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RESEARCH ARTICLE

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Security distributed state estimation for nonlinear networked systems against DoS attacks

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Summary

This paper is concerned with security distributed state estimation for nonlinear networked systems against denial-of-service attacks. By taking the effects of resource constraints into consideration, an event-triggered scheme and a quantization mechanism are employed to alleviate the burden of network. A mathematical model of distributed state estimation is constructed for nonlinear networked systems against denial-of-service attacks. Sufficient conditions ensuring the exponential stability of the estimation error systems are obtained by utilizing the Lyapunov stability theory. The explicit expressions of the designed state estimators are acquired in terms of the linear matrix inequalities. Finally, a numerical example is used to testify the feasibility of the proposed method.

KEYWORDS

distributed state estimation, DoS attacks, event-triggered scheme (ETS), nonlinear networked systems, quantization mechanism

1 | INTRODUCTION

Over the past decades, the nonlinear networked systems have captured enormous attention for its successful applications in different fields such as industrial automation and intelligent traffic, etc. In order to describe nonlinear networked systems as accurately as possible, various models are established for approximating these nonlinear systems. Among these models, Takagi-Sugeno (T-S) fuzzy model is considered as an effective mathematical tool to approximate the nonlinear networked systems with a series of linear systems. In view of this, the stability analysis of the T-S fuzzy systems has been paid great attention.[1-6] Besides, enormous scholars are devoted to studying the state estimation problem for nonlinear networked systems. For instance, in the work of Hu et al,[7] the authors investigated the estimation problem for nonlinear dynamical networks with sensor delays. Hu et al[8] studied the fault estimation for time-varying nonlinear systems with sensor saturations. Especially, the research of distributed state estimation for nonlinear networked systems is extremely attractive.[9-12] For example, by taking time delays into consideration, Li et al[12] investigated distributed estimation problem for state-saturated systems.

Due to the insertion of network, it brings many advantages to the systems such as convenience, high efficiency, and low cost.[13,14] Nevertheless, with the rapid expansion of the sampled signals, the disadvantages of the network resource constraints have been exposed such as packet dropouts,[15] time delay,[16] chance constraints,[17] stochastic nonlinearities,[18] etc. To alleviate the impact of resource constraints, various event-triggered mechanisms are employed.[19-21] It needs to point out that the event-triggered scheme (ETS) proposed in the work of Yue et al[21] depends on the discrete supervision of system state, where the signals are transmitted when they reach the triggering threshold. There are considerable academic achievements[22-29] referring to the ETS mentioned in the work of Yue et al.[21] Based on the contribution of the work of the aforementioned authors,[21] a hybrid-triggered scheme was employed to networked control systems with cyber-attacks in the works of Liu et al.[27,29] In addition, distributed ETS is also widely applied to various systems such as multiagent systems[30] and sensor networks.[31] Another method of alleviating the burden of communication resources is quantization mechanism,[32-36] which also plays a significant role in reducing redundant data. By taking the effect of quantization into consideration, the envelope-constrained filtering problem was addressed for nonlinear systems in the work of Ma et al.[37] In the work of Zheng et al,[38] the authors investigated the stabilization of sliding mode control for uncertain linear systems by employing the quantization mechanism.

The networked systems, which have the open signal transmission channels, are vulnerable to the multifarious attacks. For this reason, the security problems of networked systems have attracted ever-increasing attention from considerable scholars. Generally speaking, there are some common attacks including deception attacks,[39-43] replay attacks,[44] denial-of-service (DoS) attacks,[45-48] and so on. Deception attacks are a class of attacks that the attackers try to exchange normal data with false data or malicious data. By considering the effect of deception attacks, Wang et al[41] investigated the filtering problem for discrete-time delayed systems. Different from deception attacks, replay attacks are a series of attacks that the attackers record the sampled data and exchange the normal transmitted data with the recorded data. In the work of Liu et al,[44] by taking the impact of replay attacks into consideration, the authors studied the control problem for state-dependent uncertain systems. Unlike the former two kinds of attacks, DoS attacks can interrupt the process of data transmission by taking up network communication resources. For instance, Wu et al[45] addressed the resilient control problem for cyber-physical systems with DoS attacks. In the work of Chen et al,[48] the authors investigated the event-based robust stabilization of uncertain networked control systems subjected to DoS attacks.

Inspired by the aforementioned literature, the main purpose of this paper is to study security distributed estimation problem for nonlinear networked systems against DoS attacks. The main contributions of this paper are summarized as follows.

1. An ETS and a quantization are adopted to nonlinear networked systems while taking the effects of resource constraints and nonperiodic DoS attacks into consideration.
2. A T-S fuzzy model is constructed for nonlinear networked systems by considering the ETS, quantization mechanism, and nonperiodic DoS attacks.
3. Based on the constructed model, the criteria for considered nonlinear networked systems stability are derived by means of Lyapunov stability theory. Moreover, the primary state estimator gains and coupling gains are acquired simultaneously in terms of linear matrix inequality techniques.

Notation. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n \times m$ real matrices. $\mathbb{N}$ denotes the natural number. $\|\cdot\|$ represents the Euclidean vector norm or the induced matrix 2-norm as appropriate. I is the identity matrix of appropriate dimension. The superscript T denotes the matrix transposition. $sym\{X\}$ denotes the sum of matrix X and its transposed matrix X^T. $\mathcal{V} = \{1, 2, \cdots, m\}$ and $\mathcal{R} = \{1, 2, \cdots, r\}$ represent a set of positive integers, respectively. $\mathcal{L}_2[0, \infty)$ stands for the space of square-integrable vector functions defined on $[0, \infty)$.

2 | PROBLEM STATEMENT AND MODELING

2.1 | System description

Consider the following system, which can be described by a T-S fuzzy model with some simple linear systems

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} l_i(\pi(x))\{A_i x(t) + B_i \omega(t)\} \\ z(t) = \sum_{i=1}^{r} l_i(\pi(x)) L_i x(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ denotes the state vector; $z(t) \in \mathbb{R}^n$ represents the output measurement of the systems; $\omega(t) \in \mathcal{L}_2$ represents the external disturbance; r is regarded as the number of IF-THEN rules; $\pi(x)$ is the vector of premise variables and measurable; $l_i(\pi(x))$ denotes the normalized membership function satisfying $l_i(\pi(x)) \geq 0$, $\sum_{i=1}^{r} l_i(\pi(x)) = 1$; A_i, B_i, and L_i are known matrices with appropriate dimensions.

The output measurement of the sth sensor can be given as follows:

$$y_s(t) = \sum_{i=1}^{r} l_i(\pi(x)) C_i x(t), \tag{2}$$

where $s \in \mathcal{V}$; $y_s(t) \in \mathbb{R}^n$ denotes the output measurement of sth sensor; C_i is a constant matrix with compatible dimensions.

2.2 | Distributed state estimators

The main purpose of the paper is to design suitable distributed state estimators for nonlinear networked systems against DoS attacks.

Consider an estimator network consisting of m event-based state estimators, which can be expressed by a directed weighted graph $G = \{T, U, W\}$. $T \in \mathcal{V}$ represents the decentralized state estimators; $U \in T \times T$ is the set of edges. The weight of each associated edge in the graph is represented by an adjacent matrix $W = [w_{sm}]$. $w_{sm} = 1$ means the edge $(s, m) \in U$ and $w_{sm} = 0$ denotes the edge $(s, m) \notin U$. A directed edge from sth state estimator to mth state estimator is represented as an ordered pair $(s, m) \in U$, which means that the m_{th} estimator can obtain the information from s_{th} estimator, but not necessarily vice versa. The matrix $\kappa = \text{diag}\{\sum w_{1s}, \sum w_{2s}, \cdots, \sum w_{ms}\}$ is regarded to represent the degree matrix of graph G. The matrix $\Lambda = \kappa - W$ denotes the Laplacian matrix of the directed graph.

By taking the impacts of ETS and quantization mechanism into consideration, the sth state estimator to be designed in this paper is proposed as follows:

$$\begin{cases} \dot{\hat{x}}_s(t) = \sum_{j=1}^{r} l_j(\pi(\hat{x})) \left\{ A_j \hat{x}_s(t) + K_{sj}(\tilde{y}_s(t) - \hat{y}_s(t)) + D_{sj} \sum_{s=1}^{m} w_{sj}(\hat{y}_s(t) - \hat{y}_m(t)) \right\} \\ \hat{z}_s(t) = \sum_{j=1}^{r} l_j(\pi(\hat{x}) L_j \hat{x}_s(t) \\ \hat{y}_s(t) = \sum_{j=1}^{r} l_j(\pi(\hat{x}) C_j \hat{x}_s(t), \end{cases} \tag{3}$$

where $s \in \mathcal{V}$; $\hat{x}_s(t) \in \mathbb{R}^n$ denotes the state estimation of sth estimator; $y_s(t)$ is the output measurement of the sth sensor; $\tilde{y}_s(t) \in \mathbb{R}^n$ is the real input of $y_s(t)$; $\hat{y}_s(t) \in \mathbb{R}^n$ is the estimation of $y_s(t)$; $\hat{z}_s(t) \in \mathbb{R}^n$ is the estimation of $z_s(t)$; $\pi(\hat{x})$ represents the fuzzy premise variable; $l_i(\pi(\hat{x}))$ denotes the normalized membership function satisfying $l_i(\pi(\hat{x})) \geq 0$, $\sum_{i=1}^{r} l_i(\pi(\hat{x})) = 1$;

K_{sj} and D_{sj} are the s_{th} state estimator gains and coupling gains to be determined; A_j, C_j, and L_j are known matrices with appropriate dimensions.

