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Event-based control for networked T-S fuzzy cascade control systems with quantization and cyber attacks

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Abstract

This paper investigates the event-based control for networked T-S fuzzy cascade control systems with quantization and cyber attacks. In order to solve the problem of limited communication resources, an event-triggered scheme and a quantization mechanism are adopted, which can effectively reduce the burden of communication and save the network resources of the system. By considering the influence of cyber attacks, a newly quantized T-S fuzzy model for networked cascade control systems (NCCSs) under the event-triggered scheme is established. By using the Lyapunov stability theory, sufficient conditions guaranteeing the asymptotical stability of networked T-S fuzzy cascade control systems are obtained. In addition, the controller gains are derived by solving a set of linear matrix inequalities. Finally, a numerical example is presented to verify the validity of the proposed method.

1. Introduction

T-S fuzzy model is a modeling method based on fuzzy logic for nonlinear system, in which a series of local linear systems smoothly linked together by fuzzy membership functions is

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used to describe nonlinear systems [1]. As a mathematical tool, T-S fuzzy model has been proved to be an effective method to approximate any smooth nonlinear systems [2,3]. Later it becomes an effective way to model and analyze the complex dynamic systems [4,5]. Therefore, much attention has been paid to the T-S fuzzy systems and a lot of remarkable results have been studied about the control synthesis of T-S fuzzy systems [6–12]. For instance, the problem of secure control for T-S fuzzy systems with limited communication resources is investigated in [6]. By considering the infinite-distributed delay and actuator faults, the problem of reliable H_∞ control for discrete-time T-S fuzzy systems is researched in [7]. The stabilization of a networked T-S fuzzy control system is studied under the consideration of various possible faults/failures of its multisensors and multiactuators in [8]. For a class of networked T-S fuzzy systems, a discrete event-triggered communication scheme is firstly proposed in [9]. For a class of T-S fuzzy interconnected systems, the fault estimation problem is studied in [10]. Moreover, the authors of [11] firstly establish network-based T-S fuzzy dynamic positioning system models for the unmanned marine vehicle. In the existing research, some scholars assume that the designed filter or controller share the same membership functions. But at present, with the development of the communication technology, the network is gradually applied to various fields. It is more accurate to use the imperfect premise matching method for networked control systems due to the network-induced delay [13]. For example, a T-S fuzzy model is investigated for active suspension systems with uncertainties in [14]. In [15], relaxed stability and stabilization conditions for T-S fuzzy systems with asynchronous grades of membership are investigated. Under imperfect premise matching, the stability analysis and performance design for T-S fuzzy system is studied in [16]. The problem of an event-based non-parallel distribution compensation control for T-S fuzzy systems is addressed by considering the limited communication resources and the imperfect premise matching in [17]. Moreover, with the consideration of packet dropouts and actuator failure, the fuzzy control problem is studied for nonlinear networked systems in [18].

Recently, networked control systems (NCSs) have become a research hotspot of process control. With the introduction of the network, the NCSs have many benefits such as low cost, convenient maintenance and large flexibility. So many researchers have conducted in-depth research on modeling, stability and controller design of NCSs. However, in practical industrial process control, cascade control system is one of the most widely used control systems except single loop feedback control system. Recently more and more attention has been paid to NCCSs which have two control loops. The primary loop is responsible for the stability of the system, and the secondary loop can eliminate quadric disturbance effectively and rapidly. As a special networked control system, NCCSs are the same as NCSs with the characteristics of low cost and easy maintenance, simple installation and large flexibility. For this reason, cascade control has been widely applied in many industrial areas and has received wide attention [19–22]. For instance, the problem of passivity-based H_∞ controller design for a class of NCCSs with random packet dropouts is investigated in [20]. The single loop feedback control shows that the non-zero disturbance of the system has some defects on the output of the system, thus a new singular networked cascade control system model is proposed by considering the network-induced delay and packet dropout in [21]. In [22], by taking actuator saturations and cyber-attacks into consideration, the H_∞ control problem is studied for a kind of NCCSs. To save the limited resources, a hybrid-triggered scheme is adopted for the NCCSs in [22]. In addition, a new event-triggered scheme is studied for NCCS in [19]. And with the consideration of stochastic nonlinearities and actuator failures, the authors in [19] investigate the control problem for NCCS. Although there exist many researches on cascade control system, there are few results that take resource limitation and cyber attacks into consideration. At the same time, considering that cascade control system cannot be completely represented by a linear system in practical application, we decide to study the control problem for T-S fuzzy cascade control systems.

In NCSs, the bandwidth of the network is limited, how to save the network bandwidth has been widely discussed [23,24]. Over the past decades, the most commonly used transmitted scheme is time-triggered scheme by which the sampled data is transmitted to the communication network at a fixed time interval. If the variation of the measurement is small, the periodic transmission of sensor measurements may result in the redundancy of the data transmitted in the communication network [25]. For the purpose of saving the communication network bandwidth, researchers begin to study the event-triggered scheme and apply it to many fields such as multi-agent systems [26–30]. Different from the time-triggered scheme, by using the event-triggered scheme, the sampled signals are transmitted only when they satisfy the predetermined conditions. Researchers first propose a dynamic event triggering mechanism [31]. However, it requires a special hardware device to monitor the state of the system in real time, which brings great influence on the cost and complexity of the system. Based on dynamic event triggering mechanism, the self-trigger scheme which replaces the hardware devices with software programs is proposed in [32]. In order to solve the problem of Zeno in the dynamic continuous event-triggered system, the authors of [33] propose a discrete event-triggered scheme, which only supervises the difference between the states sampled in discrete instants and does not require any additional hardware devices. Based on the event-triggered scheme proposed in [33], researchers have carried out extensive research on the issue of event-triggered control synthesis. For instance, by adopting the event-triggered scheme proposed in [33], the event-based H_∞ control problem for discrete-time nonlinear NCSs with unreliable communication links is studied in [34]. On the basis of the event-triggered scheme proposed in [33], the authors of [35] pay attention to the event-based control problem for the fuzzy Markovian jump systems. Inspired by Yue et al. [33], the authors of Ref. [36] propose an improved static event-triggered scheme for a kind of nonlinear systems and study the probability-constrained filtering problem. Moreover, the event-triggered scheme proposed in [33] is applied to a class of network nonlinear interconnected systems in [37]. Motivated by work Yue et al. [33], considering the merits and drawbacks of event-triggered scheme and time-triggered scheme, the authors in [49] propose a hybrid-triggered mechanism for networked control systems. The applications of this mechanism in T-S fuzzy systems [6] and networked cascade control systems [22] are also studied.

In addition, quantization mechanism is another method to save network bandwidth, which can effectively compress the transmission data and reduce the transmission burden of the communication network. Therefore, the research on quantizers has attracted the attention of many researchers [38–40]. In the existing publications, there are mainly two types of quantizers. One is the linear quantizers. For instance, by adopting a linear quantizer, a new controller design methodology which relies on the possibility of changing the sensitivity of the quantizer is proposed for linear time-invariant control systems in [38]. The other is logarithmic quantizers. For example, for networked control systems with a logarithmic quantizer, an event-based control design problem is investigated in [39]. In addition, the authors of Ref. [40] investigate the problem of event-triggered control for the singular systems with logarithmic quantizer.

Although the introduction of the network brings many benefits to the control system, it brings some problems to the system such as packet loss [18], time delay [41], nonlinear disturbance [19] and other issues [42]. Meanwhile, because of the openness of the communication network, the networked control system is more vulnerable to various kinds of cyber attacks. As described in [43], the cyber-attacks are roughly divided into three types according to the physical implementation: denial of service (DoS) attacks, replay attacks and deception attacks. DoS attacks reduce the system performance by preventing the signal or information from reaching the destination. The replay attacks repeat transmitted data maliciously to paralyze the control system. Different from the DoS attacks and replay attacks, deception attacks achieve the goal of manipulating the system by injecting deception information into the system measurements or the control actions. In view of the above situation, the problem of system control synthesis under cyber attacks has been paid attention by many researchers [44–48]. With the consideration of both random DoS attacks and deception attacks, a event-based control problem for a discrete-time system is investigated in [44]. In [45], considering the impact of stochastic cyber attacks, the problem of H_∞ filtering for networked systems under hybrid-triggered communication scheme is studied. In view of the influence of cyber attacks, a decentralized event-triggered H_∞ control for neural networks with cyber attacks is addressed in [46]. By considering cyber-attacks and lossy sensors, an observer-based event-triggering consensus control for discrete-time multiagent systems is investigated in [47]. In [48], with the consideration of DoS attacks, resilient event-triggered consensus control is studied for nonlinear multi-agent systems.

Motivated by the mentioned results above, this paper firstly focuses on quantized controller design for networked T-S fuzzy cascade control systems with event-triggered scheme and cyber attacks. The main contributions of this paper are as follows.

1) A new T-S fuzzy model for NCCSs is firstly established by considering the event-triggered scheme, quantization and cyber attacks.
2) Based on the established system model, the sufficient conditions stabilizing the system are obtained by using Lyapunov stability theory. In addition, the gains of controllers are obtained.

   This paper is organized as follows. The problem formulation and the necessary lemmas are present in Section 2. Based on the Lyapunov stability theory, the main results are obtained in Section 3. In Section 4, an illustrative example is given to show the performance of the controller. Concluding remarks are provided in Section 5.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $\mathbb{R}^{n\times n}$ denotes the set of $n\times n$ real matrices. Superscripts "T" and "-1" stand for matrix transposition and matrix inverse, respectively. $sym\{R\}$ donates the sum of the matrix R and its transposition R^T. $R>0$ for $R\in\mathbb{R}^{n\times n}$ represents that R is real symmetric positive definite matrix. I and 0 represent identity matrix and zero matrix with compatible dimension. * stands for a term that is induced by symmetry. The set $\{1, 2, \ldots, r\}$ is represented as S.

