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Event-Triggered State Estimation for T–S Fuzzy Neural Networks with Stochastic Cyber-Attacks

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Abstract This paper is mainly concerned with event-triggered state estimation for Takagi–Sugeno (T–S) fuzzy neural networks subjected to stochastic cyber-attacks. An event-triggered scheme is utilized to decide whether the sampled data should be delivered or not. By taking the influence of the cyber-attacks into consideration, a T–S fuzzy model for the state estimation of neural networks is established with the event-triggered scheme. Through the utilization of Lyapunov stability theory and linear matrix inequality (LMI) techniques, the sufficient conditions are derived which can ensure the stability of estimator error systems. In addition, the gains of the estimator are acquired in the form of LMIs. Finally, a simulated example is presented to illustrate the effectiveness of the proposed method.

Keywords Event-triggered scheme · T–S fuzzy neural networks · Stochastic cyber-attacks · State estimation

1 Introduction

Since the 1980s of the twentieth century, due to the unique nonlinear adaptive information processing capability, neural networks have received considerable attentions and have been widely used in the fields of automatic control [1–5], pattern recognition [6–8], intelligent robots [9] and so on. Plenty of scholars devote themselves to studying different classes of neural networks. From the control perspective, different issues on neural networks have been investigated, such as H_∞ filter design [10], the robust stability [11, 12], synchronization [13] and state estimator design [14]. For example, in [15], the temporal properties problems for neural networks by using support vector regression are

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addressed. With the consideration of stochastic cyber-attacks, the problem of H_∞ filter design for neural networks is investigated in [16]. In [17], the authors propose a dissipativity-based state estimation method for static neural networks with time-varying delay. The authors in [18] concentrate on the reduced-order state estimation problem for a class of neural networks with the help of an integral inequality. H_∞ filter design for a class of neural network systems with event-triggered communication scheme and quantization is discussed in [10]. Quantized state estimation for neural networks with cyber-attacks is studied in [19].

As a mathematical tool, T–S fuzzy model is widely used in the analysis and modeling of nonlinear systems [20–25]. For instance, network-based fault-tolerant control is addressed for a T–S fuzzy system in [22]. By using the smooth membership functions, the authors in [26] are concerned with the adaptive control for the multi-input and multi-output system via T–S fuzzy model. For the T–S fuzzy systems with time delays, the fault detection problem is investigated in [27]. A newly slack variable technique is proposed in [28] which can effectively improve the relaxation quality of control synthesis for discrete-time T–S fuzzy systems. T–S fuzzy models can help not only ordinary nonlinear systems but also neural networks to carry out control analysis. As a fusion of fuzzy logic and neural networks, fuzzy neural networks have the advantages of both and avoid some shortcomings. Therefore, it has become one of the hot spots in the research of intelligent control. By using the wavelet fuzzy neural network, a robust adaptive sliding-mode control is studied in [29]. For the estimation of the unknown functions in the dynamics fractional-order chaotic systems, the self-evolving nonsingleton type-2 fuzzy neural networks are presented in [30]. The state estimation of T–S fuzzy neural networks with Markovian jumping parameters via sampled-data control is investigated in [31]. In addition, with the consideration of probabilistic actuator faults and leakage terms, robust reliable control for T–S fuzzy neural networks is addressed in [32].

In the past few years, the restrictions of energy and bandwidth in networked control systems have stirred remarkable interests. For solving the problem, different communication transmission mechanisms have been proposed. Under time-triggered scheme, the signals are sampled and transmitted at a fixed interval, which brings great burden to the network. In order to overcome the shortcomings of the time-triggered scheme, event-triggered scheme is adopted in [14]. Up to the present, various event-triggered schemes have been proposed, one of which is an event-triggered scheme relying on discrete supervision of the system states. This event-triggered scheme firstly proposed in [33] only supervises the difference between the states sampled in discrete instants and the data can be transmitted only when they satisfy the given conditions. A low transmission frequency and decrease in the release times of the sensor are regarded as the main advantages of the event-triggered scheme. Based on [33], different issues have been addressed under different event-triggered schemes and a great many research results have been published [34, 35]. For example, the authors in [36] address the problem of adaptive model-based event-triggered control for an uncertain continuous system. In [37], the event-triggered-based control is investigated for discrete-time T–S fuzzy systems. Motivated by the work in [33], the distributed event-triggered scheme is adopted to design H_∞ filter for sensor networks [35]. On the basis of the work in [33], the authors propose a hybrid-driven communication scheme which can further improve the system performance and reduce the transmission of sampled signals at the same time in [38]. Based on the hybrid-driven communication scheme in [38], a H_∞ filter design method is proposed for hybrid-driven networked systems with stochastic cyber-attacks in [39]. Under hybrid-driven communication scheme in [38], the authors in [40] focus on the resilient observer-based output feedback controller design for hybrid-triggered networked T–S fuzzy systems.

Nowadays, the channels in control systems are connected via communication network, which results that the systems are vulnerable to suffer from cyber-attacks. In general, the cyber-attacks are classified into three kinds including deception attacks, denial of service (DoS) attacks and eavesdropping attacks. Deception attacks degrade system performance via replacing normal data by pretending to be trusted parties. DoS attacks send plenty of spam or jamming information to destroy service system. Different from the above attacks, eavesdropping attacks bring about information leakage by methods of network monitoring, illegal access to data and password files. Recently, the control synthesis of the system against cyber-attacks has been a hot topic. For example, the remote estimator over a multi-channel network is investigated for cyber-physical system under the DoS attacks in [41]. The load frequency control is studied for multi-area power systems with energy-limited DoS attacks in [42]. By taking the influence of the deception attacks into account, the authors in [43] adopt an event-triggered mechanism against false data injection (deception) attack on distributed network. Under the influence of deception attacks, quadratic cost criterion safety control problem is addressed for a class of discrete-time stochastic nonlinear systems in [44]. A method is proposed in [45], which is capable of detecting compromised sensor networks vulnerable to DoS attacks. Moreover, with the consideration of stochastic cyber-attacks, quantized stabilization is discussed for T–S fuzzy systems under hybrid-triggered mechanism in [46]. In view of the cyber-attacks and lossy sensors, the observer-based event-triggering consensus control problem is studied for a class of discrete-time multi-agent systems in [47]. To the best of our knowledge, the event-triggered state estimation for T–S fuzzy neural networks with stochastic cyber-attacks has not been investigated, which still remains as a challenging problem.

The rest of this paper is organized as follows. In Sect. 2, the problem under consideration is presented and system modeling is described. In Sect. 3, the sufficient conditions guaranteeing the stability of the system are acquired and the desired estimator gains are accurately derived. In Sect. 4, a practical paradigm is supplied to prove the usefulness of designed estimator.

Notation R^m and $R^{m\times n}$ represent the m-dimensional Euclidean space, and the set of $m\times n$ real matrices; the superscript T denotes matrix transposition; I is the identity matrix with appropriate dimension; $\mathbb{E}$ is the expectation operator; the notation $X>0$ (respectively, $X\geq 0$), for $X\in R^{m\times m}$ stands for that the matrix X is real symmetric positive definite (respectively, positive semi-definite). For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ represents a symmetric matrix, where $*$ refers to the entries implied by symmetry. The set $\{1,2,\ldots,r\}$ is represented as L.

2 System Description

A T–S fuzzy model with r plant rules is exhibited, which can describe the following neural networks.

IF $\eta_1(x(t))$ is H_1^i and $\eta_q(x(t))$ is H_q^i, THEN

$$\begin{cases} \dot{x}(t) = -A_i x(t) + W_i g(x(t)) + V_i g(x(t-\phi(t))) \\ y(t) = C_i x(t) \end{cases} \tag{1}$$

where r is the number of IF-THEN rules, $x(t)=[x_1,x_2,\ldots,x_m]^T\in R^m$ is the state vector of network and $y(t)=[y_1,y_2,\ldots,y_m]^T\in R^m$ is the measurement output. $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\ldots,g_m(x_m(t))]^T$ denotes the neuron activation function, $\phi(t)$ represents the time-varying delay satisfying $0\leq\phi(t)\leq\phi_M$, ϕ_M is the upper bound of $\phi(t)$ and H_v^i $(i\in L, v=1,2,\ldots,q)$ denotes the fuzzy sets, $\eta_v(x(t))$ are fuzzy premise variables; for simplicity, $\eta(x)$ is used to represent $\eta_v(x(t))$ and $\eta(x)=[\eta_1(x),\eta_2\ (x),\ldots,\eta_q\ (x)]$. $A_i=\mathrm{diag}\{a_{i1},a_{i2}\ldots a_{im}\}>0$ is a constant real matrix; W_i and V_i are the connection weight matrix and the delayed connection weight matrix, respectively; C_i is the parameter matrix with appropriate dimensions.