Remark 1. As is known to all, the results of estimating sensor measurement by individual state estimator are not always accurate and perfect. In order to get more accurate estimation, the information interaction among the estimators should be taken into account. In this paper, the distributed state estimators are designed for nonlinear networked systems by considering the information interaction among the adjacent estimators.

2.3 | Resource constraints

In this section, by taking the effect of resource constraints into consideration, an ETS and a quantization mechanism are employed to alleviate the pressure of communication network.

2.3.1 | Event-triggered scheme

In order to save the limited network resources, an ETS is adopted to determine whether the latest sampled data should be delivered or not. As shown in Figure 1, the event generators are placed between the sensors and the quantizers.

Supposing that $t_k^s h$ represents the instant of the latest sampled data from the sth sensor, then the next transmitted instant $t_{k+1}^s h$ can be obtained as follows:

$$t_{k+1}^s h = t_k^s h + \min\left\{ l^s h | e_k^s\left(t_k^s h\right)^T \Omega_s e_k^s\left(t_k^s h\right) > \sigma_s \bar{y}_s\left(t_k^s h + l^s h\right)^T \Omega_s \bar{y}_s\left(t_k^s h + l^s h\right)\right\}, \tag{4}$$

where $s \in \mathcal{V}$; h denotes the constant sampling period; Ω_s is a positive symmetric matrix; $\sigma_s \in [0, 1)$ is the trigger parameters of the s_{th} generator; $t_k^s h + l^s h$ represents the current sampled instants; $e_k^s(t_k^s h) = \bar{y}_s(t_k^s h) - \bar{y}_s(t_k^s h + l^s h)$ represents the threshold error; $\bar{y}_s(t_k^s h)$ is the latest transmitted data; $\bar{y}_s(t_k^s h + l^s h)$ is the current sampling data. Whether the latest sampled data $\bar{y}(t_k^s h + l^s h)$ is delivered or not is determined by the following condition:

$$e_k^{s^T}\left(t_k^s h\right) \Omega_s e_k^s\left(t_k^s h\right) < \sigma_s \bar{y}_s^T\left(t_k^s h + l^s h\right) \Omega_s \bar{y}_s\left(t_k^s h + l^s h\right). \tag{5}$$

If the latest sampled data $\bar{y}_s(t_k^s h + l^s h)$ satisfies the inequality (5), it would not be delivered.

Remark 2. In the sight of triggering condition (5), the event-triggered parameter σ_s can determine the frequency of releasing sampled data. Especially, when trigger parameters $\sigma_s = 0 (s \in \mathcal{V})$, it denotes that the ETS reduces to time-triggered scheme. The set of transmitted instants $\{t_0 h, t_1 h, t_2 h, \cdots\} \subseteq \{0, h, 2h, \cdots\}$ gets completely released.

Then, the transmission data $\check{y}_s(t)$ under ETS can be expressed as

$$\check{y}_s(t) = y_s\left(t - \tau_k^s(t)\right) + e_k^s(t). \tag{6}$$

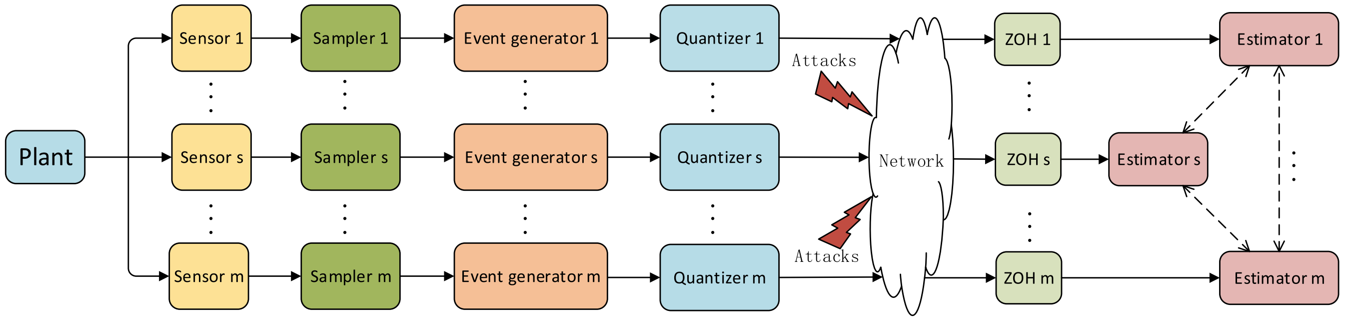

FIGURE 1 The structure of state estimation for nonlinear networked systems against denial-of-service attacks [Colour figure can be viewed at wileyonlinelibrary.com]

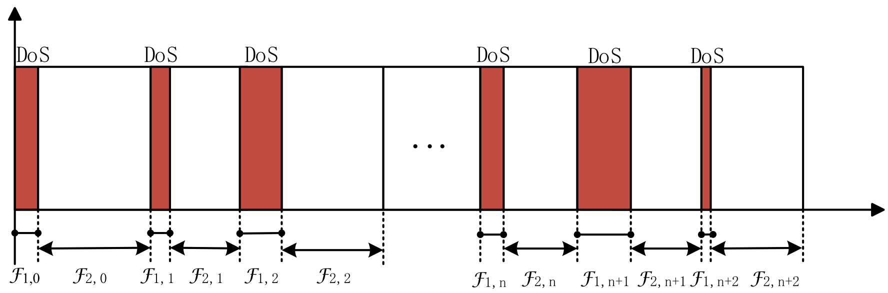

FIGURE 2 The evolution of the nonperiodic denial-of-service (DoS) jamming attacks [Colour figure can be viewed at wileyonlinelibrary.com]

2.3.2 | Quantization mechanism

In order to further save the bandwidth of communication network, a quantization mechanism is adopted. In Figure 1, the logarithmic quantizers are placed between the event generators and state estimators connected by network. The s_{th} logarithmic quantizer can be defined as follows:

$$r^s(x_v) = \begin{cases} h_v^s, x_v^s > 0, \frac{h_v^s}{1+b^s} < x_v^s < \frac{h_v^s}{1-b^s} \\ 0, x_v^s = 0 \\ -r^s(-x_v), x_v^s < 0, \end{cases} \tag{7}$$

where $s \in \mathcal{V}$; $b^s = \frac{1-\vartheta^s}{1+\vartheta^s}$; $0 < \vartheta < 1$; ϑ^s is regarded as quantization density of $r^s(\cdot)$; the quantized set is represented by $H^s = \{\pm h_v^s : h_v^s = \vartheta_v^s h_0^s, v = \pm 1, \pm 2, \cdots\} \cup \{\pm h_0^s\} \cup \{0\}, h_0 > 0$.

Define that

$$r^s(x) = \text{diag}\left\{r_1^s(x_1), r_2^s(x_2), \cdots, r_m^s(x_m)\right\}. \tag{8}$$

For the symmetrical matrix $r_v^s(\cdot)(v = 1, 2, \cdots, m)$, the equality $r_v^s(-x_v) = -r_v^s(x_v)$ holds. The logarithmic quantizer $r_v^s(\cdot)$ can be described as

$$r_v^s(x_v) = \left(I + \Delta_{r_v}^s(x_v)\right) x_v^s, \tag{9}$$

where $|\Delta_{r_v}^s(x_v^s)| \leq b_v^s$, $\Delta_{r_v}^s$ is utilized to represent $\Delta_{r_v}^s(x_v)$ to simplify the calculation process. Then, the following equality can be obtained:

$$r^s(x) = \left(I + \Delta_r^s\right) x^s, \tag{10}$$

where $\Delta_r = \text{diag}\{\Delta_{r_1}^1, \Delta_{r_2}^2, \cdots, \Delta_{r_m}^m\}$; according to the equalities (3) to (10), the signals transmitted from the sth quantizers can be acquired as

$$\tilde{y}_s(t) = (I + \Delta_r)\check{y}_s(t). \tag{11}$$

2.4 | Denial-of-service attacks

As shown in Figure 1, the sampled data are transmitted from the sensors to the state estimators via network, where the DoS attacks may occur. The attackers launch DoS attacks to prevent the sampled data from arriving their destinations by jamming network channels. As depicted in Figure 2, red parts denote the jamming time intervals of DoS attacks when normal data transmission is denied. White parts represent the sleeping time intervals of DoS attacks when sampled data can be successfully delivered by communication network.

