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Hybrid-driven H_∞ filter design for T–S fuzzy systems with quantization

Jinliang Liu [a,b,*], Lili Wei [a], Jie Cao [a], Shumin Fei [c]

[a] College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, PR China
[b] Collaborative Innovation Center for Modern Grain Circulation and Safety, Nanjing, Jiangsu 210023, PR China
[c] School of Automation, Southeast University, Nanjing, Jiangsu 210096, PR China

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ABSTRACT

This paper is mainly concerned with hybrid-driven H_∞ filtering for a class of Takagi–Sugeno (T–S) fuzzy systems with quantization. To reduce the redundancy of transmission data and save the network bandwidth, a hybrid-driven scheme and a logarithmic quantizer are introduced in this paper. Firstly, by taking the effect of hybrid-driven scheme and quantization into consideration, a mathematical H_∞ filter model for T–S fuzzy systems is constructed. Secondly, by applying Lyapunov stability theory, sufficient conditions for asymptotical stabilization of desired system are obtained. Moreover, an explicit algorithm for H_∞ filter design is presented with the help of linear matrix inequality (LMI) techniques. Finally, numerical and physical simulations show the usefulness of the proposed filter design approach.

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1. Introduction

Nowadays, more and more researchers pay attention to the further study of T–S fuzzy systems because of the convenience and accuracy in modeling for complex nonlinear systems [1–3]. As is well known, T–S fuzzy model is a powerful tool which can convert control design problems of nonlinear system into linear system with a systematic framework [4–6]. In recent years, an ever increasing interest in the filter or controller design for T–S fuzzy systems has been witnessed. For example, in [7], an adaptive fuzzy output feedback tracking control problem for nonstrict-feedback switched nonlinear systems is investigated. The authors in [8] propose a novel delay partitioning method for the stability analysis of T–S fuzzy systems with interval time-varying delays and nonlinear perturbations. In [9], the authors investigate an adaptive event-triggered communication scheme for a class of networked T–S fuzzy control systems. In [10], the problem of $\ell_2 - \ell_\infty$ filter design for T–S fuzzy systems with multiple time-varying delays is investigated.

During the last decades, the time-triggered scheme (periodic sampling) plays an important role in NCSs. With this trigger method, even in the condition of the worst network environment such as external disturbances, limited capacity and channel bandwidth, the desired performance can be guaranteed by fixing the appropriate sampling interval [11]. It is acceptable to sample with a periodic interval from the perspective of system analysis and design, however, with the development of network, the drawback of the time-triggered method exposes gradually, which does not consider the effective use of limited network resources [12]. In order to solve these problems such as packet dropouts, transmission delays resulting from the limited capacity of the network, event-triggered scheme and quantization method have been proposed. First of all, lots of scholars have put forward different kinds of event-triggered schemes to alleviate the networked burden. One of

* Corresponding author at: College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, PR China.
E-mail address: liujinliang@vip.163.com (J. Liu).

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the novel event-triggered schemes is proposed by the authors in [13] for system analysis and controller design, in which the system states are sampled in discrete instants. Due to the efficiency in alleviating the networked burden, there are lots of researches focusing on the investigations of NCSs [14–16] based on the novel event-triggered scheme raised in [13]. For instances, reference [17] is concerned with the problem of event-triggered output feedback control of Markovian jump systems with quantization. In [18], the matter of H_∞ filter design for neural networked systems with event-triggered scheme and quantization is investigated. The authors in [19] address the issue of a decentralized event-triggering scheme for large scale systems. In [20], an event-triggered H_∞ controller is designed for discrete-time nonlinear systems with unreliable communication links. In addition, the introduction of quantizer can also save the energy of network transmission. As an indispensable step in the process of information transfer, quantization can convert the discrete signals into digital signals which is characteristic for its high anti-interference performance and compression ratios during the transmission [21]. Therefore, quantization is becoming a heat topic in the investigations of NCSs. By means of quantized output feedback, the authors in [22] concentrate on the issue of stabilizing a nonlinear system. In [23], the problem of event-triggered controller design for NCSs with both state and control input quantization is dealt with. In [24], an H_∞ filter is designed for a class of discrete time systems with quantization based on the fuzzy models. In [25], the authors consider the output feedback stabilization in linear systems by taking the event-triggered sampling method and dynamic quantization into account.

Motivated by the event-triggered scheme proposed in [13], the authors in [26] firstly propose a novel hybrid-driven scheme which consists of time-triggered scheme and event-triggered scheme to analyze the stability of NCSs. Based on the hybrid-driven scheme mentioned above, the authors in [27] concentrate on the study of H_∞ filtering with cyber attacks. The problem of hybrid-driven-based filter design for neural networks subject to deception attacks is investigated in [28]. In [29], the issue of reliable controller design for hybrid-driven T–S fuzzy systems with probabilistic actuator faults and nonlinear perturbations is addressed. Inspired by the theories mentioned above, this paper is concerned with hybrid-driven H_∞ filter design for a class of T–S fuzzy systems with quantization. In order to reduce the redundancy of transmitted data effectively, a hybrid-driven scheme, which is governed by a Bernoulli random variable, is adopted in this paper. To optimize the networked systems, a logarithmic quantizer is applied which can make the transmitted signals account for less bandwidth. To the best of our knowledge, no relevant results have been reported for hybrid-driven filter design of T–S fuzzy systems with quantization, which motivates the current work.

The remainder of this paper is organized as follows. In Section 2, a mathematical model of filter error system with hybrid-driven scheme and quantization is formulated. In Section 3, sufficient conditions to guarantee the asymptotical stabilization of filter error system are obtained and the design algorithm for desired filter parameters is presented. Numerical and physical examples are given in Section 4 to demonstrate the feasibility of designed filter with the help of MATLAB.

Notation: R^n and $R^{n\times m}$ denote the n-dimensional Euclidean space, and the set of $n \times m$ real matrices; the superscript T stands for matrix transposition; I is the identity matrix of appropriate dimension; $sym\{X\}$ denotes the sum of matrix X and its transposed matrix X^T; S denotes a set of positive integers; $\mathbb{E}\{X\}$ represents the mathematical expectation of X; $X > 0$, for $X \in R^{n\times n}$ means that the matrix X is real symmetric positive definite. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

2. Problem statement and preliminaries

Consider the nonlinear system represented by the following T–S fuzzy system with *ith* rule:
IF $\theta_1(t)$ is W_1^i and $\cdots$ and $\theta_g(t)$ is W_g^i , THEN

$$\begin{cases} \dot{x}(t) = A_i x(t) + A_{di} x(t-\tau(t)) + A_{\omega i}\omega(t) \\ y(t) = C_i x(t) \\ z(t) = L_i x(t) \end{cases} \tag{1}$$

where $(i = 1, 2, \ldots, r)$, $r \in S$ is the number of IF-THEN rules. $x(t) \in R^n$, $y(t) \in R^m$ and $z(t) \in R^s$ are the state vector, output vector and the signal to be estimated, respectively. $\theta(t)$ denotes the vector of fuzzy premise variables, $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_g(t)]$, W_p^i $(p = 1, 2, \ldots, g)$ are fuzzy sets. A_i, A_{di}, $A_{\omega i}$, C_i, L_i are parameter matrices with appropriate dimensions, $\omega(t) \in L_2\,[0, \infty)$ denotes the exogenous disturbance signal, $\tau(t)$ is a time-varying delay taking values on the interval $[\tau_m, \tau_M]$, where τ_m and τ_M are positive real numbers.

By using center-average defuzzifier, product interference and singleton fuzzifier, the fuzzy system (1) can be obtained as follows:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} h_i(\theta(t))\{A_i x(t) + A_{di} x(t-\tau(t)) + A_{\omega i}\omega(t)\} \\ y(t) = \sum_{i=1}^{r} h_i(\theta(t)) C_i x(t) \\ z(t) = \sum_{i=1}^{r} h_i(\theta(t)) L_i x(t) \end{cases} \tag{2}$$

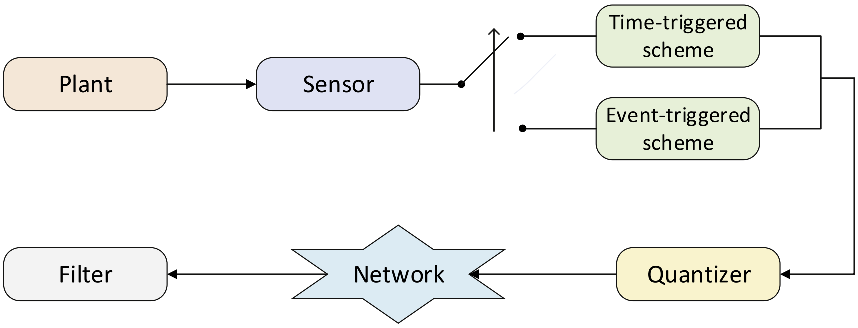

Fig. 1. The structure of hybrid-driven H_∞ filtering with quantization.

where $h_i(\theta(t)) = \frac{\mu_i(\theta(t))}{\sum_{i=1}^{r}\mu_i(\theta(t))}$, $\mu_i(\theta(t)) = \prod_{p=1}^{g} W_p^i(\theta_p(t))$, $W_p^i(\theta_p(t))$ is the grade membership value of $\theta_p(t)$. $h_i(\theta(t))$ denotes the normalized membership function satisfying $h_i(\theta(t)) \geq 0$, $\sum_{i=1}^{r} h_i(\theta(t)) = 1$.

As shown in Fig. 1, a hybrid-driven scheme, which consists of time-triggered scheme and event-triggered scheme, is introduced to save the networked resources. When the system (2) is under the networked environments with quality-of-service constraints, the time-triggered scheme is chosen for data transmission which can sample data periodically. In order to save the network resources and improve the system performance, the event-triggered scheme is applied between the sensor and the quantizer, which can decide whether the newly sampled data should be transmitted or not. Once $y(s_k h)$ is transmitted, the next transmission instant via event-triggered scheme can be described as following equality [30].

$$s_{k+1}h = s_k h + \min_{l \geqslant 1}\{lh | e_k^T(s_k h)\Omega e_k(s_k h) > \sigma y^T(s_k h + lh)\Omega y(s_k h + lh)\} \quad (3)$$

where $e_k(s_k h) = y(s_k h) - y(s_k h + lh)$ denotes threshold error, h represents the sampling period, $\{s_k h | s_k \in S\}$ represents the set of transmitted instants, $\{s_k h + lh | l = 1, 2, \ldots\}$ denotes the current sampling instants, $\Omega > 0$ and $\sigma \in [0, 1)$ denote the trigger parameters.

Remark 1. According to the event-triggered judgment algorithm (3), the set of releasing instants is $\{s_1 h, s_2 h, \ldots\} \subseteq \{h, 2h, \ldots\}$, initial value is $s_0 h = 0$. Moreover, the next triggered instant $s_{k+1}h$ is determined by trigger parameters and the threshold error $e_k(s_k h)$.

In this paper, by borrowing the quantized theory from references [31,32], the logarithmic quantizer $q(\cdot)$ is applied to quantize the measurements $y(t)$. Define $q(y) = \begin{bmatrix} q_1(y_1) & q_2(y_2) & \ldots & q_m(y_m)\end{bmatrix}^T$, for each $q_s(y_s)$ $s = 1, 2, \ldots, m$, the set of quantized levels can be described by

$$U_s = \{\pm u_v^{(s)} : u_v^{(s)} = \rho_s^v u_0^{(s)}, v = \pm 1, \pm 2, \ldots\} \cup \{\pm u_0^{(s)}\} \cup \{0\}, 0 < \rho_s < 1, u_0^{(s)} > 0 \quad (4)$$

The logarithmic quantizer $q_s(y_s)$ is given by

$$q_s(y_s) = \begin{cases} u_v^{(s)}, & \frac{u_v^{(s)}}{1+\delta} < y_s \leq \frac{u_v^{(s)}}{1-\delta}, y_s > 0 \\ 0, & y_s = 0 \\ -q(-y_s), & y_s < 0 \end{cases} \quad (5)$$

where $\delta = \frac{1-\rho}{1+\rho}$, ρ is quantization density.

