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Distributed event-triggered state estimators design for sensor networked systems with deception attacks

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Abstract: This study addresses the issue of distributed event-triggered H_∞ state estimators subject to deception attacks for sensor networked systems. A decentralised event-triggered scheme (ETS) is introduced to determine whether the sampling data of each sensor is transmitted or not, respectively. In this scheme, each sensor node is independent to decide to deliver the local measurement output through the corresponding ETS. Due to the insertion of the network, the effect of the deception attacks along with time delay and packet dropouts are considered in this study. A novel estimator network is established to realise the estimation of the decoupling output measurements and coupling intercommunication measurements. Firstly, a distributed event-triggered H_∞ estimating system with deception attacks is constructed in a mathematical model. Secondly, sufficient conditions are derived, which can ensure the stability of the designed H_∞ estimating error systems and the related parameters of the desired distributed estimators are presented in an accurate form. Finally, a simulated example is given to demonstrate the effectiveness of the designed event-triggered distributed H_∞ state estimator systems under the deception attacks.

1 Introduction

The sensor networks, which are distributed spatially to monitor physical or environmental conditions, can realise the autonomous data sampling such as temperature, sound, and pressure [1–3]. Hundreds or thousands of sensor nodes constitute the sensor networks. Due to the convenience, flexibility and low cost of large numbers of sensors, the sensor networks play an important role in a wide domain of the applications such as environmental monitoring, traffic system, manufacturing automation, and object tracking systems [4]. Hence, an ever-increasing interest has gained in the research field and large amounts of fruitful achievements have been achieved in the recent year. For example, in [5], the authors proposed a novel distributed information-weighted Kalman consensus filter algorithm for sensor networks, which can effectively solve the challenging issues such as poor local sensor node estimation. The authors of [6] applied a leader-following-based Kalman filtering algorithm for sensor networks with communication delays. The authors of [7] proposed distributed algorithms to deal with the distributed energy resource coordination problem over multiple time periods.

The sensor networks discussed above early are based on the periodic sampling (time-triggered scheme), in which all the sampled data is delivered through the network [8, 9]. The adoption of a time-triggered scheme can generate a lot of redundancy if the sampling period is short, which can result in network congestion [10–13]. To overcome the shortcomings of the time-triggered scheme, the event-triggering strategy is proposed by many scholars such as Tabuada [14], Lunze [15], and Yue [16], to reduce the number of transmitted packets and alleviate the communication load. For example, in [16], the authors proposed a novel event-triggered scheme (ETS) in which a set threshold is used to determine whether the current signals are delivered to the network or not. In comparison with the ETS proposed in [15], where the ETS needs to obtain the continuous states to judge whether the current state violates the ETS or not, however, the advantage of this ETS [16] can realise the supervision of system states in discrete instants. Based on the proposed triggered scheme in [16], large numbers of research studies have achieved the different results of the control fields [17, 18], e.g. the authors of [19] designed the non-fragile state estimator for delayed neural networks with an ETS to save the limited resource. To reduce the network burden, the authors of [20] focused on designing a non-parallel distribution compensation controller for Takagi–Sugeno fuzzy systems. The event-triggered communication mechanism was applied in [21] to investigate the fuzzy filtering for a class of nonlinear networked control systems. The literature above is based on the centralised ETS, nonetheless, due to the complexity of the network, there are lots of control devices distributed physically such as sensors and estimators. Suppose that large numbers of sensors sample data through an ETS, it may result in the collapse of this scheme. To overcome this scenario, a distributed ETS has increased the investigating interests, which can trigger the partial information independently through the preset event-triggering condition. In [22], the authors proposed a distributed event-triggered communication optimisation for economic dispatch to reduce information exchange requirements in smart grids. In [23], the problem of distributed event-triggered cooperative control was investigated for frequency and voltage stability in the microgrid. By applying an adaptive distributed event-based scheme in [24], the authors studied the fuzzy control problem for uncertain nonlinear multi-agent systems in the strict feedback form. Motivated by the aforementioned results, this study applies a decentralised event-triggered communication mechanism to decide whether the sampled data of each corresponding sensor is delivered through the network or not.

In recent years, the insertion of the network has brought about countless advantages such as high transmission rate, low control cost, and flexible communication approaches [25, 26]. However, with the rapid development of the network, the risk and vulnerability have been emerging gradually [27], especially in the aspect of network security. One of the factors threatening the network security is cyber-attacks. As described in most references [28, 29], the cyber-attacks can be divided into three categories, denial of service (DoS) [30, 31], replay attacks [32, 33], and

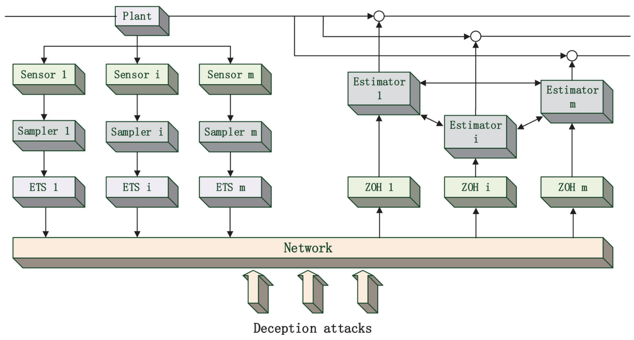

Fig. 1 *Structure of distributed event-triggered state estimators with deception attacks*

deception attacks [34, 35]. The main purpose of the cyber-attacks is to decline the normal performance of the networked system and collapse the stability of the system platform. In this study, the discussed cyber-attacks are classed as deception attacks. By injecting false data into the normal data during the transmission, deception attacks intend to damage the normal transmission to obtain the secret information or fabricate the data [36]. Due to the huge threat brought by deception attacks, there are numerous scholars interested in investigating the deception attacks and the satisfactory results have been obtained. The authors of [37] dealt with the problem of distributed recursive filters for a class of discrete time-delayed systems by taking the uniform quantisation and deception attacks into account. In [38], a novel event-triggered distributed state estimator algorithm was proposed to defend against the false data injection attack in wireless sensor networks. The false data injection attacks were detected by the resilient attack detection estimators, which can also provide locally reliable state estimations in [39].

Motivated by the aforementioned discussions, this study is concerned with the problem of distributed event-triggered H_∞ state estimation under the deception attacks for sensor networked systems. Large numbers of sensors construct the sensor networks. The sampled data from each sensor is determined independently by the corresponding ETS to be delivered or not. The main contributions of this study can be summarised as follows. (i) To reduce the transmitted data in the network, each sensor is equipped with an independent ETS, which can determine whether the current sampled data is sent to the network or not. (ii) Due to the threat of the cyber security, the influence of deception attacks is firstly considered in the design of distributed H_∞ state estimators, whose occurring probability is governed by a Bernoulli variable. (iii) Distributed H_∞ estimators are formulated to estimate the output measurements from the decentralised ETS and the inner information from adjacent estimators. This study firstly realises the co-design of the distributed H_∞ state estimator by considering the decentralised ETS and stochastic deception attacks.

The rest of the paper is organised as follows. In Section 2, a mathematical model of H_∞ distributed estimating error systems is constructed by considering both decentralised ETSs and deception attacks into consideration. In Section 3, sufficient conditions which can guarantee the stability of the augmented system are derived by using Lyapunov stability theory and linear matrix inequality (LMI) techniques. Thus, the estimated gains and coupling parameters are presented in an explicit form. A simulated example is given to demonstrate the effectiveness of the designed H_∞ distributed event-triggered estimators in Section 4. Finally, a conclusion is given to make a summarisation.

Notation: R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n\times m$ real matrices; the superscript "T" stands for matrix transposition; I is the identity matrix of appropriate dimension; $\otimes$ denotes Kronecker product; the notation $X>0$, for $X\in R^{n\times n}$ means that the matrix X is real symmetric positive definite. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C\end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entry implied by symmetry.

