & N_5^T & N_6^T & 0_{1\times 8} \end{bmatrix}, \\
M^T &= \begin{bmatrix} M_1^T & 0_{1\times 3} & M_5^T & 0_{1\times 9} \end{bmatrix}, \\
S^T &= \begin{bmatrix} 0_{1\times 6} & S_7^T & S_8^T & 0_{1\times 6} \end{bmatrix} \\
Z^T &= \begin{bmatrix} Z_1^T & 0_{1\times 5} & Z_7^T & 0_{1\times 7} \end{bmatrix}, \\
V^T &= \begin{bmatrix} 0_{1\times 8} & V_9^T & V_{10}^T & 0_{1\times 4} \end{bmatrix}, \\
W^T &= \begin{bmatrix} W_1^T & 0_{1\times 7} & W_9^T & 0_{1\times 5} \end{bmatrix}
\end{aligned}$$

Proof. Choose the following Lyapunov functional candidate as

$$V(t) = V_1(\bar{x}_t) + V_2(\bar{x}_t) + V_3(\bar{x}_t) \tag{24}$$

where

$$\begin{aligned}
V_1(\bar{x}_t) &= \bar{x}^T(t)P\bar{x}(t) \\
V_2(\bar{x}_t) &= \int_{t-\phi_m}^{t} \bar{x}^T(s)Q_1\bar{x}(s)ds + \int_{t-\phi_M}^{t} \bar{x}^T(s)Q_2\bar{x}(s)ds \\
&\quad + \int_{t-\tau_M}^{t} \bar{x}^T(s)Q_3\bar{x}(s)ds + \int_{t-\eta_M}^{t} \bar{x}^T(s)Q_4\bar{x}(s)ds \\
&\quad + \int_{t-d_M}^{t} \bar{x}^T(s)Q_5\bar{x}(s)ds \\
V_3(\bar{x}_t) &= \int_{t-\phi_M}^{t-\phi_m}\int_s^t \dot{\bar{x}}^T(v)R_1\dot{\bar{x}}(v)dvds + \phi_m\int_{t-\phi_m}^{t}\int_s^t \dot{\bar{x}}^T(v)R_2\dot{\bar{x}}(v)dvds \\
&\quad + \int_{t-\tau_M}^{t}\int_s^t \dot{\bar{x}}^T(v)R_3\dot{\bar{x}}(v)dvds \\
&\quad + \int_{t-\eta_M}^{t}\int_s^t \dot{\bar{x}}^T(v)R_4\dot{\bar{x}}(v)dvds + \int_{t-d_M}^{t}\int_s^t \dot{\bar{x}}^T(v)R_5\dot{\bar{x}}(v)dvds
\end{aligned}$$

Apply the infinitesimal operator (Definition 1) for $V_k(t)$ $(k = 1, 2, 3)$ and take expectation on it, one can get:

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_1(\bar{x}_t)\} &= 2\bar{x}^T(t)P[\bar{A}\bar{x}(t) + \bar{W}_0\bar{g}(H\bar{x}(t)) + \bar{W}_1\bar{g}(H\bar{x}(t-\phi(t))) \\
&\quad + \bar{\theta}_1\bar{\alpha}\bar{B}\bar{x}(t-\tau(t)) + \bar{\theta}_1\bar{\alpha}_1\bar{B}\bar{x}(t-\eta(t)) + \bar{\theta}_1\bar{\alpha}_1\bar{C}e_k(t) \\
&\quad + \bar{\theta}\bar{D}f(H\bar{x}(t-d(t)))]
\end{aligned} \tag{25}$$

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_2(\bar{x}_t)\} &= \bar{x}^T(t)(Q_1+Q_2+Q_3+Q_4+Q_5)\bar{x}(t) \\
&\quad - \bar{x}^T(t-\phi_m)Q_1\bar{x}(t-\phi_m) \\
&\quad - \bar{x}^T(t-\phi_M)Q_2\bar{x}(t-\phi_M) - \bar{x}^T(t-\tau_M)Q_3\bar{x}(t-\tau_M) \\
&\quad - \bar{x}^T(t-\eta_M)Q_4\bar{x}(t-\eta_M) - \bar{x}^T(t-d_M)Q_5\bar{x}(t-d_M)
\end{aligned} \tag{26}$$

$$\begin{aligned}
\mathbb{E}\{\mathcal{L}V_3(\bar{x}_t)\} &= \mathbb{E}\{\dot{\bar{x}}^T(t)\tilde{R}\dot{\bar{x}}(t)\} - \int_{t-\phi_M}^{t-\phi_m}\dot{\bar{x}}^T(s)R_1\dot{\bar{x}}(s)ds \\
&\quad - \phi_m\int_{t-\phi_m}^{t}\dot{\bar{x}}^T(s)R_2\dot{\bar{x}}(s)ds - \int_{t-\tau_M}^{t}\dot{\bar{x}}^T(s)R_3\dot{\bar{x}}(s)ds \\
&\quad - \int_{t-\eta_M}^{t}\dot{\bar{x}}^T(s)R_4\dot{\bar{x}}(s)ds - \int_{t-d_M}^{t}\dot{\bar{x}}^T(s)R_5\dot{\bar{x}}(s)ds
\end{aligned} \tag{27}$$

in which

$$\begin{aligned}
\mathbb{E}(\dot{\bar{x}}^T(t)\tilde{R}\dot{\bar{x}}(t)) &= \mathcal{A}^T\tilde{R}\mathcal{A} + \bar{\theta}_1^2\rho_2^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 \\
&\quad + \rho_2^2\mathcal{B}_2^T\tilde{R}\mathcal{B}_2 + \rho_1^2\rho_2^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 + \rho_2^2F_c^T\bar{D}^T\tilde{R}\bar{D}F_c
\end{aligned} \tag{28}$$

where

$$\begin{aligned}
\mathcal{A} &= \bar{A}\bar{x}(t) + \bar{W}_0\bar{g}(H\bar{x}(t)) + \bar{W}_1\bar{g}(H\bar{x}(t-\phi(t))) + \bar{\theta}_1\bar{\alpha}\bar{B}\bar{x}(t-\tau(t)) \\
&\quad + \bar{\theta}_1\bar{\alpha}_1\bar{B}\bar{x}(t-\eta(t)) + \bar{\theta}_1\bar{\alpha}_1\bar{C}e_k(t) + \bar{\theta}\bar{D}f(H\bar{x}(t-d(t))) \\
\mathcal{B}_1 &= \bar{B}\bar{x}(t-\tau(t)) - \bar{B}\bar{x}(t-\eta(t)) - \bar{C}e_k(t), \\
\mathcal{B}_2 &= \bar{B}\bar{\alpha}e(t-\tau(t)) + \bar{B}\bar{\alpha}_1e(t-\eta(t)) + \bar{C}\bar{\alpha}_1e_k(t) \\
\tilde{R} &= (\phi_M - \phi_m)R_1 + \phi_m^2R_2 + \tau_MR_3 + \eta_MR_4 + d_MR_5, \\
F_c &= f(H\bar{x}(t-d(t)))
\end{aligned}$$

From Assumption 1, which is the limited condition of cyber-attacks, we can obtain

$$\bar{\theta}\bar{x}^T(t-d(t))H^TG^TGH\bar{x}(t-d(t)) - \bar{\theta}f^T(H\bar{x}(t-d(t))f(H\bar{x}(t-d(t))) \geq 0 \tag{29}$$

By utilizing the free-weighting matrices method [46,47], it can be obtained that:

$$2\xi^T(t)L\left[\bar{x}(t-\phi_m) - \bar{x}(t-\phi(t)) - \int_{t-\phi(t)}^{t-\phi_m}\dot{\bar{x}}(s)d_s\right] = 0 \tag{30}$$

$$2\xi^T(t)Y\left[\bar{x}(t-\phi(t)) - \bar{x}(t-\phi_M) - \int_{t-\phi_M}^{t-\phi(t)}\dot{\bar{x}}(s)d_s\right] = 0 \tag{31}$$

$$2\xi^T(t)M\left[\bar{x}(t) - \bar{x}(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)d_s\right] = 0 \tag{32}$$

$$2\xi^T(t)N\left[\bar{x}(t-\tau(t)) - \bar{x}(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot{\bar{x}}(s)d_s\right] = 0 \tag{33}$$

$$2\xi^T(t)Z\left[\bar{x}(t) - \bar{x}(t-\eta(t)) - \int_{t-\eta(t)}^{t}\dot{\bar{x}}(s)d_s\right] = 0 \tag{34}$$

$$2\xi^T(t)S\left[\bar{x}(t-\eta(t)) - \bar{x}(t-\eta_M) - \int_{t-\eta_M}^{t-\eta(t)}\dot{\bar{x}}(s)d_s\right] = 0 \tag{35}$$

$$2\xi^T(t)W\left[\bar{x}(t) - \bar{x}(t-d(t)) - \int_{t-d(t)}^{t}\dot{\bar{x}}(s)d_s\right] = 0 \tag{36}$$

$$2\xi^T(t)V\left[\bar{x}(t-d(t)) - \bar{x}(t-d_M) - \int_{t-d_M}^{t-d(t)}\dot{\bar{x}}(s)d_s\right] = 0 \tag{37}$$

where L, Y, N, M, Z, S, W, V are matrices with appropriate dimensions, and $\xi^T(t)$ is defined as follows

$$\begin{aligned}
\xi^T(t) &= \begin{bmatrix} \xi_1^T(t) & \xi_2^T(t) & \xi_3^T(t) \end{bmatrix} \\
\xi_1^T(t) &= \begin{bmatrix} \bar{x}^T(t) & \bar{x}^T(t-\phi_m) & \bar{x}^T(t-\phi(t)) & \bar{x}^T(t-\phi_M) & \bar{x}^T(t-\tau(t)) \end{bmatrix} \\
\xi_2^T(t) &= \begin{bmatrix} \bar{x}^T(t-\tau_M) & \bar{x}^T(t-\eta(t) & \bar{x}^T(t-\eta_M) & \bar{x}^T(t-d(t) \\ \bar{x}^T(t-d_M) & \bar{g}^T(H\bar{x}(t)) \end{bmatrix} \\
\xi_3^T(t) &= \begin{bmatrix} \bar{g}^T(H\bar{x}(t-\phi(t))) & e_k^T(t) & f^T(H\bar{x}(t-d(t))) \end{bmatrix}
\end{aligned}$$

By using Lemma 2, we have

$$\begin{aligned}
-2\xi^T(t)L\int_{t-\phi(t)}^{t-\phi_m}\dot{\bar{x}}(s)ds &\leq (\phi(t)-\phi_m)\xi^T(t)LR_1^{-1}L^T\xi(t) \\
&\quad + \int_{t-\phi(t)}^{t-\phi_m}\dot{\bar{x}}^T(s)R_1\dot{\bar{x}}(s)ds
\end{aligned} \tag{38}$$

$$\begin{aligned}
-2\xi^T(t)Y\int_{t-\phi_M}^{t-\phi(t)}\dot{\bar{x}}(s)ds &\leq (\phi_M-\phi(t))\xi^T(t)YR_1^{-1}Y^T\xi(t) \\
&\quad + \int_{t-\phi_M}^{t-\phi(t)}\dot{\bar{x}}^T(s)R_1\dot{\bar{x}}(s)ds
\end{aligned} \tag{39}$$

$$-2\xi^T(t)M\int_{t-\tau(t)}^{t}\dot{\bar{x}}(s)ds \le \tau(t)\xi^T(t)MR_3^{-1}M^T\xi(t) + \int_{t-\tau(t)}^{t}\dot{\bar{x}}^T(s)R_3\dot{\bar{x}}(s)ds \tag{40}$$

$$-2\xi^T(t)N\int_{t-\tau_M}^{t-\tau(t)}\dot{\bar{x}}(s)ds \le (\tau_M-\tau(t))\xi^T(t)NR_3^{-1}N^T\xi(t) + \int_{t-\eta_M}^{t-\eta(t)}\dot{\bar{x}}^T(s)R_3\dot{\bar{x}}(s)ds \tag{41}$$

$$-2\xi^T(t)Z\int_{t-\eta(t)}^{t}\dot{\bar{x}}(s)ds \le \eta(t)\xi^T(t)ZR_4^{-1}Z^T\xi(t) + \int_{t-\eta(t)}^{t}\dot{\bar{x}}^T(s)R_4\dot{\bar{x}}(s)ds \tag{42}$$

$$-2\xi^T(t)S\int_{t-\eta_M}^{t-\eta(t)}\dot{\bar{x}}(s)ds \le (\eta_M-\eta(t))\xi^T(t)SR_4^{-1}S^T\xi(t) + \int_{t-\eta_M}^{t-\eta(t)}\dot{\bar{x}}^T(s)R_4\dot{\bar{x}}(s)ds \tag{43}$$

$$-2\xi^T(t)W\int_{t-d(t)}^{t}\dot{\bar{x}}(s)ds \le d(t)\xi^T(t)WR_5^{-1}W^T\xi(t) + \int_{t-d(t)}^{t}\dot{\bar{x}}^T(s)R_5\dot{\bar{x}}(s)ds \tag{44}$$

$$-2\xi^T(t)V\int_{t-d_M}^{t-d(t)}\dot{\bar{x}}(s)ds \le (d_M-d(t))\xi^T(t)VR_5^{-1}V^T\xi(t) + \int_{t-d_M}^{t-d(t)}\dot{\bar{x}}^T(s)R_5\dot{\bar{x}}(s)ds \tag{45}$$

