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Multi-sensors-based security control for T-S fuzzy systems over resource-constrained networks

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Abstract

This paper investigates the problem of multi-sensors-based security control for Takagi-Sugeno (T-S) fuzzy system over resource-constrained networks. To decrease the effects of resource-constrained networks, a distributed event-triggered scheme and a quantization mechanism are applied to lighten the burden of network transmission. By taking the influences of actuator saturation and denial-of-service (DoS) attacks into consideration, a novel mathematical model for T-S fuzzy system is constructed. Then, sufficient conditions ensuring the exponential stability of T-S fuzzy system are derived by using the Lyapunov stability theory. Moreover, the security controller gains for T-S fuzzy system are obtained by solving a set of linear matrix inequalities (LMIs). Finally, the feasibility of the proposed approach is demonstrated through simulation examples.

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1. Introduction

T-S fuzzy model, which is firstly proposed in [1], has become a powerful method to describe the nonlinear systems by using IF-THEN rules. In particular, the T-S model-based approach can integrate the fuzzy logic theory and the linear systems theory into a unified framework to approximate a certain class of nonlinear systems [2]. In recent years, T-S fuzzy systems have widely practical applications in different fields, such as automatic train operation system [3], helicopter system [4], two-wheel drive robot [5] and so on. Therefore, considerable attentions have been paid to T-S fuzzy systems and fruitful research results have been reported in the literature [6,7]. For example, the problem of quantized stabilization is investigated in [8] for T-S fuzzy systems with hybrid-triggered mechanism and stochastic cyber-attacks. The authors in [9] address the controller synthesis issue for switched T-S fuzzy systems with time delay, uncertainties and process disturbances. On the other hand, actuator saturation has received increasing interests [10–12]. Physical actuators are usually affected by input saturation due to physical constraints. It is well known that the presence of actuator saturation may affect the performance of the T-S fuzzy systems and even leads to instability of control systems. Therefore, the stabilization of T-S fuzzy systems with actuator saturation have been studied by many researchers [13,14]. For example, in order to handle the effect of actuator saturation, the issue of output feedback robust stabilization for T-S fuzzy systems is studied in [14]. In [15], the authors investigate hybrid-driven-based H_∞ control for networked cascade control systems with actuator saturations.

The introduction of the communication networks brings many advantages such as efficient data transmission and faster processing speeds of control systems. Nevertheless, it also brings some challenging problems such as packet dropouts, packet disordering, network-induced delay and cyber-attacks. Due to the physical limitations of the network, network bandwidth is always limited. Hence, how to effectively save the limited resources has been extensively studied [16–18]. In the past few years, event-triggered scheme is widely adopted to reduce the burden of communication [19–21]. There are several common types of event-triggered scheme such as dynamic event triggering scheme, self-triggered scheme, discrete event-triggered scheme and so on [22]. One of the event-triggered schemes is proposed in [23] which can be implemented by the sampled system state of discrete instants. The main idea of the proposed event-triggered scheme in [23] is that whether the sampled signals are transmitted or not depends on the predefined event-triggered condition. Motivated by the work in [23], researchers have made extensive studies on the event-triggered control problem [24–27]. For instance, based on the event-triggered scheme in [23], a new event-triggered scheme is introduced in [28] for networked cascade control system under stochastic nonlinearities. On the basis of the event-triggered scheme in [23], the decentralized event-triggered H_∞ control for neural networks is studied in [29]. The controller design problem for networked systems with hybrid-triggered scheme and cyber-attacks is investigated in [30]. With the increasing focus on distributed sensor networks, the problem of control systems with distributed event-triggered scheme has become a hot issue [31,32]. For example, the distributed event-triggered output-feedback controller is designed in [32] for large-scale fuzzy systems over a sensor network. The distributed state estimation is investigated in [33] for nonlinear networked systems with DoS attacks. In addition to the aforementioned event-triggered scheme, the quantization mechanism is another useful method to reduce the pressure of transmission. The so-called quantization mechanism can convert a continuous/discrete signal into a fewer discrete values, which can reduce the scale of the signal transmission. In view of this, the stabilization problem of control systems with quantization has been paid much attention by many researchers [34,35]. For example, the observer-based event-triggered output control problem with quantization is investigated in [35], which employs the dynamic uniform quantizer to improve the communication efficiency. By considering the quantization and cyber-attacks, the distributed recursive filtering problem is studied in [36] for discrete time-delayed system. To the best of our knowledge, the problem of event-based control for T-S fuzzy systems with quantization has not been fully investigated, which is one of the motivation in this paper.

In the last decade, due to the openness of the network and the diversity of attack method, security problems have become one of the important factors affecting the stable operation of the control systems. Cyber-attacks are considered to be one of the factors threatening the network security. In generally, cyber-attacks include three main types, namely, DoS attacks, covert attacks and deception attacks [37,38]. In particular, the DoS attacks are mainly intended to interrupt data transmission, which can degrade the closed-loop control system performance and cause instability of the control system [39]. In view of the above situation, the analysis and design of control system with DoS attacks has attracted wide attentions in recent years [40–43]. For instance, based on a new state error-dependent switched system model, the event-based controller is designed in [43] for networked control systems with periodic DoS jamming attacks. In [44], considering the influence of DoS attacks, the authors address the event-based robust stabilization problem of uncertain networked control systems with quantization. The resilient control problem is investigated in [45] for networked control system under DoS attacks. Motivated by this, the objective in this paper is to design a controller to restrain the impact of the periodic DoS jamming signals while ensuring the stability of T-S fuzzy systems.

Summarizing the discussions above, this paper aims to deal with the multi-sensors-based security control for T-S fuzzy systems over resource-constrained networks. The main contributions of this paper are shown as follows:

1) A distributed event-triggered scheme and a quantization mechanism are introduced to save the limited communication resources in resource-constrained networks while the influence of the jamming signal is also considered;
2) Different from some existing results ([8,33]), a more realistic T-S fuzzy model is firstly formulated by considering distributed event-triggered scheme and quantization mechanism. The influences of actuator saturation and periodic DoS jamming attacks are tackled simultaneously in the model;
3) Based on the constructed model, the Lyapunov stability theory is exploited to investigate the exponential stability of control system.

The remainder of this paper is organized as follows. In Section 2, a mathematical T-S fuzzy model for multi-sensors-based control system is established with quantization, DoS attacks and actuator saturation. In Section 3, the sufficient conditions which can guarantee the stability of T-S fuzzy system are obtained. Moreover, the desired controller gains are presented. Two examples are given in Section 4 to demonstrate the efficiency of designed T-S fuzzy system. Conclusion is presented in Section 5.

Notation: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ stand for the n-dimensional Euclidean space and the set of $n\times m$ dimensional real matrices, respectively. I is the identity matrix with appropriate dimension and $\mathbb{N}$ is the set of natural numbers. For matrix X^T denotes the transposition for matrix X. The notation $X>0$ means that the matrix X is real symmetric positive definite. For a matrix B and two symmetric matrices A and C, $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ denotes a symmetric matrix, where * is used as the entries implied by symmetric.

2. Problem formulation and modeling

2.1. System description

In this paper, the physical plant is described by the following T-S fuzzy model with ith rule:

IF $\vartheta_1(x(t))$ is G_1^i and...and $\vartheta_l(x(t))$ is G_l^i, THEN

$$\dot{x}(t) = A_i x(t) + B_i \bar{u}(t) \tag{1}$$

where $G_v^i (i = 1, 2, \ldots, r, v = 1, 2, \ldots, l)$ is the fuzzy sets, $\vartheta_v(x(t))$ denotes the vector of fuzzy premise variables, r represents the number of IF-THEN rules. For the sake of simplicity, $\vartheta_v(x)$ is used to represent $\vartheta_v(x(t))$, $\vartheta_v(x) = [\vartheta_1(x), \vartheta_2(x), \ldots, \vartheta_l(x)]$. A_i and B_i are known constant real matrices with appropriate dimensions. $x(t) \in \mathbb{R}^n$ denotes the state vector and $\bar{u}(t) \in \mathbb{R}^m$ is the saturating control input. The saturating input $\bar{u}(t)$ is defined as $\bar{u}(t) = sat(u(t))$, in which $u(t) \in \mathbb{R}^m$ represents the control input vector and $sat(\cdot)$: $\mathbb{R}^m \to \mathbb{R}^m$ denotes a saturation function to be designed later.

By using center-average defuzzifier, product inference and singleton fuzzifier, system (1) can be rewritten as follows

$$\dot{x}(t) = \sum_{i=1}^{r} \varsigma_i(\vartheta(x))[A_i x(t) + B_i \bar{u}(t)] \tag{2}$$

where $\varsigma_i(\vartheta(x)) = \frac{\prod_{v=1}^{l} G_v^i(\vartheta_v(x))}{\sum_{i=1}^{r} \prod_{v=1}^{l} G_v^i(\vartheta_v(x))}$, $G_v^i(\vartheta_v(x))$ denotes the grade of the membership function of $\vartheta_v(x)$ and $\varsigma_i(\vartheta(x))$ is the normalized membership function satisfying

$$\varsigma_i(\vartheta(x)) \geq 0, \sum_{i=1}^{r} \varsigma_i(\vartheta(x)) = 1 \tag{3}$$

The main purpose of this paper is to design a multi-sensors-based controller for T-S fuzzy system. The infrastructure of such a T-S fuzzy system over resource-constrained networks is shown in Fig. 1. An event-trigger generator and a quantizer are employed in each sensor.

Throughout this paper, the following assumptions are used:

Assumption 1. The information transmission in communication network is assumed to be affected by the occurrence of DoS attacks.

Assumption 2. As shown in Fig. 1, the distributed sensors are time-driven with a constant sampling period h. The sampled signals are transmitted from sensors to the corresponding event-trigger generators. The latest transmitted instants of the pth event-trigger generator are denoted by $t_k^p h (p = 1, 2, \ldots, m, k = 1, 2, \ldots)$.

Assumption 3. The quantizer, controller and actuator are event-driven. The released signals from event-trigger generators are then quantized by the corresponding quantizers, which are further sent to the controller. The control output can be transmitted to the actuator via communication network.

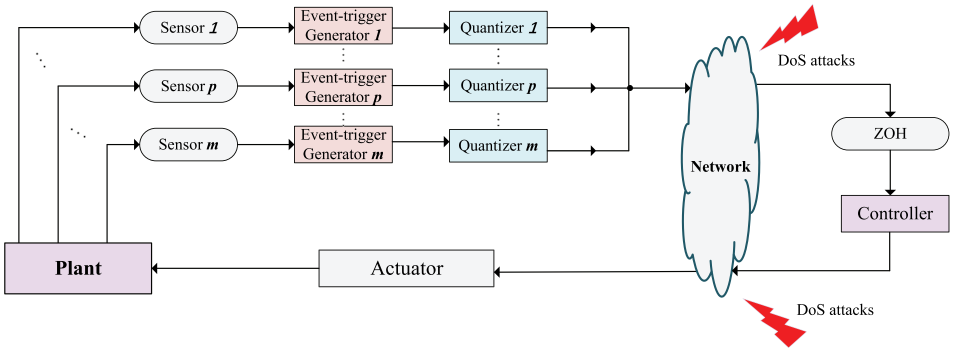

Fig. 1. The structure of multi-sensors-based security control for T-S fuzzy system over resource-constrained networks.

In this paper, we will design the following controller with jth rules.
IF $\vartheta_1(\hat{x}(t))$ is H_1^j,..., and $\vartheta_q(\hat{x}(t))$ is H_q^j, THEN

$$u(t) = K_j\hat{x}(t) \tag{4}$$

where $H_w^j(j = 1, 2, \ldots, r, w = 1, 2, \ldots, q)$ denotes the fuzzy sets and $\vartheta_w(\hat{x}(t))$ denotes the premise variables. K_j represents the desired controller parameters to be determined. $\hat{x}(t)$ is the real input of controller. For simplicity, $\vartheta_w(\hat{x})$ is used to represent $\vartheta_w(\hat{x}(t))$ and $\vartheta_w(\hat{x}) = [\vartheta_1(\hat{x}), \vartheta_2(\hat{x}), \ldots, \vartheta_q(\hat{x})]$.

The controller rule for defuzzified output of Eq. (4) can be rewritten in the following form

$$u(t) = \sum_{j=1}^{r} g_j(\vartheta(\hat{x}))K_j\hat{x}(t) \tag{5}$$

where $g_j(\vartheta(\hat{x})) = \frac{\prod_{w=1}^{q} H_w^j(\vartheta_w(\hat{x}))}{\sum_{j=1}^{r}\prod_{w=1}^{q} H_w^j(\vartheta_w(\hat{x}))}$, $H_w^j(\vartheta_w(\hat{x}))$ denotes the membership value of $\vartheta_w(\hat{x})$ in H_w^j. Therefore, we have

$$g_j(\vartheta(\hat{x})) \geq 0, \sum_{j=1}^{r} g_j(\vartheta(\hat{x})) = 1 \tag{6}$$

Remark 1. This paper extends the traditional analysis and design of T-S fuzzy system to the more complex T-S fuzzy system over resource-constrained networks. Under the environment of resource-constrained networks, the impacts of actuator saturation and DoS attacks are taken into account. Moreover, a mathematical model of T-S fuzzy system is presented that integrates distributed event-triggered parameters, quantization parameters and DoS jamming signals in a unified framework, which is more general than the existing model with single sensor node [6,8,30].

