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ORIGINAL PAPER

Resilient observer-based control for networked nonlinear T–S fuzzy systems with hybrid-triggered scheme

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Abstract In this paper, the resilient observer-based output feedback controller is designed for a class of networked T–S fuzzy systems under a hybrid-triggered scheme and mismatched membership functions. In order to improve network bandwidth utilization, a hybrid-triggered scheme is introduced between the state observer and the controller, which is based on a switching between periodic sampling and an event-triggered scheme. Considering the inaccurate implementation of the parameters of the observer-based controller, a novel hybrid-triggered T–S fuzzy model is constructed. Sufficient conditions are established for the augmented fuzzy systems by using Lyapunov theory and the linear matrix inequality techniques. Furthermore, the observer-based controller gains are designed in terms of linear matrix inequalities. Finally, a numerical example is provided to demonstrate the usefulness of the proposed method.

Keywords Event-triggered scheme · Fuzzy systems · Observer-based control · Networked control systems (NCSs)

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1 Introduction

During the past few decades, Takagi–Sugeno (T–S) fuzzy systems have been proved to be an effective approach to approximate any smooth nonlinear systems in the form of IF–THEN rules [1–5]. A majority of remarkable literature has been published about the stability analysis and control synthesis via T–S fuzzy systems [6–8]. Most of the publications make an assumption that the designed controller or filter shares the same membership functions with the related T–S fuzzy systems. However, due to the network-induced delay in networked control systems, it is more realistic to use the imperfect premise matching method [9,10]. Recently, T–S fuzzy control with mismatched membership functions has received considerable research interests [11–14].

In many practical systems, complete measurement of the system states is always infeasible to be obtained. Considering these difficulties, much attention has been devoted to the controller design that only requires partial state variables in recent years [15–22]. For example, in [15], the output feedback control is investigated for decentralized guaranteed cost stabilization of large-scale discrete-delay systems. In [17], the robust observer-based passive control method is developed for uncertain singular time-delay systems with actuator saturation. In [18], the non-fragile observer-based H_∞ control approach is presented for stochastic time-delay systems. In [19], the problem of simultaneous design of observers and controllers is considered for a class of piecewise affine systems with signal quantization. Considering imperfect communication links and parameter uncertainties, the fuzzy observer-based controller design is discussed in [20] for networked control systems (NCSs). Notice that the aforementioned results about observer-based control approaches may be not applicable to NCSs with constrained network bandwidth. This is the first motivation to design observer-based control for nonlinear systems with limited communication resources.

NCSs have advantages in improving control efficiency [23–27] and reducing maintenance cost [28,29]. Considerable efforts have been devoted to NCSs. However, the network bandwidth or sensor power sources are limited, which may result in overall system performance degradation. Over the past decades, periodic sampling and event-triggered control are main control strategies to deal with these problems. Periodical transmission of sensor measurement may result in data redundance, especially when the variation of the sensor measurement is small. While the event-triggered scheme can improve data transmission efficiency, under which the sampled data can be transmitted into communication link only when it violates predefined threshold in an event generator [30,31]. Therefore, event-triggered control is advantageous over periodic sampling in avoiding unnecessary transmission and waste of network resources. Much efforts have been dedicated to developing event-triggered schemes [32–37]. For instance, the adaptive control is considered in [34] for a class of event-trigger-based uncertain nonlinear systems. The authors in [32] propose a new event-triggered scheme and investigate event-triggered H_∞ controller design for NCSs. However, when the imperfect networked environment encounters random changes, neither periodic sampling nor the event-triggered scheme is suitable to determine when or how frequently the sensor should transmit the sampled data. Very recently, a hybrid-triggered scheme is proposed in [38] to deal with this problem. Note that the results of hybrid-triggered control for nonlinear systems are still very limited, which motivates the current work.

Inspired by the above observations, we focus on resilient observer-based control for networked T–S fuzzy systems under the hybrid-triggered scheme. The contributions of this paper are shown as follows. (1) The hybrid-triggered scheme is adopted to save limited network resources and achieve better performance, which is implemented based on randomly switching between the periodic sampling and the event-triggered scheme. (2) Different from the existing publications, the control law is updated according to transmitted observer states. In order to reflect the controller implementation uncertainties, the variation of the gains is taken into account. (3) By employing the Lyapunov functional approach, the sufficient conditions are obtained which can guarantee the asymptotical stability of the system. Furthermore, by solving a set of linear matrix inequalities, the gains of observer-based controller are designed for the discussed closed-loop system.

This paper is organized as follows. Section 2 introduces the problem of resilient observer-based control for hybrid-triggered T–S fuzzy systems. The main results are presented in Sect. 4. A numerical example is employed to verify usefulness of the proposed method in Sect. 5. Finally, the conclusion is drawn in Sect. 5.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n \times m$ real matrices, respectively; the superscript T stands for matrix transposition; I is the identity matrix of appropriate dimension; the notation $X > 0$ (respectively, $X \geq 0$), for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, positive semi-definite); $Prob\{X\}$ denotes probability of event X to occur; $rank(C)$ denotes the rank of the matrix C; $\mathcal{E}$ denotes the expectation operator; for a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2 System description

In this paper, a T–S fuzzy dynamic model is used to describe a nonlinear NCS. The ith rule of the fuzzy model is expressed as follows:

Plant Rule i: IF $\theta_1(x)$ is M_1^i, ..., and $\theta_g(x)$ is M_g^i

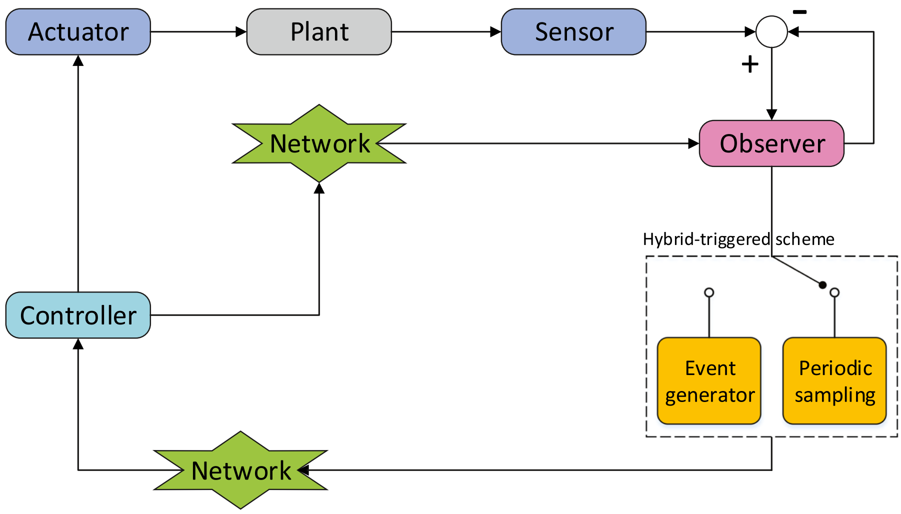

Fig. 1 Block diagram of the networked T–S fuzzy system

Then

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where M_q^i $(i = 1, \ldots, r, q = 1, 2, \ldots, g)$ is the fuzzy set, $\theta_q(x)$ $(q = 1, 2, \ldots, g)$ denotes the premise variable, $x(t) \in \mathbb{R}^n$ is the state vector, and $y(t) \in \mathbb{R}^m$ is the measurable output, $u(t) \in \mathbb{R}^u$ is the control input. A_i, B_i and C_i are known real constant matrices with appropriate dimensions. By using a center-average defuzzifier, product fuzzy inference, and a singleton fuzzifier, system (1) can be inferred as

$$\begin{cases} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(\theta(x))[A_i x(t) + B_i u(t)] \\ y(t) = \sum\limits_{i=1}^{r} h_i(\theta(x)) C_i x(t) \end{cases} \tag{2}$$

where $\theta(x) = \left[\theta_1(x), \theta_2(x), \ldots, \theta_g(x)\right], h_i(\theta(x)) = \frac{\mu_i(\theta(x))}{\sum_{i=1}^{r} \mu_i(\theta(x))}$ and $\mu_i(\theta(x)) = \prod_{m=1}^{g} M_m^i(\theta_m(x))$, $M_m^i(\theta_m(x))$ is the membership value of $\theta_m(x)$ in M_m^i, $h_i(\theta(x)) \geq 0, \sum_{i=1}^{r} h_i(\theta(x)) = 1$.