Since $q_s(\cdot)$ is symmetrical, $q_s(-y_s) = -q_s(y_s)$. Then, logarithmic quantizer $q_s(y_s)$ can be described as

$$q_s(y_s) = \left[I + \Delta_{q_s}(y_s)\right] y_s \quad (6)$$

where $|\Delta_{q_s}(y_s)| \leq \delta_s$. For the sake of simplicity, $\Delta_{q_s}(y_s)$ is used to represent Δ_{q_s}.

Then, the following equality can be obtained

$$q(y) = (I + \Delta_q)y \quad (7)$$

where $\Delta_q = diag\{\Delta_{q_1}, \Delta_{q_2}, \ldots, \Delta_{q_m}\}$.

Based on the definition (5) and equality (7), the newly sampled measurements via quantizer can be expressed by the following equation

$$y_q(t) = (I + \Delta_q)y(t) \quad (8)$$

As shown in Fig. 1, on account of communication network between the quantizer and the filter, the network-induced delays should be taken into consideration. Therefore, the quantized output measurements $y_q(t)$ should be replaced by $\hat{y}(t)$, which is the actual data available in the filter. In the following discussion, $\hat{y}(t)$ will be described in detail.

When the system (2) is under the "time-triggered scheme", define $\eta(t) = t - s_k h$ for $t \in [s_k h + \tau_{s_k}, s_{k+1}h + \tau_{s_{k+1}})$, where τ_{s_k} ($\forall k \in S$) is used to represent the network-induced delay. Similar to [13,23], $\eta(t)$ is supposed to be bounded satisfying $0 \le \tau_{s_k} \le \eta(t) \le s_{k+1}h - s_k h + \tau_{s_{k+1}} \triangleq \eta_M$. Then, the data $y_1(t)$ through time-triggered scheme and quantization can be written as

$$y_1(t) = (I + \Delta_q)y(t - \eta(t)), t \in [s_k h + \tau_{s_k}, s_{k+1}h + \tau_{s_{k+1}}) \tag{9}$$

where $\eta(t) \in [0, \eta_M]$.

When the system (2) is under the "event-triggered scheme", similar to [33], a zero-order-hold (ZOH) is introduced to store the latest transmitted data. The holding interval of ZOH is defined as $[s_k h + \tau_{s_k}, s_{k+1}h + \tau_{s_{k+1}}) = \bigcup_{j=0}^{d} \bigwedge_j$, where $\bigwedge_j = [s_k h + jh + \tau_{s_k+j}, s_k h + jh + h + \tau_{s_k+j+1})$, $j = 0, 1, \ldots, d$ and $d = s_{k+1} - s_k - 1$. Let $d(t) = t - s_k h - jh$, $0 \le \tau_{s_k+j} \le d(t) \le \tau_{s_k+j+1} + h \triangleq d_M$, then, the data $y_2(t)$ via event-triggered scheme (3) and quantization can be described by

$$y_2(t) = (I + \Delta_q)\left[y(t - d(t)) + e_k(t)\right], t \in [s_k h + \tau_{s_k}, s_{k+1}h + \tau_{s_{k+1}}) \tag{10}$$

where $d(t) \in [0, d_M]$.

Based on Eqs. (9) and (10), the actual input $\hat{y}(t)$ available in the filter can be written as

$$\begin{aligned}\hat{y}(t) &= \alpha(t)y_1(t) + (1 - \alpha(t))y_2(t) \\ &= \alpha(t)(I + \Delta_q)y(t - \eta(t)) + (1 - \alpha(t))(I + \Delta_q)\left[y(t - d(t)) + e_k(t)\right]\end{aligned} \tag{11}$$

where $\alpha(t)$ is a random variable satisfying Bernoulli distribution, and its mathematical expectation can be written as $\mathbb{E}\{\alpha(t)\} = \bar{\alpha}$. μ^2 is utilized to represent the mathematical variance of $\alpha(t)$.

Remark 2. Different from traditional event-triggered scheme or time-triggered scheme, the hybrid-driven scheme is determined by a variable $\alpha(t)$ satisfying Bernoulli distribution. When "$\alpha(t) = 1$", Eq. (11) can be written as "$\hat{y}(t) = (I + \Delta_q)y(t - \eta(t))$" which means that the "time-triggered scheme" is selected for data transmission. Otherwise, the "event-triggered scheme" is activated and Eq. (11) can be written as "$\hat{y}(t) = (I + \Delta_q)\left[y(t - d(t)) + e_k(t)\right]$".

This paper aims to design a hybrid-driven H_∞ filter for a class of T–S fuzzy system with quantization. The premise variable of system (2) is $\theta(t)$. Due to the presence of network-induced delay, it is reasonable to use the latest lagged premise variable $\theta(s_k h)$ in the filter design. *jth* rule is expressed in the following IF-THEN rule:

IF $\theta_1(s_k h)$ is M_1^j and $\cdots$ and $\theta_g(s_k h)$ is M_d^j, THEN

$$\begin{cases}\dot{x}_f(t) = A_{fj}(t)x_f(t) + B_{fj}(t)\hat{y}(t) \\ z_f(t) = C_{fj}(t)x_f(t)\end{cases} \tag{12}$$

where $x_f(t) \in R^n$, $z_f(t) \in R^s$ are the state and output of the filter, respectively. $\hat{y}(t)$ is the real input of the filter. $A_{fj} \in R^{n\times n}$, $B_{fj} \in R^{n\times m}$, $C_{fj} \in R^{p\times n}$ are the filter parameters to be determined.

The defuzzified output of (12) is referred by

$$\begin{cases}\dot{x}_f(t) = \sum_{j=1}^{r} h_j(\theta(s_k h))\{A_{fj}x_f(t) + B_{fj}\hat{y}(t)\} \\ z_f(t) = \sum_{j=1}^{r} h_j(\theta(s_k h))C_{fj}x_f(t)\end{cases} \tag{13}$$

where $r \in S$, $h_j(\theta(s_k h)) = \frac{o_j(\theta(s_k h))}{\sum_{j=1}^{r} o_j(\theta(s_k h))}$, $o_j(\theta(s_k h)) = \prod_{q=1}^{d} M_q^j(\theta_q(s_k h))$, $M_q^j(\theta_q(s_k h))$ is the grade membership value of $\theta_q(s_k h)$. $h_j(\theta(s_k h))$ denotes the normalized membership function satisfying $h_j(\theta(s_k h)) \ge 0$, $\sum_{j=1}^{r} h_j(\theta(s_k h)) = 1$, $\hat{y}(t) = \sum_{i=1}^{r} h_i(\theta(t))\left[\alpha(t)(I + \Delta_q)C_i x(t - \eta(t)) + (1 - \alpha(t))(I + \Delta_q)(C_i x(t - d(t)) + e_k(t))\right]$.

Remark 3. According to equality (13), the premise variable $\theta(t)$ adopted in system (2) cannot synchronously arrive at the filter, which implies $\theta(s_k h) \neq \theta(t)$. However, notice that there exists network before the filter in Fig. 1. On account of the effect of network-induced delay, the premise variable $\theta(s_k h)$ in (13) is assumed to have relationship with the premise variable $\theta(t)$ of system (2).

Define

$$e(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}, \tilde{z}(t) = z(t) - z_f(t)$$

Based on Eqs. (2) and (13), the filter error system can be rewritten as

$$\begin{cases}\dot{e}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(s_kh))\{\bar{A}e(t)+\bar{A}_dHe(t-\tau(t))+\alpha(t)\bar{B}_1He(t-\eta(t))\\ \quad+(1-\alpha(t))[\bar{B}_1He(t-d(t))+\bar{B}_2e_k(t)]+\bar{A}_\omega\omega(t)\}\\ \tilde{z}(t)=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(\theta(t))h_j(\theta(s_kh))\bar{L}e(t)\end{cases} \tag{14}$$

$$\bar{A}=\begin{bmatrix}A_i & 0\\ 0 & A_{fj}\end{bmatrix},\bar{A}_d=\begin{bmatrix}A_{di}\\ 0\end{bmatrix},\bar{B}_1=\begin{bmatrix}0\\ B_{fj}(I+\Delta_q)C_i\end{bmatrix},\bar{B}_2=\begin{bmatrix}0\\ B_{fj}(I+\Delta_q)\end{bmatrix}$$
$$\bar{A}_\omega=\begin{bmatrix}A_{\omega i}\\ 0\end{bmatrix},\bar{L}=\begin{bmatrix}L_i & -C_{fj}\end{bmatrix},H=\begin{bmatrix}I & 0\end{bmatrix}$$

In the following, an assumption and three lemmas are introduced which are important in deriving the main results.

Assumption 1 ([34,35]). Suppose that the asynchronous errors of premise variables satisfy the following inequality

$$|h_j(\theta(t))-h_j(\theta(s_kh))|\le\varpi_l,j,l=1,2,\ldots,r \tag{15}$$

where $\varpi_l>0$ denotes the additive bounds.

Lemma 1 ([36]). *For any vectors x, $y\in R^n$, and positive definite matrix $Q\in R^{n\times n}$, the following inequality holds*

$$2x^Ty\le x^TQx+y^TQ^{-1}y \tag{16}$$

Lemma 2 ([37]). *Suppose $\tau(t)\in[\tau_m,\tau_M]$, $d(t)\in[0,d_M]$, $\eta(t)\in[0,\eta_M]$, Ξ_1, Ξ_2, Ξ_3, Ξ_4, Ξ_5, Ξ_6 and Ω are matrices with appropriate dimensions, then*

$$(\tau(t)-\tau_m)\Xi_1+(\tau_M-\tau(t))\Xi_2+d(t)\Xi_3+(d_M-d(t))\Xi_4+\eta(t)\Xi_5+(\eta_M-\eta(t))\Xi_6+\Omega<0 \tag{17}$$

if and only if

$$\begin{cases}(\tau_M-\tau_m)\Xi_1+d_M\Xi_3+\eta_M\Xi_5+\Omega<0, & (\tau_M-\tau_m)\Xi_2+d_M\Xi_3+\eta_M\Xi_5+\Omega<0,\\ (\tau_M-\tau_m)\Xi_1+d_M\Xi_4+\eta_M\Xi_5+\Omega<0, & (\tau_M-\tau_m)\Xi_2+d_M\Xi_4+\eta_M\Xi_5+\Omega<0,\\ (\tau_M-\tau_m)\Xi_1+d_M\Xi_3+\eta_M\Xi_6+\Omega<0, & (\tau_M-\tau_m)\Xi_2+d_M\Xi_3+\eta_M\Xi_6+\Omega<0,\\ (\tau_M-\tau_m)\Xi_1+d_M\Xi_4+\eta_M\Xi_6+\Omega<0, & (\tau_M-\tau_m)\Xi_2+d_M\Xi_4+\eta_M\Xi_6+\Omega<0.\end{cases} \tag{18}$$

Lemma 3 ([38]). *Given matrices U, G and W of appropriate dimensions and U is symmetrical, then*

$$U+GF(t)W+W^TF^T(t)G^T<0 \tag{19}$$

for any $F(t)$ satisfying $F^T(t)F(t)\le I$, if and only if there exists a parameter $\epsilon>0$ such that

$$U+\epsilon^{-1}GG^T+\epsilon W^TW<0 \tag{20}$$

3. Main results

In this section, sufficient conditions ensuring the stability of systems (14) and design algorithm for H_∞ filtering will be obtained by using Lyapunov stability theory and LMI techniques.