2.2. Design of distributed event-triggered scheme

In order to save the limited network resources and reduce the burden of network, a distributed event-triggered scheme is adopted. The event-trigger generator is employed in each sensor side, which is shown in Fig. 1. Then the next transmitted instant $t_{k+1}^p h$ of the pth sensor can be described as

$$t_{k+1}^p h = t_k^p h + \inf_{j^p \geq 1} \left\{ j^p h | e_k^p\left(t_k^p h\right)^T \Omega_p e_k^p\left(t_k^p h\right) > \sigma_p^2 x^p\left(t_k^p h + j^p h\right)^T \Omega_p x^p\left(t_k^p h + j^p h\right) \right\} \tag{7}$$

where $j^p = 1, 2, \ldots, t_{k+1}^p - t_k^p - 1$, $\Omega_p > 0$ denotes a weighting matrix, $\sigma_p \in [0, 1)$ $(p = 1, 2, \ldots, m)$ is a design parameter and $t_k^{j^p} h = t_k^p h + j^p h$ denotes the current sampling instant. The following $e_k^p(t_k^p h)$ is defined as the threshold error between the latest transmitted instant $t_k^p h$ and the current sampling instant $t_k^{j^p} h$ of the pth sensor

$$e_k^p(t_k^p h) = x^p\left(t_k^p h\right) - x^p\left(t_k^p h + j^p h\right) \tag{8}$$

According to (7), for all sensors, the following event-triggered condition [30] is derived

$$e_k^T(t_k h) \Omega e_k(t_k h) > \sigma^2 x^T(t_k h + jh) \Omega x(t_k h + jh) \tag{9}$$

where $\sigma = diag\{\sigma_1, \sigma_2, \ldots, \sigma_m\}$, $\Omega = diag\{\Omega_1, \Omega_2, \ldots, \Omega_m\}$, $e_k(t_k h) = [\left(e_k^1\left(t_k^1 h\right)\right)^T, \ldots, \left(e_k^p\left(t_k^p h\right)\right)^T, \ldots, \left(e_k^m\left(t_k^m h\right)\right)^T]^T$, $x(t_k h + jh) = [\left(x^1\left(t_k^1 h + j^1 h\right)\right)^T, \ldots, \left(x^m\left(t_k^m h + j^m h\right)\right)^T]^T$.

Remark 2. Different from the traditional control system which receives sampled signal from single sensor, the controller communicates with sensors through distributed sensor networks in this paper. In view of this, the measured signals are sampled by distributed sensors from different geographic space. It is worth noting that whether the sampled data of each sensor is transmitted or not depends on the different event-triggered conditions. The sampled signals will be transmitted to the corresponding quantizer only when the sampled data satisfies the event-triggered condition (9).

2.3. Logarithmic quantizers

As shown in Fig. 1, the logarithmic quantizers are employed to further save the limited resources. The quantizers can be described by $f^p[\cdot](p = 1, 2, \ldots, m)$, and the set of quantized levels can be illustrated [36]

$$\mathbb{Z}^p = \left\{\pm z_s^p : z_s^p = (\rho^p)^s z_0^p, s = 0, \pm 1, \pm 2, \ldots\right\} \cup \left\{\pm z_0^p\right\} \cup \{0\} \tag{10}$$

where ρ^p denotes the quantization density of the pth sensor side and $0 < \rho^p < 1$, $z_0^p > 0$.

The quantizer $f^p[\cdot](p = 1, 2, \ldots, m)$ is defined as follows

$$f^p(x_s^p) = \begin{cases} z_s^p, & x_s^p > 0, \frac{z_s^p}{1+\kappa^p} < x_s^p < \frac{z_s^p}{1-\kappa^p} \\ 0, & x_s^p = 0 \\ -f^p(-x^p), & x_s^p < 0 \end{cases} \tag{11}$$

where $\kappa^p = \frac{1-\rho^p}{1+\rho^p} < 1$.

Because $f^p[\cdot](p = 1, 2, \ldots, m)$ is symmetrical, $f^p(-x_s) = -f^p(x_s)$. The logarithmic quantizer $f^p(\cdot)$ can be described as

$$f^p(x_s^p) = \left(I + \Delta_q^p(x_s^p)\right) x_s^p \tag{12}$$

where $|\Delta_q^p(x_s^p)| \leq \kappa_s^p$. For the sake of simplicity, we use Δ_q^p to represent $\Delta_q^p(x_s^p)$.

For all sensors, it seems naturally to express the real sampled data via quantizer as follow

$$x_q(t) = \big(I + \Delta_q\big)\bar{x}(t) \tag{13}$$

where $\Delta_q = diag\{\Delta^1_{q^1}, \Delta^2_{q^2}, \cdots, \Delta^m_{q^m}\}$, $\Delta^p_{q^p} \in [-\kappa^p, \kappa^p]$, $(p = 1, 2, \ldots, m)$, $\bar{x}(t)$ is the released measurements via distributed event-triggered scheme.

Remark 3. The quantization mechanism aims at further reducing the use of network bandwidth. The existing quantizers can be classified into two types, namely, static quantizers, dynamic quantizers [41]. Specifically, static quantizers are a class of time-invariant quantizers with fixed quantization levels, which include logarithmic quantizers, hysteresis quantizers and uniform quantizers [41]. The quantizer (11) considered in this paper is the logarithmic quantizer whose quantization levels are linear in logarithmic scale. Moreover, all quantizers attached the corresponding event-trigger generator are geographically distributed.

2.4. DoS attacks model

As shown in Fig. 1, the released signals are delivered via the communication network, it may be subject to the jamming signals which are defined as the periodic DoS attacks in this paper. Inspired by [43], the periodic jamming signal can be described as follows

$$\mathbb{S}_{DoS}(t) = \begin{cases} 1, & t \in [nT, nT + L_n) \\ 0, & t \in [nT + L_n, nT + T) \end{cases} \tag{14}$$

where $n \in \mathbb{N}$ denotes the period number, $T \in \mathbb{R}_{>0}$ is action-period of jammer and $L_n \in \mathbb{R}_{>0}$ is the period which the jammer is sleeping. The time interval $[nT, nT + L_n)$ represents the set of time instants where the DoS attacks are off. Moreover, the communication is abnormal in time interval $[nT + L_n, nT + T)$ while the jamming signals are active. Therefore, suppose that there exists a real sclar $L_n^{\min}$ such that $L_n^{\min} \le L_n < T < +\infty$.

By taking the influence of the periodic DoS attacks into consideration, the control signal is intercepted over the active intervals $\cup_{n\in\mathbb{N}}[nT + L_n, nT + T)$ of jamming signals, which cause the distributed event-triggered condition (9) cannot be satisfied. Then, it is necessary to modify the above distributed event-triggered scheme to adapt to the periodic DoS attacks. In view of this, suppose that the jammer is maintaining a “worst-case jamming scenario” [43], namely, $L_n = L_n^{\min}$ for all $n \in \mathbb{N}$.

Then the real input of controller can be expressed as

$$\hat{x}(t) = \mathbb{S}_{DoS}(t)x_q(t) \tag{15}$$

Remark 4. In the presence of the periodic DoS attacks, the attacker’s goal is to destabilize control system by jamming the communication on sensor-to-controller and controller-to-actuator channels. According to (14), under the active interval $[nT + L_n, nT + T)$ of periodic DoS attacks, the measurement signals of each quantizer cannot be successfully sent to the controller, meanwhile the control output cannot be successfully transmitted to the actuator too. In view of this, we consider the case that the periodic jamming signals simultaneously block both sensor-to-controller and controller-to-actuator communication channels.

2.5. Actuator saturation

In this paper, $\bar{u}(t)$ is the control input given by $\bar{u}(t) = sat(u(t))$, and $sat(u(t)) = [sat(u_1(t)), sat(u_2(t)), \cdots, sat(u_m(t))]^T$. the saturation nonlinearity function $sat(\cdot)$ can be defined as follows [46]

$$sat(u_s) \triangleq \begin{cases} \iota_s, & u_s > \iota_s \\ u_s, & -\iota_s \le u_s \le \iota_s, \quad s = 1, 2, \dots, m \\ -\iota_s, & u_s < -\iota_s \end{cases} \tag{16}$$

Then, the saturating input $\bar{u}(t)$ can be represented by a linear $u(t)$ and a nonlinear $\eta(u(t))$, namely

$$\bar{u}(t) = u(t) - \eta(u(t)) \tag{17}$$

where $\eta(u(t)) = [\eta(u_1(t)), \eta(u_2(t)), \dots, \eta(u_m(t))]^T \in \mathbb{R}^m$, and $\eta(u(t))$ denotes the dead-zone nonlinear function.

According to Eq. (16), there exists a restraint coefficient $\varepsilon \in (0, 1)$, such that $\varepsilon \Sigma_{s=1}^{m} u_s(t) \ge \Sigma_{s=1}^{m} \eta_s^2(u(t))$ holds, namely

$$\varepsilon u^T(t)u(t) \ge \eta^T(u(t))\eta(u(t)) \tag{18}$$

By combining Eqs. (5), (13), (15) and (17), the real control input $\bar{u}(t)$ under the DoS attacks Eq. (14) can be expressed as

$$\bar{u}(t) = \begin{cases} \sum_{j=1}^{r} g_j\{K_j(I + \Delta_q)\bar{x}(t) - \eta(u(t))\}, & t \in [nT, nT + L_n) \cap [t_{k,n+1}h, t_{k+1,n+1}h) \\ 0, & t \in [nT + L_n, nT + T) \end{cases} \tag{19}$$

where $t_{k,n}h$ is the set of control update instants $(t_{0,n+1}h = nT)$ in the $(n+1)$th jammer action period, which are generated by the distributed event-triggered scheme to be designed later. To simplify representation, g_j is used to represent $g_j(\vartheta(\hat{x}))$ and ς_i is abbreviated to represent $\varsigma_i(\vartheta(x))$. The released signal under the designed distributed event-triggered scheme can be represented as the set $\bar{x}(t_{k,n+1}h)$, $k \in \{1, 2, \dots, k(n)\} \triangleq \mathbb{D}(n)$ with $n \in \mathbb{N}$ and $k(n) = \sup\{k \in \mathbb{N} | (nT + L_n) \ge t_{k,n+1}h\}$.

Remark 5. According to Eqs. (16) and (18), the restraint coefficient ε is dependent on two factors, one is ι_s and the other is u_s. Similar to [46], when $u_s > \iota_s$, $\eta(u_s(t))^2 = (u_s - \iota_s)^2 \le \varepsilon u_s^2$. That is to say, $\varepsilon \ge \left(1 - (\iota_s/|u_s|^2_{\max})\right)^2$, where ι_s is the maximum allowable range for actuator output and $|u_s|_{\max}$ denotes the maximum amplitude of control input $u(t)$.

2.6. Modeling of networked T-S fuzzy multi-sensors system under the periodic DoS attacks

In this paper, to simplify representation, define $\mathcal{O}_{1,n} = [nT, nT + L_n)$, $\mathcal{O}_{2,n} = [nT + L_n, nT + T)$, $k \le \mathbb{D}(n)$, $\mathcal{C}_{k,n} = [t_{k,n+1}h, t_{k+1,n+1}h)$.

When the periodic DoS attacks are absent, the event-triggering instant can be expressed as follows

$$t_{k,n+1}h = \left\{t_{k_j}h \text{ satisfying (9) } | \ t_{k_j}h \in \mathcal{O}_{1,n}\right\} \cup \{nT + T\} \tag{20}$$

where n, j, $t_{k_j}h$, $k_j \in \mathbb{N}$, k represents the number of triggering times in the $(n+1)$th jammer action period.

Remark 6. According to Eq. (20), it is worth emphasizing that the triggering instant $t_{k,n+1}h$ generated by Eq. (7) in the $(n+1)$th DoS attacks action period lies in either the interval

$\mathcal{O}_{1,n}$ or $(nT+T)$. In particular, if the event-triggered condition is violated during $\mathcal{O}_{1,n}$, we can get $\{t_{k_j}h \text{ satisfying (9) } \mid t_{k_j}h \in \mathcal{O}_{1,n}\} = \varnothing$, then, the triggering instant only occurs at the $(nT+T)$.

Similar to [43], the event intervals $\mathcal{C}_{k,n}(\forall k \in \mathbb{D}(n))$ can be decomposed as

$$\mathcal{C}_{k,n} = \cup_{j=1}^{\lambda_{k,n}}\left[t_{k,n+1}h + (j-1)h, t_{k,n+1}h + jh\right) \cup \left[t_{k,n+1}h + \lambda_{k,n}h, t_{k+1,n+1}h\right) \tag{21}$$

where $\lambda_{k,n} \triangleq \sup\left\{j \in \mathbb{N} | t_{k,n+1}h + jh < t_{k+1,n+1}h\right\}$.