2. System description

This paper is concerned with the problem of event-based H_∞ controller design for networked T-S fuzzy cascade control systems with quantization and cyber attacks. As is shown in Fig. 1, one can see the structure of the system which includes two loops, a primary loop and a secondary loop. A network is assumed to interposed in the primary loop between the quantizer *Quantizer* and the primary controller *Controller*1.

Consider the networked T-S fuzzy cascade control system with r plant rules.

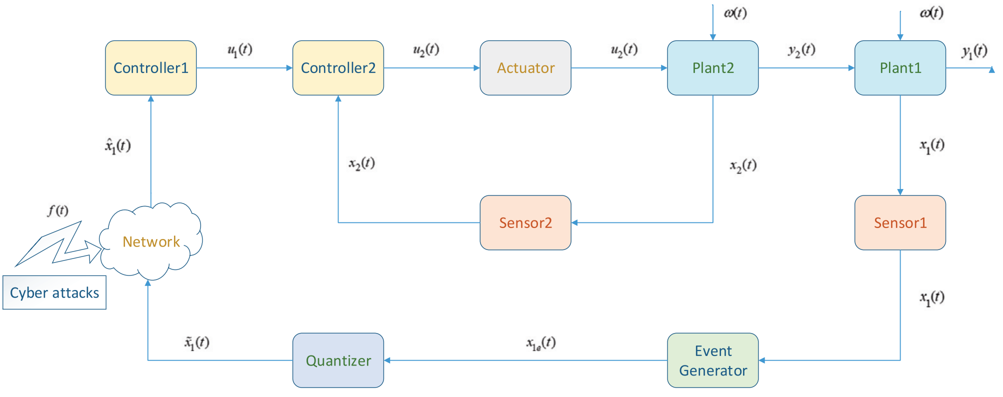

Fig. 1. The structure of the networked cascade control systems under cyber attacks.

*Plant*1 rule i: IF $k_1(x_1(t))$ is F_1^i and $\cdots$ and $k_n(x_1(t))$ is F_n^i, THEN

$$Plant1:\begin{cases}\dot{x}_1(t)=A_{1i}x_1(t)+B_{1i}y_2(t)\\ y_1(t)=C_{1i}x_1(t)+D_{1i}\omega(t)\end{cases} \tag{1}$$

where $x_1(t)\in\mathbb{R}^n$ denotes the state vector and $y_1(t)\in\mathbb{R}^n$ denotes the measurement output of *Plant*1. $y_2(t)$ is the measurement output of *Plant*2. $F_v^i(i\in S, v=1,2,\ldots,n)$ denote the fuzzy sets, $k_v(x_1(t))$ are fuzzy premise variables. For the sake of simplicity, we use $k_v(x_1)$ to represent $k_v(x_1(t))$ and $k_v(x_1)=[k_1(x_1),k_2(x_1),\ldots,k_n(x_1)]$. A_{1i}, B_{1i}, C_{1i}, and D_{1i} are the known constant matrices of appropriate dimensions.

By using center-average defuzzifier, product interference and singleton fuzzifier, the fuzzy system (1) can be inferred as follows:

$$Plant1:\begin{cases}\dot{x}_1(t)=\sum\limits_{i=1}^{r}g_i(k(x_1))[A_{1i}x_1(t)+B_{1i}y_2(t)]\\ y_1(t)=\sum\limits_{i=1}^{r}g_i(k(x_1))[C_{1i}x_1(t)+D_{1i}\omega(t)]\end{cases} \tag{2}$$

where $g_i(k(x_1))$ are the normalized membership functions satisfying that $g_i(k(x_1))=\frac{\prod_{v=1}^{n}F_v^i(k_v(x_1))}{\sum_{i=1}^{r}\prod_{v=1}^{n}F_v^i(k_v(x_1))}\geqslant 0$, $\sum_{i=1}^{r}g_i(k(x_1))=1$ with $F_v^i(k_v(x_1))$ representing the grade membership of premise variable $k_v(x_1)$.

With the introduction of the network, the premise variable $g_i(k(x_1))$ is not matching, we consider the rule of jth controller model.

*Controller*1 rule j: IF $k_1(\hat{x}_1(t))$ is W_1^j and $\cdots$ and $k_n(\hat{x}_1(t))$ is W_n^j, THEN

$$u_1(t)=K_{1j}\hat{x}_1(t) \tag{3}$$

where K_{1j} are the controller gains, $\hat{x}_1(t)$ is the final input of the *Controller*1. $W_v^j(j\in S, v=1,2,\ldots,n)$ denote the fuzzy sets, $k_v(\hat{x}_1(t))$ are fuzzy premise variables. $k_v(\hat{x}_1)$ is used to represent $k_v(\hat{x}_1(t))$ and $k_v(\hat{x}_1)=[k_1(\hat{x}_1),k_2(\hat{x}_1),\ldots,k_n(\hat{x}_1)]$.

The output of *Controller*1 can be rewritten as follows:

$$u_1(t)=\sum_{j=1}^{r}h_j(k(\hat{x}_1))K_{1j}\hat{x}_1(t) \tag{4}$$

where $h_j(k(\hat{x}_1))$ are the normalized membership functions satisfying that $h_j(k(\hat{x}_1)) = \frac{\prod_{v=1}^{n} W_v^j k(\hat{x}_1)}{\sum_{j=1}^{r}\prod_{v=1}^{n} W_v^j(k_v(\hat{x}_1))} \geqslant 0$, $\sum_{l=1}^{r} h_j(k(\hat{x}_1)) = 1$ with $W_v^j(k_v(\hat{x}_1))$ representing the grade membership of premise variable $k_v(\hat{x}_1)$.

*Plant*2 rule l: IF $k_1(x_2(t))$ is M_1^l and $\cdots$ and $k_n(x_2(t))$ is M_n^l, THEN

$$Plant2: \begin{cases} \dot{x}_2(t) = A_{2l}x_2(t) + A_{3l}x_2(t-\theta(t)) + B_{2l}u_2(t) + B_{3l}\omega(t) \\ y_2(t) = C_{2l}x_2(t) + D_{2l}\omega(t) \end{cases} \tag{5}$$

where $x_2(t)$ denotes the state vector of *Plant*2. $\theta(t) \in (0, \theta_M]$ is a state-delay variable of *Plant*2. $M_v^l(l \in S, v = 1, 2, \ldots, n)$ denote the fuzzy sets, $k_v(x_2(t))$ are fuzzy premise variables. $k_v(x_2)$ is used to represent $k_v(x_2(t))$ and $k_v(x_2) = [k_1(x_2), k_2(x_2), \ldots, k_n(x_2)]$. A_{2l}, A_{3l}, B_{2l}, B_{3l}, C_{2l}, and D_{2l} are given constant matrices of appropriate dimensions.

Then by defuzzing the system (5), we can obtain

$$Plant2: \begin{cases} \dot{x}_2(t) = \sum_{l=1}^{r} v_l(k(x_2))[A_{2l}x_2(t) + A_{3l}x_2(t-\theta(t)) + B_{2l}u_2(t) + B_{3l}\omega(t)] \\ y_2(t) = \sum_{l=1}^{r} v_l(k(x_2))[C_{2l}x_2(t) + D_{2l}\omega(t)] \end{cases} \tag{6}$$

where $v_l(k(x_2))$ are the normalized membership functions satisfying that $v_l(k(x_2)) = \frac{\prod_{v=1}^{n} M_v^l(k_v(x_2))}{\sum_{l=1}^{r}\prod_{v=1}^{n} M_v^l(k_v(x_2))} \geqslant 0$, $\sum_{l=1}^{r} v_l(k(x_2)) = 1$ with $M_v^l(k_v(x_2))$ representing the grade membership of premise variable $k_v(x_2)$.

*Controller*2 rule m: IF $k_1(x_2(t))$ is M_1^m and $\cdots$ and $k_n(x_2(t))$ is M_n^m, THEN

$$u_2(t) = u_1(t) + K_{2m}x_2(t) \tag{7}$$

where K_{2m} are the controller gains, $M_v^m(m \in S, v = 1, 2, \cdots, n)$ denote the fuzzy sets, $k_v(x_2(t))$ are fuzzy premise variables. $k_v(x_2)$ is used to represent $k_v(x_2(t))$ and $k_v(x_2(t)) = [k_1(x_2(t)), k_2(x_2(t)), \cdots, k_n(x_2(t))]$.

The *Controller*2 rule for defuzzified output of Eq. (7) can be rewritten as follows:

$$u_2(t) = \sum_{m=1}^{r} v_m(k(x_2))[u_1(t) + K_{2m}x_2(t)] \tag{8}$$

where $v_m(k(x_2(t)))$ are the normalized membership functions satisfying that $v_m(k(x_2)) = \frac{\prod_{v=1}^{n} M_v^m k(x_2)}{\sum_{m=1}^{r}\prod_{v=1}^{n} M_v^m(k_v(x_2))} \geqslant 0$, $\sum_{m=1}^{r} v_m(k(x_2)) = 1$.

Remark 1. As is shown in Fig. 1, the networked T-S fuzzy cascade control systems have one more loop than the general networked control system, and the extra loop can eliminate the disturbance effectively and rapidly. Due to the existence of the secondary loop, the interference entering the secondary loop can be controlled in advance, thus the influence of the interference on the primary loop can be reduced. To the best of our knowledge, the event-based H_∞ control problem for the networked T-S fuzzy cascade control systems with quantization and cyber attacks is firstly investigated in this paper.

