



Research article

State estimation for cyber–physical systems with limited communication resources, sensor saturation and denial-of-service attacks

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HIGHLIGHTS

- The estimator design is proposed for CPSs with limited communication resources, sensor saturation and DoS attacks.
- The event-triggered scheme and quantization mechanism are adopted to relieve the effects of limited communication resources.
- A novel mathematical model of estimating CPSs with limited communication resources, sensor saturation and DoS attacks is constructed.
- A new criterion for stabilization analysis of CPSs is obtained and the estimator gains are derived.

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ABSTRACT

This paper addresses the issue of the state estimation for cyber–physical systems (CPSs) with limited communication resources, sensor saturation and denial-of-service (DoS) attacks. In order to conveniently handle nonlinear term in CPSs, a Takagi–Sugeno (T–S) fuzzy model is borrowed to approximate it. The event-triggered scheme and quantization mechanism are introduced to relieve the effects brought by limited communication resources. By taking the influence of sensor saturation and DoS attacks into account, a novel mathematical model of state estimation for CPSs is constructed with limited communication resources. By using the Lyapunov stability theory, the sufficient conditions, which can ensure the system exponentially stable, are derived. Moreover, the explicit expressions of the event-based estimator gains are obtained in the form of linear matrix inequalities (LMIs). At last, a simulated example is provided for illustrating the effectiveness of the proposed method.

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1. Introduction

Cyber–physical systems (CPSs) are regarded as a class of the multi-dimensional complex systems which integrate the physical environment, network and computational capability. Recently, the modeling and analysis of CPSs have been paid widespread attentions because of the wide spectrum of applications in areas such as transportation networks, healthcare and power generation, which leads to numbers of publications. For example, the authors in [1] investigate the transportation-cyber–physical systems which are applied in the vehicle-to-infrastructure-based communications to avert traffic disasters and traffic jams. In [2], the nonlinear CPSs

with multiple stochastic incomplete measurements are discussed where a T–S fuzzy model is used for approximating the physical process. T–S fuzzy model is firstly proposed in [3], which can be used to approximate nonlinear systems by a set of local linear systems smoothly connected by fuzzy membership functions. Since T–S fuzzy model is an effective method of modeling nonlinear systems, considerable attentions have been paid by scholars and lots of achievements have been acquired [4–8]. For instance, a class of T–S fuzzy systems is studied with the consideration of non-fragile distributed H_∞ filtering problem [4]. With the assistance of the T–S fuzzy model, the authors propose the algorithm of H_∞ filtering design for networked systems in [5]. The authors in [8] address the issue of continuous-time T–S fuzzy systems by using the time derivatives. Motivated by this work, T–S fuzzy model which can be utilized to approximate nonlinear systems by a series of local linear systems is borrowed to describe CPSs in this paper.

Due to the extensive attentions paid to CPSs, the problem of state estimation for CPSs has aroused considerable interests. Lots

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of scholars devote themselves into investigating the issue and numerous results are available. In [9], the state estimation of the wireless CPSs subjected to DoS attacks is investigated. The authors in [10] focus on the state estimation of CPSs with DoS attacks occurring in a wireless channel. A finite time event-triggered supervised estimator is designed to estimate linear CPSs under the event-driven scheme [11]. Apart from the above statement about estimating CPSs, the issue of sensor saturation has also fascinated plenty of academics for the reason that sensor saturation is likely to undermine the stability of CPSs to some degree. It need to be pointed out that the physical process is measured by a series of sensors where there always exists sensor saturation. In fact, many efforts have been devoted into researching sensor saturation for several years and massive achievements are obtained [12,13]. By taking the sensor saturations into account, the distributed H_∞ filter is designed in [12] to estimate sensor networks. With the consideration of randomly occurring sensor saturation, the authors in [13] investigate the problem of the H_∞ estimator design of complex networks. It needs to be pointed out that the problem of the state estimation for CPSs has received wide attentions in the recent years, however, the research of estimating CPSs with sensor saturation has not been fully done yet, which initially motivates this paper.

In CPSs, owing to the limitation of bandwidth and communication resources, it may bring some problems which can degrade the system performance. How to handle the problems of limited communication resources has been widely discussed. During the past years, the periodic sampling (time-triggered scheme) is a frequently-used signal processing method, where the signals are sampled and transmitted at a fixed interval. However, the periodic sampling brings the issue of redundant data transmission, which results in the great network load. For the purpose of relieving the burden of communication network, various event-triggered schemes are proposed [14–16]. One of the famous event-triggered schemes is an event-triggered scheme relying on discrete supervision of the system states, which is firstly proposed in [16]. This event-triggered scheme only supervises the difference between the states sampled in discrete instants and the data can be transmitted only when they satisfy the given condition. Based on [16], numerous researches with event-triggered scheme are carried out during the last decade [17–28]. For instance, the networked systems with multiple sensor distortions is investigated in [22] by utilizing the event-triggered scheme mentioned [16]. Inspired by [16], the authors propose an adaptive event-triggered communication mechanism where the assumption of the prior work is relaxed in [23]. On the basis of the event-triggered scheme presented in [16], the authors in [25] propose a new discrete event-triggered transmission protocol for networked control systems (NCSs) with delays and noise. Motivated by the work in [16], the resilient event-triggering communication scheme is proposed to improve the performance of NCSs with periodic DoS attacks in [27]. Based on the event-triggered scheme proposed in [16], the hybrid-driven scheme is proposed in [28], which combines the advantages of the time-triggered scheme and the event-triggered scheme. The authors in [29] address the problem of H_∞ filter design for neural networks with the hybrid-driven scheme. By adopting the hybrid-driven scheme, the issue of H_∞ control for networked cascade control systems is addressed in [30]. Besides, the hybrid-driven scheme proposed in [28] is utilized to study the T–S fuzzy systems [5,31,32]. In addition to the event-triggered mechanism as stated above, the quantization mechanism is also adopted to relieve the influence of limited communication resources [33–40]. In [33], the authors use quantization mechanism to alleviate network transmission load of NCSs. A quantization mechanism is utilized to relieve the network bandwidth burden of Markovian jump systems in [36]. By using the quantization mechanism, the

authors in [40] design the robust fuzzy-model-based filter for networked systems. Motivated by the above research, the event-triggered scheme and quantization mechanism are introduced in this paper to alleviate the effects of limited communication resources.

It is well known that the introduction of the network has brought many advantages to CPSs, however, it also may lead to the common communication constraints such as packet dropouts, limited bandwidth and time delay [41]. In addition, the network may be susceptible to cyber attacks due to the openness of communication networks [42]. In general, there are three kinds of cyber attacks including injection attacks, deception attacks and DoS attacks [29]. The injection attacks aim to replace the transmitted signal with the false data when the data is transmitted via the network. The deception attacks replace normal data by pretending to be trusted parties to degrade system performance. Different from the above attacks, the purpose of the DoS attacks is to make efforts to block data transmission. Recently, the cyber attacks have been paid considerable attentions by researchers and a great many results have been available in the literature. For instance, considering the influence of false data injection attacks, the issue of estimating the NCSs is addressed in [43]. From an attacker's point of view, the optimal attacking region is determined while a false data injection attack takes place in a power grid [44]. The problem of H_∞ filter design for neural networks is investigated under the consideration of deception attacks [29]. By considering the effects of DoS attacks, the robust stabilization of NCSs with event-triggered scheme and quantization is studied in [26]. In [45], CPSs with DoS attacks are studied where the data is transmitted over the network from sensors to remote estimators. The event-triggered control model for CPSs subjected to DoS attacks is constructed in [46]. To the best of our knowledge, with the consideration of DoS attacks, the issue of state estimation for CPSs with limited communication resources and sensor saturation has been paid few attentions, which is the second motivation of this paper.

The rest of this paper is organized as follows. In Section 2, the mathematical models of considered problems are presented, respectively and system modeling is given. In Section 3, the sufficient conditions are acquired which can guarantee the exponential stability of the system and the desired estimator gains are accurately derived. In Section 4, a simulated example is supplied to illustrate the usefulness of designed estimator.

Notation. $\mathbb{R}_{>0}$, \mathbb{R}^m and $\mathbb{R}^{m \times n}$ represent the set of positive real scalars, the m -dimensional Euclidean space and the set of $m \times n$ real matrices, respectively; the superscript T denotes matrix transposition; I is the identity matrix with appropriate dimension; for $X \in \mathbb{R}^{m \times m}$, the notation $X > 0$ represents that the matrix X is real symmetric positive definite. For a matrix B and two symmetric matrices A and C , $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ stands for a symmetric matrix, where $*$ refers to the entries implied by symmetry. $\mathcal{L}_2[0, \infty)$ denotes the space of square-integrable vector functions defined on $[0, \infty)$. \mathcal{N} denotes the set of non-negative integers. For a scalar $\epsilon > 0$, $\|\mathbf{x}_t\|_\epsilon \triangleq \sup_{-\epsilon \leq \theta \leq 0} \{\|\dot{\mathbf{x}}(t + \theta)\|, \|\mathbf{x}(t + \theta)\|\}$, where $\mathbf{x}(t)$ is a continuous function $t_{k_2, n}$ denotes that $t_{k_2, n}$ belongs to time interval $[t_{k_1, n}, t_{k_2, n})$.

2. Problem formulation and modeling

2.1. System description

The framework of event-triggered state estimation for CPSs under sensor saturations, quantization and DoS attacks is shown in Fig. 1, where the data transmission between the quantizer and estimator is realized over the network with DoS attacks. In the following, we will give the mathematical model of the physical

plant, event-triggered scheme, sensor saturation, quantizer and DoS attacks, respectively.

Consider the physical process which can be described as the following continuous-time nonlinear system:

$$\dot{x}(t) = f(x(t)) + Bw(t) \quad (1)$$

where $x(t) \in \mathbb{R}^m$ is the state vector; $w(t)$ denotes the disturbance input vector belonging to $\mathcal{L}_2[0, \infty)$; B is the known system matrix with appropriate dimensions; $f(x(t))$ is a smooth function with $f(0) = 0$.

In order to conveniently deal with the nonlinear function $f(x(t))$ in (1), $f(x(t))$ is assumed to be approximated by T–S fuzzy model here. By using the T–S fuzzy model based on fuzzy IF–THEN rules, the i th rule of fuzzy linear model is given as follows which can describe the above system (1).

IF $h_1(x(t))$ is H_1^i and $h_l(x(t))$ is H_l^i , THEN

$$\dot{x}(t) = A_i x(t) + B_i w(t) \quad (2)$$

where $i \in \mathfrak{R} = \{1, 2, \dots, r\}$, r is the number of IF–THEN rules; H_v^i ($i \in \mathfrak{R}$, $v = 1, 2, \dots, l$) denote the fuzzy sets; $h_v(x(t))$ are fuzzy premise variables; for simplicity, $h_v(x(t)) = [h_1(x), h_2(x), \dots, h_l(x)]$ is abbreviated as $h(x)$; A_i and B_i are constant matrices.

By the utilization of a center-average defuzzifier, product interference and singleton fuzzifier, the system (2) can be expressed as follows:

$$\dot{\hat{x}}(t) = \sum_{i=1}^r \varphi_i(h(x)) [A_i \hat{x}(t) + B_i w(t)] \quad (3)$$

where $\varphi_i(h(x)) = \frac{\theta_i(h(x))}{\sum_{i=1}^r \theta_i(h(x))}$, $\theta_i(h(x)) = \prod_{v=1}^l \theta_v^i(h_v(x))$, $\theta_v^i(h_v(x))$ stands for the grade membership value of $h_v(x)$. $\varphi_i(h(x))$ is the normalized membership function which satisfies $\varphi_i(h(x)) \geq 0$, $\sum_{i=1}^r \varphi_i(h(x)) = 1$.

Remark 1. T–S fuzzy model firstly proposed in [3] can be utilized to approximate nonlinear systems by many local linear systems which are smoothly linked together through fuzzy membership functions. As a mathematical tool, T–S fuzzy model has been demonstrated to be an effective method of modeling numerous nonlinear systems [2,23,31]. In this paper, T–S fuzzy model is borrowed to describe the CPSs with limited communication resources under the consideration of sensor saturation and DoS attacks.

For the local sensor, give the measurement model as follows

$$\begin{cases} y(t) = \sum_{i=1}^r \varphi_i(h(x)) C_i x(t) \\ z(t) = \sum_{i=1}^r \varphi_i(h(x)) L_i x(t) \end{cases} \quad (4)$$

where $y(t) \in \mathbb{R}^{m_2}$ is the ideal measurement; $z(t) \in \mathbb{R}^{m_3}$ is the signal to be estimated; C_i and L_i are constant matrices with appropriate dimensions.

2.2. State estimation

The purpose of the paper is designing the estimator to estimate the CPSs with limited communication resources, sensor saturation and DoS attacks. As shown in Fig. 1, an event generator is located between the sensor and the quantizer which is used for determining whether the sampled data should be delivered to the quantizer or not. In addition to the event generator, the quantizer is utilized to save the limited communication resources. At the same time, the influences of sensor saturation and DoS attacks in the network are both considered.

Consider the following state estimator:

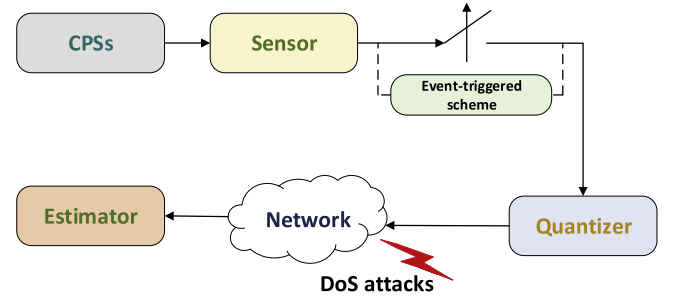


Fig. 1. The structure of the state estimation for CPSs with DoS attacks.

IF $h_1(\hat{x}(t))$ is G_1^j and $h_\ell(\hat{x}(t))$ is G_ℓ^j , THEN

$$\begin{cases} \dot{\hat{x}}(t) = A_j \hat{x}(t) + K_j (\tilde{y}(t) - \hat{y}(t)) \\ \hat{y}(t) = C_j \hat{x}(t) \\ \hat{z}(t) = L_j \hat{x}(t) \end{cases} \quad (5)$$

where $\hat{x}(t) \in \mathbb{R}^m$ is the estimated state vector and K_j denote the state estimator parameters to be determined, $\tilde{y}(t)$ is the actual input of the state estimator and $\hat{y}(t)$ is the estimation of $y(t)$, $\hat{z}(t) \in \mathbb{R}^{m_3}$ is the estimation of $z(t)$. G_l^j ($j \in \mathfrak{R}$, $l = 1, 2, \dots, \varrho$) denote the fuzzy sets, $h_l(\hat{x}(t))$ are premise variables. For simplicity, $h(\hat{x})$ is used to represent $h_l(\hat{x}(t))$ and $h_l(\hat{x}) = [h_1(\hat{x}), h_2(\hat{x}), \dots, h_\ell(\hat{x})]$. A_j are C_j are the parameter matrices with appropriate dimensions.