By utilizing center-average defuzzifier, product interference and singleton fuzzifier, fuzzy system (1) is inferred as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r}\varphi_i(\eta(x))[-A_i x(t) + W_i g(x(t)) \\ \qquad + V_i g(x(t-\phi(t)))] \\ y(t) = \sum_{i=1}^{r}\varphi_i(\eta(x))C_i x(t) \end{cases} \tag{2}$$

where $\varphi_i(\eta(x))=\frac{\mu_i(\eta(x))}{\sum_{i=1}^{r}\mu_i(\eta(x))}$, $\mu_i(\eta(x))=\prod_{v=1}^{q}\mu_v^i(\eta_v(x))$, $\mu_v^i(\eta_v(x))$ represents the grade membership value of $\eta_v(x)$. $\varphi_i(\eta(x))$ is the normalized membership function which satisfies $\varphi_i(\eta(x))\geq 0, \sum_{i=1}^{r}\varphi_i(\eta(x))=1$ for $i\in L$.

In this paper, the estimator design is investigated for T–S fuzzy neural networks with event-triggered scheme and stochastic cyber-attacks. As is shown in Fig. 1, an event generator is located between the sensor and the estimator, which is utilized to decide whether the sampled signals should be transmitted into the network. The influence of stochastic cyber-attacks in the network is also taken into account.

Since there exists network-induced delay, the premise variable $\varphi_i(\eta(x))$ of system (2) is mismatched with the one in fuzzy estimator. The state estimator model is designed as follows

IF $\eta_1(\hat{x}(t))$ is G_1^j and $\eta_p(\hat{x}(t))$ is G_p^j, THEN

$$\begin{cases} \dot{\hat{x}}(t) = -A_j\hat{x}(t) + K_j(\tilde{y}(t)-\hat{y}(t)) \\ \hat{y}(t) = C_j\hat{x}(t) \end{cases} \tag{3}$$

where $\hat{x}(t)\in R^m$ is the estimated state vector and K_j denote the state estimator parameters to be determined, $\tilde{y}(t)$ is the actual input of the state estimator and $\hat{y}(t)$ is the estimation of $y(t)$. $G_\ell^j(j\in L, \ell=1,2,\ldots,p)$ denotes the fuzzy sets, and $\eta_\ell(\hat{x}(t))$ is premise variables. For simplicity, $\eta(\hat{x})$ is used to represent $\eta_\ell(\hat{x}(t))$ and $\eta(\hat{x})=[\eta_1(\hat{x}),\eta_2(\hat{x}),\ldots,\eta_p(\hat{x})]$. A_j are C_j are the parameter matrices with appropriate dimensions.

Then, the fuzzy state estimator can be described as follows.

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{j=1}^{r}h_j(\eta(\hat{x}))\left[-A_j\hat{x}(t) + K_j(\tilde{y}(t)-\hat{y}(t))\right] \\ \hat{y}(t) = \sum_{j=1}^{r}h_j(\eta(\hat{x}))C_j\hat{x}(t) \end{cases} \tag{4}$$

where $h_j(\eta(\hat{x}))=\frac{\varrho_j(\eta(\hat{x}))}{\sum_{j=1}^{r}\varrho_j(\eta(\hat{x}))}$, $\varrho_j(\eta(\hat{x}))=\prod_{\ell=1}^{p}G_\ell^j(\eta_\ell(\hat{x}))$, $G_\ell^j(\eta_\ell(\hat{x}))$ denotes the grade membership value of $\eta_\ell(\hat{x})$. $h_j(\eta(\hat{x}))$ presents the normalized membership function satisfies $h_j(\eta(\hat{x}))\geq 0, \sum_{j=1}^{r}h_j(\eta(\hat{x}))=1$.

Remark 1 As a powerful mathematical tool, T–S fuzzy model can provide an effective way of representing complex nonlinear systems by a series of simple local linear dynamic systems with their linguistic description. In this paper, the T–S fuzzy model is adopted to investigate state estimation of the delayed neural networks with event-triggered scheme and stochastic cyber-attacks shown in Fig. 1.

Motivated by [10], an event generator is designed at the sensor side, and the sampling data will not be delivered when they satisfy following inequality (5):

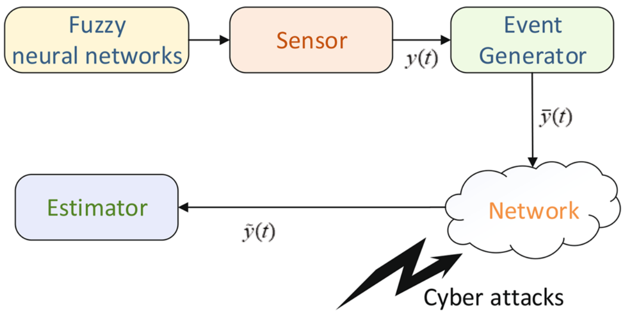

Fig. 1 The structure of the event-triggered neural networks with stochastic cyber-attacks

$$e_k^T(t)\Omega e_k(t) \leq \sigma y^T(t_k h + \lambda h)\Omega y(t_k h + \lambda h) \tag{5}$$

where $\sigma \in [0, 1)$, $\Omega > 0$ and $\lambda = 1, 2, \ldots$. The threshold error is $e_k(t) = y(t_k h) - y(t_k h + \lambda h)$.

It should be noted that once the sample signal is released into network at instant $t_k h$, then, it will be forwarded to the state estimator at instant $t_k h + \tau_{t_k}$, in which τ_{t_k} is network-induced communication delay. Similar to what is depicted in [48], the interval $[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}})$ can be divided into several subintervals. Suppose that there exists a constant ς which satisfies $[t_k h + \tau_{t_k},\ t_{k+1}h + \tau_{t_{k+1}}) = \bigcup_{\lambda=1}^{\varsigma} \Pi_\lambda$, where $\Pi_\lambda = [t_k h + \lambda h + \tau_{t_k+\lambda},\quad t_k h + \lambda h + h + \tau_{t_k+\lambda+1}]$, $\lambda = \{1, 2, \ldots, \varsigma\}$, $\varsigma = t_{k+1} - t_k - 1$. Define $\tau(t) = t - t_k h - \lambda h$, $0 \leq \tau_{t_k} \leq \tau(t) \leq h + \tau_{t_{k+\lambda+1}} \triangleq \tau_M$.

Then, the sampled signals via event-triggered scheme can be expressed as follows.

$$\bar{y}(t) = y(t - \tau(t)) + e_k(t) \tag{6}$$

Remark 2 In inequality (5), σ is a positive parameter, which can decide the frequency of event-triggered scheme. When $\sigma = 0$, $\bar{y}(t) = y(t)$, it indicates that all sampled data are delivered. When $\sigma \in (0, 1)$, it means that the measurement data will be transmitted via the network only when they exceed the threshold in (5) and the transmitted signal is $\bar{y}(t) = y(t - \tau(t)) + e_k(t)$.

The use of communication network makes neural networks vulnerable to the malicious signals, and the threats of the cyber-attacks can not be neglected. The malicious signals may substitute the normal transmitted data when the networks suffer from stochastic cyber-attacks $f(y(t))$. In this paper, the transmitted signals are assumed to be attacked by cyber-attacks randomly; then, the real signal received by the state estimator is

$$\tilde{y}(t) = \theta(t)f(y(t - d(t))) + (1 - \theta(t))\bar{y}(t) \tag{7}$$

where $d(t) \in [0, d_M]$, d_M is the upper bound of $d(t)$; $\theta(t) \in \{0, 1\}$ and $\theta(t)$ is a Bernoulli variable with the following statistical properties.

$$\mathbb{E}\{\theta(t)\} = \bar{\theta}, \mathbb{E}\left\{(\theta(t) - \bar{\theta})^2\right\} = \bar{\theta}(1 - \bar{\theta}) = \rho^2$$

where $\bar{\theta}$ denotes the expectation of $\theta(t)$ and ρ^2 is utilized to represent the mathematical variance of $\theta(t)$.