The main objective of this paper is to design an observer-based controller for the system (2). The observer model is presented as follows:

Observer Rule j: IF $\theta_1(x)$ is $M_1^j, \ldots,$ and $\theta_g(x)$ is M_g^j

Then

$$\begin{aligned} \dot{\hat{x}}(t) &= A_j \hat{x}(t) + B_j u(t) \\ &\quad + (L_j + \Delta L_j)(y(t) - C_j \hat{x}(t)) \end{aligned} \tag{3}$$

where $\hat{x}(t)$ is the estimation of the state vector $x(t)$, L_j is the observer gain to be designed later. $\Delta L_j = M_{1j} \Delta_1(t) N_{1j}$ represents the uncertainties in the observer gain, in which $\Delta_1^T(t)\Delta_1(t) \leq I$, M_{1j} and N_{1j} are the known matrices with appropriate dimensions. The global model of the observer is deduced as

$$\begin{aligned} \dot{\hat{x}}(t) = \sum_{j=1}^{r} & h_j(\theta(x))[A_j \hat{x}(t) + B_j u(t) \\ & + (L_j + \Delta L_j)(y(t) - C_j \hat{x}(t))] \end{aligned} \tag{4}$$

As shown in Fig. 1, the observer and the controller are connected by a communication network with resource constraints. A hybrid-triggered scheme is introduced at the observer side to improve the efficiency of the communication network. Whether the observer state is transmitted or not will be determined by a switching strategy between the periodic sampling and the event-triggered scheme.

For ease of exposition, we make the following assumptions:

(1) The state of the observer is sampled at a constant period h. $\{sh \mid s \in \mathbb{N}\}$ represents the set of sampled instants.

(2) The set of the transmission instants is denoted by $\{t_k h \mid t_k \in \mathbb{N}\}$, which is determined by the sampled state $\hat{x}(t_k h)$. $\hat{x}(t_k h)$ represents the signal which arrives at the controller successfully.

(3) τ_{t_k} represents the communication delays between the observer and the controller. $\tau_{t_k} \in [0, \bar{\tau})$, $\bar{\tau}$ is the upper bound of τ_{t_k}.

It should be noticed that the premise variables between the observer and the controller may be asynchronous because of the existing network-induced communication delays. With consideration of the mismatched membership functions, the hybrid-triggered scheme and the implementation of the observer-based controller, the controller can be designed as follows:

Controller Rule l: IF $\omega_1(\hat{x})$ is $W_1^l, \ldots,$ and $\omega_p(\hat{x})$ is W_p^l

Then

$$u(t) = (K_l + \Delta K_l)\hat{x}(t_k h), \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{5}$$

where K_l are the controller gains to be determined later. The fuzzy controller is given by

$$u(t) = \sum_{l=1}^{r} m_l(\omega(\hat{x}))(K_l + \Delta K_l)\hat{x}(t_k h), \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{6}$$

where $\omega(\hat{x}) = \left[\omega_1(x), \omega_2(x), \ldots, \omega_g(x)\right]$, $m_l(\omega(\hat{x})) = \frac{o_l(\omega(\hat{x}))}{\sum_{l=1}^{r} o_l(\omega(\hat{x}))}$ and $o_l(\omega(\hat{x})) = \prod_{d=1}^{p} W_d^l(\omega_d(\hat{x}))$, $W_d^l(\omega_d(\hat{x}))$ is the membership value of $\omega_d(\hat{x})$ in W_d^l, $o_l(\omega(\hat{x})) \geq 0$, $\sum_{l=1}^{r} o_l(\omega(\hat{x})) = 1$.

If the observer state $\hat{x}(t)$ are transmitted through the network periodically, similar to [38], define $\tau(t) = t - t_k h$, the control law in (6) can be rewritten as

$$u(t) = \sum_{l=1}^{r} m_l(\omega(\hat{x}))(K_l + \Delta K_l)\hat{x}(t - \tau(t)), \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{7}$$

In periodic sampling, the limited network bandwidth is unavoidable waste, under which the sampled signals are still transmitted into the network even when the system states have little change. For energy-saving purpose, event-triggered scheme is employed as an alternative transmission strategy. The event-triggered condition is designed as follows:

$$[\hat{x}(t_k h + sh) - \hat{x}(t_k h)]^T W_1[\hat{x}(t_k h + sh) - \hat{x}(t_k h)] \leq \sigma \hat{x}^T(t_k h + sh) W_2 \hat{x}(t_k h + sh) \tag{8}$$

where $\hat{x}(t_k h)$ is the latest transmitted observer state, $s = 0, 1, 2 \ldots, \sigma \in [0, 1)$, W_1 and W_2 are matrices with appropriate dimensions. h is the sampling constant period. The holding interval $\Omega = [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$ can be described as $\Omega = \bigcup_{s=0}^{s=t_{k+1}-t_k-1} \Omega_s$, in which $\Omega_s = [t_k h + sh + \tau_{t_{k+s}}, t_k h + sh + h + \tau_{t_{k+s+1}})$, $s = 0, 1, \ldots, t_{k+1} - t_k - 1$. Set $e_k(t) = \hat{x}(t_k h) - \hat{x}(t_k h + sh)$, $d(t) = t - t_k h - sh$, $d(t)$ satisfies $0 \leq \tau_{t_k} \leq d(t) \leq h + \tau_{t_{k+s+1}} \trianglerighteq d_M$. Then, (6) can be represented as

$$u(t) = \sum_{l=1}^{r} m_l(\omega(\hat{x}))(K_l + \Delta K_l)[\hat{x}(t - d(t)) + e_k(t)], \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{9}$$

In this paper, under the hybrid-triggered scheme, the real control output can be presented as a switching between (7) and (9). $\alpha(t)$ is used to describe the switching law in the hybrid-triggered scheme. We can derive

$$u(t) = \sum_{l=1}^{r} m_l(\omega(\hat{x}))(K_l + \Delta K_l) [(1 - \alpha(t))\left(\hat{x}(t - d(t)) + e_k(t)\right) + \alpha(t)\hat{x}(t - \tau(t))], \quad t \in [t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}) \tag{10}$$

where K_l are the controller gains to be determined, ΔK_l denote the norm-bounded parameter uncertainty. $\Delta K_l = M_{2l}\Delta_2(t)N_{2l}$, in which M_{2l} and N_{2l} are the given constant matrices, $\Delta_2(t)$ satisfies $\Delta_2^T(t)\Delta_2(t) \leq I$.