2 System description

In this study, there is an estimator network consisting of m state estimators which can be represented by a directed weighted graph $G=(V,E,W)$, where V are the vertices of the graph numbered $1,2,\dots,m$, $E\in V\times V$ are edges of graph G, $W=[w_{ij}]$ denotes the weight of each associated edge, which can be represented by the adjacency matrix. An ordered pair $(i,j)\in E$ denotes a directed edge from state estimator i to state estimator j, it means that estimator i can obtain information from estimator j, but the reverse is false. If $(i,j)\in E$, then $w_{ij}=1$; otherwise, $w_{ij}=0$. The degree matrix of graph G is defined as $\chi=\mathrm{diag}\{\Sigma w_{1j},\Sigma w_{2j},\dots,\Sigma w_{mj}\}$. $\Xi=\chi-W$ is used to represent the Laplacian matrix of the directed graph [40].

Consider the following linear system:

$$\begin{cases}\dot{x}(t)=Ax(t)+B\omega(t),\\ z(t)=Lx(t),\end{cases}\tag{1}$$

where $x(t)\in R^n$ and $z(t)\in R^m$ are the state vector and the output to be estimated by state estimators, respectively. $w(t)\in R^p$ denotes the external perturbance satisfying $\omega(t)\in L_2[0,\infty)$. A, B and L are known matrices with appropriate dimensions.

The output measurement of the ith sensor can be represented by

$$y_i(t)=Cx_i(t),\quad i=1,2,\dots,m,\tag{2}$$

where $y_i(t)\in R^q$, C is the known matrix with appropriate dimensions.

In this study, the problem of distributed event-triggered state estimation with deception attacks is investigated. As is shown in Fig. 1, there are m sensors distributed in different space, the output measurement $y_i(t)$ of each sensor is delivered to its corresponding state estimator through the ETS and the network. m state estimators construct the estimation network in which each state estimator can receive information from other estimators. The adoption of the ETS is to alleviate the burden of the network, which can determine whether the current sampled data is delivered or not. Similar to [16], for the sampled data from the ith sensor, the judgement of the ETS is introduced as follows:

$$\begin{aligned}&e_{ki}^{\mathrm T}(t_kh)\Omega_ie_{ki}(t_kh)\le y_i^{\mathrm T}(t_kh+lh)\sigma_i\Omega_iy_i(t_kh+lh),\\&i=1,2,\dots,m,\end{aligned}\tag{3}$$

where h denotes the sampling period, $t_kh\subseteq\{t_0h,t_1h,t_2h,\dots\}$ are release times, $t_0=0$ is the initial time, $\sigma_i\in[0,1)$, $\Omega_i>0$ and $l=1,2,\dots$. The threshold error $e_{ki}(t_kh)=y_i(t_kh+lh)-y_i(t_kh)$.

Remark 1: In the light of event-triggered algorithm (3), σ_i is a positive parameter, which can determine the frequency of event-trigger, when $\sigma_i=0$, it means that the scheme is time-triggered, all the sampled data are released. Different from the event-triggered condition in [16], σ_i is one of the triggered parameters of the distributed ETSs in this study, which can make its own decision on whether the current signals are transmitted through the network or not. The introduction of the decentralised ETSs can effectively make full use of the networked resources.

By taking the analogous sampling method in [41], some uncertainties including network-induced delay, packet dropouts, and external perturbance are all considered in this study. However, when the output measurements are delivered through the network, it may be subject to the malicious signals, which are defined as deception attacks in this study. By injecting the false data into the normal transmission, deception attacks aim to paralyse the networked system and destroy the stability of the platform. Therefore, it is necessary to consider the effect of the deception attacks when the distributed state estimators are designed for networked sensor systems. In this study, the nonlinear function $f(y(t))$ is utilised to represent the deception attacks.

According to Fig. 1, the zero-order-holder (ZOH) can maintain the current instant until the next instant in the process of signal transmission. To analyse easily, similar to [20], for the holding interval $[t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}})$ affected by ZOH, it is divided into several subintervals. To describe the holding zone Λ of ZOH, suppose there exists a constant ϖ satisfying

$$\Lambda = [t_k h + \tau_{t_k}, t_{k+1} h + \tau_{t_{k+1}}) = \bigcup_{l=0}^{\varpi} \Lambda_l,$$

where

$$\Lambda_l = [t_k h + lh + \tau_{t+l}, t_k h + lh + h + \tau_{t+l+1}],$$

$$l = \{1, 2, \ldots, \varpi\}, \varpi = t_{k+1} - t_k - 1 .$$

Define $\tau(t) = t - t_k h - lh$, $0 \le \tau_{t_k} \le \tau(t) \le h + \tau_{t_{k+l+1}} \triangleq \tau_M$. Define $d(t) = t - t_k h$, $d(t) \in [0, d_M]$, where d_M is the upper bound of $d(t)$. Then, the output measurement, which is transmitted to the state estimator, can be described as follows:

$$\begin{aligned} \bar{y}_i(t) = {} & \alpha(t) f_i(y(t - d(t))) \\ & + (1 - \alpha(t))(y_i(t - \tau_i(t)) - e_{ki}(t)), \end{aligned} \tag{4}$$

where $\alpha(t) \in \{0, 1\}$, $e_{ki}(t) = y_i(t - \tau_i(t)) - y_i(t)$. $\bar{\alpha}$ and ρ^2 are utilised to represent the expectation and the mathematical variance of $\alpha(t)$, respectively. $f_i(y(t))$ represents the deception attacks to the corresponding output measurements through the ith ETS.

Remark 2: Inspired by [28, 37], the random Bernoulli variable $\alpha(t)$ is used to govern the deception attacks. When $\alpha(t) = 0$, (4) can be written as $\bar{y}_i(t) = y_i(t - \tau_i(t)) - e_{ki}(t)$ it means that the data is transmitted without deception attacks; otherwise, $\bar{y}_i(t) = f_i(y(t - d(t)))$, the normal signals are substituted by false data.

Remark 3: It is noted that the deception attacks $f(y(t))$ is a nonlinear function related to $y(t)$ instead of $y_i(t)$. As is shown in Fig. 1, some of the transmitted signals through the decentralised ETS may be delivered to the networked simultaneously, and these signals may suffer from the same malicious signals randomly.

This study aims to design the distributed state estimators over sensor networks. Then, the ith state estimator can be described as follows:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + K_i(\bar{y}_i(t) - \hat{y}_i(t)) \\ \qquad + D_i \sum_{j=1}^{m} w_{ij}(\hat{y}_j(t) - \hat{y}_i(t)), \\ \hat{z}_i(t) = L\hat{x}_i(t), \\ \hat{y}_i(t) = C\hat{x}_i(t), \end{cases} \tag{5}$$

where $t \in \Lambda_l$, $i = 1, 2, \ldots, m$. $\hat{x}_i(t)$, $\hat{y}_i(t)$ and $\hat{z}_i(t)$ are the estimations of $x_i(t)$, $y_i(t)$ and $z_i(t)$, respectively. K_i and D_i are the decoupling and coupling gains of the ith estimator to be determined.