Remark 3 It needs to be pointed out that the cyber-attacks discussed in this paper belong to stochastic attacks, which corrupt the data transmission in the way of replacing the normal signals with the attack signals randomly. Bernoulli variable $\theta(t)$ in (7) is utilized to depict the appearance probability of the random cyber-attacks. When $\theta(t) = 0$, $\tilde{y}(t) = \bar{y}(t)$, the data are transmitted without stochastic attacks. When $\theta(t) = 1$, the false signals $\tilde{y}(t) = f(y(t - d(t)))$ substitute for the original transmission signals.

Remark 4 In recent years, cyber-attacks have received increasing research interests, which aim at damaging the system performance and reducing the reliability of the network [34, 35]. It should be pointed out that the problem of state estimator design for neural networks can be investigated by considering different types of cyber-attacks, such as DoS attacks, deception attacks and reply attacks. Due to page limitation, in this paper, the state estimator for neural network is designed only considering the randomly occurring deception attacks whose aim is to degrade the system stability by injecting false data on the state estimator inputs.

Define $e(t) = x(t) - \hat{x}(t)$; then, by combining (2), (4), (6) and (7), one can obtain the estimation error

$$\begin{aligned}\dot{e}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \varphi_i(\eta(x))h_j(\eta(\hat{x}))\big[\big(A_j - A_i + K_jC_j\big)x(t) \\ - \big(A_j + K_jC_j\big)e(t) - (1 - \theta(t))K_jC_ix(t - \tau(t)) \\ - (1 - \theta(t))K_je_k(t) - \theta(t)K_jf(y(t - d(t))) \\ + W_ig(x(t)) + V_ig(x(t - \phi(t)))\big]\end{aligned} \tag{8}$$

To simplify representation, $\varphi_i(\eta(x))$ and $h_j(\eta(\hat{x}))$ are abbreviated as φ_i and h_j, respectively.

Define

$$\begin{aligned}\bar{x}(t) &= \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T \\ \bar{A}_{ij} &= \begin{bmatrix} -A_i & 0 \\ A_j - A_i + K_jC_j & -(A_j + K_jC_j) \end{bmatrix} \\ \bar{B}_{ij} &= \begin{bmatrix} 0 & 0 \\ -K_jC_i & 0 \end{bmatrix}, \bar{W}_i = \begin{bmatrix} W_i \\ W_i \end{bmatrix}, \bar{V}_i = \begin{bmatrix} V_i \\ V_i \end{bmatrix} \\ H_1 &= [I \quad 0], H_2 = [0 \quad I]^T\end{aligned}$$

Then, we have

$$\begin{aligned}\dot{\bar{x}}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} \varphi_ih_j\big[\bar{A}_{ij}\bar{x}(t) + \bar{W}_i\bar{g}(H_1\bar{x}(t)) \\ + \bar{V}_i\bar{g}(H_1\bar{x}(t - \phi(t))) + (1 - \theta(t))\bar{B}_{ij}\bar{x}(t - \tau(t)) \\ - \theta(t)H_2K_jf(C_iH_1\bar{x}(t - d(t))) \\ - (1 - \theta(t))H_2K_je_k(t)\big]\end{aligned} \tag{9}$$

In the following, some important assumptions and lemma are introduced, which assist us in deriving the main results.

Assumption 1 [10] The neuron activation function is assumed to satisfy the following condition:

$$\left[g(x)-U_1x\right]^T\left[g(x)-U_2x\right]\leq 0 \tag{10}$$

where the real constant matrices U_1, U_2 satisfy $U_2-U_1\geq 0$.

Assumption 2 [39, 46] The cyber-attacks function $f(y(t))$ is presumed to satisfy the following condition

$$\| f(y(t)) \|_2 \leq \| Fy(t) \|_2 \tag{11}$$

where F is a given constant matrix which represents the upper bound of cyber-attacks.

Lemma 1 [49] Assume $\tau(t)\in[0,\tau_M]$, for any constant matrices $X\in\mathbb{R}^{m\times m}$ and $S\in\mathbb{R}^{m\times m}$ satisfying $\begin{bmatrix} X & * \\ S & X\end{bmatrix}\geq 0$, the following inequality holds:

$$-\tau_M\int_{t-\tau_M}^{t}\dot{x}^T(s)X\dot{x}(s)\mathrm{d}s\leq\begin{bmatrix} x(t)\\ x(t-\tau(t))\\ x(t-\tau_M)\end{bmatrix}^T \begin{bmatrix} -X* & * & \\ X+S & -2X-S-S^T & * \\ -S & X+S & -X\end{bmatrix}\begin{bmatrix} x(t)\\ x(t-\tau(t))\\ x(t-\tau_M)\end{bmatrix} \tag{12}$$

3 Main Results

Theorem 1 For the given positive parameters $\bar{\theta}$, time delays $\phi_M>0$, $\tau_M>0$, $d_M>0$, trigger parameter σ, matrices K_j and F, system (9) is stable if there exist matrices Ω, $P>0$, $Q_l>0$, $R_l>0$, $S_l>0$ $(l=1,2,3)$ with appropriate dimensions and parameters $\alpha>0$, $\beta>0$, such that the following inequalities hold with $h_j-\zeta_j\varphi_j\geq 0$ for all $i,j\in L$.

$$\Phi_{ij}-Z_i<0; \tag{13}$$

$$\zeta_i\Phi_{ii}-\zeta_iZ_i+Z_i<0; \tag{14}$$

$$\zeta_j\Phi_{ij}+\zeta_i\Phi_{ji}-\zeta_jZ_i-\zeta_iZ_j+Z_i+Z_j<0(i<j); \tag{15}$$

$$\begin{bmatrix} R_l & * \\ S_l & R_l\end{bmatrix}\geq 0,(l=1,2,3). \tag{16}$$

where

$$\Phi_{ij}=\begin{bmatrix}\psi_{11}* & * & \\ \psi_{21} & \psi_{22} & * \\ \psi_{31} & 0 & \psi_{33}\end{bmatrix} \tag{17}$$

in which

$$\psi_{11}=\begin{bmatrix}\Upsilon_1* & * & * & \\ \Upsilon_2 & \Upsilon_3* & * & \\ \Upsilon_4 & 0 & \Upsilon_5 & * \\ 0 & \Upsilon_6 & 0 & -I\end{bmatrix}$$

$$\Upsilon_1=\begin{bmatrix}\Psi_1* & * & \\ R_1+S_1 & \Psi_2 & * \\ -S_1 & R_1+S_1 & -Q_1-R_1\end{bmatrix}$$

$$\Psi_1=P\bar{A}_{ij}+\bar{A}_{ij}^TP+Q_1+Q_2+Q_3-R_1-R_2-R_3-\alpha\bar{U}_1$$

$$\Psi_2=-2R_1-S_1-S_1^T-\beta\bar{U}_1$$

$$\Upsilon_2=\begin{bmatrix}\bar{\theta}_1\bar{B}_{ij}^T+R_2+S_2 & 0 & 0\\ -S_2 & 0 & 0\\ R_3+S_3 & 0 & 0\\ -S_3 & 0 & 0\end{bmatrix}$$

$$\Upsilon_3=\begin{bmatrix}\Psi_3* & * & * & \\ R_2+S_2 & -Q_2-R_2* & * & \\ 0 & 0 & \Psi_4 & * \\ 0 & 0 & \Psi_5 & -Q_3-R_3\end{bmatrix}$$

$$\Psi_3=-2R_2-S_2-S_2^T+\sigma H_1^TC_i\Omega C_iH_1$$

$$\Psi_4=-2R_3-S_3-S_3^T,\Psi_5=R_3+S_3$$

$$\Upsilon_4=\begin{bmatrix}\bar{W}_i^TP-\alpha\bar{U}_2^T & 0 & 0\\ \bar{V}_i^T & -\beta\bar{U}_2^T & 0\\ -\bar{\theta}_1K_j^TH_2^TP & 0 & 0\\ -\bar{\theta}K_j^TH_2^TP & 0 & 0\end{bmatrix}$$

$$\Upsilon_5=\mathrm{diag}\{-\alpha I,-\beta I,-\Omega,-\bar{\theta}I\}$$

$$\Upsilon_6=\begin{bmatrix}0 & 0 & \sqrt{\bar{\theta}}FH_1 & 0\end{bmatrix}$$

$$\psi_{21}=\begin{bmatrix}\Upsilon_7 & \Upsilon_8 & \Upsilon_9 & \Upsilon_{10}\end{bmatrix}$$

