Finite-Time H_∞ Filtering for State-Dependent Uncertain Systems With Event-Triggered Mechanism and Multiple Attacks

Jinliang Liu, Meng Yang, Xiangpeng Xie, Chen Peng, *Senior Member, IEEE*, and Huaicheng Yan

***Abstract*—This paper is concerned with finite-time H_∞ filtering problem for networked state-dependent uncertain systems with event-triggered mechanism and multiple attacks, which consists of deception attacks, denial-of-service (DoS) attacks and replay attacks. A novel multiple attacks model is firstly proposed for state-dependant uncertain systems with consideration of the randomly occurring replay attacks, DoS attacks and deception attacks in a unified framework. In order to save the limited resource, an event-triggered mechanism is used to ease the communication burden in real communication environment. Then, by using Lyapunov-Krasocskii stability theory and linear matrix inequality techniques, sufficient conditions guaranteeing the exponentially mean-square finite-time boundedness of filtering error systems are obtained. Moreover, the explicit expression is derived for the parameters of the desired finite-time filter. Finally, two illustrative examples are employed to demonstrate the validity and applicability of the proposed theoretical approach in electronic circuits.**

***Index Terms*—Finite-time boundedness, state-dependent uncertain system, event-triggered mechanism, multiple attacks, electronic circuits.**

I. Introduction

IN THE last few years, state-dependent uncertain systems, as a kind of nonlinear systems, have been well investigated due to its widespread applications in electronic circuits systems [1], mechanical systems [2] and spring damping systems [3]. In general, actual systems are usually encountered with uncertainties due to system parametric variations, dynamic perturbations and disturbance. A lot of significant results have been developed on various issues of state-dependent uncertain systems [4]–[6]. For example, in [5], the event-driven H_∞ control problem is investigated for state-dependent uncertain system in the sense of input-to-state stability. Based on the convex optimization, the event-triggered robust filter design problem is discussed in [6] for a class of state-dependent uncertain systems with sampling and network transmission delay. In [7], the problem of robust control for state-dependent uncertain systems is considered.

On another frontier, the studies on finite-time stability have received an enormous amount of attention [8]–[11]. Compared with the Lyapunov asymptotic or exponential stability, the finite-time stability implies that the system states must be kept within specified bounds in the finite-time interval. Specially, in practical situations, the dynamic behavior of the systems is not allowed to exceed prescribed bounds during a fixed time interval. Therefore, based on the finite-time stability, extensive researches have been conducted on the finite-time boundedness [12]–[14]. For instance, the authors in [14] investigate the problem of finite-time filtering for wireless networked multirate systems with fading channels. In [15], the problem of finite-horizon filtering is considered for nonlinear time-delayed systems with an energy harvesting sensor. The authors in [16] address the finite-time energy-to-peak filtering problem for Markov jump systems with memory packet dropouts. Owing to its practical point of view, the study on the finite-time filter design problem for state-dependent uncertain systems has great theoretical and practical significance, which is the first motivation of our investigation.

With the development of network technology and control theory, networked systems have been increasingly emphasized [17]–[21]. However, the use of communication networks not only brings many advantages, but also introduces some challenging problems. To mention a few, the sampling with a fixed period may bring unnecessary data transmission; the finite property of network bandwidth usually results network-induced delays and packet dropouts; and cyber-attacks inevitably occur when the measurement signals are transmitted though the unreliable communication network. In order to overcome the problem of limited communication resource, a great number of results on event-triggered mechanism have been reported over the past few years [22]–[24]. For example, the authors in [24] are concerned with the problem of event-triggered dissipative filtering for discrete-time singular

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J. Liu is with the College of Information Engineering, Nanjing University of Finance and Economics, Nanjing 210023, China, and also with the College of Automation Electronic Engineering, Qingdao University of Science and Technology, Qingdao 266061, China (e-mail: liujinliang@vip.163.com).

M. Yang is with the College of Information Engineering, Nanjing University of Finance and Economics, Nanjing 210023, China (e-mail: mengyang9505@163.com).

X. Xie is with the Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210023, China (e-mail: xiexiangpeng1953@163.com).

C. Peng is with the School of Mechatronics Engineering and Automation, Shanghai University, Shanghai 200072, China (e-mail: c.peng@i.shu.edu.cn).

H. Yan is with the Key Laboratory of Advanced Control and Optimization for Chemical Process of Ministry of Education, East China University of Science and Technology, Shanghai 200237, China (e-mail: hcyan@ecust.edu.cn).

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neural networks with Markovian jump parameters. In [25], the quantized stabilization for Takagi-Sugeno fuzzy systems is investigated with stochastic cyber-attacks and hybrid-triggered mechanism. The problems of event-triggered mixed H_∞ and passive filter design are discussed in [26] for discrete-time singular stochastic network systems with Markovian jump parameters. So far, the event-based finite-time filter design problem has not brought to the enough attention for state-dependent uncertain systems, which is another motivation of this paper.

As we all know, cyber-attacks are one of the most significant factors threatening the system security. In general, three main types of cyber-attacks, namely replay attacks, DoS attacks and deception attacks, have been researched extensively [27]–[29]. For example, linear quadratic Gaussian controller for cyber-physical systems is designed in [30], where the replay attacks randomly take place. The hybrid-driven-based H_∞ controller design problem is investigated in [31] for networked cascade control systems with deception attacks. Event-based security controller design problem is addressed in [32] for state-dependent uncertain systems with hybrid-attacks. However, many studies on system security are limited to the single cyber-attacks, e.g. either replay attacks or deception attacks. Up to the author's knowledge, finite-time filter design for networked systems with randomly occurring multiple attacks remains an open issue, which motivates this study.

Inspired by the above observations, this paper focuses on the design of a finite-time H_∞ filter for state-dependant uncertain systems with event-triggered mechanism and multiple attacks. The primary contributions of this paper can be highlighted as follows:

1) An event-based H_∞ filter design problem is, for the first time, investigated for a class of state-dependant uncertain systems with randomly occurring uncertainty and multiple attacks over a finite-time boundedness;
2) A new multiple attacks model for state-dependent uncertain systems is firstly proposed to consider the influences of randomly occurring DoS attacks, replay attacks and deception attacks;
3) An explicit expression of the desired filter parameters is obtained. The design filter can be resilient to the randomly occurring multiple attacks, which consists of deception attacks, DoS attacks, replay attacks.

Notation: $\mathbb{N}$ and $\mathbb{R}^n$ represent the set of all non-negative integers and the n-dimensional Euclidean space, respectively. For a matrix S, S^T denotes its transposition. $S > 0$ ($S \geq 0$) means that S is real symmetric positive definite. For any a matrix P and two symmetric matrices S, A, $\begin{bmatrix} S & * \\ P & A \end{bmatrix}$ stands for a symmetric matrix, where $*$ represents the entries implied by symmetry.

II. System Description

In this paper, a finite-time H_∞ filtering problem is investigated for networked state-dependent uncertain systems with event-triggered mechanism and multiple attacks. We additionally assume that the sensor measurements sifted by the event-triggered mechanism are transmitted to the filter over a unreliable communication network. The framework of the networked state-dependent uncertain systems is provided in Fig.1.

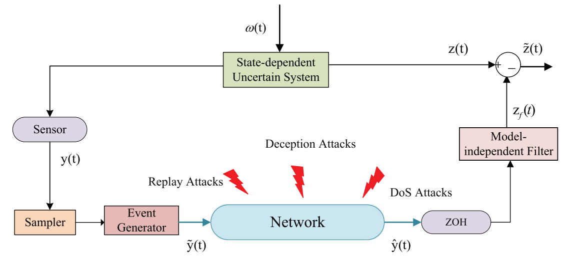

Fig. 1. The structure of finite-time H_∞ filtering for state-dependent uncertain systems with event-triggered mechanism and multiple attacks.

A. Model Description

This paper considers the continuous-time uncertain system with state-dependent uncertainties in the following form

$$\begin{cases} \dot{x}(t) = A(\alpha(x(t), \epsilon(t)))x(t) + B(\alpha(x(t), \epsilon(t)))\omega(t) \\ y(t) = C(\alpha(x(t), \epsilon(t)))x(t) + D(\alpha(x(t), \epsilon(t)))\omega(t) \\ z(t) = L\,(\alpha\,(x(t), \epsilon(t)))\,x(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^{n_x}$ represents the system state, $\omega(t) \in \mathfrak{L}_2[0, +\infty)$ is the disturbance input, $y(t) \in \mathbb{R}^{n_y}$ and $z(t) \in \mathbb{R}^{n_z}$ denote the measured output and the signal to be estimated, respectively; $\alpha\,(x(t), \epsilon(t)) \in \mathbb{R}^{n_\alpha}$ describes the system uncertainty which involves the time-varying parameters $\epsilon(t) \in \mathbb{R}^{n_\epsilon}$ and the state-dependent parametric perturbations. Furthermore, $\alpha_i\left(x^i(t), \epsilon^i(t)\right) \in \mathbb{R}^{n_\alpha}$ represents the ith complement of $\alpha\,(x(t), \epsilon(t))$; $\epsilon^i(t)$ and $x^i(t)$ denote vectors whose entries are the elements of $\epsilon(t)$ and $x(t)$; $A\,(\alpha\,(x(t), \epsilon(t)))$, $B\,(\alpha\,(x(t), \epsilon(t)))$, $C\,(\alpha\,(x(t), \epsilon(t)))$, $D\,(\alpha\,(x(t), \epsilon(t)))$ and $L\,(\alpha\,(x(t), \epsilon(t)))$ are given by the convex polytopic set as follows

$$\begin{aligned} \mathbb{X} = \Big\{ & X\,(\alpha\,(x(t), \epsilon(t)))\mid X\,(\alpha\,(x(t), \epsilon(t))) \\ & = \sum_{i=1}^{m} \left(\alpha_i\left(x^i(t), \epsilon^i(t)\right)\right) X_i, \\ & \qquad X_i \in \{A_i, B_i, C_i, D_i, L_i\}\Big\} \end{aligned} \tag{2}$$

where $\alpha\,(x(t), \epsilon(t))$ satisfies

$$\begin{aligned} \mathcal{D}_\alpha = \Big\{ & \alpha\,(x(t), \epsilon(t)) \mid \sum_{i=1}^{m} \alpha_i\left(x^i(t), \epsilon^i(t)\right) = 1, \\ & \alpha_i\left(x^i(t), \epsilon^i(t)\right) \geq 0, i \in \jmath_m = \{1, 2, \ldots, m\}\Big\} \end{aligned} \tag{3}$$

in which A_i, B_i, C_i, D_i and L_i $(i \in \jmath_m)$ are known real matrices with appropriate dimensions. Then, the reference system model (1) is given by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{m} \alpha_i\left(x^i(t), \epsilon^i(t)\right)[A_i x(t) + B_i \omega(t)] \\ y(t) = \sum_{i=1}^{m} \alpha_i\left(x^i(t), \epsilon^i(t)\right)[C_i x(t) + D_i \omega(t)] \\ z(t) = \sum_{i=1}^{m} \alpha_i\left(x^i(t), \epsilon^i(t)\right)[L_i x(t)] \end{cases} \tag{4}$$

Remark 1: In recent years, the state-dependent uncertain systems have been extensively studied due to its practical applications, especially in electronic circuit systems, as well as mechanical systems and flight control systems. In this paper, a novel mathematical model for state-dependent uncertain systems is proposed with consideration of the impacts of randomly occurring replay attacks, DoS attacks, and deception attacks in a unified framework. Furthermore, the proposed model is applied to the simulation example of electronic circuit systems.

The goal of this paper is to design the following model-independent filter

$$\begin{cases} \dot{x}_f(t) = A_f x_f(t) + B_f \hat{y}(t) \\ z_f(t) = C_f x_f(t) \end{cases} \tag{5}$$

where $x_f(t) \in \mathbb{R}^{n_x}$ represents the filter state vector, $\hat{y}(t) \in \mathbb{R}^{n_y}$ and $z_f(t) \in \mathbb{R}^{n_z}$ denote the filter real input and the estimation of $z(t)$, respectively; A_f, B_f and C_f denote the filter parameter to be designed later.

B. The Event-Triggered Mechanism

For the purpose of reducing the network burden, the sampled signals are sifted by the event generator before entering the network transmission channels. For further exposition, the following event-triggered criterion [34] is adopted to determine whether the current measured data will be updated or not

$$e_k^T(t)\Omega e_k(t) \le \sigma^2 y^T(t_k h + lh)\,\Omega y(t_k h + lh) \tag{6}$$

where Ω denotes a positive definite matrix, $e_k(t) = y(t_k h) - y(t_k h + lh)$; $y(t_k h)$ and $y(t_k h + lh)$ are used to represent the latest transmitted data and the current sampled output, respectively; $t_k h$ denotes the triggered instants, h is the sample period and kh is sampled instants, $l = 1, 2, \ldots, t_{k+1} - t_k - 1$, $\sigma \in [0, 1)$ represents a threshold coefficient and the next transmitted instant $t_{k+1}h$ is defined as

$$t_{k+1}h = t_k h + \inf_{l\ge 1}\Big\{lh | e_k^T(t_k h)\ \Omega e_k(t_k h) \le \sigma^2 y^T(t_k h + lh)\ \Omega y(t_k h + lh)\Big\} \tag{7}$$

Remark 2: In order to improve the utilization of communication bandwidth, an event-triggered mechanism is introduced between the sensor and the communication network. As can be seen from (6), the event generator will never sent out the sampled data to the communication network until the specific criterion (6) is violated. Since the sampled output that satisfies inequality (6) will not be transmitted during $[t_k h, t_{k+1} h)$, so the burden of networked communication is significantly alleviated.

C. Approach of Modeling Multiple Attacks

In this paper, suppose that the attackers are capable to launch replay attacks, deception attacks and DoS attacks with a certain probability of success. The framework of networked system is shown in Fig.1, in which the data transmission on sensor-to-filter channel is affected by multiple attacks. Moreover, under the multiple attacks, effective signals sifted by the event generator may be interfered, forged, modified and even replayed.

Remark 3: It is worth emphasizing that the cyber-attacks may block the transmission of signals and lead to deterioration of system performance. However, few approaches are proposed in the literature for modeling system with two or more kinds of cyber-attacks. In view of this, the objective of the proposed multiple attacks model is to provide a stochastic allocation strategy, in which the randomly occurring replay attacks, deception attacks and DoS attacks are considered in a unified framework.