Considering the condition of event-triggered scheme (5), we can obtain that

$$\sigma\bar{x}^T(t-\eta(t))H^TC^T\Omega CH\bar{x}(t-\eta(t)) - e_k^T(t)\Omega e_k(t) \ge 0 \tag{46}$$

By using Lemma 1, notice that:

$$-\phi_m\int_{t-\phi_m}^{t}\dot{\bar{x}}^T(s)R_2\dot{\bar{x}}(s)ds \le \begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-\phi_m)\end{bmatrix}^T\begin{bmatrix}-R_2 & R_2\\ R_2 & -R_2\end{bmatrix} \times \begin{bmatrix}\bar{x}(t)\\ \bar{x}(t-\phi_m)\end{bmatrix} \tag{47}$$

By employing Assumption 2, we obtain that:

$$\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H\bar{x}(t))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & *\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H\bar{x}(t))\end{bmatrix} \le 0, \tag{48}$$

where $\bar{U}_1 = H^T\hat{U}_1H$, $\bar{U}_2 = -H^T\hat{U}_2$, $\hat{U}_1 = \frac{U_1^TU_2+U_2^TU_1}{2}$, $\hat{U}_2 = \frac{U_1^T+U_2^T}{2}$. So for the parameters $\alpha > 0$, $\beta > 0$, it is easy to get:

$$-\alpha\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H\bar{x}(t))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & *\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ \bar{g}(H\bar{x}(t))\end{bmatrix} \ge 0, \tag{49}$$

$$-\beta\begin{bmatrix}\bar{x}(t-\phi(t))\\ \bar{g}(H\bar{x}(t-\phi(t)))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & *\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t-\phi(t))\\ \bar{g}(H\bar{x}(t-\phi(t)))\end{bmatrix} \ge 0, \tag{50}$$

Combine (24)–(50), we can obtain that

$$\begin{aligned}\mathbb{E}\{\mathcal{L}V(\bar{x}(t))\} \le\ & \xi^T(t)(\Omega_{11}+\Gamma+\Gamma^T)\xi(t) + (\phi(t)-\phi_m)\xi^T(t)\\ & \times LR_1^{-1}L^T\xi(t) + (\phi_M-\phi(t))\xi^T(t)YR_1^{-1}Y^T\xi(t)\\ & + \tau(t)\xi^T(t)MR_3^{-1}M^T\xi(t) + (\tau_M-\tau(t))\xi^T(t)\\ & \times NR_3^{-1}N^T\xi(t) + \eta(t)\xi^T(t)ZR_4^{-1}Z^T\xi(t)\\ & + (\eta_M-\eta(t))\xi^T(t)SR_4^{-1}S^T\xi(t)\\ & + d(t)\xi^T(t)WR_5^{-1}W^T\xi(t)\\ & + (d_M-d(t))\xi^T(t)VR_5^{-1}V^T\xi(t)\\ & + \mathcal{A}^T\tilde{R}\mathcal{A} + \bar{\theta}_1^2\rho_1^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1 + \rho_1^2\mathcal{B}_2^T\tilde{R}\mathcal{B}_2 + \rho_1^2\rho_2^2\mathcal{B}_1^T\tilde{R}\mathcal{B}_1\\ & + \rho_2^2f^T(x(t-d(t)))\bar{B}_f^T\tilde{R}\bar{B}_ff(x(t-d(t)))\end{aligned} \tag{51}$$

By applying Lemma 4 and Schur complement, combining Eq. (23) and Eq. (51), it can be concluded that $\mathbb{E}\{\mathcal{L}(V(t))\} < 0$ can be ensured. □

According to the stability conditions given in Theorem 1, the design method of gain matrix K will be given in the following theorem.

Theorem 2. *For given positive parameters: the bounds of time-delay ϕ_m, ϕ_M, τ_M, η_M and d_M, the event triggered parameter σ, the expectations of Bernoulli variables $\bar{\theta}$ and $\bar{\alpha}$, a constant matrix G, m_1, m_2, ϵ_k $(k = 1, 2, 3, 4, 5)$, if there exist matrices $P_1 > 0$, $P_2 > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$ $(k = 1, 2, 3, 4, 5)$, $\Omega > 0$, $\tilde{L}$, $\tilde{Y}$, $\tilde{M}$, $\tilde{N}$, $\tilde{Z}$, $\tilde{S}$, $\tilde{W}$, $\tilde{V}$ with appropriate dimension and parameters $\alpha > 0$, $\beta > 0$, the estimating error system (14) is asymptotically stable if the following LMIs hold:*

$$\tilde{\Omega}(s) = \begin{bmatrix}\tilde{\Omega}_{11}+\tilde{\Gamma}+\tilde{\Gamma}^T & * & * & * & *\\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * & * & *\\ \tilde{\Omega}_{31} & 0 & \tilde{\Omega}_{33} & * & *\\ \tilde{\Omega}_{41}(s) & 0 & 0 & \tilde{\Omega}_{44} & *\\ \tilde{\Omega}_{51} & \tilde{\Omega}_{52} & \tilde{\Omega}_{53} & 0 & \tilde{\Omega}_{55}\end{bmatrix} < 0,$$

$$s = 1, 2, 3, \cdots, 16 \tag{52}$$

where

$$\tilde{\Omega}_{11} = \begin{bmatrix}\tilde{\Gamma}_1 & * & *\\ \tilde{\Gamma}_2 & \tilde{\Gamma}_3 & *\\ \tilde{\Gamma}_4 & 0 & \tilde{\Gamma}_5\end{bmatrix}, \quad \Gamma = \begin{bmatrix}\tilde{\Phi}_1 & \tilde{\Phi}_2\end{bmatrix},$$

$$\tilde{\Phi}_1 = \begin{bmatrix}\tilde{M}+\tilde{Z}+\tilde{W} & \tilde{L} & -\tilde{L}+\tilde{Y} & -\tilde{Y} & -\tilde{M}+\tilde{N}\end{bmatrix}$$

$$\tilde{\Phi}_2 = \begin{bmatrix}-\tilde{N} & -\tilde{Z}+\tilde{S} & -\tilde{S} & -\tilde{W}+\tilde{V} & -\tilde{V} & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\tilde{\Gamma}_1 = \begin{bmatrix}\tilde{\Lambda}_1 & * & * & * & *\\ \tilde{R}_2 & -\tilde{Q}_1-\tilde{R}_2 & * & * & *\\ 0 & 0 & \tilde{\Lambda}_2 & * & *\\ 0 & 0 & 0 & -\tilde{Q}_2 & *\\ \tilde{\Lambda}_3 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\tilde{\Gamma}_2 = \begin{bmatrix}0 & 0 & 0 & 0 & 0\\ \tilde{\Lambda}_4 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}, \quad \tilde{\Gamma}_4 = \begin{bmatrix}\tilde{\Lambda}_5 & 0 & 0 & 0 & 0\\ \tilde{\Lambda}_6 & 0 & \tilde{\Lambda}_7 & 0 & 0\\ \tilde{\Lambda}_8 & 0 & 0 & 0 & 0\\ \tilde{\Lambda}_9 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\tilde{\Gamma}_3 = diag\{-\tilde{Q}_3, \psi_1, -\tilde{Q}_4, \psi_2, -\tilde{Q}_5\},$$

$$\tilde{\Gamma}_5 = diag\{-\alpha I, -\beta I, -\Omega, -\bar{\theta}I\}, \quad \psi_1 = \begin{bmatrix}\sigma C^T\Omega C & 0\\ 0 & 0\end{bmatrix}$$

$$\psi_2 = \begin{bmatrix}\bar{\theta}G^TG & 0\\ 0 & 0\end{bmatrix}, \quad \tilde{\Lambda}_1 = \psi_3 + \psi_3^T + +\tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 + \tilde{Q}_4 + \tilde{Q}_5 - \tilde{R}_2 - \psi_4,$$

$$\psi_3 = \begin{bmatrix}-P_1A & 0\\ Y_1C & -P_2A-Y_1C\end{bmatrix}, \quad \psi_4 = \begin{bmatrix}\alpha\frac{U_1^TU_2+U_2^TU_1}{2} & 0\\ 0 & 0\end{bmatrix},$$

$$\tilde{\Lambda}_2 = \begin{bmatrix}-\beta\frac{U_1^TU_2+U_2^TU_1}{2} & 0\\ 0 & 0\end{bmatrix}$$

$$\tilde{\Lambda}_3 = \begin{bmatrix}0 & -\bar{\theta}_1\bar{\alpha}C^TY_1^T\\ 0 & 0\end{bmatrix}, \quad \tilde{\Lambda}_4 = \begin{bmatrix}0 & -\bar{\theta}_1\bar{\alpha}_1C^TY_1^T\\ 0 & 0\end{bmatrix},$$

$$\tilde{\Lambda}_5 = \begin{bmatrix}W_0^TP_1+\alpha\frac{U_1+U_2}{2} & W_0^TP_2\end{bmatrix}$$