Define the estimation error $e_i(t) = x_i(t) - \hat{x}_i(t)$, substitute (4) into (5), $\dot{e}_i(t)$ can be described as follows:

$$\begin{aligned} \dot{e}_i(t) = {} & (A - K_i C)e_i(t) + K_i C x_i(t) + B\omega_i(t) - \alpha(t)K_i \\ & f_i(y(t - d(t))) - (1 - \alpha(t))K_i C x_i(t - \tau_i(t)) \\ & + (1 - \alpha(t))K_i e_{ki}(t) - D_i \sum_{j=1}^{m} w_{ij}(\hat{y}_j(t) - \hat{y}_i(t)) . \end{aligned} \tag{6}$$

Set $\xi(t) = \left[e^{\mathrm{T}}(t) \quad x^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, $\tilde{z}(t) = z(t) - \hat{z}(t)$, the estimation error systems can be expressed as follows:

$$\begin{cases} \dot{\xi}(t) = \tilde{A}\xi(t) + \tilde{B}\omega(t) + (1 - \alpha(t))KH_2 e_k(t) \\ \qquad - (1 - \alpha(t))K\bar{C}H_2 H_1 \xi(t - \tau(t)) \\ \qquad - \alpha(t)KH_2 f(\bar{C}H_1 \xi(t - d(t))), \\ \tilde{z}(t) = \tilde{L}\xi(t), \end{cases} \tag{7}$$

where

$$\begin{aligned} & x(t) = [x_1^{\mathrm{T}}(t) \quad x_2^{\mathrm{T}}(t) \quad \ldots \quad x_m^{\mathrm{T}}(t)]^{\mathrm{T}}, \\ & \hat{x}(t) = [\hat{x}_1^{\mathrm{T}}(t) \quad \hat{x}_2^{\mathrm{T}}(t) \quad \ldots \quad \hat{x}_m^{\mathrm{T}}(t)]^{\mathrm{T}}, \\ & \omega(t) = [\omega_1^{\mathrm{T}}(t) \quad \omega_2^{\mathrm{T}}(t) \quad \ldots \quad \omega_m^{\mathrm{T}}(t)]^{\mathrm{T}}, \\ & e_k(t) = [e_{k1}^{\mathrm{T}}(t) \quad e_{k2}^{\mathrm{T}}(t) \quad \ldots \quad e_{km}^{\mathrm{T}}(t)]^{\mathrm{T}}, \\ & f(y(t - d(t))) = \begin{bmatrix} f_1(y(t - d(t))) \\ f_2(y(t - d(t))) \\ \cdots \\ f_m^{\mathrm{T}}(y(t - d(t))) \end{bmatrix}, \\ & \tilde{A} = \begin{bmatrix} \bar{A} - K\bar{C} + D\Xi\bar{C} & K\bar{C} - D\Xi\bar{C} \\ 0 & \bar{A} \end{bmatrix}, \\ & B = \begin{bmatrix} \bar{B} \\ \bar{B} \end{bmatrix}, \quad H_1 = [0 \quad I], \quad H_2 = \begin{bmatrix} I \\ 0 \end{bmatrix}, \\ & K = \mathrm{diag}\{K_1, \ K_2, \ldots, K_m\}, \\ & D = \mathrm{diag}\{D_1, \ D_2, \ldots, D_m\}, \\ & \bar{A} = A \otimes I_m, \quad \bar{B} = B \otimes I_m, \quad \bar{C} = C \otimes I_m, \\ & \bar{L} = L \otimes I_m, \quad \tilde{L} = [\bar{L} \quad 0], \\ & \Xi = \begin{bmatrix} \sum_{j=1}^{m} w_{1j} & -w_{12} & \ldots & -w_{1m} \\ -w_{21} & \sum_{j=1}^{m} w_{2j} & \ldots & -w_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ -w_{m1} & -w_{m2} & \ldots & \sum_{j=1}^{m} w_{mj} \end{bmatrix}. \end{aligned}$$

An assumption and some important lemmas are introduced as follows.

Assumption 1: The deception attacks $f(y(t))$ introduced above are assumed to satisfy the following constraints [42, 43]:

$$\| f(y(t)) \|_2 \ \le \ \| My(t) \|_2 , \tag{8}$$

where M is the upper bound of the deception attacks.

Lemma 1: Suppose $\tau(t) \in [0, \tau_M]$, $d(t) \in [0, d_M]$, Ξ_1, Ξ_2, Ξ_3, Ξ_4 and Ω are matrices with appropriate dimensions [44], then

$$\begin{aligned} & \tau(t)\Xi_1 + (\tau_M - \tau(t))\Xi_2 + d(t)\Xi_3 \\ & + (d_M - d(t))\Xi_4 + \Omega < 0, \end{aligned} \tag{9}$$

if and only if

$$\begin{cases} \tau_M \Xi_1 + d_M \Xi_3 + \Omega < 0, \\ \tau_M \Xi_2 + d_M \Xi_3 + \Omega < 0, \\ \tau_M \Xi_1 + d_M \Xi_4 + \Omega < 0, \\ \tau_M \Xi_2 + d_M \Xi_4 + \Omega < 0. \end{cases} \tag{10}$$

Lemma 2: Suppose that $\tau(t)$ satisfies $0 < \tau(t) \le \bar{\tau}$. For any matrices $X \in R^{n\times n}$ and $U \in R^{n\times n}$ that satisfy $\begin{bmatrix} X & U \\ U^{\mathrm{T}} & X \end{bmatrix} \ge 0$, the following inequality holds [45]:

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t}\dot{\xi}^{T}(s)X\dot{\xi}(s)ds \le \begin{bmatrix}\xi(t)\\ \xi(t-\tau(t))\\ \xi(t-\bar{\tau})\end{bmatrix}^{\mathrm{T}} \begin{bmatrix} -X & * & * \\ X^{\mathrm{T}}-U^{\mathrm{T}} & -2X+U+U^{\mathrm{T}} & * \\ U^{\mathrm{T}} & X^{\mathrm{T}}-U^{\mathrm{T}} & -X \end{bmatrix}\begin{bmatrix}\xi(t)\\ \xi(t-\tau(t))\\ \xi(t-\bar{\tau})\end{bmatrix}. \tag{11}$$

3 State estimator design

Theorem 1: If there exists $P > 0$, $Q_k > 0$, $R_k > 0$, $U_k > 0$ $(k = 1, 2)$ with appropriate dimensions, the estimation error system can be asymptotically stable with the given positive scalars $\bar{\alpha}$, τ_M, d_M, γ, σ_i $(i = 1, 2, \ldots, m)$, matrices K, D, Ξ and M, such the inequalities hold

$$\begin{bmatrix} \Upsilon_{11} & * & * & * & * \\ \Upsilon_{21} & -I & * & * & * \\ \Upsilon_{31} & 0 & -P & * & * \\ \Upsilon_{41} & 0 & 0 & \Upsilon_{44} & * \\ \Upsilon_{51} & 0 & 0 & 0 & \Upsilon_{55} \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} R_k & * \\ U_k & R_k \end{bmatrix} > 0, \quad (k = 1, 2). \tag{13}$$