The replay attacks may replace the normal transmission signals at arbitrary instants. This means that the normal transmission data can be replaced by the past signals, and the real input under replay attacks is

$$y_1(t) = \theta(t) y_r(t) + (1 - \theta(t))\,\tilde{y}(t) \tag{8}$$

where $\theta(t)$ represents a Bernoulli variable and $\theta(t) \in \{0, 1\}$. Define $\tilde{\theta}(t) = \theta(t) - \bar{\theta}$, $\mathbb{E}\left[\tilde{\theta}(t)\right] = 0$, $\mathbb{E}\left[\tilde{\theta}^2(t)\right] = \rho_1^2$ and ρ_1^2 denotes the mathematical variance of $\theta(t)$. Note that $y_r(t) = \tilde{y}(t_r)$, $\tilde{y}(t_r)$ denotes the injected past signals recorded by attacker at instant t_r, and $\tilde{y}(t)$ is the transmitted data via event-triggered mechanism.

Remark 4: The action of replay attacks is divided into two stages: the replay attacker records the normal transmission signals during the first stage, and injects the captured signal as malicious signal into the system in the second stage. In view of this, from (8), it is assumed that the attacker successfully records some past signals and inserts them into the system such that $y_r(t) = \tilde{y}(t_r)$ when $\theta(t) = 1$.

The deception attacks are defined as the arbitrary modification of the integrity data. It is assumed that $f(y(t))$ is a nonlinear function denoting the deception attacks. When the network suffers form the replay attacks and deception attacks, the signals arriving at the filter can be expressed as

$$y_2(t) = \beta(t) f(y(t)) + (1 - \beta(t))\, y_1(t) \tag{9}$$

where $\beta(t)$ denotes a Bernoulli variable and $\beta(t) \in \{0, 1\}$; $\tilde{\beta}(t) = \beta(t) - \bar{\beta}$, $\mathbb{E}\left[\tilde{\beta}(t)\right] = 0$, $\mathbb{E}\left[\tilde{\beta}^2(t)\right] = \rho_2^2$ and ρ_2 denotes the mathematical variance of $\beta(t)$.

The following assumption for deception attacks is needed:

Assumption 1 (Deception Attacks [35]): For given real constant matrix G, the deception attacks $f(x(t))$ satisfies

$$\| f(x(t)) \|_2 \le \| Gx(t) \|_2 \tag{10}$$

where G represents the upper bound of nonlinearity.

Remark 5: In order to describe the stochastic switching rule between the two different cyber-attacks, the Bernoulli variables $\theta(t)$ and $\beta(t)$ are considered. It is worth noting that the Bernoulli variables $\theta(t)$ and $\beta(t)$ are independent of each other. When $\beta(t) = 1$, the malicious signals sent by the deception attackers can replace the original signals. Otherwise, when $\beta(t) = 0$, the systems are subject to the replay attacks.

The DoS attacks are modeled as the phenomenon that causes blocking of the communication channels. Suppose that $\{\mathcal{H}_n\}_{n\in\mathbb{N}}$ and h_0 represent the sequence of the DoS attacks off/on transitions and the start interval of attackers, respectively. Then, the n_{th} sleeping time interval are determined by

$$\mathcal{H}_n = [h_n, h_n + \Delta_n) \tag{11}$$

where $\Delta_n \in \mathbb{R}_{\geq 0}$ denotes the length of n_{th} DoS sleeping period and satisfies $h_n + \Delta_n < h_{n+1}$.

Similar to [36], let $t_1 \in \mathbb{R}$, $t_2 \in \mathbb{R}$ and $t_1 < t_2$, for each interval $[t_1, t_2]$

$$\Lambda_s(t_1, t_2) = \bigcup_{n\in\mathbb{N}} \mathcal{H}_n \bigcap [t_1, t_2] \tag{12}$$

$$\Lambda_a(t_1, t_2) = [t_1, t_2] \backslash \Lambda_s(t_1, t_2) \tag{13}$$

where $\Lambda_s(t_1, t_2)$ is the sets of time instant when the DoS attacks is sleeping for each interval $[t_1, t_2]$; $\Lambda_a(t_1, t_2) = \{t \mid t \in [t_1, t_2], t \notin \Lambda_s(t_1, t_2)\}$ denotes the time instant for rejecting communication and it's the subset of $[t_1, t_2]$.

Several important assumptions are provided in the following, which will be helpful to develop the DoS attacks.

Assumption 2 (DoS Frequency [36]): Let $n(t)$ be the total number of DoS attacks sleep/active transitions over interval $[t_1, t_2)$. There exist $b_1, \omega_D \in \mathbb{R}$ such that

$$n(t) \leq b_1 + \frac{t_2 - t_1}{\omega_D} \tag{14}$$

for all $t_1, t_2 \in \mathbb{R}_{\geq 0}$ with $t_1 > 0$, $t_2 > 0$.

Assumption 3 (DoS Duration [37]): There exists a unified lower bound $\Delta_{\min}$ and a unified upper bound $b_{\max}$ such that

$$\Delta_{\min} \geq \inf_{n\in\mathbb{N}} \{\Delta_n\}, b_{\max} \geq \sup_{n\in\mathbb{N}} \{h_n - h_{n-1} - \Delta_n\} \tag{15}$$

By considering the types of cyber-attacks, the following multiple attack model is adopted

$$\begin{aligned}\hat{y}(t) = \Upsilon_y(t) y_2(t) = \Upsilon_y(t) \big\{ \beta(t) C_j f(y(t)) \\ + (1 - \beta(t)) \left[\theta(t) y_r(t) + (1 - \theta(t)) \tilde{y}(t)\right] \big\}\end{aligned} \tag{16}$$

where $\Upsilon_y(t) = \begin{cases} 1, t \in \Lambda_s(0, t) \\ 0, t \in \Lambda_a(0, t). \end{cases}$

Remark 6: It can be seen from (16) that when $\Upsilon_y(t) = 0$, the system suffers from DoS attacks; when $\Upsilon_y(t) = 1$, $\theta(t) = 0$ and $\beta(t) = 1$, the system is subject to deception attacks; if $\Upsilon_y(t) = 1$, $\theta(t) = 0$ and $\beta(t) = 0$, the cyber-attacks are not occur. In (16), a novel multiple attacks model is proposed which consists of replay attacks, deception attacks and DoS attacks. Specially, the proposed model can represent whether the systems are subject to different types of attacks, i.e. a single attacks, two types of attacks, or even three types of attacks.

Remark 7: The order of multiple attacks considered in this paper is replay attacks, deception attacks, DoS attacks. In practical application, considering that the order of multiple attacks is unknown, we can use the attacks detection method to identify the order and type of multiple attacks. Then, the appropriate multiple attacks model is established though the above mentioned way.

D. Problem Statement

In Fig.1, an event-triggered mechanism is adopted in the state-dependent uncertain system affected by exogenous disturbances. By considering the impact of DoS attacks, the triggered signals may be lost and the event-triggered criterion (6) needs to be modified. Under the action of event generator, the triggering instant is given by

$$t_{k_n} h = \left\{ t_{k,l} h \text{ violating (6)} \mid t_{k,l} h \in \Lambda_s(0, t) \right\} \cup \{h_n\} \tag{17}$$

where $t_{k,l} h \in \mathbb{N}$, k denotes the number of release times in the nth DoS period, $k \in \mathcal{T}_n$, $\mathcal{T}_n = \{1, 2, \ldots, k_n\}$ with $n \in \mathbb{N}$ and $k_n = \sup\{k \in \mathbb{N} | (h_{n-1} + \Delta_{n-1}) \geq t_{k_n} h\}$.

Remark 8: Different from some existing results on event-triggered mechanism [25], [26], an event-triggered mechanism is proposed in this paper with consideration of DoS attacks dependent. In other words, the proposed event-triggered mechanism is associated with not only triggering parameters but also DoS attacks parameters.

Inspired by [37], set

$$\Sigma_s(0, t) = \cup_{k=1}^{k_n} \left\{ I_k^n \cap \Lambda_s(0, t) \right\} \tag{18}$$

where $I_k^n = \cup_{l=1}^{S_{k_n}} [t_{k_n} h + (l-1)h, t_{k_n} h + lh)$, $S_{k_n} = \sup\{l \in \mathbb{N} \mid t_{k_n} h + lh < t_{(k+1)_n} h\}$, I_k^n denotes the time interval and $l \in \{1, 2, \ldots, S_{k_n}\}$.

Then $\Lambda_s(0, t) = \cup_{k=1}^{k_n} \cup_{l=1}^{S_{k_n}} \{\Lambda_s(0, t) \cap [t_{k_n} h + (l-1)h, t_{k_n} h + lh)\}$. For $k \in \mathcal{T}_n$ with $n \in \mathbb{N}$, the network-induced delay function $\tau_{k_n}(t)$ and the error vector $e_{k_n}(t)$ are given by

$$\begin{cases} \tau_{k_n}(t) = t - t_{k_n} h - (S_{k_n} - 1)h \\ e_{k_n}(t) = x\left(t_{k_n} h\right) - x\left(t_{k_n} h + (S_{k_n} - 1)h\right) \end{cases} \tag{19}$$

with $t \in \Sigma_s(0, t) \cap [t_{k_n} h + (S_{k_n} - 1)h, t_{(k+1)_n} h)$. Based on (17) and (19), the event-triggered condition is designed as

$$e_{k_n}^T(t) \Omega e_{k_n}(t) \leq \sigma^2 y^T\left(t - \tau_{k_n}(t)\right) \Omega y\left(t - \tau_{k_n}(t)\right) \tag{20}$$

The output measurement via event-triggered mechanism can be represented as

$$\tilde{y}(t) = y\left(t - \tau_{k_n}(t)\right) + e_{k_n}(t) \tag{21}$$

with $t \in \bar{\Lambda}_s(0, t)$, $\bar{\Lambda}_s(0, t) = \Sigma_s(0, t) \cap I_k^n$, $k \in \mathcal{T}_n$.

According to (4), (5), (16), (21), and denote $e(t) = \begin{bmatrix} x(t) \\ x_f(t) \end{bmatrix}$, $\tilde{z}(t) = z(t) - z_f(t)$, $\bar{\omega}(t) = \begin{bmatrix} \omega(t) \\ \omega(t - \tau(t)) \end{bmatrix}$, $\alpha_i = \alpha_i\left(x^i(t), \epsilon^i(t)\right)$, $\hat{\alpha}_j = \hat{\alpha}_j(x^j(t - \tau_{k_n}(t)), \epsilon^j(t - \tau_{k_n}(t)))$, the filtering error system can be obtained by

$$\begin{cases} \dot{e}(t) = \begin{cases} \displaystyle\sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \hat{\alpha}_j \Big\{ \tilde{A}_{1i} e(t) + \tilde{B}_i \bar{\omega}(t) + \beta(t) \\ \times \tilde{B}_f f(y(t)) + (1 - \beta(t)) \theta(t) \Big[\tilde{B}_f e_{k_n}(t_r) \\ + \tilde{B}_j H \bar{\omega}(t_r) + \tilde{A}_{2j} H e\left(t - \tau_{k_n}(t_r)\right) \Big] \\ + (1 - \beta(t))(1 - \theta(t)) \Big[\tilde{B}_f e_{k_n}(t) \\ + \tilde{B}_j H \bar{\omega}(t) + \tilde{A}_{2j} H e\left(t - \tau_{k_n}(t)\right) \Big] \Big\}, \\ \qquad t \in \bar{\Lambda}_s(0, t) \\ \displaystyle\sum_{i=1}^{m} \alpha_i \left[\tilde{A}_{1i} x(t) \right], \quad t \in \Lambda_a(0, t) \end{cases} \\ \tilde{z}(t) = \displaystyle\sum_{i=1}^{m} \sum_{j=1}^{m} \alpha_i \hat{\alpha}_j \left[\tilde{C}_i e(t) \right] \\ e(t) = \phi(t), \quad t \in [-h, 0] \end{cases} \tag{22}$$

where $\tilde{A}_{1i}(t) = \begin{bmatrix} A_i & 0 \\ 0 & A_f \end{bmatrix}$, $\tilde{B}_i(t) = \begin{bmatrix} B_i & 0 \\ 0 & 0 \end{bmatrix}$, $\tilde{B}_f = \begin{bmatrix} 0 \\ B_f \end{bmatrix}$, $\tilde{A}_{2j}(t) = \begin{bmatrix} 0 \\ B_f C_j \end{bmatrix}$, $\tilde{B}_j(t) = \begin{bmatrix} 0 \\ B_f D_j \end{bmatrix}$, $\tilde{C}_i(t) = \begin{bmatrix} L_i & -C_f \end{bmatrix}$, $H = \begin{bmatrix} I & 0 \end{bmatrix}$, $\phi(t)$ denotes the initial condition of $e(t)$.

Before proceeding, a definition is expressed as follows:

Definition 1 [38], [39]: For given positive scalars a, b, T, $\mathbb{W}$, $c_2 > c_1$, d and a positive matrix R, if (23) holds, the filtering error system (22) is said to be exponentially mean-square finite-time bounded (EMFTB) with respect to (c_1, c_2, R, T, $\mathbb{W}$, ϕ).

$$\begin{cases} \sup_{-h\le s\le 0}\left\{\mathbb{E}[e^T(s)Re(s)]\right\} \le c_1^2 \\ \quad \Rightarrow \mathbb{E}[e^T(s)Re(s)] \le c_2^2, \\ \forall \bar{\omega}(t) \in \mathbb{W} = \left\{\bar{\omega}(t) \mid \bar{\omega}^T(t)\bar{\omega}(t) \le d\right\}, \forall t \in [0, T] \\ ae^{-bt}\|\phi\|^2 \ge \mathbb{E}\left[\|e(t)\|^2|\phi\right], \forall t \ge 0 \end{cases} \tag{23}$$

III. Main Results

In this section, based on Lyapunov-Krasocskii stability theory and linear matrix inequality techniques, the sufficient conditions are derived to guarantee the EMFTB for filtering error system (22). The main results are presented by the following theorems.