$$\tilde{\Lambda}_6 = \begin{bmatrix} W_0^T P_1 & W_0^T P_2 \end{bmatrix}, \quad \tilde{\Lambda}_7 = \beta \frac{U_1^T + U_2^T}{2},$$
$$\tilde{\Lambda}_8 = \begin{bmatrix} 0 & -\bar{\theta}_1\bar{\alpha}_1 Y_1^T \end{bmatrix}, \quad \tilde{\Lambda}_9 = \begin{bmatrix} 0 & -\bar{\theta} C^T Y_1^T \end{bmatrix}$$
$$\tilde{\Omega}_{21} = \begin{bmatrix} \tilde{\Gamma}_6 & \tilde{\Gamma}_7 & \tilde{\Gamma}_8 & \tilde{\Gamma}_9 \\ 0 & \tilde{\Gamma}_{10} & 0 & \tilde{\Gamma}_{11} \\ 0 & \tilde{\Gamma}_{12} & 0 & \tilde{\Gamma}_{13} \end{bmatrix}, \quad \tilde{\Omega}_{31} = \begin{bmatrix} 0 & \tilde{\Gamma}_{14} & 0 & \tilde{\Gamma}_{15} \\ 0 & 0 & 0 & \tilde{\Gamma}_{16} \end{bmatrix},$$
$$\tilde{\Omega}_{22} = diag\{\tilde{\Upsilon}, \tilde{\Upsilon}, \tilde{\Upsilon}\}, \quad \tilde{\Omega}_{33} = diag\{\tilde{\Upsilon}, \quad \tilde{\Upsilon}\}$$
$$\tilde{\Upsilon} = diag\{-2\epsilon_1 P + \epsilon_1^2 \tilde{R}_1, -2\epsilon_2 P + \epsilon_2^2 \tilde{R}_2, -2\epsilon_3 P + \epsilon_3^2 \tilde{R}_3, -2\epsilon_4 P + \epsilon_4^2 \tilde{R}_4, -2\epsilon_5 P + \epsilon_5^2 \tilde{R}_5\}$$
$$\tilde{\Gamma}_6 = \begin{bmatrix} \kappa\psi_5 & 0 & 0 & 0 \\ \phi_m\psi_5 & 0 & 0 & 0 \\ \sqrt{\tau_M}\psi_5 & 0 & 0 & 0 \\ \sqrt{\eta_M}\psi_5 & 0 & 0 & 0 \\ \sqrt{d_M}\psi_5 & 0 & 0 & 0 \end{bmatrix},$$
$$\tilde{\Gamma}_7 = \begin{bmatrix} \bar{\theta}_1\bar{\alpha}\kappa\psi_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\kappa\psi_6 & 0 \\ \bar{\theta}_1\bar{\alpha}\phi_m\psi_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\phi_m\psi_6 & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{\tau_M}\psi_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\psi_6 & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{\eta_M}\psi_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\psi_6 & 0 \\ \bar{\theta}_1\bar{\alpha}\sqrt{d_M}\psi_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\psi_6 & 0 \end{bmatrix}$$
$$\tilde{\Gamma}_8 = \begin{bmatrix} 0 & 0 & \kappa\psi_7 & \kappa\psi_8 \\ 0 & 0 & \phi_m\psi_7 & \phi_m\psi_8 \\ 0 & 0 & \sqrt{\tau_M}\psi_7 & \sqrt{\tau_M}\psi_8 \\ 0 & 0 & \sqrt{\eta_M}\psi_7 & \sqrt{\eta_M}\psi_8 \\ 0 & 0 & \sqrt{d_M}\psi_7 & \sqrt{d_M}\psi_8 \end{bmatrix},$$
$$\tilde{\Gamma}_9 = \begin{bmatrix} \bar{\theta}_1\bar{\alpha}_1\kappa\psi_9 & \bar{\theta}\kappa\psi_{10} \\ \bar{\theta}_1\bar{\alpha}_1\phi_m\psi_9 & \bar{\theta}\phi_m\psi_{10} \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\psi_9 & \bar{\theta}\sqrt{\tau_M}\psi_{10} \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\psi_9 & \bar{\theta}\sqrt{\eta_M}\psi_{10} \\ \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\psi_9 & \bar{\theta}\sqrt{d_M}\psi_{10} \end{bmatrix}$$
$$\tilde{\Gamma}_{10} = \begin{bmatrix} \bar{\theta}_1\rho_1\kappa\psi_6 & 0 & -\bar{\theta}_1\rho_1\kappa\psi_6 & 0 \\ \bar{\theta}_1\rho_1\phi_m\psi_6 & 0 & -\bar{\theta}_1\rho_1\phi_m\psi_6 & 0 \\ \bar{\theta}_1\rho_1\sqrt{\tau_M}\psi_6 & 0 & -\bar{\theta}_1\rho_1\sqrt{\tau_M}\psi_6 & 0 \\ \bar{\theta}_1\rho_1\sqrt{\eta_M}\psi_6 & 0 & -\bar{\theta}_1\rho_1\sqrt{\eta_M}\psi_6 & 0 \\ \bar{\theta}_1\rho_1\sqrt{d_M}\psi_6 & 0 & -\bar{\theta}_1\rho_1\sqrt{d_M}\psi_6 & 0 \end{bmatrix},$$
$$\tilde{\Gamma}_{11} = \begin{bmatrix} -\bar{\theta}_1\rho_1\kappa\psi_9 & 0 \\ -\bar{\theta}_1\rho_1\phi_m\psi_9 & 0 \\ -\bar{\theta}_1\rho_1\sqrt{\tau_M}\psi_9 & 0 \\ -\bar{\theta}_1\rho_1\sqrt{\eta_M}\psi_9 & 0 \\ -\bar{\theta}_1\rho_1\sqrt{d_M}\psi_9 & 0 \end{bmatrix}$$
$$\tilde{\Gamma}_{12} = \begin{bmatrix} \bar{\alpha}\rho_2\kappa\psi_6 & 0 & \bar{\alpha}_1\rho_2\kappa\psi_6 & 0 \\ \bar{\alpha}\rho_2\phi_m\psi_6 & 0 & \bar{\alpha}_1\rho_2\phi_m\psi_6 & 0 \\ \bar{\alpha}\rho_2\sqrt{\tau_M}\psi_6 & 0 & \bar{\alpha}_1\rho_2\sqrt{\tau_M}\psi_6 & 0 \\ \bar{\alpha}\rho_2\sqrt{\eta_M}\psi_6 & 0 & \bar{\alpha}_1\rho_2\sqrt{\eta_M}\psi_6 & 0 \\ \bar{\alpha}\rho_2\sqrt{d_M}\psi_6 & 0 & \bar{\alpha}_1\rho_2\sqrt{d_M}\psi_6 & 0 \end{bmatrix},$$
$$\tilde{\Gamma}_{13} = \begin{bmatrix} \bar{\alpha}_1\rho_2\kappa\psi_9 & 0 \\ \bar{\alpha}_1\rho_2\phi_m\psi_9 & 0 \\ \bar{\alpha}_1\rho_2\sqrt{\tau_M}\psi_9 & 0 \\ \bar{\alpha}_1\rho_2\sqrt{\eta_M}\psi_9 & 0 \\ \bar{\alpha}_1\rho_2\sqrt{d_M}\psi_9 & 0 \end{bmatrix}$$
$$\tilde{\Gamma}_{14} = \begin{bmatrix} \kappa\rho_1\rho_2\psi_6 & 0 & -\kappa\rho_1\rho_2\psi_6 & 0 \\ \phi_m\rho_1\rho_2\psi_6 & 0 & -\phi_m\rho_1\rho_2\psi_6 & 0 \\ \sqrt{\tau_M}\rho_1\rho_2\psi_6 & 0 & -\sqrt{\tau_M}\rho_1\rho_2\psi_6 & 0 \\ \sqrt{\eta_M}\rho_1\rho_2\psi_6 & 0 & -\sqrt{\eta_M}\rho_1\rho_2\psi_6 & 0 \\ \sqrt{d_M}\rho_1\rho_2\psi_6 & 0 & -\sqrt{d_M}\rho_1\rho_2\psi_6 & 0 \end{bmatrix},$$
$$\tilde{\Gamma}_{15} = \begin{bmatrix} -\kappa\rho_1\rho_2\psi_9 & 0 \\ -\phi_m\rho_1\rho_2\psi_9 & 0 \\ -\sqrt{\tau_M}\rho_1\rho_2\psi_9 & 0 \\ -\sqrt{\eta_M}\rho_1\rho_2\psi_9 & 0 \\ -\sqrt{d_M}\rho_1\rho_2\psi_9 & 0 \end{bmatrix}, \quad \tilde{\Gamma}_{16} = \begin{bmatrix} 0 & \kappa\rho_2\psi_{10} \\ 0 & \phi_m\rho_2\psi_{10} \\ 0 & \sqrt{\tau_M}\rho_2\psi_{10} \\ 0 & \sqrt{\eta_M}\rho_2\psi_{10} \\ 0 & \sqrt{d_M}\rho_2\psi_{10} \end{bmatrix}$$
$$\psi_5 = \begin{bmatrix} -P_1 A & 0 \\ Y_1 C & -(P_2 A + Y_1 C) \end{bmatrix}, \quad \psi_6 = \begin{bmatrix} 0 & 0 \\ -Y_1 C & 0 \end{bmatrix},$$
$$\psi_7 = \begin{bmatrix} P_1 W_0 \\ P_2 W_0 \end{bmatrix}$$
$$\psi_8 = \begin{bmatrix} P_1 W_1 \\ P_2 W_1 \end{bmatrix}, \quad \psi_9 = \begin{bmatrix} 0 \\ -Y_1 \end{bmatrix}, \quad \psi_{10} = \begin{bmatrix} 0 \\ -Y_1 C \end{bmatrix}$$
$$\tilde{\Omega}_{41}(1) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(2) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$
$$\tilde{\Omega}_{41}(3) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(4) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$
$$\tilde{\Omega}_{41}(5) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(6) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$
$$\tilde{\Omega}_{41}(7) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(8) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$
$$\tilde{\Omega}_{41}(9) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(10) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$
$$\tilde{\Omega}_{41}(11) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(12) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$
$$\tilde{\Omega}_{41}(13) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(14) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{S}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$
$$\tilde{\Omega}_{41}(15) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{41}(16) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{\eta_M}\tilde{Z}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$
$$\tilde{\Omega}_{44} = diag\{-\tilde{R}_1, -\tilde{R}_3, -\tilde{R}_4, -\tilde{R}_5\},$$
$$\tilde{Y}^T = \begin{bmatrix} 0_{1\times 2} & \tilde{Y}_3^T & \tilde{Y}_4^T & 0_{1\times 10} \end{bmatrix},$$
$$\tilde{L}^T = \begin{bmatrix} 0 & \tilde{L}_2^T & \tilde{L}_3^T & 0_{1\times 11} \end{bmatrix}$$
$$\tilde{N}^T = \begin{bmatrix} 0_{1\times 4} & \tilde{N}_5^T & \tilde{N}_6^T & 0_{1\times 8} \end{bmatrix},$$
$$\tilde{M}^T = \begin{bmatrix} \tilde{M}_1^T & 0_{1\times 3} & \tilde{M}_5^T & 0_{1\times 9} \end{bmatrix},$$
$$\tilde{S}^T = \begin{bmatrix} 0_{1\times 6} & \tilde{S}_7^T & \tilde{S}_8^T & 0_{1\times 6} \end{bmatrix}$$
$$\tilde{Z}^T = \begin{bmatrix} \tilde{Z}_1^T & 0_{1\times 5} & \tilde{Z}_7^T & 0_{1\times 7} \end{bmatrix},$$
$$\tilde{V}^T = \begin{bmatrix} 0_{1\times 8} & \tilde{V}_9^T & \tilde{V}_{10}^T & 0_{1\times 4} \end{bmatrix},$$
$$\tilde{W}^T = \begin{bmatrix} \tilde{W}_1^T & 0_{1\times 7} & \tilde{W}_9^T & 0_{1\times 5} \end{bmatrix}$$

$$\tilde{\Omega}_{51}=\begin{bmatrix}\Pi_1 & 0 & 0\\ m_1\Pi_2 & 0 & 0\\ \Pi_3 & 0 & 0\\ 0 & m_2\Pi_4 & m_2\Pi_5\end{bmatrix},\quad \tilde{\Omega}_{52}=\begin{bmatrix}\Pi_6 & \Pi_7 & \Pi_8\\ 0 & 0 & 0\\ \Pi_9 & \Pi_{10} & \Pi_{11}\\ 0 & 0 & 0\end{bmatrix},$$

$$\tilde{\Omega}_{53}=\begin{bmatrix}\Pi_{12} & 0\\ 0 & 0\\ \Pi_{13} & 0\\ 0 & 0\end{bmatrix}$$

$$\tilde{\Omega}_{55}=diag\left\{\frac{-m_1}{\delta_q^2}I, -m_1I, \frac{-m_2}{\delta_q^2}I, -m_2I\right\},$$

$$\Pi_1=\begin{bmatrix}-\bar{\theta}_1\bar{\alpha}\Xi_1 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Pi_2=\begin{bmatrix}0 & 0 & 0 & 0 & \Xi_2\end{bmatrix}$$

$$\Pi_3=\begin{bmatrix}-\bar{\theta}_1\bar{\alpha}_1\Xi_1 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\Pi_4=\begin{bmatrix}0 & \Xi_2 & 0 & 0 & 0\end{bmatrix},\quad \Pi_5=\begin{bmatrix}0 & 0 & I & 0\end{bmatrix}$$

$$\Pi_6=\begin{bmatrix}-\kappa\bar{\theta}_1\bar{\alpha}\Xi_1 & -\tau_m\bar{\theta}_1\bar{\alpha}\Xi_1 & -\sqrt{\tau_M}\bar{\theta}_1\bar{\alpha}\Xi_1 & -\sqrt{\eta_M}\bar{\theta}_1\bar{\alpha}\Xi_1 & -\sqrt{d_M}\bar{\theta}_1\bar{\alpha}\Xi_1\end{bmatrix}$$

$$\Pi_7=\begin{bmatrix}-\kappa\bar{\theta}_1\rho_1\Xi_1 & -\tau_m\bar{\theta}_1\rho_1\Xi_1 & -\sqrt{\tau_M}\bar{\theta}_1\rho_1\Xi_1 & -\sqrt{\eta_M}\bar{\theta}_1\rho_1\Xi_1 & -\sqrt{d_M}\bar{\theta}_1\rho_1\Xi_1\end{bmatrix}$$

$$\Pi_8=\begin{bmatrix}-\kappa\rho_2\bar{\alpha}\Xi_1 & -\tau_m\rho_2\bar{\alpha}\Xi_1 & -\sqrt{\tau_M}\rho_2\bar{\alpha}\Xi_1 & -\sqrt{\eta_M}\rho_2\bar{\alpha}\Xi_1 & -\sqrt{d_M}\rho_2\bar{\alpha}\Xi_1\end{bmatrix}$$

$$\Pi_9=\begin{bmatrix}-\kappa\bar{\theta}_1\bar{\alpha}_1\Xi_1 & -\tau_m\bar{\theta}_1\bar{\alpha}_1\Xi_1 & -\sqrt{\tau_M}\bar{\theta}_1\bar{\alpha}_1\Xi_1 & -\sqrt{\eta_M}\bar{\theta}_1\bar{\alpha}_1\Xi_1 & -\sqrt{d_M}\bar{\theta}_1\bar{\alpha}_1\Xi_1\end{bmatrix}$$

$$\Pi_{10}=\begin{bmatrix}\kappa\bar{\theta}_1\rho_1\Xi_1 & \tau_m\bar{\theta}_1\rho_1\Xi_1 & \sqrt{\tau_M}\bar{\theta}_1\rho_1\Xi_1 & \sqrt{\eta_M}\bar{\theta}_1\rho_1\Xi_1 & \sqrt{d_M}\bar{\theta}_1\rho_1\Xi_1\end{bmatrix}$$

$$\Pi_{11}=\begin{bmatrix}-\kappa\rho_2\bar{\alpha}_1\Xi_1 & -\tau_m\rho_2\bar{\alpha}_1\Xi_1 & -\sqrt{\tau_M}\rho_2\bar{\alpha}_1\Xi_1 & -\sqrt{\eta_M}\rho_2\bar{\alpha}_1\Xi_1 & -\sqrt{d_M}\rho_2\bar{\alpha}_1\Xi_1\end{bmatrix}$$

$$\Pi_{12}=\begin{bmatrix}-\kappa\rho_1\rho_2\Xi_1 & -\tau_m\rho_1\rho_2\Xi_1 & -\sqrt{\tau_M}\rho_1\rho_2\Xi_1 & -\sqrt{\eta_M}\rho_1\rho_2\Xi_1 & -\sqrt{d_M}\rho_1\rho_2\Xi_1\end{bmatrix}$$

$$\Pi_{13}=\begin{bmatrix}\kappa\rho_1\rho_2\Xi_1 & \tau_m\rho_1\rho_2\Xi_1 & \sqrt{\tau_M}\rho_1\rho_2\Xi_1 & \sqrt{\eta_M}\rho_1\rho_2\Xi_1 & \sqrt{d_M}\rho_1\rho_2\Xi_1\end{bmatrix}$$

$$\Xi_1=\begin{bmatrix}0 & Y_1^T\end{bmatrix},\quad \Xi_2=\begin{bmatrix}0 & C\end{bmatrix}$$

Moreover, if the above conditions are feasible, the gain matrix K of the state estimator is given by $K=P_2^{-1}Y_1$.