$$\Upsilon_7=\begin{bmatrix}\phi_MR_1\bar{A}_{ij} & 0 & 0\\ \tau_MR_2\bar{A}_{ij} & 0 & 0\\ d_MR_3\bar{A}_{ij} & 0 & 0\end{bmatrix}$$

$$\Upsilon_8=\begin{bmatrix}\phi_M\bar{\theta}_1R_1\bar{B}_{ij} & 0 & 0 & 0\\ \tau_M\bar{\theta}_1R_2\bar{B}_{ij} & 0 & 0 & 0\\ d_M\bar{\theta}_1R_3\bar{B}_{ij} & 0 & 0 & 0\end{bmatrix}$$

$$\Upsilon_9=\begin{bmatrix}\phi_MR_1\bar{W}_i & \phi_MR_1\bar{V}_i\\ \tau_MR_2\bar{W}_i & \tau_MR_2\bar{V}_i\\ d_MR_3\bar{W}_i & d_MR_3\bar{V}_i\end{bmatrix}$$

$$\Upsilon_{10}=\begin{bmatrix}-\phi_M\bar{\theta}_1R_1H_2K_j & -\phi_M\bar{\theta}R_1H_2K_j & 0\\ -\tau_M\bar{\theta}_1R_2H_2K_j & -\tau_M\bar{\theta}R_2H_2K_j & 0\\ -d_M\bar{\theta}_1R_3H_2K_j & -d_M\bar{\theta}R_3H_2K_j & 0\end{bmatrix}$$

$$\psi_{31}=\begin{bmatrix}0_{3\times 3} & \Upsilon_{11} & 0_{3\times 2} & \Upsilon_{12}\end{bmatrix}$$

$$\Upsilon_{11}=\begin{bmatrix}\phi_M\rho R_1\bar{B}_{ij} & 0 & 0 & 0\\ \tau_M\rho R_2\bar{B}_{ij} & 0 & 0 & 0\\ d_M\rho R_3\bar{B}_{ij} & 0 & 0 & 0\end{bmatrix}$$

$$\Upsilon_{12}=\begin{bmatrix}-\phi_M\rho R_1H_2K_j & \phi_M\rho R_1H_2K_j & 0\\ -\tau_M\rho R_2H_2K_j & \tau_M\rho R_2H_2K_j & 0\\ -d_M\rho R_3H_2K_j & d_M\rho R_3H_2K_j & 0\end{bmatrix}$$

$$\psi_{22}=\psi_{33}=\mathrm{diag}\{-R_1,-R_2,-R_3\}$$

$$\rho=\sqrt{\bar{\theta}(1-\bar{\theta})},\bar{\theta}_1=1-\bar{\theta}$$

Proof The following Lyapunov–Krasovskii functional is constructed for system (9) [50]:

$$V(\bar{x}_t) = V_1(\bar{x}_t) + V_2(\bar{x}_t) + V_3(\bar{x}_t) \tag{18}$$

where

$$V_1(\bar{x}_t) = \bar{x}^T(t)P\bar{x}(t)$$

$$V_2(\bar{x}_t) = \int_{t-\phi_M}^{t} \bar{x}^T(s)Q_1\bar{x}(s)\mathrm{d}s + \int_{t-\tau_M}^{t} \bar{x}^T(s)Q_2\bar{x}(s)\mathrm{d}s + \int_{t-d_M}^{t} \bar{x}^T(s)Q_3\bar{x}(s)\mathrm{d}s$$

$$V_3(\bar{x}_t) = \int_{t-\phi_M}^{t}\int_{s}^{t} \dot{\bar{x}}^T(v)R_1\dot{\bar{x}}(v)\mathrm{d}v\mathrm{d}s + \int_{t-\tau_M}^{t}\int_{s}^{t} \dot{\bar{x}}^T(v)R_2\dot{\bar{x}}(v)\mathrm{d}v\mathrm{d}s + \int_{t-d_M}^{t}\int_{s}^{t} \dot{\bar{x}}^T(v)R_3\dot{\bar{x}}(v)\mathrm{d}v\mathrm{d}s$$

where $P > 0$, $Q_l > 0$, $R_l > 0$ $(l = 1, 2, 3)$.

By taking the derivative and mathematical expectation of equations above, then we can obtain

$$\begin{aligned}\mathbb{E}\{\dot{V}_1(\bar{x}_t)\} = &\sum_{i=1}^{r}\varphi_i\sum_{j=1}^{r}h_j 2\bar{x}^T(t)P\big[\bar{A}_{ij}\bar{x}(t) \\ &+ \bar{W}_i g(H_1\bar{x}(t)) + \bar{V}_i g(H_1\bar{x}(t-\phi(t))) \\ &- \theta(t)H_2K_j f(C_iH_1\bar{x}(t-d(t))) \\ &+ (1-\theta(t))\bar{B}_{ij}\bar{x}(t-\tau(t)) \\ &- (1-\theta(t))H_2K_j e_k(t)\big]\end{aligned} \tag{19}$$

$$\begin{aligned}\mathbb{E}\{\dot{V}_2(\bar{x}_t)\} = &\bar{x}^T(t)(Q_1+Q_2+Q_3)\bar{x}(t) \\ &- \bar{x}^T(t-\phi_M)Q_1\bar{x}(t-\phi_M) \\ &- \bar{x}^T(t-\tau_M)Q_2\bar{x}(t-\tau_M) \\ &- \bar{x}^T(t-d_M)Q_3\bar{x}(t-d_M)\end{aligned} \tag{20}$$

$$\begin{aligned}\mathbb{E}\{\dot{V}_3(\bar{x}_t)\} = &\sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j\mathbb{E}\{\dot{\bar{x}}(t)^T(\phi_M^2R_1 \\ &+ \tau_M^2R_2 + d_M^2R_3)\dot{\bar{x}}(t)\} \\ &- \int_{t-\phi_M}^{t}\dot{\bar{x}}^T(s)R_1\dot{\bar{x}}(s)\mathrm{d}s \\ &- \int_{t-\tau_M}^{t}\dot{\bar{x}}^T(s)R_2\dot{\bar{x}}(s)\mathrm{d}s \\ &- \int_{t-d_M}^{t}\dot{\bar{x}}^T(s)R_3\dot{\bar{x}}(s)\mathrm{d}s\end{aligned} \tag{21}$$

Notice that

$$\begin{aligned}&\mathbb{E}\big\{(\dot{\bar{x}}(t)^T(\phi_M^2R_1 + \tau_M^2R_2 + d_M^2R_3)\dot{\bar{x}}(t)\big\} \\ &\quad = \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j\Big(\mathcal{A}_{ij}^T\tilde{R}\mathcal{A}_{ij} + \rho^2\mathcal{B}_{ij}^T\tilde{R}\mathcal{B}_{ij}\Big)\end{aligned}$$

where

$$\begin{aligned}\mathcal{A}_{ij} = &\bar{A}_{ij}\bar{x}(t) + \bar{\theta}_1\bar{x}(t-\tau(t)) - \bar{\theta}_1H_2K_je_k(t) \\ &- \bar{\theta}H_2K_jf(C_iH_1\bar{x}(t-d(t))) + \bar{W}_ig(H_1\bar{x}(t)) \\ &+ \bar{V}_ig(H_1\bar{x}(t-\phi(t))) \\ \mathcal{B}_{ij} = &\bar{B}_{ij}\bar{x}(t-\tau(t)) - H_2K_je_k(t) \\ &+ H_2K_jf(C_iH_1\bar{x}(t-d(t))) \\ \tilde{R} = &\phi_M^2R_1 + \tau_M^2R_2 + d_M^2R_3\end{aligned}$$