Then, the fuzzy state estimator can be described as follows.

$$\begin{cases} \dot{\hat{x}}(t) = \sum_{j=1}^r \psi_j(h(\hat{x})) [A_j \hat{x}(t) + K_j (\tilde{y}(t) - \hat{y}(t))] \\ \hat{y}(t) = \sum_{j=1}^r \psi_j(h(\hat{x})) C_j \hat{x}(t) \\ \hat{z}(t) = \sum_{j=1}^r \psi_j(h(\hat{x})) L_j \hat{x}(t) \end{cases} \quad (6)$$

where $\psi_j(h(\hat{x})) = \frac{\varsigma_j(h(\hat{x}))}{\sum_{j=1}^r \varsigma_j(h(\hat{x}))}$, $\varsigma_j(h(\hat{x})) = \prod_{l=1}^{\varrho} G_l^j(h_l(\hat{x}))$, $G_l^j(h_l(\hat{x}))$ denotes the grade membership value of $h_l(\hat{x})$. $\psi_j(h(\hat{x}))$ which presents the normalized membership function satisfy $\psi_j(h(\hat{x})) \geq 0$, $\sum_{j=1}^r \psi_j(h(\hat{x})) = 1$.

Remark 2. From (3) and (6), one can see that the premise variable $\psi_j(h(\hat{x}))$ in (6) is assumed to be different from $\varphi_i(h(x))$ in (3), which will be handled in the next section. Based on the estimator model (6), the problem of state estimation for CPSs is firstly addressed with limited communication resources, sensor saturation and DoS attacks in this paper.

2.3. Sensor saturation

In this paper, sensor saturation is taken into account, which can be described by a saturation function $sat(\vartheta) = [sat(\vartheta_1), sat(\vartheta_2), \dots, sat(\vartheta_s)]^T \in \mathbb{R}^s$ and the definition of each element for $sat(\vartheta)$ is given as follows [12]:

$$sat(\vartheta_i) = \begin{cases} \rho_i, & \vartheta_i > \rho_i \\ \vartheta_i, & -\rho_i < \vartheta_i < \rho_i, i = 1, 2, \dots, s \\ -\rho_i, & \vartheta_i < -\rho_i \end{cases} \quad (7)$$

where ρ_i represents the threshold value of i th sensor saturation.

In addition, the saturation function can be decomposed into a linear part and a nonlinear part, that is $sat(\vartheta) = \vartheta - \phi(\vartheta)$. Then, there exists a real number $\varepsilon \in (0, 1)$ such that

$$\varepsilon \vartheta^T \vartheta \geq \phi^T(\vartheta) \phi(\vartheta) \quad (8)$$

where $\phi(\vartheta)$ is a nonlinear function.

As results, the real output of sensor node with sensor saturation is

$$\bar{y}(t) = \text{sat}(y(t)) = y(t) - \phi(y(t)) \quad (9)$$

holds for the constraint that for $\varepsilon \in (0, 1)$,

$$\varepsilon y(t)^T y(t) \geq \phi^T(y(t)) \phi(y(t)) \quad (10)$$

Remark 3. It should be noted that the saturation considered in this paper belongs to sensor saturation. The behavior of sensor saturation usually occurs in physical system when the system output value measured by a sensor is larger than its dynamic range owing to the physical/technological restrictions. In order to describe the influence of sensor saturation quantitatively, a saturation function $\text{sat}(\vartheta)$ is introduced which can be divided into two parts, namely, a linear part ϑ and a nonlinear part $\phi(\vartheta)$.

2.4. Event-triggered scheme

In order to alleviate the impact of limited communication resources, the event-triggered scheme is adopted in this paper, which is shown in Fig. 1. Whether the sampled data is delivered or not depends on the given event-triggering condition which is presented as follows [47].

$$e^T(t_k h) \Omega e(t_k h) \leq \sigma \bar{y}^T(t_k h + ah) \Omega \bar{y}(t_k h + ah) \quad (11)$$

where $e(t_k h) = \bar{y}(t_k h) - \bar{y}(t_k h + ah)$, h is the sampling period, $t_k h$ denotes the latest transmitting instant, $\sigma \in [0, 1)$ is a design parameter, $\Omega > 0$ and $a = 1, 2, \dots$. Then the next triggering instant $t_{k+1} h$ can be expressed by

$$t_{k+1} h = t_k h + \min_{a \in \mathcal{N}} \{ ah | e^T(t_k h) \Omega e(t_k h) > \sigma \bar{y}^T(t_k h + ah) \Omega \bar{y}(t_k h + ah) \} \quad (12)$$

Remark 4. In inequality (11), σ is a positive parameter, which can determine the frequency of event-triggered scheme. When $\sigma = 0$, it indicates that all sampled data are delivered. When $\sigma \in (0, 1)$, it means that the measurement data will be transmitted via the network only when they exceed the threshold in (11). It should be pointed out that the event-triggered algorithm (12) is firstly proposed in [16], where the difference between the states sampled in discrete instants is supervised. In this paper, the event-triggered scheme is adopted to investigate the issue of the quantized state estimation for CPSs with sensor saturation and DoS attacks.

2.5. Signal quantization

For the purpose of further relieving the effects brought by limited communication resources, a quantization mechanism is introduced to save communication resources. In Fig. 1, the logarithmic quantizer $q(\cdot)$ is located between the sensor and the estimator, which is defined as follows to quantize the quantizer input $y_e(t)$ [2,12].

$$q(y_i) = \begin{cases} z_i, & y_i > z_i, \frac{z_i}{1+\delta_i} < y_i < \frac{z_i}{1-\delta_i} \\ 0, & y_i = 0 \\ -q(-y_i), & y_i < 0 \end{cases} \quad (13)$$

where $\delta_i = \frac{1-r_i}{1+r_i}$, r_i is quantization density of $q(\cdot)$. Besides, the set of quantized levels can be described by

$$Z = \{\pm z_i : z_i = r_i z_0, i = \pm 1, \pm 2, \dots\} \cup \{\pm z_0\} \cup \{0\}, 0 < r_i < 1, z_0 > 0. \quad (14)$$

Define

$$q(y) = \text{diag}\{q_1(y_1), q_2(y_2), \dots, q_n(y_n)\} \quad (15)$$

Due to the symmetry of logarithmic quantizer, $q_i(-y_i) = -q_i(y_i)$, then we have

$$q_i(y_i) = (1 + \Delta_{q_i}(y_i)) y_i \quad (16)$$

where $|\Delta_{q_i}(y_i)| \leq \delta_i$. For simplicity, we use Δ_{q_i} to denote $\Delta_{q_i}(y_i)$, it yields that

$$q(y) = (I + \Delta_q) y \quad (17)$$

where $\Delta_q = \text{diag}\{\Delta_{q_1}, \Delta_{q_2}, \dots, \Delta_{q_n}\}$.

Then the transmitted data via quantizer can be expressed as follows:

$$y_q(t) = y_q(y_e(t)) = (I + \Delta_q) y_e(t) \quad (18)$$

Remark 5. The purpose of introducing quantization is to further save network bandwidth and alleviate the effects of limited communicated resources. In general, there are two kinds of quantizers, that is, dynamic quantizers (e.g. uniform quantizers) and static quantizers (e.g. logarithmic quantizers) [48]. The quantizer (13) adopted in this paper is a logarithmic quantizer which can quantize the input $y_e(t)$ to save limited bandwidth and energy consumption.

2.6. DoS attacks

As shown in Fig. 1, we consider the effects of cyber attacks in this paper, which are assumed to be periodic DoS attacks described by the following DoS jamming signal [26]:

$$F_{\text{DoS}}(t) = \begin{cases} 1, & t \in [(n-1)T, (n-1)T + T_{\text{sleep}}) \\ 0, & t \in [(n-1)T + T_{\text{sleep}}, nT) \end{cases} \quad (19)$$

where $n \in \mathcal{N}$ stands for the period number; $T \in \mathbb{R}_{>0}$ represents the period of the jammer; T_{sleep} stands for the sleeping time of the jammer, which is with minimum boundary T_{sleep}^{\min} in every period T , namely, $T_{\text{sleep}}^{\min} \leq T_{\text{sleep}} < T$. Therefore, in a period T , $[0, T_{\text{sleep}})$ represents the interval when DoS jamming attack is off and $[T_{\text{sleep}}, T)$ denotes the interval when the jamming signal is active. That is to say, the communication is successful in $\cup_{n \in \mathcal{N}} [(n-1)T, (n-1)T + T_{\text{sleep}})$ while is blocked in $\cup_{n \in \mathcal{N}} [(n-1)T + T_{\text{sleep}}, nT)$. It is worthy of being mentioned that T_{sleep} has no need to keep time-invariant for Pulse-Width Modulated jamming as described in [26]. Therefore, we might as well presume that $T_{\text{sleep}} = T_{\text{sleep}}^{\min}$ over which the event-triggered scheme (20) maintain a ‘‘worse-case jamming scenario’’.

Under the consideration of periodic DoS attacks, the communication is denied over the time intervals $\cup_{n \in \mathcal{N}} [(n-1)T + T_{\text{sleep}}^{\min}, nT)$ when the event-triggering condition (11) is no longer satisfied. Motivated by [27], a novel event-triggered mechanism is introduced which is resilient towards the periodic DoS attacks (19).

Assumption 1. The cyber attacks discussed in this paper belong to the periodic DoS attacks. Further, with the consideration of the periodic DoS attacks (19), the event-triggered instant is defined as follows [25]:

$$t_{k,n} h = \{nT\} \cup \{t_{k_a} h \text{ satisfying (11)}\} \\ t_{k_a} h \in [(n-1)T, (n-1)T + T_{\text{sleep}}^{\min}) \} \quad (20)$$

where $k_a(a \in \mathcal{N})$, k represents the number of triggering times occurring in a th jammer action period.

Remark 6. In recent years, DoS attacks have received considerable attentions, which can bring out packet dropouts and reduce system performance. Many scholars devote themselves into the analysis and design of various systems subjected to DoS attacks [27,49]. Up

to now, there are no publications for estimating CPSs with event-triggered scheme and DoS attacks. In this paper, the co-design of both the event-triggered scheme (11) and the periodic DoS attacks (19) for CPSs with sensor saturation and quantization is given in the next subsection, which is obviously different from the problems investigated in existing publications.

Remark 7. In inequality (11), the event-triggering condition $e^T(t_k h) \Omega e(t_k h) \leq \sigma \bar{y}^T(t_k h + ah) \Omega \bar{y}(t_k h + ah)$ holds for $\forall t_k, k \in \mathcal{N}$ without the consideration of DoS attacks. Nevertheless, in (20), this triggering condition can hold only in the intervals $\cup_{n \in \mathcal{N}} [(n-1)T, (n-1)T + T_{sleep}^{min}]$ rather than $\forall t_k, k \in \mathcal{N}$ due to the existence of the periodic DoS attacks (19). Especially, the communication is blocked in the intervals $\cup_{n \in \mathcal{N}} [(n-1)T + T_{sleep}^{min}, nT]$ where there is no event happening, then the triggering instant is only the right endpoint nT .

2.7. Modeling of CPSs with periodic DoS attacks

To simplify representation, for $n \in \mathcal{N}$, denote $\mathcal{D}_{1,n} \triangleq [(n-1)T, (n-1)T + T_{sleep}^{min}]$, $\mathcal{D}_{2,n} \triangleq [(n-1)T + T_{sleep}^{min}, nT]$, $\mathcal{F}_{k,n} \triangleq [t_{k,n}h, t_{k+1,n}h]$, $k \in \{0, 1, \dots, k(n)\} \triangleq \mathcal{G}(n)$, $t_{0,n}h \triangleq (n-1)T (h < T)$, and

$$k(n) = \sup \{k \in \mathcal{N} | t_{k,n}h \leq (n-1)T + T_{sleep}\}$$

Then the time interval $\mathcal{F}_{k,n}$ can be divided into

$$\mathcal{F}_{k,n} = \left\{ \cup_{\zeta=1}^{o_{k,n}} [t_{k,n}h + (\zeta-1)h, t_{k,n}h + \zeta h] \cup [t_{k,n}h + \zeta_{k,n}h, t_{k,n}h] \right\} \quad (21)$$

where $k \in \mathcal{G}(n)$, $n \in \mathcal{N}$, and $o_{k,n} \triangleq \inf \{\zeta \in \mathcal{N} | t_{k,n}h + \zeta h \geq t_{k,n+1}h\}$.