Remark 1 It should be pointed out that the fluctuations or drafts on the parameters of the observer-based controller are unavoidable. The small uncertainties on the controller gains should be considered in controller implementation. Therefore, the resilient observer-based control problem is necessary to be considered.

Defining the estimation error $\eta(t) = x(t) - \hat{x}(t)$. Substitute (10) into (2) and (4), we can get

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{l=1}^{r} h_i(\theta(t))m_l(\omega(\hat{x})) \{A_i x(t) + B_i K_l \mathcal{X}(t)\} \tag{11}$$

$$\dot{\hat{x}}(t) = \sum_{j=1}^{r}\sum_{l=1}^{r} h_j(\theta(t))m_l(\omega(\hat{x})) \{\Pi_{ij}\hat{x}(t) + (L_j + \Delta L_j)C_i\eta(t) + B_j(K_l + \Delta K_l)\mathcal{X}(t)\} \tag{12}$$

$$\dot{\eta}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(t))h_j(\theta(t))m_l(\omega(\hat{x})) \left\{(A_i - \Pi_{ij})\hat{x}(t) + [A_i - (L_j + \Delta L_j)C_i]\eta(t) + (B_i - B_j)(K_l + \Delta K_l)\mathcal{X}(t)\right\} \tag{13}$$

where $\Pi_{ij} = A_j + (L_j + \Delta L_j)(C_i - C_j)$, $\mathcal{X}(t) = \bar{\mathcal{X}}(t) + (\alpha(t) - \bar{\alpha})(\hat{x}(t - \tau(t)) - \hat{x}(t - d(t)) -$

$e_k(t)), \bar{\mathcal{X}}(t) = (1-\bar{\alpha})\left(\hat{x}(t-d(t)) + e_k(t)\right) + \bar{\alpha}\hat{x}(t-\tau(t))$.

Remark 2 It is noted that Bernoulli distributed random variable $\alpha(t)$ is introduced to model the switching law of periodic sampling and the event-triggered scheme in the hybrid-triggered scheme. The sojourn probability $\bar{\alpha}$ can be obtained through the statistical method in [38].

The aim of this paper is to design L_j and K_l for the stability of system (11), (12) and (13). The following lemmas are useful in deriving our main results.

Lemma 1 [39] *Consider the augmented system with $\tau(t)$ that satisfies $0 < \tau(t) \le \bar{\tau}$. For any matrices $X \in \mathbb{R}^{n\times n}$ and $U \in \mathbb{R}^{n\times n}$ that satisfy $\begin{bmatrix} X & U \\ U^T & X \end{bmatrix} \ge 0$, the following inequality holds:*

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t} \dot{\xi}^T(s)X\dot{\xi}(s) \le \begin{bmatrix} \xi(t) \\ \xi(t-\tau(t)) \\ \xi(t-\bar{\tau}) \end{bmatrix}^T \begin{bmatrix} -X & * & * \\ X^T - U^T & -2X + U + U^T & * \\ U^T & X^T - U^T & -X \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t-\tau(t)) \\ \xi(t-\bar{\tau}) \end{bmatrix} \tag{14}$$

Lemma 2 [40] *Given matrices $F_1 = F_1^T$, F_2 and F_3 of appropriate dimensions, we have $F_1 + F_3\Delta(t)F_2 + F_2^T\Delta^T(t)F_3^T < 0$ for all $\Delta(t)$ satisfying $\Delta^T(t)\Delta(t) \le I$, if and only if there exists a positive scalar $\varepsilon < 0$, such that $F_1 + \varepsilon^{-1}F_3F_3^T + \varepsilon F_2^T F_2 < 0$.*

Lemma 3 [41] *For full rank matrix $rank(C) = m$, $C \in \mathbb{R}^{m\times n}$, the singular value decomposition (SVD) for C can be described as $C = O\left[S\ 0\right]V^T$, where $O\cdot O^T = I$ and $V\cdot V^T = I$. Let $X > 0$, $M \in \mathbb{R}^{m\times m}$, $N \in \mathbb{R}^{n\times n}$. Then, there exists $\bar{X}$ such that $CX = \bar{X}C$ if and only if $X = Vdiag\{M, N\}V^T$.*

3 Main results

3.1 Stability analysis

In this section, we are in position to give sufficient conditions for the stability of the discussed hybrid-triggered T–S fuzzy system by using Lyapunov–Krasovskii functional approach.

Theorem 1 *For given positive scalars $\bar{\alpha}, \tau_M, d_M, \sigma$, and the membership functions satisfying $m_l(\omega(\hat{x})) - \rho_l h_l(\theta(x)) \ge 0$ $(0 < \rho_l \le 1)$, the augmented system (12) and (13) with the resilient fuzzy control (10) are asymptotically stable if there exist positive matrices $P > 0, Q_1 > 0, Q_2 > 0, R_1 > 0$, $R_2 > 0$, $W_1 > 0$, $W_2 > 0$, U_1, U_2 and V with compatible dimensions satisfying the following inequalities $(i, j, l = 1, \ldots, r)$,*

$$\rho_l\Xi^{ijl} + \rho_j\Xi^{ilj} - \rho_j\mathcal{V} - \rho_l\mathcal{V} + 2\mathcal{V} < 0 \tag{15}$$

$$\Xi^{ijl} + \Xi^{ilj} - 2\mathcal{V} < 0 \tag{16}$$

$$\begin{bmatrix} R_1 & * \\ U_1 & R_1 \end{bmatrix} > 0 \tag{17}$$

$$\begin{bmatrix} R_2 & * \\ U_2 & R_2 \end{bmatrix} > 0 \tag{18}$$

where

$$\Xi^{ijl} = \begin{bmatrix} \Xi_{11}^{ijl} & * & * \\ \Xi_{21}^{ijl} & \Xi_{22} & * \\ \Xi_{31}^{ijl} & 0 & \Xi_{33} \end{bmatrix},$$

$$\mathcal{V} = \begin{bmatrix} V \\ 0 \\ 0 \end{bmatrix},$$

$$\Xi_{11}^{ijl} = \begin{bmatrix} \Xi_{111}^{ijl} & * \\ \Xi_{112}^{ijl} & \Xi_{113}^{ijl} \end{bmatrix}$$

$$\Xi_{111}^{ijl} = \begin{bmatrix} \Gamma_1^{ijl} & * & * & * \\ \Gamma_2^{ijl} & -2R_1 + U_1 + U_1^T & * & * \\ U_1 & R_1 - U_1 & -Q_1 - R_1 & * \\ \Gamma_3^{ijl} & 0 & 0 & \Gamma_4 \end{bmatrix}$$

$$\Xi_{112}^{ijl} = \begin{bmatrix} U_2 & 0 & 0 & R_2 - U_2 \\ \Gamma_5^{ijl} & \bar{\alpha}\Gamma_6^{ijl} & 0 & (1-\bar{\alpha})\Gamma_6^{ijl} \\ \Gamma_8^{ijl} & 0 & 0 & 0 \end{bmatrix}$$

$$\Xi_{113}^{ijl} = \begin{bmatrix} -Q_2 - R_2 & * & * \\ 0 & \Gamma_7^{ijl} + \Gamma_7^{ijl^T} & * \\ 0 & (1-\bar{\alpha})\Gamma_6^{ijl^T} & -W_1 \end{bmatrix}$$