Let

$$\begin{cases} \mathcal{H}_{k,n}^{j} = \left[t_{k,n+1}h + (j-1)h, t_{k,n+1}h + jh\right) \\ \mathcal{H}_{k,n}^{\lambda_{k,n}+1} = \left[t_{k,n+1}h + \lambda_{k,n}h, t_{k+1,n+1}h\right) \end{cases} \tag{22}$$

Note that

$$\mathcal{O}_{1,n} = \cup_{k=0}^{k(n)}\left\{\mathcal{C}_{k,n} \cap \mathcal{O}_{1,n}\right\} \subseteq \cup_{k=0}^{k(n)}\mathcal{C}_{k,n} \tag{23}$$

By combining Eqs. (21)–(23), the interval $\mathcal{O}_{1,n}$ can be expressed as

$$\mathcal{O}_{1,n} = \cup_{k=0}^{k(n)} \cup_{j=1}^{\lambda_{k,n}+1} \Theta_{k,n}^{j} \tag{24}$$

where $\Theta_{k,n}^{j} = \mathcal{H}_{k,n}^{j} \cap \mathcal{O}_{1,n}$.

Therefore, for the pth sensor, two piecewise functions can be expressed as follows

$$\tau_{k,n}^{p}(t) = \begin{cases} t - t_{k,n+1}^{p}h, & t \in \Theta_{k,n}^{1} \\ t - t_{k,n+1}^{p}h - h, & t \in \Theta_{k,n}^{2} \\ \vdots & \\ t - t_{k,n+1}^{p}h - \lambda_{k,n}h, & t \in \Theta_{k,n}^{\lambda_{k,n}+1} \end{cases} \tag{25}$$

and

$$e_{k,n}^{p}(t) = \begin{cases} 0, & t \in \Theta_{k,n}^{1} \\ x^{p}\left(t_{k,n+1}^{p}h\right) - x^{p}\left(t_{k,n+1}^{p}h + h\right), & t \in \Theta_{k,n}^{2} \\ \vdots & \\ x^{p}\left(t_{k,n+1}^{p}h\right) - x^{p}\left(t_{k,n+1}^{p}h + \lambda_{k,n}h\right), & t \in \Theta_{k,n}^{\lambda_{k,n}+1} \end{cases} \tag{26}$$

where $\tau_{k,n}(t) \in [0,\ h)$, $t \in \mathcal{C}_{k,n} \cap \mathcal{O}_{1,n}$.

According to the inequality (9) and the definition of $\tau_{k,n}^{p}(t)$ and $e_{k,n}^{p}(t)$, for all sensors, the distributed event-triggered condition is given as

$$e_{k,n}^{T}(t)\Omega e_{k,n}(t) > \sigma^{2}x^{T}\left(t - \tau_{k,n}(t)\right)\Omega x\left(t - \tau_{k,n}(t)\right) \tag{27}$$

where $e_{k,n}(t) = [(e_{k,n}^{1}(t))^{T}, \ldots, (e_{k,n}^{m}(t))^{T}]^{T}$, $x(t - \tau_{k,n}(t)) = [(x^{1}(t - \tau_{k,n}^{1}(t)))^{T}, \ldots, (x^{m}(t - \tau_{k,n}^{m}(t)))^{T}]^{T}$.

Then the successfully event-triggered sampled state $\bar{x}(t)$ can be described as

$$\bar{x}(t) = x\big(t - \tau_{k,n}(t)\big) + e_{k,n}(t), t \in \mathcal{C}_{k,n} \cap \mathcal{O}_{k,n} \tag{28}$$

By combining Eqs. (2), (28) and (19), the T-S fuzzy multi-sensors system can be expressed as

$$\begin{cases} \dot{x}(t) = \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} \varsigma_i g_j \big\{A_i x(t) + \big(I + \Delta_q\big)B_i K_j\big[x(t - \tau_{k,n}(t)) + e_{k,n}(t)\big] - B_i\eta(u(t))\big\}, \\ \qquad t \in \mathcal{O}_{1,n} \cap \mathcal{C}_{k,n} \\ \sum_{i=1}^{r} \varsigma_i\{A_i x(t)\}, \qquad t \in \mathcal{O}_{2,n} k \in \mathbb{D}(n) \end{cases} \\ x(t) = \phi(t), t \in [-h, 0] \end{cases} \tag{29}$$

where $\phi(t)$ denotes the supplemented initial condition of the state $x(t)$ on $[-h, 0]$.

Remark 7. By taking the impacts of quantization, actuator saturation and DoS jamming attacks into consideration, sufficient conditions are derived to co-design both security controller and distributed event-triggered parameters for T-S fuzzy system. To the best of our knowledge, the multi-sensors-based security control problem for T-S fuzzy system with quantization, DoS attacks and actuator saturation is firstly investigated in this paper.

To proceed with, a definition and a lemma are introduced, which are needed in the proof of main results.

Definition 1 [47]**.** The zero solution of Eq. (29) is said to be globally exponentially stable (GES), if there exist two scalars $a > 0$ and $b > 0$, such that for any solution $x(t)$ with the initial functions ϕ, one has $a\|\phi\|^2 e^{-bt} \geq \|x(t)\|^2$, $\forall t \geq 0$.

Lemma 1 [48]**.** *For any vectors $x, y \in \mathbb{R}^n$, and positive definite matrix $Q \in \mathbb{R}^{n\times n}$, the following inequality holds:*

$$2x^T y \leq x^T Qx + y^T Q^{-1} y. \tag{30}$$

3. Main results

In this paper, sufficient conditions are presented to ensure GES of the T-S fuzzy system (29) under the periodic DoS attacks. The research results are stated as follows.

Theorem 1. *For given positive parameter ε, matrix K_j and a jamming signal $\mathbb{S}_{DoS}(t)$ with the known parameters T and L_n. If for some known scalars α_s, μ_s, σ, χ_i^s and h, there exist matrices $P_s > 0$, $Q_s > 0$, $R_s > 0$, $Z_s > 0$, $\Omega > 0$, $N_i^s > 0$ and matrices M_s, W_s, S_s, $s \in \{1, 2\}$ with appropriate dimensions, such that the following inequalities hold with $g_j - \chi_j\varsigma_j \geq 0$ for all i, j=1, 2,..., r.*

$$\Gamma_{ij}^s - N_i^s < 0 \tag{31}$$

$$\chi_i^s\Gamma_{ii}^s - \chi_i^s N_i^s + N_i^s < 0 \tag{32}$$

$$\chi_i^s\Gamma_{ij}^s + \chi_j^s\Gamma_{ji}^s - \chi_i^s N_j^s - \chi_j^s N_i^s + N_i^s + N_j^s < 0(i < j) \tag{33}$$

$$Q_s \leq \mu_{3-s}Q_{3-s}, R_s \leq \mu_{3-s}R_{3-s}\, Z_s \leq \mu_{3-s}Z_{3-s} \tag{34}$$

$$P_1 \leq \mu_2 P_2, P_2 \leq \mu_1 e^{2(\alpha_1+\alpha_2)h} P_1 \tag{35}$$

$$0 < \rho = 2\alpha_1 L_n - 2\alpha_2(T - L_n) - 2(\alpha_1+\alpha_2)h - \ln(\mu_1\mu_2) \tag{36}$$

where

$$\Gamma_{ij}^s = \begin{bmatrix} \check{\Psi}_{11}^s & * & * \\ \Psi_{21}^s & \Psi_{22}^s & * \\ \Psi_{31}^s & 0 & \Psi_{33}^s \end{bmatrix}, \check{\Psi}_{11}^s = \check{\Psi}_{11}^s + \Pi_s + \Pi_s^T$$

$$\check{\Psi}_{11}^1 = \begin{bmatrix} \Phi_{11}^1 & * & * & * & * \\ \Phi_{21}^1 & \sigma^2\Omega & * & * & * \\ 0 & 0 & \Phi_{33}^1 & * & * \\ \Phi_{41}^1 & 0 & 0 & -\Omega & * \\ -B_i^T P & 0 & 0 & 0 & -I \end{bmatrix}, \check{\Psi}_{11}^2 = \begin{bmatrix} \Phi_{11}^2 & * & * \\ 0 & 0 & * \\ 0 & 0 & \Phi_{33}^2 \end{bmatrix}$$

$$\Phi_{11}^1 = 2\alpha_1 P_1 + P_1 A_i + A_i^T P_1 + Q_1, \Phi_{21}^1 = \Phi_{41}^1 = (I+\Delta_q)K_j^T B_i^T P_1, \Phi_{33}^1 = -e^{-2\alpha_1 h} Q_1$$

$$\Psi_{21}^1 = \sqrt{h}\begin{bmatrix} W_{ij1}^T & 0 \\ S_{ij1}^T & 0 \\ M_{ij1}^T & 0 \end{bmatrix}, \Psi_{21}^2 = \sqrt{h}\begin{bmatrix} W_{ij2}^T \\ S_{ij2}^T \\ M_{ij2}^T \end{bmatrix}$$

$$\Psi_{31}^1 = \begin{bmatrix} \sqrt{h}P_1A_i & \sqrt{h}(I+\Delta_q)P_1B_iK_j & 0 & \sqrt{h}(I+\Delta_q)P_1B_iK_j & -P_1B_i \\ \sqrt{h}P_1A_i & \sqrt{h}(I+\Delta_q)P_1B_iK_j & 0 & \sqrt{h}(I+\Delta_q)P_1B_iK_j & -P_1B_i \\ 0 & \sqrt{\varepsilon}(I+\Delta_q)K_j & 0 & \sqrt{\varepsilon}(I+\Delta_q)K_j & 0 \end{bmatrix}$$

$$\Psi_{22}^1 = -e^{-2\alpha_1 h} diag\{R_1, R_1, Z_1\}, \Psi_{33}^1 = diag\{-P_1R_1^{-1}P_1, -P_1Z_1^{-1}P_1, -I\}$$

$$\Phi_{11}^2 = -2\alpha_2 P_2 + P_2A_i + A_i^T P_2 + Q_2, \Phi_{33}^2 = -e^{2\alpha_2 h}Q_2, \Psi_{22}^2 = diag\{-R_2, -R_2, -Z_2\}$$

$$\Psi_{31}^2 = \sqrt{h}\begin{bmatrix} P_2A_i & 0 & 0 \\ P_2A_i & 0 & 0 \end{bmatrix}, \Psi_{33}^2 = diag\{-P_2R_2^{-1}P_2, -P_2Z_2^{-1}P_2\}$$

$$\Pi_1 = \begin{bmatrix} M_{ij1} + W_{ij1} & -W_{ij1} + S_{ij1} & -M_{ij1} - S_{ij1} & 0 \end{bmatrix}$$

$$\Pi_2 = \begin{bmatrix} M_{ij2} + W_{ij2} & -W_{ij2} + S_{ij2} & -M_{ij2} - S_{ij2} \end{bmatrix}$$

$$M_{ij1} = \begin{bmatrix} M_{ij11}^T & M_{ij12}^T & M_{ij13}^T & M_{ij14}^T \end{bmatrix}^T, M_{ij2} = \begin{bmatrix} M_{ij21}^T & M_{ij22}^T & M_{ij23}^T \end{bmatrix}^T$$

$$W_{ij1} = \begin{bmatrix} W_{ij11}^T & W_{ij12}^T & W_{ij13}^T & W_{ij14}^T \end{bmatrix}^T, W_{ij2} = \begin{bmatrix} W_{ij21}^T & W_{ij22}^T & W_{ij23}^T \end{bmatrix}^T$$

$$S_{ij1} = \begin{bmatrix} S_{ij11}^T & S_{ij12}^T & S_{ij13}^T & S_{ij14}^T \end{bmatrix}^T, S_{ij2} = \begin{bmatrix} S_{ij21}^T & S_{ij22}^T & S_{ij23}^T \end{bmatrix}^T$$

then the T-S fuzzy system (29) with DoS attacks are GES with decay rate $\epsilon \triangleq \frac{\rho}{2T}$.