Set

$$\begin{cases} G_{k,n}^\zeta = [t_{k,n}h + (\zeta-1)h, t_{k,n}h + \zeta h], \\ \zeta \in \{1, 2, \dots, o_{k,n} - 1\} \\ G_{k,n}^{o_{k,n}} = [t_{k,n}h + (o_{k,n}-1)h, t_{k+1,n}h] \end{cases} \quad (22)$$

Note that

$$\mathcal{D}_{1,n} = \cup_{k=0}^{\mathcal{G}(n)} \{\mathcal{F}_{k,n} \cap \mathcal{D}_{1,n}\} \subseteq \cup_{k=0}^{\mathcal{G}(n)} \mathcal{F}_{k,n} \quad (23)$$

By combining (21), (22) and (23), the interval $\mathcal{D}_{1,n}$ can be represented as

$$\mathcal{D}_{1,n} = \cup_{k=0}^{\mathcal{G}(n)} \cup_{\zeta=1}^{o_{k,n}} \{G_{k,n}^\zeta \cap \mathcal{D}_{1,n}\} \quad (24)$$

Let $O_{k,n}^\zeta = G_{k,n}^\zeta \cap \mathcal{D}_{1,n}$, then we have $\mathcal{D}_{1,n} = \cup_{k=0}^{\mathcal{G}(n)} \cup_{\zeta=1}^{o_{k,n}} O_{k,n}^\zeta$. Therefore, the definitions of two piecewise functions are given as follows:

$$d_{k,n}(t) = \begin{cases} t - t_{k,n}h, & t \in O_{k,n}^1 \\ t - t_{k,n}h - h, & t \in O_{k,n}^2 \\ \vdots \\ t - t_{k,n}h - (o_{k,n} - 1)h, & t \in O_{k,n}^{o_{k,n}} \end{cases}$$

and

$$e_{k,n}(t) = \begin{cases} 0, & t \in O_{k,n}^1 \\ \bar{y}(t_{k,n}h) - \bar{y}(t_{k,n}h + h), & t \in O_{k,n}^2 \\ \vdots \\ \bar{y}(t_{k,n}h) - \bar{y}(t_{k,n}h + (o_{k,n} - 1)h), & t \in O_{k,n}^{o_{k,n}} \end{cases}$$

According to the definitions of $d_{k,n}(t)$ and $e_{k,n}(t)$, it yields that $d_{k,n}(t) \in [0, h]$, $t \in \mathcal{F}_{k,n} \cap \mathcal{D}_{1,n}$. Then the event-triggered sampled state $\bar{y}(t_{k,n}h)$ can be described as:

$$y_e(t) = \bar{y}(t_{k,n}h) = e_{k,n}(t) + \bar{y}(t - d_{k,n}(t)) \quad (25)$$

Define $e(t) = x(t) - \hat{x}(t)$, $\bar{z}(t) = z(t) - \hat{z}(t)$, then by combining (3), (4), (6), (9), (18) and (25), the estimation error system is obtained

$$\dot{e}(t) = \begin{cases} \sum_{i=1}^r \sum_{j=1}^r \varphi_i(h(x)) \psi_j(h(\hat{x})) [(A_i - A_j + K_j C_j)x(t) + (A_j - K_j C_j)e(t) - K_j(I + \Delta_q)C_i x(t - d_{k,n}(t)) + K_j(I + \Delta_q)\phi(C_i x(t - d_{k,n}(t)))] \\ + B_i w(t) - K_j(I + \Delta_q)e_{k,n}(t), \\ t \in \mathcal{D}_{1,n} \cap \mathcal{F}_{k,n}, k \in \mathcal{G}(n) \\ \sum_{i=1}^r \sum_{j=1}^r \varphi_i(h(x)) \psi_j(h(\hat{x})) [(A_i - A_j + K_j C_j)x(t) + (A_j - K_j C_j)e(t) + B_i w(t)], \\ t \in \mathcal{D}_{2,n}, n \in \mathcal{N} \end{cases} \quad (26)$$

$$\bar{z}(t) = \sum_{i=1}^r \sum_{j=1}^r \varphi_i(h(x)) \psi_j(h(\hat{x})) [(L_i - L_j)x(t) + L_j e(t)] \quad (27)$$

In order to simplify representation, $\varphi_i(h(x))$, $\psi_j(h(\hat{x}))$ are abbreviated as φ_i and ψ_j , respectively.

Define

$$\begin{aligned} \eta(t) &= [x^T(t) \quad e^T(t)]^T, \bar{z}(t) = [z^T(t) \quad \bar{z}^T(t)]^T \\ \bar{A}_{ij} &= \begin{bmatrix} A_i & 0 \\ A_i - A_j + K_j C_j & A_i - K_j C_j \end{bmatrix}, \bar{B}_i = \begin{bmatrix} B_i \\ B_i \end{bmatrix} \\ \bar{C}_{ij} &= \begin{bmatrix} 0 & 0 \\ -K_j(I + \Delta_q)C_i & 0 \end{bmatrix}, \bar{V}_j = \begin{bmatrix} 0 \\ K_j(I + \Delta_q) \end{bmatrix} \\ \bar{E}_j &= \begin{bmatrix} 0 \\ -K_j(I + \Delta_q) \end{bmatrix}, \bar{L}_{ij} = \begin{bmatrix} L_i & 0 \\ L_i - L_j & L_j \end{bmatrix} \\ H &= [I \quad 0] \end{aligned}$$

Then the Eqs. (26) and (27) can be expressed as

$$\dot{\eta}(t) = \begin{cases} \sum_{i=1}^r \sum_{j=1}^r \varphi_i \psi_j [\bar{A}_{ij} \eta(t) + \bar{B}_i w(t) + \bar{C}_{ij} \eta(t - d_{k,n}(t)) + \bar{E}_j e_{k,n}(t) + \bar{V}_j \phi(C_i H \eta(t - d_{k,n}(t)))] \\ t \in \mathcal{D}_{1,n} \cap \mathcal{F}_{k,n}, k \in \mathcal{G}(n) \\ \sum_{i=1}^r \sum_{j=1}^r \varphi_i \psi_j [\bar{A}_{ij} \eta(t) + \bar{B}_i w(t)], \\ t \in \mathcal{D}_{2,n}, n \in \mathcal{N} \end{cases} \quad (28)$$

$$\bar{z}(t) = \sum_{i=1}^r \sum_{j=1}^r \varphi_i \psi_j \bar{L}_{ij} \eta(t) \quad (29)$$

In order to conveniently represent the Eqs. (28) and (29), we introduce the following variable

$$\beta(t) = \begin{cases} 1, & t \in [-h, 0] \cup (\cup_{n \in \mathcal{N}} \mathcal{D}_{1,n}) \\ 2, & t \in \cup_{n \in \mathcal{N}} \mathcal{D}_{2,n} \end{cases}$$

For $\beta(t) = m \in \{1, 2\}$ and $n \in \mathcal{N}$, define

$$t_{m,n} = \begin{cases} (n-1)T, & m = 1 \\ (n-1)T + T_{sleep}^{min}, & m = 2 \end{cases}$$

which means that $\mathcal{D}_{m,n} = [t_{m,n}, t_{3-m,n+m-1})$, $\beta(t_{m,n}) = m$, $\beta(t_{m,n}^-) = 3 - m$.

According to the definition of $\beta(t)$, the Eqs. (28) and (29) can be expressed as

$$\begin{cases} \dot{\eta}(t) = \sum_{i=1}^r \sum_{j=1}^r \varphi_i \psi_j [\bar{A}_{ij} \eta(t) + \bar{B}_i w(t) \\ + \bar{C}_{ij}^m \eta(t - d_{k,n}(t)) + \bar{E}_j^m e_{k,n}(t) \\ + \bar{V}_j^m \phi(C_i H \eta(t - d_{k,n}(t)))] \\ m \in \{1, 2\}, t \in [t_{m,n}, t_{3-m,n+m-1}) \\ \tilde{z}(t) = \sum_{i=1}^r \sum_{j=1}^r \varphi_i \psi_j \bar{L}_{ij} \eta(t) \end{cases} \quad (30)$$

where $\bar{C}_{ij}^1 = \bar{C}_{ij}$, $\bar{V}_j^1 = \bar{V}_j$, $\bar{E}_j^1 = \bar{E}_j$, $\bar{C}_{ij}^2 = \bar{V}_j^2 = \bar{E}_j^2 = 0$; $\eta(t) = \varrho(t)$, $t \in [-h, 0]$; $\varrho(t)$ is the supplemented initial condition of the state $\eta(t)$ on $[-h, 0]$, $\varrho(0) \triangleq \varrho_0$.

Now, the event-triggered estimator design for CPS with sensor saturation, quantization and DoS attacks studied in this paper can be formulated as follows: given a scalar $\gamma > 0$, the system (30) satisfies the following conditions:

(I) The system (30) is exponentially stable if there exist two positive scalars $\nu_1 > 0$ and $\nu_2 > 0$ such that $\forall t \geq 0, \|\eta(t)\| \leq \nu_1 \|\varrho\|_h e^{-\nu_2 t}$ for $w(t) = 0$.

(II) The system (30) has L_2 -gain less than $\gamma > 0$, namely, when $\varrho = 0$, the estimator output $\tilde{z}(t)$ satisfies $\|\tilde{z}(t)\|_2 \leq \gamma \|\varrho(t)\|_2$ for all $w(t) \in \mathcal{L}_2[0, +\infty)$.

In the following, a lemma is presented to assist us in obtaining the main results.

Lemma 1 ([50]). For matrices $H_1 = H_1^T, H_2$ and H_3 with appropriate dimensions, $H_1 + H_2 \Delta(t) H_3 + H_3^T \Delta(t) H_2^T < 0$ for any $\Delta(t)$ satisfying $\Delta(t)^T \Delta(t) \leq I$, if and only if there exists a parameter $m_1 > 0$ such that $H_1 + m_1 H_2^T H_2 + m_1^{-1} H_3 H_3^T < 0$.

3. Main results

The performance of estimator error system (30) is discussed and the main results are exhibited in the following three theorems with their corresponding proofs. In Theorem 1, the sufficient conditions guaranteeing the system (30) exponentially stable are derived. Then the system performance of CPSs subjected to the external disturbance $w(t)$ is discussed in Theorem 2. Based on the analysis in Theorems 1 and 2, the gains of state estimator are obtained in Theorem 3.

Theorem 1. Given the sequence $\{nT\}_{n \in \mathcal{N}}$, the period T , T_{sleep}^{min} in $F_{DoS}(t)$ (19), and the estimator gain matrix K_j . The system (30) is exponentially stable, if for some prescribed scalars $\alpha_1 > 0, \alpha_2 > 0, \mu_1 > 0, \mu_2 > 0, \sigma \in (0, 1)$ and $h > 0$, there exist symmetric matrices $\Omega, P_1 > 0, P_2 > 0, Q_1 > 0, Q_2 > 0, R_1 > 0, R_2 > 0, Z_1 > 0, Z_2 > 0$ and matrices $D_1, D_2, E_1, E_2, S_1, S_2$ with appropriate dimensions such that the following inequalities hold with $\psi_j - \rho_j \psi_j \geq 0, i, j \in \mathfrak{X}$ and $m \in \{1, 2\}$.

$$\Psi_{ij}^m - W_i < 0; \quad (31)$$

$$\rho_i \Psi_{ii}^m - \rho_i W_i + W_i < 0; \quad (32)$$

$$\begin{aligned} \rho_j \Psi_{ij}^m + \rho_i \Psi_{ji}^m - \rho_j W_i - \rho_i W_j + W_i \\ + W_j < 0 (i < j). \end{aligned} \quad (33)$$

$$P_1 \leq \mu_2 P_2, P_2 \leq \mu_1 e^{2(\alpha_1 + \alpha_2)h} P_1 \quad (34)$$

$$\begin{cases} Q_m \leq \mu_{3-m} Q_{3-m} \\ R_m \leq \mu_{3-m} R_{3-m} \\ Z_m \leq \mu_{3-m} Z_{3-m} \end{cases} \quad (35)$$

$$\delta > 0 \quad (36)$$

where

$$\Psi_{ij}^1 = \begin{bmatrix} \Theta_{11}^1 & * & * & * & * & * \\ \sqrt{h} E_1^T & \Theta_{22}^1 & * & * & * & * \\ \sqrt{h} S_1^T & 0 & \Theta_{33}^1 & * & * & * \\ \sqrt{h} D_1^T & 0 & 0 & \Theta_{44}^1 & * & * \\ \sqrt{h} R_1 \bar{A}^1 & 0 & 0 & 0 & -R_1 & * \\ \sqrt{h} Z_1 \bar{A}^1 & 0 & 0 & 0 & 0 & -Z_1 \end{bmatrix} < 0$$

$$\Psi_{ij}^2 = \begin{bmatrix} \Theta_{11}^2 & * & * & * & * & * \\ \sqrt{h} E_2^T & \Theta_{22}^2 & * & * & * & * \\ \sqrt{h} S_2^T & 0 & \Theta_{33}^2 & * & * & * \\ \sqrt{h} D_2^T & 0 & 0 & \Theta_{44}^2 & * & * \\ \sqrt{h} R_2 \bar{A}^2 & 0 & 0 & 0 & -R_2 & * \\ \sqrt{h} Z_2 \bar{A}^2 & 0 & 0 & 0 & 0 & -Z_2 \end{bmatrix} < 0$$

$$\Theta_{11}^1 = \Sigma_{11} + \Sigma_1 + \Sigma_1^T, \Theta_{21}^1 = \Sigma_{21} + \Sigma_2 + \Sigma_2^T$$

$$\Theta_{22}^1 = \Theta_{33}^1 = -e^{-2\alpha_1 h} R_1, \Theta_{22}^2 = \Theta_{33}^2 = -R_2$$

$$\Theta_{44}^1 = -e^{-2\alpha_1 h} Z_1, \Theta_{44}^2 = -Z_2$$

$$\bar{A}^1 = [\bar{A}_{ij} \ \bar{C}_{ij} \ 0 \ \bar{E}_j \ \bar{V}_j], \bar{A}^2 = [\bar{A}_{ij} \ 0 \ 0]$$

$$\Sigma_{11} = \begin{bmatrix} \nu_1 & * & * & * & * \\ \bar{C}_{ij}^T P_1 & \nu_2 & * & * & * \\ 0 & 0 & \nu_3 & * & * \\ \bar{E}_j^T P_1 & \sigma \Omega \bar{H} & 0 & \sigma \Omega - \Omega & * \\ \bar{V}_j^T P_1 & -\sigma \Omega \bar{H} & 0 & -\sigma \Omega & \sigma \Omega - I \end{bmatrix}$$

$$\nu_1 = P_1 \bar{A}_{ij} + \bar{A}_{ij}^T P_1 + Q_1 + 2\alpha_1 P_1, \bar{H} = [C_i \ 0]$$

$$\nu_2 = \sigma \bar{H}^T \Omega \bar{H} + \varepsilon \bar{H}^T \bar{H}, \nu_3 = -e^{-2\alpha_1 h} Q_1$$

$$\Sigma_{21} = \begin{bmatrix} \nu_4 & * & * \\ 0 & 0 & * \\ 0 & 0 & -e^{2\alpha_2 h} Q_2 \end{bmatrix}$$

$$\nu_4 = P_2 \bar{A}_{ij} + \bar{A}_{ij}^T P_2 + Q_2 - 2\alpha_2 P_2$$

$$\Sigma_1 = [D_1 + E_1 \quad -E_1 + S_1 \quad -D_1 - S_1 \quad 0 \quad 0]$$

$$\Sigma_2 = [D_2 + E_2 \quad -E_2 + S_2 \quad -D_2 - S_2]$$

$$D_1 = [D_{11}^T \ D_{12}^T \ D_{13}^T]^T, D_2 = [D_{21}^T \ D_{22}^T \ D_{23}^T]^T$$

$$E_1 = [E_{11}^T \ E_{12}^T \ E_{13}^T]^T, E_2 = [E_{21}^T \ E_{22}^T \ E_{23}^T]^T$$

$$S_1 = [S_{11}^T \ S_{12}^T \ S_{13}^T]^T, S_2 = [S_{21}^T \ S_{22}^T \ S_{23}^T]^T$$

$$\begin{aligned} \delta = 2\alpha_1 T_{sleep}^{min} - 2\alpha_2 (T - T_{sleep}^{min}) - 2(\alpha_1 + \alpha_2)h \\ - \ln(\mu_1 \mu_2) \end{aligned}$$

Then the system (30) under (19) is exponentially stable with decay rate $\omega = \frac{\delta}{2T}$.