A. Analysis of Finite-Time Filtering H_∞ Performance

Theorem 1: For given positive DoS parameters ϖ, ω_D, $\Delta_{\min}$, $b_{\max}$, η_s, ς_s, positive parameters μ_i, $\bar{\beta}$, $\bar{\theta}$, $\rho_s (s = 1, 2)$, matrix G, sampling period h, triggering parameter σ, scalars set (c_1, c_2, R, T, d) satisfying Definition 1, the system (22) is EMFTB, if there exist positive scalars $\lambda_q(q = 1, 2, 3, 4, 5, 6)$, matrices $P_s > 0$, $S_{ks} > 0$, $W_{ks} > 0$, $Z_{ks} > 0$, $L_i^s > 0$, $\Omega > 0$, U_{ks} and $V_{ks}(s = 1, 2)$ with appropriate dimensions, such that for any $i, j, k \in \jmath_m$, the following inequalities hold with $\hat{\alpha}_j - \mu_j\alpha_j \ge 0$

$$\Psi_{ijk}^s - L_i^s < 0 \tag{24}$$

$$\mu_i\Psi_{iik}^s - \mu_i L_i^s + L_i^s < 0 \tag{25}$$

$$\mu_j\Psi_{ijk}^s + \mu_i\Psi_{jik}^s - \mu_j L_i^s - \mu_i L_j^s + L_i^s + L_j^s < 0 \quad (i < j) \tag{26}$$

$$\begin{bmatrix} W_{ks} & U_{ks} \\ * & W_{ks} \end{bmatrix} \ge 0, \quad \begin{bmatrix} Z_{ks} & V_{ks} \\ * & Z_{ks} \end{bmatrix} \ge 0 \tag{27}$$

$$S_{ks} \le \varsigma_{3-s}S_{3-ks}, W_{ks} \le \varsigma_{3-s}W_{3-ks}, \; Z_{ks} \le \varsigma_{3-s}Z_{3-ks} \tag{28}$$

$$P_1 \le \varsigma_2 \; P_2, P_2 \le \varsigma_1 \; e^{2(\eta_1+\eta_2)h}P_1, \quad \mathcal{I} > 0 \tag{29}$$

$$\Pi c_1^2 + \lambda_6 d^2 < c_2^2 e^{(-1)^s 2\eta_s T}\lambda_1 \tag{30}$$

$$\lambda_1 R < P_i < \lambda_2 R, \quad S_{ks} < \lambda_3 R, \; W_{ks} < \lambda_4 R, \; Z_{ks} < \lambda_5 R \tag{31}$$

where the matrix Ψ_{ijk}^s is shown in Appendix A.

Proof: See Appendix B. ∎

In Theorem 1, sufficient conditions which ensure the EMFTB of the system (22) have been presented. Then, by using the same approach in Theorem 1, sufficient conditions will be given for the H_∞ finite-time bounded of the systems (22) in the following theorem.

Theorem 2: For given positive DoS parameters ϖ, ω_D, $\Delta_{\min}$, $b_{\max}$, η_s, ς_s, positive parameters μ_i, $\bar{\beta}$, $\bar{\theta}$, $\rho_s(s = 1, 2)$, matrix G, sampling period h, trigger parameter σ, scalars set (c_1, R, T, d) satisfying Definition 1, the system (22) is EMFTB with H_∞ performance, if there exist positive scalars $\lambda_q(q = 1, 2, 3, 4, 5)$, H_∞ performance level γ, c_2, matrices $W_{ks} > 0$, $Z_{ks} > 0$, $P_s > 0$, $S_{ks} > 0$, $L_i^s > 0$, $\Omega > 0$, U_{ks}, $V_{ks}(s = 1, 2)$ with appropriate dimensions, such that for any $i, j, k \in \jmath_m$, the following inequalities and (27)-(31) hold with $\hat{\alpha}_j - \mu_j\alpha_j \ge 0$

$$\bar{\Psi}_{ijk}^s - L_i^s < 0 \tag{32}$$

$$\mu_i\bar{\Psi}_{iik}^s - \mu_i L_i^s + L_i^s < 0 \tag{33}$$

$$\mu_j\bar{\Psi}_{ijk}^s + \mu_i\bar{\Psi}_{jik}^s - \mu_j L_i^s - \mu_i L_j^s + L_i^s + L_j^s < 0 \quad (i < j) \tag{34}$$

where

$$\bar{\Psi}_{ijk}^1 = \begin{bmatrix} F_{11ijk}^1 & * & * & * & * \\ \tilde{C}_i & -I & * & * & * \\ F_{21ijk}^1 & 0 & F_{22k}^1 & * & * \\ F_{31ijk}^1 & 0 & 0 & F_{33k}^1 & * \\ F_{41k}^1 & 0 & 0 & 0 & F_{44k}^1 \end{bmatrix}$$

$$\bar{\Psi}_{ijk}^2 = \begin{bmatrix} \Theta_{11ik}^2 & * & * & * & * & * \\ \Theta_{21k}^2 & \Theta_{22k}^2 & * & * & * & * \\ \Theta_{31k}^2 & \Theta_{32k}^2 & \Theta_{33k}^2 & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * \\ \tilde{C}_i & 0 & 0 & 0 & -I & * \\ \Theta_{51i}^2 & 0 & 0 & \Theta_{54k}^2 & 0 & \Theta_{55k}^2 \end{bmatrix}$$

and other variables involved in the above matrix are given in Theorem 1.

Proof: See Appendix C. ∎

In Theorem 2, sufficient conditions are provided to guarantee the filtering error system being EMFTB with H_∞ performance level γ. Then, according to Theorem 2, the design algorithm of finite-time filter are given in the following theorem.

B. Finite-Time H_∞ Filtering Design

Theorem 3: For given positive DoS parameters ϖ, ω_D, $\Delta_{\min}$, $b_{\max}$, η_s, ς_s, positive parameters ρ_s, ε_{s1}, ε_{s2} $(s = 1, 2)$, $\bar{\beta}$, $\bar{\theta}$, μ_i, matrix G, sampling period h, triggering parameter σ, scalars set (c_1, R, T, d) satisfying Definition 1, the system (22) is EMFTB with H_∞ performance, if there exist positive scalars $\lambda_q(q = 1, 2, 3, 4, 5)$, H_∞ performance level γ, c_2, matrices $P_{1s} > 0$, $P_{2s} > 0$, $\bar{P}_{3s} > 0$, $\hat{W}_{ks} > 0$, $\hat{Z}_{ks} > 0$, $L_i^s > 0$, $\hat{A}_f$, $\hat{B}_f$, $\hat{C}_f$, $\hat{S}_{ks} > 0$, $\Omega > 0$, $\hat{U}_{ks}$, $\hat{V}_{ks}$, $M_s(s = 1, 2)$ with appropriate dimensions, such that for any $i, j, k \in \jmath_m$, the following linear matrix inequalities and (29)-(31) hold with $\hat{\alpha}_j - \mu_j\alpha_j \ge 0$

$$\check{\Psi}_{ijk}^s - L_i^s < 0 \tag{35}$$

$$\mu_i\check{\Psi}_{iik}^s - \mu_i L_i^s + L_i^s < 0 \tag{36}$$

$$\mu_j\check{\Psi}_{ijk}^s + \mu_i\check{\Psi}_{jik}^s - \mu_j L_i^s - \mu_i L_j^s + L_i^s + L_j^s < 0 \quad (i < j) \tag{37}$$

$$P_{1s} - \bar{P}_{3s} > 0 \tag{38}$$

$$M_s \hat{S}_{ks} M_s^T \leq \varsigma_{3-s} \hat{S}_{3-ks}, \quad M_s \hat{W}_{ks} M_s^T \leq \varsigma_{3-s} \hat{W}_{3-ks}, \quad M_s \hat{Z}_{ks} M_s^T \leq \varsigma_{3-s} \hat{Z}_{3-ks} \tag{39}$$

where the matrix $\check{\Psi}_{ijk}^{s}$ is given in Appendix D.

Moreover, if the above inequalities have a feasible solution, the filter parameters are given by

$$\begin{cases} A_f = \hat{A}_f \bar{P}_{31}^{-1} \\ B_f = \hat{B}_f \\ C_f = \hat{C}_f \bar{P}_{31}^{-1} \end{cases} \tag{40}$$

Proof: See Appendix E. ∎

IV. Simulation Examples

In this section, two practical engineering examples are given to demonstrate the feasibility and the applicability of the proposed results.

Example 1: Consider the tunnel diode circuit system. The model is borrowed from [7] and its state equation is governed by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} \frac{1}{R_L C} - \frac{0.01x_1^2(t) + 0.002}{C} & \frac{1}{C} \\ \frac{1}{2L} & -\frac{R_E}{L} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix} \omega(t) \tag{41}$$

where $x_1(t) \in [-5, 5]$, $C = 25mF$, $L = 10H$, $R_E = 180\Omega$ and $R_L = 2k\Omega$.

Let the scalars $\Delta_{\min} = 4$, $\varsigma_1 = 1.01$, $\varsigma_2 = 1.01$, $\eta_1 = 0.04$, $\eta_2 = 0.4$, $\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22} = 4$, the finite-time boundedness conditions (c_1, d, T, R)=(0.001, 0.3, 3, I), the parameter of event-triggered scheme $\sigma^2 = 0.4$, $\alpha = 0.75$, $\hat{\alpha} = 0.95$ which can guarantee $\hat{\alpha}_j - \mu_j \alpha_j \geq 0$. The deception attack function is $f(y(t)) = -tanh(0.15y(t))$. Using the Assumption 1, the nonlinearity upper bound is given as $G = 0.15$.

By utilizing Theorem 3 with $\bar{\theta} = 0.1$, $\bar{\beta} = 0.5$, $b_{\max} = 0.35$, we can get $\gamma = 7.7797$, $c_2 = 12.01$, and

$$\bar{P}_{31} = \begin{bmatrix} 31.69 & 30.15 \\ 30.14 & 198.65 \end{bmatrix}, \quad \hat{A}_f = \begin{bmatrix} -357.54 & 341.71 \\ -598.58 & -621.47 \end{bmatrix}$$

$$\hat{B}_f = \begin{bmatrix} -70.2072 \\ -26.5292 \end{bmatrix}, \quad \hat{C}_f = \begin{bmatrix} 0.1186 & -0.2603 \end{bmatrix} \tag{42}$$

According to (40) and (42), the parameters of the model-independent filter are computed as

$$A_f = \begin{bmatrix} -15.0960 & 4.0111 \\ -18.5942 & -0.3067 \end{bmatrix}, \quad B_f = \begin{bmatrix} -70.2072 \\ -26.5292 \end{bmatrix}$$

$$C_f = \begin{bmatrix} 0.0058 & -0.0022 \end{bmatrix} \tag{43}$$

Set the initial state $x(t) = [-0.05, -0.02]^T$, $x_f(t) = [0, 0]^T$ and the sampling period $h = 0.02s$, the external disturbance is designed by

$$\omega(t) = \begin{cases} 0.1 & 5 \leq t \leq 7 \\ -0.1 & 10 \leq t \leq 13 \\ 0 & else. \end{cases}$$

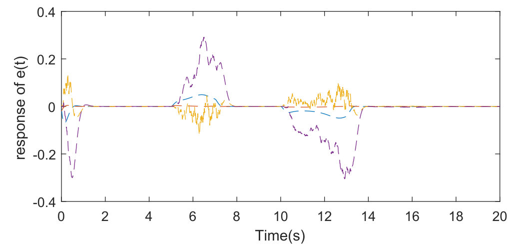

Fig. 2. State response of $e(t)$ in Example 1.

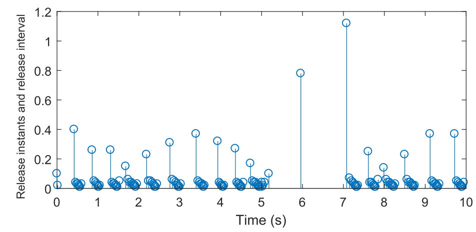

Fig. 3. Release intervals of Example 1.

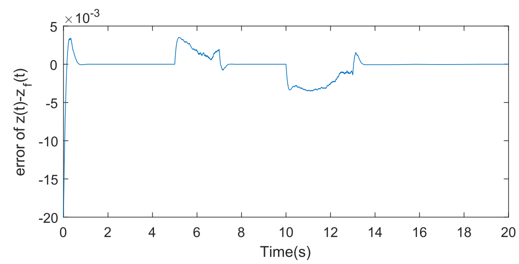

Fig. 4. Response of $\tilde{z}(t)$ in Example 1.

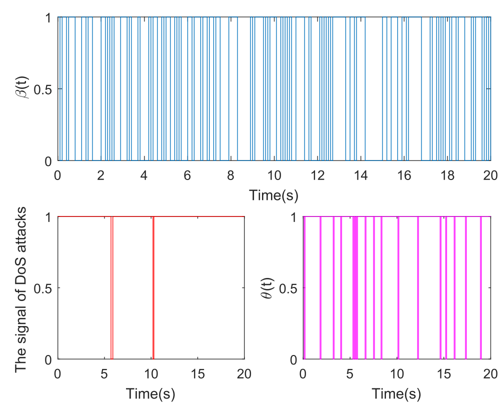

Fig. 5. Bernoulli distribution variables $\beta(t)$, $\theta(t)$ and sequence of DoS attacks in Example 1.

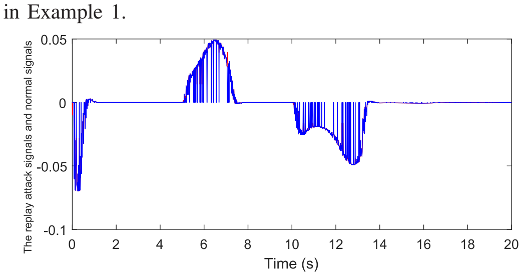

Fig. 6. Comparison between replay attack signals and normal transmission signals of Example 1.

Simulation results are depicted in Fig.2-Fig.6. The response of $e(t)$ is plotted in Fig.2. The release instants and interval of the event-triggered mechanism can be seen from Fig.3, and the limited communication resources are saved by 83%. Fig.4 shows the response of $\tilde{z}(t)$. In Fig.5, the evolution of Bernoulli

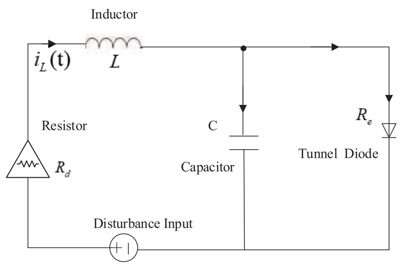

Fig. 7. The structure of tunnel diode circuit in Example 2.

variables $\beta(t)$ and $\theta(t)$ are plotted and the sequence of DoS attacks is depicted. Fig.6 illustrates comparison result between the data suffer from replay attacks and normal transmission data. It can conclude from Fig.2 that the filter design method is feasible.