The following expression is used to represent the behavior of DoS attacks[49]:

$$\beta(t) = \begin{cases} 1, t \in \mathcal{F}_{1,n} \\ 0, t \in \mathcal{F}_{2,n}, \end{cases} \tag{12}$$

where the set $\mathcal{F}_{1,n} \triangleq [w_{n-1} + \ell_{n-1}, w_n)$ represents that the normal data transmission is denied. The set $\mathcal{F}_{2,n} \triangleq [w_n, w_n + \ell_n)$ denotes that the data transmission is allowed without DoS attacks. w_n represents the length of nth DoS time interval; ℓ_n denotes the length of nth normal data transmission time interval. Meanwhile, $w_{n-1} + \ell_{n-1}$ and w_n ought to satisfy the condition $w_n > w_{n-1} + \ell_{n-1}$.

Define $\beta(t) = q \in \{1, 2\}$, and

$$t_{q,n} = \begin{cases} w_{n-1} + \ell_{n-1}, & q = 1 \\ w_n, & q = 2 \end{cases}. \tag{13}$$

Then, $\mathcal{F}_{1,n}$ and $\mathcal{F}_{2,n}$ can be summarized as $\mathcal{F}_{q,n} = [t_{q,n}, t_{3-q,n+q-1})$.

The transmission signal under DoS attacks can be described as

$$x_D(t) = \begin{cases} 0, & t \in \cup_{n\in N}\mathcal{F}_{1,n} \\ \tilde{y}_s(t), & t \in [-h, 0] \cup \left(\cup_{n\in N}\mathcal{F}_{2,n}\right). \end{cases} \tag{14}$$

In order to facilely analysis the DoS attacks, two assumptions are given as follows.

Assumption 1 (See the work of Chen et al[49]). For the interval $[0, t)$, $n(t)$ are used to denote the number of DoS off/on transitions $n(t) = \text{card} \quad \{n \in \mathbb{N} | w_n \in [0, t)\}$, where the card represents the number of elements in interval $\mathcal{F}_{1,n}$. For given $\partial > 0$, $\tau_a > 0$, the frequency of DoS attacks $n(t)$ over $0 \leq t$ satisfies the following condition:

$$n(t) \leq \partial + \frac{t}{\tau_a}. \tag{15}$$

Assumption 2 (See the work of Chen et al[49]). Set $\iota_{\max}$ as the uniform upper bound of DoS active interval length. One can get the following constraint:

$$\sup_{n\in\mathbb{N}}\{w_n - w_{n-1} - \ell_{n-1}\} \leq \iota_{\max}. \tag{16}$$

Set $\ell_{\min}$ as the uniform lower bound of DoS sleeping interval length. One can derive the following condition:

$$\inf_{n\in\mathbb{N}}\{\ell_n \geq \ell_{\min}\}. \tag{17}$$

Remark 3. In the existing research, great attention has been paid on the effect of periodic DoS attacks. However, when the hackers launch the DoS attacks, they might not follow certain manners or rules. There is full of uncertainties such as nonperiodic behavior or irregular probability distribution. Therefore, it is more meaningful and realistic to investigate the influence of nonperiodic DoS attacks. This paper is concerned with the security problem of state estimation for nonlinear networked systems against nonperiodic DoS attacks.

Remark 4. In order to guarantee the stabiliaty of the system, it is necessary that communication network could recover for a while after the attackers stop launching DoS attacks. By taking the actual situation into consideration, the frequency of DoS attacks should satisfy the restricted condition in Assumptions 1. Meanwhile, the duration of DoS attacks can not continue too long. The parameters ln and $w_n - w_{n-1} - l_{n-1}$ should ensure that there is no overlap between the former time interval's finish time and the latter time interval's start time.

2.5 | Modeling state estimation under DoS attacks

In this section, by taking DoS attacks into consideration, a model of distributed state estimation for nonlinear networked systems is established with some mathematical derivation.

Under nonperiodic DoS attacks, the event-triggered instant in (4) can be improved as follows:

$$t^s_{k,n}h = \left\{ t^s_{k_d} \text{ satisfying } (4) | t^s_{k_d}h \in \mathcal{F}_{2,n} \right\} \cup \{w_n\}, \tag{18}$$

where n, $t^s_{k_d}$, k_d, $d \in \mathbb{N}$, k represents the number of triggering time occurring in nth jammer action period, $k \in \{1, \cdots, k(n)\} \triangleq \varrho(n)$, and $k(n) = \sup\{k \in \mathbb{N} | t^s_{k,n}h \leq w_{n-1} + \ell_{n-1}\}$.

For $n \in \mathbb{N}$, defining $\mathcal{Z}_{k,n} \triangleq [t^s_{k,n}h, t^s_{k+1,n}h)$, $\varsigma_{k,n} \triangleq \sup\{\varpi \in \mathbb{N} | t^s_{k+1,n}h > t^s_{k,n}h + \varpi h, \varpi = 1, 2, \cdots\}$, the time interval $Z_{k,n}$ can be expressed as follows:

$$\mathcal{Z}_{k,n} = \cup_{\varpi=1}^{\varsigma_{k,n+1}} \mathcal{E}^{\varpi}_{k,n}, \tag{19}$$

where

$$\begin{cases} \mathcal{E}^{\varpi}_{k,n} = \left[t^s_{k,n}h + (\varpi - 1)h, t^s_{k,n}h + \varpi h\right), \varpi \in \{1, 2, \cdots, \varsigma_{k,n}\} \\ \mathcal{E}^{\varsigma_{k,n+1}}_{k,n} = \left[t^s_{k,n}h + \varsigma_{k,n}h, t^s_{k+1,n}h\right). \end{cases} \tag{20}$$

Note that

$$\mathcal{F}_{2,n} = \cup_{k=1}^{\varrho(n)} \left\{\mathcal{Z}_{k,n} \cap \mathcal{F}_{2,n}\right\} \subseteq \cup_{k=1}^{\varrho(n)} \mathcal{Z}_{k,n}. \tag{21}$$

Based on (19) to (21), the interval $\mathcal{F}_{2,n}$ can be represented as

$$\mathcal{F}_{2,n} = \cup_{k=1}^{\varrho(n)} \cup_{\varpi=1}^{\varsigma_{k,n}} \left\{\mathcal{E}^{\varpi}_{k,n} \cap \mathcal{F}_{2,n}\right\}. \tag{22}$$

Setting $\mathcal{R}^{\varpi}_{k,n} = \{\mathcal{E}^{\varpi}_{k,n} \cap \mathcal{F}_{2,n}\}$, we can get $\mathcal{F}_{2,n} = \cup_{k=1}^{\varrho(n)} \cup_{\varpi=1}^{\varsigma_{k,n}} \mathcal{R}^{\varpi}_{k,n}$.

For $n \in \mathbb{N}$, $k \in \varrho(n)$, two piecewise functions of sth sensor can be derived as follows:

$$\tau^s_{k,n}(t) = \begin{cases} t - t^s_{k,n}h, t \in \mathcal{R}^1_{k,n} \\ t - t^s_{k,n}h - h, t \in \mathcal{R}^2_{k,n} \\ \vdots \\ t - t^s_{k,n}h - (\varsigma_{k,n} - 1)h, t \in \mathcal{R}^{\varsigma_{k,n}}_{k,n} \end{cases}$$

and

$$e^s_{k,n}(t) = \begin{cases} 0, t \in \mathcal{R}^1_{k,n} \\ \bar{y}_s\left(t^s_{k,n}h\right) - \bar{y}_s\left(t^s_{k,n}h + h\right), t \in \mathcal{R}^2_{k,n} \\ \vdots \\ \bar{y}_s\left(t^s_{k,n}h\right) - \bar{y}_s\left(t^s_{k,n}h + \left(\varsigma_{k,n} - 1\right)h\right), t \in \mathcal{R}^{\varsigma_{k,n}}_{k,n}. \end{cases}.$$

Based on the definitions of $\tau^s_{k,n}(t)$ and $e^s_{k,n}(t)$, it yields that $\tau^s_{k,n}(t) \in [0, h)$, $t \in \mathcal{Z}_{k,n} \cap \mathcal{F}_{2,n}$.