Proof. Choose the Lyapunov-Krasovskii functional as follows

$$V_{\beta(t)} = x^T(t)P_{\beta(t)}x(t) + \int_{t-h}^{t} x^T(s)\mathrm{e}(\cdot)Q_{\beta(t)}x(s)ds + \int_{-h}^{0}\int_{t+v}^{t} \dot{x}^T(s)\mathrm{e}(\cdot)R_{\beta(t)}\dot{x}(s)dsdv$$
$$+ \int_{-h}^{0}\int_{t+v}^{t} \dot{x}^T(s)\mathrm{e}(\cdot)Z_{\beta(t)}\dot{x}(s)dsdv \tag{37}$$

where $Q_{\beta(t)} > 0$, $Z_{\beta(t)} > 0$, $P_{\beta(t)} > 0$, $\alpha_{\beta(t)} > 0$, $\mathrm{e}(\cdot) \triangleq e^{2(-1)^{\beta(t)}\alpha_{\beta(t)}(t-s)}$, and the variable $\beta(t)$ is defined as follows

$$\beta(t) = \begin{cases} 1, & t \in [-h, 0] \cup \left(\cup_{n\in\mathbb{N}}\mathcal{O}_{1,n}\right) \\ 2, & t \in \cup_{n\in\mathbb{N}}\mathcal{O}_{2,n} \end{cases}$$

When $\beta(t) = 1$, by taking the derivative of $V_1(t)$ for $t \in \mathcal{O}_{1,n} \cap \mathcal{C}_{1,n}$, it yields that

$$\dot{V}_1(t) \le -2\alpha_1 V_1(t) + 2\alpha_1 x^T(t)P_1x(t) + 2x^T(t)P_1\dot{x}(t) + x^T(t)Q_1x(t) + h\dot{x}^T(t)(R_1 + Z_1)\dot{x}(t)$$
$$- x^T(t-h)e^{-2\alpha_1 h}Q_1x(t-h) + \int_{t-h}^{t} \dot{x}^T(s)e^{-2\alpha_1 h}Z_1\dot{x}(s)ds$$
$$+ \int_{t-\tau_{k,n}(t)}^{t} \dot{x}^T(s)e^{-2\alpha_1 h}R_1\dot{x}(s)ds + \int_{t-h}^{t-\tau_{k,n}(t)} \dot{x}^T(s)e^{-2\alpha_1 h}R_1\dot{x}(s)ds \tag{38}$$

in which

$$h\dot{x}^T(t)(R_1 + Z_1)\dot{x}(t) = h\mathcal{A}_0^T(R_1 + Z_1)\mathcal{A}_0 + h\mathcal{A}_1^T(R_1 + Z_1)\mathcal{A}_1 \tag{39}$$

where $\mathcal{A}_0 = A_ix(t)$, $\mathcal{A}_1 = \left(I + \Delta_q\right)B_i\left[\left(x(t - \tau_{k,n}(t)\right) + e_{k,n}(t)\right] - B_i\eta(u(t))$.

By employing the free-weighting matrices method [49], one can get

$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \varsigma_i g_j \xi_1^T(t)M_{ij1}\left[x(t) - x(t-h) - \int_{t-h}^{t}\dot{x}(s)ds\right] = 0,$$
$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \varsigma_i g_j \xi_1^T(t)W_{ij1}\left[x(t) - x(t-\tau_{k,n}(t)) - \int_{t-\tau_{k,n}(t)}^{t}\dot{x}(s)ds\right] = 0,$$
$$2\sum_{i=1}^{r}\sum_{j=1}^{r} \varsigma_i g_j \xi_1^T(t)S_{ij1}\left[x(t-\tau_{k,n}(t)) - x(t-h) - \int_{t-h}^{t-\tau_{k,n}(t)}\dot{x}(s)ds\right] = 0. \tag{40}$$

where M_{ij}, S_{ij}, W_{ij} are matrices with appropriate dimensions, $\xi^T(t) = \left[\xi_1^T(t) \quad I \quad I \quad I \quad I \quad I \quad I\right]$, $\xi_1^T(t) = \left[\xi_2^T(t) \quad e_{k,n}^T(t) \quad \eta^T(t)\right]$, $\xi_2^T(t) = \left[x^T(t) \quad x^T\left(t - \tau_{k,n}(t)\right) \quad x^T(t-h)\right]$.

According to Lemma 1 and apply (40), the following inequalities are obtained

$$-2\sum_{i=1}^{r}\sum_{j=1}^{r} \varsigma_i g_j \xi_1^T(t)M_{ij1}\int_{t-h}^{t}\dot{x}(s)ds \le \sum_{i=1}^{r}\sum_{j=1}^{r} \varsigma_i g_j h e^{2\alpha_1 h}\xi_1^T(t)M_{ij1}Z_1^{-1}M_{ij1}^T\xi_1(t)$$
$$+ \int_{t-h}^{t}\dot{x}^T(s)e^{-2\alpha_1 h}Z_1\dot{x}(s)ds$$

$$-2\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j \xi_1^T(t)W_{ij1}\int_{t-\tau_{k,n}(t)}^{t}\dot{x}(s)ds \le \sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j h e^{2\alpha_1 h}\xi_1^T(t)W_{ij1}R_1^{-1}W_{ij1}^T\xi_1(t) + \int_{t-\tau_{k,n}(t)}^{t}\dot{x}^T(s)e^{-2\alpha_1 h}R_1\dot{x}(s)ds$$

$$-2\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j \xi_1^T(t)S_{ij1}\int_{t-h}^{t-\tau_{k,n}(t)}\dot{x}(s)ds \le \sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j h e^{2\alpha_1 h}\xi_1^T(t)S_{ij1}R_1^{-1}S_{ij1}^T\xi_1(t) + \int_{t-h}^{t-\tau_{k,n}(t)}\dot{x}^T(s)e^{-2\alpha_1 h}R_1\dot{x}(s)ds \tag{41}$$

According to the event-triggered condition (27), the following inequality is derived

$$e_{k,n}^T(t)\Omega e_{k,n}(t) > \sigma^2 x^T\big(t-\tau_{k,n}(t)\big)\Omega x\big(t-\tau_{k,n}(t)\big) \tag{42}$$

By recalling the constraint of actuator saturation (18), one can get that

$$\varepsilon u^T(t)u(t) - \eta^T(u(t))\eta(u(t)) \ge 0. \tag{43}$$

From Eq. (19), the following equality can be obtained

$$\varepsilon u^T(t)u(t) = \sum_{j=1}^{r} g_i\big(\varepsilon \mathcal{B}_1^T K_j^T K_j \mathcal{B}_1\big) \tag{44}$$

where $\mathcal{B}_1 = (I+\Delta_q)\big[x(t-\tau_{k,n}(t)) + e_{k,n}(t)\big]$.

By combining Eqs. (38)–(44), the inequality (38) can be rewritten as

$$\dot{V}_1(t) \le -2\alpha_1 V_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j \xi_1^T(t)\Big[\check{\Psi}_{11}^1 + he^{2\alpha_1 h}M_{ij1}Z_1^{-1}M_{ij1}^T + he^{2\alpha_1 h}W_{ij1}R_1^{-1}W_{ij1}^T + he^{2\alpha_1 h}S_{ij1}R_1^{-1}S_{ij1}^T + h\mathcal{A}_1^T(R_1+Z_1)\mathcal{A}_1 + h\mathcal{A}_0^T(R_1+Z_1)\mathcal{A}_0 + \varepsilon \mathcal{B}_1^T K_j^T K_j \mathcal{B}_1\Big]\xi_1(t) \tag{45}$$

By using Schur complement, one can get that

$$\dot{V}_1(t) \le -2\alpha_1 V_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j\big[\xi^T(t)\Gamma_{ij}^1\xi(t)\big] \tag{46}$$

Similar to [50], a slack matrix N_i^1 is introduced

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i(\varsigma_j - g_j)N_i^1 = \sum_{i=1}^{r}\varsigma_i\left(\sum_{j=1}^{r}\varsigma_j - \sum_{j=1}^{r}g_j\right)N_i^1 = \sum_{i=1}^{r}\varsigma_i(1--1)N_i^1 = 0$$

where $N_i^1 = (N_i^1)^T \in \mathbb{R}^{n\times n} > 0$, $i = 1, 2, \ldots, r$, are arbitrary matrices. Then, the inequality

(45) can be rewritten as

$$\begin{aligned}\dot{V}_1(t) &\le \sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j\left[\xi^T(t)\Gamma_{ij}^1\xi(t)\right]-2\alpha_1 V_1(t)\\ &=\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i\left(\varsigma_j-g_j+\chi_j\varsigma_j-\chi_j\varsigma_j\right)\xi^T(t)N_i^1\xi(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j\left[\xi^T(t)\Gamma_{ij}^1\xi(t)\right]-2\alpha_1 V_1(t)\\ &=\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i\varsigma_j\xi^T(t)\left(\chi\Gamma_{ij}^1-\chi_j N_i^1+N_i^1\right)\xi(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i\left(g_j-\chi_j\varsigma_j\right)\xi^T(t)\left(\Gamma_{ij}^1-N_i^1\right)\xi(t)-2\alpha_1 V_1(t)\end{aligned} \tag{47}$$

From Eq. (47), one can get

$$\dot{V}_1(t)\le -2\alpha_1 V_1(t)+\sum_{i=1}^{r}\varsigma_i^2\xi^T(t)\left(\chi_i\Gamma_{ii}^1-\chi_i N_i^1+N_i^1\right)\xi(t)+\sum_{i=1}^{r}\varsigma_i\left(g_j-\chi_j\varsigma_j\right)\xi^T(t)$$
$$\left(\Gamma_{ij}^1-N_i^1\right)\xi(t)+\sum_{i=1}^{r}\sum_{i<j}\xi^T(t)\left(\chi_j\Gamma_{ij}^1+\chi_i\Gamma_{ji}^1-\chi_j N_i^1-\chi_i N_j^1+N_i^1+N_j^1\right)\xi(t) \tag{48}$$

According to the inequalities (31)–(33) and (48), one can obtain

$$\dot{V}_1(t)\le -2\alpha_1 V_1(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j\left[\xi^T(t)\Gamma_{ij}^1\xi(t)\right]<0 \tag{49}$$

with $g_j-\chi_j\varsigma_j\ge 0$ for all j.

When $\beta(t)=2$, by taking derivation on $V_2(t)$ for $t\in\left[t_{2,n},t_{1,n+1}\right)$, we can get

$$\dot{V}_2(t)\le 2\alpha_2 V_2(t)+\sum_{i=1}^{r}\sum_{j=1}^{r}\varsigma_i g_j\xi_2^T(t)\left[\check{\Psi}_{11}^2+h\mathcal{A}_0^T(R_2+Z_2)\mathcal{A}_0\right]\xi_2(t) \tag{50}$$

By using the Schur complement and apply Eqs. (31)–(33), we deduce that $\dot{V}_2(t)\le 2\alpha_2 V_2(t)$.

Next, construct a Lyapunov functional candidate as: $V(t)=V_{\beta(t)}$, $\beta(t)\in\{1,2\}$. From Theorem 1, the following inequalities can be obtained

$$V(t)\le\begin{cases}e^{-2\alpha_1(t-t_{1,n})}V_1\left(t_{1,n}\right), t\in\left[t_{1,n},t_{2,n}\right)\\ e^{2\alpha_2(t-t_{2,n})}V_2\left(t_{2,n}\right), t\in\left[t_{2,n},t_{1,n+1}\right)\end{cases} \tag{51}$$

where $t_{s,n}=\begin{cases}nT, & s=1\\ nT+L_n, & s=2\end{cases}$.

Similar to [43], from Eqs. (34)–(36), it can be obtained that

$$\mu_2 V_2\left(t_{1,n}^-\right)-V_1\left(t_{1,n}\right)\ge 0,\ \mu_1 e^{2(\alpha_1+\alpha_2)h}V_2\left(t_{2,n}^-\right)-V_2\left(t_{2,n}\right)\ge 0 \tag{52}$$

Consider that for any given $t\ge 0$, there exists a non-negative integer $n\in\mathbb{N}$ such that $t\in[t_{1,n}, t_{2,n})$ or $t\in\left[t_{2,n},t_{1,n+1}\right)$. For $t\in[t_{1,n}, t_{2,n})$, it follows from Eqs. (51) and (52) that

$$V_1(t)\le\mu_2 e^{-2\alpha_1\left(t-t_{1,n}^-\right)}V_2\left(t_{1,n}^-\right)\le(\mu_1\mu_2)^n e^{d+2(\alpha_1+\alpha_2)n}V_1\left(t_{1,0}\right) \tag{53}$$

where $d = -2\alpha_1[(t - t_{1,n}) + (t_{2,n-1} - t_{1,n-1}) + \cdots + (t_{1,0} - t_{2,0})] + 2\alpha_2[(t_{1,n} - t_{2,n-1}) + (t_{1,n-1} - t_{2,n-2}) + \cdots + (t_{1,1} - t_{2,0})]$.