Proof. The following time-varying Lyapunov functional is constructed for system (30):

$$\begin{aligned} V_{\beta(t)}(t) = \eta^T(t) P_{\beta(t)} \eta(t) + \int_{t-h}^t \eta^T(t) g(\cdot) Q_{\beta(t)} \eta(t) dsd\theta \\ + \int_{-h}^0 \int_{t+\theta}^t \eta^T(t) g(\cdot) R_{\beta(t)} \eta(t) dsd\theta \\ + \int_{-h}^0 \int_{t+\theta}^t \eta^T(t) g(\cdot) Z_{\beta(t)} \eta(t) dsd\theta \end{aligned} \quad (37)$$

where $P_{\beta(t)} > 0, Q_{\beta(t)} > 0, R_{\beta(t)} > 0, Z_{\beta(t)} > 0$ and $g(\cdot) \triangleq e^{(-1)^{\beta(t)} 2\alpha_{\beta(t)}(t-s)}$.

In the following, we will discuss the cases of $\beta(t) = 1$ and $\beta(t) = 2$, respectively.

When $\beta(t) = 1$, for $\forall k \in \mathcal{G}(n), n \in \mathcal{N}$, calculating the derivation of $V_1(t)$ along the trajectories of the system (30) with $m = 1$, by utilizing the free weighting matrix method [31], we obtain that

$$\begin{aligned} \dot{V}_1(t) \leq & -2\alpha_1 V_1(t) + 2\alpha_1 \eta^T(t) P_1 \eta(t) + \eta^T(t) Q_1 \eta(t) \\ & - \eta^T(t-h) e^{-2\alpha_1 h} Q_1 \eta(t-h) \\ & + h \dot{\eta}^T(t) (R_1 + Z_1) \dot{\eta}(t) \\ & - \int_{t-h}^t \dot{\eta}^T(s) e^{-2\alpha_1 h} Z_1 \dot{\eta}(s) ds \\ & - \int_{t-d_{k,n}(t)}^t \dot{\eta}^T(s) e^{-2\alpha_1 h} R_1 \dot{\eta}(s) ds \\ & - \int_{t-h}^{t-d_{k,n}(t)} \dot{\eta}^T(s) e^{-2\alpha_1 h} R_1 \dot{\eta}(s) ds \\ & + 2\xi^T(t) [D_1 \Phi_1 + E_1 \Phi_2 + S_1 \Phi_3] \xi(t) \end{aligned} \quad (38)$$

where $\xi(t) = [\eta^T(t) \ \eta^T(t-d_{k,n}(t)) \ \eta^T(t-h) \ e_{k,n}^T(t) \ \phi^T(\bar{H}\eta(t-d_{k,n}(t))) \ I \ I \ I \ I \ I]^T$ and $\mathbf{0} = \Phi_1 \triangleq \eta(t) - \eta(t-h) - \int_{t-h}^t \dot{\eta}(x) dx$, $\mathbf{0} = \Phi_2 \triangleq \eta(t) - \eta(t-d_{k,n}(t)) - \int_{t-d_{k,n}(t)}^t \dot{\eta}(x) dx$, $\mathbf{0} = \Phi_3 \triangleq \eta(t-d_{k,n}(t)) - \eta(t-h) - \int_{t-h}^{t-d_{k,n}(t)} \dot{\eta}(x) dx$. Then it is easy to obtain that

$$\begin{aligned} \dot{V}_1(t) \leq & -2\alpha_1 V_1(t) + \xi^T(t) [\Theta_{11}^1 + hE_1 e^{2\alpha_1 h} R_1^{-1} E_1^T \\ & + hS_1 e^{2\alpha_1 h} R_1^{-1} S_1^T + hD_1 e^{2\alpha_1 h} Z_1^{-1} D_1^T \\ & + h\chi_1^T (R_1 + Z_1) \chi_1] \xi(t) \end{aligned} \quad (39)$$

where $\chi_1 = \bar{A}^1$.

Through the utilization of Schur complement of matrix $\Psi_{ij}^1 < 0$, we have $\Theta_{11}^1 + hE_1 e^{2\alpha_1 h} R_1^{-1} E_1^T + hS_1 e^{2\alpha_1 h} R_1^{-1} S_1^T + hD_1 e^{2\alpha_1 h} Z_1^{-1} D_1^T + h\chi_1^T (R_1 + Z_1) \chi_1 < 0$, then we can get $\dot{V}_1(t) \leq -2\alpha_1 V_1(t)$ holds for $t \in [t_{1,n}, t_{2,n})$.

When $\beta(t) = 2$, calculating the derivation of $V_2(t)$ for $t \in [t_{2,n}, t_{1,n+1})$ and utilizing the similar method as above, it yields that

$$\begin{aligned} \dot{V}_2(t) \leq & 2\alpha_2 V_2(t) + \xi^T(t) [\Theta_{11}^2 + hE_2 R_2^{-1} E_2^T \\ & + hS_2 R_2^{-1} S_2^T + hD_2 Z_2^{-1} D_2^T \\ & + h\chi_2^T (R_2 + Z_2) \chi_2] \xi(t) \end{aligned} \quad (40)$$

where $\chi_2 = \bar{A}^2$. By utilizing $\Psi_{ij}^2 < 0$, we deduce that $\dot{V}_2(t) \leq 2\alpha_2 V_2(t)$.

Similar to the analysis method in [26], along the trajectories of the system (30), we can obtain that for $t \in [t_{m,n}, t_{3-m,n+m-1})$, $m \in \{1, 2\}$,

$$V_m(t) \leq e^{2(-1)^m \alpha_m (t-t_{m,n})} V_m(t_{m,n}). \quad (41)$$

Based on the above analysis, construct a time-varying Lyapunov functional: $V(t) = V_{\beta(t)}(t)$, $\beta(t) \in \{1, 2\}$. Therefore, we have

$$V(t) \leq \begin{cases} e^{-2\alpha_1(t-t_{1,n})}, & t \in [t_{1,n}, t_{2,n}) \\ e^{2\alpha_2(t-t_{2,n})}, & t \in [t_{2,n}, t_{1,n+1}) \end{cases} \quad (42)$$

According to (41), we can easily obtain that

$$\begin{cases} V_1(t_{1,n}) \leq \mu_2 V_2(t_{1,n}^-) \\ V_2(t_{2,n}) \leq \mu_1 e^{2(\alpha_1 + \alpha_2)h} V_1(t_{2,n}^-) \end{cases} \quad (43)$$

If $t \in [t_{1,n}, t_{2,n})$, it follows from (42) and (43) that

$$\begin{aligned} V(t) & \leq \mu_2 e^{-2\alpha_1(t-t_{1,n})} V_2(t_{1,n}^-) \\ & \vdots \\ & \leq e^{-\partial n} V_1(t_{1,0}) = e^{-\partial n} V_1(0) \end{aligned} \quad (44)$$

Notice that $t < t_{2,n} = nT + T_{sleep}^{min}$, namely, $n > \frac{t-T_{sleep}^{min}}{T}$, combining it with (44), we obtain

$$V(t) \leq V_1(0) e^{\frac{\partial T_{sleep}^{min}}{T}} e^{-\frac{\partial}{T} t} \quad (45)$$

If $t \in [t_{2,n}, t_{1,n+1})$, it follows from the similar analysis as above that

$$V(t) \leq \frac{V_1(0)}{\mu_2} e^{-\frac{\partial}{T} t} \quad (46)$$

At present, define $c_1 = \max\{e^{\frac{\partial T_{sleep}^{min}}{T}}, \frac{1}{\mu_2}\}$, $c_2 = \min\{\partial_{min}(P_1)\}$, $c_3 = \max\{\partial_{min}(P_i)\}$ and $c_4 = c_3 + h\partial_{max}(Q_1) + \frac{h^2}{2}\partial_{max}(R_1 + Z_1)$, it follows from (45) and (46) that

$$V(t) \leq c_1 e^{-\frac{\partial}{T} t} V_1(0), \forall t \geq 0 \quad (47)$$

From the definition of $V(t)$ in (37), we have

$$V(t) \geq c_2 \|\eta(t)\|^2, V_1(0) \leq \|\varrho_0\|_h^2 \quad (48)$$

Combining (47) and (48), one has

$$\|\eta(t)\| \leq \sqrt{\frac{c_1 c_4}{c_2}} e^{-\frac{\partial}{2T} t} \|\varrho_0\|_h, \forall t \geq 0 \quad (49)$$

Define $\omega = \frac{\partial}{2T}$, (49) can be rewritten as

$$\|\eta(t)\| \leq \sqrt{\frac{c_1 c_4}{c_2}} e^{-\omega t} \|\varrho_0\|_h, \forall t \geq 0 \quad (50)$$

Similar to the analysis in [31], consider a slack matrix W_i that

$$\begin{aligned} & \sum_{i=1}^r \sum_{j=1}^r \varphi_i (\varphi_j - \psi_j) W_i \\ & = \sum_{i=1}^r \varphi_i \left(\sum_{j=1}^r \varphi_j - \sum_{j=1}^r \psi_j \right) W_i \\ & = 0 \end{aligned}$$

where $W_i = W_i^T$, $i \in \mathfrak{R}$, are arbitrary matrices, then according to (31)-(33), it yields that

$$\dot{V}_1(t) \leq \sum_{i=1}^r \sum_{j=1}^r \varphi_i \psi_j \xi^T(t) \Psi_{ij}^1 \xi(t) < 0$$

With $\psi_j - \rho_j \varphi_j \geq 0$ for any $j \in \mathfrak{R}$, there is a scalar $\delta_1 > 0$ which satisfies the inequality $\dot{V}_1(t) \leq -\delta_1 \|\xi(t)\|^2$ for $\xi(t) \neq 0$, then $\dot{V}_1(t) < 0$. Besides, using the same method, one has that $\dot{V}_2(t) < 0$. Therefore, the system (30) is exponentially stable with decay rate ω . That completes the proof. ■

In Theorem 1, the sufficient conditions are derived which can guarantee the exponential stability of the system (30). In the view of Theorem 1, the analysis on system performance of estimator system (30) with the disturbance $w(t)$ is given in Theorem 2.

Theorem 2. For the known sequence $\{nT\}_{n \in \mathcal{N}}$, the period T , T_{sleep}^{min} in $F_{DoS}(t)$ (19), and the estimator gain matrix K_j and scalar γ , the system (30) is exponentially stable with decay rate ω given in Theorem 1, if for parameters $\alpha_1 > 0$, $\alpha_2 > 0$, $h > 0$, triggering parameter $\sigma \in (0, 1)$, there exist symmetric matrices Ω , $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $Z_1 > 0$, $Z_2 > 0$ and matrices D_1 , D_2 , E_1 , E_2 , S_1 , S_2 with appropriate dimensions such that (34)–(36) and the following inequalities hold with $\psi_j - \rho_j \varphi_j \geq 0$, $i, j \in \mathfrak{R}$ and $m \in \{1, 2\}$.

$$\Upsilon_{ij}^m - \tilde{W}_i < 0; \quad (51)$$

$$\rho_i \Upsilon_{ii}^m - \rho_i \tilde{W}_i + \tilde{W}_i < 0; \quad (52)$$

$$\begin{aligned} & \rho_j \Upsilon_{ij}^m + \rho_i \Upsilon_{ji}^m - \rho_j \tilde{W}_i - \rho_i \tilde{W}_j + \tilde{W}_i \\ & + \tilde{W}_j < 0 (i < j). \end{aligned} \tag{53}$$

where

$$\begin{aligned} \Upsilon_{ij}^1 &= \begin{bmatrix} \Lambda_{11}^1 & * & * & * & * & * & * \\ \sqrt{h}E_1^T & \Lambda_{22}^1 & * & * & * & * & * \\ \sqrt{h}S_1^T & 0 & \Lambda_{33}^1 & * & * & * & * \\ \sqrt{h}D_1^T & 0 & 0 & \Lambda_{44}^1 & * & * & * \\ \sqrt{h}R_1\bar{A}^1 & 0 & 0 & 0 & -R_1 & * & * \\ \sqrt{h}Z_1\bar{A}^1 & 0 & 0 & 0 & 0 & -Z_1 & * \\ \Lambda_{71}^1 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} \\ \Upsilon_{ij}^2 &= \begin{bmatrix} \Lambda_{11}^2 & * & * & * & * & * & * \\ \sqrt{h}E_2^T & \Lambda_{22}^2 & * & * & * & * & * \\ \sqrt{h}S_2^T & 0 & \Lambda_{33}^2 & * & * & * & * \\ \sqrt{h}D_2^T & 0 & 0 & \Lambda_{44}^2 & * & * & * \\ \sqrt{h}R_2\bar{A}^2 & 0 & 0 & 0 & -R_2 & * & * \\ \sqrt{h}Z_2\bar{A}^2 & 0 & 0 & 0 & 0 & -Z_2 & * \\ \Lambda_{71}^2 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix} \end{aligned}$$

in which

$$\begin{aligned} \Lambda_{11}^1 &= \Pi_{11} + \Pi_1 + \Pi_1^T, \Lambda_{11}^2 = \Pi_{21} + \Pi_2 + \Pi_2^T \\ \Lambda_{22}^1 &= \Lambda_{33}^1 = -e^{-2\alpha_1 h} R_1, \Lambda_{22}^2 = \Lambda_{33}^2 = -R_2 \\ \Lambda_{44}^1 &= -e^{-2\alpha_1 h} Z_1, \Lambda_{44}^2 = -Z_2 \\ \Lambda_{71}^1 &= [\bar{L}_{ij} \ 0 \ 0 \ 0 \ 0 \ 0], \Lambda_{71}^2 = [\bar{L}_{ij} \ 0 \ 0 \ 0] \\ \bar{A}^1 &= [\bar{A}_{ij} \ \bar{C}_{ij} \ 0 \ \bar{B}_i \ \bar{E}_j \ \bar{V}_j] \\ \bar{A}^2 &= [\bar{A}_{ij} \ 0 \ 0 \ \bar{B}_i] \end{aligned}$$

$$\Pi_{11} = \begin{bmatrix} v_1 & * & * & * & * & * & * \\ \bar{C}_{ij}^T P_1 & v_2 & * & * & * & * & * \\ 0 & 0 & v_3 & * & * & * & * \\ \bar{B}_i^T P_1 & 0 & 0 & -\gamma^2 I & * & * & * \\ \bar{E}_j^T P_1 & \sigma \Omega \bar{H} & 0 & 0 & \sigma \Omega - \Omega & * & * \\ \bar{V}_j^T P_1 & -\sigma \Omega \bar{H} & 0 & 0 & -\sigma \Omega & v_5 & * \end{bmatrix}$$

$$\begin{aligned} v_1 &= P_1 \bar{A}_{ij} + \bar{A}_{ij}^T P_1 + Q_1 + 2\alpha_1 P_1 \\ v_2 &= \sigma \bar{H}^T \Omega \bar{H} + \varepsilon \bar{H}^T \bar{H}, v_3 = -e^{-2\alpha_1 h} Q_1 \\ v_5 &= \sigma \Omega - I, \bar{H} = [C_i \ 0] \end{aligned}$$

$$\Pi_{21} = \begin{bmatrix} v_4 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & -e^{2\alpha_2 h} Q_2 & * \\ \bar{B}_i^T P_2 & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\begin{aligned} v_4 &= P_2 \bar{A}_{ij} + \bar{A}_{ij}^T P_2 + Q_2 - 2\alpha_2 P_2 \\ \Pi_1 &= [D_1 + E_1 \ -E_1 + S_1 \ -D_1 - S_1 \ 0 \ 0 \ 0] \\ \Pi_2 &= [D_2 + E_2 \ -E_2 + S_2 \ -D_2 - S_2 \ 0] \\ D_1 &= [D_{11}^T \ D_{12}^T \ D_{13}^T]^T, D_2 = [D_{21}^T \ D_{22}^T \ D_{23}^T]^T \\ E_1 &= [E_{11}^T \ E_{12}^T \ E_{13}^T]^T, E_2 = [E_{21}^T \ E_{22}^T \ E_{23}^T]^T \\ S_1 &= [S_{11}^T \ S_{12}^T \ S_{13}^T]^T, S_2 = [S_{21}^T \ S_{22}^T \ S_{23}^T]^T \end{aligned}$$