where

$$\Upsilon_{11} = \begin{bmatrix} \Phi_{11} & * & * & * & * & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * & * & * & * & * \\ U_1^T & \Phi_{32} & \Phi_{33} & * & * & * & * & * \\ \Phi_{41} & 0 & 0 & \Phi_{44} & * & * & * & * \\ U_2^T & 0 & 0 & \Phi_{54} & \Phi_{55} & * & * & * \\ \Phi_{61} & 0 & 0 & 0 & 0 & -\alpha I & * & * \\ \Phi_{62} & 0 & 0 & 0 & 0 & 0 & -\Omega & * \\ \tilde{B}^T P & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Phi_{11} = P\tilde{A} + \tilde{A}^T P + Q_1 + Q_2 - R_1 - R_2,$$
$$\Phi_{21} = -\alpha_1 H_1^T H_2^T \bar{C}^T K^T P + R_1^T - U_1^T,$$
$$\Phi_{22} = -2R_1 + U_1 + U_1^T + \sigma H_1^T \bar{C}^T \Omega \bar{C} H_1,$$
$$\Phi_{32} = R_1^T - U_1^T, \Phi_{33} = -Q_1 - R_1,$$
$$\Phi_{41} = R_2^T - U_2^T, \Phi_{44} = -2R_2 + U_2 + U_2^T,$$
$$\Phi_{54} = R_2^T - U_2^T, \Phi_{55} = -Q_2 - R_2,$$
$$\Phi_{61} = -\alpha H_2^T K^T P, \Phi_{62} = -\alpha_1 H_2^T K^T P,$$
$$\Upsilon_{21} = \begin{bmatrix} \tilde{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Upsilon_{31} = \begin{bmatrix} 0 & 0 & 0 & \sqrt{\alpha} PM\bar{C}H_1 & 0 & 0 & 0 & 0 \end{bmatrix},$$
$$\Upsilon_{44} = \Upsilon_{55} = diag\{-PR_1^{-1}P, -PR_2^{-1}P\}$$
$$\Upsilon_{41} = \begin{bmatrix} \Upsilon_{411} & \Upsilon_{412} \end{bmatrix},$$
$$\Upsilon_{411} = \begin{bmatrix} \tau_M P\tilde{A} & -\bar{\alpha}_1 \tau_M PK\bar{C}H_2H_1 & 0 & 0 \\ d_M P\tilde{A} & -\bar{\alpha}_1 d_M PK\bar{C}H_2H_1 & 0 & 0 \end{bmatrix},$$
$$\Upsilon_{412} = \begin{bmatrix} 0 & -\bar{\alpha}\tau_M PKH_2 & \bar{\alpha}_1 \tau_M PKH_2 & 0 \\ 0 & -\bar{\alpha} d_M PKH_2 & \bar{\alpha}_1 d_M PKH_2 & 0 \end{bmatrix},$$
$$\Upsilon_{51} = \begin{bmatrix} \Upsilon_{511} & \Upsilon_{512} \end{bmatrix},$$
$$\Upsilon_{511} = \begin{bmatrix} 0 & \rho\tau_M PK\bar{C}H_2H_1 & 0 & 0 \\ 0 & \rho d_M PK\bar{C}H_2H_1 & 0 & 0 \end{bmatrix},$$
$$\Upsilon_{512} = \begin{bmatrix} 0 & -\rho\tau_M PKH_2 & -\rho\tau_M PKH_2 & 0 \\ 0 & -\rho d_M PKH_2 & -\rho d_M PKH_2 & 0 \end{bmatrix}.$$

Proof: Consider the following Lyapunov–Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t), \tag{14}$$

where

$$\begin{aligned} V_1(t) &= \zeta^{\mathrm{T}}(t)P\zeta(t), \\ V_2(t) &= \int_{t-\tau_M}^{t} \zeta^{\mathrm{T}}(s)Q_1\zeta(s)\,\mathrm{d}s + \int_{t-d_M}^{t}\zeta^{\mathrm{T}}(s)Q_2\zeta(s)\,\mathrm{d}s, \\ V_3(t) &= \tau_M\int_{t-\tau_M}^{t}\int_{s}^{t}\dot{\zeta}^{\mathrm{T}}(v)R_1\dot{\zeta}(v)\,\mathrm{d}v\,\mathrm{d}s \\ &\quad + d_M\int_{t-d_M}^{t}\int_{s}^{t}\dot{\zeta}^{\mathrm{T}}(v)R_2\dot{\zeta}(v)\,\mathrm{d}v\,\mathrm{d}s, \\ &\quad P > 0, \quad Q_k > 0, \quad R_k > 0 \quad (k = 1, 2). \end{aligned}$$

By taking the derivative and mathematical expectation of (14), the following equations are obtained:

$$\mathbb{E}\{\dot{V}_1(t)\} = 2\xi^{\mathrm{T}}(t)P\mathscr{A}, \tag{15}$$

$$\begin{aligned} \mathbb{E}\{\dot{V}_2(t)\} = {} & \xi^{\mathrm{T}}(t)(Q_1 + Q_2)\xi(t) - \xi^{\mathrm{T}}(t-\tau_M)Q_1 \\ & \xi(t-\tau_M) - \xi^{\mathrm{T}}(t-d_M)Q_2\xi(t-d_M), \end{aligned} \tag{16}$$

$$\begin{aligned} \mathbb{E}\{\dot{V}_3(t)\} = {} & \mathbb{E}\{\dot{\xi}^{\mathrm{T}}(t)(\tau_M^2 R_1 + d_M^2 R_2)\dot{\xi}(t)\} - \tau_M \int_{t-\tau_M}^{t} \dot{\xi}^T(s) \\ & R_1\dot{\xi}(s)ds - d_M\int_{t-d_M}^{t}\dot{\xi}^T(s)R_2\dot{\xi}(s)ds. \end{aligned} \tag{17}$$

With the calculation of $\mathbb{E}\{\dot{\xi}^{\mathrm{T}}(t)(\tau_M^2 R_1 + d_M^2 R_2)\dot{\xi}(t))\}$ in (17) above, the following equality can be gained:

$$\mathbb{E}\{\dot{\xi}^{\mathrm{T}}(t)(\tau_M^2 R_1 + d_M^2 R_2)\dot{\xi}(t))\} = \mathscr{A}^{\mathrm{T}}\tilde{R}\mathscr{A} + \rho^2\mathscr{B}^{\mathrm{T}}\tilde{R}\mathscr{B}, \tag{18}$$

where

$$\begin{aligned} \mathscr{A} &= \tilde{A}\xi(t) + \tilde{B}\omega(t) - \bar{\alpha}KH_2 f(\bar{C}H_1\xi(t-d(t))) \\ &\quad -(1-\bar{\alpha})[K\bar{C}H_2H_1\xi(t-\tau(t)) - KH_2e_k(t)], \\ \mathscr{B} &= K\bar{C}H_2H_1\xi(t-\tau(t)) \\ &\quad -KH_2e_k(t) - KH_2 f(\bar{C}H_1\xi(t-d(t))), \\ \tilde{R} &= \tau_M^2 R_1 + d_M^2 R_2. \end{aligned}$$

According to the event-triggered judgement algorithm (3), the following condition is derived:

$$\begin{aligned} & \xi^{\mathrm{T}}(t-\tau(t))H_1^{\mathrm{T}}C^{\mathrm{T}}\sigma\Omega CH_1\xi(t-\tau(t)) \\ & -e_k^{\mathrm{T}}(t)\Omega e_k(t) \ge 0, \end{aligned} \tag{19}$$

where $\sigma = \mathrm{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$, $\Omega = \mathrm{diag}\{\Omega_1, \Omega_2, \ldots, \Omega_m\}$.

Considering the definition of deception attacks (4), then the following inequality can be obtained:

$$\begin{aligned}&\bar{\alpha}\xi^{\mathrm{T}}(t-d(t))H_1^{\mathrm{T}}\bar{C}^T M^T P M\bar{C}H_1\xi(t-d(t))\\&\quad-\bar{\alpha}f^{\mathrm{T}}(CH_1\xi(t-d(t)))f(CH_1\xi(t-d(t)))\ge 0.\end{aligned}\tag{20}$$

Define that

$$\begin{aligned}\zeta^{\mathrm{T}}(t)&=\begin{bmatrix}\zeta_1^{\mathrm{T}}(t) & \zeta_2^{\mathrm{T}}(t) & \zeta_3^{\mathrm{T}}(t)\end{bmatrix},\\ \zeta_1^{\mathrm{T}}(t)&=\begin{bmatrix}\xi^{\mathrm{T}}(t) & \xi^{\mathrm{T}}(t-\tau(t)) & \xi^{\mathrm{T}}(t-\tau_M)\end{bmatrix},\\ \zeta_2^{\mathrm{T}}(t)&=\begin{bmatrix}\xi^{\mathrm{T}}(t-d(t)) & \xi^{\mathrm{T}}(t-d_M)\end{bmatrix},\\ \zeta_3^{\mathrm{T}}(t)&=\begin{bmatrix}f^{\mathrm{T}}(\bar{C}H_1\xi(t-d(t))) & e_k^{\mathrm{T}}(t) & \omega^{\mathrm{T}}(t)\end{bmatrix}.\end{aligned}$$