Notice that $nT = t_{1,n} \le t \le t_{2,n} = \left(nT + L_n^{\min}\right)$, namely, $n > \frac{t - L_n^{\min}}{T}$. By combining (52) and (53), we have

$$V(t) \le e^{\frac{\rho L_n}{T}t} e^{-\frac{\rho}{T}t} V_1(0) \tag{54}$$

Similarly, for $t \in \left[t_{2,n}, t_{2,n+1}\right)$, Eq. (54) can be rewritten as

$$V(t) \le e^{-\frac{\rho}{T}t} \frac{V_1(0)}{\mu_2} \tag{55}$$

Then, define $\mathcal{M}_0 = \max\left\{e^{\left[\frac{\rho L_n}{T}/T\right]}, \frac{1}{\mu_2}\right\}$, $\mathcal{M}_1 = \min\{\lambda_{\min} P_s\}$, $\mathcal{M}_3 = \mathcal{M}_1 + h\lambda_{\min}(Q_s) + \frac{h^2}{2}\lambda_{\min}(R_s + Z_s)$, from Eqs. (54) and (55), one can get

$$V(t) \le \mathcal{M}_0 e^{-\frac{\rho}{T}t} V_1(0) \tag{56}$$

It can be derived from the definition of $V(t)$ that

$$V(t) \le \mathcal{M}_1 \parallel x(t) \parallel^2, V_1(t) \le \mathcal{M}_3 \|\phi\|_h^2 \tag{57}$$

By combining (56) and (57), one can obtain

$$\parallel x(t) \parallel \le \sqrt{\frac{\mathcal{M}_0 \mathcal{M}_3}{\mathcal{M}_1}} \mathcal{M}_0 e^{-\epsilon t} \|\phi\|_h, \forall t \ge 0 \tag{58}$$

Based on the Definition 1 and Eq. (58), we can derive that the T-S fuzzy system (29) are GES with decay rate ϵ if Eqs. (31)–(36) hold.

This completes the proof. □

In Theorem 1, sufficient conditions which can guarantee the GES of the T-S fuzzy system (29) have been obtained. However, due to the existence of nonlinear terms such like $(I + \triangle q)B_i K_j$ in Theorem 1, it cannot be solved through LMIs. Furthermore, the nonlinear terms will be treated in the following section and the desired controller gains will be given.

Theorem 2. *For given a jamming signal $\mathbb{S}_{DoS}(t)$ with known parameters T and L_n, positive parameters ε, α_s, μ_s, h, o_s, ϱ_s, ψ_{1s}, ψ_{2s}, ψ_{3s}, trigger parameter σ, quantized parameter δ, the T-S fuzzy system are GES if there exist positive matrices $X_s > 0$, $\tilde{P}_s > 0$, $\tilde{Q}_s > 0$, $\tilde{R}_s > 0$, $\tilde{Z}_s > 0$, $\bar{N}_i^s > 0 (s = 1, 2)$, Ω, Y_j and matrices $\bar{M}_s$, $\bar{W}_s$, $\bar{S}_s (s = 1, 2)$ with appropriate dimensions, then the following LMIs hold with $g_j - \chi_j \varsigma_j \ge 0$ for all $i, j = 1, 2, \ldots, r$.*

$$\bar{\Gamma}_{ij}^s - \bar{N}_i^s < 0 \tag{59}$$

$$\chi_i^s \bar{\Gamma}_{ii}^s - \chi_i^s \bar{N}_i^s + \bar{N}_i^s < 0 \tag{60}$$

$$\chi_i^s \bar{\Gamma}_{ij}^s + \chi_j^s \bar{\Gamma}_{ji}^s - \chi_i^s \bar{N}_j^s - \chi_j^s \bar{N}_i^s + \bar{N}_i^s + \bar{N}_j^s < 0 (i < j) \tag{61}$$

$$\begin{bmatrix} -\mu_2 X_2 & * \\ X_2 & -X_1 \end{bmatrix} \le 0, \begin{bmatrix} -\mu_1 e^{2(\alpha_1 + \alpha_2)h} X_1 & * \\ X_1 & -X_2 \end{bmatrix} \le 0 \tag{62}$$

$$\begin{bmatrix} -\mu_{3-s}\tilde{Q}_s & * \\ X_{3-s} & \psi_{1s}^2\tilde{Q}_s - 2\psi_{1s}X_s \end{bmatrix} \le 0, \begin{bmatrix} -\mu_{3-s}\tilde{R}_s & * \\ X_{3-s} & \psi_{2s}^2\tilde{R}_s - 2\psi_{2s}X_s \end{bmatrix} \le 0,$$
$$\begin{bmatrix} -\mu_{3-s}\tilde{Z}_s & * \\ X_{3-s} & \psi_{3s}^2\tilde{Z}_s - 2\psi_{3s}X_s \end{bmatrix} \le 0 \quad (63)$$

where

$$\bar{\Gamma}_{ij}^1 = \begin{bmatrix} \bar{\tilde{\Psi}}_{11}^1 & * & * & * & * \\ \bar{\Psi}_{21}^1 & \bar{\Psi}_{22}^1 & * & * & * \\ \bar{\Psi}_{31}^1 & 0 & \bar{\Psi}_{33}^1 & * & * \\ \bar{\Psi}_{41}^1 & 0 & 0 & -I & * \\ \bar{\Psi}_{51}^1 & 0 & \bar{\Xi}_1 & \bar{\Xi}_2 & \bar{\Psi}_{55}^1 \end{bmatrix}, \bar{\tilde{\Psi}}_{11}^1 = \bar{\Psi}_{11}^1 + \bar{\Pi}_1 + \bar{\Pi}_1^T$$

$$\bar{\Psi}_{11}^1 = \begin{bmatrix} \bar{\Phi}_{11}^1 & * & * & * & * \\ Y_j^T B_i^T & \sigma^2\bar{\Omega} & * & * & * \\ 0 & 0 & -e^{-2\alpha_1 h}\tilde{Q}_1 & * & * \\ Y_j^T B_i^T & 0 & 0 & -\bar{\Omega} & * \\ -B_i^T & 0 & 0 & 0 & -I \end{bmatrix}, \bar{\Phi}_{11}^1 = A_iX_1 + X_1A_i^T + 2\alpha_1X_1 + \tilde{Q}_1$$

$$\bar{\Psi}_{22}^1 = diag\{-e^{-2\alpha_1 h}\tilde{R}_1, -e^{-2\alpha_1 h}\tilde{R}_1, -e^{-2\alpha_1 h}\tilde{Z}_1\}, \bar{\Psi}_{21}^1 = \sqrt{h}\begin{bmatrix} \bar{W}_{ij1}^T & 0 \\ \bar{S}_{ij1}^T & 0 \\ \bar{M}_{ij1}^T & 0 \end{bmatrix}$$

$$\bar{\Psi}_{31}^1 = \begin{bmatrix} \sqrt{h}A_iX_1 & \sqrt{h}B_iY_j & 0 & \sqrt{h}B_iY_j & -B_i \\ \sqrt{h}A_iX_1 & \sqrt{h}B_iY_j & 0 & \sqrt{h}B_iY_j & -B_i \end{bmatrix}, \bar{\Psi}_{33}^1 = diag\{-2\varrho_1X_1 + \varrho_1^2R_1, -2\varrho_2X_1 + \varrho_2^2Z_1\}$$

$$\bar{\Psi}_{41}^1 = \sqrt{\varepsilon}\begin{bmatrix} 0 & Y_j & 0 & Y_j & 0 \end{bmatrix}, \bar{\Psi}_{51}^1 = \begin{bmatrix} \sqrt{m_1}\delta B_i^T & 0 & 0 & 0 & 0 \\ 0 & Y_j & 0 & 0 & Y_j \end{bmatrix}$$

$$\bar{\Xi}_1 = \begin{bmatrix} \delta\sqrt{hm_1}B_i^T & \delta\sqrt{hm_1}B_i^T \\ 0 & 0 \end{bmatrix}, \bar{\Xi}_2 = \begin{bmatrix} \delta\sqrt{m_1\varepsilon} \\ 0 \end{bmatrix}, \bar{\Psi}_{55}^1 = \begin{bmatrix} -I & * \\ 0 & -m_1I \end{bmatrix}$$

$$\bar{\Gamma}_{ij}^2 = \begin{bmatrix} \bar{\tilde{\Psi}}_{11}^2 & * & * \\ \bar{\Psi}_{21}^2 & \bar{\Psi}_{22}^2 & * \\ \bar{\Psi}_{31}^2 & 0 & \bar{\Psi}_{33}^2 \end{bmatrix}, \bar{\tilde{\Psi}}_{11}^2 = \bar{\Psi}_{11}^2 + \bar{\Pi}_2 + \bar{\Pi}_2^T, \bar{\Psi}_{11}^2 = \begin{bmatrix} \bar{\Phi}_{11}^2 & * & * \\ 0 & 0 & * \\ 0 & 0 & -e^{2\alpha_2 h}\tilde{Q}_2 \end{bmatrix}$$

$$\bar{\Phi}_{11}^2 = A_iX_2 + X_2A_i^T - 2\alpha_2X_2 + \tilde{Q}_2, \bar{\Psi}_{22}^2 = diag\{-e^{2\alpha_2 h}\tilde{R}_2, -e^{2\alpha_2 h}\tilde{R}_2, -e^{2\alpha_2 h}\tilde{Z}_2\}$$

$$\bar{\Psi}_{21}^2 = \sqrt{h}\begin{bmatrix} \bar{W}_{ij2}^T & 0 \\ \bar{S}_{ij2}^T & 0 \\ \bar{M}_{ij2}^T & 0 \end{bmatrix}, \bar{\Psi}_{31}^2 = \sqrt{h}\begin{bmatrix} A_iX_2 & 0 & 0 \\ A_iX_2 & 0 & 0 \end{bmatrix}, \bar{\Psi}_{33}^2 = \begin{bmatrix} -2o_1X_2 + o_1^2R_2 & * \\ 0 & -2o_2X_2 + o_2^2Z_2 \end{bmatrix}$$

$$\bar{\Pi}_1 = \begin{bmatrix} \bar{M}_{ij1} + \bar{W}_{ij1} & -\bar{W}_{ij1} + \bar{S}_{ij1} & -\bar{M}_{ij1} - \bar{S}_{ij1} & 0 \end{bmatrix}$$

$$\bar{\Pi}_2 = \begin{bmatrix} \bar{M}_{ij2} + \bar{W}_{ij2} & -\bar{W}_{ij2} + \bar{S}_{ij2} & -\bar{M}_{ij2} - \bar{S}_{ij2} \end{bmatrix}$$

$$\bar{M}_{ij1} = \begin{bmatrix} \bar{M}_{ij11}^T & \bar{M}_{ij12}^T & \bar{M}_{ij13}^T & \bar{M}_{ij14}^T \end{bmatrix}^T, \bar{M}_{ij2} = \begin{bmatrix} \bar{M}_{ij21}^T & \bar{M}_{ij22}^T & \bar{M}_{ij23}^T \end{bmatrix}^T$$

$$\bar{W}_{ij1} = \begin{bmatrix} \bar{W}_{ij11}^T & \bar{W}_{ij12}^T & \bar{W}_{ij13}^T & \bar{W}_{ij14}^T \end{bmatrix}^T, \bar{W}_{ij2} = \begin{bmatrix} \bar{W}_{ij21}^T & \bar{W}_{ij22}^T & \bar{W}_{ij23}^T \end{bmatrix}^T$$

$$\bar{S}_{ij1} = \begin{bmatrix} \bar{S}_{ij11}^T & \bar{S}_{ij12}^T & \bar{S}_{ij13}^T & \bar{S}_{ij14}^T \end{bmatrix}^T, \bar{S}_{ij2} = \begin{bmatrix} \bar{S}_{ij21}^T & \bar{S}_{ij22}^T & \bar{S}_{ij23}^T \end{bmatrix}^T$$

Moreover, based on the feasible conditions above, the controller gains for T-S fuzzy system can be given by

$$K_j = Y_j X_1^{-1} \tag{64}$$

Proof. Due to the existence of quantization, the matrix Γ_{ij}^s in Eq. (31) with $i = 1$ can be rewritten as the following form

$$\Gamma_{ij}^1 = \Lambda_{ij} + sym\{Q_C^T \triangle q Q_J\}. \tag{65}$$

where

$$\Lambda_{ij} = \begin{bmatrix} \tilde{\tilde{\Psi}}_{11}^1 & * & * & * & * & * \\ \tilde{\Psi}_{ij}^{21} & \tilde{\Psi}_{ij}^{22} & * & * & * & * \\ \tilde{\Psi}_{ij}^{31} & 0 & \tilde{\Psi}_{ij}^{33} & * & * & * \\ \tilde{\Psi}_{ij}^{41} & 0 & 0 & -I & * & * \\ \tilde{\Psi}_{ij}^{51} & 0 & 0 & 0 & -I & * \\ \tilde{\Psi}_{ij}^{61} & 0 & 0 & 0 & 0 & -I \end{bmatrix}, \tilde{\tilde{\Psi}}_{11}^1 = \tilde{\Psi}_{11}^1 + \Pi_1 + \Pi_1^T$$

$$\tilde{\Psi}_{11}^1 = \begin{bmatrix} \tilde{\Phi}_{11}^1 & * & * & * & * \\ K_j^T B_i^T P_1 & \sigma^2 \Omega & * & * & * \\ 0 & 0 & -e^{-2\alpha_1 h} Q_1 & * & * \\ K_j^T B_i^T P_1 & 0 & 0 & -\Omega & * \\ -B_i^T P & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\tilde{\Phi}_{11}^1 = 2\alpha_1 P_1 + P_1 A_i + A_i^T P_1 + Q_1, \tilde{\Psi}_{22}^1 = -e^{-2\alpha_1 h} diag\{R_1, R_1, Z_1\}$$