The triggering condition of s_{th} event generator also can be acquired

$$e^{s\,T}_{k,n}\left(t^s_{k,n}h\right)\Omega e^s_{k,n}\left(t^s_{k,n}h\right) < \sigma \bar{y}^T_s\left(t^s_{k,n}h + l^s_n h\right)\Omega \bar{y}_s\left(t^s_{k,n}h + l^s_n h\right). \tag{23}$$

Combining (6) and (23), the signal transmitted from the event generator can be expressed as

$$\bar{y}_s\left(t^s_{k,n}h\right) = y_s\left(t - \tau^s_{k,n}(t)\right) + e^s_{k,n}(t). \tag{24}$$

By combining (11), (14), and (24), one can get the final input measurement of sth estimator

$$\tilde{y}_s(t) = \begin{cases} 0, \quad t \in \cup_{n \in N}\mathcal{F}_{1,n} \\ (I + \Delta_r)\left[y_s\left(t - \tau^s_{k,n}(t)\right) + e^s_{k,n}(t)\right], t \in [-h, 0] \cup \left(\cup_{n \in N}\mathcal{F}_{2,n}\right). \end{cases} \tag{25}$$

Setting $e_s(t) = x(t) - \hat{x}_s(t)$, by combining (1), (3), and (25), the estimation error $\dot{e}_s(t)$ can be acquired as follows:

$$\dot{e}_s(t) = \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x})) \left\{ \left(A_i - A_j + K_{sj}C_j - D_{sj}\sum_{s=1}^{m} w_{sj}C_j \right) x(t) + B_i\omega(t) \right. \\ \qquad \left. + \left(A_j - K_{sj}C_j + D_{sj}\sum_{s=1}^{m} w_{sj}C_j \right) e_s(t) \right\}, \quad t \in \mathcal{F}_{1,n} \\ \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x})) \left\{ \left(A_i - A_j + K_{sj}C_j - D_{sj}\sum_{s=1}^{m} w_{sj}C_j \right) x(t) + B_i\omega(t) \right. \\ \qquad -K_{sj}(I + \Delta_r)\left(C_i x\left(t - \tau_{k,n}^s(t)\right) + e_{k,n}^s(t) \right) \\ \qquad \left. + \left(A_j - K_{sj}C_j + D_{sj}\sum_{s=1}^{m} w_{sj}C_j \right) e_s(t) \right\}, \quad t \in \mathcal{F}_{2,n} \cap \mathcal{Z}_{k,n}. \end{cases} \tag{26}$$

Based on the aforementioned analysis, setting $e(t) = \check{x}(t) - \hat{x}(t)$, $\hat{x}(t) = [\hat{x}_1^T(t) \cdots \hat{x}_m^T(t)]^T$, $\check{x}(t) = [x^T(t) \cdots x^T(t)]^T$, $\bar{\omega}(t) = [\omega^T(t) \cdots \omega^T(t)]^T$, $e(t) = [e_1^T(t) \cdots e_m^T(t)]^T$, $e_{k,n}(t) = [e_{k,n}^{1\ T}(t) \cdots e_{k,n}^{m\ T}(t)]^T$, the estimation error $\dot{e}(t)$ can be obtained

$$\dot{e}(t) = \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x})) \left\{ \left(\bar{A}_i - \bar{A}_j + K_j\bar{C}_j - D_j\Lambda\bar{C}_j \right) \check{x}(t) + \bar{B}_i\bar{\omega}(t) \right. \\ \qquad \left. + \left(\bar{A}_j - K_j\bar{C}_j + D_j\Lambda\bar{C}_j \right) e(t) \right\}, \quad t \in \mathcal{F}_{1,n} \\ \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x})) \left\{ \left(\bar{A}_i - \bar{A}_j + K_j\bar{C}_j - D_j\Lambda\bar{C}_j \right) \check{x}(t) + \bar{B}_i\bar{\omega}(t) \right. \\ \qquad -K_j(I + \Delta_r)\left(\bar{C}_i\check{x}(t - \tau_{k,n}(t)) + e_{k,n}(t) \right) \\ \qquad \left. + \left(\bar{A}_j - K_j\bar{C}_j + D_j\Lambda\bar{C}_j \right) e(t) \right\}, \quad t \in \mathcal{F}_{2,n} \cap \mathcal{Z}_{k,n}, \end{cases} \tag{27}$$

where

$$\begin{aligned} &\bar{A}_i = I_m \otimes A_i, \bar{B}_i = I_m \otimes B_i, \bar{C}_i = I_m \otimes C_i, \bar{L}_i = I_m \otimes L_i \\ &\bar{A}_j = I_m \otimes A_j, \bar{C}_j = I_m \otimes C_j, \bar{L}_j = I_m \otimes L_j \\ &K_j = \mathrm{diag}\{K_{1j}, K_{2j}, \cdots, K_{mj}\}, D_j = \mathrm{diag}\{D_{1j}, D_{2j}, \cdots, D_{mj}\} \\ &\Lambda = \begin{bmatrix} \Sigma_{s=1}^m w_{1s} & -w_{12} & \cdots & -w_{1m} \\ -w_{21} & \Sigma_{s=1}^m w_{2s} & \cdots & -w_{2m} \\ \cdots & \cdots & \cdots & \cdots \\ -w_{m1} & -w_{m2} & -w_{m3} & \Sigma_{s=1}^m w_{ms} \end{bmatrix}. \end{aligned}$$

Defining $\bar{x}(t) = \left[\check{x}^T(t) \ \ e^T(t) \right]^T$, $\tilde{z}(t) = z(t) - \hat{z}(t)$, the estimation error system can be expressed as

$$\begin{cases} \dot{\bar{x}}(t) = \begin{cases} \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x})) \left(\bar{A}_{ij}\bar{x}(t) + \bar{B}_{ij}\bar{\omega}(t) \right), \quad t \in \mathcal{F}_{1,n} \\ \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x})) \left\{ \bar{A}_{ij}\bar{x}(t) + \bar{B}_{ij}\bar{\omega}(t) - (I + \Delta_r)\left[\bar{C}_{ij}\bar{x}(t - \tau_{k,n}(t)) \right. \right. \\ \qquad \left. \left. + H_2K_je_{k,n}(t) \right] \right\}, \quad t \in \mathcal{F}_{2,n} \cap \mathcal{Z}_{k,n} \end{cases} \\ \tilde{z}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\tilde{L}_{ij}\bar{x}(t) \\ \bar{x}(t) = \varphi(t), t \in [-h, 0], \end{cases} \tag{28}$$

where $H_1 = [I \ \ 0]$, $H_2 = [0 \ \ I]^T$, $\tilde{L}_{ij} = \left[\bar{L}_i - \bar{L}_j \ \ \bar{L}_j \right]$, $\bar{B}_{ij} = \left[\bar{B}_i \ \ \bar{B}_i \right]^T \bar{A}_{ij} = \begin{bmatrix} \bar{A}_i & 0 \\ \bar{A}_i - \bar{A}_j + K_j\bar{C}_j + D_j\Lambda\bar{C}_j & \bar{A}_j - K_j\bar{C}_j - D_j\Lambda\bar{C}_j \end{bmatrix}$,
$\bar{C}_{ij} = \begin{bmatrix} 0 & 0 \\ K_j\bar{C}_i & 0. \end{bmatrix}$

Before giving the main results, a definition and two lemmas are introduced.

Definition 1 (See the work of Chen et al[48]). The zero solution of (28) is said to be exponentially stable if there exist two scalars $F > 0$ and $\epsilon > 0$ such that $\|\bar{x}(t)\| \leq F\|\varphi\|_h e^{-\epsilon t}$, $\forall t \geq 0$.d

Lemma 1 (See the work of Liu et al[23]). *Consider the augmented system with $\tau(t)$, which satisfies $0 \leq \tau(t) \leq \bar{\tau}$. For all of constant matrices $R \in \mathbb{R}^{n\times n}$ and $U \in \mathbb{R}^{n\times n}$ satisfying $\begin{bmatrix} R & * \\ U & R \end{bmatrix} \geq 0$, the following inequality holds:*

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t} \dot{x}^T(s)R\dot{x}(s)ds \leq \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\bar{\tau}) \end{bmatrix}^T \begin{bmatrix} -R & * & * \\ R-U & -2R+U+U^T & * \\ U & R-U & -R \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\tau(t)) \\ x(t-\bar{\tau}). \end{bmatrix}$$

Lemma 2 (See the work of Hu and Yue[50]). *For given appropriately dimensioned matrices H_1, H_2 and a symmetric matrix A, the inequality $A + sym\{H_1\Delta(k)H_2\} < 0$ holds for all $\Delta(k)$ satisfying $\Delta^T(k)\Delta(k) \leq I$ if and only if there exists a positive scalar $d_1 > 0$ such that $A + d_1H_1^TH_1 + d_1^{-1}H_2H_2^T < 0$.*