By Lemma 2, we can obtain that

$$-\tau_M\int_{t-\tau_M}^{t}\dot{\xi}^T(s)R_1\dot{\xi}(s)ds\le\zeta_1^{\mathrm{T}}(t)F_1\zeta_1(t),\tag{21}$$

$$-d_M\int_{t-d_M}^{t}\dot{\xi}^T(s)R_2\dot{\xi}(s)ds\le\zeta_4^{\mathrm{T}}(t)F_2\zeta_4(t),\tag{22}$$

where

$$\begin{aligned}\zeta_4^{\mathrm{T}}(t)&=\begin{bmatrix}\xi^{\mathrm{T}}(t) & \xi^{\mathrm{T}}(t-d(t)) & \xi^{\mathrm{T}}(t-d_M)\end{bmatrix},\\ F_1&=\begin{bmatrix}-R_1 & * & *\\ R_1^{\mathrm{T}}-U_1^{\mathrm{T}} & -2R_1+U_1+U_1^{\mathrm{T}} & *\\ U_1^{\mathrm{T}} & R_1^{\mathrm{T}}-U_1^{\mathrm{T}} & -R_1\end{bmatrix},\\ F_2&=\begin{bmatrix}-R_2 & * & *\\ R_2^{\mathrm{T}}-U_2^{\mathrm{T}} & -2R_2+U_2+U_2^{\mathrm{T}} & *\\ U_2^{\mathrm{T}} & R_2^{\mathrm{T}}-U_2^{\mathrm{T}} & -R_2\end{bmatrix}.\end{aligned}$$

By combining (14)–(22), then we have

$$\begin{aligned}&\mathbb{E}\{\dot{V}(t)+\tilde{z}^{\mathrm{T}}\tilde{z}-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\}\\&\le 2\xi^{\mathrm{T}}(t)P\mathscr{A}+\xi^{\mathrm{T}}(t)(Q_1+Q_2)\xi(t)\\&\quad-\xi^{\mathrm{T}}(t-\tau_M)Q_1\xi(t-\tau_M)-\xi^{\mathrm{T}}(t-d_M)Q_2\xi(t-d_M)\\&\quad+\mathscr{A}^{\mathrm{T}}\tilde{R}\mathscr{A}+\rho^2\mathscr{B}^{\mathrm{T}}\tilde{R}\mathscr{B}+\xi^{\mathrm{T}}(t)\tilde{L}^{\mathrm{T}}\tilde{L}\xi(t)-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\\&\quad+\xi^{\mathrm{T}}(t-\tau(t))H_1^{\mathrm{T}}C^{\mathrm{T}}\sigma\Omega CH_1\xi(t-\tau(t))\\&\quad+\bar{\alpha}\xi^{\mathrm{T}}(t-d(t))H_1^{\mathrm{T}}\bar{C}^T M^T PM\bar{C}H_1\xi(t-d(t))\\&\quad-\bar{\alpha}f^{\mathrm{T}}(CH_1\xi(t-d(t)))f(CH_1\xi(t-d(t)))\\&\quad+\zeta_1^{\mathrm{T}}(t)F_1\zeta_1+\zeta_4^{\mathrm{T}}(t)F_2\zeta_4-e_k^{\mathrm{T}}(t)\Omega e_k(t)\\&\le\zeta^{\mathrm{T}}(t)\Psi\zeta(t),\end{aligned}\tag{23}$$

where $\Psi=\Upsilon_{11}+\Upsilon_{21}^{\mathrm{T}}I\Upsilon_{21}+\Upsilon_{31}^{\mathrm{T}}P^{-1}\Upsilon_{31}-\Upsilon_{41}^{\mathrm{T}}\Upsilon_{44}^{-1}\Upsilon_{41}-\Upsilon_{51}^{\mathrm{T}}\Upsilon_{55}^{-1}\Upsilon_{51}$. Υ_{11}, Υ_{21}, Υ_{31}, Υ_{41}, Υ_{44}, Υ_{51} and Υ_{55} are defined in the Theorem 1. It can be obtained that

$$\mathbb{E}\{\dot{V}(t)+\tilde{z}^{\mathrm{T}}\tilde{z}-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\}\le\zeta^{\mathrm{T}}(t)\Phi\zeta(t).\tag{24}$$

Similar to [46, 47], by applying Schur complement and Lemma 1, if (24) holds and $\Psi<0$, there exists a scalar $\lambda>0$ such that $\mathbb{E}\{\dot{V}(t)+\tilde{z}^{\mathrm{T}}\tilde{z}-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\}\le-\lambda\parallel\zeta(t)\parallel^2$ for $\zeta(t)\ne 0$, then $\mathbb{E}\{\dot{V}(t)+\tilde{z}^{\mathrm{T}}\tilde{z}-\gamma^2\omega^{\mathrm{T}}(t)\omega(t)\}<0$, it indicates that the system is asymptotically stable. The proof is completed. □

In Theorem 1, it drives the sufficient conditions, which can guarantee the stability of the system (7). On the basis of Theorem 1, the design problem of the distributed estimators will be dealt with in Theorem 2.

Theorem 2: For the given occurring probability of deception-attacks $\bar{\alpha}$, time-delay bounds τ_M and d_M, trigger scalars σ_i $(i=1,2,\dots,m)$, γ, ϵ_k $(k=1,2)$ and matrices Ξ, M, the estimator error distributed system (7) is asymptotically stable if there exists matrices $\bar{P}_i>0$, $\bar{K}_i$, $\bar{D}_i$ $(i=1,2,\dots,m)$, $Q_k>0$, $R_k>0$, $U_k>0$ $(k=1,2)$ with appropriate dimensions, then, the following LMIs hold:

$$\begin{bmatrix}\bar{\Upsilon}_{11} & * & * & * & *\\ \bar{\Upsilon}_{21} & -I & * & * & *\\ \bar{\Upsilon}_{31} & 0 & -\bar{P} & * & *\\ \bar{\Upsilon}_{41} & 0 & 0 & \bar{\Upsilon}_{44} & *\\ \bar{\Upsilon}_{51} & 0 & 0 & 0 & \bar{\Upsilon}_{55}\end{bmatrix}<0,\tag{25}$$

$$\begin{bmatrix}R_k & *\\ U_k & R_k\end{bmatrix}>0,\quad(k=1,2).\tag{26}$$

where

$$\bar{\Upsilon}_{11}=\begin{bmatrix}\bar{\Phi}_{11} & * & * & * & * & * & * & *\\ \bar{\Phi}_{21} & \bar{\Phi}_{22} & * & * & * & * & * & *\\ U_1^T & \Phi_{32} & \Phi_{33} & * & * & * & * & *\\ \Phi_{41} & 0 & 0 & \Phi_{44} & * & * & * & *\\ U_2^T & 0 & 0 & \Phi_{54} & \Phi_{55} & * & * & *\\ \bar{\Phi}_{61} & 0 & 0 & 0 & 0 & -\alpha I & * & *\\ \bar{\Phi}_{62} & 0 & 0 & 0 & 0 & 0 & -\Omega & *\\ \hat{B} & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I\end{bmatrix},$$