When the signal is sampled by *Sensor*1 in Fig. 1, the selection of communication transmission mechanism will have a great influence on the signal transmission. In order to reduce unnecessary data transmission, an event-triggered scheme [33] is introduced into the system. By adopting the event-triggered scheme, the system will transmit the periodically sampled measurements only when the measurements violate the triggering condition as

$$e_k^T(t)\Omega e_k(t) \leqslant \beta x_1^T(t_kh + jh)\Omega x_1(t_kh + jh) \tag{9}$$

where the state error $e_k(t) = x_1(t_k h) - x_1(t_k h + jh)$, $\Omega > 0$, $\beta \in [0, 1)$, $t_k h$ stands for the latest transmitting instant, h is the sampling period, and $j = 1, 2, \ldots$. Define $t_k^j h = t_k h + jh$ stand for the current sampling instant, then the sequence of transmitting instant can be expressed as follows:

$$t_{k+1}h = t_k h + \inf_{j \geqslant 1}\left\{jh | e_k^T(t)\Omega e_k(t) > \beta x_1^T\left(t_k^j h\right)\Omega x_1\left(t_k^j h\right)\right\} \tag{10}$$

Similar to the analysis in [33], one can divide the interval $[t_k h + \mu_{t_k}, t_{k+1}h + \mu_{t_{k+1}})$ into several subintervals, which can be expressed as $[t_k h + \mu_{t_k}, t_{k+1}h + \mu_{t_{k+1}}) = \bigcup_{j=0}^{\theta}[t_k^j h + \mu_{t_k}, t_k^j h + h + \mu_{t_{k+1}})$. μ_{t_k} are the corresponding network-induced delay, and $\theta = t_{k+1} - t_k - 1$. Define $d(t) = t - t_k^j h$, and it is easy to get the range of $d(t)$ as $0 \leqslant \mu_{t_k} \leqslant d(t) \leqslant h + d_{t_{k+j+1}} \triangleq d_M$.

Then the sampled signals can be described as follows:

$$x_{1e}(t) = x_1(t - d(t)) + e_k(t) \tag{11}$$

For the sake of further reducing the network burden, as shown in Fig. 1, a quantizer is added between the event triggering mechanism and the network. A logarithmic quantizer [49] is adopted in this paper and the set of quantization levels M of the logarithmic quantizer has the following form:

$$M = \{\pm m_b : m_b = \rho_b m_0, b = \pm 1, \pm 2, \ldots\} \cup \{\pm m_0\} \cup \{0\}, m_0 > 0, 0 < \rho < 1 \tag{12}$$

ν is quantization resolution of $q(\cdot)$. More precisely, the quantization law of $q(\cdot)$ is defined as follows:

$$q(x_b) = \begin{cases} m_b, & x_b > 0, \frac{m_b}{1+\delta} < x_b < \frac{m_b}{1-\delta} \\ 0, & x_b = 0 \\ -q(-x_b), & x_b < 0 \end{cases} \tag{13}$$

where $\delta = \frac{1-\nu}{1+\nu}$.

Because $q_b(\cdot)(i = 1, 2, \ldots m)$ is symmetrical, so we can easily get that $q_b(-x_b) = -q_b(x_b)$. The logarithmic quantizer $q_b(\cdot)$ can be described as

$$q_b(x_b) = (I + \Delta_{q_b}(x_b))x_b \tag{14}$$

where $|\Delta_{q_b}(x_b)| \leq \delta_b$. For simplicity, we use Δ_{q_b} to represent $\Delta_{q_b}(x_b)$.

Define that

$$q(x) = diag\{q_1(x_1), q_2(x_2), \ldots, q_m(x_m)\} \tag{15}$$

then we can obtain

$$q(x) = (I + \Delta_q)x \tag{16}$$

where $\Delta_q = diag\{\Delta_{q_1}, \Delta_{q_2}, \ldots, \Delta_{q_m}\}$.

By combining Eqs. (11) and (16), the transmitted signals can be expressed as follows.

$$\tilde{x}_1(t) = (I + \Delta_q)x_{1e}(t) \tag{17}$$

As shown in Fig. 1, the signals are transmitted from the event generator to the controller through a network. Due to the openness of the network, the networked T-S fuzzy cascade control system is more vulnerable to the cyber attacks. Cyber attacks not only affect the transmission of measurement signals in the system, but also affect the stability of the system.

In this paper, we assume that the system suffer from the cyber attacks which can be expressed by a nonlinear functions $f(x_1(t))$, and the cyber attacks $f(x_1(t))$ are assumed to satisfy Assumption 1. On the basis of [49,51], the signal $\hat{x}_1(t)$ shown in Fig. 1 can be described as follows.

$$\hat{x}_1(t) = \alpha(t) f(x_1(t - \tau(t))) + (1 - \alpha(t))\tilde{x}_1(t) \tag{18}$$

where $\alpha(t) \in [0, 1]$ is a Bernoulli random variable to govern the cyber attacks, and $\mathbb{E}\{\alpha(t)\} = \bar{\alpha}$, $\mathbb{E}\{(\alpha(t) - \bar{\alpha})^2\} = \rho^2$. $\tau(t) \in [0, \tau_M]$ denotes the corresponding time-delay of cyber attacks.

Remark 2. The cyber attacks discussed in this paper belong to deception attacks, which are assumed that they completely replace the original transmission data with the attack signals [47]. To be more specific, when the cyber attacks occur, the attack signal $f(x_1(t))$ will completely replace the original transmission signal $\tilde{x}_1(t)$ in Fig. 1.

Remark 3. The Bernoulli random variable $\alpha(t)$ in Eq. (18) is utilized to govern the cyber attacks. When $\alpha(t) = 1$, Eq. (18) can be rewritten as $\hat{x}_1(t) = f(x_1(t - \tau(t)))$, this means that the cyber attacks occur and the original transmitted signal is replaced by the cyber attack signals as $f(x_1(t - \tau(t)))$. Otherwise, when $\alpha(t) = 0$, Eq. (18) can be rewritten as $\hat{x}_1(t) = \tilde{x}_1(t)$, it means that the systems do not suffer the cyber attacks and the transmitted data is normal.

By combining Eqs. (2), (4), (6), (8) and (18), we can get the discussed system as follows.

$$\begin{cases} \dot{x}_1(t) = \sum_{i=1}^{r}\sum_{l=1}^{r} g_i v_l [A_{1i}x_1(t) + B_{1i}C_{2l}x_2(t) + B_{1i}D_{2l}\omega(t)] \\ \dot{x}_2(t) = \sum_{j=1}^{r}\sum_{v=1}^{r}\sum_{m=1}^{r} h_j v_l v_m \big[A_{2l}x_2(t) + A_{3l}x_2(t - \theta(t)) + B_{2l}(K_{1j}(\alpha(t) f(x_1(t - \tau(t))) \\ \quad +(1 - \alpha(t))(I + \Delta_q)(x_1(t - d(t)) + e_k(t))) + K_{2m}x_2(t)) + B_{3l}\omega(t)\big] \\ y_1(t) = \sum_{i=1}^{r} g_i [C_{1i}x_1(t) + D_{1i}\omega(t)] \end{cases} \tag{19}$$

In order to facilitate obtaining the desired results, two lemmas and an assumption are introduced.

Lemma 1 [22]**.** *Suppose* $0 < d(t) \le d_M$, $x(t) \in \mathbb{R}^n$ *and there exist matrices* $R \in \mathbb{R}^{n\times n}$ *and* $M \in \mathbb{R}^{n\times n}$ *such that* $\begin{bmatrix} R & * \\ M & R \end{bmatrix} \ge 0$. *Then the following inequality holds:*

$$-\eta_M \int_{t-\eta_M}^{t} \dot{x}^T(s) R \dot{x}(s) ds \le \begin{bmatrix} x(t) \\ x(t - \eta(t)) \\ x(t - \eta_M) \end{bmatrix}^T \begin{bmatrix} -R & * & * \\ R + M & -2R - M - M^T & * \\ -M & R + M & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t - \eta(t)) \\ x(t - \eta_M) \end{bmatrix} \tag{20}$$

Lemma 2 [50]**.** *For matrices* $A = A^T$, B *and* C *with appropriate dimensions,* $A + B\Delta(t)C + C^T\Delta(t)B^T < 0$ *for any* $\Delta(t)$ *satisfying* $\Delta(t)^T\Delta(t) \le I$, *if and only if there exists a parameter* $m_1 > 0$ *such that* $A + m_1^{-1}CC^T + m_1 B^T B < 0$.

Assumption 1 [49,51]**.** Deception attacks $f(x)$ are assumed to satisfy the following condition.

$$\| f(x) \|_2 \le \| Fx \|_2 \tag{21}$$

where F is a constant matrix standing for the upper bound of the nonlinearity.