3 | MAIN RESULTS

Theorem 1. *For given positive scalars α_1, α_2, μ_1, and μ_2; sampling period h; trigger parameter σ; DoS parameters ∂, τ_a, ℓ_{min}, and $\imath_{max}$, quantized parameter b; matrices Λ, K_j, and D_j, the system (28) is exponentially stable with decay rate ν, if there exist positive matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $Z_1 > 0$, $Z_2 > 0$, $\Omega > 0$, U_1, U_2, W_1, W_2 with compatible dimensions, such that the following inequalities hold with $l_j(\pi(\hat{x})) - \chi_j l_j(\pi(x)) \geq 0$:*

$$\Upsilon_{ij}^q - \zeta_i < 0(i, j \in \mathcal{R}; q = 1, 2) \tag{29}$$

$$\chi_i\Upsilon_{ii}^q - \chi_i\zeta_i + \zeta_i < 0(i \in \mathcal{R}; q = 1, 2) \tag{30}$$

$$\chi_j\Upsilon_{ij}^q + \chi_i\Upsilon_{ji}^q - \chi_j\zeta_i - \chi_i\zeta_j + \zeta_i + \zeta_j < 0(i < j; i, j \in \mathcal{R}; q = 1, 2) \tag{31}$$

$$P_1 \leq \mu_2 P_2 \tag{32}$$

$$P_2 \leq \mu_1 e^{2(\alpha_1+\alpha_2)h} P_1 \tag{33}$$

$$\begin{cases} Q_1 \leq \mu_2 Q_2, Q_2 \leq \mu_1 Q_1 \\ R_1 \leq \mu_2 R_2, R_2 \leq \mu_1 R_1 \\ Z_1 \leq \mu_2 Z_2, Z_2 \leq \mu_1 Z_1 \end{cases} \tag{34}$$

$$0 < \delta = \frac{2\alpha_1(\ell_{\min} - h) - 2\alpha_2(\imath_{\max} + h) - ln(\mu_1\mu_2)}{\tau_a}, \tag{35}$$

where

$$\Upsilon_{ij}^{q} = \begin{bmatrix} \Xi_{ij}^{q} & * \\ \mathcal{M}_1^q & \mathcal{M}_2^q \end{bmatrix}, \Xi_{ij}^{q} = \begin{bmatrix} \Theta_{11}^q & * & * & * \\ \Theta_{21}^q & \Theta_{22}^q & * & * \\ \Theta_{31}^q & \Theta_{32}^q & \Theta_{33}^q & * \\ \Theta_{41}^q & 0 & 0 & \Theta_{44}^q \end{bmatrix}$$

$$\Theta_{11}^1 = P_1\bar{A}_{ij} + \bar{A}_{ij}^T P_1^T + 2\alpha_1 P_1 + Q_1 + f_1 R_1 + f_1 Z_1, f_1 = -\frac{1}{h}e^{-2\alpha_1 h}$$

$$\Theta_{21}^1 = f_1(R_1 - U_1 + Z_1 - W_1), \Theta_{22}^1 = f_1\left(-2R_1 + U_1 + U_1^T - 2Z_1 + W_1 + W_1^T\right)$$

$$\Theta_{31}^1 = f_1(U_1 + W_1), \Theta_{32}^1 = f_1(R_1 - U_1 + Z_1 - W_1), \Theta_{33}^1 = f_1(R_1 + Z_1 + hQ_1)$$

$$\Theta_{41}^1 = \Theta_{44}^1 = 0, \mathcal{M}_1^1 = \left[\sqrt{h}P_1\bar{A}_{ij} \ \ 0 \ \ 0 \ \ 0\right], \mathcal{M}_2^1 = -P_1(R_1)^{-1}P_1 - P_1(Z_1)^{-1}P_1$$

$$\Theta_{11}^2 = P_2\bar{A}_{ij} + \bar{A}_{ij}^T P_2^T - 2\alpha_2 P_2 + Q_2 + f_2 R_2 + f_2 Z_2, f_2 = \frac{1}{h}e^{2\alpha_2 h}$$

$$\Theta_{21}^2 = f_2(R_2 - U_2 + Z_2 - W_2) - (I + \Delta_r)\bar{C}_{ij}^T P_2^T, \Theta_{32}^2 = f_2(R_2 - U_2 + Z_2 - W_2)$$

$$\Theta_{22}^2 = f_2\left(-2R_2 + U_2 + U_2^T - 2Z_2 + W_2 + W_2^T\right) + \sigma H_1^T \bar{C}_i^T \Omega \bar{C}_i H_1, \Theta_{44}^2 = -\Omega$$

$$\Theta_{31}^2 = f_2(U_2 + W_2), \Theta_{33}^2 = f_2(R_2 + Z_2 + hQ_2), \Theta_{41}^2 = -(I + \Delta_r)K_j^T H_2^T P_2^T,$$

$$\mathcal{M}_1^2 = \left[\mathcal{M}_{11}^2 \ \ \mathcal{M}_{12}^2 \ \ 0 \ \ \mathcal{M}_{14}^2\right], \mathcal{M}_{11}^2 = \sqrt{h}P_2\bar{A}_{ij}, \mathcal{M}_{12}^2 = \sqrt{h}(I + \Delta_r)P_2\bar{C}_{ij}$$

$$\mathcal{M}_{14}^2 = \sqrt{h}(I + \Delta_r)P_2 H_2 K_j, \mathcal{M}_2^2 = -P_2(R_2)^{-1}P_2 - P_2(Z_2)^{-1}P_2.$$

Proof. Construct the following Lyapunov function for the system (28):

$$V_q(t) = \bar{x}^T(t)P_q\bar{x}(t) + \int_{t-h}^{t} \bar{x}^T(s)\mathcal{B}(\cdot)Q_q\bar{x}(s)ds + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{\bar{x}}^T(s)\mathcal{B}(\cdot)R_q\dot{\bar{x}}(s)dsd\theta + \int_{-h}^{0}\int_{t+\theta}^{t} \dot{\bar{x}}^T(s)\mathcal{B}(\cdot)Z_q\dot{\bar{x}}(s)dsd\theta, \tag{36}$$

where $Q_q > 0$, $R_q > 0$, $Z_q > 0$, $\alpha_q > 0$, symmetric matrix $P_q > 0$, and $\mathcal{B}(\cdot) = e^{2(-1)^q\alpha_q(t-s)} (q = 1, 2)$.

Next, the proof of Theorem 1 is demonstrated with $q = 1$ and $q = 2$, respectively.

When $q = 1$, the derivation of $V_1(t)$ can be expressed as

$$\begin{aligned} \dot{V}_1(t) \leq\ & 2\bar{x}^T(t)P_1\dot{\bar{x}}(t) + \bar{x}^T(t)Q_1\bar{x}(t) - \bar{x}^T(t-h)e^{-2\alpha_1 h}Q_1\bar{x}(t-h) + h\dot{\bar{x}}^T(t)(R_1 + Z_1)\dot{\bar{x}}(t) \\ & - \int_{t-h}^{t} \dot{\bar{x}}^T(s)e^{-2\alpha_1 h}R_1\dot{\bar{x}}(s)ds - \int_{t-h}^{t} \dot{\bar{x}}^T(s)e^{-2\alpha_1 h}Z_1\dot{\bar{x}}(s)ds - 2\alpha_1\int_{t-h}^{t} \bar{x}^T(s)\mathcal{B}(\cdot)Q_1\bar{x}(s)ds \\ & - 2\alpha_1\int_{-h}^{0}\int_{t+\theta}^{t} \dot{\bar{x}}^T(s)\mathcal{B}(\cdot)R_1\dot{\bar{x}}(s)dsd\theta - 2\alpha_1\int_{-h}^{0}\int_{t+\theta}^{t} \dot{\bar{x}}^T(s)\mathcal{B}(\cdot)Z_1\dot{\bar{x}}(s)dsd\theta - 2\alpha_1\bar{x}^T(t)P_1\bar{x}(t). \end{aligned} \tag{37}$$

By combining (37) and Lemma 1, the following inequality can be derived:

$$\begin{aligned} \dot{V}_1(t) \leq\ & -2\alpha_1 V_1(t) + 2\bar{x}^T(t)P_1(t)\dot{\bar{x}}(t) + \bar{x}^T(t)Q_1(t)\bar{x}(t) + h\dot{\bar{x}}^T\left(R_1(t) + Z_1(t)\right)\dot{\bar{x}}(t) \\ & + 2\alpha_1\bar{x}^T(t)P_1\bar{x}(t) - e^{-2\alpha_1 h}\bar{x}^T(t-h)Q_1(t)\bar{x}(t-h) + \frac{1}{h}e^{-2\alpha_1 h}\mathcal{X}\mathcal{G}_1\mathcal{X}^T + \frac{1}{h}e^{-2\alpha_1 h}\mathcal{X}\mathcal{H}_1\mathcal{X}^T, \end{aligned} \tag{38}$$

where

$$\mathcal{X} = \left[\bar{x}(t)\ \bar{x}(t-\tau_{k,n}(t))\ \bar{x}(t-h)\right]$$

$$\mathcal{G}_1 = \begin{bmatrix} -R_1 & * & * \\ R_1 - U_1 & -2R_1 + U_1 + R_1^T & * \\ U_1 & R_1 - U_1 & -R_1 \end{bmatrix}$$

$$\mathcal{H}_1 = \begin{bmatrix} -Z_1 & * & * \\ Z_1 - W_1 & -2Z_1 + W_1 + Z_1^T & * \\ W_1 & Z_1 - W_1 & -Z_1 \end{bmatrix}.$$

By using the Schur complement, (38) can be described as

$$\dot{V}_1(t) \le -2\alpha_1 V_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\eta^T(t)\Upsilon_{ij}^1\eta(t), \tag{39}$$

where $\eta(t) = \left[\bar{x}^T(t)\ \bar{x}^T(t-\tau_{k,n}(t))\ \bar{x}^T(t-h)\ e_{k,n}{}^T(t)\ I\right]^T$.