Theorem 1. *For given time delays τ_m, τ_M, d_M, η_M, trigger parameter σ, value ϖ_l $(l=1,\ldots,r)$ and $\bar{\alpha}$, system (14) is asymptotically stable with an H_∞ disturbance attenuation level γ, if there exist matrices $P>0$, $Q_k>0$, $R_k>0$ $(k=1,2,3,4)$, $\Omega>0$, Π, M_{ij}, N_{ij}, T_{ij}, S_{ij}, W_{ij} and V_{ij} with appropriate dimensions satisfying*

$$\Phi^{ii}<0,i=1,2,\ldots,r \tag{21}$$

$$\Phi^{ij}+\Phi^{ji}<0,i,j=1,2,\ldots,r,(i<j) \tag{22}$$

$$\Xi^{ij}+\Pi>0,i,j=1,2,\ldots,r \tag{23}$$

where

$$\Xi^{ij}=\begin{bmatrix}\Omega_{11}^{ij} & * & * & *\\ \Omega_{21}^{ij} & \Omega_{22}^{ij} & * & *\\ \Omega_{31}^{ij} & 0 & \Omega_{33}^{ij} & *\\ \Omega_{41}^{ij}(s) & 0 & 0 & \Omega_{44}^{ij}\end{bmatrix},(s=1,\ldots,8)$$

$$
\begin{aligned}
\Omega_{11}^{ij} &= \begin{bmatrix}
\Gamma_{ij1} & * & * & * & * & * & * & * & * & * \\
R_2 & \Gamma_{ij2} & * & * & * & * & * & * & * & * \\
H^T\bar{A}_d{}^T P & M_{ij3}-M_{ij2}^T & \Gamma_{ij3} & * & * & * & * & * & * & * \\
0 & 0 & N_{ij4}-N_{ij3}^T & \Gamma_{ij4} & * & * & * & * & * & * \\
\Gamma_{ij9} & 0 & 0 & 0 & \Gamma_{ij5} & * & * & * & * & * \\
0 & 0 & 0 & 0 & V_{ij6}-V_{ij5}^T & \Gamma_{ij6} & * & * & * & * \\
\Gamma_{ij0} & 0 & 0 & 0 & 0 & 0 & \Gamma_{ij7} & * & * & * \\
0 & 0 & 0 & 0 & 0 & 0 & S_{ij8}-S_{ij7}^T & \Gamma_{ij8} & * & * \\
\bar{\alpha}_1\bar{B}_2{}^T P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Omega & * \\
\bar{A}_\omega^T P & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I
\end{bmatrix} \\
\Gamma_{ij1} &= P\bar{A}+\bar{A}^TP+Q_1+Q_2+Q_3+Q_4-R_2+T_{ij1}+T_{ij1}^T+W_{ij1}+W_{ij1}^T \\
\Gamma_{ij2} &= -Q_1-R_2+M_{ij2}+M_{ij2}^T,\ \Gamma_{ij3}=-M_{ij3}-M_{ij3}^T+N_{ij3}+N_{ij3}^T \\
\Gamma_{ij4} &= -Q_2-N_{ij4}-N_{ij4}^T,\ \Gamma_{ij5}=-W_{ij5}-W_{ij5}^T+V_{ij5}+V_{ij5}^T \\
\Gamma_{ij6} &= -Q_4-V_{ij6}-V_{ij6}^T,\ \Gamma_{ij7}=\sigma H^TC_i^T\Omega C_iH-T_{ij7}-T_{ij7}^T+S_{ij7}+S_{ij7}^T \\
\Gamma_{ij8} &= -Q_3-S_{ij8}-S_{ij8}^T,\ \Gamma_{ij9}=\bar{\alpha}H^T\bar{B}_1{}^TP+W_{ij5}-W_{ij1}^T \\
\Gamma_{ij0} &= \bar{\alpha}_1H^T\bar{B}_1{}^TP+T_{ij7}-T_{ij1},\ \Phi^{ij}=\Xi^{ij}+\sum_{l=1}^{r}\varpi_l(\Xi^{il}+\Pi) \\
\Omega_{21}^{ij} &= \begin{bmatrix}
\bar{L} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sqrt{\tau_{21}}P\bar{A} & 0 & \sqrt{\tau_{21}}P\bar{A}_dH & 0 & \bar{\alpha}\sqrt{\tau_{21}}P\bar{B}_1H & 0 & \bar{\alpha}_1\sqrt{\tau_{21}}P\bar{B}_1H & 0 & \bar{\alpha}_1\sqrt{\tau_{21}}P\bar{B}_2 & \sqrt{\tau_{21}}P\bar{A}_\omega \\
\tau_mP\bar{A} & 0 & \tau_mP\bar{A}_dH & 0 & \bar{\alpha}\tau_mP\bar{B}_1H & 0 & \bar{\alpha}_1\tau_mP\bar{B}_1H & 0 & \bar{\alpha}_1\tau_mP\bar{B}_2 & \tau_mP\bar{A}_\omega \\
\sqrt{d_M}P\bar{A} & 0 & \sqrt{d_M}P\bar{A}_dH & 0 & \bar{\alpha}\sqrt{d_M}P\bar{B}_1H & 0 & \bar{\alpha}_1\sqrt{d_M}P\bar{B}_1H & 0 & \bar{\alpha}_1\sqrt{d_M}P\bar{B}_2 & \sqrt{d_M}P\bar{A}_\omega \\
\sqrt{\eta_M}P\bar{A} & 0 & \sqrt{\eta_M}P\bar{A}_dH & 0 & \bar{\alpha}\sqrt{\eta_M}P\bar{B}_1H & 0 & \bar{\alpha}_1\sqrt{\eta_M}P\bar{B}_1H & 0 & \bar{\alpha}_1\sqrt{\eta_M}P\bar{B}_2 & \sqrt{\eta_M}P\bar{A}_\omega
\end{bmatrix} \\
\Omega_{31}^{ij} &= \begin{bmatrix}
0 & 0 & 0 & 0 & \mu\sqrt{\tau_{21}}P\bar{B}_1H & 0 & \mu\sqrt{\tau_{21}}P\bar{B}_1H & 0 & \mu\sqrt{\tau_{21}}P\bar{B}_2 & 0 \\
0 & 0 & 0 & 0 & \mu\tau_mP\bar{B}_1H & 0 & \mu\tau_mP\bar{B}_1H & 0 & \mu\tau_mP\bar{B}_2 & 0 \\
0 & 0 & 0 & 0 & \mu\sqrt{d_M}P\bar{B}_1H & 0 & \mu\sqrt{d_M}P\bar{B}_1H & 0 & \mu\sqrt{d_M}P\bar{B}_2 & 0 \\
0 & 0 & 0 & 0 & \mu\sqrt{\eta_M}P\bar{B}_1H & 0 & \mu\sqrt{\eta_M}P\bar{B}_1H & 0 & \mu\sqrt{\eta_M}P\bar{B}_2 & 0
\end{bmatrix} \\
\Omega_{41}^{ij}(1) &= \begin{bmatrix}\sqrt{\tau_{21}}M_{ij}^T\\ \sqrt{d_M}T_{ij}^T\\ \sqrt{\eta_M}W_{ij}^T\end{bmatrix},\ \Omega_{41}^{ij}(2)=\begin{bmatrix}\sqrt{\tau_{21}}M_{ij}^T\\ \sqrt{d_M}T_{ij}^T\\ \sqrt{(\eta_M)}V_{ij}^T\end{bmatrix},\ \Omega_{41}^{ij}(3)=\begin{bmatrix}\sqrt{\tau_{21}}M_{ij}^T\\ \sqrt{d_M}S_{ij}^T\\ \sqrt{\eta_M}W_{ij}^T\end{bmatrix},\ \Omega_{41}^{ij}(4)=\begin{bmatrix}\sqrt{\tau_{21}}M_{ij}^T\\ \sqrt{d_M}S_{ij}^T\\ \sqrt{\eta_M}V_{ij}^T\end{bmatrix} \\
\Omega_{41}^{ij}(5) &= \begin{bmatrix}\sqrt{\tau_{21}}N_{ij}^T\\ \sqrt{d_M}T_{ij}^T\\ \sqrt{\eta_M}W_{ij}^T\end{bmatrix},\ \Omega_{41}^{ij}(6)=\begin{bmatrix}\sqrt{\tau_{21}}N_{ij}^T\\ \sqrt{d_M}T_{ij}^T\\ \sqrt{\eta_M}V_{ij}^T\end{bmatrix},\ \Omega_{41}^{ij}(7)=\begin{bmatrix}\sqrt{\tau_{21}}N_{ij}^T\\ \sqrt{d_M}S_{ij}^T\\ \sqrt{\eta_M}W_{ij}^T\end{bmatrix},\ \Omega_{41}^{ij}(8)=\begin{bmatrix}\sqrt{\tau_{21}}N_{ij}^T\\ \sqrt{d_M}T_{ij}^T\\ \sqrt{\eta_M}V_{ij}^T\end{bmatrix} \\
\sqrt{\tau_{21}} &= \sqrt{\tau_M-\tau_m},\ \bar{\alpha}_1=1-\bar{\alpha},\ \Omega_{22}^{ij}=diag\{-I,-PR_1^{-1}P,-PR_2^{-1}P,-PR_3^{-1}P,-PR_4^{-1}P\} \\
\Omega_{33}^{ij} &= diag\{-PR_1^{-1}P,-PR_2^{-1}P,-PR_3^{-1}P,-PR_4^{-1}P\},\ \Omega_{44}^{ij}=diag\{-R_1,-R_3,-R_4\},\ \bar{L}=\begin{bmatrix}L_i & -C_{fj}\end{bmatrix} \\
M_{ij}^T &= \begin{bmatrix}0 & M_{ij2}^T & M_{ij3}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix},\ N_{ij}^T=\begin{bmatrix}0 & 0 & N_{ij3}^T & N_{ij4}^T & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix} \\
T_{ij}^T &= \begin{bmatrix}T_{ij1}^T & 0 & 0 & 0 & 0 & 0 & T_{ij7}^T & 0 & 0 & 0\end{bmatrix},\ S_{ij}^T=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & S_{ij7}^T & S_{ij8}^T & 0 & 0\end{bmatrix} \\
W_{ij}^T &= \begin{bmatrix}W_{ij1}^T & 0 & 0 & 0 & W_{ij5}^T & 0 & 0 & 0 & 0 & 0\end{bmatrix},\ V_{ij}^T=\begin{bmatrix}0 & 0 & 0 & 0 & V_{ij5}^T & V_{ij6}^T & 0 & 0 & 0 & 0\end{bmatrix}
\end{aligned}
$$

Proof. By referring to Lyapunov functional construction in references [39–41], a Lyapunov function can be built as follows

$$V(t)=V_1(t)+V_2(t)+V_3(t) \tag{24}$$

where

$$
\begin{aligned}
V_1(t) &= e^T(t)Pe(t) \\
V_2(t) &= \int_{t-\tau_m}^{t} e^T(s)Q_1e(s)ds+\int_{t-\tau_M}^{t} e^T(s)Q_2e(s)ds+\int_{t-d_M}^{t} e^T(s)Q_3e(s)ds+\int_{t-\eta_M}^{t} e^T(s)Q_4e(s)ds
\end{aligned}
$$

$$V_3(t) = \int_{t-\tau_M}^{t-\tau_m}\int_s^t \dot{e}^T(v)R_1\dot{e}(v)dvds + \tau_m\int_{t-\tau_m}^{t}\int_s^t \dot{e}^T(v)R_2\dot{e}(v)dvds + \int_{t-d_M}^{t}\int_s^t \dot{e}^T(v)R_3\dot{e}(v)dvds + \int_{t-\eta_M}^{t}\int_s^t \dot{e}^T(v)R_4\dot{e}(v)dvds$$

and $P>0$, $Q_k>0$, $R_k>0$ $(k=1,2,3,4)$.