$$\Gamma_1^{ijl} = P\Pi_{ij} + \Pi_{ij}^T P + Q_1 + Q_2 - R_1 - R_2$$

$$\Gamma_2^{ijl} = \bar{\alpha}(K_l + \Delta K_l)^T B_j^T P + R_1 - U_1$$

$$\Gamma_3^{ijl} = (1-\bar{\alpha})(K_l + \Delta K_l)^T B_j^T P + R_2 - U_2,$$
$$\Gamma_4 = -2R_2 + U_2 + U_2^T + \sigma W_2$$
$$\Gamma_5^{ijl\,T} = P(L_j + \Delta L_j)C_i + (A_i - \Pi_{ij})^T P,$$
$$\Gamma_6^{ijl} = P(B_i - B_j)(K_l + \Delta K_l)$$
$$\Gamma_7^{ijl} = PA_i - P(L_j + \Delta L_j)C_i,$$
$$\Gamma_8^{ijl} = (1-\bar{\alpha})(K_l + \Delta K_l)^T B_j^T P$$
$$\Xi_{21}^{ijl} = \begin{bmatrix} \tau_M \mathcal{F}^{ijl} \\ d_M \mathcal{F}^{ijl} \end{bmatrix}, \quad \Xi_{31}^{ijl} = \begin{bmatrix} \tau_M \mathcal{G}^{ijl} \\ d_M \mathcal{G}^{ijl} \end{bmatrix},$$
$$\Xi_{22} = diag\{-PR_1^{-1}P, -PR_2^{-1}P\}$$
$$\Xi_{33} = \Xi_{22},$$
$$\mathcal{F}^{ijl} = [\mathcal{F}_1^{ijl} \quad P(L_j + \Delta L_j)C_i \quad (1-\bar{\alpha})PB_j(K_l + \Delta K_l)]$$
$$\mathcal{F}_1^{ijl} = [\Pi_{ij} \quad \bar{\alpha}PB_j(K_l + \Delta K_l) \quad 0 \quad (1-\bar{\alpha}) PB_j(K_l + \Delta K_l) \quad 0]$$
$$\mathcal{G}^{ijl} = [0 \quad \delta PB_j(K_l + \Delta K_l) \quad 0 - \delta PB_j (K_l + \Delta K_l) 0 0 - \delta PB_j(K_l + \Delta K_l)]$$

Proof The Lyapunov functional candidate is chosen as follows:

$$\begin{aligned} V(t) = {} & \hat{x}^T(t)P\hat{x}(t) + \eta^T(t)P\eta(t) \\ & + \int_{t-\tau_M}^{t} \hat{x}^T(s)Q_1\hat{x}(s)ds \\ & + \int_{t-d_M}^{t} \hat{x}^T(s)Q_2\hat{x}(s)ds \\ & + \tau_M \int_{t-\tau_M}^{t} \int_{s}^{t} \dot{\hat{x}}^T(v)R_1\dot{\hat{x}}(v)dvds \\ & + d_M \int_{t-d_M}^{t} \int_{s}^{t} \dot{\hat{x}}^T(v)R_2\dot{\hat{x}}(v)dvds \end{aligned} \tag{19}$$

Calculating derivative on $V(t)$ and taking expectation on them, we have

$$\begin{aligned} \mathcal{E}\{\dot{V}(t)\} = {} & 2\hat{x}^T(t)P\mathcal{E}\{\dot{\hat{x}}(t)\} + 2\eta^T(t)P\mathcal{E}\{\dot{\eta}(t)\} \\ & + \hat{x}^T(t)(Q_1 + Q_2)\hat{x}(t) \\ & - \hat{x}^T(t-\tau_M)Q_1\hat{x}(t-\tau_M) \\ & - \hat{x}^T(t-d_M)Q_2\hat{x}(t-d_M) \\ & + \mathcal{E}\{\dot{\hat{x}}^T(t)(\tau_M^2 R_1 + d_M^2 R_2)\dot{\hat{x}}(t)\} \\ & - \tau_M \int_{t-\tau_M}^{t} \dot{\hat{x}}^T(s)R_1\dot{\hat{x}}(s)ds \\ & - d_M \int_{t-d_M}^{t} \dot{\hat{x}}^T(s)R_2\dot{\hat{x}}(s)ds \end{aligned} \tag{20}$$

Note that

$$\mathcal{E}\{\dot{\hat{x}}(t)\} = \sum_{j=1}^{r}\sum_{l=1}^{r} h_j(\theta(x))m_l(\omega(\hat{x}))\mathcal{A}^{ijl} \tag{21}$$

$$\begin{aligned} \mathcal{E}\{\dot{\eta}(t)\} = {} & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(x))h_j(\theta(x))m_l(\omega(\hat{x})) \\ & \big\{(A_i - \Pi_{ij})\hat{x}(t) \\ & + [A_i - (L_j + \Delta L_j)C_i]\eta(t) \\ & + (B_i - B_j)(K_l + \Delta K_l)\bar{\mathcal{X}}(t)\big\} \end{aligned} \tag{22}$$

$$\begin{aligned} & \mathcal{E}\{\dot{\hat{x}}^T(t)(\tau_M^2 R_1 + d_M^2 R_2)\dot{\hat{x}}(t)\} \\ & = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(x))h_j(\theta(x))m_l(\omega(\hat{x})) \\ & \quad \Big\{(\mathcal{A}^{ijl})^T\mathcal{R}\mathcal{A}^{ijl} + \delta^2(\mathcal{B}^{ijl})^T\mathcal{R}\mathcal{B}^{ijl}\Big\} \end{aligned} \tag{23}$$

where $\mathcal{A}^{ijl} = \Pi_{ij}\hat{x}(t) + (L_j + \Delta L_j)C_i(\eta(t) + \hat{x}(t)) + B_j(K_l + \Delta K_l)\bar{\mathcal{X}}(t)$, $\mathcal{B}^{ijl} = B_j(K_l + \Delta K_l)\bar{\mathcal{X}}(t)$, $\mathcal{R} = \tau_M^2 R_1 + d_M^2 R_2$.

By Lemma 1, from (17) and (18), we obtain

$$\begin{aligned} & -\tau_M \int_{t-\tau_M}^{t} \dot{\hat{x}}^T(s)R_1\dot{\hat{x}}(s)ds \\ & \le \xi_1^T(t) \begin{bmatrix} -R_1 & * & * \\ R_1 - U_1 & -2R_1 + U_1 + U_1^T & * \\ U_1 & R_1 - U_1 & -R_1 \end{bmatrix} \xi_1(t) \end{aligned} \tag{24}$$

$$\begin{aligned} & -d_M \int_{t-d_M}^{t} \dot{\hat{x}}^T(s)R_2\dot{\hat{x}}(s)ds \\ & \le \xi_2^T(t) \begin{bmatrix} -R_2 & * & * \\ R_2 - U_2 & -2R_2 + U_2 + U_2^T & * \\ U_2 & R_2 - U_2 & -R_2 \end{bmatrix} \xi_2(t) \end{aligned} \tag{25}$$

where

$$\xi_1^T(t) = \begin{bmatrix} \hat{x}^T(t) & \hat{x}^T(t-\tau(t)) & \hat{x}^T(t-\tau_M) \end{bmatrix}$$
$$\xi_2^T(t) = \begin{bmatrix} \hat{x}^T(t) & \hat{x}^T(t-d(t)) & \hat{x}^T(t-d_M) \end{bmatrix}$$

From (8), we can obtain

$$e_k^T(t)W_1e_k(t) \le \sigma\hat{x}^T(t-d(t))W_2\hat{x}(t-d(t)) \tag{26}$$

Define $\xi^T(t) = [\xi_1^T(t)\hat{x}^T(t-d(t))\hat{x}^T(t-d_M) \ \eta^T(t) \quad e_k^T(t)]$

Recalling (20–26), we derive

$$\mathcal{E}\{\dot{V}(t)\} \le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(x))h_j(\theta(x))m_l(\omega(\hat{x}))$$

$$\left\{\xi^T(t)\Theta^{ijl}\xi(t)\right\} \tag{27}$$

in which $\Theta^{ijl} = \Xi_{11}^{ijl} + (\mathcal{F}^{ijl})^T\mathcal{R}\mathcal{F}^{ijl} + (\mathcal{G}^{ijl})^T\mathcal{R}\mathcal{G}^{ijl}$.