3. Main results

Theorem 1. *For given positive scalars $\bar{\alpha}$, ρ, time delay d_M, τ_M, θ_M, trigger parameter β, disturbance parameter γ, membership parameter ϱ_m, matrices F, K_{1j}, K_{2m}, the argument system (19) is asymptotically stable if there exist matrices $\Omega > 0$, $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, M, N, and V with appropriate dimensions satisfying the following inequalites with $v_m - \varrho_m g_m \geqslant 0$ $(i, j, l, m \in S)$*

$$\Upsilon_{ijlm} - L_i < 0 \tag{22}$$

$$\varrho_i \Upsilon_{ijli} - \varrho_i L_i + L_i < 0 \tag{23}$$

$$\varrho_m \Upsilon_{ijlm} + \varrho_i \Upsilon_{mjli} - \bar{\varrho}_m L_i - \bar{\varrho}_i L_m < 0, \ (i < m) \tag{24}$$

where

$$\Upsilon_{ijlm} = \begin{bmatrix} \Pi_{11} & * & * & * \\ \Pi_{21} & -I & * & * \\ \Pi_{31} & 0 & \Pi_{33} & * \\ \Pi_{41} & 0 & 0 & \Pi_{44} \end{bmatrix}, \Pi_{11} = \begin{bmatrix} \Phi_{11} & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ \Phi_{31} & 0 & \Phi_{33} & * \\ \Phi_{41} & 0 & \Phi_{43} & \Phi_{44} \end{bmatrix},$$

$$\Phi_{11} = \begin{bmatrix} \Gamma_1 & * & * \\ R_1 + V & \Gamma_2 & * \\ -V & R_1 + V & -Q_1 - R_1 \end{bmatrix}, \Gamma_1 = P_1 A_{1i} + A_{1i}^T P_1 + Q_1 + Q_2 - R_1 - R_2,$$

$$\Gamma_2 = -2R_1 - V - V^T + \beta\Omega,$$

$$\Phi_{21} = \begin{bmatrix} R_2 + M & 0 & 0 \\ -M & 0 & 0 \end{bmatrix}, \Phi_{22} = \begin{bmatrix} -2R_2 - M - M^T & * \\ R_2 + M & -Q_2 - R_2 \end{bmatrix},$$

$$\Phi_{31} = \begin{bmatrix} C_{2l}^T B_{1i}^T P_1 & (1 - \bar{\alpha})(I + \Delta_q) P_2 B_{2l} K_{1j} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{33} = \begin{bmatrix} \Gamma_3 & * & * \\ \Gamma_4 & -2R_3 - N - N^T & * \\ -N & R_3 + N & -Q_3 - R_3 \end{bmatrix},$$

$$\Gamma_3 = P_2 A_{2l} + A_{2l}^T P_2 + Q_3 - R_3 + K_{2m}^T B_{2l}^T P_2 + P_2 B_{2l} K_{2m}, \Gamma_4 = A_{3l}^T P_2 + R_3 + N,$$

$$\Phi_{41} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ D_{2l}^T B_{1i}^T P_1 & 0 & 0 \end{bmatrix}, \Phi_{43} = \begin{bmatrix} (1 - \bar{\alpha})(I + \Delta_q) K_{1j}^T B_{2l}^T P_2 & 0 & 0 \\ \bar{\alpha} K_{1j}^T B_{2l}^T P_2 & 0 & 0 \\ B_{3l}^T P_2 & 0 & 0 \end{bmatrix},$$

$$\Phi_{44} = diag\{-\Omega, -\bar{\alpha}I, -\gamma^2 I\}, \Pi_{21} = \begin{bmatrix} 0 & 0 & 0 & \sqrt{\bar{\alpha}}F & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_{31} = \begin{bmatrix} \Gamma_5 & 0 & \Gamma_6 & \Gamma_7 \end{bmatrix}, \Gamma_5 = \begin{bmatrix} d_M P_1 A_{1i} & 0 & 0 \\ \tau_M P_1 A_{1i} & 0 & 0 \end{bmatrix},$$

$$\Gamma_6 = \begin{bmatrix} d_M P_1 B_{1i} C_{2l} & 0 & 0 \\ \tau_M P_1 B_{1i} C_{2l} & 0 & 0 \end{bmatrix}, \Gamma_7 = \begin{bmatrix} 0 & 0 & d_M P_1 B_{1i} D_{2l} \\ 0 & 0 & \tau_M P_1 B_{1i} D_{2l} \end{bmatrix},$$

$$\Pi_{33} = diag\{-P_1R_1^{-1}P_1, -P_1R_2^{-1}P_1\}, \Pi_{41} = \begin{bmatrix}\Gamma_8 & 0 & \Gamma_9 & \Gamma_{10}\end{bmatrix},$$

$$\Gamma_8 = \begin{bmatrix} 0 & (1-\bar{\alpha})(I+\Delta_q)\theta_M P_2 B_{2l} K_{1j} & 0 \\ 0 & \rho(I+\Delta_q)\theta_M P_2 B_{2l} K_{1j} & 0 \\ C_{1i} & 0 & 0 \end{bmatrix},$$

$$\Gamma_9 = \begin{bmatrix} \theta_M P_2 A_{2l} + \theta_M P_2 B_{2l} K_{2m} & \theta_M P_2 A_{3l} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma_{10} = \begin{bmatrix} (1-\bar{\alpha})(I+\Delta_q)\Gamma_{11} & \bar{\alpha}\Gamma_{11} & \theta_M P_2 B_{3l} \\ \rho(I+\Delta_q)\Gamma_{11} & -\rho\Gamma_{11} & 0 \\ 0 & 0 & D_{1i} \end{bmatrix}, \Gamma_{11} = \theta_M P_2 B_{2l} K_{1j}, \bar{\varrho}_m = \varrho_m - 1,$$

$$\Pi_{44} = diag\left\{-P_2R_3^{-1}P_2, -P_2R_3^{-1}P_2, -I\right\}.$$

Proof. Choose the Lyapunov–Krasovskii functional candidate for system as

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{25}$$

where

$$V_1(t) = x_1^T(t)P_1x_1(t) + x_2^T(t)P_2x_2(t)$$

$$V_2(t) = \int_{t-d_M}^{t} x_1^T(s)Q_1x_1(s)ds + \int_{t-\tau_M}^{t} x_1^T(s)Q_2x_1(s)ds + \int_{t-\theta_M}^{t} x_2^T(s)Q_3x_2(s)ds$$

$$V_3(t) = d_M\int_{t-d_M}^{t}\int_{s}^{t} \dot{x}_1^T(v)R_1\dot{x}_1(v)dvds + \tau_M\int_{t-\tau_M}^{t}\int_{s}^{t} \dot{x}_1^T(v)R_2\dot{x}_1(v)dvds + \theta_M\int_{t-\theta_M}^{t}\int_{s}^{t} \dot{x}_2^T(v)R_3\dot{x}_2(v)dvds$$

and $P = diag\{P_1, P_2\} > 0,\ Q_i > 0,\ R_i > 0(i = 1, 2, 3)$.

By taking the derivative and mathematical expectation of equations above, then we can obtain

$$\mathbb{E}\{\dot{V}_1(t)\} = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_ih_jv_lv_m 2\left[x_1^T(t)P_1\dot{x}_1(t) + x_2^T(t)P_2\mathbb{E}\{\dot{x}_2(t)\}\right] \tag{26}$$

$$\begin{aligned}\mathbb{E}\{\dot{V}_2(t)\} = {} & x_1^T(t)(Q_1+Q_2)x_1(t) + x_2^T(t)Q_3x_2(t) - x_1^T(t-d_M)Q_1x_1(t-d_M) \\ & - x_1^T(t-\tau_M)Q_2x_1(t-\tau_M) - x_2^T(t-\theta_M)Q_3x_2(t-\theta_M)\end{aligned} \tag{27}$$

$$\begin{aligned}\mathbb{E}\left\{\dot{V}_3(t)\right\} = {} & \sum_{i=1}^{r}\sum_{j=1}^{r} g_ih_j\Big\{\mathbb{E}\{\dot{x}_1^T(t)(d_M^2R_1+\tau_M^2R_2)\dot{x}_1(t)\} + \mathbb{E}\{\dot{x}_2^T(t)\theta_M^2R_3\dot{x}_2(t)\} \\ & -d_M\int_{t-d_M}^{t}\dot{x}_1^T(s)R_1\dot{x}_1(s)ds - \tau_M\int_{t-\tau_M}^{t}\dot{x}_1^T(s)R_2\dot{x}_1(s)ds - \theta_M\int_{t-\theta_M}^{t}\dot{x}_2^T(s)R_3\dot{x}_2(s)ds\Big\}\end{aligned} \tag{28}$$

Notice that

$$\mathbb{E}\{\dot{x}_2^T(t)\theta_M^2 R_3\dot{x}_2(t)\} = \sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} h_j v_l v_m \{\mathcal{A}^T\theta_M^2 R_3\mathcal{A} + \rho^2\mathcal{B}^T\theta_M^2 R_3\mathcal{B}\} \tag{29}$$

where $\mathcal{A} = A_{2l}x_2(t) + A_{3l}x_2(t-\theta(t)) + B_{2l}K_{2m}x_2(t) + B_{3l}\omega(t) + \bar{\alpha}B_{2l}K_{1j}f(x_1(t-\tau(t))) + (1-\bar{\alpha})(I+\Delta_q)B_{2l}K_{1j}[x_1(t-d(t)) + e_k(t)]$, $\mathcal{B} = (I+\Delta_q)B_{2l}K_{1j}[x_1(t-d(t)) + e_k(t)]$.