$$\tilde{\Psi}_{21}^1 = \sqrt{h} \begin{bmatrix} W_{ij1}^T & 0 \\ S_{ij1}^T & 0 \\ M_{ij1}^T & 0 \end{bmatrix}, \tilde{\Psi}_{31}^1 = \begin{bmatrix} \sqrt{h} P_1 A_i & \bar{\Xi}_1 & 0 & \bar{\Xi}_1 & -P_1 B_i \\ \sqrt{h} P_1 A_i & \bar{\Xi}_1 & 0 & \bar{\Xi}_1 & -P_1 B_i \\ 0 & \sqrt{\varepsilon} K_j & 0 & \sqrt{\varepsilon} K_j & 0 \end{bmatrix}$$

$$\tilde{\Psi}_{33}^1 = diag\{-P_1 R_1^{-1} P_1, -P_1 Z_1^{-1} P_1, -I\}, Q_C = \begin{bmatrix} P_1 B_i & 0_{0\times 7} & \sqrt{h} P_1 B_i & \sqrt{h} P_1 B_i & \sqrt{\varepsilon} \end{bmatrix}$$

$$Q_J = \begin{bmatrix} 0 & K_j & 0 & K_j & 0_{0\times 7} \end{bmatrix}, \Pi_1 = \begin{bmatrix} M_{ij1} + W_{ij1} & -W_{ij1} + S_{ij1} & -M_{ij1} - S_{ij1} & 0 \end{bmatrix}$$

$$M_{ij1} = \begin{bmatrix} M_{ij11}^T & M_{ij12}^T & M_{ij13}^T & M_{ij14}^T \end{bmatrix}^T, \bar{\Xi}_1 = \sqrt{h} P_1 B_i K_j$$

$$W_{ij1} = \begin{bmatrix} W_{ij11}^T & W_{ij12}^T & W_{ij13}^T & W_{ij14}^T \end{bmatrix}^T, S_{ij1} = \begin{bmatrix} S_{ij11}^T & S_{ij12}^T & S_{ij13}^T & S_{ij14}^T \end{bmatrix}^T$$

From Eq. (65), there exists scalar $m_1 > 0$ such that

$$\Lambda_{ij} + m_1 Q_C^T \triangle_q^2 Q_C + m_1^{-1} Q_J^T Q_J < 0 \tag{66}$$

Notice that $\triangle_q^2 < \delta^2 I$, the inequality (66) can be rewritten as

$$\Lambda_{ij} + m_1\delta^2 Q_C^T Q_C + m_1^{-1} Q_J^T Q_J < 0 \tag{67}$$

For any positive scalars ϱ_s, o_s, due to

$$\begin{aligned}\left(R_s - \varrho_s^{-1}P_s\right)R_s^{-1}\left(R_s - \varrho_s^{-1}P_s\right) \geq \quad & 0\\ \left(Z_s - o_s^{-1}P_s\right)Z_s^{-1}\left(Z_s - o_s^{-1}P_s\right) \geq \quad & 0, \; (s=1,2)\end{aligned} \tag{68}$$

Form Eq. (68), one can get

$$\begin{aligned}&-P_1R_1^{-1}P_1 \leq -2\varrho_1P_1 + \varrho_1^2R_1, -P_2R_2^{-1}P_2 \leq -2\varrho_2P_2 + \varrho_2^2R_2\\ &-P_1Z_1^{-1}P_1 \leq -2o_1P_1 + o_1^2Z_1, -P_2Z_2^{-1}P_2 \leq -2o_2P_2 + o_2^2Z_2\end{aligned} \tag{69}$$

Define $\mathcal{J}_1 = diag\{\underbrace{X_1, X_1, \dots, X_1}_{4}, I, \underbrace{X_1, X_1, \dots, X_1}_{5}, I, I, I\}$, $\mathcal{J}_2 = diag\{\underbrace{X_2, X_2, \dots, X_2}_{8}\}$, $X_s = P_s^{-1}$, $\tilde{Z}_s = X_sZ_sX_s^T$, $\tilde{R}_s = X_sR_sX_s^T$, $\tilde{Q}_s = X_sQ_sX_s^T$, $\tilde{\Omega} = X_s\Omega X_s^T$, $\bar{W}_{ijs} = X_sW_{ijs}X_s^T$, $\bar{S}_{ijs} = X_sS_{ijs}X_s^T$, $\bar{M}_{ijs} = X_sM_{ijs}X_s^T$, $\bar{N}_i^s = X_sN_i^sX_s^T\,(s=1,2)$, $Y_j = K_jX_1$. According to Eq. (69), by replacing $-P_sR_s^{-1}P_s$, $-P_sZ_s^{-1}P_s$ in Eq. (65) with $-2\varrho_sP_s + \varrho_s^2R_s$, $-2o_sP_s + o_s^2Z_s(s=1,2)$, respectively, and pre- and post- multiplying $\mathcal{J}_1$ and its transpose on both sides of Eq. (65), then we can obtain the inequalities (59) with $s=1$. Similarly, pre- and post- multiplying $\mathcal{J}_2$ and its transpose on both sides of Eq. (65), the inequalities (59) with $s=2$ can be obtained as well. That is, one can see that Eqs.(59)–(63) ensure Eqs. (31)–(36) hold. Then if Eqs. (31)–(36) hold, similar to Theorem 1, $\sum_{i=1}^r\sum_{j=1}^r \varsigma_i\left(\varsigma_j - g_j\right)\bar{N}_i^1 = 0$, the system (29) is GES. According to $Y_j = K_jX_1$, the desired controller gains can be expressed as Eq.(64).

This completes the proof. □

Remark 8. The calculating cost increased with the amount of decision variables, which is one of important problem leading to conservatism of the result. Fortunately, the feasible solutions to LMIs (59)–(63) are obtained by using the off-line calculation methods. Furthermore, with the continuous development of computer hardware, it will further reduce the impact of the conservativeness of the results, which is no longer a fatal problem.

4. Numerical examples

In this section, two examples are given to illustrate the usefulness of the proposed controller design method for T-S fuzzy system with quantization, DoS attacks and actuator saturation.

Example 1. Consider the following system [51]:

Rule 1:IF $\vartheta_1(x)$ is $\pm\pi$, THEN

$$\dot{x}(t) = A_1x(t) + B_1\bar{u}(t)$$

Rule 2:IF $\vartheta_2(x)$ is ± 0, THEN

$$\dot{x}(t) = A_2x(t) + B_2\bar{u}(t)$$

where

$$A_1 = \begin{bmatrix} -2.10 & 0.10 \\ 1 & -2.11 \end{bmatrix}, A_2 = \begin{bmatrix} -1.9 & 0 \\ -0.21 & -1.1 \end{bmatrix}, B_1 = \begin{bmatrix} 0.1 \\ -0.8 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ -1.2 \end{bmatrix}$$

$$\varsigma_1(\vartheta(x)) = \left(1 - \frac{1}{1 + e^{-3(\vartheta(x)-0.5\pi)}}\right), \left(\frac{1}{1 + e^{-3(\vartheta(x)+0.5\pi)}}\right), g_2(\vartheta(x)) = 1 - \varsigma_1(\vartheta(x))$$

The fuzzy controller is shown as follows:

Rule 1:IF $\vartheta_1(\hat{x})$ is $\pm\pi$, THEN

$$u(t) = K_1 x_q(t)$$

Rule 2:IF $\vartheta_2(\hat{x})$ is ± 0, THEN

$$u(t) = K_2 x_q(t)$$

The membership functions are given by:

$$\varsigma_1(\vartheta_1(\hat{x})) = 0.99 e^{\frac{-\vartheta(\hat{x})^2}{2\times 1.5^2}}, g_2(\vartheta_1(\hat{x})) = 1 - \varsigma_1(\vartheta_1(\hat{x}))$$

Suppose there are three sensors with its own quantization mechanism and event-triggered scheme. Set the sleeping time of the jammer $L_n = 1.78s$, $T = 2s$, the parameters of event-triggered scheme $\sigma_1^2 = 0.7$, $\sigma_2^2 = 0.3$, $\sigma_3^2 = 0.2$, $\chi_1 = 0.75$, $\chi_2 = 0.95$ which can guarantee $g_j - \bar{\chi}_j \varsigma_j \geq 0$. Let $\mu_1 = 1.03$, $\mu_2 = 1.03$, $\alpha_1 = 0.15$, $\alpha_2 = 0.4$, $o_1 = o_2 = 8$, $\varrho_1 = \varrho_2 = 8$ and the saturation parameter $\varepsilon = 0.7$.

By solving Theorem 2 using MATLAB, we can obtain the following parameters

$$Y_1 = \begin{bmatrix} -0.1823 & 2.3703 \end{bmatrix}, Y_2 = \begin{bmatrix} -0.5039 & 36.0145 \end{bmatrix}, X_1 = \begin{bmatrix} -0.0012 & -0.00017 \\ -0.0002 & -0.0013 \end{bmatrix} \tag{70}$$

According to Eq. (70), the controller gains can be figured out from Eq. (64) in Theorem (2) as follows:

$$K_1 = \begin{bmatrix} 0.0002 & -0.0021 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0005 & -0.0319 \end{bmatrix} \tag{71}$$

The initial states of system (29) are given by $x_1(0) = x_2(0) = \begin{bmatrix} 1 & -2 \end{bmatrix}^T$ and the sampling period is taken as $h = 0.1s$. The following simulation results from Figs. 2–7can be depicted. Figs. 2–4 show the release time intervals of three trigger schemes, respectively. Fig. 5 presents the state response of $x(t)$. Fig. 6 shows the signal of periodic DoS attacks. The saturation control signal $sat(u(t))$ is depicted in Fig. 7 with red solid line while the original control signal is with blue dashed. In Fig. 8, the output of quantizer is described by red line while the original signal is depicted by yellow line. The simulation results from $t \in [0, 30]$ demonstrate that the T-S fuzzy system subject to periodic DoS attacks are stable and the designed algorithm is feasible.

Example 2. Consider the following flexible joint robot arm model [52]:

$$I_1 \ddot{\theta} + \text{mglsin}(\theta_1) + k(\theta_1 - \theta_2), \tag{72}$$

$$I_2 \ddot{\theta} + b\theta_2 + k(\theta_2 - \theta_1) = u. \tag{73}$$

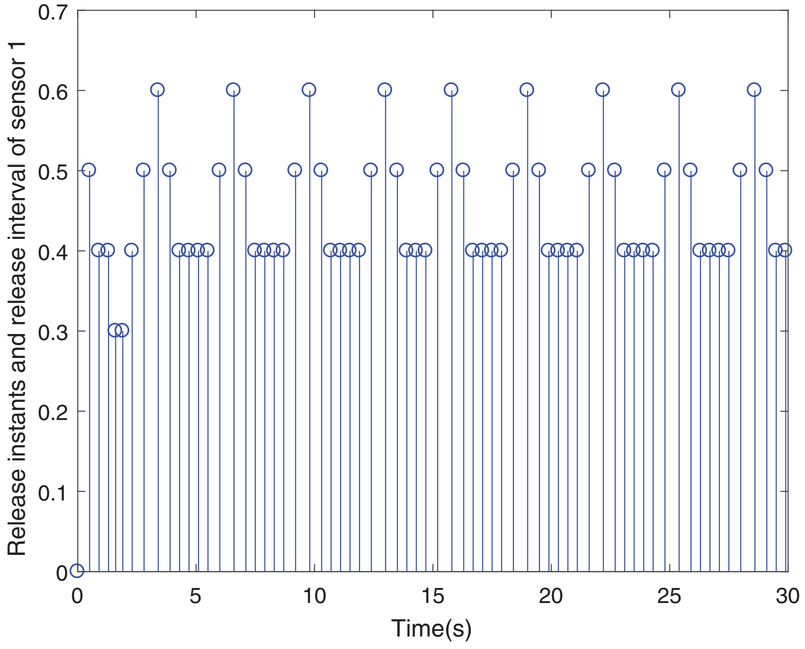

Fig. 2. Release intervals of sensors 1 with $T = 2s$ and $L_n = 1.78$.

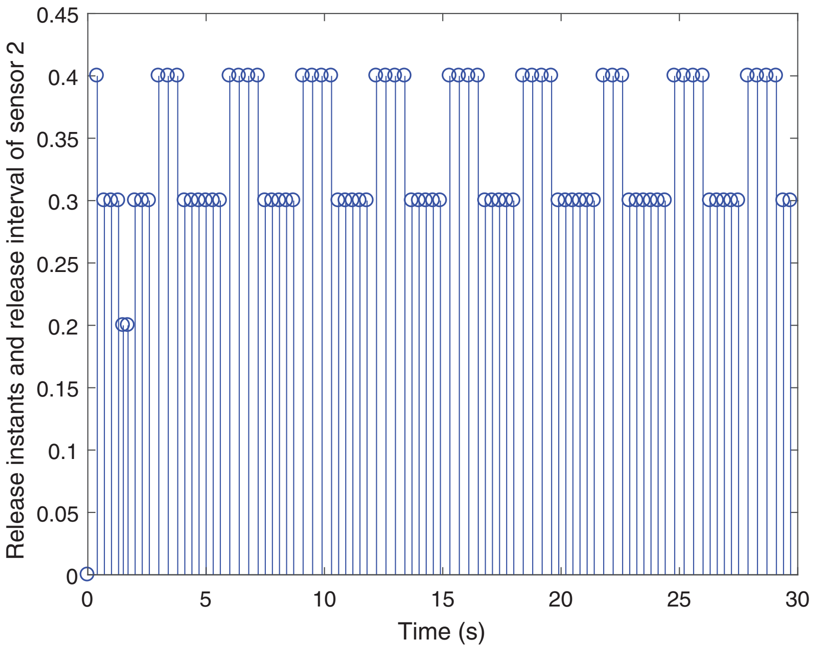

Fig. 3. Release intervals of sensors 2 with $T = 2s$ and $L_n = 1.78$.