By using Lemma 1, we have

$$-\phi_M\int_{t-\phi_M}^{t}\dot{\bar{x}}^T(s)R_1\dot{\bar{x}}(s)\mathrm{d}s \le \xi_1^T(t)\Phi_1(t)\xi_1(t) \tag{22}$$

$$-\tau_M\int_{t-\tau_M}^{t}\dot{\bar{x}}^T(s)R_2\dot{\bar{x}}(s)\mathrm{d}s \le \xi_2^T(t)\Phi_2(t)\xi_2(t) \tag{23}$$

$$-d_M\int_{t-d_M}^{t}\dot{\bar{x}}^T(s)R_3\dot{\bar{x}}(s)\mathrm{d}s \le \xi_3^T(t)\Phi_3(t)\xi_3(t) \tag{24}$$

where

$$\begin{aligned}\xi_1^T(t) &= \begin{bmatrix}\bar{x}^T(t) & \bar{x}^T(t-\phi(t)) & \bar{x}^T(t-\phi_M)\end{bmatrix} \\ \xi_2^T(t) &= \begin{bmatrix}\bar{x}^T(t) & \bar{x}^T(t-\tau(t)) & \bar{x}^T(t-\tau_M)\end{bmatrix} \\ \xi_3^T(t) &= \begin{bmatrix}\bar{x}^T(t) & \bar{x}^T(t-d(t)) & \bar{x}^T(t-d_M)\end{bmatrix} \\ \Phi_l(t) &= \begin{bmatrix}-R_l* & * & \\ R_l+S_l & -2R_l-S_l-S_l^T & * \\ -S_l & R_l+S_l & -R_l\end{bmatrix} \\ &(l = 1, 2, 3)\end{aligned}$$

Considering condition (5), the following inequality is obtained.

$$\begin{aligned}&\sigma\bar{x}^T(t-\tau(t))H_1^TC_i^T\Omega C_iH_1\bar{x}(t)(t-\tau(t)) \\ &\quad - e_k^T(t)\Omega e_k(t) \ge 0\end{aligned} \tag{25}$$

From Assumption 1, we have

$$\begin{bmatrix}\bar{x}(t) \\ g(H_1\bar{x}(t))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & * \\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t) \\ g(H_1\bar{x}(t))\end{bmatrix} \le 0 \tag{26}$$

where $\bar{U}_1 = \frac{H_1^T U_1^T U_2 H_1 + H_1^T U_2^T U_1 H_1}{2}$, $\bar{U}_2 = -\frac{H_1^T U_1^T + H_1^T U_2^T}{2}$. For the parameters $\alpha > 0$, $\beta > 0$, it is easy to get:

$$-\alpha\begin{bmatrix}\bar{x}(t)\\ g(H_1\bar{x}(t))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & *\\ \bar{U}_2 & I\end{bmatrix}\begin{bmatrix}\bar{x}(t)\\ g(H_1\bar{x}(t))\end{bmatrix} \geq 0 \tag{27}$$

$$\begin{aligned}&-\beta\begin{bmatrix}\bar{x}(t)\\ g(H_1\bar{x}(t-d(t)))\end{bmatrix}^T\begin{bmatrix}\bar{U}_1 & *\\ \bar{U}_2 & I\end{bmatrix}\\ &\times\begin{bmatrix}\bar{x}(t)\\ g(H_1\bar{x}(t-d(t)))\end{bmatrix} \geq 0\end{aligned} \tag{28}$$

Based on condition (11) in Assumption 2, one can get

$$\begin{aligned}&\bar{\theta}\bar{x}^T(t-d(t))H_1^T F^T F H_1 \bar{x}(t-d(t)) -\\ &\bar{\theta} f^T(C_i H_1 \bar{x}(t-d(t))) f(C_i H_1 \bar{x}(t-d(t))) \geq 0\end{aligned} \tag{29}$$

By combining (18)–(29) and using Schur complement, then we have

$$\begin{aligned}&\mathbb{E}\{\dot{V}(\bar{x}(t))\}\\ &\quad = \mathbb{E}\{\dot{V}_1(\bar{x}(t))\} + \mathbb{E}\{\dot{V}_2(\bar{x}(t))\} + \mathbb{E}\{\dot{V}_3(\bar{x}(t))\}\\ &\quad \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j\left[\psi_{11} + \psi_{21}^T\psi_{22}\psi_{21} + \psi_{31}^T\psi_{33}\psi_{31}\right]\xi(t)\\ &\quad = \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j \xi(t)^T \Phi_{ij}\xi(t)\end{aligned} \tag{30}$$

where $\xi(t) = [\bar{x}(t), \bar{x}(t-\phi(t)), \bar{x}(t-\phi_M), \bar{x}(t-\tau(t)), \bar{x}(t-\tau_M), \bar{x}(t-d(t)), \bar{x}(t-d_M), \bar{g}(H_1\bar{x}(t)), \bar{g}(H_1\bar{x}(t-\phi(t))), e_k(t), f(C_i H_1 \bar{x}(t-d(t))), I, I, I, I, I, I, I]^T$.

Similar to the analysis in [46], consider a slack matrix Z_i that

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i(\varphi_j - h_j)Z_i\\ &\quad = \sum_{i=1}^{r}\varphi_i\left(\sum_{j=1}^{r}\varphi_j - \sum_{j=1}^{r}h_j\right)Z_i\\ &\quad = 0\end{aligned} \tag{31}$$

where $Z_i = Z_i^T \in R^{n\times n} > 0$, $i = 1, 2, \ldots, r$, are arbitrary matrices. We can get that

$$\begin{aligned}&\mathbb{E}\{\dot{V}(x(t))\}\\ &\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j \xi^T(t)\Phi_{ij}\xi(t)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j \xi^T(t)\Phi_{ij}\xi(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i\\ &\quad \xi^T(t)\left(\varphi_j - h_j + \zeta_j\varphi_j - \zeta_j\varphi_j\right)Z_i\xi(t)\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i\varphi_j\xi^T(t)\left(\zeta_j\Phi_{ij} - \zeta_j Z_i + Z_i\right)\xi(t)\\ &\quad + \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i\left(h_j - \zeta_j\varphi_j\right)\xi^T(t)\left(\Phi_{ij} - Z_i\right)\xi(t)\\ &\leq \sum_{i=1}^{r}\varphi_i^2\xi^T(t)(\zeta_i\Phi_{ii} - \zeta_i Z_i + Z_i)\xi(t)\\ &\quad + \sum_{i=1}^{r}\varphi_i\left(h_j - \zeta_j\varphi_j\right)\xi^T(t)\left(\Phi_{ij} - Z_i\right)\xi(t)\\ &\quad + \sum_{i=1}^{r}\sum_{i<j}\xi^T(t)\left(\zeta_j\Phi_{ij} + \zeta_i\Phi_{ji}\right.\\ &\quad \left.-\zeta_j Z_i - \zeta_i Z_j + Z_i + Z_j\right)\xi(t)\end{aligned} \tag{32}$$

According to (13)–(15) and (32), it yields that

$$\mathbb{E}\{\dot{V}(x(t))\} \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j \xi^T(t)\Phi_{ij}\xi(t) < 0 \tag{33}$$

With $h_j - \zeta_j\varphi_j \geq 0$ for any $j \in L$, there is a scalar $\delta > 0$ which satisfies the inequality $\mathbb{E}\{\dot{V}(x(t))\} \leq -\delta \parallel \xi(t) \parallel^2$ for $\xi(t) \neq 0$; then, $\mathbb{E}\{\dot{V}(x(t))\} < 0$. This completes the proof. □

In Theorem 1, the sufficient conditions which can guarantee the stability of system (9) are obtained. Based on Theorem 1, an estimator design method is proposed and the parameters of the estimator are given in Theorem 2.

Theorem 2 For given positive parameters $\bar{\theta}$, $\phi_M > 0$, $\tau_M > 0$, $d_M > 0$, and ϵ_l $(l = 1, 2, 3)$, trigger parameter σ, matrix F, system (9) with stochastic cyber-attack is stable if there exist positive matrices $P_1 > 0$, $P_2 > 0$, $\hat{Q}_l > 0$, $\hat{R}_l > 0$, $\hat{S}_l > 0$ $(l = 1, 2, 3)$, Ω and Y_j with appropriate dimensions and parameters $\alpha > 0$, $\beta > 0$, then the following LMIs hold with $h_j - \zeta_j\varphi_j \geq 0$ for all $i, j \in L$.