By taking the derivative and expectation on $V(t)$, the following results can be obtained

$$\mathbb{E}\{\dot{V}_1(t)\} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))2e^T(t)P\mathcal{A} \tag{25}$$

$$\mathbb{E}\{\dot{V}_2(t)\} = e^T(t)(Q_1+Q_2+Q_3+Q_4)e(t) - e^T(t-\tau_m)Q_1e(t-\tau_m) - e^T(t-\tau_M)Q_2e(t-\tau_M) - e^T(t-d_M)Q_3e(t-d_M) - e^T(t-\eta_M)Q_4e(t-\eta_M) \tag{26}$$

$$\mathbb{E}\{\dot{V}_3(t)\} = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\mathcal{A}^T\tilde{R}\mathcal{A} + \mu^2\mathcal{B}^T\tilde{R}\mathcal{B} - \int_{t-\tau_M}^{t-\tau_m}\dot{e}^T(s)R_1\dot{e}(s)ds - \tau_m\int_{t-\tau_m}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds - \int_{t-d_M}^{t}\dot{e}^T(s)R_3\dot{e}(s)d_s - \int_{t-\eta_M}^{t}\dot{e}^T(s)R_4\dot{e}(s)ds \tag{27}$$

where $\mathcal{A} = \bar{A}e(t) + \bar{A}_dHe(t-\tau(t)) + \bar{\alpha}\bar{B}_1He(t-\eta(t)) + (1-\bar{\alpha})[\bar{B}_1He(t-d(t)) + \bar{B}_2e_k(t)] + \bar{A}_\omega\omega(t)$, $\mathcal{B} = \bar{B}_1He(t-\eta(t)) - \bar{B}_1He(t-d(t)) - \bar{B}_2e_k(t)$, $\tilde{R} = (\tau_M-\tau_m)R_1 + \tau_m^2R_2 + d_MR_3 + \eta_MR_4$, $P>0$, $Q_k>0$, $R_k>0$ $(k=1,2,3,4)$.

Notice that

$$-\tau_m\int_{t-\tau_m}^{t}\dot{e}^T(s)R_2\dot{e}(s)ds \le \begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix}^T \begin{bmatrix} -R_2 & R_2 \\ R_2 & -R_2\end{bmatrix}\begin{bmatrix} e(t) \\ e(t-\tau_m)\end{bmatrix} \tag{28}$$

Apply the free-weighting matrices method [42,43], it can be obtained that

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\xi^T(t)M_{ij}\left[e(t-\tau_m) - e(t-\tau(t)) - \int_{t-\tau(t)}^{t-\tau_m}\dot{e}(s)d_s\right] = 0 \tag{29}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\xi^T(t)N_{ij}\left[e(t-\tau(t)) - e(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)d_s\right] = 0 \tag{30}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\xi^T(t)T_{ij}\left[e(t) - e(t-d(t)) - \int_{t-d(t)}^{t}\dot{e}(s)d_s\right] = 0 \tag{31}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\xi^T(t)S_{ij}\left[e(t-d(t)) - e(t-d_M) - \int_{t-d_M}^{t-d(t)}\dot{e}(s)d_s\right] = 0 \tag{32}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\xi^T(t)W_{ij}\left[e(t) - e(t-\eta(t)) - \int_{t-\eta(t)}^{t}\dot{e}(s)d_s\right] = 0 \tag{33}$$

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\xi^T(t)V_{ij}\left[e(t-\eta(t)) - e(t-\eta_M) - \int_{t-\eta_M}^{t-\eta(t)}\dot{e}(s)d_s\right] = 0 \tag{34}$$

where N_{ij}, M_{ij}, T_{ij}, S_{ij}, W_{ij}, V_{ij} are matrices with appropriate dimensions, and

$$\xi^T(t) = \begin{bmatrix}\xi_1^T(t) & \xi_2^T(t)\end{bmatrix}$$
$$\xi_1^T(t) = \begin{bmatrix}e^T(t) & e^T(t-\tau_m) & e^T(t-\tau(t)) & e^T(t-\tau_M) & e^T(t-\eta(t))\end{bmatrix}$$
$$\xi_2^T(t) = \begin{bmatrix}e^T(t-\eta_M) & e^T(t-d(t)) & e^T(t-d_M) & e_k^T(t) & \omega^T(t)\end{bmatrix}$$

By Lemma 1, the following inequalities can be derived

$$-2\xi^T(t)M_{ij}\int_{t-\tau(t)}^{t-\tau_m}\dot{e}(s)ds \le (\tau(t)-\tau_m)\xi^T(t)M_{ij}R_1^{-1}M_{ij}^T\xi(t) + \int_{t-\tau(t)}^{t-\tau_m}\dot{e}^T(s)R_1\dot{e}(s)ds \tag{35}$$

$$-2\xi^T(t)N_{ij}\int_{t-\tau_M}^{t-\tau(t)}\dot{e}(s)ds \le (\tau_M-\tau(t))\xi^T(t)N_{ij}R_1^{-1}N_{ij}^T\xi(t) + \int_{t-\tau_M}^{t-\tau(t)}\dot{e}^T(s)R_1\dot{e}(s)ds \tag{36}$$

$$-2\xi^T(t)T_{ij}\int_{t-d(t)}^{t}\dot{e}(s)ds \le d(t)\xi^T(t)T_{ij}R_3^{-1}T_{ij}^T\xi(t) + \int_{t-d(t)}^{t}\dot{e}^T(s)R_3\dot{e}(s)ds \tag{37}$$

$$-2\xi^T(t)S_{ij}\int_{t-d_M}^{t-d(t)}\dot{e}(s)ds \le (d_M - d(t))\xi^T(t)S_{ij}R_3^{-1}S_{ij}^T\xi(t) + \int_{t-d_M}^{t-d(t)}\dot{e}^T(s)R_3\dot{e}(s)ds \quad (38)$$

$$-2\xi^T(t)W_{ij}\int_{t-\eta(t)}^{t}\dot{e}(s)ds \le \eta(t)\xi^T(t)W_{ij}R_4^{-1}W_{ij}^T\xi(t) + \int_{t-\eta(t)}^{t}\dot{e}^T(s)R_4\dot{e}(s)ds \quad (39)$$

$$-2\xi^T(t)V_{ij}\int_{t-\eta_M}^{t-\eta(t)}\dot{e}(s)ds \le (\eta_M - \eta(t))\xi^T(t)S_{ij}R_4^{-1}V_{ij}^T\xi(t) + \int_{t-\eta_M}^{t-\eta(t)}\dot{e}^T(s)R_4\dot{e}(s)ds \quad (40)$$

Combining (25)–(40), it can be obtained that

$$\begin{aligned}&\mathbb{E}\{\dot{V}(t) - \gamma^2 w^T(t)w(t) + \tilde{z}^T(t)\tilde{z}(t)\}\\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(s_kh))\Big\{\xi^T(t)\Omega_{11}^{ij}\xi(t) + \mathcal{A}^T\tilde{R}\mathcal{A} + \mu^2\mathcal{B}^T\tilde{R}\mathcal{B} + e^T(t)\bar{L}^T\bar{L}e(t) + (\tau(t)-\tau_m)\xi^T(t)M_{ij}R_1^{-1}M_{ij}^T\xi(t)\\ &+ (\tau_M - \tau(t))\xi^T(t)N_{ij}R_1^{-1}N_{ij}^T\xi(t) + d(t)\xi^T(t)T_{ij}R_3^{-1}T_{ij}^T\xi(t) + (d_M - d(t))\xi^T(t)S_{ij}R_3^{-1}S_{ij}^T\xi(t)\\ &+ \eta(t)\xi^T(t)W_{ij}R_4^{-1}W_{ij}^T\xi(t) + (\eta_M - \eta(t))\xi^T(t)V_{ij}R_4^{-1}V_{ij}^T\xi(t)\Big\}\end{aligned}$$

By using well-known Schur complement theory and Lemma 2, one can easily see that $\mathbb{E}\{\dot{V}(t) - \gamma^2 w^T(t)w(t) + \tilde{z}^T(t)\tilde{z}(t)\} < 0$ can be ensured by

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))\, h_j(\theta(s_kh))\, \Xi^{ij} < 0 \quad (41)$$

The slack matrix Π is introduced to relax the design results

$$\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))\left(h_j(\theta(s_kh)) - h_j(\theta(t))\right)\Pi = 0 \quad (42)$$

Since $\Xi^{ij} + \Pi > 0$ in (23), then, by substituting (42) into (41) and applying Assumption 1, one can get that

$$\begin{aligned}&\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))\left[h_j(\theta(t))\Xi^{ij} + (h_j(\theta(s_kh)) - h_j(\theta(t)))(\Xi^{ij} + \Pi)\right]\\ &\le \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\theta(t))h_j(\theta(t))\left[\Xi^{ij} + \sum_{l=1}^{r}\varpi_l(\Xi^{il} + \Pi)\right]\\ &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i^2(\theta(t))\,\Phi^{ii} + \sum_{i=1}^{r}\sum_{i<j\le S} h_i(\theta(t))\,h_j(\theta(t))\left(\Phi^{ij} + \Phi^{ji}\right)\end{aligned} \quad (43)$$

where $\Phi^{ij} = \Xi^{ij} + \sum_{l=1}^{r}\varpi_l(\Xi^{il} + \Pi)$.

According to (21)–(23), it is easily obtained that the inequality (41) holds, which can deduce that $\mathbb{E}\{\dot{V}(t) - \gamma^2 w^T(t)w(t) + \tilde{z}^T(t)\tilde{z}(t)\} < 0$. This completes the proof.

Through Theorem 1, sufficient conditions have been obtained which can guarantee the stability of system (14). The main difficulty is how to deal with the nonlinear terms of $H^T B_{fj}^T(I + \Delta_q)^T P$ and $B_{fj}^T(I + \Delta_q)^T P$ in (21)–(23) and Theorem 2 will be introduced to solve this problem.