Proof. Choose a time-varying Lyapunov functional (37) for system (30), given $\forall k \in \mathcal{G}(n), n \in \mathcal{N}$, for $t \in \mathcal{D}_{1,n}$, the following inequality is got by the similar proof in Theorem 1:

$$\begin{aligned} \dot{V}_1(t) &\leq -2\alpha_1 V_1(t) + \bar{\xi}^T(t) [\Theta_{11}^1 + hE_1 e^{2\alpha_1 h} R_1^{-1} E_1^T \\ &+ hS_1 e^{2\alpha_1 h} R_1^{-1} S_1^T + hD_1 e^{2\alpha_1 h} Z_1^{-1} D_1^T \end{aligned}$$

$$\begin{aligned} &+ h\chi_1^T (R_1 + Z_1)\chi_1] \bar{\xi}(t) \\ &- \bar{z}^T(t)\bar{z}(t) + \gamma^2 w^T(t)w(t) \end{aligned} \tag{54}$$

where $\chi_1 = \bar{A}^1, \bar{\xi}(t) = [\eta^T(t) \ \eta^T(t - d_{k,n}(t)) \ \eta^T(t - h) \ w^T(t) \ e_{k,n}^T(t) \ \phi^T(\bar{H}\eta(t - d_{k,n}(t))) \ I \ I \ I \ I \ I \ I]^T$. According to Υ_{ij}^1 , it yields that

$$\dot{V}_1(t) + 2\alpha_1 V_1(t) + \bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t) \leq 0$$

For $t \in \mathcal{D}_{2,n}$, calculating the derivation of $V_2(t)$, it follows from the similar analysis as above, then we can obtain

$$\dot{V}_2(t) - 2\alpha_2 V_2(t) + \bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t) \leq 0$$

Consequently, for all $w(t) \in \mathcal{L}_2[0, \infty)$, we get

$$\begin{aligned} \dot{V}_m(t) + 2\alpha_m (-1)^{m-1} V_m(t) + \bar{z}^T(t)\bar{z}(t) \\ - \gamma^2 w^T(t)w(t) \leq 0, t \in [t_{m,n}, t_{3-m,n+m-1}) \end{aligned} \tag{55}$$

At the same time, we note that the inequality (36) is equivalent to

$$\frac{1}{\mu_1} e^{2\alpha_1 T_{sleep} - 2(\alpha_1 + \alpha_2)h} - \mu_2 e^{2\alpha_2(T - T_{sleep}^{min})} > 0 \tag{56}$$

Due to $T_{sleep}^{min} \leq T_{sleep} \leq T$, we have

$$\begin{aligned} & \frac{1}{\mu_1} e^{2\alpha_1 T_{sleep} - 2(\alpha_1 + \alpha_2)h} - \mu_2 e^{2\alpha_2(T - T_{sleep})} \\ & \geq \frac{1}{\mu_1} e^{2\alpha_1 T_{sleep}^{min} - 2(\alpha_1 + \alpha_2)h} - \mu_2 e^{2\alpha_2(T - T_{sleep}^{min})} \end{aligned} \tag{57}$$

Combining (56) and (57), one has

$$\frac{1}{\mu_1} e^{2\alpha_1 T_{sleep} - 2(\alpha_1 + \alpha_2)h} - \mu_2 e^{2\alpha_2(T - T_{sleep})} > 0 \tag{58}$$

Then, for given $t \in [0, (n + 1)T), t \in \mathcal{N}$, it follows from the zero-initial condition that

$$\begin{aligned} & \sum_{k=0}^n \left(\int_{kT}^{kT+T_{sleep}} [\dot{V}_1(t) + 2\alpha_1 V_1(t)] dt \right. \\ & \left. + \int_{kT+T_{sleep}}^{(k+1)T} [\dot{V}_2(t) - 2\alpha_2 V_2(t)] dt \right) > 0 \end{aligned} \tag{59}$$

Integrating (59) from 0 to $(n + 1)T$ with zero initial condition, we can obtain

$$\int_0^{(n+1)T} [\bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t)] dt < 0$$

Letting $n \rightarrow \infty$, it yields that

$$\int_0^\infty \bar{z}^T(t)\bar{z}(t) dt \leq \int_0^\infty \gamma^2 w^T(t)w(t) dt \tag{60}$$

which means that $\|\bar{z}(t)\|_2 \leq \gamma \|w(t)\|_2$ holds for all $w(t) \in \mathcal{L}_2[0, \infty)$. Similar to the analysis in Theorem 1, consider a slack matrix \tilde{W}_i that

$$\begin{aligned} & \sum_{i=1}^r \sum_{j=1}^r \varphi_i(\varphi_j - \psi_j) \tilde{W}_i \\ &= \sum_{i=1}^r \varphi_i \left(\sum_{j=1}^r \varphi_j - \sum_{j=1}^r \psi_j \right) \tilde{W}_i \\ &= 0 \end{aligned}$$

where $\tilde{W}_i = \tilde{W}_i^T (i \in \mathfrak{R})$ are arbitrary matrices, then combining (51)–(53), we have $\dot{V}_m(t) + \bar{z}^T(t)\bar{z}(t) - \gamma^2 w^T(t)w(t) < 0 (m = 1, 2)$. Therefore, the system (30) is exponentially stable under DoS attacks (19) and the external disturbance $w(t)$. That completes the proof. ■

Based on the analysis of **Theorems 1** and **2**, the gains of state estimator for estimating CPSs with limited communication resources, sensor saturation and periodic DoS attacks are derived in **Theorem 3**.

Theorem 3. Given the sequence $\{nT\}_{n \in \mathcal{N}}$, the period T , T_{sleep}^{\min} in $F_{\text{DoS}}(t)$ (19), and the known positive scalars $\delta, \gamma, \sigma \in (0, 1)$, $h > 0$, $\alpha_1 > 0, \alpha_2 > 0, \mu_1, \mu_2$, the system (30) is exponentially stable with decay rate ω given in **Theorem 1**, if there exist symmetric matrices $\hat{\Omega}, P_1 = \text{diag}\{P_{11}, P_{12}\} > 0, P_2 > 0, \hat{Q}_1 > 0, \hat{Q}_2 > 0, \hat{R}_1 > 0, \hat{R}_2 > 0, \hat{Z}_1 > 0, \hat{Z}_2 > 0$ and matrices $\hat{D}_1, \hat{D}_2, \hat{E}_1, \hat{E}_2, \hat{S}_1, \hat{S}_2$ and Y_j with appropriate dimensions and scalars $e_{11}, e_{21}, e_{12}, e_{22}, e_3$ such that (36) and the following LMIs hold with $\psi_j - \rho_j \psi_j \geq 0, i, j \in \mathfrak{X}$ and $m \in \{1, 2\}$.

$$\Phi_{ij}^m - \hat{W}_i < 0; \tag{61}$$

$$\rho_i \Phi_{ii}^m - \rho_i \hat{W}_i + \hat{W}_i < 0; \tag{62}$$

$$\rho_j \Phi_{ij}^m + \rho_i \Phi_{ji}^m - \rho_j \hat{W}_i - \rho_i \hat{W}_j + \hat{W}_j + \hat{W}_j < 0 (i < j). \tag{63}$$

$$\begin{bmatrix} -\mu_2 P_2 & * \\ P_2 & -P_1 \end{bmatrix} \leq 0 \tag{64}$$

$$\begin{bmatrix} -\mu_1 e^{2(\alpha_1 + \alpha_2)h} P_2 & * \\ P_1 & -P_2 \end{bmatrix} \leq 0 \tag{65}$$

$$\begin{bmatrix} -\mu_{3-m} \hat{Q}_{3-m} & * \\ I & -\hat{Q}_m \end{bmatrix} \leq 0 \tag{66}$$

$$\begin{bmatrix} -\mu_{3-m} \hat{R}_{3-m} & * \\ P_{3-m} & e_{1m}^2 \hat{R}_m - 2e_{1m} P_m \end{bmatrix} \leq 0 \tag{67}$$

$$\begin{bmatrix} -\mu_{3-m} \hat{Z}_{3-m} & * \\ P_{3-m} & e_{2m}^2 \hat{Z}_m - 2e_{2m} P_m \end{bmatrix} \leq 0 \tag{68}$$

where

$$\Phi_{ij}^1 = \begin{bmatrix} \theta_{11}^1 & * & * \\ \theta_{21}^1 & \theta_{22}^1 & * \\ \theta_{31}^1 & \theta_{32}^1 & \theta_{33}^1 \end{bmatrix}$$

$$\Phi_{ij}^2 = \begin{bmatrix} \theta_{11}^2 & * & * \\ \theta_{21}^2 & \theta_{22}^2 & * \\ \theta_{31}^2 & 0 & -I \end{bmatrix}$$

$$\theta_{11}^1 = \Xi_{11} + \Xi_1 + \Xi_1^T, \theta_{11}^2 = \Xi_{21} + \Xi_2 + \Xi_2^T$$

$$\theta_{21}^1 = [\sqrt{h} \hat{E}_1 \quad \sqrt{h} \hat{S}_1 \quad \sqrt{h} \hat{S}_1 \bar{A}^1 \quad \sqrt{h} \hat{R}_1 \bar{A}^1 \quad \sqrt{h} \hat{Z}_1]^T$$

$$\theta_{21}^2 = [\sqrt{h} \hat{E}_2 \quad \sqrt{h} \hat{S}_2 \quad \sqrt{h} \hat{S}_2 \bar{A}^2 \quad \sqrt{h} \hat{R}_2 \bar{A}^2 \quad \sqrt{h} \hat{Z}_2]^T$$

$$\theta_{22}^1 = \text{diag} \{ \Gamma_{22}^1, \Gamma_{33}^1, \Gamma_{44}^1, \Gamma_{55}^1, \Gamma_{66}^1 \}$$

$$\theta_{22}^2 = \text{diag} \{ \Gamma_{22}^2, \Gamma_{33}^2, \Gamma_{44}^2, \Gamma_{55}^2, \Gamma_{66}^2 \}$$

$$\theta_{31}^1 = \begin{bmatrix} \bar{L}_{ij} & 0 & 0 & 0 & 0 & 0 \\ \delta P_1^T & 0 & 0 & 0 & 0 & 0 \\ 0 & F_{19} & 0 & 0 & F_{111} & F_{112} \end{bmatrix}$$

$$\theta_{32}^1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sqrt{h} \delta \hat{R}_1^T \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\theta_{31}^2 = [\bar{L}_{ij} \quad 0 \quad 0 \quad 0]$$

$$\theta_{33}^1 = \begin{bmatrix} -I & * & * \\ \sqrt{h} \delta \hat{Z}_1^T & \Gamma_{88}^1 & * \\ 0 & 0 & \Gamma_{99}^1 \end{bmatrix}$$

$$\Gamma_{22}^1 = \Gamma_{33}^1 = -e^{-2\alpha_1 h} \hat{R}_1, \Gamma_{22}^2 = \Gamma_{33}^2 = -\hat{R}_2$$

$$\Gamma_{44}^1 = -e^{-2\alpha_1 h} \hat{Z}_1, \Gamma_{44}^2 = -\hat{Z}_2$$

$$\bar{A}^1 = [F_{11} \quad F_{19} \quad 0 \quad F_{110} \quad F_{111} \quad F_{112}]$$

$$\bar{A}^2 = [F_{21} \quad 0 \quad 0 \quad F_{23}], \Gamma_{55}^1 = -2e_{11} P_1 + e_{21}^2 \hat{R}_1$$

$$\Gamma_{55}^2 = -2e_{12} P_2 + e_{12}^2 \hat{R}_2, \Gamma_{66}^1 = -2e_{21} P_1 + e_{21}^2 \hat{Z}_1$$

$$\Gamma_{66}^2 = -2e_{22} P_2 + e_{22}^2 \hat{Z}_2$$

$$\Gamma_{71}^1 = [\bar{L}_{ij} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \Gamma_{71}^2 = [\bar{L}_{ij} \quad 0 \quad 0 \quad 0]$$

$$\Gamma_{81} = [\delta P_1^T \quad 0 \quad 0 \quad 0 \quad 0 \quad 0], \Gamma_{88}^1 = -m_1 I$$

$$\Gamma_{91} = [0 \quad F_{19} \quad 0 \quad 0 \quad F_{111} \quad F_{112}]$$

$$\Gamma_{99}^1 = -m_1^{-1} e_3 P_1 + m_1^{-1} e_3^2 I$$

$$\Xi_{11} = \begin{bmatrix} \vartheta_1 & * & * & * & * & * \\ F_{12} & F_{13} & * & * & * & * \\ 0 & 0 & \vartheta_2 & * & * & * \\ F_{14} & 0 & 0 & -\gamma^2 I & * & * \\ F_{15} & F_{16} & 0 & 0 & \sigma \hat{\Omega} - \hat{\Omega} & * \\ F_{17} & F_{18} & 0 & 0 & -\sigma \hat{\Omega} & \vartheta_3 \end{bmatrix}$$

$$\vartheta_1 = F_{11} + F_{11}^T + \hat{Q}_1 + 2\alpha_1 P_1$$

$$\vartheta_2 = -e^{-2\alpha_1 h} \hat{Q}_1, \vartheta_3 = \sigma \hat{\Omega} - I$$