Proof. Define $P=diag\{P_1,P_2\}$, $P_2K=Y_1$ and $\Lambda=diag\{I,\Psi_1,\Psi_2,I\}$ where

$$\Psi_1=diag\{\bar{\Upsilon},\bar{\Upsilon},\bar{\Upsilon}\}\quad \Psi_2=diag\{\bar{\Upsilon},\bar{\Upsilon}\}$$
$$\bar{\Upsilon}=diag\{PR_1^{-1},PR_2^{-1},PR_3^{-1},PR_4^{-1},PR_5^{-1}\}$$

Multiplying Λ and Λ^T on both sides of (23), respectively. Due to $(R_k-\epsilon_k^{-1}P)R_k^{-1}(R_k-\epsilon_k^{-1}P)\geq 0, (k=1,2,3,4,5)$, we have $-PR_k^{-1}P\leq -2\epsilon_kP+\epsilon_k^2R_k$.

Substitute $-PR_k^{-1}P$ with $-2\epsilon_kP+\epsilon_k^2R_k$ into $\bar{\Upsilon}$, we have

$$\Xi(s)=\begin{bmatrix}\bar{\Omega}_{11}+\tilde{\Gamma}+\tilde{\Gamma}^T & * & * & *\\ \bar{\Omega}_{21} & \tilde{\Omega}_{22} & * & *\\ \bar{\Omega}_{31} & 0 & \tilde{\Omega}_{33} & *\\ \tilde{\Omega}_{41}(s) & 0 & 0 & \tilde{\Omega}_{44}\end{bmatrix}<0,$$
$$s=1,2,3,\cdots,16 \tag{53}$$

where

$$\bar{\Omega}_{11}=\begin{bmatrix}\bar{\Gamma}_1 & * & *\\ \bar{\Gamma}_2 & \tilde{\Gamma}_3 & *\\ \bar{\Gamma}_4 & 0 & \tilde{\Gamma}_5\end{bmatrix},$$

$$\bar{\Gamma}_1=\begin{bmatrix}\tilde{\Lambda}_1 & * & * & * & *\\ \tilde{R}_2 & -\tilde{Q}_1-\tilde{R}_2 & * & * & *\\ 0 & 0 & \tilde{\Lambda}_2 & * & *\\ 0 & 0 & 0 & -\tilde{Q}_2 & *\\ \bar{\Lambda}_3 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\bar{\Gamma}_2=\begin{bmatrix}0 & 0 & 0 & 0 & 0\\ \bar{\Lambda}_4 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}$$

$$\bar{\Gamma}_4=\begin{bmatrix}\tilde{\Lambda}_5 & 0 & 0 & 0 & 0\\ \tilde{\Lambda}_6 & 0 & \tilde{\Lambda}_7 & 0 & 0\\ \bar{\Lambda}_8 & 0 & 0 & 0 & 0\\ \tilde{\Lambda}_9 & 0 & 0 & 0 & 0\end{bmatrix},$$

$$\bar{\Lambda}_3=\begin{bmatrix}0 & \bar{\theta}_1\bar{\alpha}(I+\Delta_q)C^TY_1^T\\ 0 & 0\end{bmatrix},\quad \bar{\Lambda}_4=\begin{bmatrix}0 & \bar{\theta}_1\bar{\alpha}_1(I+\Delta_q)C^TY_1^T\\ 0 & 0\end{bmatrix}$$

$$\bar{\Lambda}_8=\begin{bmatrix}0 & \bar{\theta}_1\bar{\alpha}_1(I+\Delta_q)C^TY_1^T\end{bmatrix},$$

$$\bar{\Omega}_{21}=\begin{bmatrix}\tilde{\Gamma}_6 & \bar{\Gamma}_7 & \tilde{\Gamma}_8 & \bar{\Gamma}_9\\ 0 & \bar{\Gamma}_{10} & 0 & \bar{\Gamma}_{11}\\ 0 & \bar{\Gamma}_{12} & 0 & \bar{\Gamma}_{13}\end{bmatrix},\quad \bar{\Omega}_{31}=\begin{bmatrix}0 & \bar{\Gamma}_{14} & 0 & \bar{\Gamma}_{15}\\ 0 & 0 & 0 & \tilde{\Gamma}_{16}\end{bmatrix}$$

$$\bar{\Gamma}_7=\begin{bmatrix}\bar{\theta}_1\bar{\alpha}\kappa\bar{\psi}_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\kappa\bar{\psi}_6 & 0\\ \bar{\theta}_1\bar{\alpha}\phi_m\bar{\psi}_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\phi_m\bar{\psi}_6 & 0\\ \bar{\theta}_1\bar{\alpha}\sqrt{\tau_M}\bar{\psi}_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\bar{\psi}_6 & 0\\ \bar{\theta}_1\bar{\alpha}\sqrt{\eta_M}\bar{\psi}_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\bar{\psi}_6 & 0\\ \bar{\theta}_1\bar{\alpha}\sqrt{d_M}\bar{\psi}_6 & 0 & \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\bar{\psi}_6 & 0\end{bmatrix},$$

$$\bar{\Gamma}_9=\begin{bmatrix}\bar{\theta}_1\bar{\alpha}_1\kappa\bar{\psi}_9 & \bar{\theta}\kappa\psi_{10}\\ \bar{\theta}_1\bar{\alpha}_1\phi_m\bar{\psi}_9 & \bar{\theta}\phi_m\psi_{10}\\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\tau_M}\bar{\psi}_9 & \bar{\theta}\sqrt{\tau_M}\psi_{10}\\ \bar{\theta}_1\bar{\alpha}_1\sqrt{\eta_M}\bar{\psi}_9 & \bar{\theta}\sqrt{\eta_M}\psi_{10}\\ \bar{\theta}_1\bar{\alpha}_1\sqrt{d_M}\bar{\psi}_9 & \bar{\theta}\sqrt{d_M}\psi_{10}\end{bmatrix}$$

$$\bar{\Gamma}_{10}=\begin{bmatrix}\bar{\theta}_1\rho_1\kappa\bar{\psi}_6 & 0 & -\bar{\theta}_1\rho_1\kappa\bar{\psi}_6 & 0\\ \bar{\theta}_1\rho_1\phi_m\bar{\psi}_6 & 0 & -\bar{\theta}_1\rho_1\phi_m\bar{\psi}_6 & 0\\ \bar{\theta}_1\rho_1\sqrt{\tau_M}\bar{\psi}_6 & 0 & -\bar{\theta}_1\rho_1\sqrt{\tau_M}\bar{\psi}_6 & 0\\ \bar{\theta}_1\rho_1\sqrt{\eta_M}\bar{\psi}_6 & 0 & -\bar{\theta}_1\rho_1\sqrt{\eta_M}\bar{\psi}_6 & 0\\ \bar{\theta}_1\rho_1\sqrt{d_M}\bar{\psi}_6 & 0 & -\bar{\theta}_1\rho_1\sqrt{d_M}\bar{\psi}_6 & 0\end{bmatrix},$$

$$\bar{\Gamma}_{11}=\begin{bmatrix}-\bar{\theta}_1\rho_1\kappa\bar{\psi}_9 & 0\\ -\bar{\theta}_1\rho_1\phi_m\bar{\psi}_9 & 0\\ -\bar{\theta}_1\rho_1\sqrt{\tau_M}\bar{\psi}_9 & 0\\ -\bar{\theta}_1\rho_1\sqrt{\eta_M}\bar{\psi}_9 & 0\\ -\bar{\theta}_1\rho_1\sqrt{d_M}\bar{\psi}_9 & 0\end{bmatrix}$$

$$\bar{\Gamma}_{12} = \begin{bmatrix} \bar{\alpha}\rho_2\kappa\bar{\psi}_6 & 0 & \bar{\alpha}_1\rho_2\kappa\bar{\psi}_6 & 0 \\ \bar{\alpha}\rho_2\phi_m\bar{\psi}_6 & 0 & \bar{\alpha}_1\rho_2\phi_m\bar{\psi}_6 & 0 \\ \bar{\alpha}\rho_2\sqrt{\tau_M}\bar{\psi}_6 & 0 & \bar{\alpha}_1\rho_2\sqrt{\tau_M}\bar{\psi}_6 & 0 \\ \bar{\alpha}\rho_2\sqrt{\eta_M}\bar{\psi}_6 & 0 & \bar{\alpha}_1\rho_2\sqrt{\eta_M}\bar{\psi}_6 & 0 \\ \bar{\alpha}\rho_2\sqrt{d_M}\bar{\psi}_6 & 0 & \bar{\alpha}_1\rho_2\sqrt{d_M}\bar{\psi}_6 & 0 \end{bmatrix},$$

$$\bar{\Gamma}_{13} = \begin{bmatrix} \bar{\alpha}_1\rho_2\kappa\bar{\psi}_9 & 0 \\ \bar{\alpha}_1\rho_2\phi_m\bar{\psi}_9 & 0 \\ \bar{\alpha}_1\rho_2\sqrt{\tau_M}\bar{\psi}_9 & 0 \\ \bar{\alpha}_1\rho_2\sqrt{\eta_M}\bar{\psi}_9 & 0 \\ \bar{\alpha}_1\rho_2\sqrt{d_M}\bar{\psi}_9 & 0 \end{bmatrix}$$

$$\bar{\Gamma}_{14} = \begin{bmatrix} \kappa\rho_1\rho_2\bar{\psi}_6 & 0 & -\kappa\rho_1\rho_2\bar{\psi}_6 & 0 \\ \phi_m\rho_1\rho_2\bar{\psi}_6 & 0 & -\phi_m\rho_1\rho_2\bar{\psi}_6 & 0 \\ \sqrt{\tau_M}\rho_1\rho_2\bar{\psi}_6 & 0 & -\sqrt{\tau_M}\rho_1\rho_2\bar{\psi}_6 & 0 \\ \sqrt{\eta_M}\rho_1\rho_2\bar{\psi}_6 & 0 & -\sqrt{\eta_M}\rho_1\rho_2\bar{\psi}_6 & 0 \\ \sqrt{d_M}\rho_1\rho_2\bar{\psi}_6 & 0 & -\sqrt{d_M}\rho_1\rho_2\bar{\psi}_6 & 0 \end{bmatrix},$$

$$\bar{\Gamma}_{15} = \begin{bmatrix} -\kappa\rho_1\rho_2\bar{\psi}_9 & 0 \\ -\phi_m\rho_1\rho_2\bar{\psi}_9 & 0 \\ -\sqrt{\tau_M}\rho_1\rho_2\bar{\psi}_9 & 0 \\ -\sqrt{\eta_M}\rho_1\rho_2\bar{\psi}_9 & 0 \\ -\sqrt{d_M}\rho_1\rho_2\bar{\psi}_9 & 0 \end{bmatrix}$$

$$\bar{\psi}_6 = \begin{bmatrix} 0 & 0 \\ -Y_1C(I+\Delta_q) & 0 \end{bmatrix}, \quad \bar{\psi}_9 = \begin{bmatrix} 0 \\ -Y_1(I+\Delta_q) \end{bmatrix}$$

The Eq. (53) can be rewritten as

$$\Xi(s) = \tilde{\Xi}(s) + H_1^T\Delta_q H_2 + H_2^T\Delta_q H_1 + H_3^T\Delta_q H_4 + H_4^T\Delta_q H_3 < 0 \tag{54}$$

where

$$\tilde{\Xi}(s) = \begin{bmatrix} \tilde{\Omega}_{11}+\tilde{\Gamma}+\tilde{\Gamma}^T & * & * & * \\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * & * \\ \tilde{\Omega}_{31} & 0 & \tilde{\Omega}_{33} & * \\ \tilde{\Omega}_{41}(s) & 0 & 0 & \tilde{\Omega}_{44} \end{bmatrix}$$

$$H_1 = \begin{bmatrix} \Pi_1 & 0 & 0 & \Pi_6 & \Pi_7 & \Pi_8 & \Pi_{12} & 0 \end{bmatrix},$$
$$H_2 = \begin{bmatrix} \Pi_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$H_3 = \begin{bmatrix} \Pi_3 & 0 & 0 & \Pi_9 & \Pi_{10} & \Pi_{11} & \Pi_{13} & 0 \end{bmatrix},$$
$$H_4 = \begin{bmatrix} 0 & \Pi_4 & \Pi_5 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Apply Lemma 3, there exists $m_1 > 0$, $m_2 > 0$, one can get the following inequality:

$$\Xi(s) \le \tilde{\Xi}(s) + m_1H_1^TH_1 + m_1^{-1}\Delta_q^2H_2^TH_2 + m_2H_3^TH_3 + m_2^{-1}\Delta_q^2H_4^TH_4 \tag{55}$$

notice that, $\Delta_q^2 < \delta_q^2$.