Theorem 2. *For given time delays τ_m, τ_M, d_M, η_M, trigger parameter σ, quantized parameter δ, scalars γ, $\bar{\alpha}$, ε_k $(k = 1, 2, 3, 4)$ and ϖ_l $(l = 1, \ldots, r)$, system (14) is asymptotically stable if there exist variables $m_1 > 0$, $n_1 > 0$ and matrices $P_1 > 0$, $\bar{P}_3 > 0$, $\bar{Q}_k > 0$, $\bar{R}_k > 0$, $\Omega > 0$, $\bar{\Pi}$, $\bar{A}_{fj}$, B_{fj}, $\bar{C}_{fj}$, $\bar{M}_{ij}$, $\bar{N}_{ij}$, $\bar{T}_{ij}$, $\bar{S}_{ij}$, $\bar{W}_{ij}$, $\bar{V}_{ij}$ with appropriate dimensions, such that the following LMIs hold.*

$$\bar{\Phi}^{ii} < 0, i = 1, 2, \ldots, r \quad (44)$$

$$\bar{\Phi}^{ij} + \bar{\Phi}^{ji} < 0, i, j = 1, 2, \ldots, r, (i < j) \quad (45)$$

$$\bar{\Xi}^{ij} + \bar{\Pi} > 0, i, j = 1, 2, \ldots, r \quad (46)$$

$$P_1 - \bar{P}_3 > 0 \quad (47)$$

where

$$\bar{\Xi}^{ij} = \begin{bmatrix} \Phi_{11}^{ij} & * & * & * & * & * & * & * \\ \Phi_{21}^{ij} & \Phi_{22}^{ij} & * & * & * & * & * & * \\ \Phi_{31}^{ij} & 0 & \Phi_{33}^{ij} & * & * & * & * & * \\ \Phi_{41}^{ij}(s) & 0 & 0 & \Phi_{44}^{ij} & * & * & * & * \\ \Phi_{51}^{ij} & \Phi_{52}^{ij} & \Phi_{53}^{ij} & 0 & -m_2 I & * & * & * \\ \Phi_{61}^{ij} & 0 & 0 & 0 & 0 & -m_1 I & * & * \\ \Phi_{71}^{ij} & \Phi_{72}^{ij} & \Phi_{73}^{ij} & 0 & 0 & 0 & -n_2 I & * \\ \Phi_{81}^{ij} & 0 & 0 & 0 & 0 & 0 & 0 & -n_1 I \end{bmatrix}, (s = 1, \ldots, 8)$$

$$\Phi_{11}^{ij} = \begin{bmatrix} \bar{\Gamma}_{ij1} & * & * & * & * & * & * & * & * & * \\ \bar{R}_2 & \bar{\Gamma}_{ij2} & * & * & * & * & * & * & * & * \\ \Upsilon_{ij2} & \bar{M}_{ij3} - \bar{M}_{ij2}^T & \bar{\Gamma}_{ij3} & * & * & * & * & * & * & * \\ 0 & 0 & \bar{N}_{ij4} - \bar{N}_{ij3}^T & \bar{\Gamma}_{ij4} & * & * & * & * & * & * \\ \bar{\Gamma}_{ij9} & 0 & 0 & 0 & \bar{\Gamma}_{ij5} & * & * & * & * & * \\ 0 & 0 & 0 & 0 & \bar{V}_{ij6} - \bar{V}_{ij5}^T & \bar{\Gamma}_{ij6} & * & * & * & * \\ \bar{\Gamma}_{ij0} & 0 & 0 & 0 & 0 & 0 & \bar{\Gamma}_{ij7} & * & * & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \bar{S}_{ij8} - \bar{S}_{ij7}^T & \bar{\Gamma}_{ij8} & * & * \\ \bar{\alpha}_1 \Upsilon_{ij4} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Omega & * \\ \Upsilon_{ij5} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\bar{\Gamma}_{ij1} = \Upsilon_{ij1} + \Upsilon_{ij1}^T + \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3 + \bar{Q}_4 + -\bar{R}_2 + \bar{T}_{ij1} + \bar{T}_{ij1}^T + \bar{W}_{ij1} + \bar{W}_{ij1}^T, \Upsilon_{ij1} = \begin{bmatrix} P_1 A_i & \bar{A}_{fj} \\ \bar{P}_3 A_i & \bar{A}_{fj} \end{bmatrix}$$

$$\bar{\Gamma}_{ij2} = -\bar{Q}_1 - \bar{R}_2 + \bar{M}_{ij2} + \bar{M}_{ij2}^T, \bar{\Gamma}_{ij3} = -\bar{M}_{ij3} - \bar{M}_{ij3}^T + \bar{N}_{ij3} + \bar{N}_{ij3}^T, \bar{\Gamma}_{ij4} = -\bar{Q}_2 - \bar{N}_{ij4} - \bar{N}_{ij4}^T$$

$$\bar{\Gamma}_{ij5} = -\bar{W}_{ij5} - \bar{W}_{ij5}^T + \bar{V}_{ij5} + \bar{V}_{ij5}^T, \bar{\Gamma}_{ij6} = -\bar{Q}_4 - \bar{V}_{ij6} - \bar{V}_{ij6}^T, \bar{\Phi}^{ij} = \bar{\Xi}^{ij} + \sum_{l=1}^{r} \varpi_l (\bar{\Xi}^{il} + \bar{\Pi})$$

$$\bar{\Gamma}_{ij7} = \Upsilon_{ij8} - \bar{T}_{ij7} - \bar{T}_{ij7}^T + \bar{S}_{ij7} + \bar{S}_{ij7}^T, \Upsilon_{ij4} = \begin{bmatrix} \bar{B}_{fj} & \bar{B}_{fj} \end{bmatrix}, \Upsilon_{ij5} = \begin{bmatrix} A_{wi}^T P_1 & A_{wi}^T \bar{P}_3 \end{bmatrix}$$

$$\bar{\Gamma}_{ij8} = -\bar{Q}_3 - \bar{S}_{ij8} - \bar{S}_{ij8}^T, \bar{\Gamma}_{ij9} = \bar{\alpha} \Upsilon_{ij3} + \bar{W}_{ij5} - \bar{W}_{ij1}^T, \bar{\Gamma}_{ij0} = \bar{\alpha}_1 \Upsilon_{ij4} + \bar{T}_{ij7} - \bar{T}_{ij1}, \Upsilon_{ij6} = \begin{bmatrix} L_i & -\bar{C}_{fj} \end{bmatrix}$$

$$\Upsilon_{ij2} = \begin{bmatrix} A_{di}^T P_1 & A_{di}^T \bar{P}_3 \\ 0 & 0 \end{bmatrix}, \Upsilon_{ij3} = \begin{bmatrix} C_i^T \bar{B}_{fj} & C_i^T \bar{B}_{fj} \\ 0 & 0 \end{bmatrix}, \Upsilon_{ij7} = \begin{bmatrix} \sigma C_i^T \Omega C_i & 0 \\ 0 & 0 \end{bmatrix}, \bar{P} = \begin{bmatrix} P_1 & \bar{p}_3 \\ \bar{P}_3 & \bar{p}_3 \end{bmatrix}$$

$$\Phi_{21}^{ij} = \begin{bmatrix} \Upsilon_{ij6} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \sqrt{\tau_{21}} \Upsilon_{ij1} & 0 & \sqrt{\tau_{21}} \Upsilon_{ij2}^T & 0 & \bar{\alpha} \sqrt{\tau_{21}} \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \sqrt{\tau_{21}} \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \sqrt{\tau_{21}} \Upsilon_{ij4}^T & \sqrt{\tau_{21}} \Upsilon_{ij5}^T \\ \tau_m \Upsilon_{ij1} & 0 & \tau_m \Upsilon_{ij2}^T & 0 & \bar{\alpha} \tau_m \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \tau_m \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \tau_m \Upsilon_{ij4}^T & \tau_m \Upsilon_{ij5}^T \\ \sqrt{d_M} \Upsilon_{ij1} & 0 & \sqrt{d_M} \Upsilon_{ij2}^T & 0 & \bar{\alpha} \sqrt{d_M} \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \sqrt{d_M} \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \sqrt{d_M} \Upsilon_{ij4}^T & \sqrt{d_M} \Upsilon_{ij5}^T \\ \sqrt{\eta_M} \Upsilon_{ij1} & 0 & \sqrt{\eta_M} \Upsilon_{ij2}^T & 0 & \bar{\alpha} \sqrt{\eta_M} \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \sqrt{\eta_M} \Upsilon_{ij3}^T & 0 & \bar{\alpha}_1 \sqrt{\eta_M} \Upsilon_{ij4}^T & \sqrt{\eta_M} \Upsilon_{ij5}^T \end{bmatrix}$$

$$\Phi_{31}^{ij} = \begin{bmatrix} 0 & 0 & 0 & 0 & \mu \sqrt{\tau_{21}} \Upsilon_{ij3}^T & 0 & \mu \sqrt{\tau_{21}} \Upsilon_{ij3}^T & 0 & \mu \sqrt{\tau_{21}} \Upsilon_{ij4}^T & 0 \\ 0 & 0 & 0 & 0 & \mu \tau_m \Upsilon_{ij3}^T & 0 & \mu \tau_m \Upsilon_{ij3}^T & 0 & \mu \tau_m \Upsilon_{ij4}^T & 0 \\ 0 & 0 & 0 & 0 & \mu \sqrt{d_M} \Upsilon_{ij3}^T & 0 & \mu \sqrt{d_M} \Upsilon_{ij3}^T & 0 & \mu \sqrt{d_M} \Upsilon_{ij4}^T & 0 \\ 0 & 0 & 0 & 0 & \mu \sqrt{\eta_M} \Upsilon_{ij3}^T & 0 & \mu \sqrt{\eta_M} \Upsilon_{ij3}^T & 0 & \mu \sqrt{\eta_M} \Upsilon_{ij4}^T & 0 \end{bmatrix}$$

$$\Phi_{22}^{ij} = diag\{-I, -2\varepsilon_1 \bar{P} + \varepsilon_1^2 \bar{R}_1, -2\varepsilon_2 \bar{P} + \varepsilon_2^2 \bar{R}_2, -2\varepsilon_3 \bar{P} + \varepsilon_3^2 \bar{R}_3, -2\varepsilon_4 \bar{P} + \varepsilon_4^2 \bar{R}_4\}$$

$$\Phi_{33}^{ij} = diag\{-2\varepsilon_1 \bar{P} + \varepsilon_1^2 \bar{R}_1, -2\varepsilon_2 \bar{P} + \varepsilon_2^2 \bar{R}_2, -2\varepsilon_3 \bar{P} + \varepsilon_3^2 \bar{R}_3, -2\varepsilon_4 \bar{P} + \varepsilon_4^2 \bar{R}_4\}$$

$$\Phi_{41}^{ij}(1)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{M}_{ij}^T\\ \sqrt{d_M}\bar{T}_{ij}^T\\ \sqrt{\eta_M}\bar{W}_{ij}^T\end{bmatrix},\Phi_{41}^{ij}(2)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{M}_{ij}^T\\ \sqrt{d_M}\bar{T}_{ij}^T\\ \sqrt{(\eta_M)}\bar{V}_{ij}^T\end{bmatrix},\Phi_{41}^{ij}(3)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{M}_{ij}^T\\ \sqrt{d_M}\bar{S}_{ij}^T\\ \sqrt{\eta_M}\bar{W}_{ij}^T\end{bmatrix},\Phi_{41}^{ij}(4)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{M}_{ij}^T\\ \sqrt{d_M}\bar{S}_{ij}^T\\ \sqrt{\eta_M}\bar{V}_{ij}^T\end{bmatrix}$$

$$\Phi_{41}^{ij}(5)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{N}_{ij}^T\\ \sqrt{d_M}\bar{T}_{ij}^T\\ \sqrt{\eta_M}\bar{W}_{ij}^T\end{bmatrix},\Phi_{41}^{ij}(6)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{N}_{ij}^T\\ \sqrt{d_M}\bar{T}_{ij}^T\\ \sqrt{\eta_M}\bar{V}_{ij}^T\end{bmatrix},\Phi_{41}^{ij}(7)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{N}_{ij}^T\\ \sqrt{d_M}\bar{S}_{ij}^T\\ \sqrt{\eta_M}\bar{W}_{ij}^T\end{bmatrix},\Phi_{41}^{ij}(8)=\begin{bmatrix}\sqrt{\tau_{21}}\bar{N}_{ij}^T\\ \sqrt{d_M}\bar{T}_{ij}^T\\ \sqrt{\eta_M}\bar{V}_{ij}^T\end{bmatrix}$$