Applying Lemma 1, it can be obtained that

$$-d_M\int_{t-d_M}^{t}\dot{x}_1^T(s)R_1\dot{x}_1(s)ds \leqslant \xi_d^T(t)\psi_V\xi_d(t) \tag{30}$$

$$-\tau_M\int_{t-\tau_M}^{t}\dot{x}_1^T(s)R_2\dot{x}_1(s)ds \leqslant \xi_\tau^T(t)\psi_M\xi_\tau(t) \tag{31}$$

$$-\theta_M\int_{t-\theta_M}^{t}\dot{x}_2^T(s)R_3\dot{x}_2(s)ds \leqslant \xi_\theta^T(t)\psi_N\xi_\theta(t) \tag{32}$$

where

$$\xi_d(t) = \begin{bmatrix} x_1^T(t) & x_1^T(t-d(t)) & x_1^T(t-d_M)\end{bmatrix}^T, \xi_\tau(t) = \begin{bmatrix} x_1^T(t) & x_1^T(t-\tau(t)) & x_1^T(t-\tau_M)\end{bmatrix}^T,$$
$$\xi_\theta(t) = \begin{bmatrix} x_2^T(t) & x_2^T(t-\theta(t)) & x_2^T(t-\theta_M)\end{bmatrix}^T,$$
$$\psi_V = \begin{bmatrix} -R_1 & * & * \\ R_1+V & -2R_1-V-V^T & * \\ -V & R_1+V & -R_1\end{bmatrix}, \psi_M = \begin{bmatrix} -R_2 & * & * \\ R_2+M & -2R_2-M-M^T & * \\ -M & R_2+M & -R_2\end{bmatrix},$$
$$\psi_N = \begin{bmatrix} -R_3 & * & * \\ R_3+N & -2R_3-N-N^T & * \\ -N & R_3+N & -R_3\end{bmatrix}.$$

By taking the event-triggered limiting conditions (9), it can be seen that

$$\beta x_1^T(t-d(t))\Omega x_1(t-d(t)) - e_k^T(t)\Omega e_k(t) > 0 \tag{33}$$

Considering Eq. (21) in the Assumption1 of the cyber attacks, one can obtain the inequality as follows.

$$\bar{\alpha}x_1^T(t-\tau(t))F^TFx_1(t-\tau(t)) - \bar{\alpha}f^T(x_1(t-\tau(t)))f(x_1(t-\tau(t))) \geqslant 0 \tag{34}$$

By combining Eqs. (25)–(34) and utilizing Schur complements, it yields

$$\begin{aligned}
&\mathbb{E}\{\dot{V}(t)\} + y_1^T(t)y_1(t) - \gamma^2\omega^T(t)\omega(t)\\
&\quad = \mathbb{E}\{\dot{V}_1(t)\} + \mathbb{E}\{\dot{V}_2(t)\} + \mathbb{E}\{\dot{V}_3(t)\} + x_1^T(t-d(t))\Omega x_1(t-d(t)) - e_k^T(t)\Omega e_k(t)\\
&\qquad + \bar{\alpha}x_1^T(t-\tau(t))F^TFx_1(t-\tau(t)) - \bar{\alpha}f^T(x_1(t-\tau(t)))f(x_1(t-\tau(t)))\\
&\qquad + y_1^T(t)y_1(t) - \gamma^2\omega^T(t)\omega(t)\\
&\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l v_m \psi^T(t)\Upsilon_{ijlm}\psi(t)
\end{aligned} \tag{35}$$

where $\psi(t) = [x_1^T(t), x_1^T(t-d(t)), x_1^T(t-d_M), x_1^T(t-\tau(t)), x_1^T(t-\tau_M), x_2^T(t), x_2^T(t-\theta(t)), x_2^T(t-\theta_M), e_k^T(t), f^T(x_1(t-\tau(t))), \omega^T(t), I, I, I, I, I, I]^T$.

For the convenience of analysis, similar to the proof in [17], a slack matrix L_i is introduced

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l (g_m - v_m) L_i = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} g_i h_j v_l \left(\sum_{m=1}^{r} g_m - \sum_{m=1}^{r} v_m \right) L_i = 0 \tag{36}$$

where $L_i = L_i^T \in R^{n\times n} > 0$ $(i \in S)$ are arbitrary matrices.

Then we can obtain the following equation from Eqs. (35) and (36):

$$\begin{aligned}\Phi \le & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l v_m \psi^T(t)\Upsilon_{ijlm}\psi(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l (g_m - v_m \\ & + \varrho_m g_m - \varrho_m g_m)\psi^T(t) L_i \psi(t) \\ = & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l g_m \psi^T(t)(\varrho_i \Upsilon_{ijlm} - \bar{\varrho}_i L_i)\psi(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l (v_m \\ & - \varrho_m g_m)\psi^T(t)(\Upsilon_{ijlm} - L_i)\psi(t)\end{aligned} \tag{37}$$

For further analysis, Eq. (37) can be rewritten as

$$\begin{aligned}\Phi \le & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} g_i^2 h_j v_l \psi^T(t)(\varrho_i \Upsilon_{ijli} - \varrho_i L_i + L_i)\psi(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l (v_m \\ & - \varrho_m g_m)\psi^T(t)(\Upsilon_{ijlm} - L_i)\psi(t) \\ & + \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{i<m} g_i h_j v_l v_m \psi^T(t)(\varrho_m \Upsilon_{ijlm} + \varrho_i \Upsilon_{mjli} - \bar{\varrho}_m L_i - \bar{\varrho}_i L_m)\psi(t)\end{aligned} \tag{38}$$

Then with the conditions (22)–(24), one can get

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l v_m \psi^T(t)\Upsilon_{ijlm}\psi(t) < 0 \tag{39}$$

for all $i, j, l, m \in S$.

With $v_m - \varrho_m g_m \geqslant 0$ for all $m \in S$, there exists a scalar ϵ such that $\mathbb{E}\{\dot{V}(t)\} \le -\epsilon ||\psi(t)||^2$ for $\psi(t) \neq 0$, then $\mathbb{E}\{\dot{V}(t)\} < 0$, and the system is asymptotically stable. That completes the proof. □

In Theorem 1, the sufficient conditions which can guarantee the stability of the networked T-S fuzzy cascade control system (19) have been obtained. However, it can not deal with linear matrix inequality techniques because of the existence of nonlinear terms such like $(1-\bar{\alpha})(I+\Delta_q)K_{1j}^T B_{2l}^T P_2$. Therefore, the following section deals with the nonlinear terms and obtains the controller gains.

Theorem 2. *For given positive scalars $\bar{\alpha}$, ρ, d_M, τ_M, θ_M, e_1, e_2, e_3, e_4, disturbance parameter γ, trigger parameter β, membership parameter ϱ_m, quantized parameter ν, matrices F, the system (19) is asymptotically stable if there exist matrices $X_1 > 0$, $X_2 > 0$, Y_{1j}, Y_{2m}, $\hat{Q}_1 > 0$, $\hat{Q}_2 > 0$, $\hat{Q}_3 > 0$, $\hat{R}_1 > 0$, $\hat{R}_2 > 0$, $\hat{R}_3 > 0$, $\hat{\Omega}$, $\hat{M}$, $\hat{N}$, and $\hat{V}$ with appropriate dimensions, such that the following linear matrix inequalities hold with $v_m - \varrho_m g_m \geqslant 0$ for all i, j, l,*

$m \in S$

$$\bar{\Upsilon}_{ijlm} - \bar{L}_i < 0 \tag{40}$$

$$\varrho_i \bar{\Upsilon}_{ijli} - \varrho_i \bar{L}_i + \bar{L}_i < 0 \tag{41}$$

$$\varrho_m \bar{\Upsilon}_{ijlm} + \varrho_i \bar{\Upsilon}_{mjli} - \bar{\varrho}_m \bar{L}_i - \bar{\varrho}_i \bar{L}_m < 0, \, (i < m) \tag{42}$$

where

$$\bar{\Upsilon}_{ijlm} = \begin{bmatrix} \bar{\Pi}_{11} & * & * & * & * \\ \bar{\Pi}_{21} & -I & * & * & * \\ \bar{\Pi}_{31} & 0 & \bar{\Pi}_{33} & * & * \\ \bar{\Pi}_{41} & 0 & 0 & \bar{\Pi}_{44} & * \\ \bar{\Pi}_{51} & 0 & 0 & \bar{\Pi}_{54} & \bar{\Pi}_{55} \end{bmatrix}, \bar{\Pi}_{11} = \begin{bmatrix} \bar{\Phi}_{11} & * & * & * \\ \bar{\Phi}_{21} & \bar{\Phi}_{22} & * & * \\ \bar{\Phi}_{31} & 0 & \bar{\Phi}_{33} & * \\ \bar{\Phi}_{41} & 0 & \bar{\Phi}_{43} & \bar{\Phi}_{44} \end{bmatrix},$$

$$\bar{\Phi}_{11} = \begin{bmatrix} \bar{\Gamma}_1 & * & * \\ \hat{R}_1 + \hat{V} & \bar{\Gamma}_2 & * \\ -\hat{V} & \hat{R}_1 + \hat{V} & -\hat{Q}_1 - \hat{R}_1 \end{bmatrix}, \bar{\Gamma}_1 = A_{1i}X_1 + X_1 A_{1i}^T + \hat{Q}_1 + \hat{Q}_2 - \hat{R}_1 - \hat{R}_2,$$

$$\bar{\Gamma}_2 = -2\tilde{R}_1 - \tilde{V} - \tilde{V}^T + \beta\tilde{\Omega}, \bar{\Phi}_{21} = \begin{bmatrix} \hat{R}_2 + \hat{M} & 0 & 0 \\ -\hat{M} & 0 & 0 \end{bmatrix},$$

$$\bar{\Phi}_{22} = \begin{bmatrix} -2\hat{R}_2 - \hat{M} - \hat{M}^T & * \\ \hat{R}_2 + \hat{M} & -\hat{Q}_2 - \hat{R}_2 \end{bmatrix}, \bar{\Phi}_{31} = \begin{bmatrix} X_2 C_{2l}^T B_{1i}^T & (1-\bar{\alpha})B_{2l}Y_{1j} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Phi}_{33} = \begin{bmatrix} \bar{\Gamma}_3 & * & * \\ \bar{\Gamma}_4 & -2\hat{R}_3 - \hat{N} - \hat{N}^T & * \\ -\hat{N} & \hat{R}_3 + \hat{N} & -\hat{Q}_3 - \hat{R}_3 \end{bmatrix}, \bar{\Gamma}_3 = A_{2l}X_2 + X_2 A_{2l}^T + \hat{Q}_3 - \hat{R}_3 + B_{2l}Y_{2m} + Y_{2m}^T B_{2l}^T,$$