$$\hat{\Phi}_{ij} - \hat{Z}_i < 0; \tag{34}$$

$$\zeta_i\hat{\Phi}_{ii} - \zeta_i\hat{Z}_i + \hat{Z}_i < 0; \tag{35}$$

$$\zeta_j\hat{\Phi}_{ij} + \zeta_i\hat{\Phi}_{ji} - \zeta_j\hat{Z}_i - \zeta_i\hat{Z}_j + \hat{Z}_i + \hat{Z}_j < 0 (i < j); \tag{36}$$

$$\begin{bmatrix} \hat{R}_l & * \\ \hat{S}_l & \hat{R}_l \end{bmatrix} \geq 0, (l = 1, 2, 3). \quad (37)$$

where

$$\hat{\Phi}_{ij} = \begin{bmatrix} \hat{\psi}_{11}* & * & \\ \hat{\psi}_{21} & \hat{\psi}_{22} & * \\ \hat{\psi}_{31} & 0 & \hat{\psi}_{33} \end{bmatrix} \quad (38)$$

in which

$$\hat{\psi}_{11} = \begin{bmatrix} \hat{\Upsilon}_1* & * & * & \\ \hat{\Upsilon}_2 & \hat{\Upsilon}_3* & * & \\ \hat{\Upsilon}_4 & 0 & \hat{\Upsilon}_5 & * \\ 0 & \hat{\Upsilon}_6 & 0 & -I \end{bmatrix}$$

$$\hat{\Upsilon}_1 = \begin{bmatrix} \hat{\Psi}_1* & * & \\ \hat{R}_1 + \hat{S}_1 & \hat{\Psi}_2 & * \\ -\hat{S}_1 & \hat{R}_1 + \hat{S}_1 & -\hat{Q}_1 - \hat{R}_1 \end{bmatrix}$$

$$\hat{\Psi}_1 = \Gamma_1 + \Gamma_1^T + \hat{Q}_1 + \hat{Q}_2 + \hat{Q}_3 - \hat{R}_1 - \hat{R}_2 - \hat{R}_3 + \Gamma_2$$

$$\hat{\Psi}_2 = -2\hat{R}_1 - \hat{S}_1 - \hat{S}_1^T + \Gamma_3$$

$$\hat{\Upsilon}_2 = \begin{bmatrix} \Gamma_4 + \hat{R}_2 + \hat{S}_2 & 0 & 0 \\ -\hat{S}_2 & 0 & 0 \\ \hat{R}_3 + \hat{S}_3 & 0 & 0 \\ -\hat{S}_3 & 0 & 0 \end{bmatrix}$$

$$\hat{\Upsilon}_3 = \begin{bmatrix} \hat{\Psi}_3* & * & * & \\ \hat{R}_2 + \hat{S}_2 & -\hat{Q}_2 - \hat{R}_2* & * & \\ 0 & 0 & \hat{\Psi}_4 & * \\ 0 & 0 & \hat{\Psi}_5 & -\hat{Q}_3 - \hat{R}_3 \end{bmatrix}$$

$$\hat{\Psi}_3 = -2\hat{R}_2 - \hat{S}_2 - \hat{S}_2^T + \Gamma_5$$

$$\hat{\Psi}_4 = -2\hat{R}_3 - \hat{S}_3 - \hat{S}_3^T, \hat{\Psi}_5 = \hat{R}_3 + \hat{S}_3$$

$$\hat{\Upsilon}_4 = \begin{bmatrix} \Gamma_6 & 0 & 0 \\ \Gamma_7 & \Gamma_8 & 0 \\ \Gamma_9 & 0 & 0 \\ \Gamma_{10} & 0 & 0 \end{bmatrix}$$

$$\hat{\Upsilon}_5 = \mathrm{diag}\{-\alpha I, -\beta I, -\hat{\Omega}, -\bar{\theta} I\}$$

$$\hat{\Upsilon}_6 = [0 \quad 0 \quad \Gamma_{11} \quad 0]$$

$$\hat{\psi}_{21} = \begin{bmatrix} \hat{\Upsilon}_7 & \hat{\Upsilon}_8 & \hat{\Upsilon}_9 & \hat{\Upsilon}_{10} \end{bmatrix}$$

$$\hat{\Upsilon}_7 = \begin{bmatrix} \phi_M \Gamma_1 & 0 & 0 \\ \tau_M \Gamma_1 & 0 & 0 \\ d_M \Gamma_1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Upsilon}_8 = \begin{bmatrix} \phi_M \Gamma_{12} & 0 & 0 & 0 \\ \tau_M \Gamma_{12} & 0 & 0 & 0 \\ d_M \Gamma_{12} & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\Upsilon}_9 = \begin{bmatrix} \phi_M \Gamma_{13} & \phi_M \Gamma_{14} \\ \tau_M \Gamma_{13} & \tau_M \Gamma_{14} \\ d_M \Gamma_{13} & d_M \Gamma_{14} \end{bmatrix}$$

$$\hat{\Upsilon}_{10} = \begin{bmatrix} -\phi_M \bar{\theta}_1 \Gamma_{15} & -\phi_M \bar{\theta} \Gamma_{15} & 0 \\ -\tau_M \bar{\theta}_1 \Gamma_{15} & -\tau_M \bar{\theta} \Gamma_{15} & 0 \\ -d_M \bar{\theta}_1 \Gamma_{15} & -d_M \bar{\theta} \Gamma_{15} & 0 \end{bmatrix}$$

$$\hat{\psi}_{31} = \begin{bmatrix} 0_{3\times 3} & \hat{\Upsilon}_{11} & 0_{3\times 2} & \hat{\Upsilon}_{12} \end{bmatrix}$$

$$\hat{\Upsilon}_{11} = \begin{bmatrix} \phi_M \rho \Gamma_{12} & 0 & 0 & 0 \\ \tau_M \rho \Gamma_{12} & 0 & 0 & 0 \\ d_M \rho \Gamma_{12} & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\Upsilon}_{12} = \begin{bmatrix} -\phi_M \rho \Gamma_{15} & \phi_M \rho \Gamma_{15} & 0 \\ -\tau_M \rho \Gamma_{15} & \tau_M \rho \Gamma_{15} & 0 \\ -d_M \rho \Gamma_{15} & d_M \rho \Gamma_{15} & 0 \\ c & & \end{bmatrix}$$

$$\hat{\psi}_{22} = \hat{\psi}_{33} = \mathrm{diag}\{-2\epsilon_1 P + \epsilon_1^2 \hat{R}_1, -2\epsilon_2 P + \epsilon_2^2 \hat{R}_2, - 2\epsilon_3 P + \epsilon_3^2 \hat{R}_3\}, P = \mathrm{diag}\{P_1, P_2\}$$

$$\rho = \sqrt{\bar{\theta}(1 - \bar{\theta})}, \bar{\theta}_1 = 1 - \bar{\theta}$$

$$\Gamma_1 = \begin{bmatrix} -P_1 A_i & 0 \\ P_2 A_j - P_2 A_i + Y_j C_j & -P_2 A_j - Y_j C_j \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} -\frac{\alpha}{2}(U_1^T U_2 + U_2^T U_1) & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Gamma_3 = \begin{bmatrix} -\frac{\beta}{2}(U_1^T U_2 + U_2^T U_1) & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Gamma_4 = \begin{bmatrix} 0 & -\bar{\theta}_1 C_j^T Y_j^T \\ 0 & 0 \end{bmatrix}, \Gamma_5 = \begin{bmatrix} \sigma C_i^T \Omega C_i & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Gamma_6 = \begin{bmatrix} W_i^T P_1 + \frac{\alpha}{2}(U_1^T + U_2^T) & W_i^T P_2 \end{bmatrix}$$

$$\Gamma_7 = \begin{bmatrix} V_i^T P_1 & V_i^T P_2 \end{bmatrix}, \Gamma_8 = \begin{bmatrix} \frac{\beta}{2}(U_1^T + U_2^T) & 0 \end{bmatrix}$$

$$\Gamma_9 = \begin{bmatrix} 0 & -\bar{\theta}_1 Y_j^T \end{bmatrix}, \Gamma_{10} = \begin{bmatrix} 0 & -\bar{\theta} Y_j^T \end{bmatrix}$$

$$\Gamma_{11} = \begin{bmatrix} \sqrt{\bar{\theta}} F & 0 \end{bmatrix}, \Gamma_{12} = \begin{bmatrix} 0 & 0 \\ -Y_j C_i & 0 \end{bmatrix}$$

$$\Gamma_{13} = \begin{bmatrix} P_1 W_i \\ P_2 W_i \end{bmatrix}, \Gamma_{14} = \begin{bmatrix} P_1 V_i \\ P_2 V_i \end{bmatrix}, \Gamma_{15} = \begin{bmatrix} 0 \\ -Y_j \end{bmatrix}$$

Moreover, the state estimator gains are achieved as follows.