$$\begin{aligned}
\bar{M}_{ij}^T &= \begin{bmatrix}0 & \bar{M}_{ij2}^T & \bar{M}_{ij3}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \bar{N}_{ij}^T = \begin{bmatrix}0 & 0 & \bar{N}_{ij3}^T & \bar{N}_{ij4}^T & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
\bar{T}_{ij}^T &= \begin{bmatrix}\bar{T}_{ij1}^T & 0 & 0 & 0 & 0 & 0 & \bar{T}_{ij7}^T & 0 & 0 & 0\end{bmatrix}, \bar{S}_{ij}^T = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & \bar{S}_{ij7}^T & \bar{S}_{ij8}^T & 0 & 0\end{bmatrix}\\
\bar{W}_{ij}^T &= \begin{bmatrix}\bar{W}_{ij1}^T & 0 & 0 & 0 & \bar{W}_{ij5}^T & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \bar{V}_{ij}^T = \begin{bmatrix}0 & 0 & 0 & 0 & \bar{V}_{ij5}^T & \bar{V}_{ij6}^T & 0 & 0 & 0 & 0\end{bmatrix}\\
\Phi_{44}^{ij} &= diag\{-\bar{R}_1, -\bar{R}_3, -\bar{R}_4\}, \Phi_{51}^{ij} = \begin{bmatrix}\bar{\alpha}\Psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
\Phi_{52}^{ij} &= \begin{bmatrix}0 & \bar{\alpha}\sqrt{\tau_{21}}\Psi & \bar{\alpha}\tau_m\Psi & \bar{\alpha}\sqrt{d_M}\Psi & \bar{\alpha}\sqrt{\eta_M}\Psi\end{bmatrix}, m_2 = \frac{1}{m_1}\\
\Phi_{53}^{ij} &= \begin{bmatrix}\mu\sqrt{\tau_{21}}\Psi & \mu\tau_m\Psi & \mu\sqrt{d_M}\Psi & \mu\sqrt{\eta_M}\Psi\end{bmatrix}, n_2 = \frac{1}{n_1}\\
\Phi_{61}^{ij} &= \begin{bmatrix}0 & 0 & 0 & 0 & \Psi_1 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \Phi_{71}^{ij} = \begin{bmatrix}\bar{\alpha}_1\Psi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
\Phi_{72}^{ij} &= \begin{bmatrix}0 & \bar{\alpha}_1\sqrt{\tau_{21}}\Psi & \bar{\alpha}_1\tau_m\Psi & \bar{\alpha}_1\sqrt{d_M}\Psi & \bar{\alpha}_1\sqrt{\eta_M}\Psi\end{bmatrix}\\
\Phi_{73}^{ij} &= \begin{bmatrix}\mu\sqrt{\tau_{21}}\Psi & \mu\tau_m\Psi & \mu\sqrt{d_M}\Psi & \mu\sqrt{\eta_M}\Psi\end{bmatrix}\\
\Phi_{81}^{ij} &= \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & \Psi_1 & 0 & H & 0\end{bmatrix}, \Psi_1 = \begin{bmatrix}C_i & 0\end{bmatrix}, \Psi = \begin{bmatrix}\delta\bar{B}_{fj}^T & \delta\bar{B}_{fj}^T\end{bmatrix}
\end{aligned}$$

Moreover, if the conditions above are feasible, the parameter matrices of the filter are given by

$$\begin{cases}A_{fj} = \bar{A}_{fj}\bar{P}_3^{-1}\\ B_{fj} = \bar{B}_{fj} \qquad j = 1, 2, \ldots, r.\\ C_{fj} = \bar{C}_{fj}\bar{P}_3^{-1},\end{cases} \tag{48}$$

Proof. First of all, there exists an equation as following

$$\Xi^{ij} = \Sigma^{ij} + sym\{H_B^T\Delta_q H_K\} + sym\{H_C^T\Delta_q H_J\} \tag{49}$$

where

$$\Sigma^{ij} = \begin{bmatrix}\Omega_{11}^{ij} & * & * & \\ \Omega_{21}^{ij} & \Omega_{22}^{ij} & * & \\ \Omega_{31}^{ij} & 0 & \Omega_{33}^{ij} & \\ \Omega_{41}^{ij}(s) & 0 & 0 & \Omega_{44}^{ij}\end{bmatrix}, \breve{C} = \begin{bmatrix}C_i & 0\end{bmatrix}, \breve{B} = \begin{bmatrix}B_{fj}P_2\\ B_{fj}P_3\end{bmatrix}$$

$$\begin{aligned}
H_B &= \begin{bmatrix}H_{B1} & H_{B2} & H_{B3} & 0 & 0\end{bmatrix}, H_K = \begin{bmatrix}H_{K1} & H_{K2} & 0 & 0\end{bmatrix}\\
H_C &= \begin{bmatrix}H_{C1} & H_{C2} & H_{C3} & 0 & 0\end{bmatrix}, H_J = \begin{bmatrix}H_{J1} & H_{J2} & 0 & 0\end{bmatrix}\\
H_{B1} &= \begin{bmatrix}\delta\bar{\alpha}\breve{B}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
H_{B2} &= \begin{bmatrix}0 & \bar{\alpha}\delta\sqrt{\tau_{21}}\breve{B}^T & \bar{\alpha}\delta\tau_m\breve{B}^T & \bar{\alpha}\delta\sqrt{d_M}\breve{B}^T & \bar{\alpha}\delta\sqrt{\eta_M}\breve{B}^T\end{bmatrix}\\
H_{B3} &= \begin{bmatrix}\mu\delta\sqrt{\tau_{21}}\breve{B}^T & \mu\delta\tau_m\breve{B}^T & \mu\delta\sqrt{d_M}\breve{B}^T & \mu\delta\sqrt{\eta_M}\breve{B}^T & 0\end{bmatrix}\\
H_{K1} &= \begin{bmatrix}0 & 0 & 0 & 0 & \breve{C} & 0 & 0 & 0 & 0 & 0\end{bmatrix}, H_{K2} = \begin{bmatrix}0_{1\times 10}\end{bmatrix}\\
H_{C1} &= \begin{bmatrix}\delta\bar{\alpha}_1\breve{B}^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}\\
H_{C2} &= \begin{bmatrix}0 & \bar{\alpha}_1\delta\sqrt{\tau_{21}}\breve{B}^T & \bar{\alpha}_1\delta\tau_m\breve{B}^T & \bar{\alpha}_1\delta\sqrt{d_M}\breve{B}^T & \bar{\alpha}_1\delta\sqrt{\eta_M}\breve{B}^T\end{bmatrix}\\
H_{C3} &= \begin{bmatrix}\mu\delta\sqrt{\tau_{21}}\breve{B}^T & \mu\delta\tau_m\breve{B}^T & \mu\delta\sqrt{d_M}\breve{B}^T & \mu\delta\sqrt{\eta_M}\breve{B}^T & 0\end{bmatrix}\\
H_{J1} &= \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & \breve{C} & 0 & I & 0\end{bmatrix}, H_{J2} = \begin{bmatrix}0_{1\times 10}\end{bmatrix}
\end{aligned}$$

By using Lemma 3, the following inequality can be acquired.

$$\Sigma^{ij} + m_1 H_B^T\Delta_q^2 H_B + m_1^{-1}H_K^T H_K + n_1 H_C^T\Delta_q^2 H_C + n_1^{-1}H_J^T H_J < 0 \tag{50}$$

that is

$$\Sigma^{ij} + m_1\delta^2 H_B^T H_B + m_1^{-1} H_K^T H_K + n_1\delta^2 H_C^T H_C + n_1^{-1} H_J^T H_J < 0 \tag{51}$$

Due to $(R_k - \varepsilon_k^{-1}P)R_k^{-1}(R_k - \varepsilon_k^{-1}P) \geq 0, (k = 1, 2, 3, 4)$, it can be obtained that $-PR_k^{-1}P \leq -2\varepsilon_k P + \varepsilon_k^2 R_k$.

Apply Schur complement on inequality (51) and substitute $-PR_k^{-1}P$ with $-2\varepsilon_k P + \varepsilon_k^2 R_k$ into (21), then, $\bar{\Sigma}^{ij} + \bar{\Sigma}^{ji} < 0$ can be derived.

$$\bar{\Sigma}^{ij} = \begin{bmatrix} \tilde{\Omega}_{11}^{ij} & * & * & * & * & * & * & * \\ \tilde{\Omega}_{21}^{ij} & \bar{\Omega}_{22}^{ij} & * & * & * & * & * & * \\ \tilde{\Omega}_{31}^{ij} & 0 & \Omega_{33}^{ij} & * & * & * & * & * \\ \Omega_{41}^{ij}(s) & 0 & 0 & \Omega_{44}^{ij} & * & * & * & * \\ \tilde{\Omega}_{51}^{ij} & \tilde{\Omega}_{52}^{ij} & \tilde{\Omega}_{53}^{ij} & 0 & -m_2 I & * & * & * \\ \tilde{\Omega}_{61}^{ij} & 0 & 0 & 0 & 0 & -m_1 I & * & * \\ \tilde{\Omega}_{71}^{ij} & \tilde{\Omega}_{72}^{ij} & \tilde{\Omega}_{73}^{ij} & 0 & 0 & 0 & -n_2 I & * \\ \tilde{\Omega}_{81}^{ij} & 0 & 0 & 0 & 0 & 0 & 0 & -n_1 I \end{bmatrix}, (s = 1, \ldots, 8)$$

$$\bar{\Omega}_{22}^{ij} = diag\{-I, -2\varepsilon_1 P + \varepsilon_1^2 R_1, -2\varepsilon_2 P + \varepsilon_2^2 R_2, -2\varepsilon_3 P + \varepsilon_3^2 R_3, -2\varepsilon_4 P + \varepsilon_4^2 R_4\}$$

Since $\bar{P}_3 > 0$, there exist P_2 and $P_3 > 0$ satisfying $\bar{P}_3 = P_2 P_3^{-1} P_2^T$.

Define

$$P = \begin{bmatrix} P_1 & P_2 \\ P_2^T & P_3 \end{bmatrix}, J = \begin{bmatrix} I & 0 \\ 0 & P_2 P_3^{-1} \end{bmatrix}, \digamma = diag\{\underbrace{J, \ldots, J}_{8}, I, I, I, \underbrace{J, \ldots, J}_{11}, I, I, I, I\}$$

By Schur complement, $P>0$ is equivalent to $P_1 - \bar{P}_3 > 0$. Multiply (45) by $\digamma$ from the left side and its transpose from the right side, and define $\bar{P} = JPJ^T = \begin{bmatrix} P_1 & \bar{P}_3 \\ \bar{P}_3 & \bar{P}_3 \end{bmatrix}$, $\bar{Q}_k = JQ_kJ^T$, $\bar{R}_k = JR_kJ^T (k = 1, 2, 3, 4)$, $\bar{M}_{ijv1} = JM_{ijv1}J^T$, $\bar{N}_{ijv2} = JN_{ijv2}J^T$, $\bar{T}_{ijv3} = JT_{ijv3}J^T$, $\bar{S}_{ijv4} = JS_{ijv4}J^T$, $\bar{W}_{ijv5} = JW_{ijv4}J^T$, $\bar{V}_{ijv4} = JV_{ijv6}J^T$, $(v_1 = 2, 3; v_2 = 3, 4; v_3 = 1, 7; v_4 = 7, 8; v_5 = 1, 5; v_6 = 5, 6)$, then, the inequalities (44)–(46) can be derived. Define variables

$$\begin{cases} \bar{A}_{fj} = \hat{A}_{fj}\bar{P}_3, \hat{A}_{fj} = P_2^T A_{fj} P_2^{-T} \\ \bar{B}_{fj} = P_2^T B_{fj} \\ \bar{C}_{fj} = \hat{C}_{fj}\bar{P}_3, \hat{C}_{fj} = C_{fj} P_2^{-T} \end{cases} \tag{52}$$

Based on the descriptions above, the filter parameters (A_{fj}, B_{fj}, C_{fj}) can be substituted by $(P_2^{-T}\hat{A}_{fj}P_2^T, P_2^{-T}\bar{B}_{fj}, \hat{C}_{fj}P_2^T)$ in (12), then, the filter model can be rewritten as

$$\begin{cases} \dot{x}_f(t) = P_2^{-T}\hat{A}_{fj}P_2^T x_f(t) + P_2^{-T}\bar{B}_{fj}\hat{y}(t) \\ z_f(t) = \hat{C}_{fj}P_2^T x_f(t) \end{cases} \tag{53}$$

Define $\hat{x}(t) = P_2^T x_f(t)$, then, equality (53) can be rewritten as follows.

$$\begin{cases} \dot{\hat{x}}(t) = \hat{A}_{fj}\hat{x}(t) + \bar{B}_{fj}\hat{y}(t) \\ z_f(t) = \hat{C}_{fj}\hat{x}(t) \end{cases} \tag{54}$$

That is $(\hat{A}_{fj}, \bar{B}_{fj}, \hat{C}_{fj})$ can be chosen as the filter parameters.