$$\begin{aligned}\bar{\Phi}_{11}&=\hat{A}+\hat{A}^T+Q_1+Q_2-R_1-R_2,\\ \bar{\Phi}_{21}&=-\bar{\alpha}_1\hat{C}+R_1^T-U_1^T,\\ \bar{\Phi}_{22}&=-2R_1+U_1+U_1^T+\hat{\Omega},\\ \bar{\Phi}_{61}&=-\bar{\alpha}\hat{K},\bar{\Phi}_{62}=\bar{\alpha}_1\hat{K},\\ \hat{A}&=\begin{bmatrix}\bar{P}\bar{A}-\bar{K}\bar{C}+\bar{D}\Xi\bar{C} & \bar{K}\bar{C}-\bar{D}\Xi\bar{C}\\ 0 & \bar{P}\bar{A}\end{bmatrix},\\ \hat{C}&=\begin{bmatrix}0 & 0\\ \bar{C}^T\bar{K}^T & 0\end{bmatrix},\hat{\Omega}=\begin{bmatrix}0 & 0\\ 0 & \sigma\bar{C}^T\Omega\bar{C}\end{bmatrix},\\ \hat{K}&=\begin{bmatrix}\bar{K}^T & 0\end{bmatrix},\hat{B}=\begin{bmatrix}\bar{B}^T\bar{P} & \bar{B}^T\bar{P}\end{bmatrix},\\ \bar{P}&=\mathrm{diag}\{\bar{P}_1,\bar{P}_2,\dots,\bar{P}_m\},\\ \bar{K}&=\mathrm{diag}\{\bar{K}_1,\bar{K}_2,\dots,\bar{K}_m\},\\ \bar{D}&=\mathrm{diag}\{\bar{D}_1,\bar{D}_2,\dots,\bar{D}_m\},\\ \bar{\Upsilon}_{44}&=\bar{\Upsilon}_{55}=\mathrm{diag}\{-2\epsilon_1\bar{P}+\epsilon_1^2R_1,-2\epsilon_2\bar{P}+\epsilon_2^2R_2\},\\ \bar{\Upsilon}_{41}&=\begin{bmatrix}\bar{\Upsilon}_{411} & \bar{\Upsilon}_{412}\end{bmatrix},\\ \bar{\Upsilon}_{411}&=\begin{bmatrix}\tau_M P\hat{A} & -\bar{\alpha}_1\tau_M\hat{C}^T & 0 & 0\\ d_M P\hat{A} & -\bar{\alpha}_1 d_M\hat{C}^T & 0 & 0\end{bmatrix},\\ \bar{\Upsilon}_{412}&=\begin{bmatrix}0 & -\tau_M\bar{\alpha}\hat{K}^T & \tau_M\bar{\alpha}_1\hat{K}^T & 0\\ 0 & -d_M\bar{\alpha}\hat{K}^T & d_M\bar{\alpha}_1\hat{K}^T & 0\end{bmatrix},\\ \bar{\Upsilon}_{51}&=\begin{bmatrix}\bar{\Upsilon}_{511} & \bar{\Upsilon}_{512}\end{bmatrix},\\ \bar{\Upsilon}_{511}&=\begin{bmatrix}0 & \rho\tau_M\hat{C}^T & 0 & 0\\ 0 & \rho d_M\hat{C}^T & 0 & 0\end{bmatrix},\\ \bar{\Upsilon}_{512}&=\begin{bmatrix}0 & -\rho\tau_M\hat{K}^T & -\rho\tau_M\hat{K}^T & 0\\ 0 & -\rho d_M\hat{K}^T & -\rho d_M\hat{K}^T & 0\end{bmatrix}.\end{aligned}$$

Moreover, the decoupling and coupling gains of the ith state estimator are obtained.

$$K_i = \bar{P}_i^{-1}\bar{K}_i, \quad D_i = \bar{P}_i^{-1}\bar{D}_i \quad (i = 1, 2, \ldots, m). \tag{27}$$

Proof: Define

$$\begin{aligned}\bar{P} &= P = \mathrm{diag}\{\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_m\},\\ \bar{K} &= \bar{P}K = \mathrm{diag}\{\bar{K}_1, \bar{K}_2, \ldots, \bar{K}_m\},\\ \bar{D} &= \bar{P}D = \mathrm{diag}\{\bar{D}_1, \bar{D}_2, \ldots, \bar{D}_m\}.\end{aligned}$$

By applying the definitions of $\bar{P}$, $\bar{K}$, $\bar{D}$ into inequality matrix (12), we can obtain the following equivalent inequality matrix:

$$\begin{bmatrix} \bar{\Upsilon}_{11} & * & * & * & * \\ \bar{\Upsilon}_{21} & -I & * & * & * \\ \bar{\Upsilon}_{31} & 0 & -\bar{P} & * & * \\ \bar{\Upsilon}_{41} & 0 & 0 & \tilde{\Upsilon}_{44} & * \\ \bar{\Upsilon}_{51} & 0 & 0 & 0 & \tilde{\Upsilon}_{55} \end{bmatrix} < 0, \tag{28}$$

where $\tilde{\Upsilon}_{44} = \tilde{\Upsilon}_{55} = \mathrm{diag}\{-\bar{P}R_1^{-1}\bar{P}, -\bar{P}R_2^{-1}\bar{P}\}$.

Due to

$$(R_k - \epsilon_k^{-1}\bar{P})R_k^{-1}(R_k - \epsilon_k^{-1}\bar{P}) \geq 0, \tag{29}$$

where $k = 1, 2$.

Then, it yields that

$$-\bar{P}R_k^{-1}\bar{P} \leq -2\epsilon_k\bar{P} + \epsilon_k^2 R_k. \tag{30}$$

That is

$$\begin{aligned}-\bar{P}R_1^{-1}\bar{P} &\leq -2\epsilon_1\bar{P} + \epsilon_1^2 R_1,\\ -\bar{P}R_2^{-1}\bar{P} &\leq -2\epsilon_2\bar{P} + \epsilon_2^2 R_2.\end{aligned}$$

Next, $\tilde{\Upsilon}_{44}$, $\tilde{\Upsilon}_{55}$ in (28) are replaced by $-2\epsilon_1\bar{P} + \epsilon_1^2 R_1$ and $\leq -2\epsilon_2\bar{P} + \epsilon_2^2 R_2$, respectively. Then, we can obtain the LMIs (25).

In Theorem 1, by applying inequalities (12) and (13), we obtain

$$\mathbb{E}\{\dot{V}(t) + \tilde{z}^{\mathrm{T}}\tilde{z} - \gamma^2\omega^{\mathrm{T}}(t)\omega(t)\} < 0. \tag{31}$$

Therefore, with the help of inequalities (25), (26) and (31), the desired distributed estimation error system (7) is asymptotically stable.

Moreover, on account of the definition of $\bar{K} = \bar{P}K$ and $\bar{D} = \bar{P}D$, then, $K = \bar{P}^{-1}\bar{K}$, $D = \bar{P}^{-1}\bar{D}$.

Since $\bar{P} = \mathrm{diag}\{\bar{P}_1, \bar{P}_2, \ldots, \bar{P}_m\}$, then

$$\begin{cases} K = \mathrm{diag}\{\bar{P}_1^{-1}\bar{K}_1, \bar{P}_2^{-1}\bar{K}_2, \ldots, \bar{P}_m^{-1}\bar{K}_m\},\\ D = \mathrm{diag}\{\bar{P}_1^{-1}\bar{D}_1, \bar{P}_2^{-1}\bar{D}_2, \ldots, \bar{P}_m^{-1}\bar{D}_m\}. \end{cases} \tag{32}$$

Hence, the decoupling and coupling gains of the ith estimator can be expressed as $K_i = \bar{P}_i^{-1}\bar{K}_i$, $D_i = \bar{P}_i^{-1}\bar{D}_i$, $(i = 1, 2, \ldots, m)$. That completes the proof. □

4 Simulation examples

In this section, a practical example is given to show the feasibility of the designed distributed state estimator system.