Example 2: In order to further manifest the feasibility and priority of the proposed algorithm, a tunnel diode circuits system described in Fig.7 is investigated in this example. Its characteristics can be expressed by

$$\begin{cases} i_L(t) = C\dfrac{dV_{C(t)}}{dt} + \dfrac{1}{R_e}V_{C(t)} \\ L\dfrac{di_L(t)}{dt} = -V_{C(t)} - R_d i_L(t) + \omega(t) \end{cases}$$

where C represents the capacitor, R_d, R_e denote the linear resistance and R_e is the impedance of tunnel diode.

More specifically, R_e denotes time-varying uncertain which depends on V_e [7], and the relationship between $i_e(t)$ and V_e can be obtained in the following

$$\frac{1}{R_e} = \frac{i_e(t)}{V_e(t)} = 0.002V_e(t) + 0.01V_e^2(t)$$

Let $x_1(t) = V_C(t)$ and $x_2(t) = i_L(t)$ be the state variables, where $x_1(t) \in [s_1, s_2]$, $m_1 = \max\{s_1^2, s_2^2\}$, the tunnel diode circuit system is described by

$$\begin{bmatrix} \dot{x_1}(t) \\ \dot{x_2}(t) \end{bmatrix} = \begin{bmatrix} -\dfrac{0.002 + 0.01x_1^2(t)}{C} & \dfrac{1}{C} \\ -\dfrac{1}{L} & -\dfrac{R_d}{L} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{1}{L} \end{bmatrix} \omega(t)$$

The considered system can be obtained in polytopic form as

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} \left(\alpha_i \left(x^i(t), \epsilon\right)\right) [A_i x(t) + B_i \omega(t)] \\ y(t) = \sum_{i=1}^{2} \left(\alpha_i \left(x^i(t), \epsilon\right)\right) [C_i x(t) + D_i \omega(t)] \\ z(t) = \sum_{i=1}^{2} \left(\alpha_i \left(x^i(t), \epsilon\right)\right) [L_i x(t)] \end{cases}$$

where $\alpha_1\left(x^1(t), \epsilon\right) = \frac{x_1^2(t)}{s_1}$, $\alpha_2\left(x^2(t), \epsilon\right) = 1 - \frac{x_1^2(t)}{s_1}$ represent the uncertain parameter vector, $x(t) = [x_1(t), x_2(t)]^T$, $y(t) = V_C(t)$ and $A_1 = \begin{bmatrix} -\frac{0.002+0.01s_1}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_d}{L} \end{bmatrix}$, $A_2 = \begin{bmatrix} -\frac{s_1}{C} & \frac{1}{C} \\ -\frac{1}{L} & -\frac{R_d}{L} \end{bmatrix}$, $B_1 = B_2 = \begin{bmatrix} 0 \\ \frac{1}{L} \end{bmatrix}$, $C_1 = C_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$, $D_1 = D_2 = -1$, $L_1 = L_2 = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

TABLE I

THE γ AND c_2 WITH DIFFERENT VALUES OF $\bar{\theta}$ AND $\bar{\beta}$ IN EXAMPLE IV

Case	$\bar{\theta}$	$\bar{\beta}$	γ	c_2
Case 1	0.1	0.5	0.1592	4.1867
Case 2	0.2	0	0.1486	4.0473
Case 3	0	0	0.1513	4.0407

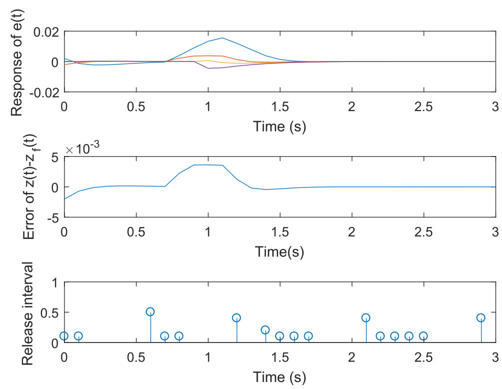

Fig. 8. State response of $e(t)$, response of $\tilde{z}(t)$ and release intervals in Case 1 of Example 2.

The corresponding circuit system parameters are given as $C = 0.04F$, $L = 1H$, $R_d = 10\Omega$ and $x_1(t) = [-5, 5]$. Applying the above parameters, the circuit system is described in a polytopic form with the following vertices:

$$A_1 = \begin{bmatrix} -6.3 & 25 \\ -1 & -10 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.05 & 25 \\ -1 & -10 \end{bmatrix}, \quad B_i = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$C_i = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad D_i = -1, \quad L_i = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Set the scalars $\Delta_{\min} = 1.78$, $\varsigma_1 = 1.01$, $\varsigma_2 = 1.01$, $\eta_1 = 0.46$, $\eta_2 = 0.4$, $\varepsilon_{11} = \varepsilon_{12} = \varepsilon_{21} = \varepsilon_{22} = 8$, the finite-time boundedness conditions (c_1, d, T, R)=(0.1, 2.5, 3, I), the parameter of event-triggered scheme $\sigma^2 = 0.4$. $\alpha = 0.75$ and $\hat{\alpha} = 0.95$ which can guarantee $\hat{\alpha}_j - \mu_j\alpha_j \geq 0$. The deception attack function is $f\,(y(t)) = -tanh\,(0.05y(t))$, and using the Assumption 1, the nonlinearity upper bound is given as $G = 0.05$. Set the initial state $x(t) = [-0.002, -0.002]^T$, $x_f(t) = [0, 0]^T$ and the sampling period $h = 0.1s$, the external disturbance is defined as $\omega(t) = \begin{cases} 0.1 & 0.75 \leq t \leq 1.15 \\ 0 & else. \end{cases}$

Then, three cases in Table I with different values of $\bar{\beta}$ and $\bar{\theta}$ are provided to show that the designed approach is useful.

Cases 1: In this case, set $\bar{\theta} = 0.1$, $\bar{\beta} = 0.5$, $b_{\max} = 0.2$, which means that replay attacks, deception attacks and DoS attacks occur simultaneously.

By solving Theorem 3, the following matrices can obtained

$$\bar{P}_{31} = \begin{bmatrix} 1.703 & 1.443 \\ 1.443 & 5.496 \end{bmatrix}, \quad \hat{A}_f = \begin{bmatrix} -10.081 & 5.817 \\ -9.953 & -12.848 \end{bmatrix}$$
$$\hat{B}_f = \begin{bmatrix} -0.6270 \\ 1.3536 \end{bmatrix}, \quad \hat{C}_f = \begin{bmatrix} 0.0321 & -0.0884 \end{bmatrix} \qquad (44)$$

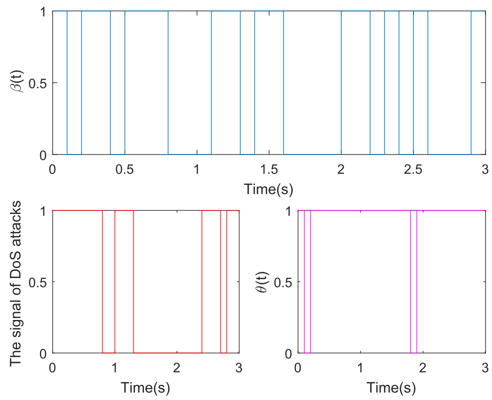

Fig. 9. $\beta(t)$, $\theta(t)$ and sequence of DoS attacks in Case 1 of Example 2.

From (40) and (44), the model-independent filter parameters are calculated as

$$A_f = \begin{bmatrix} -8.7675 & 3.3598 \\ -4.9696 & -1.0330 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.6270 \\ 1.3536 \end{bmatrix}$$
$$C_f = \begin{bmatrix} 0.0417 & -0.0270 \end{bmatrix} \tag{45}$$

Simulation results are shown in Fig.8-Fig.9. Fig.8 depicts the trajectory of $e(t)$, the response of $\tilde{z}(t)$ and the release instants under event-triggered mechanism. In Fig.8, only 53.33% measurement signals arrive at the filter. The Bernoulli variables $\beta(t)$, $\theta(t)$ and the sequence of DoS attacks are depicted in Fig.9. It can verifies from Fig.8 that the proposed filter is EMFTB.

Cases 2: In this case, let $\bar{\theta} = 0.5$, $\bar{\beta} = 0$, $b_{\max} = 0.2$, which means that only replay attacks and DoS attacks occur simultaneously.

By applying Theorem 3, one can obtain

$$\bar{P}_{31} = \begin{bmatrix} 1.590 & 1.350 \\ 1.350 & 5.141 \end{bmatrix}, \quad \hat{A}_f = \begin{bmatrix} -9.443 & 5.414 \\ -9.154 & -11.928 \end{bmatrix}$$
$$\hat{B}_f = \begin{bmatrix} -0.5086 \\ 1.4161 \end{bmatrix}, \quad \hat{C}_f = \begin{bmatrix} 0.0150 & -0.0878 \end{bmatrix} \tag{46}$$

Combine (40) in Theorem 3 and (46), the following filter parameters can obtain

$$A_f = \begin{bmatrix} -8.7937 & 3.3626 \\ -4.8733 & -1.0402 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.5086 \\ 1.4161 \end{bmatrix}$$
$$C_f = \begin{bmatrix} 0.0308 & -0.0252 \end{bmatrix} \tag{47}$$

Fig.10-Fig.11 can be obtained through the simulation. The response of $e(t)$, the trajectory of $\tilde{z}(t)$ and the release instants under event-triggered mechanism are shown in Fig.10. It can be observed in Fig.10 that the tunnel diode circuit system is EMFTB. The Bernoulli variables $\theta(t)$ for replay attacks and the sequence of DoS attacks are provided in Fig.11.

Cases 3: In this case, let $\bar{\theta} = 0$, $\bar{\beta} = 0$, $b_{\max} = 0.2$, which means that only DoS attacks occur.

Solving Theorem 3 by using MATLAB, the matrices are obtained as

$$\bar{P}_{31} = \begin{bmatrix} 1.585 & 1.342 \\ 1.342 & 5.119 \end{bmatrix}, \quad \hat{A}_f = \begin{bmatrix} -9.471 & 5.405 \\ -9.092 & -11.905 \end{bmatrix}$$
$$\hat{B}_f = \begin{bmatrix} -0.6656 \\ 1.2534 \end{bmatrix}, \quad \hat{C}_f = \begin{bmatrix} 0.0332 & -0.0880 \end{bmatrix} \tag{48}$$

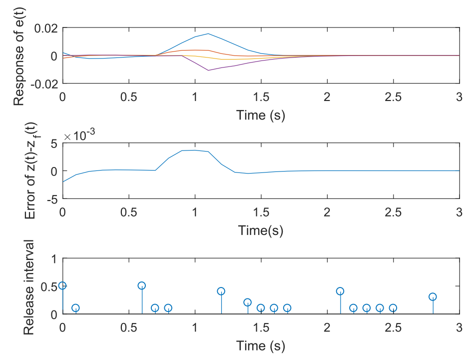

Fig. 10. State response of $e(t)$, response of $\tilde{z}(t)$ and release intervals in Case 2 of Example 2.

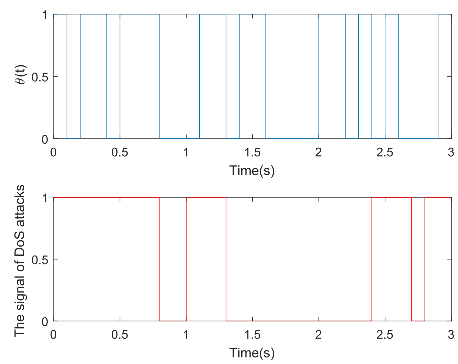

Fig. 11. $\theta(t)$ and sequence of DoS attacks in Case 2 of Example 2.

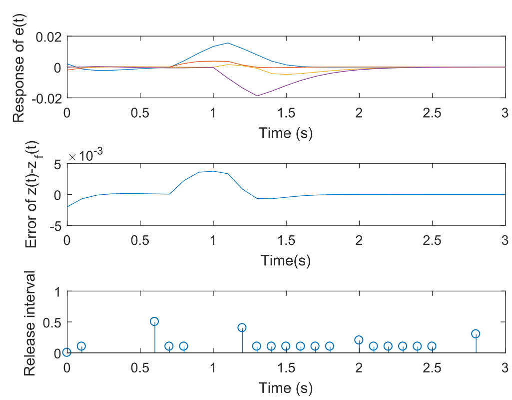

Fig. 12. State response of $e(t)$, response of $\tilde{z}(t)$ and release intervals in Case 3 of Example 2.

By combining (48) and (40), the model-independent filter parameters are given as

$$A_f = \begin{bmatrix} -8.8288 & 3.3702 \\ -4.8419 & -1.0562 \end{bmatrix}, \quad B_f = \begin{bmatrix} -0.6656 \\ 1.2534 \end{bmatrix}$$
$$C_f = \begin{bmatrix} 0.0456 & -0.0291 \end{bmatrix} \tag{49}$$

Through Fig.12, the variation of $e(t)$, the trajectory of $\tilde{z}(t)$ and the event-triggered instants and intervals can be obtained. To summarize the simulation results, the event-triggered mechanism can be reduce the network burden in the tunnel diode circuit system. Moreover, it is obvious that the filter error system for tunnel diode circuit system with multiple attacks is EMFTB.

V. Conclusion

The finite-time H_∞ filter design problem for state-dependent uncertain systems with event-triggered mechanism and multiple attacks has been investigated in this paper. An event-triggered mechanism is introduced to alleviate the load of networks. A novel multiple attacks model, referred to as "multiple attacks", is presented for the analysis of the randomly occurring deception attacks, denial-of-service attacks and replay attacks. Based on Lyapunov-Krasocskii stability theory, sufficient conditions of EMFTB are established for the filtering error systems. Besides, the explicit expression of the parameters of the desired finite-time filter are derived. Finally, two simulation examples are presented to demonstrate the usefulness and applicability of the obtained theoretical method in electronic circuits. In the future, the detection and defending against different types of cyber-attacks in state-dependent uncertain systems will be further studied.