By using Schur supplement, according to Eqs. (23) and (55), we can obtain Theorem 2 and the estimating error system (14) is asymptotically stable. According to $P_2K = Y_1$, we can get the gain matrix $K = P_2^{-1}Y_1$, the proof is complete. □

Theorem 2 gives the design method of state estimator for neural networks with hybrid triggered scheme and quantization subject to cyber attacks. In the following, two corollaries will be given which can be seen as the special cases of Theorem 2. Set the Bernoulli random variable $\alpha(t) \equiv 1$, which means that the designed state estimator is under "time triggered scheme" in Fig. 1, the augmented system (14) can be written as:

$$\begin{aligned}\dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + (1-\theta(t))\bar{B}\bar{x}(t-\tau(t)) + \theta(t)\bar{D}f(H\bar{x}(t-d(t))) \\ &+ \bar{W}_0 g(H\bar{x}(t)) + \bar{W}_1 g(H\bar{x}(t-\phi(t)))\end{aligned} \tag{56}$$

Similar to the proof of Theorem 2, we can get the following Corollary 1, in which the designed state estimator is under "time triggered scheme".

Corollary 1. *For given positive parameters: the bounds of time-delay ϕ_m, ϕ_M, τ_M, d_M, the expectation of Bernoulli variable $\bar{\theta}$, a constant matrix G, m_1, ϵ_k $(k = 1, 2, 3, 4)$, if there exist matrix $P_1 > 0$, $P_2 > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$ $(k = 1, 2, 3, 4)$, $\tilde{L}$, $\tilde{Y}$, $\tilde{M}$, $\tilde{N}$, $\tilde{W}$, $\tilde{V}$ with appropriate dimension and parameters $\alpha > 0$, $\beta > 0$, the augment system (56) is asymptotically stable if the following LMIs hold:*

$$\tilde{\Omega}(s) = \begin{bmatrix} \tilde{\Omega}_{11}+\tilde{\Gamma}+\tilde{\Gamma}^T & * & * & * \\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * & * \\ \tilde{\Omega}_{31}(s) & 0 & \tilde{\Omega}_{33} & * \\ \tilde{\Omega}_{41} & \tilde{\Omega}_{42} & 0 & \tilde{\Omega}_{44} \end{bmatrix} < 0, \ s = 1, 2, 3, \cdots, 8 \tag{57}$$

where

$$\tilde{\Omega}_{11} = \begin{bmatrix} \tilde{\Gamma}_1 & * & * \\ 0 & \tilde{\Gamma}_2 & * \\ \tilde{\Gamma}_3 & 0 & \tilde{\Gamma}_4 \end{bmatrix}, \quad \tilde{\Gamma} = \begin{bmatrix} \tilde{M}+\tilde{W} & \tilde{L} & -\tilde{L}+\tilde{Y} & -\tilde{Y} \\ -\tilde{M}+\tilde{N} & -\tilde{N} & -\tilde{W}+\tilde{V} & -\tilde{V} & 0_{1\times 3} \end{bmatrix}$$

$$\tilde{\Gamma}_1 = \begin{bmatrix} \tilde{\Lambda}_1 & * & * & * & * \\ \tilde{R}_2 & -\tilde{Q}_1-\tilde{R}_2 & * & * & * \\ 0 & 0 & \tilde{\Lambda}_2 & * & * \\ 0 & 0 & 0 & -\tilde{Q}_2 & * \\ \tilde{\Lambda}_3 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_2 = diag\{-\tilde{Q}_3, \psi_1, -\tilde{Q}_4\}, \quad \tilde{\Gamma}_3 = \begin{bmatrix} \tilde{\Lambda}_4 & 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_5 & 0 & \tilde{\Lambda}_6 & 0 & 0 \\ \tilde{\Lambda}_7 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_4 = diag\{-\alpha I, -\beta I, -\bar{\theta}I\}, \quad \psi_1 = \begin{bmatrix} \bar{\theta}G^TG & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Lambda}_1 = \psi_2 + \psi_2^T + \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 + \tilde{Q}_4 - \tilde{R}_2 - \psi_3,$$

$$\psi_2 = \begin{bmatrix} -P_1A & 0 \\ Y_1C & -P_2A - Y_1C \end{bmatrix}, \quad \psi_3 = \begin{bmatrix} \alpha\frac{U_1^TU_2+U_2^TU_1}{2} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Lambda}_2 = \begin{bmatrix} -\beta\frac{U_1^TU_2+U_2^TU_1}{2} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{\Lambda}_3 = \begin{bmatrix} 0 & -\bar{\theta}_1C^TY_1^T \\ 0 & 0 \end{bmatrix}, \quad \tilde{\Lambda}_4 = \begin{bmatrix} W_0^TP_1 + \alpha\frac{U_1+U_2}{2} & W_0^TP_2 \end{bmatrix},$$

$$\tilde{\Lambda}_5 = \begin{bmatrix} W_0^TP_1 & W_0^TP_2 \end{bmatrix}$$

$$\tilde{\Lambda}_6 = \beta\frac{U_1^T+U_2^T}{2}, \quad \tilde{\Lambda}_7 = \begin{bmatrix} 0 & -\bar{\theta}C^TY_1^T \end{bmatrix},$$

$$\tilde{\Omega}_{21} = \begin{bmatrix} \tilde{\Gamma}_5 & 0 & \tilde{\Gamma}_6 \\ \tilde{\Gamma}_7 & 0 & \tilde{\Gamma}_8 \end{bmatrix}, \quad \tilde{\Omega}_{22} = diag\{\tilde{\Upsilon}, \tilde{\Upsilon}\}$$

$$\begin{aligned}\tilde{\Upsilon} = diag\{&-2\epsilon_1P + \epsilon_1^2\tilde{R}_1, -2\epsilon_2P + \epsilon_2^2\tilde{R}_2, -2\epsilon_3P + \epsilon_3^2\tilde{R}_3, \\ &-2\epsilon_4P + \epsilon_4^2\tilde{R}_4\}\end{aligned}$$

$$\tilde{\Gamma}_5 = \begin{bmatrix} \kappa\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\kappa\psi_5 \\ \phi_m\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\phi_m\psi_5 \\ \sqrt{\tau_M}\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\sqrt{\tau_M}\psi_5 \\ \sqrt{d_M}\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\sqrt{d_M}\psi_5 \end{bmatrix},$$

$$\tilde{\Gamma}_6 = \begin{bmatrix} \kappa\psi_6 & \kappa\psi_7 & \bar{\theta}\kappa\psi_8 \\ \phi_m\psi_6 & \phi_m\psi_7 & \bar{\theta}\phi_m\psi_8 \\ \sqrt{\tau_M}\psi_6 & \sqrt{\tau_M}\psi_7 & \bar{\theta}\sqrt{\tau_M}\psi_8 \\ \sqrt{d_M}\psi_6 & \sqrt{d_M}\psi_7 & \bar{\theta}\sqrt{d_M}\psi_8 \end{bmatrix}$$

$$\tilde{\Gamma}_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & -\kappa\rho_1\psi_5 \\ 0 & 0 & 0 & 0 & -\phi_m\rho_1\psi_5 \\ 0 & 0 & 0 & 0 & -\sqrt{\tau_M}\rho_1\psi_5 \\ 0 & 0 & 0 & 0 & -\sqrt{d_M}\rho_1\psi_5 \end{bmatrix},$$

$$\tilde{\Gamma}_8 = \begin{bmatrix} 0 & 0 & \kappa\rho_1\psi_8 \\ 0 & 0 & \phi_m\rho_1\psi_8 \\ 0 & 0 & \sqrt{\tau_M}\rho_1\psi_8 \\ 0 & 0 & \sqrt{d_M}\rho_1\psi_8 \end{bmatrix}$$

$$\psi_4 = \begin{bmatrix} -P_1A & 0 \\ Y_1C & -(P_2A+Y_1C) \end{bmatrix}, \quad \psi_5 = \begin{bmatrix} 0 & 0 \\ -Y_1C & 0 \end{bmatrix},$$

$$\psi_6 = \begin{bmatrix} P_1W_0 \\ P_2W_0 \end{bmatrix}, \quad \psi_7 = \begin{bmatrix} P_1W_1 \\ P_2W_1 \end{bmatrix}, \quad \psi_8 = \begin{bmatrix} 0 \\ -Y_1C \end{bmatrix}$$

$$\tilde{\Omega}_{31}(1) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{31}(2) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$

$$\tilde{\Omega}_{31}(3) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{31}(4) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\tilde{\Omega}_{31}(5) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{31}(6) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$

$$\tilde{\Omega}_{31}(7) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \tilde{\Omega}_{31}(8) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\tau_M}\tilde{W}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\tilde{Y}^T = \begin{bmatrix} 0_{1\times 2} & \tilde{Y}_3^T & \tilde{Y}_4^T & 0_{1\times 7} \end{bmatrix},$$
$$\tilde{L}^T = \begin{bmatrix} 0 & \tilde{L}_2^T & \tilde{L}_3^T & 0_{1\times 8} \end{bmatrix},$$
$$\tilde{N}^T = \begin{bmatrix} 0_{1\times 4} & \tilde{N}_5^T & \tilde{N}_6^T & 0_{1\times 5} \end{bmatrix}$$
$$\tilde{M}^T = \begin{bmatrix} \tilde{M}_1^T & 0_{1\times 3} & \tilde{M}_5^T & 0_{1\times 6} \end{bmatrix},$$
$$\tilde{V}^T = \begin{bmatrix} 0_{1\times 6} & \tilde{V}_7^T & \tilde{V}_8^T & 0_{1\times 3} \end{bmatrix},$$
$$\tilde{W}^T = \begin{bmatrix} \tilde{W}_1^T & 0_{1\times 6} & \tilde{W}_7^T & 0_{1\times 4} \end{bmatrix}$$

$$\tilde{\Omega}_{41} = \begin{bmatrix} \Pi_1 & 0 & 0 \\ m_1\Pi_2 & 0 & 0 \end{bmatrix}, \quad \tilde{\Omega}_{42} = \begin{bmatrix} \Pi_3 & \Pi_4 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Omega}_{44} = diag\left\{\frac{-m_1}{\delta_q^2}I, -m_1I\right\}, \quad \Pi_1 = \begin{bmatrix} -\bar{\theta}_1\Xi_1 & 0 & 0 & 0 & 0 \end{bmatrix}$$
$$\Pi_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \Xi_2 \end{bmatrix},$$
$$\Pi_3 = \begin{bmatrix} -\kappa\bar{\theta}_1\Xi_1 & -\phi_m\bar{\theta}_1\Xi_1 & -\sqrt{\tau_M}\bar{\theta}_1\Xi_1 & -\sqrt{d_M}\bar{\theta}_1\Xi_1 \end{bmatrix}$$
$$\Pi_4 = \begin{bmatrix} \kappa\rho_1\Xi_1 & \phi_m\rho_1\Xi_1 & \sqrt{\tau_M}\rho_1\Xi_1 & \sqrt{d_M}\rho_1\Xi_1 \end{bmatrix},$$
$$\Xi_1 = \begin{bmatrix} 0 & Y_1^T \end{bmatrix}, \quad \Xi_1 = \begin{bmatrix} C & 0 \end{bmatrix}$$

Moreover, by considering the feasible conditions above, the gain matrix K of the state estimator is given by $K = P_2^{-1}Y_1$. □

Set the Bernoulli random variable $\alpha(t) \equiv 0$, which means that the designed state estimator is under "event triggered scheme" in Fig. 1, the augmented system (14) can be written as:

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + (1-\theta(t))\bar{B}\bar{x}(t-\eta(t)) + (1-\theta(t))\bar{C}e_k(t) \\ &\quad + \theta(t)\bar{D}f(H\bar{x}(t-d(t))) \\ &\quad + \bar{W}_0g(H\bar{x}(t)) + \bar{W}_1g(H\bar{x}(t-\phi(t))) \end{aligned} \tag{58}$$

By utilizing the proof method of Theorem 2, the following Corollary 2 can be obtained, in which the designed state estimator is under "event triggered scheme".