Following the method in the work of Liu et al,[6] a slack matrix ζ_i is introduced to simplify the calculation

$$\begin{aligned} &\sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))\left\{l_i(\pi(x)) - l_j(\pi(\hat{x}))\right\}\zeta_i \\ &= \sum_{i=1}^{r} l_i(\pi(x))\left(\sum_{j=1}^{r} l_j(\pi(x)) - \sum_{j=1}^{r} l_j(\pi(\hat{x}))\right)\zeta_i = 0, \end{aligned} \tag{40}$$

where $\zeta_i = \zeta_i^T \in \mathbb{R}^{n\times n} > 0 (i = 1, 2, \ldots, r,)$ are arbitrary matrices. Then, it can be obtained that

$$\begin{aligned} \dot{V}_1(t) \le& -2\alpha_1 V_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\eta^T(t)\Upsilon_{ij}^1\eta(t) \\ =& -2\alpha_1 V_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\eta^T(t)\Upsilon_{ij}^1\eta(t) \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))\left\{l_j(\pi(x)) - l_j(\pi(\hat{x})) + \chi_j l_j(\pi(x)) - \chi_j l_j(\pi(\hat{x}))\right\}\eta^T(t)\zeta_i\eta(t) \\ =& -2\alpha_1 V_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(x))\eta^T(t)\left(\chi_j\Upsilon_{ij}^1 - \chi_j\zeta_i + \zeta_i\right)\eta(t) \\ &+ \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))\left\{l_j(\pi(\hat{x})) - \chi_j l_j(\pi(x))\right\}\eta^T(t)\left(\Upsilon_{ij}^1 - \zeta_i\right)\eta(t). \end{aligned} \tag{41}$$

By combining (39) to (41), we can derive

$$\begin{aligned} \dot{V}_1(t) \le& -2\alpha_1 V_1(t) + \sum_{i=1}^{r} l_i^2(\pi(x))\eta^T(t)\left(\chi_i\Upsilon_{ii}^1 + \chi_i\Upsilon_{ji}^1 - \chi_i\zeta_i + \zeta_i\right)\eta(t) \\ &+ \sum_{i=1}^{r} l_i(\pi(x))\left\{l_j(\pi(\hat{x})) - \chi_j l_j(\pi(x))\right\}\eta^T(t)\left(\Upsilon_{ij}^1 - \zeta_i\right)\eta(t) \\ &+ \sum_{i=1}^{r}\sum_{i<j}\eta^T(t)\left(\chi_j\Upsilon_{ij}^1 + \chi_i\Upsilon_{ji}^1 - \chi_j\zeta_i - \chi_i\zeta_j + \zeta_i + \zeta_j\right)\eta(t). \end{aligned} \tag{42}$$

Based on the inequalities (29) to (31) and (42), one can acquire the following inequality:

$$\dot{V}_1(t) \leq -2\alpha_1 V_1(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\eta^T(t)\Upsilon_{ij}^1\eta(t) < 0 \tag{43}$$

with $l_j(\pi(\hat{x})) - \chi_j l_j(\pi(x)) \geq 0$ for all $j \in \mathbb{R}$; it declares that there exists a positive scalar ε_1 satisfying $\dot{V}_1(t) \leq -\varepsilon_1\|\eta(t)\|^2$ for $\eta(t) \neq 0$. Then, the inequality $\dot{V}_1(t) \leq 2\alpha_1 V_1(t)$ holds for $t \in [t_{1,n}, t_{2,n})$.

When $q = 2$, the effect of ETS should be taken into consideration. The triggering condition can be expressed as

$$e_{k,n}{}^T(t)\Omega e_{k,n}(t) < \sigma\bar{x}^T(t-\tau_{k,n}(t))C_i^T\Omega C_i\bar{x}(t-\tau_{k,n}(t)), \tag{44}$$

where $\sigma = \text{diag}\{\sigma_1, \ldots, \sigma_m\}$, $\bar{x}(t-\tau_{k,n}(t)) = \begin{bmatrix}\bar{x}_1(t-\tau_{k,n}(t)) & \ldots & \bar{x}_m(t-\tau_{k,n}(t))\end{bmatrix}^T$, $\Omega = \text{diag}\{\Omega_1, \ldots, \Omega_m\}$.

By combining (36) and (44), the derivation of $V_2(t)$ can be obtained

$$\begin{aligned}\dot{V}_2(t) \leq{}& 2\alpha_2 V_2(t) + 2\bar{x}^T(t)P_2(t)\dot{\bar{x}}(t) + \bar{x}^T(t)Q_2(t)\bar{x}(t) + h\dot{\bar{x}}^T(R_2(t)+Z_2(t))\dot{\bar{x}}(t)\\ &+ 2\alpha_2\bar{x}^T(t)P_2\bar{x}(t) - e^{2\alpha_2 h}\bar{x}^T(t-h)Q_2(t)\bar{x}(t-h) + \frac{1}{h}e^{2\alpha_2 h}\mathcal{X}\mathcal{G}_2\mathcal{X}^T + \frac{1}{h}e^{2\alpha_2 h}\mathcal{X}\mathcal{H}_2\mathcal{X}^T\\ &+ \sigma\bar{x}^T(t-\tau_{k,n}(t))C_i^T\Omega C_i\bar{x}(t-\tau_{k,n}(t)) - e_{k,n}{}^T(t)\Omega e_{k,n}(t),\end{aligned} \tag{45}$$

where

$$\mathcal{X} = \begin{bmatrix}\bar{x}(t) & \bar{x}(t-\tau_{k,n}(t)) & \bar{x}(t-h)\end{bmatrix}$$
$$\mathcal{G}_2 = \begin{bmatrix} -R_2 & * & * \\ R_2 - U_2 & -2R_2 + U_2 + R_2^T & * \\ U_2 & R_2 - U_2 & -R_2 \end{bmatrix}$$
$$\mathcal{H}_2 = \begin{bmatrix} -Z_2 & * & * \\ Z_2 - W_2 & -2Z_2 + W_2 + Z_2^T & * \\ W_2 & Z_2 - W_2 & -Z_2. \end{bmatrix}$$

Following the analysis method above, by using Schur complement theory, (45) can be described as

$$\dot{V}_2(t) \leq 2\alpha_2 V_2(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\eta^T(t)\Upsilon_{ij}^2\eta(t) < 0. \tag{46}$$

According to the inequalities (29) to (31), the inequality $\dot{V}_2(t) \leq 2\alpha_2 V_2(t)$ holds if there exists a positive ε_2 such that $\dot{V}_2(t) \leq -\varepsilon_2\|\eta(t)\|^2$ for $\eta(t) \neq 0$.

Define $V_q(t_{q,n}) = q$, $V_q(t_{q,n}^-) = 3 - q$, $q \in \{1, 2\}$. For $t \in [t_{q,n}, t_{3-q,n+q-1})$, the following inequality can be derived with the method in the work of Chen et al[48]:

$$V_q(t) \leq e^{2(-1)^q(t-t_{q,n})}V_{q,n}(t_{q,n}). \tag{47}$$

The inequality (47) can be summarized as

$$V(t) \leq \begin{cases} e^{-2\alpha_1(t-t_{1,n})}, t \in [t_{1,n}, t_{2,n}) \\ e^{2\alpha_2(t-t_{2,n})}, t \in [t_{2,n}, t_{1,n+1}). \end{cases} \tag{48}$$

According to (32) to (34), one can easily get that

$$\begin{cases} V_1(t_{1,n}) \leq \mu_2 V_2\left(t_{1,n}^-\right) \\ V_2(t_{2,n}) \leq \mu_1 e^{2(\alpha_1+\alpha_2)h}V_1\left(t_{2,n}^-\right). \end{cases} \tag{49}$$

For $t \in [t_{1,n}, t_{2,n})$, by combining the inequalities (48) and (49), one can derive

$$V_1(t) \le \mu_2 e^{-2\delta(t-t_{1,n})V_2(t_{1,n}^-)} \le e^{n(t)\times 2(\alpha_1+\alpha_2)+n(t)ln(\mu_1\mu_2)}V_1(0)e^c \le e^{b_1(t)}V_1(0), \tag{50}$$

where $c = 2\alpha_2(w_n - w_{n-1} - \ell_{n-1} - \ell_{n-2} - \cdots - \ell_0) - 2\alpha_1(\ell_{n-1} + \ell_{n-2} + \cdots + \ell_0)$, $b_1(t) = 2h(\alpha_1+\alpha_2)(\partial + \frac{t}{\tau_a}) + 2\alpha_2\imath_{\max}(\partial + \frac{t}{\tau_a}) - 2\alpha_2\ell_{\min}(\partial + \frac{t}{\tau_a}) + (\partial + \frac{t}{\tau_a})ln(\mu_1\mu_2)$.