Consider the continuous stirred tank reactor system borrowed from [48]. As is shown in Fig. 2, ψ_A and ψ_B denote the educt and product, respectively. The balance equations of the continuous stirred tank reactor are shown as follows [49]:

$$\frac{\mathrm{d}W_A}{\mathrm{d}t} = \frac{F}{V}(W_{A_0} - W_A) - g_1 W_A, \tag{33}$$

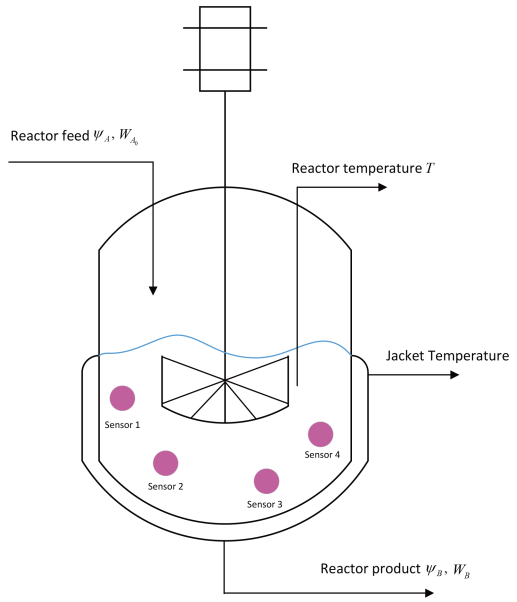

Fig. 2 *Continuous stirred tank reactor*

$$\begin{aligned}\frac{\mathrm{d}W_B}{\mathrm{d}t} &= -\frac{F}{V}W_B + g_1 W_A - g_2 W_B,\\ \frac{\mathrm{d}T}{\mathrm{d}t} &= \frac{F}{V}(T_0 - T) + \frac{g_\omega A_R}{\delta W_P V}(T_s - T)\end{aligned} \tag{34}$$

$$-\frac{g_1 W_A \Delta H_R^{AB} + g_2 W_B \Delta H_R^{BC}}{\delta W_P}, \tag{35}$$

where Tables 1 and 2 present the definitions of the variables and model parameters, respectively. Based on (33)–(35), the linearised state-space model of the continuous stirred tank reactor near the operating point can be obtained as follows:

$$\dot{x}(t) = Ax(t) + B\omega(t), \tag{36}$$

where $x(t) = [x_1(t) \quad x_2(t) \quad x_3(t)]^{\mathrm{T}}$, $x_1(t)$ denotes the concentration of the educt ψ_A at instant t, $x_2(t)$ represents the concentration of the product ψ_B at instant t, $x_3(t)$ is the reactor temperature at instant t. $B = [0 \quad 0.6474 \quad 0.6779]^{\mathrm{T}}$ and the linearised system matrix A can be derived as follows:

$$A = \begin{bmatrix} -\frac{F}{V} - g_1 & 0 & \frac{E_{A1}}{RT_s^2}g_1 W_{A_s} \\ g_1 & -\frac{F}{V} - g_2 & \beta_{23} \\ -\frac{g_1 \Delta H_R^{AB}}{\delta W_P} & -\frac{g_2 \Delta H_R^{BC}}{\delta W_P} & \beta_{33} \end{bmatrix} = \begin{bmatrix} -0.9388 & 0 & 0.0459 \\ 0.625 & -0.9388 & -0.0125 \\ -0.9335 & 2.4449 & -0.8894 \end{bmatrix},$$

$$\begin{aligned}\beta_{23} &= -\frac{E_{A1}}{RT_s^2}g_1 W_{A_s} + \frac{E_{A2}}{RT_s^2}g_2 W_{B_s},\\ \beta_{33} &= -\frac{F}{V} - \frac{g_\omega A_R}{\delta W_P V}\\ &\quad + \frac{E_{A1}g_1 W_{A_s}\Delta H_R^{AB} + E_{A2}g_2 W_{B_s}\Delta H_R^{BC}}{RT_s^2 \delta W_P}.\end{aligned}$$

Set the sampling period $T = 0.5$, the other parameters of the system are given by

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad L = [-1 \quad -1 \quad 0],$$

$$\omega(t) = \begin{cases} 1, & 5 \le t \le 10, \\ -1, & 15 \le t \le 20, \\ 0, & \text{else}. \end{cases}$$

The function of deception attacks $f(y(t)) = [-\tanh^{\mathrm{T}}(0.3y_1(t)) - \tanh^{\mathrm{T}}(0.5y_2(t)) - \tanh^{\mathrm{T}}(0.6y_3(t))]^{\mathrm{T}}$, which can be depicted in Fig. 3. According to Assumption 1, we can obtain the upper bound matrix $M = \mathrm{diag}\{0.3, 0.5, 0.6\}$ satisfying inequality (8).

As is shown in Fig. 2, there are four sensors deployed to monitor the reactor temperature, correspondingly, assume that there are four state estimators constructing the estimating network, the graph of the estimating network is represented in Fig. 4 which is borrowed from [37]. The estimating system can be described by the following matrix:

$$\Xi = \begin{bmatrix} 2 & -1 & -1 & 0 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix}.$$

Set time delays $\tau_M = 0.4$, $d_M = 0.3$, $\gamma = 1.2$ and the triggered factors of ETS $\sigma_1 = 0.9$, $\sigma_2 = 0.5$, $\sigma_3 = 0.6$, $\sigma_4 = 0.3$, let the probability of the deception attacks $\bar{\alpha} = 0.5$, it means that the distributed state estimating systems are subject to the ETS and deception attacks. With the initial condition $x(0) = [0 \quad 0 \quad 0]^{\mathrm{T}}$, the decoupling and coupling gains of state estimators are obtained by applying (27) in Theorem 2 as follows:

$$K_1 = \begin{bmatrix} 0.0007 & 0.0108 & 0.0032 \\ 0.0048 & 0.0627 & 0.0082 \\ 0.0146 & 0.2486 & 0.0612 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.0004 & 0.0273 & 0.0022 \\ -0.0024 & 0.1467 & 0.0005 \\ -0.0132 & 0.6158 & 0.0353 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 0.0026 & -0.0109 & 0.0158 \\ 0.0125 & -0.0574 & 0.0767 \\ 0.0525 & -0.2459 & 0.3484 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 0.0063 & -0.0179 & 0.0219 \\ 0.0336 & -0.1089 & 0.1207 \\ 0.1356 & -0.4282 & 0.5066 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} -0.0590 & 0.0093 & 0.0009 \\ -0.2440 & -0.2815 & 0.0564 \\ 0.3399 & 0.0607 & -0.0348 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} -0.0402 & 0.0144 & -0.0003 \\ -0.3741 & -0.2908 & 0.0675 \\ 0.5597 & 0.1856 & -0.0578 \end{bmatrix},$$

$$D_3 = \begin{bmatrix} -0.0941 & -0.0020 & 0.0056 \\ -0.4038 & -0.0624 & 0.0489 \\ 2.8974 & 0.4089 & -0.2323 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} -0.1078 & 0.0035 & 0.0069 \\ -0.3617 & -0.0071 & 0.0424 \\ 1.6207 & 0.3519 & -0.1172 \end{bmatrix}.$$

The diagrams of event-triggered instants and intervals are shown as Figs. 5–8. The response of estimation error $\tilde{z}(t)$ is reflected in Fig. 9. The occurring probability $\alpha(t)$ of deception attacks is depicted in Fig. 10. According to the above figures, one can see that the design of event-triggered distributed H_∞ state estimators with deception attacks is feasible.