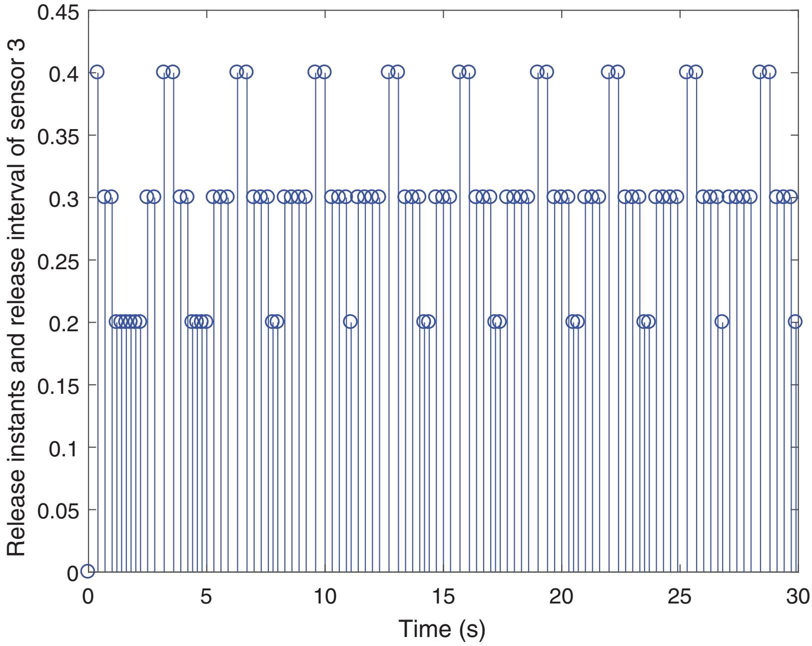

Fig. 4. Release intervals of sensors 3 with $T = 2s$ and $L_n = 1.78$.

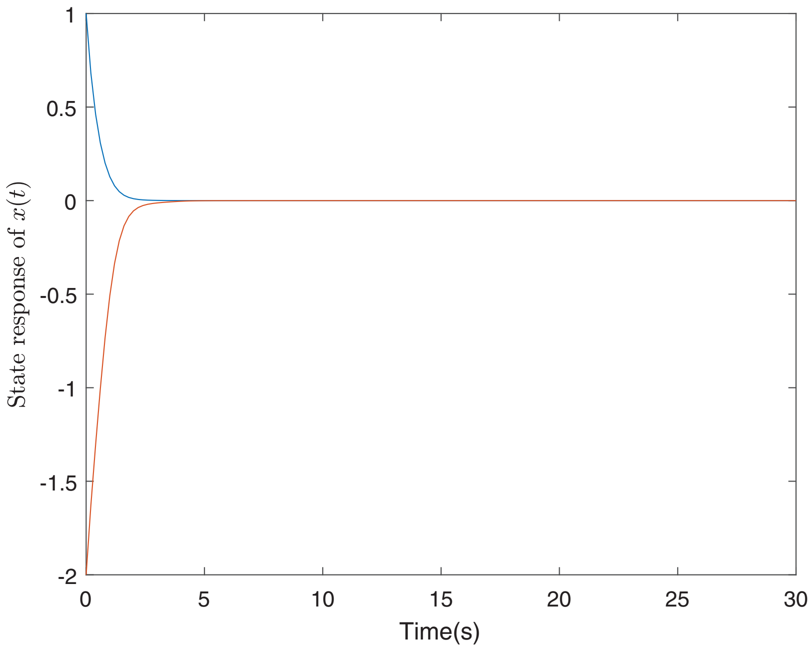

Fig. 5. State response of x(t).

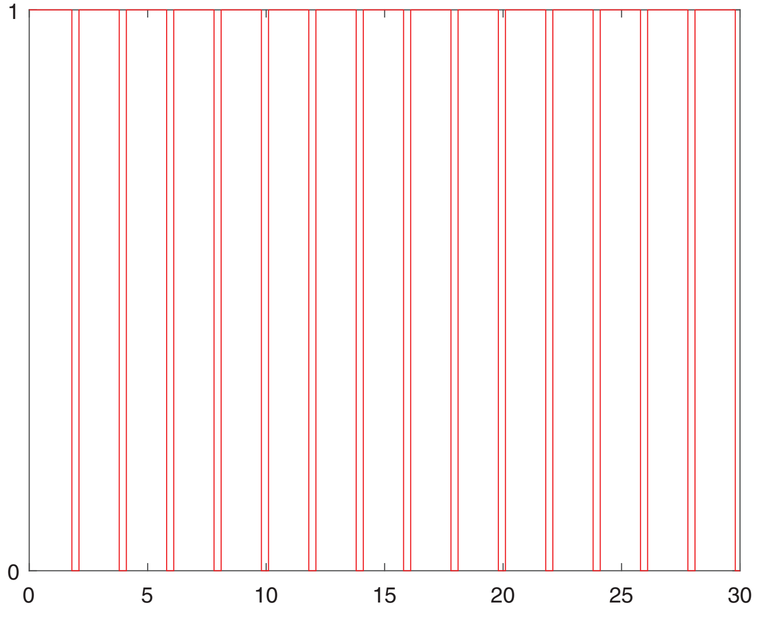

Fig. 6. DoS attacks with $T = 2s$ and $L_n = 1.78$.

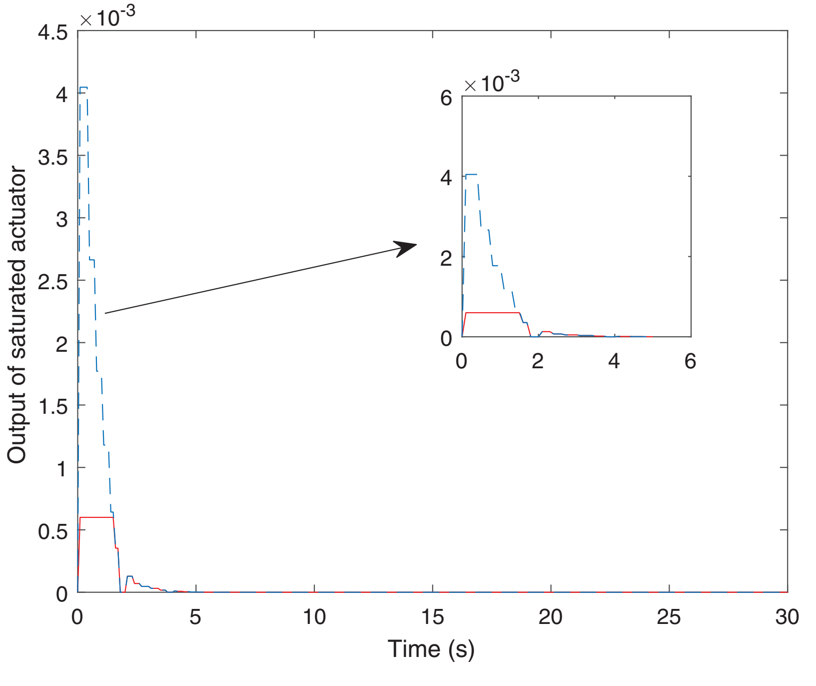

Fig. 7. The output of saturated actuator.

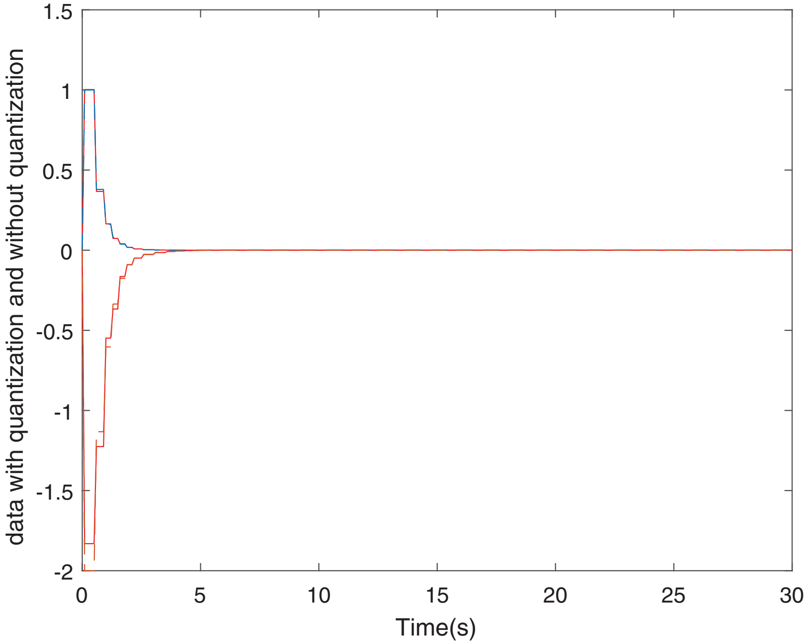

Fig. 8. Data with quantization and without quantization.

Let $x_1 = \theta_1$, $x_2 = \dot{\theta}_1$, $x_3 = \theta_2$, $x_4 = \ddot{\theta}_2$, the state equation of the system can be expressed as:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \psi_1 & 0 & \psi_2 & 0 \\ 0 & 0 & 0 & 1 \\ \psi_3 & 0 & -\psi_3 & \psi_4 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{I_2} \end{bmatrix} \bar{u}(t) \tag{74}$$

where

$$\psi_1 = -\frac{\text{mgl}}{I_1}\frac{\sin x_1(t)}{x_1(t)} - \frac{k}{I_1}, \psi_2 = \frac{k}{I_1}, \psi_3 = \frac{k}{I_2}, \psi_4 = \frac{b}{I_2}, x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) & x_3^T(t) & x_4^T(t) \end{bmatrix}^T$$

The practical system parameters are chosen as $m = 0.02kg$, $I_1 = I_2 = 1kgm^2$, $k = 0.06Nm/rad$, $l = 1m$, $b = 0.008Nms/rad$, $g = 9.78m/s^2$. The system (74) can be described by T-S fuzzy model as follows:

Rule 1:IF $x_1(t)$ is ω_1, THEN

$$\dot{x}(t) = A_1 x(t) + B_1 \bar{u}(t)$$

Rule 2:IF $x_1(t)$ is ω_2, THEN

$$\dot{x}(t) = A_2 x(t) + B_2 \bar{u}(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.2556 & 0 & 0.06 & 0 \\ 0 & 0 & 0 & 1 \\ 0.06 & 0 & -0.06 & -0.008 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -0.1845 & 0 & 0.06 & 0 \\ 0 & 0 & 0 & 1 \\ 0.06 & 0 & -0.06 & -0.008 \end{bmatrix}$$

$$B_1 = B_2 = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T, \omega_1 = 0, \omega_2 = \pm\frac{\pi}{2}$$

Set the event-triggered parameters $\sigma_1^2 = 0.5,\ \sigma_2^2 = 0.3,\ \sigma_3^2 = 0.1,\ \chi_1 = 0.75,\ \chi_2 = 0.95$ which can ensure $g_j - \bar{\chi}_j \varsigma_i \geq 0$. Let $\mu_1 = 1.03,\ \mu_2 = 1.03,\ \alpha_1 = 0.16,\ \alpha_2 = 0.4,\ o_1 = o_2 = 10,\ \varrho_1 = \varrho_2 = 10,\ T = 2s,\ L_n = 1.76s$ and the saturation parameter $\varepsilon = 0.7$.