Similar to [41], a slack matrix V with appropriate dimension is introduced

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(x))h_j(\theta(x)) [h_l(\theta(x)) - m_l(\omega(\hat{x}))]V = 0 \tag{28}$$

Substituting (28) into (27), we can get

$$\begin{aligned} \mathcal{E}\{\dot{V}(t)\} &\le \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(x))h_j(\theta(x)) \\ &\quad \Big\{m_l(\omega(\hat{x}))\xi^T(t)[\Theta^{ijl} + (h_l(\theta(x)) \\ &\quad -m_l(\omega(\hat{x})))V]\xi(t) \\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(x))h_j(\theta(x)) \\ &\quad \Big\{h_l(\theta(x))\xi^T(t)(\rho_l\Theta^{ijl} - \rho_l V + V)\xi(t) \\ &\quad +(m_l(\omega(\hat{x})) - \rho_l h_l(\theta(x)))\xi^T(t) \\ &\quad (\Theta^{ijl} - V)\xi(t)\Big\} \end{aligned} \tag{29}$$

$$\begin{aligned} &= \frac{1}{2}\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(\theta(x))h_j(\theta(x)) \\ &\quad \Big\{h_l(\theta(x))\xi^T(t)(\rho_l\Theta^{ijl} + \rho_j\Theta^{ilj} \\ &\quad -\rho_j V - \rho_l V + 2V)\xi(t) \\ &\quad +\frac{1}{2}(m_l(\omega(\hat{x})) - \rho_l h_l(\theta(x))) \\ &\quad \xi^T(t)(\Theta^{ijl} + \Theta^{ilj} - 2V)\xi(t)\Big\} \end{aligned} \tag{30}$$

With $m_l(\omega(\hat{x})) - \rho_l h_l(\theta(x)) \ge 0$, $\mathcal{E}\{\dot{V}(t)\} < 0$ is ensured by

$$\rho_l\Theta^{ijl} + \rho_j\Theta^{ilj} - \rho_j V - \rho_l V + 2V < 0 \tag{31}$$

$$\Theta^{ijl} + \Theta^{ilj} - 2V < 0 \tag{32}$$

By utilizing Schur complement, it can be concluded that (31) and (32) can be obtained from (15) and (16), respectively.

Therefore, the asymptotical stability of the system can be ensured by (15–18).

This completes the proof. □

3.2 Observer-based controller design

On the basis of Theorem 1, the following theorem provides the explicit design method of the gain matrix of observer-based controller.

Theorem 2 *Considering the membership functions* $m_l(\omega(\hat{x})) - \rho_l h_l(\theta(x)) \ge 0$ $(0 < \rho_l \le 1)$, *for given positive scalars* $\bar{\alpha}, \tau_M, d_M, \sigma, \varepsilon_i$ $(i = 1, 2, 3, 4, 5)$, *the augmented system* (12) *and* (13) *with the resilient state feedback fuzzy control* (10) *are asymptotically stable if there exist positive matrices* $X > 0$, $\bar{Q}_1 > 0$, $\bar{Q}_2 > 0$, $\bar{R}_1 > 0$, $\bar{R}_2 > 0$, $\bar{W}_1 > 0$, $\bar{W}_2 > 0$, $\bar{U}_1, \bar{U}_2$ *and* $\bar{V}$ *with compatible dimensions satisfying the following linear matrix inequalities* $(i, j, l = 1, \ldots, r)$,

$$\rho_l\bar{\Xi}^{ijl} + \rho_j\bar{\Xi}^{ilj} - \rho_j\bar{\mathcal{V}} - \rho_l\bar{\mathcal{V}} + 2\bar{\mathcal{V}} < 0 \tag{33}$$

$$\bar{\Xi}^{ijl} + \bar{\Xi}^{ilj} - 2\bar{\mathcal{V}} < 0 \tag{34}$$

$$\begin{bmatrix} \bar{R}_1 & * \\ \bar{U}_1 & \bar{R}_1 \end{bmatrix} > 0 \tag{35}$$

$$\begin{bmatrix} \bar{R}_2 & * \\ \bar{U}_2 & \bar{R}_2 \end{bmatrix} > 0 \tag{36}$$

where

$$\bar{\Xi}^{ijl} = \begin{bmatrix} \bar{\Xi}_{11}^{ijl} & * & * & * & * & * \\ \bar{\Xi}_{21}^{ijl} & \bar{\Xi}_{22} & * & * & * & * \\ \bar{\Xi}_{31}^{ijl} & 0 & \bar{\Xi}_{33} & * & * & * \\ \bar{\Xi}_{41}^{ijl} & \bar{\Xi}_{42}^{ijl} & 0 & \bar{\Xi}_{44} & * & * \\ \bar{\Xi}_{51}^{ijl} & \bar{\Xi}_{52}^{ijl} & 0 & 0 & \bar{\Xi}_{55} & * \\ \bar{\Xi}_{61}^{ijl} & 0 & \bar{\Xi}_{63}^{ijl} & 0 & 0 & \bar{\Xi}_{66} \end{bmatrix},$$

$$\bar{\mathcal{V}} = \begin{bmatrix} \bar{V} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \bar{\Xi}_{11}^{ijl} = \begin{bmatrix} \bar{\Xi}_{111}^{ijl} & * \\ \bar{\Xi}_{112}^{ijl} & \bar{\Xi}_{113}^{ijl} \end{bmatrix}$$

$$\bar{\Xi}_{11}^{ijl} = \begin{bmatrix} \bar{\Gamma}_1^{ijl} & * & * & * \\ \bar{\Gamma}_2^{ijl} & -2\bar{R}_1 + \bar{U}_1 + \bar{U}_1^T & * & * \\ \bar{U}_1 & \bar{R}_1 - \bar{U}_1 & -\bar{Q}_1 - \bar{R}_1 & * \\ \bar{\Gamma}_3^{ijl} & 0 & 0 & \bar{\Gamma}_4 \end{bmatrix}$$

$$\bar{\Xi}_{112}^{ijl} = \begin{bmatrix} \bar{U}_2 & 0 & 0 & \bar{R}_2 - \bar{U}_2 \\ \bar{\Gamma}_5^{ijl} & \bar{\alpha}\bar{\Gamma}_6^{ijl} & 0 & (1-\bar{\alpha})\bar{\Gamma}_6^{ijl} \\ \bar{\Gamma}_8^{ijl} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Xi}_{113}^{ijl} = \begin{bmatrix} -\bar{Q}_2 - \bar{R}_2 & * & * \\ 0 & \bar{\Gamma}_7^{ijl} + (\bar{\Gamma}_7^{ijl})^T & * \\ 0 & (1-\bar{\alpha})(\bar{\Gamma}_6^{ijl})^T & -\bar{W}_1 \end{bmatrix}$$