$$\Xi_{21} = \begin{bmatrix} \vartheta_4 & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & -e^{2\alpha_2 h} \hat{Q}_2 & * \\ F_{22} & 0 & 0 & -\gamma^2 I \end{bmatrix}$$

$$\vartheta_4 = F_{21} + F_{21}^T + \hat{Q}_2 - 2\alpha_2 P_2, \bar{H} = [C_i \quad 0]$$

$$\Xi_1 = [\hat{D}_1 + \hat{E}_1 \quad -\hat{E}_1 + \hat{S}_1 \quad -\hat{D}_1 - \hat{S}_1 \quad 0 \quad 0 \quad 0]$$

$$\Xi_2 = [\hat{D}_2 + \hat{E}_2 \quad -\hat{E}_2 + \hat{S}_2 \quad -\hat{D}_2 - \hat{S}_2 \quad 0]$$

$$\hat{D}_1 = [\hat{D}_{11}^T \quad \hat{D}_{12}^T \quad \hat{D}_{13}^T]^T, \hat{D}_2 = [\hat{D}_{21}^T \quad \hat{D}_{22}^T \quad \hat{D}_{23}^T]^T$$

$$\hat{E}_1 = [\hat{E}_{11}^T \quad \hat{E}_{12}^T \quad \hat{E}_{13}^T]^T, \hat{E}_2 = [\hat{E}_{21}^T \quad \hat{E}_{22}^T \quad \hat{E}_{23}^T]^T$$

$$\hat{S}_1 = [\hat{S}_{11}^T \quad \hat{S}_{12}^T \quad \hat{S}_{13}^T]^T, \hat{S}_2 = [\hat{S}_{21}^T \quad \hat{S}_{22}^T \quad \hat{S}_{23}^T]^T$$

$$P_1 = \text{diag}\{P_{11}, P_{12}\}, P_2 = \text{diag}\{P_{21}, P_{22}\}$$

$$F_{11} = \begin{bmatrix} P_{11} A_i & 0 \\ P_{12} A_i - P_{12} A_j + Y_j C_j & P_{12} A_j - Y_j C_j \end{bmatrix}$$

$$F_{12} = \begin{bmatrix} 0 & -C_i^T Y_j^T \\ 0 & 0 \end{bmatrix}, F_{13} = \begin{bmatrix} \sigma C_i^T \Omega C_i + \varepsilon C_i^T C_i & 0 \\ 0 & 0 \end{bmatrix}$$

$$F_{14} = [B_i^T P_{11} \quad B_i^T P_{12}], F_{15} = [0 \quad -Y_j^T]$$

$$F_{16} = [\sigma \Omega C_i \quad 0], F_{17} = [0 \quad Y_j^T]$$

$$F_{18} = [-\sigma \Omega C_i \quad 0], F_{19} = \begin{bmatrix} 0 & 0 \\ -Y_j C_i & 0 \end{bmatrix}$$

$$F_{110} = \begin{bmatrix} P_{11} B_i \\ P_{12} B_i \end{bmatrix}, F_{111} = \begin{bmatrix} 0 \\ -Y_j \end{bmatrix}, F_{112} = \begin{bmatrix} 0 \\ Y_j \end{bmatrix}$$

$$F_{21} = \begin{bmatrix} P_{21} A_i & 0 \\ P_{22} A_i - P_{22} A_j + Y_j C_j & P_{22} A_j - Y_j C_j \end{bmatrix}$$

$$F_{22} = [B_i^T P_{21} \quad B_i^T P_{22}], F_{23} = [P_{21} B_i \quad P_{22} B_i]$$

Moreover, the quantized state estimator gains are achieved as follows.

$$K_j = P_{12}^{-1} Y_j, j \in \mathfrak{X} \tag{69}$$

Proof. Due to the existence of quantization, Υ_{ij}^1 can be equivalently expressed as

$$\Upsilon_{ij}^1 = J_{ij} + \text{sym}\{H_E^T \Delta_q H_F\} \quad (70)$$

$$J_{ij} = \begin{bmatrix} \eta_{11}^1 & * & * & * & * & * & * \\ \sqrt{h}E_1^T & \eta_{22}^1 & * & * & * & * & * \\ \sqrt{h}S_1^T & 0 & \eta_{33}^1 & * & * & * & * \\ \sqrt{h}D_1^T & 0 & 0 & \eta_{44}^1 & * & * & * \\ \sqrt{h}R_1\bar{A}^1 & 0 & 0 & 0 & -R_1 & * & * \\ \sqrt{h}Z_1\bar{A}^1 & 0 & 0 & 0 & 0 & -Z_1 & * \\ \Lambda_{\gamma_1}^1 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}$$

where

$$\begin{aligned} \eta_{11}^1 &= \mathcal{E}_{11} + \mathcal{E}_1 + \mathcal{E}_1^T \\ \eta_{22}^1 &= \eta_{33}^1 = -e^{-2\alpha_1 h} R_1, \eta_{44}^1 = -e^{-2\alpha_1 h} Z_1 \\ \mathcal{E}_{11} &= \begin{bmatrix} \vartheta_1 & * & * & * & * & * \\ \tilde{C}_{ij}^T P_1 & \vartheta_2 & * & * & * & * \\ 0 & 0 & \vartheta_3 & * & * & * \\ \tilde{B}_i^T P_1 & 0 & 0 & -\gamma^2 I & * & * \\ \tilde{E}_j^T P_1 & \sigma \Omega \bar{H} & 0 & 0 & \sigma \Omega - \Omega & * \\ \tilde{V}_j^T P_1 & -\sigma \Omega \bar{H} & 0 & 0 & -\sigma \Omega & \vartheta_4 \end{bmatrix} \\ \vartheta_1 &= P_1 \bar{A}_{ij} + \bar{A}_{ij}^T P_1 + Q_1 + 2\alpha_1 P_1 \\ \vartheta_2 &= \sigma \bar{H}^T \Omega \bar{H} + \varepsilon \bar{H}^T \bar{H}, \vartheta_3 = -e^{-2\alpha_1 h} Q_1 \\ \vartheta_4 &= \sigma \Omega - I, \bar{H} = [C_i \ 0] \\ \mathcal{E}_1 &= [D_1 + E_1 \quad -E_1 + S_1 \quad -D_1 - S_1 \quad 0 \quad 0 \quad 0] \\ D_1 &= [D_{11}^T \quad D_{12}^T \quad D_{13}^T]^T, E_1 = [E_{11}^T \quad E_{12}^T \quad E_{13}^T]^T \\ S_1 &= [S_{11}^T \quad S_{12}^T \quad S_{13}^T]^T \\ \Gamma_{\gamma_1}^1 &= [\bar{L}_{ij} \quad 0 \quad 0 \quad 0 \quad 0 \quad 0] \\ \bar{A}^1 &= [\bar{A}_{ij} \quad \tilde{C}_{ij} \quad 0 \quad \tilde{B}_i \quad \tilde{E}_j \quad \tilde{V}_j] \\ H_E &= [P_1^T \quad 0_{1 \times 9} \quad \delta \sqrt{h} R_1^T \quad \delta \sqrt{h} Z_1^T] \\ H_F &= [0 \quad \tilde{C}_{ij} \quad 0 \quad 0 \quad \tilde{E}_j \quad \tilde{V}_j \quad 0_{1 \times 6}] \\ \tilde{C}_{ij} &= \begin{bmatrix} 0 & 0 \\ -K_j C_i & 0 \end{bmatrix}, \tilde{E}_j = \begin{bmatrix} 0 \\ K_j \end{bmatrix}, \tilde{V}_j = \begin{bmatrix} 0 \\ -K_j \end{bmatrix} \end{aligned}$$

By applying Lemma 1, we get

$$J_{ij} + m_1 H_E^T \Delta_q^2 H_E + m_1^{-1} H_F^T H_F < 0 \quad (71)$$

Notice that $\Delta_q^2 < \delta^2$, one can get

$$J_{ij} + m_1 \delta^2 H_E^T H_E + m_1^{-1} H_F^T H_F < 0 \quad (72)$$

Using the Schur complement, (72) can be rewritten as

$$\Upsilon_{ij}^1 = \begin{bmatrix} J_{ij} & * & * \\ \delta H_E & -m_1 I & * \\ H_F & 0 & -m_1^{-1} I \end{bmatrix} \quad (73)$$

Define

$$\Lambda = \text{diag}\{I, I, \dots, I, P_1 R_1^{-1}, P_1 Z_1^{-1}, I, I, P_1\}$$

$$P_1 = \text{diag}\{P_{11}, P_{12}\}, Y_j = P_{12} K_j$$

Then multiplying Λ and Λ^T on both sides of (73), respectively. Owing to $(R_m - e_m^{-1} P) R_m^{-1} (R_m - e_m^{-1} P) \geq 0$ ($m = 1, 2$), we have

$$-P R_m^{-1} P \leq -2e_m P + e_m^2 R_m \quad (74)$$

That is

$$\begin{cases} -P_1 R_1^{-1} P_1 \leq -2e_{11} P_1 + e_{11}^2 R_1 \\ -P_1 R_1^{-1} P_1 \leq -2e_{21} P_1 + e_{21}^2 R_2 \\ -P_1 I^{-1} P_1 \leq -2e_3 P_1 + e_3^2 I \end{cases}$$

Then, replace the terms $\Gamma_{55}^1, \Gamma_{66}^1$ and Γ_{99}^1 in (73) with $-2e_{11} P_1 + e_{11}^2 R_1, -2e_{21} P_1 + e_{21}^2 R_2, -2e_3 P_1 + e_3^2 I$, respectively. Then we have

$$\begin{aligned} & \dot{V}_1(t) + \bar{z}^T(t) \bar{z}(t) - \gamma^2 w^T(t) w(t) \\ & \leq \sum_{i=1}^r \sum_{j=1}^r \varphi_i \psi_j \bar{\xi}^T(t) \Phi_{ij}^1 \bar{\xi}(t) \end{aligned} \quad (75)$$

According to the Schur complement, it follows that (64) and (65) are equivalent to the two inequalities in (37), respectively. Following the preceding derivations, we can draw a conclusion that the LMIs (66)–(68) guarantee the three inequalities in (35) hold, respectively. Then, similar to the analysis in Theorem 1, consider a slack matrix \hat{W}_i that

$$\begin{aligned} & \sum_{i=1}^r \sum_{j=1}^r \varphi_i (\varphi_j - \psi_j) \hat{W}_i \\ & = \sum_{i=1}^r \varphi_i \left(\sum_{j=1}^r \varphi_j - \sum_{j=1}^r \psi_j \right) \hat{W}_i \\ & = 0 \end{aligned}$$

where $\hat{W}_i = \hat{W}_i^T$ ($i \in \mathfrak{R}$) are arbitrary matrices, and combining (61)–(63), it yields that $\dot{V}_m(t) + \bar{z}^T(t) \bar{z}(t) - \gamma^2 w^T(t) w(t) < 0$ ($m = 1, 2$). Accordingly, the system is exponentially stable. Notice that $Y_j = P_{12} K_j$, then the state estimator gains are derived as $K_j = P_{12}^{-1} Y_j$ ($j \in \mathfrak{R}$). ■

4. Simulation examples

In this section, a simulation example related to CPSs is presented to illustrate the feasibility of estimator design for CPSs, where the sensor saturation and DoS attacks are considered and the event-triggered scheme and quantization are introduced to relieve the influence brought by limited communication resources.

Consider the system (3) with the following system matrices:

$$\begin{aligned} A_1 &= \begin{bmatrix} -2.5 & 0.1 \\ 0.2 & -1.5 \end{bmatrix}, A_2 = \begin{bmatrix} -2.1 & 0.2 \\ 0.5 & -2 \end{bmatrix} \\ B_1 &= \begin{bmatrix} 1.2 \\ -0.4 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ -0.2 \end{bmatrix} \\ C_1 &= [1.2 \quad 0.8], C_2 = [0.8 \quad 0.6] \\ L_1 &= [-0.25 \quad 0.28], L_2 = [-0.2 \quad 0.3] \end{aligned}$$

In the following, we present three cases to prove the effectiveness of the designed method, namely, in case 1, the influence brought by sensor saturation and DoS attacks on CPSs is not considered and the event-triggered scheme and quantization are not used; in case 2, consider the CPSs with sensor saturation and periodic DoS attacks but the event-triggered scheme and quantization are not adopted; in case3, the event-triggered scheme and quantization are introduced for CPSs with sensor saturation and periodic DoS attacks considered in case 2.

Case 1: Set $\sigma = 0$ and $r = 0$, which means that event-triggered scheme and quantization are not used, at the same time, the effects of sensor saturation and DoS attacks in CPSs are not taken into consideration. Let $h = 0.01s, T = 3s, T_{sleep}^{min} = 2.76s, \mu_1 = \mu_2 = 1.01$, and $\alpha_1 = 0.05, \alpha_2 = 0.5$ such that inequality (36) holds. Set $e_{1m} = 1, e_{2m} = 1, e_3 = 1$ ($m = 1, 2$), $\gamma = 3.19$. Through utilizing LMI toolbox in MATLAB, we have

$$Y_1 = [-0.0933 \quad 0.0350]^T, Y_2 = [0.0759 \quad -0.0659]^T$$

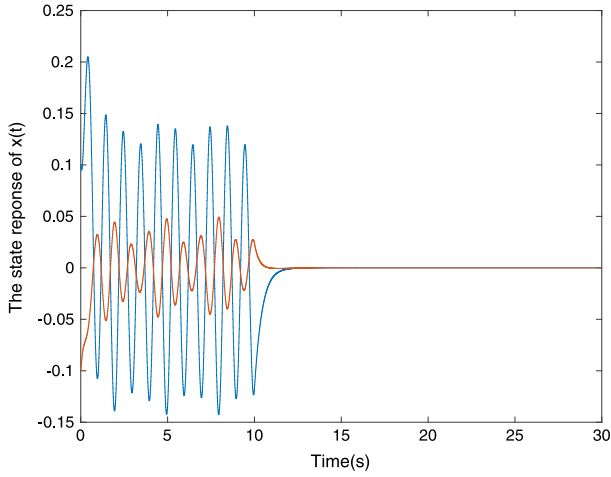


Fig. 2. The response of $x(t)$ in case 1.