Corollary 2. *For given positive parameters: the bounds of time-delay* ϕ_m, ϕ_M, η_M, *and* d_M, *the event triggered parameter* σ, *the exception of Bernoulli variable* $\bar{\theta}$, *the constant matrix G*, m_1, ϵ_k $(k = 1, 2, 3, 4)$, *if there exist matrix* $P_1 > 0$, $P_2 > 0$, $\tilde{Q}_k > 0$, $\tilde{R}_k > 0$ $(k = 1, 2, 3, 4)$, $\Omega > 0$, $\tilde{L}$, $\tilde{Y}$, $\tilde{M}$, $\tilde{N}$, $\tilde{W}$, $\tilde{V}$ *with appropriate dimension and parameters* $\alpha > 0$, $\beta > 0$, *the augment system (58) is asymptotically stable if the following LMIs hold:*

$$\tilde{\Omega}(s) = \begin{bmatrix} \tilde{\Omega}_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T & * & * & * \\ \tilde{\Omega}_{21} & \tilde{\Omega}_{22} & * & * \\ \tilde{\Omega}_{31}(s) & 0 & \tilde{\Omega}_{33} & * \\ \tilde{\Omega}_{41} & \tilde{\Omega}_{42} & 0 & \tilde{\Omega}_{44} \end{bmatrix} < 0,$$
$$s = 1, 2, 3, \cdots, 8 \tag{59}$$

where

$$\tilde{\Omega}_{11} = \begin{bmatrix} \tilde{\Gamma}_1 & * & * \\ 0 & \tilde{\Gamma}_2 & * \\ \tilde{\Gamma}_3 & 0 & \tilde{\Gamma}_4 \end{bmatrix}, \quad \tilde{\Gamma} = \begin{bmatrix} \tilde{M}+\tilde{W} & \tilde{L} & -\tilde{L}+\tilde{Y} & -\tilde{Y} \end{bmatrix}$$
$$\begin{bmatrix} -\tilde{M}+\tilde{N} & -\tilde{N} & -\tilde{W}+\tilde{V} & -\tilde{V} & 0_{1\times 4} \end{bmatrix}$$

$$\tilde{\Gamma}_1 = \begin{bmatrix} \tilde{\Lambda}_1 & * & * & * & * \\ \tilde{R}_2 & -\tilde{Q}_1-\tilde{R}_2 & * & * & * \\ 0 & 0 & \tilde{\Lambda}_2 & * & * \\ 0 & 0 & 0 & -\tilde{Q}_2 & * \\ \tilde{\Lambda}_3 & 0 & 0 & 0 & \tilde{\Lambda}_4 \end{bmatrix},$$

$$\tilde{\Gamma}_2 = diag\{-\tilde{Q}_3, \psi_1, -\tilde{Q}_4\},$$

$$\tilde{\Gamma}_3 = \begin{bmatrix} \tilde{\Lambda}_5 & 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_6 & 0 & \tilde{\Lambda}_7 & 0 & 0 \\ \tilde{\Lambda}_8 & 0 & 0 & 0 & 0 \\ \tilde{\Lambda}_9 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_4 = diag\{-\alpha I, -\beta I, -\Omega, -\bar{\theta}I\}, \quad \psi_1 = \begin{bmatrix} \bar{\theta}G^TG & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{\Lambda}_1 = \psi_2 + \psi_2^T + \tilde{Q}_1 + \tilde{Q}_2 + \tilde{Q}_3 + \tilde{Q}_4 - \tilde{R}_2 - \psi_3,$$

$$\psi_2 = \begin{bmatrix} -P_1A & 0 \\ Y_1C & -P_2A - Y_1C \end{bmatrix}$$

$$\psi_3 = \begin{bmatrix} \alpha\frac{U_1^TU_2+U_2^TU_1}{2} & 0 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Lambda}_2 = \begin{bmatrix} -\beta\frac{U_1^TU_2+U_2^TU_1}{2} & 0 \\ 0 & 0 \end{bmatrix}, \quad \tilde{\Lambda}_3 = \begin{bmatrix} 0 & -\bar{\theta}_1C^TY_1^T \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Lambda}_4 = \begin{bmatrix} \sigma C^T\Omega C & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{\Lambda}_5 = \begin{bmatrix} W_0^TP_1 + \alpha\frac{U_1+U_2}{2} & W_0^TP_2 \end{bmatrix}, \quad \tilde{\Lambda}_6 = \begin{bmatrix} W_0^TP_1 & W_0^TP_2 \end{bmatrix},$$

$$\tilde{\Lambda}_7 = \beta\frac{U_1^T+U_2^T}{2}$$

$$\tilde{\Lambda}_8 = \begin{bmatrix} 0 & -\bar{\theta}_1Y_1^T \end{bmatrix}, \quad \tilde{\Lambda}_9 = \begin{bmatrix} 0 & -\bar{\theta}C^TY_1^T \end{bmatrix},$$

$$\tilde{\Omega}_{21} = \begin{bmatrix} \tilde{\Gamma}_5 & 0 & \tilde{\Gamma}_6 \\ \tilde{\Gamma}_7 & 0 & \tilde{\Gamma}_8 \end{bmatrix}, \quad \tilde{\Omega}_{22} = diag\{\tilde{\Upsilon}, \tilde{\Upsilon}\}$$

$$\begin{aligned} \tilde{\Upsilon} &= diag\{-2\epsilon_1P + \epsilon_1^2\tilde{R}_1, -2\epsilon_2P + \epsilon_2^2\tilde{R}_2, -2\epsilon_3P + \epsilon_3^2\tilde{R}_3, \\ &\quad -2\epsilon_4P + \epsilon_4^2\tilde{R}_4\} \end{aligned}$$

$$\tilde{\Gamma}_5 = \begin{bmatrix} \kappa\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\kappa\psi_5 \\ \phi_m\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\phi_m\psi_5 \\ \sqrt{\eta_M}\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\sqrt{\eta_M}\psi_5 \\ \sqrt{d_M}\psi_4 & 0 & 0 & 0 & \bar{\theta}_1\sqrt{d_M}\psi_5 \end{bmatrix},$$

$$\tilde{\Gamma}_6 = \begin{bmatrix} \kappa\psi_6 & \kappa\psi_7 & \bar{\theta}_1\kappa\psi_8 & \bar{\theta}\kappa\psi_9 \\ \phi_m\psi_6 & \phi_m\psi_7 & \bar{\theta}_1\phi_m\psi_8 & \bar{\theta}\phi_m\psi_9 \\ \sqrt{\eta_M}\psi_6 & \sqrt{\eta_M}\psi_7 & \bar{\theta}_1\sqrt{\eta_M}\psi_8 & \bar{\theta}\sqrt{\eta_M}\psi_9 \\ \sqrt{d_M}\psi_6 & \sqrt{d_M}\psi_7 & \bar{\theta}_1\sqrt{d_M}\psi_8 & \bar{\theta}\sqrt{d_M}\psi_9 \end{bmatrix}$$

$$\tilde{\Gamma}_7 = \begin{bmatrix} 0 & 0 & 0 & 0 & -\kappa\rho_1\psi_5 \\ 0 & 0 & 0 & 0 & -\phi_m\rho_1\psi_5 \\ 0 & 0 & 0 & 0 & -\sqrt{\eta_M}\rho_1\psi_5 \\ 0 & 0 & 0 & 0 & -\sqrt{d_M}\rho_1\psi_5 \end{bmatrix},$$

$$\tilde{\Gamma}_8 = \begin{bmatrix} 0 & 0 & -\kappa\rho_1\psi_8 & \kappa\rho_1\psi_9 \\ 0 & 0 & -\phi_m\rho_1\psi_8 & \phi_m\rho_1\psi_9 \\ 0 & 0 & -\sqrt{\eta_M}\rho_1\psi_8 & \sqrt{\eta_M}\rho_1\psi_9 \\ 0 & 0 & -\sqrt{d_M}\rho_1\psi_8 & \sqrt{d_M}\rho_1\psi_9 \end{bmatrix}$$

$$\psi_4 = \begin{bmatrix} -P_1A & 0 \\ Y_1C & -(P_2A + Y_1C) \end{bmatrix},$$

$$\psi_5 = \begin{bmatrix} 0 & 0 \\ -Y_1C & 0 \end{bmatrix}, \quad \psi_6 = \begin{bmatrix} P_1W_0 \\ P_2W_0 \end{bmatrix}$$

$$\psi_7 = \begin{bmatrix} P_1W_1 \\ P_2W_1 \end{bmatrix}, \quad \psi_8 = \begin{bmatrix} 0 \\ -Y_1 \end{bmatrix}, \quad \psi_9 = \begin{bmatrix} 0 \\ -Y_1C \end{bmatrix}$$

$$\tilde{\Omega}_{31}(1) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\eta_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{31}(2) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\eta_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$

$$\tilde{\Omega}_{31}(3) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\eta_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{31}(4) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\eta_M}\tilde{N}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\tilde{\Omega}_{31}(5) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\eta_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{31}(6) = \begin{bmatrix} \kappa\tilde{Y}^T \\ \sqrt{\eta_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix},$$

$$\tilde{\Omega}_{31}(7) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\eta_M}\tilde{M}^T \\ \sqrt{d_M}\tilde{V}^T \end{bmatrix}, \quad \tilde{\Omega}_{31}(8) = \begin{bmatrix} \kappa\tilde{L}^T \\ \sqrt{\eta_M}\tilde{W}^T \\ \sqrt{d_M}\tilde{W}^T \end{bmatrix}$$

$$\tilde{Y}^T = \begin{bmatrix} 0_{1\times2} & \tilde{Y}_3^T & \tilde{Y}_4^T & 0_{1\times8} \end{bmatrix},$$

$$\tilde{L}^T = \begin{bmatrix} 0 & \tilde{L}_2^T & \tilde{L}_3^T & 0_{1\times9} \end{bmatrix},$$

$$\tilde{N}^T = \begin{bmatrix} 0_{1\times4} & \tilde{N}_5^T & \tilde{N}_6^T & 0_{1\times6} \end{bmatrix}$$

$$\tilde{M}^T = \begin{bmatrix} \tilde{M}_1^T & 0_{1\times3} & \tilde{M}_5^T & 0_{1\times7} \end{bmatrix},$$

$$\tilde{V}^T = \begin{bmatrix} 0_{1\times6} & \tilde{V}_7^T & \tilde{V}_8^T & 0_{1\times4} \end{bmatrix},$$

$$\tilde{W}^T = \begin{bmatrix} \tilde{W}_1^T & 0_{1\times5} & \tilde{W}_7^T & 0_{1\times5} \end{bmatrix}$$

$$\tilde{\Omega}_{41} = \begin{bmatrix} \Pi_1 & 0 & 0 \\ m_1\Pi_2 & 0 & m_1\Pi_3 \end{bmatrix}, \quad \tilde{\Omega}_{42} = \begin{bmatrix} \Pi_4 & \Pi_5 \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\Omega}_{44} = diag\left\{ \frac{-m_1}{\delta_q^2}I, -m_1I \right\}, \quad \Pi_1 = \begin{bmatrix} -\bar{\theta}_1\Xi_1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Pi_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & \Xi_2 \end{bmatrix}, \quad \Pi_3 = \begin{bmatrix} 0 & 0 & I & 0 \end{bmatrix}$$

$$\Pi_4 = \begin{bmatrix} -\kappa\bar{\theta}_1\Xi_1 & -\phi_m\bar{\theta}_1\Xi_1 & -\sqrt{\tau_M}\bar{\theta}_1\Xi_1 & -\sqrt{d_M}\bar{\theta}_1\Xi_1 \end{bmatrix}$$

$$\Pi_5 = \begin{bmatrix} \kappa\rho_1\Xi_1 & \phi_m\rho_1\Xi_1 & \sqrt{\tau_M}\rho_1\Xi_1 & \sqrt{d_M}\rho_1\Xi_1 \end{bmatrix},$$

$$\Xi_1 = \begin{bmatrix} 0 & Y_1^T \end{bmatrix}, \quad \Xi_2 = \begin{bmatrix} C & 0 \end{bmatrix}$$

Moreover, based on the feasible conditions above, the gain matrix K of the state estimator is given by $K = P_2^{-1}Y_1$. □

4. Numerical examples

In order to illustrate the usefulness of the proposed method in this paper, a numerical example is given as follows:

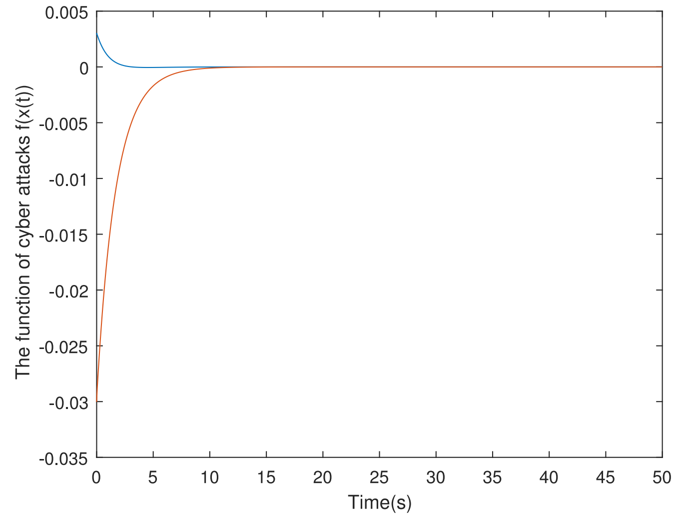


Fig. 2. The function of cyber attacks $f(x(t))$.