Based on (35), the following inequality can be expressed as:

$$V_1(t) \le e^{s_1}e^{-\delta t}V_1(0), \tag{51}$$

where $s_1 = 2(\alpha_1+\alpha_2)h + ln(\mu_1\mu_2)\partial + 2\alpha_2\imath_{\max}\partial - 2\alpha_1\ell_{\min}\partial$.

By combining (50) and (51), it yields

$$V_2(t) \le \frac{1}{\mu_2}e^{b_2(t)}V_1(0) \le \frac{V_1(0)}{\mu_2}e^{s_2}e^{-\delta(t)}, \tag{52}$$

where $b_2(t) = 2(\alpha_1+\alpha_2)h(\partial + \frac{t}{\tau_a} + 1) - 2\alpha_1\ell_{min}(\partial + \frac{t}{\tau_a} + 1) + ln(\mu_1\mu_2)(\partial + \frac{t}{\tau_a} + 1) + 2\alpha_2\imath_{max}(\partial + \frac{t}{\tau_a} + 1)$, $s_2 = (\partial+1)(2(\alpha_1+\alpha_2)h + ln(\mu_1\mu_2) + 2\alpha_2\imath_{\max} - 2\alpha_1\ell_{\min})$.

Define $\mathcal{W} = \max\{e^{s_1}, \frac{e^{s_2}}{\mu_2}\}$, $a_1 = min\{\delta_{\min}(P_q)\}$, $a_2 = \delta_{max}(P_1) + h\delta_{\max}(Q_1) + \frac{h^2}{2}\delta_{\max}(R_1+Z_1)$, and $\nu = \frac{\delta}{2}$.

Based on inequalities (51) and (52), one can get that

$$V(t) \le \mathcal{W}e^{-\delta t}V_1(0). \tag{53}$$

According to the Definition 1 and (53), the following inequalities are acquired:

$$V(t) \ge a_1\|\bar{x}(t)\|^2, V_1(0) \le \|\varphi_0\|_h^2. \tag{54}$$

Combining (53) and (54), one can obtain

$$\|\bar{x}(t)\| \le \sqrt{\frac{\mathcal{W}a_2}{a_1}}e^{-\nu t}\|\varphi_0\|_h, \forall t \ge 0, \tag{55}$$

which proves that the system (28) is exponentially stable with decay rate $\nu = \frac{\delta}{2}$. □

The sufficient conditions are obtained in Theorem 1, which can ensure the exponential stability of the system (28). Next, H_∞ performance of the estimation error system subjected to external disturbance $\omega(t)$ is studied in Theorem 2.

Theorem 2. *For given positive scalars α_1, α_2, μ_1, and μ_2; sampling period h; trigger parameter σ; H_∞ disturbance attention level γ; DoS parameters ∂, τ_a, $\ell_{\min}$, and $\imath_{\max}$; quantized parameter b; and the matrices Λ, K_j, and D_j, the estimation error system (28) is exponentially stable with decay rate ν given in Theorem 1, if there exist matrices $P_1 > 0$, $P_2 > 0$, $Q_1 > 0$, $Q_2 > 0$, $R_1 > 0$, $R_2 > 0$, $Z_1 > 0$, $Z_2 > 0$, $\Omega > 0$, U_1, U_2, W_1, W_2 with appropriate dimensions, such that (32)-(35) and the following inequalities hold with $l_j(\pi(\hat{x})) - \chi_j l_j(\pi(x)) \ge 0$:*

$$\Phi_{ij}^q - \zeta_i < 0(i,j \in \mathcal{R}; q = 1,2) \tag{56}$$

$$\chi_i\Phi_{ii}^q - \chi_i\zeta_i + \zeta_i < 0(i \in \mathcal{R}; q = 1,2) \tag{57}$$

$$\chi_j\Phi_{ij}^q + \chi_i\Phi_{ji}^q - \chi_j\zeta_i - \chi_i\zeta_j + \zeta_i + \zeta_j < 0(i<j; i,j \in \mathcal{R}; q = 1,2), \tag{58}$$

where

$$\Phi_{ij}^{q}=\begin{bmatrix}\mathcal{K}_{ij}^{q} & * \\ \mathcal{N}_1^{q} & \mathcal{N}_2^{q}\end{bmatrix}$$

$$\mathcal{K}_{ij}^{q}=\begin{bmatrix}\lambda_{11}^{q} & * & * & * & * & * \\ \lambda_{21}^{q} & \lambda_{22}^{q} & * & * & * & * \\ \lambda_{31}^{q} & \lambda_{32}^{q} & \lambda_{33}^{q} & * & * & * \\ \lambda_{41}^{q} & 0 & 0 & \lambda_{44}^{q} & * & * \\ \bar{B}_{ij}^{T}P_q^{T} & 0 & 0 & 0 & -\gamma^2 I & * \\ -\tilde{L}_{ij} & 0 & 0 & 0 & 0 & -I\end{bmatrix}$$

$$\lambda_{11}^{1}=P_1\bar{A}_{ij}+\bar{A}_{ij}^{T}P_1^{T}+2\alpha_1P_1+Q_1+f_1R_1+f_1Z_1, f_1=-\frac{1}{h}e^{-2\alpha_1h}$$

$$\lambda_{21}^{1}=f_1(R_1-U_1+Z_1-W_1), \lambda_{22}^{1}=f_1\left(-2R_1+U_1+U_1^{T}-2Z_1+W_1+W_1^{T}\right)$$

$$\lambda_{31}^{1}=f_1(U_1+W_1), \lambda_{32}^{1}=f_1(R_1-U_1+Z_1-W_1), \lambda_{33}^{1}=f_1(R_1+Z_1+hQ_1)$$

$$\lambda_{41}^{1}=\lambda_{44}^{1}=0, \mathcal{N}_1^{1}=\left[\sqrt{h}P_1\bar{A}_{ij}\ 0\ 0\ 0\right], \mathcal{N}_2^{1}=-P_1(R_1)^{-1}P_1-P_1(Z_1)^{-1}P_1$$

$$\lambda_{11}^{2}=P_2\bar{A}_{ij}+\bar{A}_{ij}^{T}P_2^{T}-2\alpha_2P_2+Q_2+f_2R_2+f_2Z_2$$

$$\lambda_{21}^{2}=f_2(R_2-U_2+Z_2-W_2)-(I+\Delta_r)\bar{C}_{ij}^{T}P_2^{T}, f_2=\frac{1}{h}e^{2\alpha_2h}$$

$$\lambda_{22}^{2}=f_h\left(-2R_2+U_2+U_2^{T}-2Z_2+W_2+W_2^{T}\right)+\sigma H_1^{T}\bar{C}_i^{T}\Omega\bar{C}_iH_1$$

$$\lambda_{31}^{2}=f_2(U_2+W_2), \lambda_{32}^{2}=f_2(R_2-U_2+Z_2-W_2), \lambda_{33}^{2}=f_2(R_2+Z_2+hQ_2)$$

$$\lambda_{41}^{2}=-(I+\Delta_r)K_j^{T}H_2^{T}P_2^{T}, \lambda_{44}^{2}=-\Omega, \mathcal{N}_1^{2}=\left[\mathcal{N}_{11}^{2}\ \mathcal{N}_{12}^{2}\ 0\ \mathcal{N}_{14}^{2}\ \mathcal{N}_{15}^{2}\ 0\right]$$

$$\mathcal{N}_{11}^{2}=\sqrt{h}P_2\bar{A}_{ij}, \mathcal{N}_{12}^{2}=\sqrt{h}(I+\Delta_r)P_2\bar{C}_{ij}, \mathcal{N}_{14}^{2}=\sqrt{h}(I+\Delta_r)P_2H_2K_j$$

$$\mathcal{N}_{15}^{2}=-\sqrt{h}(I+\Delta_r)P_2\bar{B}_{ij}, \mathcal{N}_2^{2}=-P_2(R_2)^{-1}P_2-P_2(Z_2)^{-1}P_2.$$