$$\bar{\Xi}_{44} = diag\{-\varepsilon_3 I, -\varepsilon_3 I\},$$

$$\bar{\Xi}_{55} = diag\{-\varepsilon_4 I, -\varepsilon_4 I\},$$

$$\bar{\Xi}_{66} = diag\{-\varepsilon_5 I, -\varepsilon_5 I\}$$

$$\bar{\Xi}_{51}^{ijl} = \begin{bmatrix} \varepsilon_4 M_{2l}^T B_j^T & 0 & 0 & 0 & 0 & \varepsilon_4 M_{2l}^T (B_i - B_j)^T & 0 \\ 0 & \bar{\alpha} N_{2l} X & 0 & (1-\bar{\alpha}) N_{2l} X & 0 & 0 & N_{2l} X \end{bmatrix}$$

$$\bar{\Gamma}_1^{ijl} = \bar{\Pi}_{ij} + \bar{\Pi}_{ij}^T + \bar{Q}_1 + \bar{Q}_2 - \bar{R}_1 - \bar{R}_2$$

$$\bar{\Gamma}_2^{ijl} = \bar{\alpha} Y_l^T B_j^T + R_1 - U_1,$$

$$\bar{\Pi}_{ij} = A_j X + Z_j (C_i - C_j)$$

$$\bar{\Gamma}_3^{ijl} = (1-\bar{\alpha}) Y_l^T B_j^T + \bar{R}_2 - \bar{U}_2,$$

$$\bar{\Gamma}_4 = -2\bar{R}_2 + \bar{U}_2 + \bar{U}_2^T + \sigma \bar{W}_2$$

$$(\bar{\Gamma}_5^{ijl})^T = Z_j C_i + X(A_i^T - \bar{\Pi}_{ij}^T), \bar{\Gamma}_6^{ijl} = (B_i - B_j) Y_l$$

$$\bar{\Gamma}_7^{ijl} = A_i X - Z_j C_i, \bar{\Gamma}_8^{ijl} = (1-\bar{\alpha}) Y_l^T B_j^T$$

$$\bar{\Xi}_{21}^{ijl} = \begin{bmatrix} \tau_M \bar{\mathcal{F}}^{ijl} \\ d_M \bar{\mathcal{F}}^{ijl} \end{bmatrix}, \Xi_{31}^{ijl} = \begin{bmatrix} \tau_M \bar{\mathcal{G}}^{ijl} \\ d_M \bar{\mathcal{G}}^{ijl} \end{bmatrix}$$

$$\bar{\Xi}_{22} = diag\{-2\varepsilon_1 X + \varepsilon_1^2 \bar{R}_1, -2\varepsilon_2 X + \varepsilon_2^2 \bar{R}_2\},$$

$$\bar{\Xi}_{33} = \bar{\Xi}_{22}$$

$$\bar{\Xi}_{52}^{ijl} = \begin{bmatrix} \varepsilon_4 \tau_M M_{2l}^T B_j^T & \varepsilon_4 d_M M_{2l}^T B_j^T \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Xi}_{63}^{ijl} = \begin{bmatrix} \varepsilon_5 \tau_M \delta M_{2l}^T B_j^T & \varepsilon_4 d_M \delta M_{2l}^T B_j^T \\ 0 & 0 \end{bmatrix}$$

$$\bar{\Xi}_{61}^{ijl} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & N_{2l} X & 0 & -N_{2l} X & 0 & 0 & -N_{2l} X \end{bmatrix}$$

The observer and controller gains can be derived as follows

$$K_l = Y_l X^{-1}, L_j = Z_j \bar{X}^{-1}, \bar{X}^{-1} = OSX_1^{-1} S^{-1} O^{-1} \tag{37}$$

Proof Considering

$$(R_k - \varepsilon_k^{-1} P) R_k^{-1} (R_k - \varepsilon_k^{-1} P) \geq 0 (k = 1, 2) \tag{38}$$

$$\bar{\mathcal{F}}^{ijl} = \begin{bmatrix} \bar{\Pi}_{ij} & \bar{\alpha} B_j Y_l & 0 & (1-\bar{\alpha}) B_j Y_l & 0 & Z_j C_i & (1-\bar{\alpha}) B_j Y_l \end{bmatrix}$$

$$\bar{\mathcal{G}}^{ijl} = \begin{bmatrix} 0 & \delta B_j Y_l & 0 & -\delta B_j Y_l & 0 & 0 - \delta B_j Y_l \end{bmatrix}$$

$$\bar{\Xi}_{41}^{ijl} = \begin{bmatrix} \varepsilon_3 M_{1l}^T & 0 & 0 & 0 & 0 & -\varepsilon_3 M_{1l}^T & 0 \\ N_{1l}(C_i - C_j) X & 0 & 0 & 0 & 0 & N_{1l} C_i X & 0 \end{bmatrix},$$

$$\bar{\Xi}_{42}^{ijl} = \begin{bmatrix} \varepsilon_3 \tau_M M_{1l}^T & \varepsilon_3 d_M M_{1l}^T \\ 0 & 0 \end{bmatrix}$$

we get

$$-PR_k^{-1} P \leq -2\varepsilon_k P + \varepsilon_k^2 R_k \tag{39}$$

From (39) and Ξ^{ijl} in (15), we can obtain

$$\Xi^{ijl} \leq \hat{\Xi}^{ijl} + \Upsilon_1^T \Delta_1(t) \Upsilon_2 + \Upsilon_3^T \Delta_2(t) \Upsilon_4 + \Upsilon_5^T \Delta_2(t) \Upsilon_6 \tag{40}$$

where

$$\hat{\Xi}^{ijl} = \begin{bmatrix} \hat{\Xi}_{11}^{ijl} & * & * \\ \hat{\Xi}_{21}^{ijl} & \hat{\Xi}_{22} & * \\ \hat{\Xi}_{31}^{ijl} & 0 & \hat{\Xi}_{33} \end{bmatrix}, \ \hat{\Xi}_{11}^{ijl} = \begin{bmatrix} \hat{\Xi}_{111}^{ijl} & * \\ \hat{\Xi}_{112}^{ijl} & \hat{\Xi}_{113}^{ijl} \end{bmatrix}$$

$$\hat{\Xi}_{111}^{ijl} = \begin{bmatrix} \hat{\Gamma}_1^{ijl} & * & * & * \\ \hat{\Gamma}_2^{ijl} & -2R_1 + U_1 + U_1^T & * & * \\ U_1 & R_1 - U_1 & -Q_1 - R_1 & * \\ \hat{\Gamma}_3^{ijl} & 0 & 0 & \Gamma_4 \end{bmatrix}$$

$$\hat{\Xi}_{112}^{ijl} = \begin{bmatrix} U_2 & 0 & 0 & R_2 - U_2 \\ \hat{\Gamma}_5^{ijl} & \bar{\alpha}\hat{\Gamma}_6^{ijl} & 0 & (1-\bar{\alpha})\hat{\Gamma}_6^{ijl} \\ \hat{\Gamma}_8^{ijl} & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\Gamma}_1^{ijl} = P\hat{\Pi}_{ij} + \hat{\Pi}_{ij}^T P + Q_1 + Q_2 - R_1 - R_2,$$