The parameters of system (1) are given as follows:

$$A = \begin{bmatrix} 1.05 & 0 \\ 0 & 1.05 \end{bmatrix}, \quad C = \begin{bmatrix} -0.09 & -0.02 \\ -0.09 & -0.01 \end{bmatrix},$$

$$W_0 = \begin{bmatrix} 0.3 & -0.4 \\ -0.4 & 0.3 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}, \quad U_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0 & 0.95 \end{bmatrix}$$

The neuron activation function $g(x(t))$ in (13) is defined as follows:

$$g(x(t)) = \begin{bmatrix} 0.5x_1(t) - tanh(0.2x_1(t)) + 0.2x_2(t) \\ 0.95x_2(t) - tanh(0.75x_1(t)) \end{bmatrix}$$

Suppose the function of cyber attacks $f(x(t)) = \begin{bmatrix} -tanh(0.2x_2(t)) \\ -tanh(0.02x_1(t)) \end{bmatrix}$. According to Assumption 1, when $G = diag\{0.02, \ 0.2\}$, it satisfies $||f(x(t))||_2 \le ||Gx(t)||_2$.

Choose the initial condition $x(0) = \begin{bmatrix} 0.3 & -0.3 \end{bmatrix}^T$, $\hat{x}(0) = \begin{bmatrix} 0.5 & -0.3 \end{bmatrix}^T$, $m_1 = m_2 = 0.1$, $\epsilon_k = 2$ $(k = 1, 2, 3, 4, 5)$, the quantization parameters $\rho = 0.5$, where $\delta_q = \frac{1-\rho}{1+\rho}$.

In the following, three cases will be given to illustrate the usefulness of designed state estimator subject to cyber attacks. In *Case 1*, it discusses the situation when the designed state estimator is under "time triggered scheme". Another circumstance is given in *Case 2*, in which the designed state estimator is under "event triggered scheme". *Case 3* gives a discussion when the designed state estimator is under "hybrid triggered scheme".

Case 1: Set $\alpha(t) = 1$, as is shown in Fig. 1, "time triggered scheme" is selected. For given parameters $\phi_m = 0.05$, $\phi_M = 0.25$, $\tau_M = 0.02$, $d_M = 0.15$, suppose the probability of cyber attacks $\bar{\theta} = 0.2$, the following matrices can be obtained by applying Corollary 1:

$$P_2 = \begin{bmatrix} 11.9306 & -0.4048 \\ -0.4048 & 12.3502 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} -4.0426 & -4.0177 \\ -3.8555 & 2.3243 \end{bmatrix}$$

According to $K = P_2^{-1}Y_1$, the gain matrix K of state estimator is given as follows:

$$K = \begin{bmatrix} -0.3498 & -0.3307 \\ -0.3236 & 0.1774 \end{bmatrix}$$

The function of cyber attacks is shown in Fig. 2. The occurring probability $\theta(t)$ of the stochastic cyber attacks is described in

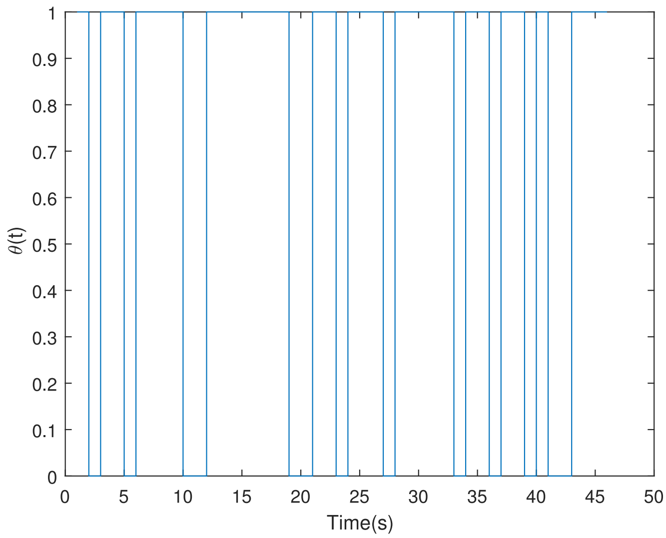


Fig. 3. Bernoulli distribution $\theta(t)$ in Case 1.

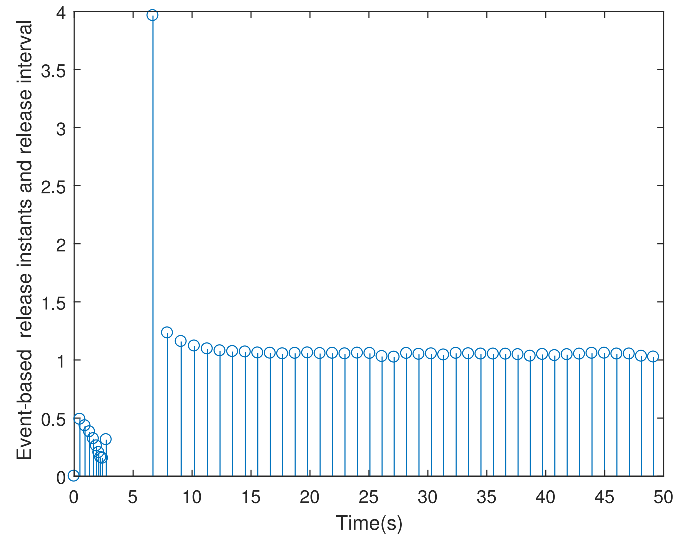


Fig. 5. Release instants and intervals in Case 2.

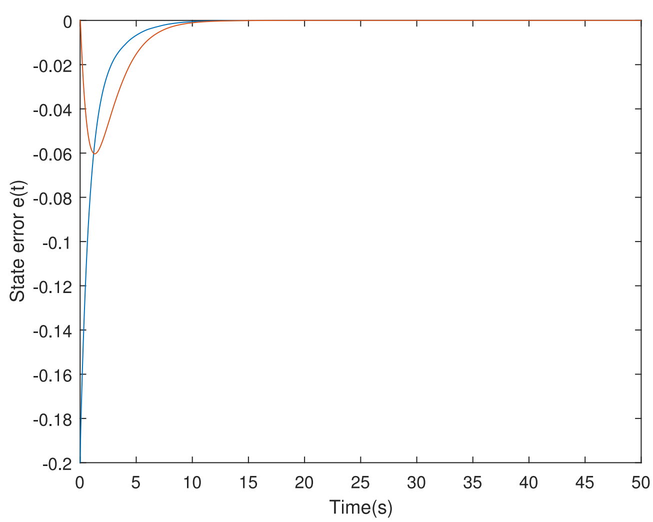


Fig. 4. The estimation error $e(t)$ in Case 1.

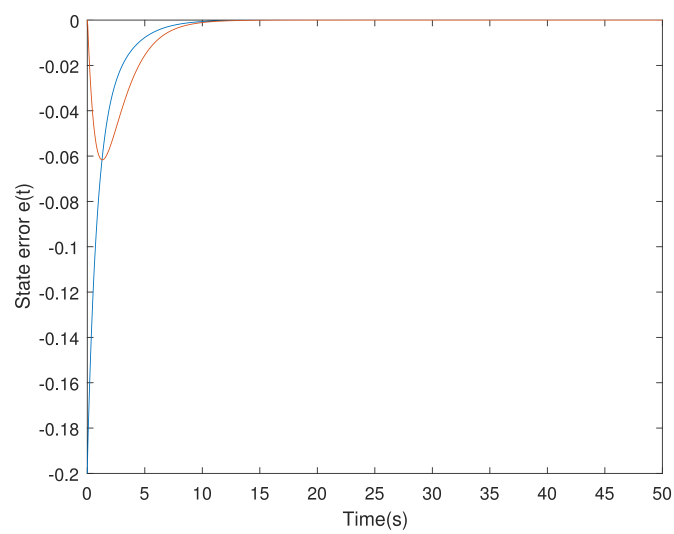


Fig. 6. The estimation error $e(t)$ in Case 2.

Fig. 3. Fig. 4 shows the state error $e(t)$ of the system when the "time triggered scheme" is selected in the hybrid triggered scheme. From the figures given above, it can be found that the proposed method is useful and the designed state estimator can satisfy the system performance.

Case 2: Set $\alpha(t) = 0$ and triggered parameter $\sigma = 0.5$, the "event triggered scheme" is selected in the hybrid triggered scheme as is shown in Fig. 1. Choose parameters $\phi_m = 0.05, \phi_M = 0.25$, $\eta_M = 0.02$, $d_M = 0.15$, let $\bar{\theta} = 0.2$, the following matrices can be derived by using Corollary 2:

$$P_2 = \begin{bmatrix} 10.1246 & -1.1065 \\ -1.1065 & 10.2122 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} 0.5283 & 0.5056 \\ 0.3659 & 0.1061 \end{bmatrix}$$

Applying the equation $K = P_2^{-1} Y_1$, the gain matrix K of state estimator is given as follows:

$$K = \begin{bmatrix} 0.0568 & 0.0517 \\ 0.0420 & 0.0160 \end{bmatrix}$$

The release instants and intervals are described in Fig. 5. The state error $e(t)$ is shown in Fig. 6 when the selecting switch turns to "event triggered scheme". According to Fig. 6, it can be easily seen that the state estimator is effective with event triggered scheme subject to cyber attacks.

Case 3: Suppose $\bar{\alpha} = 0.5$ and the event triggered parameter $\sigma = 0.5$, as is shown in Fig. 1, the designed state estimator is under the hybrid triggered scheme. For $\phi_m = 0.01, \phi_M = 0.02, \tau_M = 0.01$, $\eta_M = 0.01$, $d_M = 0.15$, set occurring probability $\bar{\theta} = 0.2$, the following matrices can be achieved by using Theorem 2:

$$P_2 = \begin{bmatrix} 15.7882 & -1.0345 \\ -1.0345 & 15.8020 \end{bmatrix}, \quad Y_1 = \begin{bmatrix} -1.2368 - 1.2377 \\ -0.3243 - 0.1615 \end{bmatrix}$$

By using the equation $K = P_2^{-1} Y_1$, the gain matrix K of state estimator is given as follows:

$$K = \begin{bmatrix} -0.0800 & -0.0794 \\ -0.0258 & -0.0154 \end{bmatrix}$$

The stochastic switch between the time triggered scheme and event triggered scheme is described by $\alpha(t)$, which is shown in Fig. 7. Fig. 8 depicts the state error $e(t)$ when the designed state estimator is under the hybrid triggered scheme. It can be found that the designed hybrid-driven-based state estimator is feasible for neural networks subject to cyber attacks.

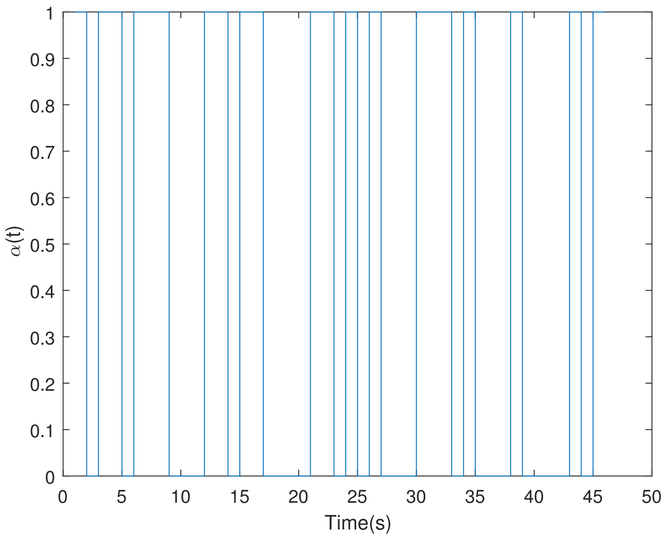


Fig. 7. Bernoulli distribution $\alpha(t)$ in Case 3.

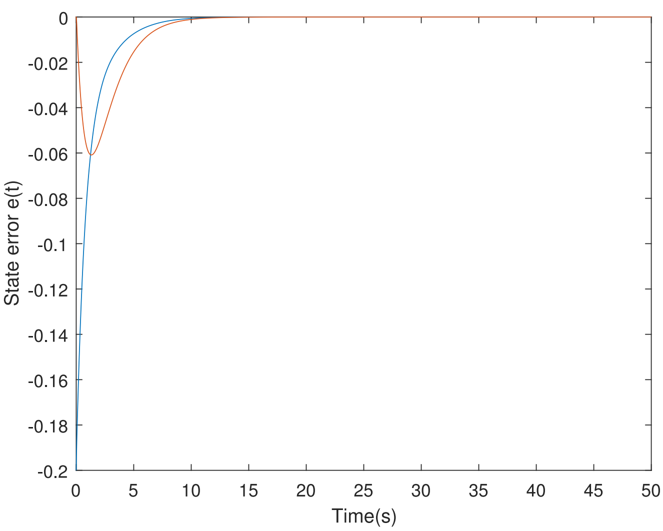


Fig. 8. The estimation error $e(t)$ in Case 3.