$$\bar{\Phi}_{41} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ D_{2l}^T B_{1i}^T & 0 & 0 \end{bmatrix}, \bar{\Phi}_{43} = \begin{bmatrix} (1-\bar{\alpha})Y_{1j}^T B_{2l}^T & 0 & 0 \\ \bar{\alpha} Y_{1j}^T B_{2l}^T & 0 & 0 \\ B_{3l}^T & 0 & 0 \end{bmatrix}, \bar{\Gamma}_4 = X_2 A_{3l}^T + \hat{R}_3 + \hat{N},$$

$$\bar{\Phi}_{44} = diag\left\{-\hat{\Omega}, -2\bar{\alpha}e_4 X_1 + \bar{\alpha}e_4^2 I, -\gamma^2 I\right\},$$

$$\bar{\Pi}_{21} = \begin{bmatrix} 0 & 0 & 0 & \sqrt{\bar{\alpha}} F X_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{31} = \begin{bmatrix} \bar{\Gamma}_5 & 0 & \bar{\Gamma}_6 & \bar{\Gamma}_7 \end{bmatrix}, \bar{\Gamma}_5 = \begin{bmatrix} d_M A_{1i} X_1 & 0 & 0 \\ \tau_M A_{1i} X_1 & 0 & 0 \end{bmatrix}, \bar{\Gamma}_6 = \begin{bmatrix} d_M B_{1i} C_{2l} X_2 & 0 & 0 \\ \tau_M B_{1i} C_{2l} X_2 & 0 & 0 \end{bmatrix},$$

$$\bar{\Gamma}_7 = \begin{bmatrix} 0 & 0 & d_M B_{1i} D_{2l} \\ 0 & 0 & \tau_M B_{1i} D_{2l} \end{bmatrix},$$

$$\bar{\Pi}_{33} = diag\left\{-2e_1 X_1 + e_1^2 R_1, -2e_2 X_1 + e_2^2 R_2\right\}, \bar{\Pi}_{41} = \begin{bmatrix} \bar{\Gamma}_8 & 0 & \bar{\Gamma}_9 & \bar{\Gamma}_{10} \end{bmatrix},$$

$$\bar{\Gamma}_8 = \begin{bmatrix} 0 & (1-\bar{\alpha})\theta_M B_{2l} Y_{1j} & 0 \\ 0 & \rho\theta_M B_{2l} Y_{1j} & 0 \\ C_{1i} X_1 & 0 & 0 \end{bmatrix}, \bar{\Gamma}_9 = \begin{bmatrix} \theta_M A_{2l} X_2 + \theta_M B_{2l} Y_{2m} & \theta_M A_{3l} X_2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Gamma}_{10} = \begin{bmatrix} (1-\bar{\alpha})\theta_M B_{2l} Y_{1j} & \bar{\alpha}\theta_M B_{2l} Y_{1j} & \theta_M B_{3l} \\ \rho\theta_M B_{2l} Y_{1j} & \rho\theta_M B_{2l} Y_{1j} & 0 \\ 0 & 0 & D_{1i} \end{bmatrix}, \bar{\Pi}_{44} = diag\left\{-2e_3 X_2 + e_3^2 R_3, -2e_3 X_2 + e_3^2 R_3, -I\right\},$$

$$\bar{\Pi}_{51} = \begin{bmatrix} \bar{\Gamma}_{11} & 0 & \bar{\Gamma}_{12} & \bar{\Gamma}_{13} \end{bmatrix}, \bar{\Gamma}_{11} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & Y_{1j} & 0 \end{bmatrix} \bar{\Gamma}_{12} = \begin{bmatrix} \sqrt{m_1}\delta(1-\bar{\alpha})B_{2l}^T & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Gamma}_{13} = \begin{bmatrix} 0 & 0 & 0 \\ Y_{1j} & 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{54} = \begin{bmatrix} \sqrt{m_1}\delta(1-\bar{\alpha})\theta_M B_{2l}^T & \sqrt{m_1}\delta\theta_M B_{2l}^T \\ 0 & 0 \end{bmatrix}, \bar{\Pi}_{55} = diag\{-I, -m_1 I\}, \bar{\varrho}_m = \varrho_m - 1.$$

Moreover, if the conditions are feasible, the gains of primary controller and secondary controller are given as following.

$$K_{1j} = Y_{1j}X_1^{-1}, K_{2m} = Y_{2m}X_2^{-1}(j, m \in S) \tag{43}$$

Proof. Due to the existence of quantization, one can obtain the following equation.

$$\Upsilon_{ijlm} = \Phi_{ijlm} + sym\{H_B^T \Delta q H_C\}. \tag{44}$$

where

$$\Phi_{ijlm} = \begin{bmatrix} \tilde{\Pi}_{11} & * & * & * \\ \Pi_{21} & -I & * & * \\ \Pi_{31} & 0 & \Pi_{33} & * \\ \tilde{\Pi}_{41} & 0 & 0 & \Pi_{44} \end{bmatrix}, \tilde{\Pi}_{11} = \begin{bmatrix} \Phi_{11} & * & * & * \\ \Phi_{21} & \Phi_{22} & * & * \\ \tilde{\Phi}_{31} & 0 & \Phi_{33} & * \\ \Phi_{41} & 0 & \tilde{\Phi}_{43} & \Phi_{44} \end{bmatrix}$$

$$\tilde{\Phi}_{31} = \begin{bmatrix} C_{2l}^T B_{1i}^T P_1 & \alpha_1 P_2 B_{2l} K_{1j} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tilde{\Phi}_{43} = \begin{bmatrix} \alpha_1 K_{1j}^T B_{2l}^T P_2 & 0 & 0 \\ \bar{\alpha} K_{1j}^T B_{2l}^T P_2 & 0 & 0 \\ B_{3l}^T P_2 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Pi}_{41} = \begin{bmatrix} \tilde{\Gamma}_8 & 0 & \Gamma_9 & \tilde{\Gamma}_{10} \end{bmatrix}, \tilde{\Gamma}_8 = \begin{bmatrix} 0 & \alpha_1\theta_M P_2 B_{2l} K_{1j} & 0 \\ 0 & \rho\theta_M P_2 B_{2l} K_{1j} & 0 \\ C_{1i} X_1 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Gamma}_{10} = \begin{bmatrix} (1-\bar{\alpha})\Gamma_{11} & \bar{\alpha}\Gamma_{11} & \theta_M P_2 B_{3l} \\ \rho\Gamma_{11} & \rho\Gamma_{11} & 0 \\ 0 & 0 & D_{1i} \end{bmatrix}, \alpha_1 = 1 - \bar{\alpha}$$

$$H_B = \begin{bmatrix} 0_{1\times 5} & \alpha_1\Gamma_{14} & 0_{1\times 8} & \alpha_1\theta_M\Gamma_{14} & \rho\theta_M\Gamma_{14} \end{bmatrix}$$

$$H_C = \begin{bmatrix} 0 & K_{1j} & 0_{1\times 6} & K_{1j} & 0_{1\times 7} \end{bmatrix}, \Gamma_{14} = B_{2l}^T P_2$$

By applying Lemma 1, one can get

$$\Phi_{ijlm} + m_1 H_B^T \Delta q^2 H_B + m_1^{-1} H_C^T H_C < 0. \tag{45}$$

Notice that $\Delta q^2 < \delta^2$, then we can obtain

$$\Phi_{ijlm} + m_1\delta^2 H_B^T H_B + m_1^{-1} H_C^T H_C < 0. \tag{46}$$

Due to $(R_k - e_k^{-1}P_1)R_k^{-1}(R_k - e_k^{-1}P_1) \geqslant 0 (k = 1, 2)$, we can easily obtain that

$$-P_1 R_k^{-1} P_1 \leq -2e_k P_1 + e_k^2 R_k. \tag{47}$$

Similarly, we have

$$\begin{cases} -P_2 R_3^{-1} P_2 \leq -2e_3 P_2 + e_3^2 R_3 \\ -P_1 I^{-1} P_1 \leq -2e_4 P_1 + e_4^2 I \end{cases} \tag{48}$$

Then by substituting the terms Π_{33} and Π_{44} in Eqs. (22)–(24) with $\tilde{\Pi}_{33} = diag\{-2e_1P_1 + e_1^2R_1, -2e_2P_1 + e_2^2R_2\}$ and $\tilde{\Pi}_{44} = diag\{-2e_3P_2 + e_3^2R_3, -2e_3P_2 + e_3^2R_3, -I\}$, one can obtain

$$\tilde{\Upsilon}_{ijlm} - \bar{L}_i < 0; \tag{49}$$

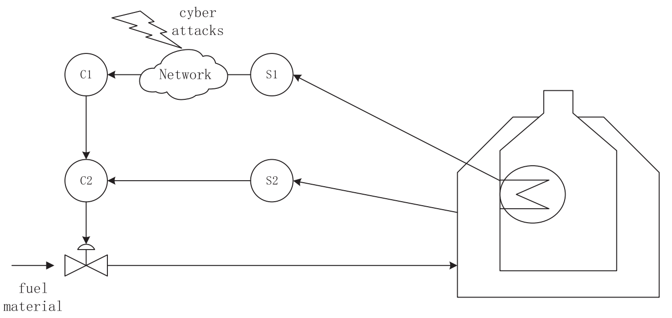

Fig. 2. The model of heating furnace (power plant boiler-turbine system).

$$\varrho_i \tilde{\Upsilon}_{ijli} - \varrho_i \bar{L}_i + \bar{L}_i < 0; \tag{50}$$

$$\varrho_m \tilde{\Upsilon}_{ijlm} + \varrho_i \tilde{\Upsilon}_{mjli} - \bar{\varrho}_m \bar{L}_i - \bar{\varrho}_i \bar{L}_m < 0, \ (i < m) \tag{51}$$

Define $X_1 = P_1^{-1}$, $X_2 = P_2^{-1}$, $\tilde{Q}_1 = X_1 Q_1 X_1$, $\tilde{Q}_2 = X_1 Q_2 X_1$, $\tilde{Q}_3 = X_2 Q_3 X_2$, $\tilde{R}_1 = X_1 R_1 X_1$, $\tilde{R}_2 = X_1 R_2 X_1$, $\tilde{R}_3 = X_2 R_3 X_2$, $\tilde{V} = X_1 V X_1$, $\tilde{M} = X_1 M X_1$, $\tilde{N} = X_2 N X_2$, $\tilde{\Omega} = X_2 \Omega X_2$, $Y_{1j} = K_{1j} X_1$, $Y_{2l} = K_{2m} X_2$, $\Lambda = diag\{X_1, X_1, X_1, X_1, X_1, X_2, X_2, X_2, X_1, X_1, I, I, X_1, X_1, X_2, X_2, I, I\}$. Pre and post-multiply both side of Eq. (49) with Λ and its transposition, we can get Eq. (40).