Appendix A
The Elements of the Matrix Ψ^s_{ijk} in Theorem 1

$$\Psi^1_{ijk} = \begin{bmatrix} F^1_{11ijk} & * & * & * \\ F^1_{21ijk} & F^1_{22k} & * & * \\ F^1_{31ijk} & 0 & F^1_{33k} & * \\ F^1_{41k} & 0 & 0 & F^1_{44k} \end{bmatrix}$$

$$\Psi^2_{ijk} = \begin{bmatrix} \Theta^2_{11ik} & * & * & * & * \\ \Theta^2_{21k} & \Theta^2_{22k} & * & * & * \\ \Theta^2_{31k} & \Theta^2_{32k} & \Theta^2_{33k} & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * \\ \Theta^2_{51i} & 0 & 0 & \Theta^2_{54k} & \Theta^2_{55k} \end{bmatrix}$$

$$F^1_{11ijk} = \begin{bmatrix} \Theta^1_{11ik} & * & * & * & * & * \\ \Theta^1_{21jk} & \Theta^1_{22k} & * & * & * & * \\ \Theta^1_{31k} & \Theta^1_{32k} & \Theta^1_{33k} & * & * & * \\ \Theta^1_{41j} & 0 & 0 & \sigma^2\ H^T\Omega H & * & * \\ \Theta^1_{51} & 0 & 0 & 0 & \Theta^1_{55} & * \\ \Theta^1_{61ij} & 0 & 0 & 0 & 0 & \Theta^1_{66} \end{bmatrix}$$

$$\Theta^1_{11ik} = 2\eta_1 P_1 + P_1\tilde{A}_{1i} + \tilde{A}^T_{1i}P_1 + S_{k1} + \frac{1}{h}(W_{k1} + Z_{k1})$$

$$\Theta^1_{21jk} = \bar{\beta}_1\bar{\theta}_1 H^T\tilde{A}^T_{2j}P_1 + \frac{1}{h}(W_{k1} - U_{k1} + Z_{k1} - V_{k1})$$

$$\Theta^1_{22k} = \sigma^2\ H^T\Omega H + \frac{1}{h}\Big(-2W_{k1} + U_{k1} + U^T_{k1} - 2Z_{k1} + V_{k1} + V^T_{k1}\Big),$$

$$\Theta^1_{32k} = \frac{1}{h}(W_{k1} - U_{k1} + Z_{k1} - V_{k1})$$

$$\Theta^1_{31k} = \frac{1}{h}(U_{k1} + V_{k1}),$$

$$\Theta^1_{33k} = -e^{-2\eta_1 h}S_{k1} - \frac{1}{h}(W_{k1} + Z_{k1}),$$

$$\Theta^1_{41j} = \bar{\beta}_1\bar{\theta}H^T\tilde{A}^T_{2j}P_1, \quad \bar{\theta}_1 = 1 - \bar{\theta}, \bar{\beta}_1 = 1 - \bar{\beta}$$

$$\Theta^1_{51} = \begin{bmatrix} \bar{\beta}_1\bar{\theta}_1\tilde{B}^T_f P_1 \\ \bar{\beta}_1\bar{\theta}\tilde{B}^T_f P_1 \end{bmatrix}, \quad \Theta^1_{66} = \begin{bmatrix} -\gamma^2 I & * \\ 0 & -\bar{\beta}I \end{bmatrix}$$

$$\Theta^1_{61ij} = \begin{bmatrix} \tilde{B}^T_i P_1 + \bar{\beta}_1\bar{\theta}H^T\tilde{B}^T_j P_1 + \bar{\beta}_1\bar{\theta}_1 H^T\tilde{B}^T_j P_1 \\ \bar{\beta}\tilde{B}^T_f P_1 \end{bmatrix}$$

$$F^1_{21ijk} = \begin{bmatrix} \Theta^1_{71} & \Theta^1_{72} & \Theta^1_{73} & \Theta^1_{74} & \Theta^1_{75} \end{bmatrix}$$

$$\Theta^1_{71} = \begin{bmatrix} \sqrt{h}P_1\tilde{A}_{1i} & \sqrt{h}\bar{\beta}_1\bar{\theta}_1 P_1\tilde{A}_{2j}H & 0 \\ \sqrt{h}P_1\tilde{A}_{1i} & \sqrt{h}\bar{\beta}_1\bar{\theta}_1 P_1\tilde{A}_{2j}H & 0 \\ 0 & -\sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{A}_{2j}H & 0 \\ 0 & -\sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{A}_{2j}H & 0 \end{bmatrix}$$

$$F^1_{31ik} = \begin{bmatrix} \Theta^1_{81} & \sqrt{h}\Theta^1_{82} & \sqrt{h}\Theta^1_{83} & \sqrt{h}\Theta^1_{84} & \Theta^1_{85} \end{bmatrix}$$

$$F^1_{41k} = \begin{bmatrix} GH & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Theta^1_{72} = \begin{bmatrix} \sqrt{h}\bar{\beta}_1\bar{\theta}P_1\tilde{A}_{2j}H & \sqrt{h}\bar{\beta}_1\bar{\theta}_1 P_1\tilde{B}_f \\ \sqrt{h}\bar{\beta}_1\bar{\theta}P_1\tilde{A}_{2j}H & \sqrt{h}\bar{\beta}_1\bar{\theta}_1 P_1\tilde{B}_f \\ \sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{A}_{2j}H & -\sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{B}_f \\ \sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{A}_{2j}H & -\sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{B}_f \end{bmatrix}$$

$$\Theta^1_{73} = \begin{bmatrix} \sqrt{h}\bar{\beta}_1\bar{\theta}P_1\tilde{B}_f \\ \sqrt{h}\bar{\beta}_1\bar{\theta}P_1\tilde{B}_f \\ \sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{B}_f \\ \sqrt{h}\bar{\beta}_1\rho_1 P_1\tilde{B}_f \end{bmatrix}, \quad \Theta^1_{75} = \begin{bmatrix} \sqrt{h}\bar{\beta}P_1\tilde{B}_f \\ \sqrt{h}\bar{\beta}P_1\tilde{B}_f \\ 0 \\ 0 \end{bmatrix}$$

$$\Theta^1_{74} = \begin{bmatrix} \sqrt{h}P_1\tilde{B}_i + \sqrt{h}\bar{\beta}_1\bar{\theta}P_1\tilde{B}_j H + \sqrt{h}\bar{\beta}_1\bar{\theta}_1 P_1\tilde{B}_j H \\ \sqrt{h}P_1\tilde{B}_i + \sqrt{h}\bar{\beta}_1\bar{\theta}P_1\tilde{B}_j H + \sqrt{h}\bar{\beta}_1\bar{\theta}_1 P_1\tilde{B}_j H \\ 0 \\ 0 \end{bmatrix}$$

$$F^1_{22k} = \text{diag}\Big\{-P_1 W^{-1}_{k1}P_1, -P_1 Z^{-1}_{k1}P_1, -P_1 W^{-1}_{k1}P_1, -P_1 Z^{-1}_{k1}P_1\Big\},$$

$$F^1_{33k} = F^1_{22k}, \quad F^1_{44k} = -I$$

$$\Theta^1_{81} = \begin{bmatrix} 0 & -\sqrt{h}\rho_1\rho_2 P_1\tilde{A}_{2j}H & 0 \\ 0 & -\sqrt{h}\rho_1\rho_2 P_1\tilde{A}_{2j}H & 0 \\ 0 & -\sqrt{h}\bar{\theta}_1\rho_2 P_1\tilde{A}_{2j}H & 0 \\ 0 & -\sqrt{h}\bar{\theta}_1\rho_2 P_1\tilde{A}_{2j}H & 0 \end{bmatrix}, \quad \Theta^1_{85} = \begin{bmatrix} 0 \\ 0 \\ \sqrt{h}\rho_2 P_1\tilde{B}_f \\ \sqrt{h}\rho_2 P_1\tilde{B}_f \end{bmatrix}$$

$$\Theta^1_{82} = \begin{bmatrix} \rho_1\rho_2 P_1\tilde{A}_{2j}H & -\rho_1\rho_2 P_1\tilde{B}_f \\ \rho_1\rho_2 P_1\tilde{A}_{2j}H & -\rho_1\rho_2 P_1\tilde{B}_f \\ -\bar{\theta}_1\rho_2 P_1\tilde{A}_{2j}H & -\bar{\theta}_1\rho_2 P_1\tilde{B}_f \\ -\bar{\theta}_1\rho_2 P_1\tilde{A}_{2j}H & -\bar{\theta}_1\rho_2 P_1\tilde{B}_f \end{bmatrix},$$

$$\Theta^1_{83} = \begin{bmatrix} \rho_1\rho_2 P_1\tilde{B}_f \\ \rho_1\rho_2 P_1\tilde{B}_f \\ -\bar{\theta}\rho_2 P_1\tilde{B}_f \\ -\bar{\theta}\rho_2 P_1\tilde{B}_f \end{bmatrix}$$

$$\Theta^1_{84} = \begin{bmatrix} 0 \\ 0 \\ -\bar{\theta}\rho_2\tilde{B}_j H - \bar{\theta}_1\rho_2 P_1\tilde{B}_j H \\ -\bar{\theta}\rho_2 P_1\tilde{B}_j H - \bar{\theta}_1\rho_2 P_1\tilde{B}_j H \end{bmatrix},$$

$$\Theta^2_{51i} = \begin{bmatrix} \sqrt{h}P_2\tilde{A}_{1i} \\ \sqrt{h}P_2\tilde{A}_{1i} \end{bmatrix}$$

$$\Theta^2_{11ik} = -2\eta_2 P_2 + P_2\tilde{A}_{1i} + \tilde{A}^T_{1i}P_2 + S_{k2} - \frac{1}{h}(W_{k1} + Z_{k2})$$

$$\Theta^2_{21k} = \frac{1}{h}(W_{k2} - U_{k2} + Z_{k2} - V_{2k}), \quad \Theta^1_{55} = diag\{-\Omega, -\Omega\}$$

$$\Theta^2_{22k} = \frac{1}{h}\Big(-2W_{k2} + U_{k2} + U^T_{k2} - 2Z_{k2} + V_{k2} + V^T_{k2}\Big)$$

$$\Theta^2_{32k} = \frac{1}{h}(W_{k2} - U_{k2} + Z_{k2} - V_{k2}),$$

$$\Theta^2_{33k} = -e^{2\eta_2 h} S_{k2} - \frac{1}{h}\left(W_{k2} + Z_{k2}\right),$$
$$\Theta^2_{31k} = \frac{1}{h}\left(U_{k2} + V_{k2}\right)$$
$$\Theta^2_{54i} = \begin{bmatrix} \sqrt{h} P_2 \tilde{B}_i \\ \sqrt{h} P_2 \tilde{B}_i \end{bmatrix}, \Theta^2_{55k} = \begin{bmatrix} -P_2 W_{k2}^{-1} P_2 & * \\ 0 & -P_2 Z_{k2}^{-1} P_2 \end{bmatrix}$$
$$\mathcal{I} = \frac{2\eta_1 \Delta_{\min} - 2(\eta_1+\eta_2)h - 2\eta_2\ b_{\max} - \ln \eta_1\eta_2}{\omega_D}$$
$$\Pi = \lambda_2 + h\lambda_3 + h^2\lambda_4 + h^2\lambda_5$$

Appendix B
The Proof of Theorem 1

Construct the Lyapunov-Krasovskii functional candidate as follows

$$V_{\Upsilon(t)}(t) = e^T(t)P_{\Upsilon(t)}e(t) + \mathcal{E}(t)e^T(s)\left(\sum_{k=1}^{m}\alpha_k S_{k\Upsilon(t)}\right) \times e(s)ds + \int_{-h}^{0}\int_{t+\varphi}^{t}\mathcal{E}(t)\dot{e}^T(s)\left(\sum_{k=1}^{m}\alpha_k \times W_{k\Upsilon(t)}\right)\dot{e}(s)dsd\varphi + \int_{-h}^{0}\int_{t+\varphi}^{t}\mathcal{E}(t)\dot{e}^T(s) \times\left(\sum_{k=1}^{m}\alpha_k Z_{k\Upsilon(t)}\right)\dot{e}(s)dsd\varphi \quad (50)$$

where $\alpha_k = \alpha_k\left(x^k(t), \epsilon^k(t)\right)$, $\mathcal{E}(t) = e^{2(-1)^{\Upsilon(t)}\eta_{\Upsilon(t)}(t-s)}$, $S_{k\Upsilon(t)}$, $W_{k\Upsilon(t)}$, $Z_{k\Upsilon(t)}$, $P_{\Upsilon(t)}$ denote symmetric positive matrices and $\Upsilon(t) = \begin{cases} 1, & t \in [h_n, h_n+\Delta_n) \\ 2, & t \in [h_n+\Delta_n, h_{n+1}). \end{cases}$

For $\Upsilon(t) = 1$, calculate the derivative and mathematical expectation of (50) along the system (22) for $i, j, k \in \jmath_m$, and then we obtain that

$$\begin{aligned}\mathbb{E}\left[\dot{V}_1(t)\right] \le& -2\eta_1 V_1(t) + 2\eta_1 e^T(t)P_1 e(t) + 2\mathbb{E}\left[e^T(t)P_1\dot{e}(t)\right] \\ & -e^{-2\eta_1 h}e^T(t-h)\left(\sum_{k=1}^{m}\alpha_k S_{k1}\right) \\ & \times e(t-h) + e^T(t)\left(\sum_{k=1}^{m}\alpha_k S_{k1}\right)e(t) \\ & + h\mathbb{E}\left[\dot{e}^T(t)\left(\sum_{k=1}^{m}\alpha_k\left(W_{k1}+Z_{k1}\right)\right)\dot{e}(t)\right] \\ & -\int_{t-h}^{t}\dot{e}^T(s)\left(\sum_{k=1}^{m}\alpha_k Z_{k1}\right)\dot{e}(s)ds \\ & -\int_{t-h}^{t}\dot{e}^T(s)\left(\sum_{k=1}^{m}\alpha_k W_{k1}\right)\dot{e}(s)ds \end{aligned} \quad (51)$$

Notice that

$$\mathbb{E}\left[\dot{e}^T(t)\mathcal{W}\dot{e}(t)\right] = \mathcal{A}_0^T\mathcal{W}\mathcal{A}_0 + \bar{\beta}_1^2\rho_1^2\mathcal{A}_2^T\mathcal{W}\mathcal{A}_2 + \rho_1^2\rho_2^2\mathcal{A}_2^T\mathcal{W}\mathcal{A}_2 + \rho_2^2\mathcal{A}_3^T\mathcal{W}\mathcal{A}_3 \quad (52)$$

where

$$\mathcal{A}_0 = \tilde{A}_{1i}e(t) + \tilde{B}_i\bar{\omega}(t) + \bar{\beta}\tilde{B}_f f\left(y(t)\right) + (1-\bar{\beta})\mathcal{A}_1,$$
$$\begin{aligned}\mathcal{A}_1 =& \bar{\theta}\left[\tilde{A}_{2j}He\left(t_r - \tau_{k_n}(t_r)\right) + \tilde{B}_f e_k(t_r) + \tilde{B}_j H\bar{\omega}(t_r)\right] \\ & +(1-\bar{\theta})\left[\tilde{A}_{2j}He\left(t-\tau_{k_n}(t)\right) + \tilde{B}_f e_k(t) + \tilde{B}_j H\bar{\omega}(t)\right],\end{aligned}$$
$$\mathcal{A}_2 = \tilde{A}_{2j}He\left(t-\tau_{k_n}(t)\right) + \tilde{B}_f e_k(t) - \tilde{A}_{2j}He\left(t_r - \tau_{k_n}(t_r)\right) - \tilde{B}_f e_k(t_r),$$
$$\mathcal{A}_3 = \tilde{B}_f f\left(x(t)\right) - \mathcal{A}_1,$$
$$\mathcal{W} = \left(\sum_{k=1}^{m}\alpha_k\left(W_{k1}+Z_{k1}\right)\right).$$