Based on Matlab toolbox and applying Theorem 2, the following parameters can be obtained

$$\begin{aligned} Y_1 &= \begin{bmatrix} 0.0023 & -0.0002 & 0.0055 & -4.6329 \end{bmatrix}, \\ Y_2 &= \begin{bmatrix} 0.1514 & 0.0011 & 0.3406 & -82.0498 \end{bmatrix} \\ X_1 &= \begin{bmatrix} -259.57 & -5.19 & -0.0103 & -0.289 \\ -5.19 & -259.30 & -0.290 & -0.007 \\ -0.0103 & -0.290 & -259.590 & -5.57 \\ -0.289 & -0.007 & -5.57 & -258.46 \end{bmatrix} \end{aligned} \tag{75}$$

By combining (64) and (75), the desired controller gains are obtained as follows

$$\begin{aligned} K_1 &= \begin{bmatrix} -0.0029 & 0.0001 & -0.0004 & 0.0179 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} -0.0009 & 0.0002 & -0.0081 & 0.3176 \end{bmatrix} \end{aligned} \tag{76}$$

Taking the sampling period $h = 0.4$s and the initial condition of the flexible joint robot arm system is chosen as $x(0) = \begin{bmatrix} 1 & 0.8 & -0.8 & -1 \end{bmatrix}^T$. The following simulation results from Figs. 10–13 can be obtained. Fig. 9 presents the state response of $x(t)$, and it illustrates that the event-based system with quantization, actuator saturation and DoS attacks is effective and stable. The release intervals of three event-triggered schemes are shown in Fig. 10, Fig. 11 and Fig. 12, respectively. The saturation control signal is shown Fig. 13 with red line while the original control signal is with blue line.

Next, the following two tables are given to demonstrate the influence of the DoS attack period T on system performance. In view of this, for given different values of T, the corresponding minimum of L_n is calculated and the optimization problem is solved as follows

$$L_n^{\min} = \min\left\{L_n^{\min} \mid L_n^{\min} \text{ satisfying } (36)\right\} \text{subject to LMIs } (59) - (63) \tag{77}$$

Set the sampling period $h = 0.1s$, the DoS parameters $\alpha_1 = 1.05$, $\alpha_2 = 0.4$, $T = 2s$, $L_n = 1.78s$, $\mu_1 = 1.03$, $\mu_2 = 1.03$, $\varrho_1 = \varrho_2 = 8$, $o_1 = o_2 = 8$, and the maximum allowable DoS attacks activity $\Upsilon = \frac{T - L_n}{T} \times 100\%$.

Based on further calculation of different parameters, Tables 1 and 2 can be obtained. Specifically, the values of the minimum $L_n^{\min}$ and the maximum allowable DoS attacks activity Υ for different T are listed in Table 1. As exhibited in the Table 2, the relationship between L_n and the decay rate ϵ is further demonstrated. According to Table 1, the percentage of Υ varies little between $T = 6$s and $T = 10$s. In addition, both the values of L_n and Υ increase gradually with the increase of T. It can be seen from Table 1 that when the action-period T

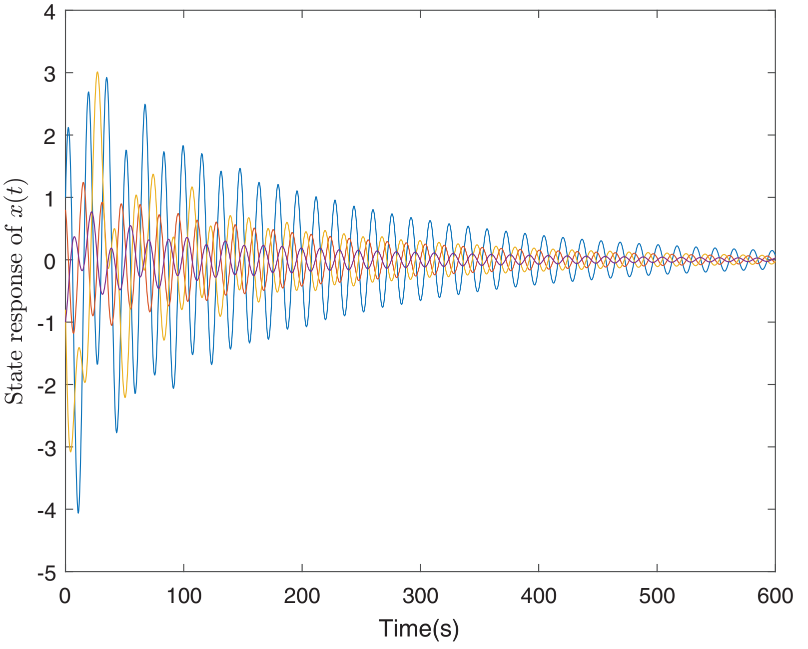

Fig. 9. State response of x(t).

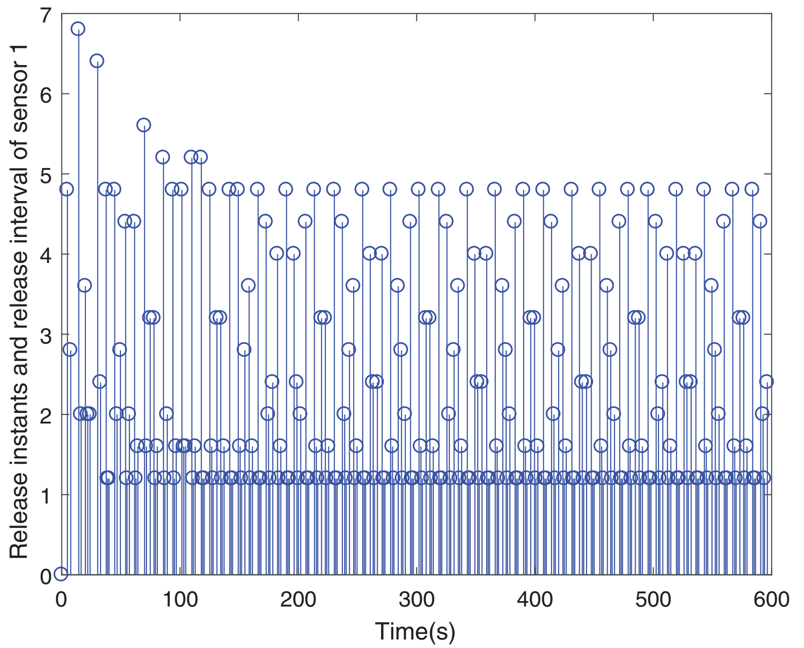

Fig. 10. Release intervals of sensors 1 with $T = 2$s and $L_n = 1.76$.

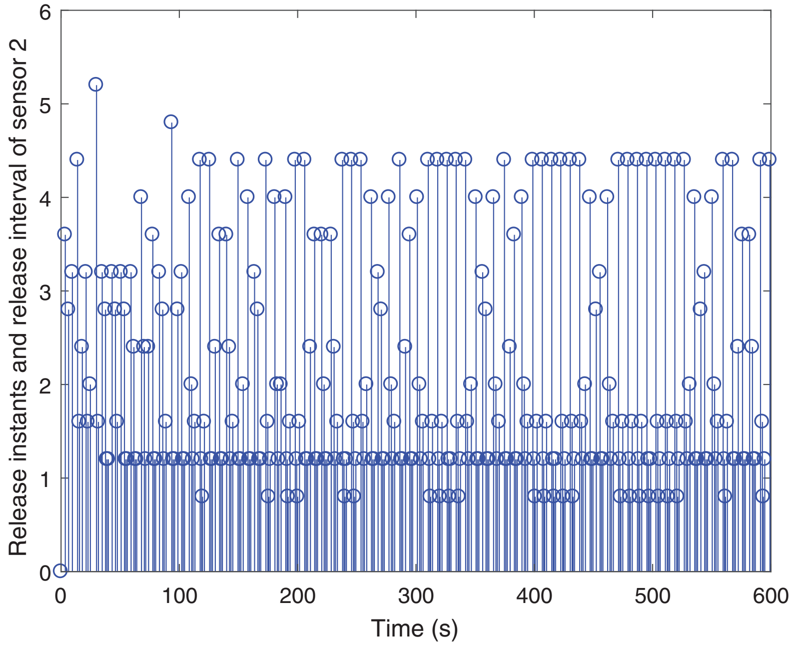

Fig. 11. Release intervals of sensors 2 with $T = 2$s and $L_n = 1.76$.

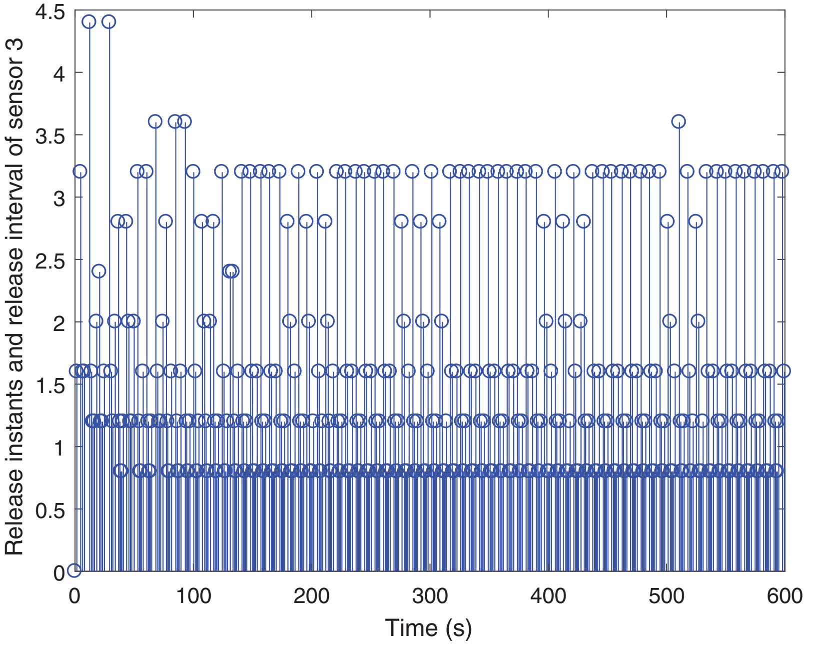

Fig. 12. Release intervals of sensors 3 with $T = 2s$ and $L_n = 1.76$.

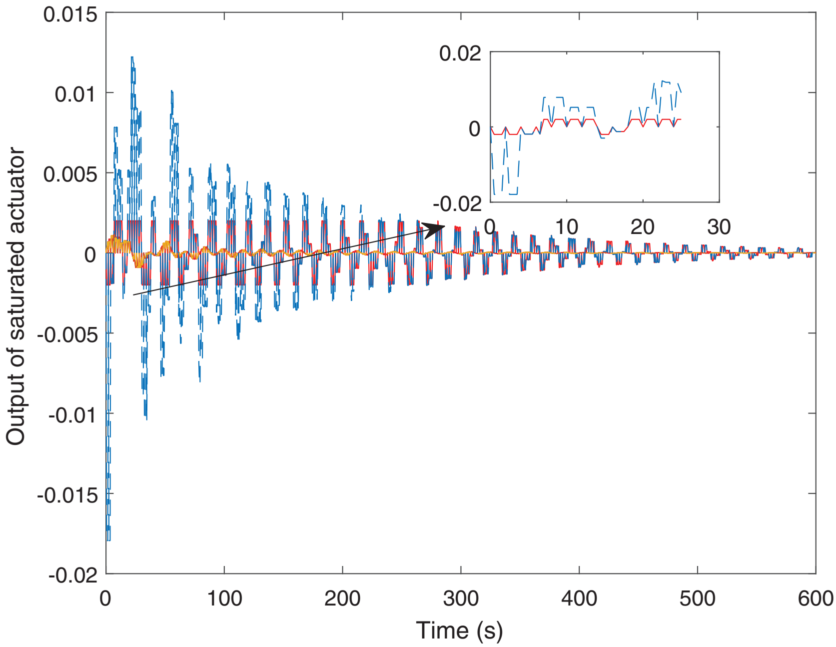

Fig. 13. The output of saturated actuator.

Table 1
Values of L_n and Υ for different T.

T	2	4	6	8	10
$L_n^{\min}$	1.75	3.48	5.18	6.90	8.62
Υ	12.5%	13%	13.6%	13.75%	13.8%

Table 2
values of λ and υ for different T.

L_n	1.75	1.78	1.82	1.86	1.95
ρ	0.1559	0.1889	0.2329	0.2769	0.3759
ϵ	0.0389	0.0472	0.5822	0.0692	0.0939

of jammer becomes larger, the sleeping period $L_n^{\min}$ of the jammer should also be larger to keep the system more stable. That is to say, compared with other control systems, the system (29) can tolerate more malicious attacks. From Table 2, it is observed that the ϵ increases with the increase of the $L_n^{\min}$. This means that in order to find a bigger value of decay rate ϵ, the values of DoS parameters $L_n^{\min}$ should be larger.

5. Conclusions

In this paper, the multi-sensors-based security control problem has been investigated for T-S fuzzy system over resource-constrained networks. A distributed event-triggered scheme and a quantization mechanism are employed to save the limited communication resources. By taking the impacts of the DoS attacks and actuator saturation into consideration, a novel T-S fuzzy model is established. On basis of the new model, sufficient conditions for the GES of the T-S fuzzy system are derived by employing Lyapunov stability theory. Moreover, the security controller gains are obtained in the form of LMIs. At last, two simulation examples are given to demonstrate the effectiveness of the proposed method. In the future, we will continue to study the analysis and synthesis for T-S fuzzy system with various type of attacks, for example: deception attacks, non-periodic DoS attacks and injection attack. Meanwhile, the hybrid cyber attacks which include two or more cyber attacks also deserve future research. In addition, the attack detection for T-S fuzzy system is an interesting question for future investigation.

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