Then similar to the analysis in Theorem 1, one can get

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l (g_m - v_m)\bar{L}_i = \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} g_i h_j v_l \left(\sum_{m=1}^{r} g_m - \sum_{m=1}^{r} v_m\right)\bar{L}_i = 0 \tag{52}$$

With the conditions (40)–(42), we can obtain

$$\tilde{\Phi} \leq \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{m=1}^{r} g_i h_j v_l v_m \psi^T(t) \tilde{\Upsilon}_{ijlm} \psi(t) < 0 \tag{53}$$

then the system is asymptotically stable. Moreover, according to $Y_{1j} = K_{1j} X_1$, $Y_{2m} = K_{2m} X_2$, the gains of primary controller and secondary controller are derived as $K_{1j} = Y_{1j} X_1^{-1}$, $K_{2m} = Y_{2m} X_2^{-1}$. This completes the proof. □

4. Simulation examples

In this section, a simulation example is given to demonstrate the effectiveness of the designed controller. The simulation example is on the basis of a power plant boiler–turbine system which is shown in Fig. 2. Heating furnace is a significant component of the steam generation process. The system contains sensors, controllers, actuator and a network. As shown in the Fig. 2, the heating furnace is connected with two sensors. The primary sensor $S1$ transmits the data of temperature and the secondary sensor $S2$ transmits the data of pressure. There exists a network in the primary loop between the primary sensor $S1$ and primary controller $C1$ which may suffer the cyber attacks. The secondary controller $C2$ controls the valve of the furnace, and then control the amount of fuel.

Consider the power plant boiler-turbine system [21,22] with the following parameters:

$$A_{11}=\begin{bmatrix}-2 & 0\\ -1 & -2\end{bmatrix}, A_{21}=\begin{bmatrix}1.3 & 1\\ 0.2 & 0\end{bmatrix}, B_{11}=\begin{bmatrix}2\\ 0.1\end{bmatrix}, A_{31}=\begin{bmatrix}0.2 & 0.1\\ 0.2 & 1\end{bmatrix}, B_{21}=\begin{bmatrix}0.2\\ 1\end{bmatrix}, B_{31}=\begin{bmatrix}-0.4\\ 0.1\end{bmatrix}$$

$$C_{11}=\begin{bmatrix}0 & 0.1\end{bmatrix}, C_{21}=\begin{bmatrix}-0.3 & 0.1\end{bmatrix}, D_{11}=0.2, D_{21}=0.1$$

$$A_{12}=\begin{bmatrix}-2 & 0\\ -1 & -2\end{bmatrix}, A_{22}=\begin{bmatrix}1.3 & 1\\ 0.4 & 0\end{bmatrix}, B_{12}=\begin{bmatrix}2\\ 0.2\end{bmatrix} A_{32}=\begin{bmatrix}0.2 & 0.1\\ 0.2 & 1\end{bmatrix}, B_{22}=\begin{bmatrix}0.2\\ 1\end{bmatrix}, B_{32}=\begin{bmatrix}-0.4\\ 0.1\end{bmatrix}$$

$$C_{12}=\begin{bmatrix}0 & 0.1\end{bmatrix}, C_{22}=\begin{bmatrix}-0.3 & 0.1\end{bmatrix}, D_{12}=0.15, D_{22}=0.1$$

The initial states of the two plants are given as $x_1=[-3 \quad 2]^T$, $x_2=[-1 \quad 1]^T$.

Assume the external disturbance as

$$\omega(t)=\begin{cases}0.8cos(2\pi t), & 15\le t\le 20;\\ 0, & otherwise.\end{cases}$$

The nonlinear signal of cyber attacks is chosen as

$$f(x_1(t))=\begin{bmatrix}-tanh(0.02x_{11}(t))\\ -tanh(0.08x_{12}(t))\end{bmatrix}$$

Moreover, by using Assumption 1, we can see that when the upper bound is set to $F=diag\{0.02, 0.08\}$, the condition (21) is satisfied.

To demonstrate the effectiveness of the designed controllers, according to whether there exist cyber attacks in the system, we give the following two cases:

case 1: Set $\alpha(t)=0$, it means that the networked T-S fuzzy cascade control system is under the network environment without cyber attacks. When $\alpha(t)=0$, the primary controller can be described as

$$u_1(t)=\sum_{j=1}^{r} h_j\big[K_{1j}((I+\Delta_q)(x_1(t-d(t))+e_k(t)))\big]$$

and the secondary controller can be described as

$$u_2(t)=\sum_{j=1}^{r}\sum_{m=1}^{r} h_j v_m\big[(K_{1j}((I+\Delta_q)(x_1(t-d(t))+e_k(t))+K_{2m}X_2(t)\big].$$

According to Theorem 2, we can obtain the following matrixes with the help of MATLAB linear matrix inequality toolbox

$$X_1=\begin{bmatrix}1.3438 & -0.0262\\ -0.0262 & 1.4144\end{bmatrix}, X_2=\begin{bmatrix}0.2079 & -0.2484\\ -0.2484 & 1.4646\end{bmatrix}$$

$$Y_{11}=\begin{bmatrix}-0.0175 & -0.0157\end{bmatrix}, Y_{21}=\begin{bmatrix}-1.3488 & -7.1742\end{bmatrix}$$

$$Y_{12}=\begin{bmatrix}-0.0305 & -0.0269\end{bmatrix}, Y_{22}=\begin{bmatrix}-1.4192 & -6.5169\end{bmatrix}$$

Then by applying Eq. (43), we can obtain the gains of the controllers

$$K_{11}=\begin{bmatrix}-0.0133 & -0.0114\end{bmatrix}, K_{21}=\begin{bmatrix}-15.4799 & -7.5243\end{bmatrix}$$

$$K_{12}=\begin{bmatrix}-0.0231 & -0.0194\end{bmatrix}, K_{22}=\begin{bmatrix}-15.2317 & -7.0334\end{bmatrix}$$

In case 1, the system does not take into account the impact of cyber attacks. The state responses of primary plant and secondary plant are shown in Figs. 3 and 4, we can obtain that

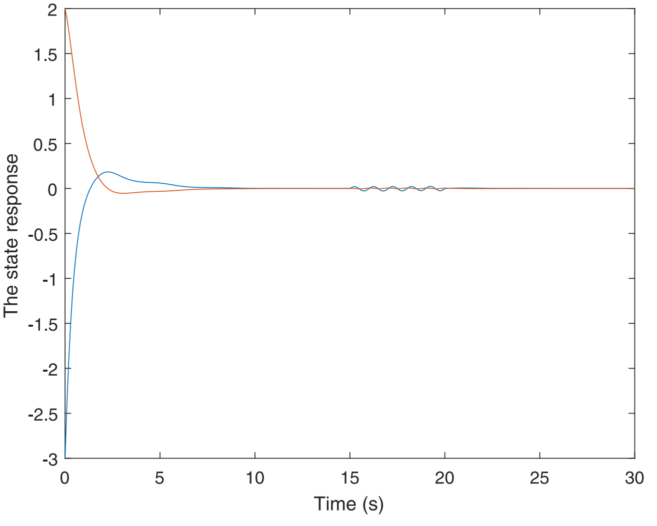

Fig. 3. The state response of *Plant*1 in case 1.

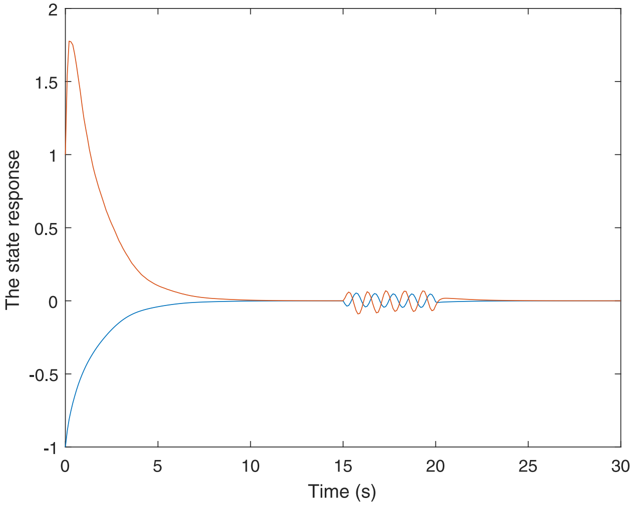

Fig. 4. The state response of *Plant*2 in case 1.

the networked T-S fuzzy cascade control system is stable when the system is under the event-triggered scheme without cyber attacks. Moreover, even in the presence of disturbances,the system can be stabilized very quickly.