From the event-triggered scheme (20), it follows

$$\sigma^2 e^T\left(t-\tau_{k_n}(t)\right)H^T\Omega He\left(t-\tau_{k_n}(t)\right) - \left(e_{k_n}(t)\right)^T\Omega e_{k_n}(t) > 0 \quad (53)$$

By recalling the Assumption 1, it can be obtained

$$\bar{\beta}e^T(t)H^TG^TGHe(t) - \bar{\beta}f^T\left(y(t)\right)f\left(y(t)\right) \ge 0 \quad (54)$$

Then, similar to [41], by using Jensens inequality to deal with the integral item in (51) and combine (50)-(54), it can ensure that the following inequality holds

$$\mathbb{E}\left[\dot{V}_1(t) - \gamma^2\bar{\omega}^T(t)\bar{\omega}(t)\right] + 2\eta_1 V_1(t) \le \xi_1^T(t)\left(\sum_{i=1}^{m}\sum_{j=1}^{m}\sum_{k=1}^{m}\alpha_i\hat{\alpha}_j\alpha_k\Psi^1_{ijk}\right)\xi_1(t) \quad (55)$$

where $\xi_1^T(t) = [e^T(t), e^T\left(t-\tau_{k_n}(t)\right), e^T(t-h), e^T(t_r - \tau_{k_n}(t_r)), \left(e_{k_n}(t)\right)^T, (e_{k_n}(t_r))^T, \bar{\omega}^T(t), f\left(y(t)\right), \underbrace{I,\ldots,I}_{8}]$.

Next, in order to build the condition of finite-time boundedness, the inequality (55) can be expressed as

$$\mathbb{E}\left[\frac{d}{dt}(e^{2\eta_1 t}V_1(t))\right] < \mathbb{E}\left[\gamma^2 e^{2\eta_1 t}\bar{\omega}^T(t)\bar{\omega}(t)\right] \quad (56)$$

Integrating (56) from 0 to t, one has that

$$\begin{aligned}\mathbb{E}\left[V_1(t)\right] &< e^{-2\eta_1 t}V_1(0) + \mathbb{E}\left[\gamma^2\int_0^t e^{-2\eta_1 t}\bar{\omega}^T(t)\bar{\omega}(t)ds\right] \\ &< e^{-2\eta_1 t}V_1(0) + \frac{\gamma^2}{\eta_1}e^{-2\eta_1 T}d^2 \end{aligned} \quad (57)$$

Due to (50), $V_1(0)$ can be transformed to

$$\begin{aligned}V_1(0) =& \int_{-h}^{0}e^{2\eta_1 s}e^T(s)\left(\sum_{k=1}^{m}\alpha_k S_{k1}\right)e(s)ds \\ & + \int_{-h}^{0}\int_{\varphi}^{0}e^{2\eta_1 s}\dot{e}^T(s)\left(\sum_{k=1}^{m}\alpha_k W_{k1}\right)\dot{e}(s)dsd\varphi \\ & + \int_{-h}^{0}\int_{\varphi}^{0}e^{2\eta_1 s}\dot{e}^T(s)\left(\sum_{k=1}^{m}\alpha_k Z_{k1}\right)\dot{e}(s)dsd\varphi \\ & + e^T(0)P_1 e(0) \le \Pi c_1^2 \end{aligned} \quad (58)$$

where $c_1^2 = \sup_{-h\le s\le 0}\left\{e^T(s)Re(s), \dot{e}^T(s)R\dot{e}(s)\right\}$, $\Pi = \lambda_{\max\{P_1\}} + he^{-2\eta_1 h}\lambda_{\max\{S_{k1}\}} + h^2e^{-2\eta_1 h}\lambda_{\max\{W_{k1}\}} + h^2e^{-2\eta_1 h}\lambda_{\max\{Z_{k1}\}}$.

Notice that

$$\mathbb{E}\left[V_1(t)\right] \ge e^T(t)P_1e(t) \ge \lambda_{\max\{P_1\}}e^T(t)Re(t) \quad (59)$$

Combining (57)-(59) leads to

$$e^T(t)Re(t) \le \frac{e^{-2\eta_1 T}\Pi c_1 - \frac{\gamma^2 d^2}{2\eta_1}e^{-2\eta_1 T}}{\lambda_{\max\{P_1\}}} \le c_2^2 \quad (60)$$

with $\lambda_6 = \frac{\gamma^2}{2\eta_1}$. Based on Definition 1 and (60), the filtering error system (22) is finite-time boundedness.

Then, according to (24)-(26), similar to the proof in [25], one can get

$$\mathbb{E}\left[\dot{V}_1(t) - \gamma^2\bar{\omega}^T(t)\bar{\omega}(t)\right] + 2\eta_1 V_1(t) < 0 \quad (61)$$

with $\hat{\alpha}_j - \mu_j\alpha_j \ge 0$ for all j.

For $\Upsilon(t) = 2$, calculating the derivative and mathematical expectation of (50) in the same way, it yields that

$$\mathbb{E}\left[\dot{V}_2(t) - \gamma^2\bar{\omega}^T(t)\bar{\omega}(t)\right] - 2\eta_2 V_2(t) \le \xi_2^T(t)\left(\sum_{i=1}^m\sum_{j=1}^m\sum_{k=1}^m \alpha_i\hat{\alpha}_j\alpha_k\Psi_{ijk}^2\right)\xi_2(t) < 0 \quad (62)$$

where $\xi_2^T(t) = \left[e^T(t)\ e^T\left(t-\tau_{k_n}(t)\right)\ e^T(t-h)\ \bar{\omega}^T(t)\right]$.

Next, similar to [37], one has from (61)-(62) that

$$\mathbb{E}[V(t)] \le \begin{cases} e^{-2\eta_1(t-h_n)}\mathbb{E}[V_1(h_n)], & t\in[h_n, h_n+\Delta_n) \\ e^{2\eta_2(t-h_n-\Delta_n)}\mathbb{E}[V_2(h_n+\Delta_n)], & t\in\left[h_n+\Delta_n, h_{n+1}\right) \end{cases} \quad (63)$$

According to (27)-(29), it can be concluded that

$$\begin{cases} \mathbb{E}[V_1(h_n)] \le \varsigma_2\mathbb{E}\left[V_2\left(h_n^-\right)\right] \\ \mathbb{E}[V_2(h_n+\Delta_n)] \le \varsigma_1 e^{2(\eta_1+\eta_2)}\mathbb{E}\left[V_1\left(h_n^- + \Delta_n^-\right)\right] \end{cases} \quad (64)$$

For any $t\in[h_n, h_n+\Delta_n)$, combining (63)-(64), it is clear that

$$\begin{aligned} \mathbb{E}[V_1(t)] &\le e^{-2\eta_1(t-h_n)}\varsigma_2\mathbb{E}\left[V_2\left(h_n^-\right)\right] \\ &\le e^{2\eta_2(h_n-h_{n-1}-\Delta_{n-1})-2\eta_1(t-h_n)}\varsigma_2 \\ &\quad\times\mathbb{E}\left[V_2\left(h_{n-1}+\Delta_{n-1}\right)\right] \\ &\le \ldots \\ &\le e^p(\varsigma_1\varsigma_2)^{n(t)}\mathbb{E}[V_1(h_0)] \end{aligned} \quad (65)$$

with $p = 2(\eta_1+\eta_2)hn(t) - 2\eta_1(\Delta_{n-1}+\Delta_{n-2}+\ldots+\Delta_1+\Delta_0) + 2\eta_2[(h_n-h_{n-1}-\Delta_{n-1}) - \Delta_{n-2} - \ldots - \Delta_1 - \Delta_0]$.

Similarly, for any $t\in\left[h_n+\Delta_n, \Delta_{n+1}\right)$, then

$$\mathbb{E}[V_2(t)] \le \frac{e^{q(n(t)+1)}}{\varsigma_2}(\varsigma_1\varsigma_2)^{(n(t)+1)}\mathbb{E}[V_1(h_0)] \quad (66)$$

where $q = 2(\eta_1+\eta_2)h - 2\eta_1\Delta_{\min} + 2\eta_2 b_{\max}$.

According to (65)-(66), and combining with the nature of DoS attacks frequency in Assumption 3, the following inequality is derived

$$\mathbb{E}[V(t)] \le \max\left\{e^{a1}, \frac{e^{a2}}{\varsigma_2}\right\}e^{-\mathcal{I}t}\mathbb{E}[V_1(t)](h_0) \quad (67)$$

where $a_1 = 2b_1(\eta_1+\eta_2)h + b_1\ln(\eta_1\eta_2) + 2\eta_2 b_{\max}b_1 - 2\eta_1\Delta_{\min}b_1, a_2 = (b_1+1)[2(\eta_1+\eta_2)h + \ln(\eta_1\eta_2) + 2\eta_2 b_{\max} - 2\eta_1\Delta_{\min}]$.

From the definition of $V(t)$, one has

$$f_1\|\phi\|_h^2 \ge \mathbb{E}[V_1(0)],\quad f_2\|e(t)\|^2 \le \mathbb{E}[V(t)] \quad (68)$$

where $f_1 = \max\{\lambda_{\max}(P_s) + h\lambda_{\max}(S_{k1}) + \frac{h^2}{2}\lambda_{\max}(Z_{k1}+W_{k1})\}$, $f_2 = \min\{\lambda_{\min}(P_s)\}$.

One has from (67)-(68) that

$$\mathbb{E}[V(t)] \le \sqrt{\frac{f_2\max\left\{e^{a_1}, \frac{e^{a_2}}{\varsigma_2}\right\}}{f_1}e^{-\frac{\mathcal{I}}{2}t}\|\phi\|_h^2} \quad (69)$$

Therefore, based on Definition 1 and (69), we can derive that if (24)-(31), the filtering error system (22) is EMFTB with decay rate $\frac{\mathcal{I}}{2}$. This completes the proof. ■

Appendix C
The Proof of Theorem 2

According to the proof of Theorem 1, it is seen from (33) that

$$\begin{cases} \mathbb{E}\left[\dot{V}_1(t) + 2\eta_1 V_1(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2\bar{\omega}^T(t)\bar{\omega}(t)\right] \le 0, & s=1 \\ \mathbb{E}\left[\dot{V}_2(t) - 2\eta_1 V_2(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2\bar{\omega}^T(t)\bar{\omega}(t)\right] \le 0, & s=2 \end{cases} \quad (70)$$

When $s = 1$, from (70), multiplying both sides by $e^{-2\eta_1 t}$ lead to

$$\mathbb{E}\left[\frac{d}{dt}e^{-2\eta_1 t}V_1(t)\right] < e^{-2\eta_1 t}\mathbb{E}\left[\gamma^2\bar{\omega}^T(t)\bar{\omega}(t) - \tilde{z}^T(t)\tilde{z}(t)\right] \quad (71)$$

Integrating (71) from 0 to T, it yields that

$$0 < \mathbb{E}\left[e^{-2\eta_1 t}V_1(t)\right] < \int_0^T \mathbb{E}\left\{e^{-2\eta_1 t}\left[\gamma^2\bar{\omega}^T(t)\bar{\omega}(t) - \tilde{z}^T(t)\tilde{z}(t)\right]\right\}dt \quad (72)$$

Then, (72) can be rewritten as

$$\begin{aligned} \mathbb{E}\left[e^{-2\eta_1 t}\int_0^T \tilde{z}^T(t)\tilde{z}(t)dt\right] &< \mathbb{E}\left[\int_0^T e^{-2\eta_1 t}\tilde{z}^T(t)\tilde{z}(t)dt\right] \\ &< \mathbb{E}\left[\gamma^2\int_0^T e^{-2\eta_1 t}\bar{\omega}^T(t)\bar{\omega}(t)dt\right] \\ &< \mathbb{E}\left[\gamma^2\int_0^T \bar{\omega}^T(t)\bar{\omega}(t)dt\right] \end{aligned} \quad (73)$$

Similarly, when $s = 2$, one has that

$$\mathbb{E}\left[e^{2\eta_2 t}\int_0^T \tilde{z}^T(t)\tilde{z}(t)dt\right] < \mathbb{E}\left[\gamma^2\int_0^T \bar{\omega}^T(t)\bar{\omega}(t)dt\right] \quad (74)$$