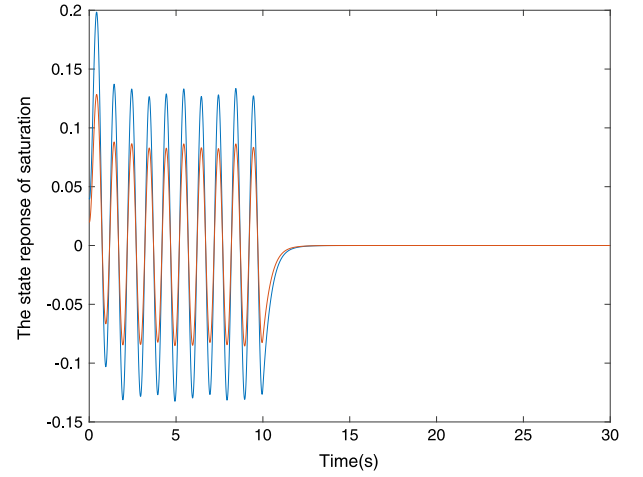


Fig. 4. The input of estimator in case 1.

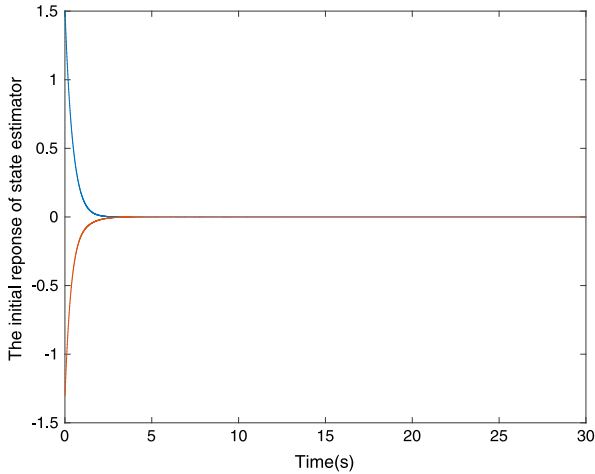


Fig. 3. The response of $\hat{x}(t)$ in case 1.

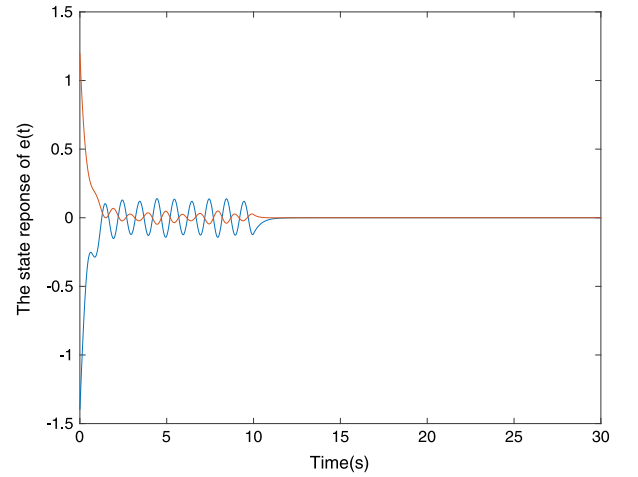


Fig. 5. The estimator error $e(t)$ in case 1.

$$P_{12} = \begin{bmatrix} 7.3523 & 0.7066 \\ 0.7066 & 9.1998 \end{bmatrix}, \Omega = 10.2249$$

Then, the state estimator gains can be figured out from equality (69) in Theorem 3 as

$$K_1 = [-0.0132 \quad 0.0048]^T, K_2 = [0.0111 \quad -0.0080]^T$$

The corresponding initial parameters of system (30) is given by $x(0) = [0.1 \quad -0.1]^T$, $\hat{x}(0) = [1.5 \quad -1.3]^T$, $w(t) = \begin{cases} 0.8 \sin(2\pi t), & 0 \leq t \leq 10, \\ 0, & \text{otherwise} \end{cases}$. Then, the results are obtained by the simulation in MATLAB. The state responses of CPSs and its estimation are exhibited in Figs. 2 and 3, respectively. Fig. 4 shows the input of estimator. The estimator error $e(t)$ is depicted in Fig. 5. It is easily obtained from the above graphs that the designed state estimator performs well.

Case 2: Set $\sigma = 0$, $r = 0$ and $\varepsilon = 0.1$, which means that the effects of sensor saturation and DoS attacks are considered while event-triggered scheme and quantization have not been introduced. Set $e_{1m} = 1$, $e_{2m} = 1$, $e_3 = 1$ ($m = 1, 2$), $\gamma = 3.19$. Let $h = 0.01s$, $T = 3s$, $T_{sleep}^{min} = 2.76s$, $\mu_1 = \mu_2 = 1.01$, and $\alpha_1 = 0.05$, $\alpha_2 = 0.5$ such that inequality (36) holds. Through the utilization of LMI toolbox in MATLAB, it yields that

$$Y_1 = [-0.0941 \quad 0.0350]^T, Y_2 = [0.0765 \quad -0.0661]^T$$

$$P_{12} = \begin{bmatrix} 7.3597 & 0.7081 \\ 0.7081 & 9.2110 \end{bmatrix}, \Omega = 10.2370$$

Then, by using the equality (69) in Theorem 3, the state estimator gains can be obtained

$$K_1 = [-0.0133 \quad 0.0048]^T, K_2 = [0.0112 \quad -0.0080]^T$$

Give corresponding initial parameters of system (30) as $x(0) = [0.1 \quad -0.1]^T$, $\hat{x}(0) = [1.5 \quad -1.3]^T$, $w(t) = \begin{cases} 0.8 \sin(2\pi t), & 0 \leq t \leq 10, \\ 0, & \text{otherwise} \end{cases}$. Then by using MATLAB, the simulation results in case 2 are presented. In Fig. 6, the output of saturated sensor is described by yellow line while the original signal is depicted by blue line. The release time intervals under the periodic DoS jamming signal are shown in Fig. 7. The state responses of CPSs and its estimation are depicted in Figs. 8 and 9, respectively. Then, the estimator input is also shown in Fig. 10 and the estimator error $e(t)$ is exhibited in Fig. 11. According to the above analysis, it is concluded that the designed state estimator for CPSs is feasible under the cases of sensor saturation and periodic DoS attacks.

Case 3: Set $\sigma = 0.2$, $r = 0.818$ and $\varepsilon = 0.1$, which means that the event-triggered scheme and quantization are utilized to alleviate the effects of limited communication resources for CPSs with sensor saturation and DoS attacks shown in case 2. Let $e_{1m} =$

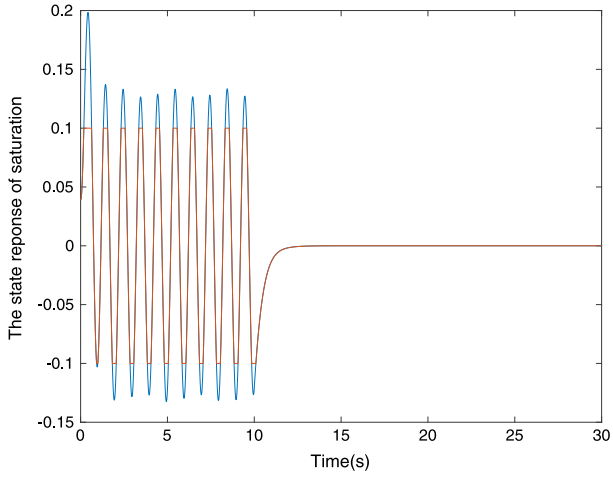


Fig. 6. The output of saturated sensor in case 2. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

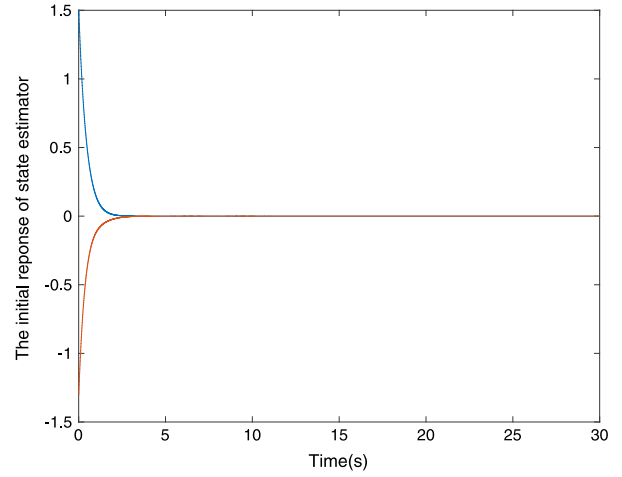


Fig. 9. The response of $\hat{x}(t)$ in case 2.

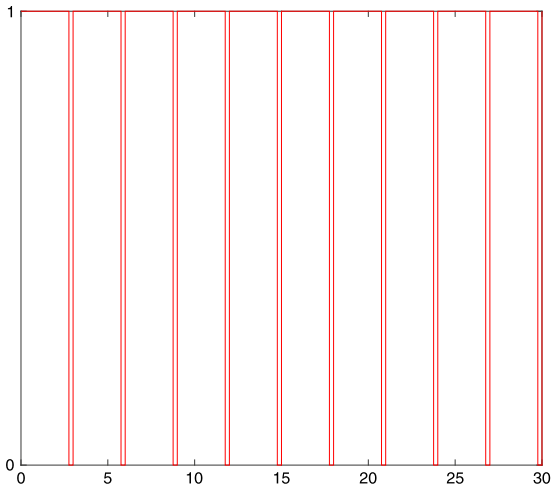


Fig. 7. The response of DoS attacks in case 2.

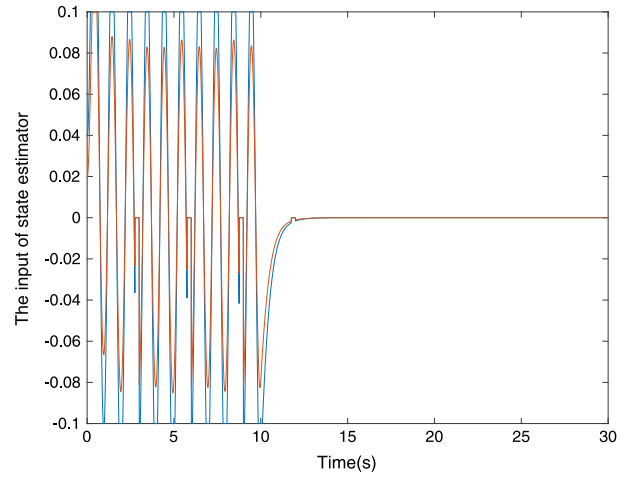


Fig. 10. The input of estimator in case 2.

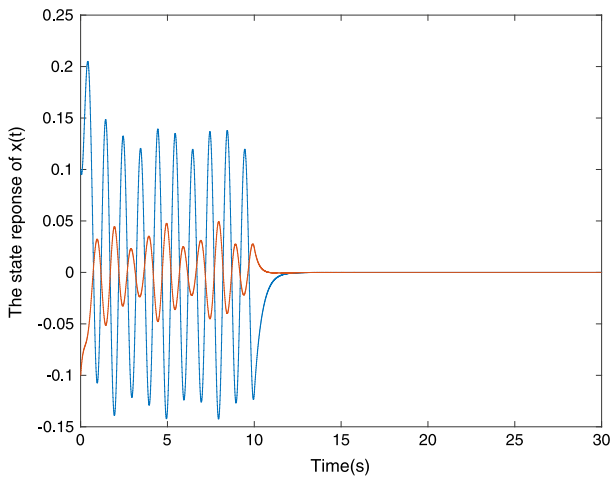


Fig. 8. The response of $x(t)$ in case 2.

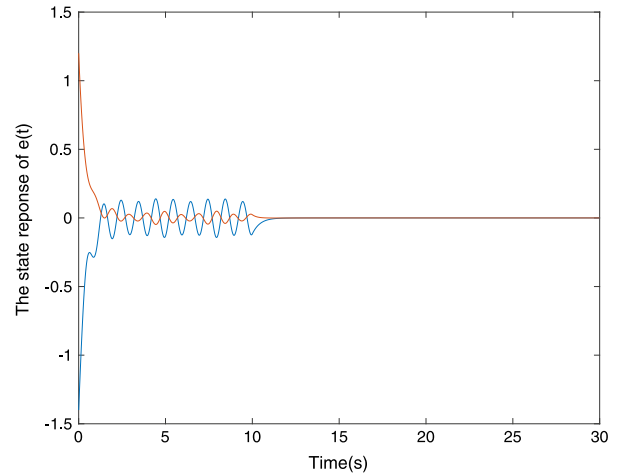


Fig. 11. The estimator error $e(t)$ in case 2.

1, $e_{2m} = 1, e_3 = 1 (m = 1, 2), \gamma = 3.19$. Set $h = 0.01s, T = 3s, T_{sleep}^{min} = 2.76s, \mu_1 = \mu_2 = 1.01$, and $\alpha_1 = 0.05, \alpha_2 = 0.5$ such that inequality (36) holds. By using LMI toolbox in MATLAB, we can get

that

$$Y_1 = [-0.0067 \quad 0.0025]^T, Y_2 = [0.0198 \quad -0.0159]^T$$

$$P_{12} = \begin{bmatrix} 6.3415 & 0.5024 \\ 0.5024 & 7.6122 \end{bmatrix}, \Omega = 1.8029$$

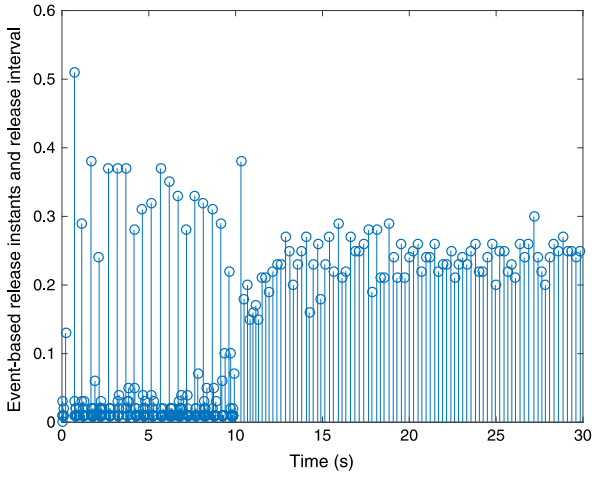


Fig. 12. Release time intervals with $T = 3$ s and $T_{sleep}^{min} = 2.76$ s in case 3.

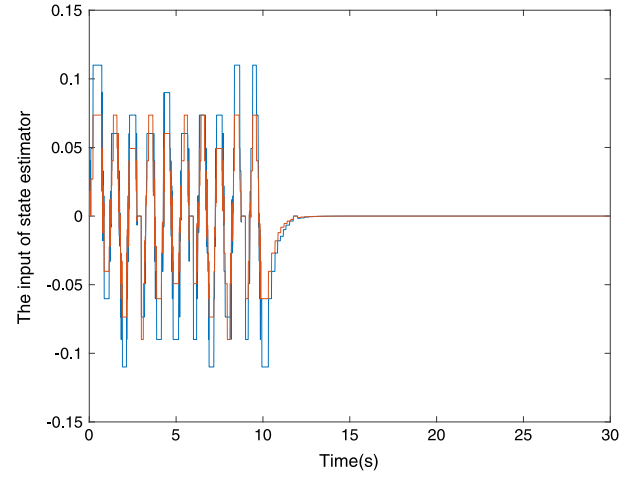


Fig. 14. The response of estimator input in case 3.