5. Conclusions

This paper is concerned with state estimation for neural networks with hybrid triggered scheme and quantization subject to cyber attacks. The hybrid triggered scheme and quantization are introduced to reduce the burden of network transmission and improve the efficiency of data communication. Two random variables which satisfy Bernoulli distribution are employed to describe the hybrid triggered scheme and the stochastic cyber attacks, respectively. By applying Lyapunov stability theory and LMI techniques, sufficient conditions can be derived which can ensure the stability of the desired estimating error system. In addition, the explicit expressions are provided for the gain matrix of the state estimator in terms of LMIs. Moreover, as the special cases of the achievements above, two corollaries are given, in which the designed state estimator is under the time triggered scheme and the event triggered scheme, respectively. Finally, an illustrative example is given to demonstrate the usefulness of the proposed method. Further work will study the effects of other kinds of cyber attacks for neural networks continuously. In addition, the cases with respect to limited data transmission rate, and encoding–decoding algorithms are also interesting, which motives our future research.

Acknowledgments

This work is partly supported by the National Key Technologies Research and Development Program of China under grant (no. 2015BAD-18B02), the National Natural Science Foundation of China (nos. 61403185, 61773218, 61473156), the Natural Science Foundation of Jiangsu Province of China (nos. BK20171481, BK20161561), Six Talent Peaks Project in Jiangsu Province (no. 2015- DZXX-21), major project supported by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China (no. 15KJA120001), Natural Science Foundation of the Jiangsu Higher Education Institutions of China (no.16KJB520017), National Science and Technology Pillar Program of Jiangsu Province of China (no.BE2016178), and a Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD).

References

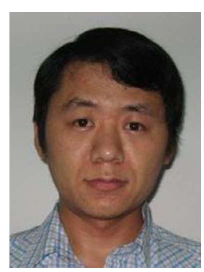
[1] Y. Zhang, Z. Wang, L. Ma, Variance-constrained state estimation for networked multi-rate systems with measurement quantization and probabilistic sensor failures, Int. J. Robust Nonlinear Control 26 (16) (2016) 3507–3523.
[2] P. Shi, Y. Zhang, R.K. Agarwal, Stochastic finite-time state estimation for discrete time-delay neural networks with Markovian jumps, Neurocomputing 151 (1) (2015) 168–174.
[3] Y. Xu, D. Zhang, State estimation with guaranteed performance for switching-type fuzzy neural networks in presence of sensor nonlinearities, Commun. Nonlinear Sci. Numer. Simul. 19 (7) (2014) 2160–2171.
[4] Z. Wang, Y. Liu, X. Liu, State estimation for jumping recurrent neural networks with discrete and distributed delays, Neural Networks 22 (1) (2009) 41–48.
[5] Z. Wang, D.W.C. Ho, Y. Liu, X. Liu, h_∞ filtering for uncertain stochastic time-delay systems with sector-bounded nonlinearities, Automatica 44 (5) (2008) 1268–1277.
[6] L. Sheng, W. Zhang, M. Gao, Relationship between Nash equilibrium strategies and H_2/H_∞ control of stochastic Markov jump systems with multiplicative noise, IEEE Trans. Autom. Control 59 (9) (2014) 2592–2597.
[7] G. Wei, H. Shu, h_∞ filtering on nonlinear stochastic systems with delay, Chaos Solitons Fract. 33 (2) (2007) 663–670.
[8] Y. Dong, E. Tian, Q.L. Han, A delay system method for designing event-triggered controllers of networked control systems, IEEE Trans. Autom. Control 58 (2) (2013) 475–481.
[9] Z. Gu, E. Tian, J. Liu, Adaptive event-triggered control of a class of nonlinear networked systems, J. Frank. Inst. 354 (9) (2017) 3854–3871.
[10] Z. Gu, L. Yang, E. Tian, H. Zhao, Event-triggered reliable h_∞ filter design for networked systems with multiple sensor distortions: a probabilistic partition approach, ISA Trans. 66 (2016) 2–9.
[11] L. Zha, J. Fang, J. Liu, E. Tian, Event-triggered non-fragile state estimation for delayed neural networks with randomly occurring sensor nonlinearity, Neurocomputing 273 (2018) 1–8.
[12] Y. Zhang, Z. Wang, L. Zou, H. Fang, Event-based finite-time filtering for multi--rate systems with fading measurements, IEEE Trans. Aerospace Electron. Syst. 53 (3) (2017) 1434–1441.
[13] J. Liu, L. Zha, J. Cao, S. Fei, Hybrid-driven-based stabilisation for networked control systems, IET Control Theory Applic. 10 (17) (2016) 2279–2285.
[14] L. Zha, J.A. Fang, J. Liu, E. Tian, Reliable control for hybrid-driven T–S fuzzy systems with actuator faults and probabilistic nonlinear perturbations, J. Frank. Inst. 354 (8) (2017) 3267–3288.
[15] J. Liu, J. Xia, E. Tian, S. Fei, Hybrid-driven-based H_∞ filter design for neural networks subject to deception attacks, Appl. Math. Comput. 320 (2018) 158–174.
[16] J. Liu, L. Wei, E. Tian, S. Fei, J. Cao, Hybrid-driven-based h_∞ filtering design for networked systems under stochastic cyber attacks, J. Frank. Inst. 354 (2017) 8490–8512.
[17] J. Liu, L. Zha, X. Xie, E. Tian, Resilient observer-based control for networked nonlinear T–S fuzzy systems with hybrid-triggered scheme, Nonlinear Dynam. 91 (3) (2017) 2049–2061.
[18] L. Zha, J.A. Fang, X. Li, J. Liu, Event-triggered output feedback h_∞ control for networked Markovian jump systems with quantizations, Nonli. Anal. Hybrid Syst. 24 (2017) 146–158.
[19] H. Yang, Y. Xu, J. Zhang, Event-driven control for networked control systems with quantization and Markov packet losses, IEEE Trans. Cybern. 47 (8) (2017) 2235–2243.
[20] L. Sheng, Z. Wang, W. Wang, F.E. Alsaadi, Output-feedback control for nonlinear stochastic systems with successive packet dropouts and uniform quantization effects, IEEE Trans. Syst. Man Cybern. Syst. 47 (7) (2017) 1181–1191.
[21] D. Ding, G. Wei, S. Zhang, Y. Liu, F.E. Alsaadi, On scheduling of deception attacks for discrete-time networked systems equipped with attack detectors, Neurocomputing 219 (2017) 99–106.
[22] D. Ding, Z. Wang, D.W.C. Ho, G. Wei, Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks, Automatica 78 (2017) 231–240.
[23] M.A. Sid, S. Chitraganti, K. Chabir, Medium access scheduling for input reconstruction under deception attacks, J. Frank. Inst. 354 (2016) 3678–3689.

[24] H. Zhang, P. Cheng, J. Wu, L. Shi, J. Chen, Online deception attack against remote state estimation, IFAC Proc. Vol. 47 (3) (2014) 128–133.
[25] W. Yang, L. Lei, C. Yang, Event-based distributed state estimation under deception attack, Neurocomputing 270 (2017) 140–151.
[26] J. Liu, J. Tang, S. Fei, Event-triggered h_∞ filter design for delayed neural network with quantization, Neural Networks 82 (2016) 39–48.
[27] L. Wang, Z. Wang, G. Wei, F.E. Alsaadi, Finite-time state estimation for recurrent delayed neural networks with component-based event-triggering protocol, IEEE Trans. Neural Networks Learn. Syst. (2017). https://doi:10.1109/TNNLS.2016.2635080.
[28] H. Rao, R. Lu, Y. Xu, C. Liu, State estimation for neural networks with jumping interval weight matrices and transmission delays, Neurocomputing 275 (2018) 909–915.
[29] Y. Liu, T. Wang, M. Chen, H. Shen, Y. Wang, D. Duan, Dissipativity-based state estimation of delayed static neural networks, Neurocomputing 247 (2017) 137–143.
[30] Y. Zhang, Z. Liu, H. Fang, H. Chen, h_∞ fault detection for nonlinear networked systems with multiple channels data transmission pattern, Inf. Sci. 221 (2013) 534–543.
[31] D. Yue, Q.L. Han, C. Peng, State feedback controller design of networked control systems, IEEE Trans. Circ. Syst. 51 (2014) 137–143.
[32] J. Liu, F. Wu, L. Zha, S. Fei, Co-design of event generator and state estimator for complex network systems with quantization, J. Frank. Inst. 353 (17) (2016) 4565–4582.
[33] F. Qu, Z. Guan, D. He, M. Chi, Event-triggered control for networked control systems with quantization and packet losses, J. Frank. Inst. 352 (3) (2015) 974–986.
[34] C. Liu, F. Hao, Dynamic output-feedback control for linear systems by using event-triggered quantization, IET Control Theory Applic. 9 (8) (2015) 1254–1263.
[35] G. Wei, Z. Wang, H. Shu, Robust filtering with stochastic nonlinearities and multiple missing measurements, Automatica 45 (3) (2009) 836–841.
[36] Z. Wang, D.W.C. Ho, Y. Liu, X. Liu, Robust h_∞ control for a class of nonlinear discrete time-delay stochastic systems with missing measurements, Automatica 45 (3) (2009) 684–691.
[37] L. Wang, Z. Wang, Q.L. Han, G. Wei, Synchronization control for a class of discrete-time dynamical networks with packet dropouts: a coding–decoding-based approach, IEEE Trans. Cybern. (2017). https://doi:10.1109/TCYB.2017.2740309.
[38] L. Wang, Z. Wang, Q.L. Han, G. Wei, Event-based variance-constrained h_∞ filtering for stochastic parameter systems over sensor networks with successive missing measurements, IEEE Trans. Cybern. 48 (3) (2017) 1007–1017.
[39] X. Mao, Exponential stability of stochastic delay interval systems with Markovian switching, Syst. Control Lett. 47 (10) (2000) 1604–1612.
[40] E. Tian, D. Yue, Decentralized control of network-based interconnected systems: a state-dependent triggering method, Int. J. Robust Nonlinear Control 25 (8) (2015) 1126–1144.
[41] N. Li, J. Hu, J. Hu, L. Li, Exponential state estimation for delayed recurrent neural networks with sampled-data, Nonlinear Dynam. 69 (69) (2012) 555–564.
[42] K. Gu, V.L. Kharitonov, J. Chen, Introduction to time-delay systems, Syst. Control Found. Applic. 72 (4) (2003) 591–597.
[43] Y. Wang, L. Xie, C.E.D. Souza, Robust control of a class of uncertain nonlinear systems, Syst. Control Lett. 19 (2) (1997) 139–149.
[44] I.R. Petersen, A stabilization algorithm for a class of uncertain linear systems, Syst. Control Lett. 8 (4) (1987) 351–357.
[45] D. Yue, E. Tian, Y. Zhang, C. Peng, Delay-distribution-dependent stability and stabilization of T–S fuzzy systems with probabilistic interval delay, IEEE Trans. Syst. 39 (2) (2009) 503–516.
[46] Y. He, M. Wu, J.H. She, G.P. Liu, Parameter-dependent Lyapunov functional for stability of time-delay systems with polytopic-type uncertainties, IEEE Trans. Autom. Control 49 (5) (2004) 828–832.
[47] D. Yue, Q.L. Han, J. Lam, Network-based robust protect H_∞ control of systems with uncertainty, Automatica 41 (6) (2005) 999–1007.

Jinliang Liu was born in Shandong Province, China, in 1980. He received the Ph.D. degree in School of Information Science and Technology at Donghua University. He was a post doctoral research associate in School of Automation, Southeast University, Nanjing, China from December 2013 to June 2016. From October 2016 to October 2017, he was a visiting researcher/scholar with the Department of Mechanical Engineering, the University of Hong Kong. Since November 2017 to January 2018, he was a visiting scholar in the Department of Electrical Engineering, Yeungnam University, South Korea. He is currently an associate professor at Nanjing University of Finance and Economics, Nanjing, China. His main research interest is networked control systems, complex dynamical networks, T–S fuzzy systems and time delay systems.

Jilei Xia was born in Jiangsu Province, China, in 1992. He received his B.S. degree from Changshu Institute of Technology in 2015. Currently, he is pursuing an M.S. degree in College of Information Engineering, Nanjing University of Finance and Economics. His research interests include networked control systems, T–S fuzzy systems, and time delay systems.

Jie Cao was born in Jiangsu Province, China, in 1969. He received his doctor's degree from Southeast University, in 2003. He is currently a professor at the Nanjing University of Finance and Economics, Nanjing, China. His main research interest is complex dynamical networks, data mining, business intelligence and recommendation system.



Engang Tian was born in Shandong Province, China, in 1980. He received the B.S. degree, M.S. degree and Ph.D. degree from Shandong Normal University, Nanjing Normal University and Donghua University, in 2002, 2005 and 2008 respectively. Since 2008, he has been with the School of Electrical and Automation Engineering, Nanjing Normal University. From February 2010 to May 2010, he was a visiting scholar with Northumbria University, Newcastle, England. From August 2011 to August 2012, he was a postdoctoral in Hong Kong Polytechnic University. Since August 2015 to August 2016, he was a visiting scholar in Brunel University, UK. His current research interests include networked control systems, T–S fuzzy systems and time delay systems.