Therefore, this completes the proof. ■

Appendix D
The Elements of the Matrix $\check{\Psi}_{ijk}^s$ in Theorem 3

$$\check{\Psi}_{ijk}^1 = \begin{bmatrix} \check{F}_{11ijk}^1 & * & * & * & * \\ \tilde{C}_i & -I & * & * & * \\ \check{F}_{21ijk}^1 & 0 & \check{F}_{22k}^1 & * & * \\ \check{F}_{31ijk}^1 & 0 & 0 & \check{F}_{33k}^1 & * \\ F_{41k}^1 & 0 & 0 & 0 & F_{44k}^1 \end{bmatrix}$$

$$\check{\Psi}^2_{ijk}=\begin{bmatrix}\check{\Theta}^2_{11ik} & * & * & * & * & * \\ \check{\Theta}^2_{21k} & \check{\Theta}^2_{22k} & * & * & * & * \\ \check{\Theta}^2_{31k} & \check{\Theta}^2_{32k} & \check{\Theta}^2_{33k} & * & * & * \\ 0 & 0 & 0 & -\gamma^2 I & * & * \\ \tilde{C}_i & 0 & 0 & 0 & -I & * \\ \check{\Theta}^2_{51i} & 0 & 0 & \check{\Theta}^2_{54k} & 0 & \check{\Theta}^2_{55k}\end{bmatrix}$$

$$\check{\digamma}^1_{11ijk}=\begin{bmatrix}\check{\Theta}^1_{11ik} & * & * & * & * & * \\ \check{\Theta}^1_{21jk} & \check{\Theta}^1_{22k} & * & * & * & * \\ \check{\Theta}^1_{31k} & \check{\Theta}^1_{32k} & \check{\Theta}^1_{33k} & * & * & * \\ \check{\Theta}^1_{41j} & 0 & 0 & \Xi_{22} & * & * \\ \check{\Theta}^1_{51} & 0 & 0 & 0 & \Theta^1_{55} & * \\ \check{\Theta}^1_{61ij} & 0 & 0 & 0 & 0 & \check{\Theta}^1_{66}\end{bmatrix}$$

$$\check{\Theta}^1_{11ik}=\Xi_{11}+\Xi^T_{11}+2\eta_1\hat{P}_1+\hat{S}_{k1}+\frac{1}{h}\left(\hat{W}_{k1}+\hat{Z}_{k1}\right)$$

$$\Xi_{11}=\begin{bmatrix}P_{11}A_i & \hat{A}_f \\ \bar{P}_{31}A_i & \hat{A}_f\end{bmatrix},\quad \Xi_{21}=\begin{bmatrix}C^T_j\hat{B}^T_f & C^T_j\hat{B}^T_f \\ 0 & 0\end{bmatrix}$$

$$\Xi_{22}=\begin{bmatrix}\sigma^2\Omega & 0 \\ 0 & 0\end{bmatrix},\quad \Xi_{31}=\begin{bmatrix}\hat{B}^T_f & \hat{B}^T_f\end{bmatrix},\check{\Theta}^1_{51}=\begin{bmatrix}\bar{\beta}_1\bar{\theta}_1\Xi_{31} \\ \bar{\beta}_1\bar{\theta}\Xi_{31}\end{bmatrix}$$

$$\Xi_{41}=\begin{bmatrix}B^T_iP_{11} & B^T_i\bar{P}_{31} \\ 0 & 0\end{bmatrix},\quad \Xi_{51}=\begin{bmatrix}D^T_j\hat{B}^T_f & D^T_j\hat{B}^T_f \\ 0 & 0\end{bmatrix}$$

$$\Xi_{61}=\begin{bmatrix}P_{12}A_i & \hat{A}_f \\ \bar{P}_{32}A_i & \hat{A}_f\end{bmatrix},\quad \Xi_{71}=\begin{bmatrix}B^T_iP_{12} & B^T_i\bar{P}_{32} \\ 0 & 0\end{bmatrix}$$

$$\check{\Theta}^1_{21jk}=\bar{\beta}_1\bar{\theta}_1\Xi_{21}+\frac{1}{h}\left(\hat{W}_{k1}-\hat{U}_{k1}+\hat{Z}_{k1}-\hat{V}_{k1}\right)$$

$$\check{\Theta}^1_{31k}=\frac{1}{h}\left(\hat{U}_{k1}+\hat{V}_{k1}\right),\quad \bar{\theta}_1=1-\bar{\theta},\bar{\beta}_1=1-\bar{\beta}$$

$$\check{\Theta}^1_{32k}=\frac{1}{h}\left(\hat{W}_{k1}-\hat{U}_{k1}+\hat{Z}_{k1}-\hat{V}_{k1}\right),$$

$$\check{\Theta}^1_{41j}=\bar{\beta}_1\bar{\theta}\Xi_{21}$$

$$\check{\Theta}^1_{22k}=\frac{1}{h}\left(-2\hat{W}_{k1}+\hat{U}_{k1}+\hat{U}^T_{k1}-2\hat{Z}_{k1}+\hat{V}_{k1}+\hat{V}^T_{k1}\right)+\Xi_{22}$$

$$\check{\Theta}^1_{33k}=-e^{-2\eta_1h}\hat{S}_{k1}-\frac{1}{h}\left(\hat{W}_{k1}+\hat{Z}_{k1}\right),\quad I_1=\mathrm{diag}\{I,I\}$$

$$\check{\Theta}^1_{61ij}=\begin{bmatrix}\Xi_{41}+(\bar{\beta}_1\bar{\theta}+\bar{\beta}_1\bar{\theta}_1)\Xi_{51} \\ \bar{\beta}\Xi_{31}\end{bmatrix},$$

$$\check{\Theta}^1_{66}=diag\{-\gamma^2I_1,-\bar{\beta}I\}$$

$$\check{\digamma}^1_{21ik}=\begin{bmatrix}\check{\Theta}^1_{71} & \check{\Theta}^1_{72} & \check{\Theta}^1_{73} & \check{\Theta}^1_{74} & \check{\Theta}^1_{75}\end{bmatrix}$$

$$\check{\Theta}^1_{71}=\sqrt{h}\begin{bmatrix}\Xi_{11} & \bar{\beta}_1\bar{\theta}_1\Xi^T_{21} & 0 \\ \Xi_{11} & \bar{\beta}_1\bar{\theta}_1\Xi^T_{21} & 0 \\ 0 & -\bar{\beta}_1\rho_1\Xi^T_{21} & 0 \\ 0 & -\bar{\beta}_1\rho_1\Xi^T_{21} & 0\end{bmatrix},\check{\Theta}^1_{75}=\sqrt{h}\begin{bmatrix}\bar{\beta}\Xi^T_{31} & 0 \\ \bar{\beta}\Xi^T_{31} & 0 \\ 0 & 0 \\ 0 & 0\end{bmatrix}$$

$$\check{\digamma}^1_{31ik}=\begin{bmatrix}\check{\Theta}^1_{81} & \check{\Theta}^1_{82} & \check{\Theta}^1_{83} & \check{\Theta}^1_{84} & \check{\Theta}^1_{85}\end{bmatrix}$$

$$\check{\Theta}^1_{72}=\sqrt{h}\begin{bmatrix}\bar{\beta}_1\bar{\theta}\Xi^T_{21} & \bar{\beta}_1\bar{\theta}_1\Xi^T_{31} \\ \bar{\beta}_1\bar{\theta}\Xi^T_{21} & \bar{\beta}_1\bar{\theta}_1\Xi^T_{31} \\ \bar{\beta}_1\rho_1\Xi^T_{21} & -\bar{\beta}_1\rho_1\Xi^T_{31} \\ \bar{\beta}_1\rho_1\Xi^T_{21} & -\bar{\beta}_1\rho_1\Xi^T_{31}\end{bmatrix},\check{\Theta}^1_{73}=\begin{bmatrix}\sqrt{h}\bar{\beta}_1\bar{\theta}\Xi^T_{31} \\ \sqrt{h}\bar{\beta}_1\bar{\theta}\Xi^T_{31} \\ \sqrt{h}\bar{\beta}_1\rho_1\Xi^T_{31} \\ \sqrt{h}\bar{\beta}_1\rho_1\Xi^T_{31}\end{bmatrix}$$

$$\check{\Theta}^1_{74}=\begin{bmatrix}\sqrt{h}\Xi^T_{41}+\sqrt{h}\bar{\beta}_1\bar{\theta}\Xi^T_{51}+\sqrt{h}\bar{\beta}_1\bar{\theta}_1\Xi^T_{51} \\ \sqrt{h}\Xi^T_{41}+\sqrt{h}\bar{\beta}_1\bar{\theta}\Xi^T_{51}+\sqrt{h}\bar{\beta}_1\bar{\theta}_1\Xi^T_{51} \\ 0 \\ 0\end{bmatrix}$$

$$\check{\Theta}^1_{81}=\begin{bmatrix}0 & -\sqrt{h}\rho_1\rho_2\Xi^T_{21} & 0 \\ 0 & -\sqrt{h}\rho_1\rho_2\Xi^T_{21} & 0 \\ 0 & -\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{21} & 0 \\ 0 & -\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{21} & 0\end{bmatrix},\check{\Theta}^1_{83}=\begin{bmatrix}\sqrt{h}\rho_1\rho_2\Xi^T_{31} \\ \sqrt{h}\rho_1\rho_2\Xi^T_{31} \\ -\sqrt{h}\bar{\theta}\rho_2\Xi^T_{31} \\ -\sqrt{h}\bar{\theta}\rho_2\Xi^T_{31}\end{bmatrix}$$

$$\check{\Theta}^1_{82}=\begin{bmatrix}\sqrt{h}\rho_1\rho_2\Xi^T_{21} & -\sqrt{h}\rho_1\rho_2\Xi^T_{31} \\ \sqrt{h}\rho_1\rho_2\Xi^T_{21} & -\sqrt{h}\rho_1\rho_2\Xi^T_{31} \\ -\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{21} & -\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{31} \\ -\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{21} & -\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{31}\end{bmatrix},\hat{P}_s=\begin{bmatrix}P_{1s} & \bar{P}_{3s} \\ \bar{P}_{3s} & \bar{P}_{3s}\end{bmatrix}$$

$$\check{\Theta}^1_{84}=\begin{bmatrix}0 \\ 0 \\ -\sqrt{h}\bar{\theta}\rho_2\Xi^T_{51}-\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{51} \\ -\sqrt{h}\bar{\theta}\rho_2\Xi^T_{51}-\sqrt{h}\bar{\theta}_1\rho_2\Xi^T_{51}\end{bmatrix},$$

$$\check{\Theta}^1_{85}=\begin{bmatrix}0 \\ 0 \\ \sqrt{h}\rho_2\Xi^T_{31} \\ \sqrt{h}\rho_2\Xi^T_{31}\end{bmatrix}$$

$$\check{\digamma}^1_{33k}=\mathrm{diag}\Big\{-2\varepsilon_{11}\hat{P}_1+\varepsilon^2_{11}\hat{W}_{k1},-2\varepsilon_{12}\hat{P}_1+\varepsilon^2_{12}\hat{Z}_{k1},$$
$$-2\varepsilon_{11}\hat{P}_1+\varepsilon^2_{11}\hat{W}_{k1},-2\varepsilon_{12}\hat{P}_1+\varepsilon^2_{12}\hat{Z}_{k1}\Big\},$$

$$\check{\digamma}^1_{44k}=\check{\digamma}^1_{33k}$$

$$\check{\Theta}^2_{11ik}=-2\eta_2\hat{P}_2+\Xi_{61}+\Xi^T_{61}+\hat{S}_{k2}-\frac{1}{h}\left(\hat{W}_{k1}+\hat{Z}_{k2}\right)$$

$$\check{\Theta}^2_{21k}=\frac{1}{h}\left(\hat{W}_{k2}-\hat{U}_{k2}+\hat{Z}_{k2}-\hat{V}_{2k}\right)$$

$$\check{\Theta}^2_{22k}=\frac{1}{h}\left(-2\hat{W}_{k2}+\hat{U}_{k2}+\hat{U}^T_{k2}-2\hat{Z}_{k2}+\hat{V}_{k2}+\hat{V}^T_{k2}\right)$$

$$\check{\Theta}^2_{31k}=\frac{1}{h}\left(\hat{U}_{k2}+\hat{V}_{k2}\right),\quad \check{\Theta}^2_{54i}=\begin{bmatrix}\sqrt{h}\Xi_{71} \\ \sqrt{h}\Xi_{71}\end{bmatrix}$$

$$\check{\Theta}^2_{32k}=\frac{1}{h}\left(\hat{W}_{k2}-\hat{U}_{k2}+\hat{Z}_{k2}-\hat{V}_{k2}\right)$$

$$\check{\Theta}^2_{33k}=-e^{2\eta_2h}\hat{S}_{k2}-\frac{1}{h}\left(\hat{W}_{k2}+\hat{Z}_{k2}\right),\check{\Theta}^2_{51i}=\begin{bmatrix}\sqrt{h}\Xi_{61} \\ \sqrt{h}\Xi_{61}\end{bmatrix}$$

$$\check{\Theta}^2_{55k}=\mathrm{diag}\{-2\varepsilon_{21}\hat{P}_2+\varepsilon^2_{21}\hat{W}_{k2},-2\varepsilon_{22}\hat{P}_2+\varepsilon^2_{22}\hat{Z}_{k2}\}$$

APPENDIX E
THE PROOF OF THEOREM 3

For any positive scalars ε_{s1} and ε_{s2}, since

$$\begin{cases}\left(W_{ks}-\varepsilon^{-1}_{s1}P_s\right)W^{-1}_{ks}\left(W_{ks}-\varepsilon^{-1}_{s1}P_s\right)\geq 0 \\ \left(Z_{ks}-\varepsilon^{-1}_{s2}P_s\right)Z^{-1}_{ks}\left(Z_{ks}-\varepsilon^{-1}_{s2}P_s\right)\geq 0,\quad (s=1,2)\end{cases}\tag{75}$$

By simple calculations, it can be seen from (75) that

$$\begin{cases}-P_sW^{-1}_{ks}P_s\leq -2\varepsilon_{s1}P_s+\varepsilon^2_{s1}W_{ks} \\ -P_sZ^{-1}_{ks}P_s\leq -2\varepsilon_{s2}P_s+\varepsilon^2_{s2}Z_{ks},(s=1,2)\end{cases}\tag{76}$$

Based on (76), substitute $-P_s W_{ks}^{-1} P_s$, $-P_s Z_{ks}^{-1} P_s$ with $-2\varepsilon_{s1} P_s + \varepsilon_{s1}^2 W_{ks}$, $-2\varepsilon_{s2} P_s + \varepsilon_{s2}^2 Z_{ks}$ into (32), respectively. For convenience, let

$$P_1 = \begin{bmatrix} P_{11} & P_{21}^T \\ P_{21} & P_{31} \end{bmatrix}, \quad J_1 = \begin{bmatrix} I & 0 \\ 0 & P_{21}^T P_{31}^{-1} \end{bmatrix}$$
$$P_2 = \begin{bmatrix} P_{12} & P_{22}^T \\ P_{22} & P_{32} \end{bmatrix}, \quad J_2 = \begin{bmatrix} I & 0 \\ 0 & P_{22}^T P_{32}^{-1} \end{bmatrix}$$

Since $\bar{P}_{3s} > 0$, there exist P_{2s} and $P_{3s} > 0$ satisfying $\bar{P}_{3s} = P_{2s}^T P_{3s}^{-1} P_{2s} (s = 1, 2)$. By applying Schur complement, $P_1 > 0$ is equivalent to $P_{11} - \bar{P}_{31} > 0$, then we can obtain (38). Furthermore, multiplying diag$\{\underbrace{J_1, \ldots, J_1}_{4}, \underbrace{I, \ldots, I}_{5}, \underbrace{J_1, \ldots, J_1}_{8}, I\}$ and its transpose on both sides of (32) with $s = 1$. Similarly, pre- and post- multiplying diag$\{J_2, J_2, J_2, I, I, J_2, J_2\}$ and its transpose on both sides of (32) with $s = 2$. Define $\hat{P}_s = J_s P_s J_s^T = \begin{bmatrix} P_{1s} & \bar{P}_{3s} \\ \bar{P}_{3s} & \bar{P}_{3s} \end{bmatrix}$, $\hat{S}_{ks} = J_s S_{ks} J_s^T$, $\hat{W}_{ks} = J_s W_{ks} J_s^T$, $\hat{Z}_{ks} = J_s Z_{k1s} J_s^T$, $\hat{U}_{ks} = J_s U_{ks} J_s^T$, $\hat{V}_{ks} = J_s V_{ks} J_s^T (s = 1, 2)$, then, one can see that (35) holds with the following variables

$$\hat{A}_f = \tilde{A}_f \bar{P}_{31}, \quad \tilde{A}_f = P_{21}^T A_f P_{21}^{-T}, \quad \hat{B}_f = P_{21}^T B_f,$$
$$\hat{C}_f = \tilde{C}_f \bar{P}_{31}, \quad \tilde{C}_f = C_f P_{21}^{-T}$$

Thus, it can be seen that for any $s = 1, 2$, $i, j, k \in J_m$, if (29)-(31) and (35)-(39) hold, the H_∞ filtering problem for state-dependant uncertain systems is solvable.