$$\hat{\Gamma}_2^{ijl} = \bar{\alpha} K_l^T B_j^T P + R_1 - U_1$$
$$\hat{\Gamma}_3^{ijl} = (1-\bar{\alpha}) K_l^T B_j^T P + R_2 - U_2,$$

$$\hat{\Pi}_{ij} = PA_j + PL_j(C_i - C_j)$$
$$(\hat{\Gamma}_5^{ijl})^T = PL_jC_i + (A_i - \hat{\Pi}_{ij})^T P,$$

$$\hat{\Gamma}_6^{ijl} = P(B_i - B_j)K_l$$

$$\hat{\Xi}_{22} = diag\{-2\varepsilon_1 P + \varepsilon_1^2 R_1, -2\varepsilon_2 P + \varepsilon_2^2 R_2\},$$

$$\hat{\Xi}_{33} = \hat{\Xi}_{22}$$

$$\hat{\Gamma}_7^{ijl} = PA_i - PL_jC_i, \ \hat{\Gamma}_8^{ijl} = (1-\bar{\alpha})K_l^T B_j^T P$$

$$\hat{\Xi}_{21}^{ijl} = \begin{bmatrix} \tau_M \hat{\mathcal{F}}^{ijl} \\ d_M \hat{\mathcal{F}}^{ijl} \end{bmatrix}, \ \Xi_{31}^{ijl} = \begin{bmatrix} \tau_M \hat{\mathcal{G}}^{ijl} \\ d_M \hat{\mathcal{G}}^{ijl} \end{bmatrix}$$

$$\hat{\mathcal{F}}^{ijl} = \begin{bmatrix} \hat{\Pi}_{ij} & \bar{\alpha}PB_jK_l & 0 & (1-\bar{\alpha})PB_jK_l & 0 & PL_jC_i & (1-\bar{\alpha})PB_jK_l \end{bmatrix}$$

$$\hat{\mathcal{G}}^{ijl} = \begin{bmatrix} 0 & \delta PB_jK_l & 0 & -\delta PB_jK_l & 0 & 0 - \delta PB_jK_l \end{bmatrix}$$

$$\Upsilon_1 = \begin{bmatrix} M_{1j}^T P & 0_{1\times 4} & -M_{1j}^T P & 0 & \tau_M M_{1j}^T P & d_M M_{1j}^T P & 0 & 0 \end{bmatrix}$$

$$\Upsilon_2 = \begin{bmatrix} N_{1j}(C_i - C_j) & 0_{1\times 4} & N_{1j}C_i & 0_{1\times 5} \end{bmatrix}$$
$$\Upsilon_3 = \begin{bmatrix} M_{2l}^T B_j^T P & 0_{1\times 4} & M_{2l}^T (B_i - B_j)^T P & 0 & \tau_M M_{2l}^T B_j^T P & d_M M_{2l}^T B_j^T P & 0 & 0 \end{bmatrix}$$
$$\Upsilon_4 = \begin{bmatrix} 0 & \bar{\alpha}N_{2l} & 0 & (1-\bar{\alpha})N_{2l} & 0 & 0 & N_{2l} & 0_{1\times 4} \end{bmatrix}$$

$$\Upsilon_5 = \begin{bmatrix} 0_{1\times 9} & \tau_M \delta M_{2l}^T B_j^T P & d_M \delta M_{2l}^T B_j^T P \end{bmatrix}$$
$$\Upsilon_6 = \begin{bmatrix} 0 & N_{2l} & 0 & -N_{2l} & 0 & 0 & -N_{2l} & 0_{1\times 4} \end{bmatrix}$$

$$\hat{\Xi}_{113}^{ijl} = \begin{bmatrix} -Q_2 - R_2 & * & * \\ 0 & \hat{\Gamma}_7^{ijl} + (\hat{\Gamma}_7^{ijl})^T & * \\ 0 & (1-\bar{\alpha})(\hat{\Gamma}_6^{ijl})^T & -W_1 \end{bmatrix}$$

By using Lemma 2, there exist positive scalars ε_3, ε_4 and ε_5, such that

$$\Xi^{ijl} \le \hat{\Xi}^{ijl} + \varepsilon_3 \Upsilon_1^T \Upsilon_1 + \varepsilon_3^{-1} \Upsilon_2^T \Upsilon_2 + \varepsilon_4 \Upsilon_3^T \Upsilon_3 + \varepsilon_4^{-1} \Upsilon_4^T \Upsilon_4 + \varepsilon_5 \Upsilon_5^T \Upsilon_5 + \varepsilon_5^{-1} \Upsilon_6^T \Upsilon_6 \tag{41}$$

Considering (41), applying the Schur complement, it is easy to obtain the following inequalities from (15) and (16),

$$\rho_l \tilde{\Xi}^{ijl} + \rho_j \tilde{\Xi}^{ilj} - \rho_j \tilde{\mathcal{V}} - \rho_l \tilde{\mathcal{V}} + 2\tilde{\mathcal{V}} < 0 \quad (42)$$

$$\tilde{\Xi}^{ijl} + \tilde{\Xi}^{ilj} - 2\tilde{\mathcal{V}} < 0 \quad (43)$$

in which

$$\tilde{\Xi}^{ijl} = \begin{bmatrix} \hat{\Xi}^{ijl} & * & * & * & * & * & * \\ \varepsilon_3 \Upsilon_1^T & -\varepsilon_3 I & * & * & * & * & * \\ \Upsilon_2 & 0 & -\varepsilon_3 I & * & * & * & * \\ \varepsilon_4 \Upsilon_3^T & 0 & 0 & -\varepsilon_4 I & * & * & * \\ \Upsilon_4 & 0 & 0 & 0 & -\varepsilon_4 I & * & * \\ \varepsilon_5 \Upsilon_5^T & 0 & 0 & 0 & 0 & -\varepsilon_5 I & * \\ \Upsilon_6 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_5 I \end{bmatrix}, \quad \tilde{\mathcal{V}} = \begin{bmatrix} V \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Define $X = P^{-1}, XR_1X = \bar{R}_1, XR_2X = \bar{R}_2, XQ_1X = \bar{Q}_1, XQ_2X = \bar{Q}_2, XU_1X = \bar{U}_1, XU_2X = \bar{U}_2, J = diag\{J_1, I, I, I, I, I, I, \}, J_1 = diag\{X, \ldots, X\}, J_1 V J_1^T = \bar{V}, Y = K_l X, Z_j = L_j \bar{X}$.

By Lemma 3, for $X = V \begin{bmatrix} X_1 & * \\ 0 & X_2 \end{bmatrix} V^T$, one can get $CX = \bar{X}C$ with $\bar{X} = OSX_1^{-1}S^{-1}O^{-1}$.

Pre-multiplying and post-multiplying (42) and (43) with J and its transpose, and taking the above definitions into consideration, we can get (33) and (34). Pre-multiplying and post-multiplying (17) and (18) with $diag\{X, X\}$, (35) and (36) can be obtained.

This completes the proof. □

Remark 3 It can be observed that the hybrid-triggered scheme in this paper combines the characteristics of periodic sampling and the event-triggered scheme. It is very convenient to apply the hybrid-triggered scheme to balance the network transmission and the system performance. The transmission amount in the communication network can be adjusted by selecting different values of $h, \bar{\alpha}$ and σ.