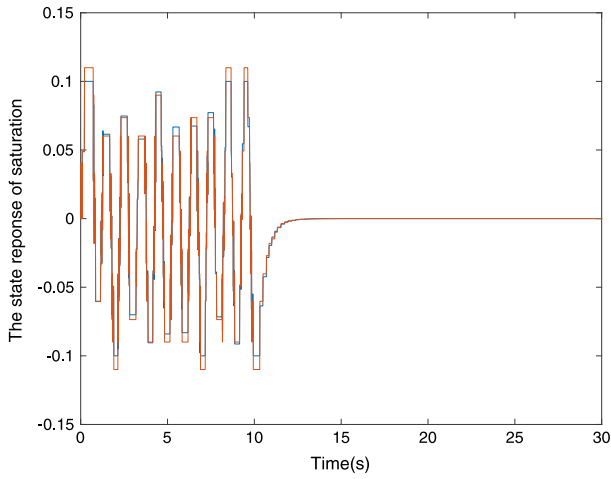


Fig. 13. The output of quantizer in case 3.

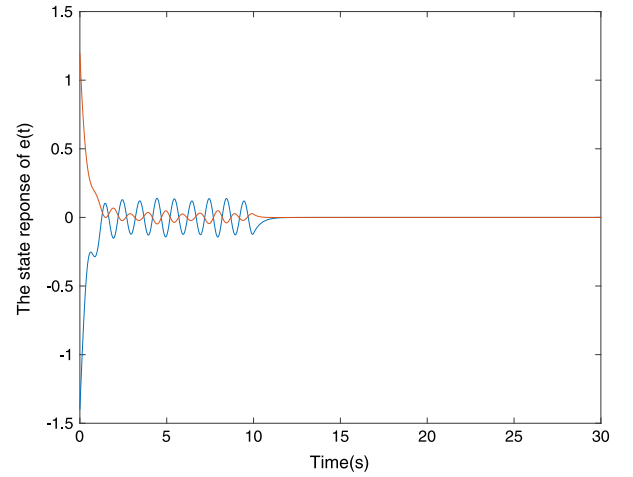


Fig. 15. The estimator error $e(t)$ in case 3.

According to the equality (69) in Theorem 3, the state estimator gains can be calculated as

$$K_1 = [-0.0011 \quad 0.0004]^T, K_2 = [0.0033 \quad -0.0023]^T$$

Give corresponding initial parameters of system (30) as $x(0) = [0.1 \quad -0.1]^T, \hat{x}(0) = [1.5 \quad -1.3]^T,$

$$w(t) = \begin{cases} 0.8 \sin(2\pi t), & 0 \leq t \leq 10, \\ 0, & \text{otherwise} \end{cases},$$

and by using MATLAB, the simulation results in case 3 are exhibited. The release time intervals are shown in Fig. 12 and the output of quantizer is depicted in Fig. 13 with yellow line while the original signal is with blue line. Further, the estimator input is also shown in Fig. 14 and the estimator error $e(t)$ is shown in Fig. 15. By comparing Fig. 10 with Fig. 14, the system performance in case 3 has been improved in comparison with the one in case 2 because of the introduction of event-triggered scheme and quantization. Based on the above analysis, the conclusions can be draw that (a) the system is exponentially stable and the gains of estimator have been obtained; (b) the event-triggered scheme and quantization can reduce the transmission of redundant data in the system.

5. Conclusions

In this paper, a problem of estimator design has been investigated for CPSs with limited communication resources, sensor

saturation and periodic DoS attacks. For the considered CPSs, a T-S fuzzy model is borrowed to described it. In order to relieve the influence of limited communication resources, the event-triggered scheme and quantization mechanism are adopted to reduce the amount of data transmission. A new mathematical model of quantized state estimation for CPSs is established, which integrates the characteristics of CPSs, sensor saturation, event-triggered scheme and periodic DoS attacks. In addition, under the assistance of Lyapunov stability theory and LMI technologies, sufficient conditions guaranteeing the exponential stability of CPSs are obtained and the gains of the estimator are acquired in terms of LMIs. Finally, the simulation example confirms the feasibility of the designed estimator for CPSs when considering limited communication resources, sensor saturation and periodic DoS attacks. In the future, we will focus on the study concerned with the non-periodic DoS attacks and the hybrid cyber attacks including two or more than two kinds of cyber attacks. Moreover, in order to further improve CPSs performance against the cyber attacks, the attack detection is also one of the problems considered in the next research.

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References

- [1] Li Y, Zhang L, Zheng H, He X, Peeta S, Zheng T, Li Y. Nonlane-discipline-based car-following model for electric vehicles in transportation-cyber-physical systems. *IEEE Trans Intell Transp Syst* 2018;19(1):38–47.
- [2] Zhang D, Song H, Yu L. Robust fuzzy-model-based filtering for nonlinear cyber-physical systems with multiple stochastic incomplete measurements. *IEEE Trans Syst Man Cybern A* 2017;47(8):1826–38.
- [3] Takagi T, Sugeno M. Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans Syst Man Cybern A* 1985;15(1):116–32.
- [4] Zhang D, Shi P, Zhang W, Yu L. Non-fragile distributed filtering for fuzzy systems with multiplicative gain variation. *Signal Process* 2016;121:102–10.
- [5] Liu J, Wei L, Cao J, Fei S. Hybrid-driven H_∞ filter design for T-S fuzzy systems with quantization. *Nonlinear Anal Hybrid Syst* 2019;31:135–52.
- [6] Chang X, Park JH, Shi P. Fuzzy resilient energy-to-peak filtering for continuous-time nonlinear systems. *IEEE Trans Fuzzy Syst* 2017;25(6):1576–88.
- [7] Chang X, Li Z, Park JH. Fuzzy generalized \mathcal{H}_2 filtering for nonlinear discrete-time systems with measurement quantization. *IEEE Trans Syst Man Cybern A* 2018;48(12):2419–30.
- [8] Xie X, Xie J, Hu S. Reducing the conservatism of stability conditions for continuous-time T-S fuzzy systems based on an extended approach. *Neurocomputing* 2016;173:1655–9.
- [9] Peng L, Cao X, Shi H, Sun C. Optimal jamming attack schedule for remote state estimation with two sensors. *J Franklin Inst B* 2018;355(14):6859–76.
- [10] Li Y, Shi L, Cheng P, Chen J, Quevedo DE. Jamming attacks on remote state estimation in cyber-physical systems: A game-theoretic approach. *IEEE Trans Automat Control* 2015;60(10):2831–6.
- [11] Ao W, Song Y, Wen C, Lai J. Finite time attack detection and supervised secure state estimation for CPSs with malicious adversaries. *Inform Sci* 2018;451–452:67–82.
- [12] Liu J, Gu Y, Cao J, Fei S. Distributed event-triggered H_∞ filtering over sensor networks with sensor saturations and cyber-attacks. *ISA Trans* 2018;81:63–75.
- [13] Ding D, Wang Z, Shen B, Shu H. H_∞ state estimation for discrete-time complex networks with randomly occurring sensor saturations and randomly varying sensor delays. *IEEE Trans Neural Netw Learn Syst* 2012;23(5):725–36.
- [14] Girard A. Dynamic triggering mechanisms for event-triggered control. *IEEE Trans Automat Control* 2015;60(7):1992–7.
- [15] Anta A, Tabuada P. To sample or not to sample: Self-triggered control for nonlinear systems. *IEEE Trans Automat Control* 2010;55(9):2030–42.
- [16] Yue D, Tian E, Han Q. A delay system method for designing event-triggered controllers of networked control systems. *IEEE Trans Automat Control* 2013;58(2):475–81.
- [17] Li H, Shi Y. Event-triggered robust model predictive control of continuous-time nonlinear systems. *Automatica* 2014;50(5):1507–13.
- [18] Liu J, Wei L, Xie X, Yue D. Distributed event-triggered state estimators design for networked sensor systems with deception attacks. *IET Control Theory Appl* 2018. <http://dx.doi.org/10.1049/iet-cta.2018.5868>.
- [19] Gu Z, Tian E, Liu J. Adaptive event-triggered control of a class of nonlinear networked systems. *J Franklin Inst B* 2017;354(9):3854–71.
- [20] Tian E, Wang Z, Zou L, Yue D. Probabilistic-constrained filtering for a class of nonlinear systems with improved static event-triggered communication. *Internat J Robust Nonlinear Control* 2018. <http://dx.doi.org/10.1002/rnc.4447>.
- [21] Shen M, Ye D, Wang Q. Event-triggered H_∞ filtering of Markov jump systems with general transition probabilities. *Inform Sci* 2017;418–419:635–51.
- [22] Gu Z, Yang L, Tian E, Zhao H. Event-triggered reliable H_∞ filter design for networked systems with multiple sensor distortions: A probabilistic partition approach. *ISA Trans* 2017;66:2–9.
- [23] Peng C, Yang M, Zhang J, Fei M, Hu S. Network-based H_∞ control for T-S fuzzy systems with an adaptive event-triggered communication scheme. *Fuzzy Sets and Systems* 2017;329:61–76.
- [24] Gu Z, Shi P, Yue D, Ding Z. Decentralized adaptive event-triggered H_∞ filtering for a class of networked nonlinear interconnected systems. *IEEE Trans Cybern* 2018. <http://dx.doi.org/10.1109/TCYB.2018.2802044>.
- [25] Hu S, Yue D, Xie X, Du Z. Event-triggered H_∞ stabilization for networked stochastic systems with multiplicative noise and network-induced delays. *Inform Sci* 2015;299:178–97.
- [26] Chen X, Wang Y, Hu S. Event-based robust stabilization of uncertain networked control systems under quantization and denial-of-service attacks. *Inform Sci* 2018;459:369–86.
- [27] Hu S, Yue D, Xie X, Chen X, Yin X. Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks. *IEEE Trans Cybern* 2018. <http://dx.doi.org/10.1109/TCYB.2018.2861834>.
- [28] Liu J, Zha L, Cao J, Fei S. Hybrid-driven-based stabilization for networked control systems. *IET Control Theory Appl* 2016;10(17):2279–85.
- [29] Liu J, Xia J, Tian E, Fei S. Hybrid-driven-based H_∞ filter design for neural networks subject to deception attacks. *Appl Math Comput* 2018;320:158–74.
- [30] Liu J, Gu Y, Xie X, Yue D, Park JH. Hybrid-driven-based H_∞ control for networked cascade control systems with actuator saturations and stochastic cyber attacks. *IEEE Trans Syst Man Cybern A* 2018. <http://dx.doi.org/10.1109/TSMC.2018.2875484>.
- [31] Liu J, Wei L, Xie X, Tian E, Fei S. Quantized stabilization for T-S fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks. *IEEE Trans Fuzzy Syst* 2018;26(6):3820–34.
- [32] Liu J, Zha L, Xie X, Tian E. Resilient observer-based control for networked nonlinear T-S fuzzy systems with hybrid-triggered scheme. *Nonlinear Dynam* 2018;91(3):2049–61.
- [33] Zhang D, Shi P, Wang Q, Yu L. Analysis and synthesis of networked control systems: A survey of recent advances and challenges. *ISA Trans* 2017;66:376–92.
- [34] Cheng J, Park JH, Cao J, Zhang D. Quantized H_∞ filtering for switched linear parameter-varying systems with sojourn probabilities and unreliable communication channels. *Inform Sci* 2018;466:289–302.
- [35] Zhang D, Cheng J, Park JH, Cao J. Robust H_∞ control for nonhomogeneous Markovian jump systems subject to quantized feedback and probabilistic measurements. *J Franklin Inst B* 2018;355(15):6992–7010.
- [36] Zha L, Fang J, Liu J, Tian E. Event-based finite-time state estimation for Markovian jump systems with quantizations and randomly occurring nonlinear perturbations. *ISA Trans* 2017;66:77–85.
- [37] Chang X, Wang Y. Peak-to-peak filtering for networked nonlinear DC motor systems with quantization. *IEEE Trans Ind Inf* 2018;14(12):5378–88.
- [38] Zhang D, Wang Q, Srinivasan D, Li H, Yu L. Asynchronous state estimation for discrete-time switched complex networks with communication constraints. *IEEE Trans Instrum Meas* 2018;29(5):1732–46.
- [39] Wang Y, Huang C, He L. Adaptive time-varying formation tracking control of unmanned aerial vehicles with quantized input. *ISA Trans* 2018. <http://dx.doi.org/10.1016/j.isatra.2018.09.013>.
- [40] Zhang D, Yang F, Yu C, Srinivasan D, Yu L. Robust fuzzy-model-based filtering for nonlinear networked systems with energy constraints. *J Franklin Inst B* 2017;354(4):1957–73.
- [41] Shi Y, Yu B. Robust mixed H_2/H_∞ control of networked control systems with random time delays in both forward and backward communication links. *Automatica* 2011;47(4):754–60.
- [42] Liu J, Tian E, Xie X, Hong L. Distributed event-triggered control for networked control systems with stochastic cyber-attacks. *J Franklin Inst B* 2018. <http://dx.doi.org/10.1016/j.jfranklin.2018.01.048>.
- [43] Hu L, Wang Z, Han Q, Liu X. State estimation under false data injection attacks: Security analysis and system protection. *Automatica* 2018;87:176–83.
- [44] Liu X, Bao Z, Lu D, Li Z. Modeling of local false data injection attacks with reduced network information. *IEEE Trans Smart Grid* 2015;6(4):1686–96.
- [45] Yuan H, Xia Y. Resilient strategy design for cyber-physical system under DoS attack over a multi-channel framework. *Inform Sci* 2018;454–455:312–27.
- [46] Sun Y, Yang G. Periodic event-triggered resilient control for cyber-physical systems under denial-of-service attacks. *J Franklin Inst B* 2018;355(13):5613–31.
- [47] Zha L, Tian E, Xie X, Gu Z, Cao J. Decentralized event-triggered H_∞ control for neural networks subject to cyber-attacks. *Inform Sci* 2018;457–458:141–55.
- [48] Gao H, Chen T. A new approach to quantized feedback control systems. *Automatica* 2008;44:534–42.
- [49] Sun H, Peng C, Zhang W, Yang T, Wang Z. Security-based resilient event-triggered control of networked control systems under denial of service attacks. *J Franklin Inst B* 2018. <http://dx.doi.org/10.1016/j.jfranklin.2018.04.001>.
- [50] Petersen IR. A stabilization algorithm for a class of uncertain linear systems. *Systems Control Lett* 1987;8(4):351–7.