Table 1 Definitions of variables in (33)–(35)

W_A	concentration of educt ψ_A
W_B	concentration of product ψ_B
W_{A_0}	low concentration of educt ψ_A
T_0	initial temperature of the reactor
F	normalised process stream inflow
V	volume flow
δ	density
W_P	heat capacity
ΔH_R^{AB}	reaction enthalpy from ψ_A to ψ_B
g_1, g_2	rate coefficients

Table 2 model parameters and main operating point

$g_0 = 1.2467 \times 10^{12}\,\mathrm{h}^{-1}$	$E_{A1}/R = E_{A2}/R = 9867.5\,\mathrm{K}$
$\Delta H_R^{AB} = 4.2\,\mathrm{KJ/mol}$	$\Delta H_R^{BC} = -11\,\mathrm{KJ/mol}$
$\delta = 0.9342\,\mathrm{kh/l}$	$W_P = 3.01\,\mathrm{kJ/kg\,K}$
$A_R = 0.215$	$V = 10.01$
$T_0 = 403.15\,\mathrm{K}$	$g_\omega = 4032\,\mathrm{kJ/h\,m^2\,K}$
$W_{A_s} = 1.235\,\mathrm{mol/l}$	$W_{A_0} = 5.1\,\mathrm{mol/l}$
$W_{B_s} = 0.9\,\mathrm{mol/l}$	$T_s = 407.29\,\mathrm{K}$
$F/V = 0.3138$	$g_1 = g_2 = 0.625$

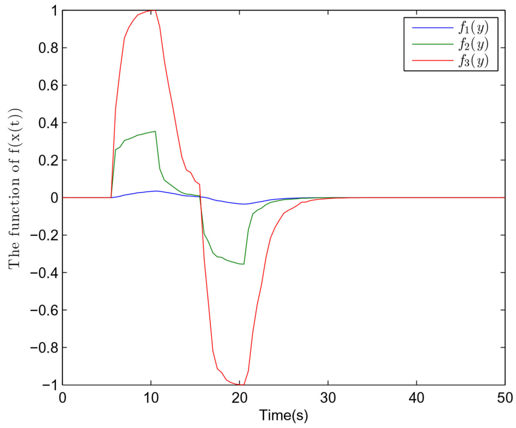

Fig. 3 *Graph of cyber-attacks $f(y(t))$*

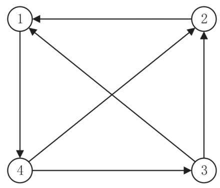

Fig. 4 *Topology of the estimator networks*

5 Conclusion

This study concentrates on the investigation of H_∞ distributed state estimating design with decentralised ETSs and deception attacks. To alleviate the burden of the transmitted burden, the distributed ETSs are introduced to determine whether the received sampled data is delivered or not. By using a random Bernoulli variable to govern the stochastic deception attacks, a mathematical model of H_∞ distributed state estimating error systems has been constructed for sensor networks. According to the Lyapunov stability theory and LMI techniques, sufficient conditions which can ensure the stability of the desired estimating error systems are derived. Thus,

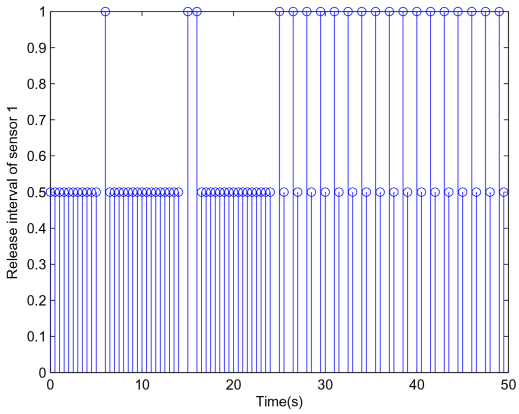

Fig. 5 *Release instants and intervals of ETC 1*

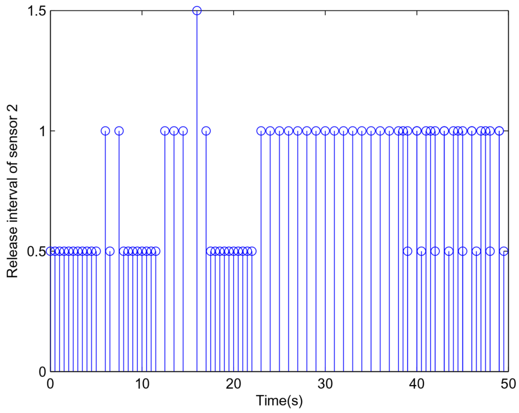

Fig. 6 *Release instants and intervals of ETC 2*

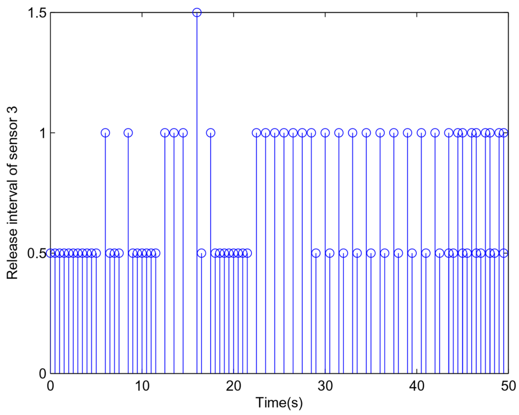

Fig. 7 *Release instants and intervals of ETC 3*

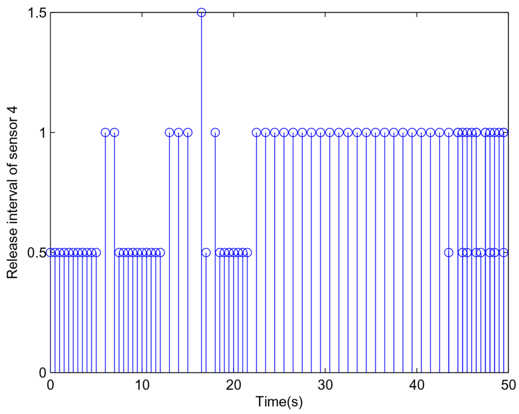

Fig. 8 *Release instants and intervals of ETC 4*

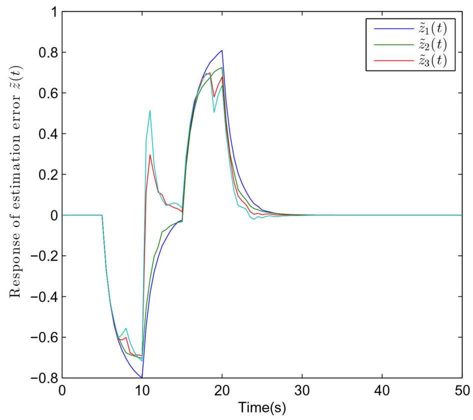

Fig. 9 *Response of estimation error* $\tilde{z}(t)$

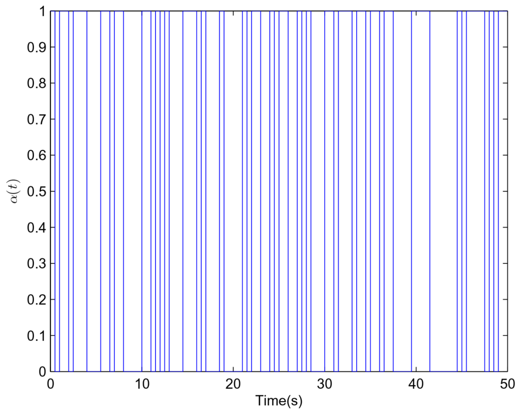

Fig. 10 *The occurring probability* $\alpha(t)$ *of deception attacks*

the designed algorithm of distributed estimators is obtained, also the estimating parameters and corresponding coupling gains are derived in an exact expression. Finally, an illustrated example is given to demonstrate the usefulness of the designed H_∞ event-triggered distributed estimators with deception attacks.

6 Acknowledgments

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