Case 2: In this case, the networked T-S fuzzy cascade control system discussed is under the network environment with cyber attacks. Assume that the occurring probability of cyber attacks is 10%, that is, $\bar{\alpha} = 0.1$. When $\bar{\alpha} = 0.1$, then

$$u_1(t) = \sum_{j=1}^{r} h_j \big[K_{1j}(\alpha(t) f(x_1(t-\tau(t))) + (1-\alpha(t))(I+\Delta_q)(x_1(t-d(t)) + e_k(t))) \big]$$

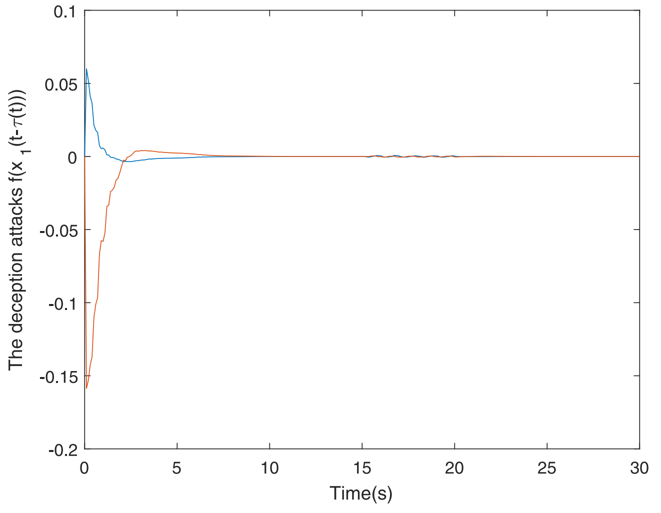

Fig. 5. The signal of cyber attacks in case 2.

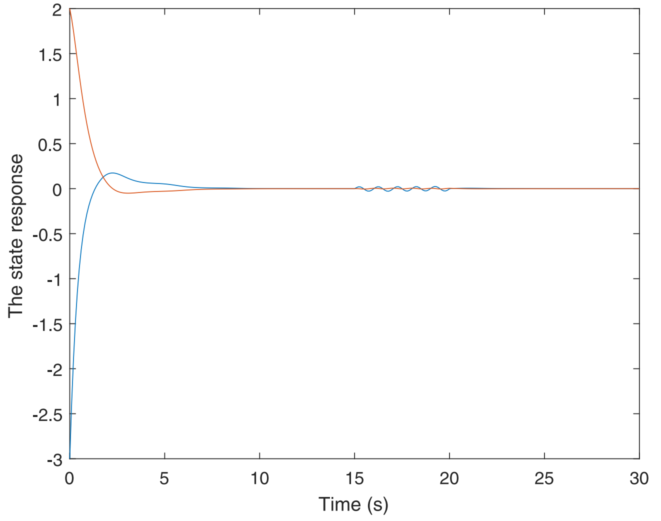

Fig. 6. The state response of *Plant*1 in case 2.

$$u_2(t) = \sum_{j=1}^{r}\sum_{m=1}^{r} h_j v_m\big[(K_{1j}\alpha(t)f(x_1(t-\tau(t))) + K_{1j}(1-\alpha(t))(I+\Delta_q)(x_1(t-d(t)) + e_k(t)) + K_{2m}X_2(t)\big]$$

Set sampling period $h = 0.1s$, the time delay $d_M = 0.5$, $\tau_M = 0.2$, $\theta_M = 0.3$, quantization density $\nu = 0.818$, trigger parameter $\beta = 0.07$, $\varrho_1 = 0.75$, $\varrho_2 = 0.95$ to satisfy the condition $v_m - \varrho_m g_m \geq 0$.

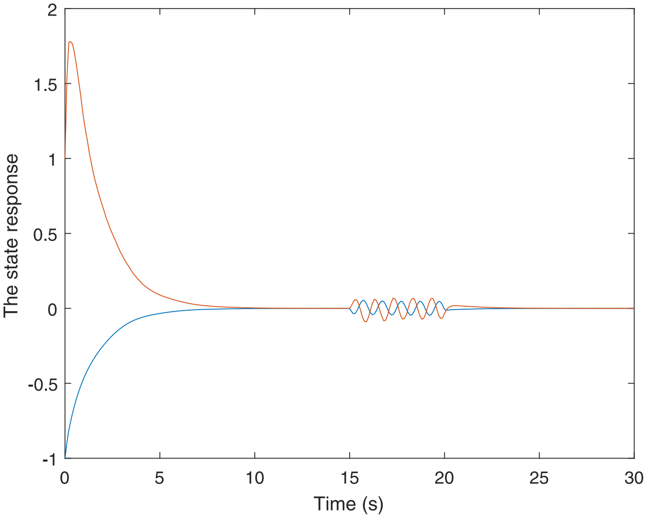

Fig. 7. The state response of *Plant*2 in case 2.

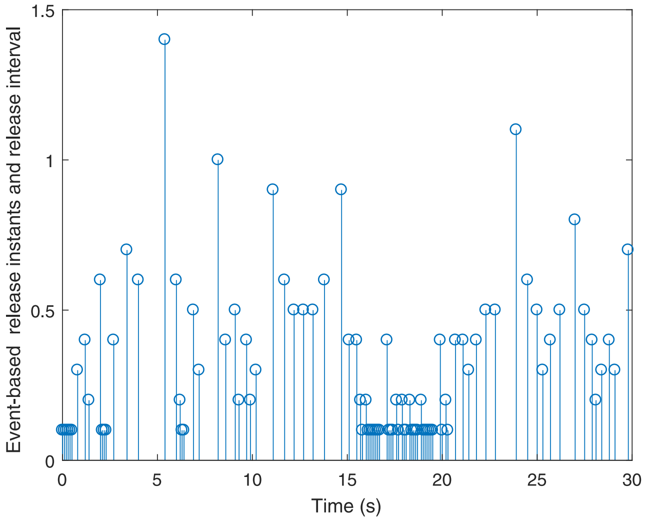

Fig. 8. Release instants and release interval in case 2.

By applying Theorem 2, we can get

$$X_1 = \begin{bmatrix} 5.7816 & -0.1054 \\ -0.1054 & 6.0229 \end{bmatrix}, X_2 = \begin{bmatrix} 0.8739 & -1.0959 \\ -1.0959 & 6.3203 \end{bmatrix}$$
$$Y_{11} = \begin{bmatrix} -0.0662 & -0.0649 \end{bmatrix}, Y_{21} = \begin{bmatrix} -5.3351 & -29.4314 \end{bmatrix}$$
$$Y_{12} = \begin{bmatrix} -0.0835 & -0.0792 \end{bmatrix}, Y_{22} = \begin{bmatrix} -5.4678 & -26.5369 \end{bmatrix}$$

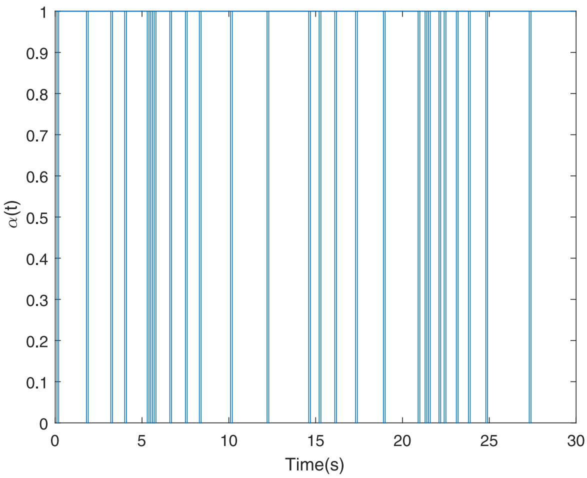

Fig. 9. Bernoulli distribution variables in Case 2.

Then we can obtain the gains of the primary controller and the secondary controller from Eq. (43)

$$K_{11} = \begin{bmatrix} -0.0117 & -0.0110 \end{bmatrix}, K_{21} = \begin{bmatrix} -15.2639 & -7.3032 \end{bmatrix}$$
$$K_{12} = \begin{bmatrix} -0.0147 & -0.0134 \end{bmatrix}, K_{22} = \begin{bmatrix} -14.7240 & -6.7517 \end{bmatrix}$$

In case 2, the operation of the networked T-S fuzzy cascade control system under certain probability cyber attack is shown in Fig. 5. The state responses of primary plant and secondary plant are shown in Figs. 6 and 7, from which we can observe that the designed method is effective even if there exists disturbance. In addition, the communication release instants and release intervals are shown in Fig. 8. From Fig. 8, we can easily see that at the beginning of the system operation, the measured data are released frequently due to the instability of the system, and the release intervals are small. When the system is stable, the release decreases instantaneously until the disturbance of 15–20 s makes the data trigger frequently again. In conclusion, compared with the time trigger mechanism, the event trigger mechanism can reduce the transmission of sampled data and consequently effectively save network bandwidth resources. The Bernoulli distribution random variables $\alpha(t)$ which is introduced to express the possibility of cyber attacks occurring can be seen in Fig. 9.

5. Conclusion

In this paper, the problem of the event-based controller design has been investigated for networked T-S fuzzy cascade control systems with quantization and cyber attacks. First of all, an event-triggered scheme and a quantization mechanism are adopted to save the communication resources. By taking the cyber attacks which are described by the Bernoulli variable into consideration, a newly T-S fuzzy model for NCCSs is established. Secondly, by using Lyapunov stability theory and linear matrix inequality technique, sufficient conditions for the stability of the networked T-S fuzzy cascade control systems are derived and the gains of controllers are obtained. Finally, a numerical example is shown to illustrate the efficiency of the designed control system. In the future, by considering the influence of different kind of cyber attacks, we will focus on the analysis and synthesis for networked T-S fuzzy cascade control systems with DoS attacks, replay attacks, etc. In addition, the attack detection for networked T-S fuzzy cascade control systems is also one of the problems we will consider.

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