$$K_j = P_2^{-1} Y_j \quad (39)$$

Proof Define

$$\Lambda = \mathrm{diag}\{\underbrace{I, I, \ldots, I}_{12}, \varpi_1, \varpi_1\}, Y_j = P_2 K_j$$

where

$$\varpi_1 = \mathrm{diag}\{PR_1^{-1}, PR_2^{-1}, PR_3^{-1}\}$$

Multiplying Λ and Λ^T on both sides of (17), respectively. we can get

$$\tilde{\Phi}_{ij} = \begin{bmatrix} \hat{\psi}_{11} & * & * \\ \tilde{\psi}_{21} & \tilde{\psi}_{22} & * \\ \tilde{\psi}_{31} & 0 & \tilde{\psi}_{33} \end{bmatrix} \tag{40}$$

where

$$\tilde{\psi}_{21} = [\,\tilde{\Upsilon}_7 \quad \tilde{\Upsilon}_8 \quad \tilde{\Upsilon}_9 \quad \tilde{\Upsilon}_{10}\,]$$

$$\tilde{\Upsilon}_7 = \begin{bmatrix} \phi_M P\bar{A}_{ij} & 0 & 0 \\ \tau_M P\bar{A}_{ij} & 0 & 0 \\ d_M P\bar{A}_{ij} & 0 & 0 \end{bmatrix}$$

$$\tilde{\Upsilon}_8 = \begin{bmatrix} \phi_M P\bar{B}_{ij} & 0 & 0 & 0 \\ \tau_M P\bar{B}_{ij} & 0 & 0 & 0 \\ d_M P\bar{B}_{ij} & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Upsilon}_9 = \begin{bmatrix} \phi_M P\bar{W}_i & \phi_M P\bar{V}_i \\ \tau_M P\bar{W}_i & \tau_M P\bar{V}_i \\ d_M P\bar{W}_i & d_M P\bar{V}_i \end{bmatrix}$$

$$\tilde{\Upsilon}_{10} = \begin{bmatrix} -\phi_M\bar{\theta}_1 PH_2K_j & -\phi_M\bar{\theta}PH_2K_j & 0 \\ -\tau_M\bar{\theta}_1 PH_2K_j & -\tau_M\bar{\theta}PH_2K_j & 0 \\ -d_M\bar{\theta}_1 PH_2K_j & -d_M\bar{\theta}PH_2K_j & 0 \end{bmatrix}$$

$$\tilde{\psi}_{31} = [\,0_{3\times3} \quad \tilde{\Upsilon}_{11} \quad 0_{3\times2} \quad \tilde{\Upsilon}_{12}\,]$$

$$\tilde{\Upsilon}_{11} = \begin{bmatrix} \phi_M\rho P\bar{B}_{ij} & 0 & 0 & 0 \\ \tau_M\rho P\bar{B}_{ij} & 0 & 0 & 0 \\ d_M\rho P\bar{B}_{ij} & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Upsilon}_{12} = \begin{bmatrix} -\phi_M\rho PH_2K_j & \phi_M\rho PH_2K_j & 0 \\ -\tau_M\rho PH_2K_j & \tau_M\rho PH_2K_j & 0 \\ -d_M\rho PH_2K_j & d_M\rho PH_2K_j & 0 \end{bmatrix}$$

$$\tilde{\psi}_{22} = \tilde{\psi}_{33} = \mathrm{diag}\{-PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\}$$

Owing to $(R_l - \epsilon_l^{-1}P)R_l^{-1}(R_l - \epsilon_l^{-1}P) \geq 0$ $(l = 1, 2, 3)$, we have

$$-PR_l^{-1}P \leq -2\epsilon_l P + \epsilon_l^2 R_l \tag{41}$$

That is

$$-PR_1^{-1}P \leq -2\epsilon_1 P + \epsilon_1^2 R_1 \tag{42}$$

$$-PR_2^{-1}P \leq -2\epsilon_2 P + \epsilon_2^2 R_2 \tag{43}$$

$$-PR_3^{-1}P \leq -2\epsilon_3 P + \epsilon_3^2 R_3 \tag{44}$$

Then, replace the terms ψ_{ij}^{22} and ψ_{ij}^{33} in (13)–(15) with $\hat{\psi}_{ij}^{22} = \mathrm{diag}\{-2\epsilon_1 P + \epsilon_1^2 R_1, -2\epsilon_2 P + \epsilon_2^2 R_2, -2\epsilon_3 P + \epsilon_3^2 R_3\}$ and $\hat{\psi}_{ij}^{33} = \mathrm{diag}\{-2\epsilon_1 P + \epsilon_1^2 R_1, -2\epsilon_2 P + \epsilon_2^2 R_2, -2\epsilon_3 P + \epsilon_3^2 R_3\}$, respectively, and then, we can obtain (34). Similar to Theorem 1, we have

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i(\varphi_j - h_j)\hat{Z}_i \\ &= \sum_{i=1}^{r}\varphi_i\left(\sum_{j=1}^{r}\varphi_j - \sum_{j=1}^{r}h_j\right)\hat{Z}_i \\ &= 0 \end{aligned} \tag{45}$$

With the help of the conditions (34)–(36) and (45), it yields that

$$\mathbb{E}\{\dot{V}(\bar{x}(t))\} \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\varphi_i h_j \xi^T(t)\hat{\Phi}_{ij}\xi(t) < 0 \tag{46}$$

Hence, the system is stable. Furthermore, according to $Y_j = P_2 K_j$, we can obtain that the state estimator gains are $K_j = P_2^{-1} Y_j$. This completes the proof. $\square$

4 Simulation Examples

In this section, a numerical example is presented to illustrate the effectiveness of the obtained results.

Consider neural network (1) with the following system matrices:

$$A_1 = \mathrm{diag}\{1.06, 1.06\}, \quad A_2 = \mathrm{diag}\{1.6, 2.3\}$$

$$W_1 = \begin{bmatrix} 0.3 & -0.42 \\ -0.42 & 0.3 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.2 & -0.32 \\ -0.32 & 0.2 \end{bmatrix}$$

$$V_1 = \begin{bmatrix} 0.3 & 0.3 \\ 0.3 & 0.3 \end{bmatrix}, \quad V_2 = \begin{bmatrix} 0.4 & 0.4 \\ 0.4 & 0.4 \end{bmatrix}$$

$$C_1 = \begin{bmatrix} -3 & -0.15 \\ -0.75 & -2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} -0.95 & -0.85 \\ -1.75 & -0.55 \end{bmatrix}$$

$$U_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.2 \end{bmatrix}, \quad U_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0 & 0.95 \end{bmatrix}$$

$$g(x(t)) = \begin{bmatrix} 0.5x_1(t) - \tanh(0.2x_1(t) + 0.2x_1(t)) \\ 0.95x_2(t) - \tanh(0.75x_1(t)) \end{bmatrix}$$

The cyber-attack function is assumed to be

$$f(y(t)) = \begin{bmatrix} -\tanh(0.02y(t)) \\ -\tanh(0.5y(t)) \end{bmatrix}$$

which satisfies inequality (25) with $F = \mathrm{diag}\{0.5, 0.02\}$. The corresponding initial parameters of system (2) are given by $x(0) = [1.7 \quad -2.6]^T, \hat{x}(0) = [0.9 \quad -1.9]^T$.

In the following, we present two cases to prove the usefulness of the designed method.

Case 1 There is no cyber-attack occurring, which means $\bar{\theta}(t) = 0$. Set event-triggered factor $\sigma = 0.4$, $\epsilon_k = 1$ $(l = 1, 2, 3)$, sample period $h = 0.1$s, the upper bound of time delays $\phi_M = 0.01$, $\tau_M = 0.01$, $d_M = 0.02$. By using LMI toolbox in MATLAB, we obtain

$$Y_1 = \begin{bmatrix} -0.0565 & -0.0720 \\ -0.0407 & -0.0188 \end{bmatrix}, Y_2 = \begin{bmatrix} 0.1040 & -0.0610 \\ -0.1182 & 0.5993 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 3.3758 & -0.0273 \\ -0.0273 & 2.9639 \end{bmatrix}, \Omega = \begin{bmatrix} 4.3839 & -0.0957 \\ -0.0957 & 4.5214 \end{bmatrix}$$

Then, according to equality (39) in Theorem 2, state estimator gains are obtained as follows.

$$K_1 = \begin{bmatrix} -0.0168 & -0.0214 \\ -0.0139 & -0.0065 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0305 & -0.0164 \\ -0.0396 & 0.2020 \end{bmatrix}$$

The state responses of the neural network and its estimation are depicted in Fig. 2. The estimator error $e(t)$ is shown in Fig. 3. It can be obtained from the above graphs that the designed state estimator performs well.