Remark 4. The increasing number of variables can lead to the larger size of LMIs, furthermore, the complexity of the approach will be added with higher number of variables and larger size of LMIs. Specifically, in this paper, Theorem 2 presents the explicit design algorithm by using LMIs techniques. The size of LMIs can be easily calculated as $(39m+5n+s)\times(39m+5n+s)$ (m, n and s are positive integers). Obviously, with the increase of variables m, n and s, the size of the LMIs is enlarged, which results in the greater complexity of the approach. The feasible solutions to LMIs in Theorem 2 is recognized to be numerically difficult to find especially for high dimensional systems.

Remark 5. The LMIs (44)–(47) of Theorem 2 are feasible for small enough delay bounds τ_M, η_M and d_M, small enough quantization error Δq and small enough trigger parameter σ. Indeed, it is more difficult to have feasible solutions if the values of τ_M, η_M and d_M are larger. The smaller quantization error Δq makes the feasible solutions of LMIs in Theorem 2 easier to be found. Whether the LMIs of Theorem 2 have feasible solutions is also dependent on the trigger parameter σ in the hybrid-triggered scheme. The frequency of data transmission is determined by σ, the larger σ is, the less data is released, then the feasible solutions of LMIs in Theorem 2 are harder to find.

4. Simulation examples

In this section, a numerical simulation and a physical example are given to demonstrate the effectiveness of designed filter.

Example 1. Consider the following T–S fuzzy system.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i(\theta(t))A_i x(t) + \sum_{i=1}^{2} h_i(\theta(t))A_{di} x(t-\tau(t)) + \sum_{i=1}^{2} h_i(\theta(t))A_{\omega i}\omega(t) \\ y(t) = \sum_{i=1}^{2} h_i(\theta(t))C_i x(t) \\ z(t) = \sum_{i=1}^{2} h_i(\theta(t))L_i x(t) \end{cases}$$

where

$$A_1 = \begin{bmatrix} -2.1 & 0.1 \\ 1 & -2 \end{bmatrix}, A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.2 & -1.1 \end{bmatrix}, A_{d1} = \begin{bmatrix} -1.1 & 0.1 \\ -0.8 & -0.9 \end{bmatrix}, A_{d2} = \begin{bmatrix} -0.9 & 0 \\ -1.1 & -1.2 \end{bmatrix}$$

$$A_{\omega 1} = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix}, A_{\omega 2} = \begin{bmatrix} 0.3 \\ 0.1 \end{bmatrix}, C_1 = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_2 = \begin{bmatrix} 0.5 & -0.6 \end{bmatrix}, L_1 = \begin{bmatrix} 1 & -0.5 \end{bmatrix}$$

$$L_2 = \begin{bmatrix} -0.2 & 0.3 \end{bmatrix}, h_1(\theta(t)) = sin^2 t, h_2(\theta(t)) = cos^2 t, \omega(t) = \begin{cases} 1, & 5 \le t \le 10 \\ -1, & 15 \le t \le 20 \\ 0, & else \end{cases}$$

Choose the initial condition $x(0) = \begin{bmatrix} 1 & -1 \end{bmatrix}^T$, $x_f(0) = \begin{bmatrix} 0.8 & -0.8 \end{bmatrix}^T$, sampling period $h = 0.1$. In this section, three cases are discussed to demonstrate the effectiveness of designed filter.

Case 1. When the system is under a non-ideal networked environment, the time-triggered scheme is adopted for data transmission. Let $\bar{\alpha} = 1$, $\tau_m = 0.1$, $\tau_M = 0.4$, $d_M = 0.3$, $\eta_M = 0.2$, $\gamma = 1.2$, $\delta = 0.818$, $\varpi_1 = \varpi_2 = 0.2$. By using LMI toolbox in MATLAB, then, the corresponding trigger matrix $\Omega = 16.8250$ and the following matrices can be derived as follows.

$$P_1 = \begin{bmatrix} 1.0614 & 0.1913 \\ 0.1913 & 1.8360 \end{bmatrix}, \bar{P}_3 = \begin{bmatrix} 0.6973 & 0.0552 \\ 0.0552 & 0.9435 \end{bmatrix}, \bar{A}_{f1} = \begin{bmatrix} -0.8859 & -0.0896 \\ 0.4625 & -1.2873 \end{bmatrix}, \bar{B}_{f1} = \begin{bmatrix} 0.0799 \\ -0.2247 \end{bmatrix}$$

$$\bar{C}_{f1} = \begin{bmatrix} -0.7631 & 0.2665 \end{bmatrix}, \bar{A}_{f2} = \begin{bmatrix} -1.0673 & -0.0159 \\ -0.2394 & -1.0438 \end{bmatrix}, \bar{B}_{f2} = \begin{bmatrix} -0.0573 \\ 0.1964 \end{bmatrix}, \bar{C}_{f2} = \begin{bmatrix} 0.1882 & -0.0751 \end{bmatrix}$$

With the application of the filter design method developed in Theorem 2, the filter parameters are obtained as

$$A_{f1} = \begin{bmatrix} -1.2689 & 0.0206 \\ 0.7750 & -1.4097 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.0799 \\ -0.2247 \end{bmatrix}, C_{f1} = \begin{bmatrix} -1.1220 & 0.3481 \end{bmatrix}$$

$$A_{f2} = \begin{bmatrix} -1.5365 & -0.0731 \\ -0.2569 & -1.0912 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0.0573 \\ 0.1964 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.2688 & -0.0953 \end{bmatrix}$$

From Figs. 2 and 3, one can see that system (14) with quantization can stay stable under the time-triggered scheme.

Case 2. Suppose that the event-triggered scheme is chosen for data transmission. Set $\bar{\alpha} = 0$ and trigger parameter $\sigma = 0.9$, $\tau_m = 0.1$, $\tau_M = 0.4$, $d_M = 0.3$, $g_M = 0.2$, $\gamma = 1.2$, $\delta = 0.818$, $\varpi_1 = \varpi_2 = 0.2$, we can get the corresponding trigger matrix $\Omega = 0.1130$ and the following matrices by using MATLAB.

$$P_1 = \begin{bmatrix} 1.0301 & 0.1654 \\ 0.1654 & 1.7212 \end{bmatrix}, \bar{P}_3 = \begin{bmatrix} 0.6831 & 0.0447 \\ 0.0447 & 0.8984 \end{bmatrix}, \bar{A}_{f1} = \begin{bmatrix} -0.9023 & -0.0718 \\ 0.4923 & -1.2508 \end{bmatrix}, \bar{B}_{f1} = \begin{bmatrix} 0.0189 \\ -0.0653 \end{bmatrix}$$

$$\bar{C}_{f1} = \begin{bmatrix} -0.7950 & 0.2612 \end{bmatrix}, \bar{A}_{f2} = \begin{bmatrix} -1.0654 & 0.0110 \\ -0.2479 & -0.9920 \end{bmatrix}, \bar{B}_{f2} = \begin{bmatrix} 0.0002 \\ 0.0238 \end{bmatrix}, \bar{C}_{f2} = \begin{bmatrix} 0.1868 & -0.0747 \end{bmatrix}$$

By applying equality (48) in Theorem 2, the filter parameters can be derived as follows.

$$A_{f1} = \begin{bmatrix} -1.3199 & -0.0143 \\ 0.8143 & -1.4328 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.0189 \\ -0.0653 \end{bmatrix}, C_{f1} = \begin{bmatrix} -1.1866 & 0.3498 \end{bmatrix}$$

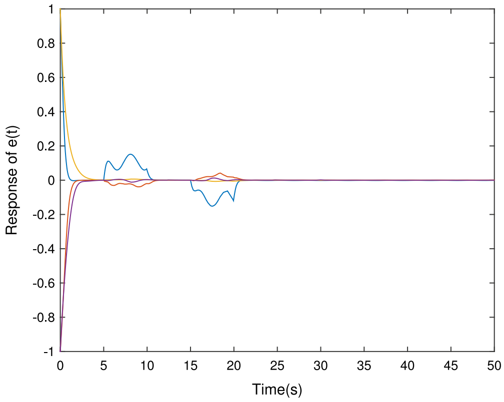

Fig. 2. Response of $e(t)$ in Case 1.

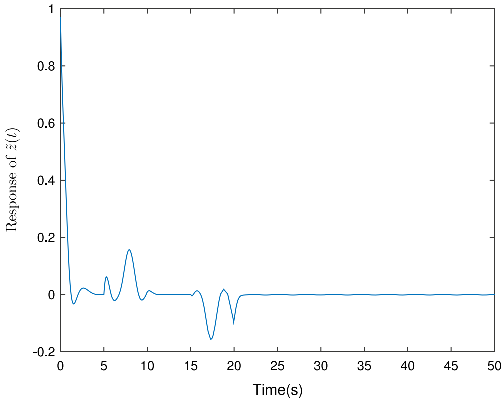

Fig. 3. Response of $\tilde{z}(t)$ in Case 1.

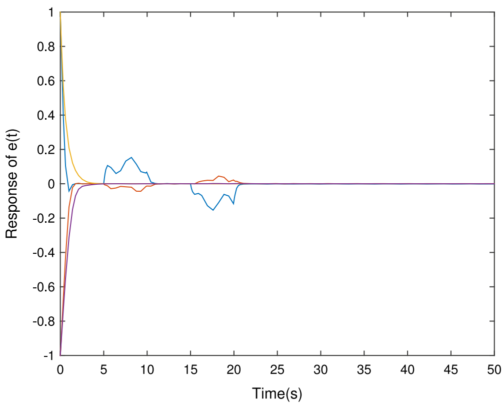

Fig. 4. Response of $e(t)$ in Case 2.

$$A_{f2} = \begin{bmatrix} -1.5655 & 0.0900 \\ -0.2916 & -1.0897 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0.0002 \\ 0.0238 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.2797 & -0.0971 \end{bmatrix}$$

From Figs. 4–6, it is easy to get that the system (14) can stay stable under the event-triggered scheme and quantization.

Case 3. When the hybrid-driven scheme is applied, set $\bar{\alpha} = 0.5$ and trigger parameter $\sigma = 0.9$, $\tau_m = 0.1$, $\tau_M = 0.4$, $d_M = 0.3$, $g_M = 0.2$, $\gamma = 1.2$, $\delta = 0.818$, $\varpi_1 = \varpi_2 = 0.2$. By applying MATLAB, the related trigger matrix $\Omega = 0.0908$ and

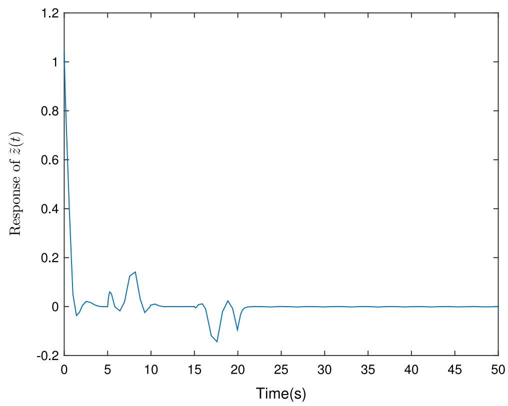

Fig. 5. Response of $\tilde{z}(t)$ in Case 2.