Remark 4 To the best of the authors' knowledge, the problem of resilient observer-based control via networked T–S fuzzy systems has not been reported in the literature. It can be seen from the following simulation part that the obtained controller design method is effective.

4 Numerical examples

In this section, a simulation example is provided to demonstrate the effectiveness of the obtained method.

The nonlinear mass–spring system is as follows:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -0.01x_1 - 0.67x_1^3 + u \end{cases} \quad (44)$$

where $x_1 \in [-1, 1]$. Similar to [8], set $h_1(x_1) = 1 - x_1^2, h_2(x_1) = 1 - h_1(x_1)$, the nonlinear system (44) can be transferred as the following T–S fuzzy model:

Rule i: x_1 is h_i, then

$$\begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t), \quad i = 1, 2 \end{cases} \quad (45)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 \\ -0.68 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 1 \\ -0.01 & 0 \end{bmatrix},$$

$$B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

By choosing $d_M = 0.1, t_M = 0.1, \bar{\alpha} = 0.4, b = 0.01, \varepsilon_i = 1 (i = 1, \ldots, 5), \rho_l = 0.8 (l = 1, 2), M_{11} = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, M_{12} = \begin{bmatrix} 0.2 \\ 0.4 \end{bmatrix}, M_{21} = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, M_{22} = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, N_{11} = 0.1, N_{12} = 0.2, N_{21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, N_{22} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.3 \end{bmatrix}$, and applying Theorem 2, we can obtain:

$$Y_1 = \begin{bmatrix} 2.0651 & -12.7919 \end{bmatrix},$$

$$Y_2 = \begin{bmatrix} -4.5906 & -10.3668 \end{bmatrix}$$

$$Z_1 = \begin{bmatrix} 1.2162 \\ 4.6035 \end{bmatrix}, Z_2 = \begin{bmatrix} 1.3320 \\ 3.3731 \end{bmatrix},$$

$$X = \begin{bmatrix} 6.2477 & -2.1553 \\ -2.1553 & 5.6193 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 33.0477 & -0.7668 \\ -0.7668 & 70.0641 \end{bmatrix},$$

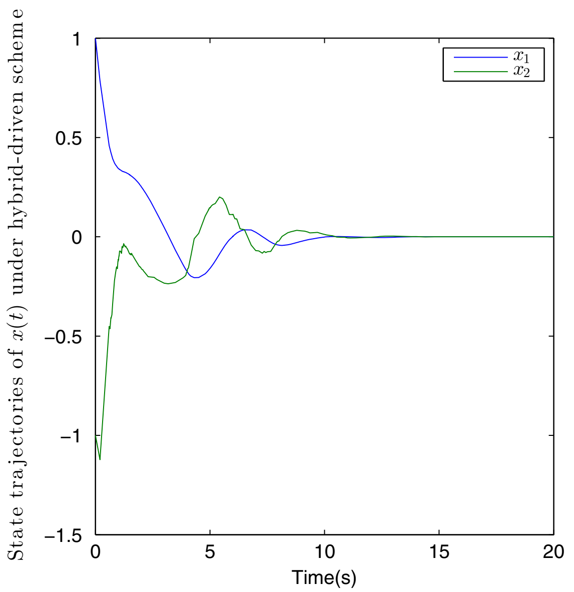

Fig. 2 State response of system (45) under hybrid-triggered scheme

$$W_2 = \begin{bmatrix} 7.7877 & 0.4772 \\ 0.4772 & 23.6749 \end{bmatrix}$$

According to (37), we derive the corresponding resilient matrices as

$$K_1 = \begin{bmatrix} -0.5241 & -2.4774 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -1.5803 & -2.4509 \end{bmatrix}$$

$$L_1 = \begin{bmatrix} 0.3239 \\ 1.2258 \end{bmatrix}, L_2 = \begin{bmatrix} 0.3547 \\ 0.8982 \end{bmatrix}$$

The initial conditions of system (45) and the estimator are chosen as

$$x_0 = \begin{bmatrix} 1 & -1 \end{bmatrix}^T, \hat{x}_0 = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$$

Some simulation results are given below. Figure 2 shows the response of the state, from which we can see that the closed-loop fuzzy system is asymptotical stable. The output of the designed controller is depicted in Fig. 3. A possible switching method of hybrid-triggered scheme is shown in Fig. 4. The simulation result demonstrates that the designed resilient observer-based controller performs well.

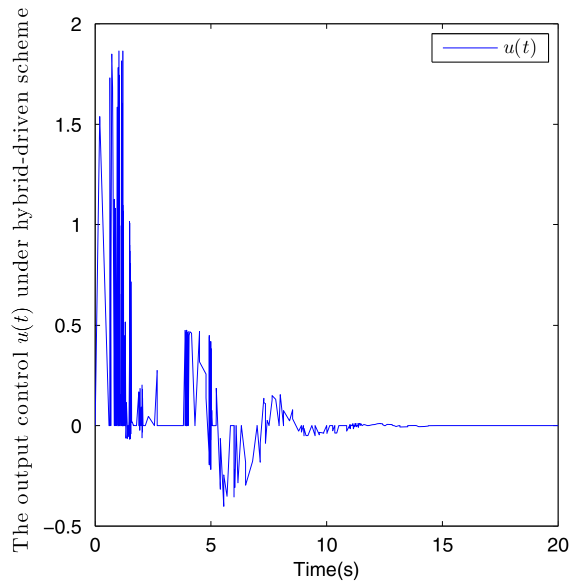

Fig. 3 The output control $u(t)$ under hybrid-triggered scheme

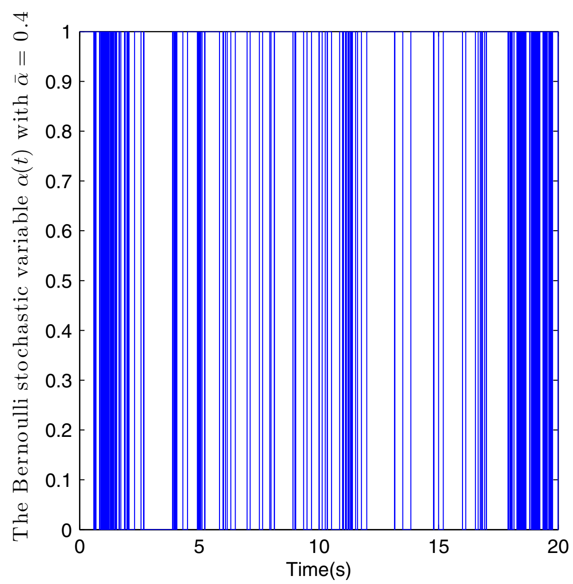

Fig. 4 The Bernoulli stochastic variable $\alpha(t)$ with $\bar{\alpha} = 0.4$

5 Conclusions

This paper focuses on resilient observer-based control for a class of T–S fuzzy systems under hybrid-triggered scheme. The data transmission strategy is governed by the adopted hybrid-driven scheme, in which the event-triggered scheme and the periodic sampling are switched stochastically. By using the Lyapunov functional approach, sufficient conditions for the asymptotical stability of the discussed system are established. Moreover, the gains of the observer-based controller are derived by solving a set of linear matrix inequalities. An illustrative example is utilized to show the usefulness of the developed approach. In our future work, the hybrid-driven scheme will be applied in more challenge and interesting investigations such as distributed control systems and cascade control systems.

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