Case 2 The influence of cyber-attacks is considered, which means $\bar{\theta} = 0.5$. Set event-triggered factor $\sigma = 0.4$, $\epsilon_k = 1$ $(l = 1, 2, 3)$, sample period $h = 0.1$s, the upper bound of time delays $\phi_M = 0.01$, $\tau_M = 0.01$, $d_M = 0.02$. With the help of the MATLAB LMI Toolbox, we can get

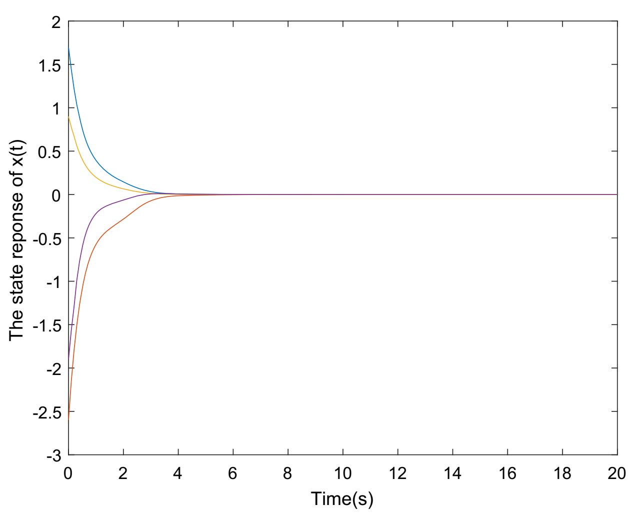

Fig. 2 Response of $x(t)$ and its estimations $\hat{x}(t)$ in case 1

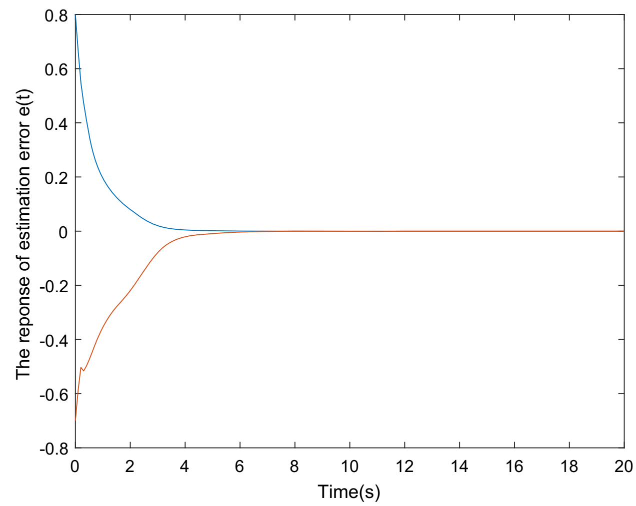

Fig. 3 The estimator error $e(t)$ in case 1

$$Y_1 = \begin{bmatrix} -0.0321 & -0.0467 \\ -0.0274 & -0.0195 \end{bmatrix}, Y_2 = \begin{bmatrix} 0.1210 & -0.1495 \\ -0.1471 & 0.7606 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 6.1439 & -0.0611 \\ -0.0611 & 5.3578 \end{bmatrix}, \Omega = \begin{bmatrix} 1.3477 & -0.7540 \\ -0.7540 & 2.5211 \end{bmatrix}$$

Then, we can obtain the gains of the state estimator according to (39).

$$K_1 = \begin{bmatrix} -0.0053 & -0.0076 \\ -0.0052 & -0.0037 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0194 & -0.0229 \\ -0.0272 & 0.1417 \end{bmatrix}$$

Simulation results in case 2 are presented. The occurrence probability of stochastic cyber-attack is shown in Fig. 4. The state responses of $x(t)$ and its estimations $\hat{x}(t)$ are shown in Fig. 5. Figure 6 denotes the release instants and intervals of event-triggered scheme, which reveals that the amount of transmitted data is reduced obviously.

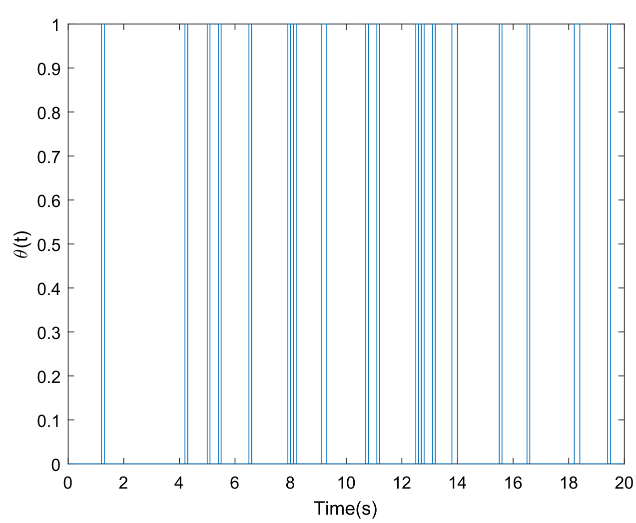

Fig. 4 The graph of $\theta(t)$ in case 2

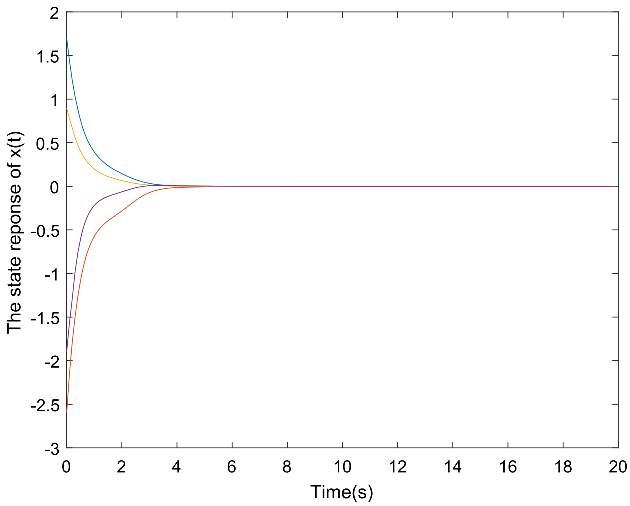

Fig. 5 Response of $x(t)$ and its estimations $\hat{x}(t)$ in case 2

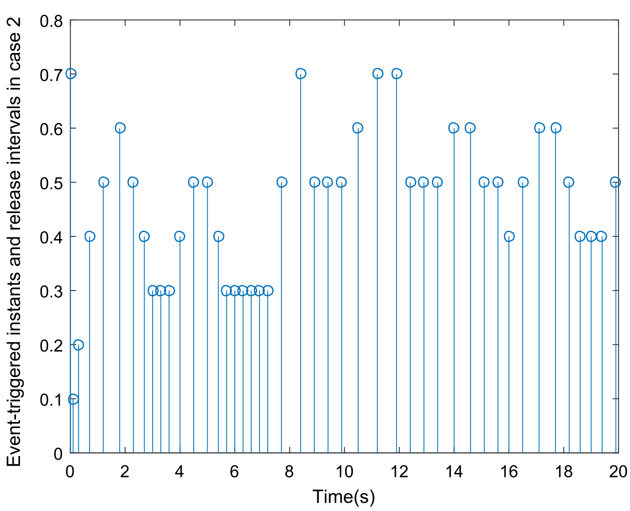

Fig. 6 Release instants and release intervals in case 2

5 Conclusion

In this paper, the state estimation for T–S fuzzy neural networks with event-triggered scheme and stochastic cyber-attacks is investigated. First of all, an event-triggered scheme is adopted to alleviate the load of network transmission. A T–S fuzzy mathematical model for estimating event-triggered neural networks with stochastic cyber-attacks is constructed. In addition, under the assistance of Lyapunov stability theory and LMIs technologies, sufficient conditions which guarantee the stability of the system are acquired and the gains of the estimator are obtained in terms of LMIs. Finally, the simulation results have demonstrated the feasibility of the designed method. In the future, in order to improve network bandwidth utilization more effectively, the control synthesis for T–S fuzzy neural networks under different kinds of triggering schemes will be studied. Moreover, we would like to consider the impacts of attacks with different kinds for T–S fuzzy neural networks, such as deception attacks and DoS attacks.

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