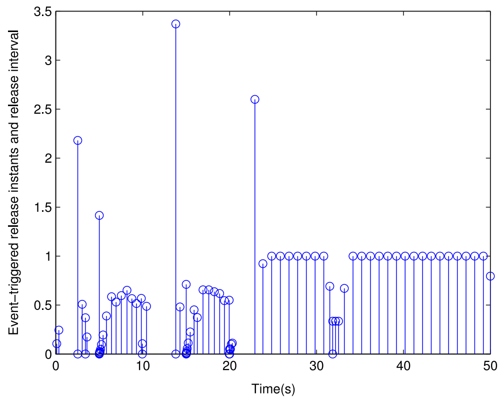

Fig. 6. Event-triggered release instants and intervals in Case 2.

the matrices can be obtained as follows.

$$P_1 = \begin{bmatrix} 1.0316 & 0.1675 \\ 0.1675 & 1.7288 \end{bmatrix}, \bar{P}_3 = \begin{bmatrix} 0.6869 & 0.0466 \\ 0.0466 & 0.9082 \end{bmatrix}, \bar{A}_{f1} = \begin{bmatrix} -0.9111 & -0.0751 \\ 0.5181 & -1.2690 \end{bmatrix}, \bar{B}_{f1} = \begin{bmatrix} 0.0316 \\ -0.0846 \end{bmatrix}$$

$$\bar{C}_{f1} = \begin{bmatrix} -0.7974 & 0.2592 \end{bmatrix}, \bar{A}_{f2} = \begin{bmatrix} -1.0812 & 0.0133 \\ -0.2510 & -1.0082 \end{bmatrix}, \bar{B}_{f2} = \begin{bmatrix} 0.0004 \\ 0.0330 \end{bmatrix}, \bar{C}_{f2} = \begin{bmatrix} 0.1877 & -0.0747 \end{bmatrix}$$

The filter parameters can be obtained by using the design algorithm (48) in Theorem 2.

$$A_{f1} = \begin{bmatrix} -1.3254 & -0.0146 \\ 0.8521 & -1.4411 \end{bmatrix}, B_{f1} = \begin{bmatrix} 0.0316 \\ -0.0846 \end{bmatrix}, C_{f1} = \begin{bmatrix} -1.1845 & 0.3463 \end{bmatrix}$$

$$A_{f2} = \begin{bmatrix} -1.5805 & 0.0959 \\ -0.2911 & -1.0951 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0.0004 \\ 0.0330 \end{bmatrix}, C_{f2} = \begin{bmatrix} 0.2798 & -0.0967 \end{bmatrix}$$

Based on Figs. 7 and 8, one can see that H_∞ filter error system (14) can stay stable with the hybrid-driven scheme and quantization.

According to Cases 1–3 illustrated above, Fig. 6 depicts the event-triggered release instants and intervals, and response of $\tilde{z}(t)$ under three different schemes are represented in Figs. 3, 5 and 8, respectively. The simulation results of discussed T–S fuzzy system with hybrid-driven scheme and quantization are shown in Figs. 2, 4 and 7 which demonstrate the stability of the filter error system (14). In other words, the burden of networked transmission can be effectively mitigated with the hybrid-driven strategy and the feasibility of the designed filter is illustrated.

Example 2. Consider a tunnel diode circuit in [44], which can be shown as follows.

$$\begin{cases} C\dot{v}_c(t) = -0.002v_c(t) - 0.01v_c(t)^3 + i_L(t) \\ y(t) = v_c(t) \\ L\dot{i}_L(t) = -v_c(t) - Ri_L(t) + w(t) \end{cases} \tag{55}$$

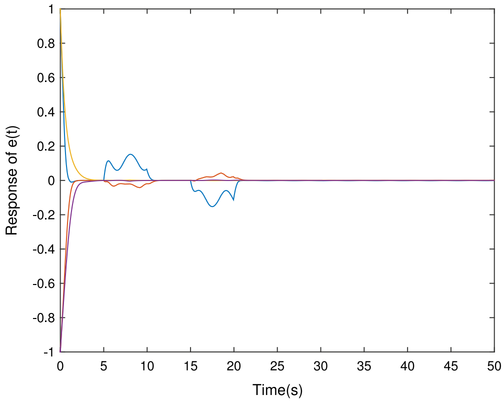

Fig. 7. Response of $e(t)$ in Case 3.

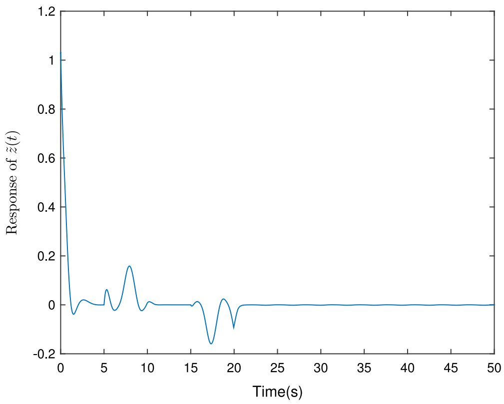

Fig. 8. Response of $\tilde{z}(t)$ in Case 3.

where $v_c(t)$ represents the voltage of the capacitor, $i_L(t)$ represents the current of the inductor, $w(t)$ represents the disturbance, $y(t)$ represents the sampled-data measurement output. Set the parameters of the capacitor, inductor and resistance $C = 20$ mF, $L = 1000$ mH and $R = 10\ \Omega$, respectively.

Suppose that $v_c(t) \in [-3, 3]$, the tunnel diode circuit can be represented in the T–S fuzzy system with $x(t) = [x_1(t) \quad x_2(t)]^T = [v_c(t) \quad i_L(t)]^T$ and $z(t) = x_1(t)$.

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} h_i(\theta(t))(A_i x(t) + A_{wi}(t)) \\ y(t) = \sum_{i=1}^{2} h_i(\theta(t)) C_i x(t) \\ z(t) = \sum_{i=1}^{2} h_i(\theta(t)) L_i x(t) \end{cases} \tag{56}$$

where

$$A_1 = \begin{bmatrix} -0.1 & 50 \\ -1 & -10 \end{bmatrix}, A_2 = \begin{bmatrix} -4.6 & 50 \\ -1 & -10 \end{bmatrix}, A_{d1} = A_{d2} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, A_{w1} = A_{w2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_1 = C_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, L1 = L2 = \begin{bmatrix} 1 & 0 \end{bmatrix}, h_1(\theta(t)) = sin^2 t, h_2(\theta(t)) = cos^2 t$$

$$\omega(t) = \begin{cases} 1, & 5 \le t \le 10 \\ -1, & 15 \le t \le 20 \\ 0, & else \end{cases}$$

Set $\bar{\alpha} = 0.1$ and triggered parameter $\sigma = 0.9$, the system (56) is under the environment with hybrid-driven scheme. Let $\tau_m = 0.1, \tau_M = 0.5, d_M = 0.5, g_M = 0.2, \gamma = 1.2, \delta = 0.818, \varpi_1 = \varpi_2 = 0.2$, then we can obtain the corresponding trigger

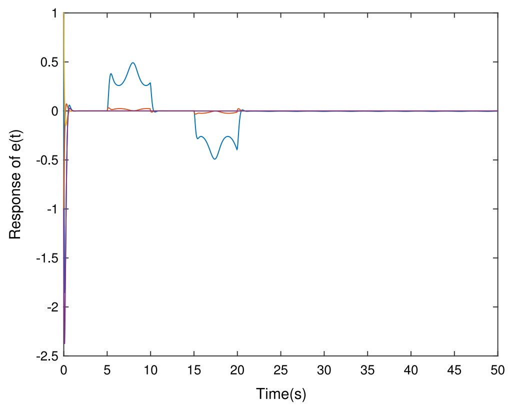

Fig. 9. Response of $e(t)$ in Example 2.

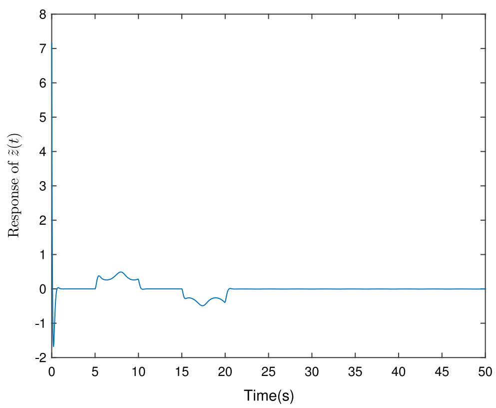

Fig. 10. Response of $\tilde{z}(t)$ in Example 2.

matrix $\Omega = 0.0022$ and the following matrices with the application of MATLAB.

$$P_1 = \begin{bmatrix} 0.2594 & 0.7660 \\ 0.7660 & 9.3286 \end{bmatrix}, \bar{P}_3 = \begin{bmatrix} 0.2147 & 0.6856 \\ 0.6856 & 8.5573 \end{bmatrix}, \bar{A}_{f1} = \begin{bmatrix} -0.7017 & 3.9675 \\ -7.1019 & -49.0094 \end{bmatrix}, \bar{B}_{f1} = \begin{bmatrix} -0.0016 \\ 0.0046 \end{bmatrix}$$

$$\bar{C}_{f1} = \begin{bmatrix} -0.9523 & -0.4430 \end{bmatrix}, \bar{A}_{f2} = \begin{bmatrix} -1.8638 & 3.3209 \\ -10.3430 & -48.8191 \end{bmatrix}, \bar{B}_{f2} = \begin{bmatrix} 0.0003 \\ 0.0058 \end{bmatrix}, \bar{C}_{f2} = \begin{bmatrix} -0.8825 & -0.1394 \end{bmatrix}$$

By using the filter design method developed in Theorem 2, the filter parameters for the tunnel diode circuit can be obtained as

$$A_{f1} = \begin{bmatrix} -6.3805 & 0.9749 \\ -19.8689 & -4.1352 \end{bmatrix}, B_{f1} = \begin{bmatrix} -0.0016 \\ 0.0046 \end{bmatrix}, C_{f1} = \begin{bmatrix} -5.7372 & 0.4079 \end{bmatrix}$$

$$A_{f2} = \begin{bmatrix} -13.3286 & 1.4560 \\ -40.2472 & -2.4802 \end{bmatrix}, B_{f2} = \begin{bmatrix} 0.0003 \\ 0.0058 \end{bmatrix}, C_{f2} = \begin{bmatrix} -5.4527 & 0.4206 \end{bmatrix}$$

When keeping all the other parameters at their default values, it can be obtained that the maximum value $\tau_M = 0.5211$ and the minimum value of interference parameter $\gamma = 1.2$ by different trials. The simulation results for the tunnel diode circuit which is approximate by T–S fuzzy model are shown in Figs. 9 and 10. According to Fig. 9, one can see that the system can be stable under the hybrid-driven scheme and Fig. 10 shows the response of $\tilde{z}(t)$. Therefore, the physical example illustrates the validity of the desired filter.

5. Conclusion

This paper investigates the problem of filter design for T–S fuzzy systems with hybrid-driven scheme and quantization. A mathematical model of filter error system is constructed by considering the influence of hybrid-driven scheme and quantization, which can alleviate the burden of networked transmission and improve the efficiency of data communication.

By using Lyapunov functional approach and LMI techniques, sufficient conditions are derived to guarantee the stability of system and explicit parameters of desired filter are obtained. Finally, illustrative examples are given to demonstrate the usefulness of desired H_∞ filter.

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