Similar to [40], on the basis of the above analysis, the parameters of filter can be written as $A_f = P_{21}^{-T} \tilde{A}_f P_{21}^T$, $B_f = P_{21}^{-T} \hat{B}_f$, $C_f = \tilde{C}_f P_{21}$. Then, the filter model (5) is designed by

$$\begin{cases} \dot{x}_f(t) = P_{21}^{-T} \tilde{A}_f P_{21} x_f(t) + P_{21}^{-T} \hat{B}_f \hat{y}(t) \\ z_f(t) = \tilde{C}_f P_{21} x_f(t) \end{cases} \tag{77}$$

Denoting $\hat{x}(t) = P_{21}^T x_f(t)$, (77) can be reformed as

$$\begin{cases} \hat{x}_f(t) = \tilde{A}_f \hat{x}(t) + \hat{B}_f \hat{y}(t) \\ z_f(t) = \tilde{C}_f \hat{x}(t) \end{cases} \tag{78}$$

That completes the proof. ■

REFERENCES

[1] X.-J. Li and G.-H. Yang, "FLS-based adaptive synchronization control of complex dynamical networks with nonlinear couplings and state-dependent uncertainties," *IEEE Trans. Cybern.*, vol. 46, no. 1, pp. 171–180, Jan. 2016.

[2] Z. Li, X. Zhao, and J. Yu, "On robust control of continuous-time systems with state-dependent uncertainties and its application to mechanical systems," *ISA Trans.*, vol. 60, pp. 12–20, Jan. 2016.

[3] G. Wang, M. Chadli, H. Chen, and Z. Zhou, "Event-triggered control for active vehicle suspension systems with network-induced delays," *J. Franklin Inst.*, vol. 356, no. 1, pp. 147–172, 2019.

[4] H. R. Karimi, "Robust delay-dependent H_∞ control of uncertain time-delay systems with mixed neutral, discrete, and distributed time-delays and Markovian switching parameters," *IEEE Trans. Circuits Syst. I: Reg. Papers*, vol. 58, no. 8, pp. 1910–1923, Aug. 2011.

[5] J. Feng, N. Li, and H. Zhao, "Event-driven H_∞ control for state-dependent uncertain systems in the sense of ISS," *IET Control Theory Appl.*, vol. 11, no. 7, pp. 962–972, 2017.

[6] J. Yu, Y. Sun, and Z. Li, "Event-based robust filter design for a class of state-dependent uncertain systems with network transmission delay under a unified framework," *Signal Process.*, vol. 148, pp. 56–57, Jul. 2018.

[7] X. Zhao, L. Zhang, P. Shi, and H. R. Karimi, "Robust control of continuous-time systems with state-dependent uncertainties and its application to electronic circuits," *IEEE Trans. Ind. Electron.*, vol. 61, no. 8, pp. 4161–4170, Aug. 2014.

[8] Y. Zhang, Y. Shi, and P. Shi, "Robust and non-fragile finite-time H_∞ control for uncertain Markovian jump nonlinear systems," *Appl. Math. Comput.*, vol. 279, pp. 125–138, Apr. 2016.

[9] Y. Zhang, C. Liu, and X. Mu, "Robust finite-time stabilization of uncertain singular Markovian jump systems," *Appl. Math. Model.*, vol. 36, pp. 5109–5121, Oct. 2012.

[10] H. Shen, F. Li, H. Yan, H. R. Karimi, and H.-K. Lam, "Finite-time event-triggered H_∞ control for T–S fuzzy Markov jump systems," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 5, pp. 3122–3135, Oct. 2018.

[11] Y. Zhang, P. Shi, and S. K. Nguang, "Observer-based finite-time H_∞ control for discrete singular stochastic systems," *Appl. Math. Lett.*, vol. 38, pp. 115–121, Dec. 2014.

[12] H. Ren, G. Zong, and H. R. Karimi, "Asynchronous finite-time filtering of networked switched systems and its application: An event-driven method," *IEEE Trans. Circuits Syst. I: Reg. Papers*, vol. 66, no. 1, pp. 391–402, Jan. 2019.

[13] H. Ren, G. Zong, and H. R. Karimi, "Asynchronous finite-time filtering of Markov jump nonlinear systems and its applications," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/TSMC.2019.2899733.

[14] Y. Zhang, Z. Wang, L. Zou, and H. Fang, "Event-based finite-time filtering for multirate systems with fading measurements," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 53, no. 3, pp. 1431–1441, Jun. 2017.

[15] B. Shen, Z. Wang, D. Wang, J. Luo, H. Pu, and Y. Peng, "Finite-horizon filtering for a class of nonlinear time-delayed systems with an energy harvesting sensor," *Automatica*, vol. 100, pp. 144–152, Feb. 2019.

[16] H. Shen, Z.-G. Wu, and J. H. Park, "Finite-time energy-to-peak filtering for Markov jump repeated scalar non-linear systems with packet dropouts," *IET Control Theory Appl.*, vol. 8, no. 16, pp. 1617–1624, 2014.

[17] S. He, W. Lyu, and F. Liu, "Robust H_∞ sliding mode controller design of a class of time-delayed discrete conic-type nonlinear systems," *IEEE Trans. Syst., Man Cybern., Syst.*, to be published, doi: 10.1109/tsmc.2018.2884491.

[18] K. Mathiyalagan, H. Su, P. Shi, and R. Sakthivel, "Exponential $\mathcal{H}_\infty$ filtering for discrete-time switched neural networks with random delays," *IEEE Trans. Cybern.*, vol. 45, no. 4, pp. 676–687, Apr. 2015.

[19] J. Liu, T. Yin, M. Shen, X. Xie, and J. Cao, "State estimation for cyber–physical systems with limited communication resources, sensor saturation and denial-of-service attacks," *ISA Trans.*, to be published, doi: 10.1016/j.isatra.2018.12.032.

[20] S. He, H. Fang, M. Zhang, F. Liu, and Z. Ding, "Adaptive optimal control for a class of nonlinear systems: The online policy iteration approach," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/tnnls.2019.2905715.

[21] K. Mathiyalagan, J. H. Park, R. Sakthivel, and S. M. Anthoni, "Robust mixed H_∞ and passive filtering for networked Markov jump systems with impulses," *Signal Process.*, vol. 101, pp. 162–173, Aug. 2014.

[22] H. Yan, H. Zhang, X. Zhan, Z. Li, and C. Yang, "Event-based H_∞ fault detection for buck converter with multiplicative noises over network," *IEEE Trans. Circuits Syst. I: Reg. Papers*, vol. 66, no. 6, pp. 2361–2370, Jun. 2019.

[23] J. Liu, Z.-G. Wu, D. Yue, and J. H. Park, "Stabilization of networked control systems with hybrid-driven mechanism and probabilistic cyber attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/tsmc.2018.2888633.

[24] Y. Zhang, P. Shi, R. K. Agarwal, and Y. Shi, "Event-based dissipative analysis for discrete time-delay singular jump neural networks," *IEEE Trans. Neural Netw. Learn. Syst.*, to be published, doi: 10.1109/tnnls.2019.2919585.

[25] J. Liu, L. Wei, X. Xie, E. Tian, and S. Fei, "Quantized stabilization for T–S fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks," *IEEE Trans. Fuzzy Syst.*, vol. 26, no. 6, pp. 3820–3834, Dec. 2018.

[26] Y. Zhang, P. Shi, R. K. Agarwal, and Y. Shi, "Event-based mixed H_∞ and passive filtering for discrete singular stochastic systems," *Int. J. Control*, to be published, doi: 10.1080/00207179.2018.1559360.

[27] X. Zhao, C. Liu, and E. Tian, "Finite-horizon tracking control for a class of stochastic systems subject to input constraints and hybrid cyber attacks," *ISA Trans.*, to be published, doi: 10.1016/j.isatra.2019.02.025.

[28] J. Liu, Y. Gu, X. Xie, D. Yue, and J. H. Park, "Hybrid-driven-based H_∞ control for networked cascade control systems with actuator saturations and stochastic cyber attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, to be published, doi: 10.1109/tsmc.2018.2875484.

[29] J. Liu, L. Wei, X. Xie, and D. Yue, "Distributed event-triggered state estimators design for sensor networked systems with deception attacks," *IET Control Theory Appl.*, to be published, doi: 10.1049/iet-cta.2018.5868.

[30] D. Ye, T.-Y. Zhang, and G. Guo, "Stochastic coding detection scheme in cyber-physical systems against replay attack," *Inf. Sci.*, vol. 481, pp. 432–444, May 2019.

[31] J. Liu, Y. Gu, L. Zha, Y. Liu, and J. Cao, "Event-triggered H_∞ load frequency control for multi-area power systems under hybrid cyber attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 49, no. 8, pp. 1665–1678, Aug. 2019.

[32] J. Liu, M. Yang, E. Tian, J. Cao, and S. Fei, "Event-based security control for state-dependent uncertain systems under hybrid-attacks and its application to electronic circuits," *IEEE Trans. Circuits Syst. I: Reg. Papers*, to be published, doi: 10.1109/tcsi.2019.2930572.

[33] Y. Sun, J. Yu, and Z. Li, "Event-triggered finite-time robust filtering for a class of state-dependent uncertain systems with network transmission delay," *IEEE Trans. Circuits Syst. I: Reg. Papers*, vol. 66, no. 3, pp. 1076–1089, Mar. 2019.

[34] D. Yue, E. Tian, and Q.-L. Han, "A delay system method for designing event-triggered controllers of networked control systems," *IEEE Trans. Autom. Control*, vol. 58, no. 2, pp. 475–481, Feb. 2013.

[35] D. Ding, Z. Wang, D. W. C. Ho, and G. Wei, "Distributed recursive filtering for stochastic systems under uniform quantizations and deception attacks through sensor networks," *Automatica*, vol. 78, pp. 231–240, Apr. 2017.

[36] C. De Persis and P. Tesi, "Input-to-state stabilizing control under denial-of-service," *IEEE Trans. Autom. Control*, vol. 60, no. 11, pp. 2930–2944, Nov. 2015.

[37] S. Hu, D. Yue, X. Xie, X. Chen, and X. Yin, "Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks," *IEEE Trans. Cybern.*, vol. 49, no. 12, pp. 4271–4281, Dec. 2019, doi: 10.1109/tcyb.2018.2861834.

[38] F. Amato and M. Ariola, "Finite-time control of discrete-time linear systems," *IEEE Trans. Autom. Control*, vol. 50, no. 5, pp. 724–729, May 2005.

[39] J.-N. Li and L.-S. Li, "Mean-square exponential stability for stochastic discrete-time recurrent neural networks with mixed time delays," *Neurocomputing*, vol. 151, pp. 790–797, Mar. 2015.

[40] C. Lin, Q.-G. Wang, T. H. Lee, and B. Chen, "H_∞ filter design for nonlinear systems with time-delay through T–S fuzzy model approach," *IEEE Trans. Fuzzy Syst.*, vol. 16, no. 3, pp. 739–746, Jun. 2008.

[41] L. Zha, J. Liu, and J. Cao, "Resilient event-triggered consensus control for nonlinear muti-agent systems with DoS attacks," *J. Franklin Inst.*, vol. 356, no. 13, pp. 7071–7090, 2019.

Jinliang Liu received the Ph.D. degree in networked control systems from Donghua University, Shanghai, China, in 2011.

From December 2013 to June 2016, he was a Post-Doctoral Research Associate with the School of Automation, Southeast University, Nanjing, China. From October 2016 to October 2017, he was a Visiting Researcher/Scholar with the Department of Mechanical Engineering, The University of Hong Kong. From November 2017 to January 2018, he was a Visiting Scholar with the Department of Electrical Engineering, Yeungnam University, South Korea. He is currently a Professor with the Nanjing University of Finance and Economics, and a Visiting Professor with the College of Automation Electronic Engineering, Qingdao University of Science and Technology. His research interests include networked control systems, complex dynamical networks, and time-delay systems.

Meng Yang received the B.Eng. degree from Nanjing Forest Police College, Nanjing, China, in 2017. She is currently pursuing the M.S. degree with the College of Information Engineering, Nanjing University of Finance and Economics.

Her research interests include networked control systems, T–S fuzzy systems, and time-delay systems.

Xiangpeng Xie received the B.S. and Ph.D. degrees in engineering from Northeastern University, Shenyang, China, in 2004 and 2010, respectively.

He is currently an Associate Professor with the Nanjing University of Posts and Telecommunications, Nanjing, China. His research interests include fuzzy modeling and control synthesis, optimization in process industries, and intelligent optimization algorithms.

Chen Peng (SM'15) received the B.Sc. and M.Sc. degrees in coal preparation and the Ph.D. degree in control theory and control engineering from the Chinese University of Mining Technology, Xuzhou, China, in 1996, 1999, and 2002, respectively.

He is currently a Professor with the School of Mechatronic Engineering and Automation, Shanghai University, Shanghai, China. His research interests include networked control systems, distributed control systems, smart grid, and intelligent control systems.

Huaicheng Yan received the B.Sc. degree in automatic control from the Wuhan University of Technology, China, in 2001, and the Ph.D. degree in control theory and control engineering from the Huazhong University of Science and Technology, China, in 2007.

He is currently a Professor with the School of Information Science and Engineering, East China University of Science and Technology, Shanghai, China. His research interests include networked control systems, multiagent systems, and stochastic systems.