Proof. Construct a Lyapunov function as follows:

$$V_q(t)=\bar{x}^{T}(t)P_q\bar{x}(t)+\int_{t-h}^{t}\bar{x}^{T}(s)\mathcal{B}(\cdot)Q_q\bar{x}(s)ds+\int_{-h}^{0}\int_{t+\theta}^{t}\dot{\bar{x}}^{T}(s)\mathcal{B}(\cdot)R_q\dot{\bar{x}}(s)dsd\theta + \int_{-h}^{0}\int_{t+\theta}^{t}\dot{\bar{x}}^{T}(s)\mathcal{B}(\cdot)Z_q\dot{\bar{x}}(s)dsd\theta. \tag{59}$$

When $q=1$, based on the similar method in Theorem 1, the following inequality can be obtained:

$$\dot{V}_1(t)+2\alpha_1V_1(t)+\tilde{z}(t)^{T}\tilde{z}(t)-\gamma^2\bar{\omega}^{T}(t)\bar{\omega}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r}l_i(\pi(x))l_j(\pi(\hat{x}))\xi^{T}(t)\left[\mathcal{K}_{ij}^{1}+h^{T}(R_1+Z_1)^{-1}h+\tilde{L}_{ij}^{T}(-I)^{-1}\tilde{L}_{ij}\right]\xi(t), \tag{60}$$

where $\xi(t)=\left[\bar{x}^{T}(t)\ \bar{x}^{T}(t-\tau_{k,n}(t))\ \bar{x}^{T}(t-h)\ e_{k,n}{}^{T}(t)\ \bar{\omega}^{T}(t)\ I\ I\right]^{T}$.

Based on equality (40), it can be derived

$$\begin{aligned}
&\dot{V}_1(t) + 2\alpha_1 V_1(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 \omega^T(t)\omega(t) \\
&\leq \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x)) l_j(\pi(\hat{x}))\xi^T(t)\Phi_{ij}^1 \xi(t) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x)) l_j(\pi(\hat{x}))\xi^T(t)\Phi_{ij}^1 \xi(t) \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x)) \left\{ l_j(\pi(x)) - l_j(\pi(\hat{x})) + \chi_j l_j(\pi(x)) - \chi_j l_j(\pi(\hat{x})) \right\} \xi^T(t)\zeta_i \xi(t) \\
&= \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x)) l_j(\pi(x))\xi^T(t)(\chi_j \Phi_{ij}^1 - \chi_j \zeta_i + \zeta_i)\xi(t) \\
&\quad + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x)) \left\{ l_j(\pi(\hat{x})) - \chi_j l_j(\pi(x)) \right\} \xi^T(t)(\Phi_{ij}^1 - \zeta_i)\xi(t) > .
\end{aligned} \tag{61}$$

According to the analysis above, (61) can be expressed as

$$\begin{aligned}
&\dot{V}_1(t) + 2\alpha_1 V_1(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 \omega^T(t)\omega(t) \\
&\leq \sum_{i=1}^{r} l_i(\pi(x)) \left\{ l_j(\pi(\hat{x})) - \chi_j l_j(\pi(x)) \right\} \xi^T(t) \left(\Phi_{ij}^1 - \zeta_i \right) \xi(t) \\
&\quad + \sum_{i=1}^{r} l_i^2(\pi(x))\xi^T(t) \left(\chi_i \Phi_{ii}^1 + \chi_i \Phi_{ji}^1 - \chi_i \zeta_i + \zeta_i \right) \xi(t) \\
&\quad + \sum_{i=1}^{r}\sum_{i<j}^{r} \xi^T(t) \left(\chi_j \Phi_{ij}^1 + \chi_i \Phi_{ji}^1 - \chi_j \zeta_i - \chi_i \zeta_j + \zeta_i + \zeta_j \right) \xi(t).
\end{aligned} \tag{62}$$

Based on the equalities (56) to (58), it can be obtained that

$$\dot{V}_1(t) \leq -2\alpha_1 V_1(t) - \tilde{z}^T(t)\tilde{z}(t) + \gamma^2 \omega^T(t)\omega(t) + \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x)) l_j(\pi(\hat{x}))\xi^T(t)\Phi_{ij}^1 \xi(t) < 0. \tag{63}$$

By utilizing Schur complement theory, (60) can be described as

$$\dot{V}_1(t) + 2\alpha_1 V_1(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 \bar{\omega}^T(t)\bar{\omega}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x)) l_j(\pi(\hat{x}))\xi^T(t)\Phi_{ij}^1 \xi(t) < 0. \tag{64}$$

According to (64), it yields the following inequality:

$$\dot{V}_1(t) + 2\alpha_1 V_1(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 \bar{\omega}^T(t)\bar{\omega}(t) \leq 0. \tag{65}$$

When $q = 2$, $t \in \mathcal{F}_{2,n}$, by using the analysis method above, one can acquire

$$\dot{V}_2(t) - 2\alpha_2 V_2(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 \bar{\omega}^T(t)\bar{\omega}(t) \leq 0. \tag{66}$$

For all $\bar{\omega}(t) \in \mathcal{L}_2[0, \infty)$, $t \in [t_{q,n}, t_{3-q,n+q-1})$, inequalities (65) and (66) can be summarized as

$$\dot{V}_q(t) + 2\alpha_q(-1)^{q-1} V_q(t) + \tilde{z}^T(t)\tilde{z}(t) - \gamma^2 \bar{\omega}^T(t)\bar{\omega}(t) \leq 0. \tag{67}$$

Meanwhile, the inequality (35) can be expressed as

$$\frac{1}{\mu_1}e^{2\alpha_1\ell_{\min}-2(\alpha_1+\alpha_2)h}-\mu_2 e^{2\alpha_2(w_n-\ell_{\min})}>0. \tag{68}$$

Based on $\ell_{\min} \leq \ell_n \leq w_{n+1}, n \in \mathbb{N}$, it yields that

$$\frac{1}{\mu_1}e^{2\alpha_1\ell_n-2(\alpha_1+\alpha_2)h}-\mu_2 e^{2\alpha_2(w_n-\ell_n)}>\frac{1}{\mu_1}e^{2\alpha_1\ell_{\min}-2(\alpha_1+\alpha_2)h}-\mu_2 e^{2\alpha_2(w_n-\ell_{\min})}. \tag{69}$$

By combining (68) and (69), it can be derived that

$$\frac{1}{\mu_1}e^{2\alpha_1\ell_n-2(\alpha_1+\alpha_2)h}-\mu_2 e^{2\alpha_2(w_n-\ell_{\min})}>0. \tag{70}$$

Consequently, for the given condition $t \in [0, w_n+\ell_n), n \in \mathbb{N}$, it follows from the zero-initial condition that

$$\sum_{k=1}^{n}\left\{\int_{w_{k-1}+\ell_{k-1}}^{w_k}\left(\dot{V}_1(t)+2\alpha_1 V_1(t)\right)dt+\int_{w_k}^{w_k+\ell_k}\left(\dot{V}_2(t)-2\alpha_2 V_2(t)\right)dt\right\}>0. \tag{71}$$

Based on the zero initial condition and (71), when $n \to \infty$, one can get that

$$\int_0^\infty \tilde{z}^T(t)\tilde{z}(t)dt \leq \int_0^\infty \gamma^2\bar{\omega}^T(t)\bar{\omega}(t)dt. \tag{72}$$

For all $\bar{\omega}(t) \in \mathcal{L}_2[0,\infty)$, the inequality $\|\tilde{z}(t)\|_2 \leq \gamma\|\bar{\omega}(t)\|_2$ holds.

By combining (56) to (58), it yields that $\dot{V}_q(t)+\tilde{z}^T(t)z(t)-\gamma^2\bar{\omega}^T(t)\bar{\omega}(t) < 0 (q=1,2)$. Thus, the estimation error system (28) is exponentially stable with DoS attacks and external disturbance $\bar{\omega}(t)$. That completes the proof. □

Based on Theorems 1 and 2, the algorithms of distributed state estimator gains and coupling gains are presented in Theorem 3.

Theorem 3. *For given positive scalars α_1, α_2, μ_1, and μ_2; sampling period h; trigger parameter σ; H_∞ disturbance attention level γ; DoS parameters ∂, τ_a, $\ell_{\min}$, and $\imath_{\max}$; quantized parameter b; and matrix Λ, the system (28) is exponentially stable with decay rate ν given in Theorem 1, if there exist matrices $\hat{Q}_1>0$, $\hat{Q}_2>0$, $\hat{R}_1>0$, $\hat{R}_2>0$, $\hat{Z}_1>0$, $\hat{Z}_2>0$, $P_1>0$, $P_2=\text{diag}\{P_{21},P_{22}\}>0$, $P_{22}^{(s)}>0 (s \in \mathcal{V})$, $\hat{\Omega}$, $\hat{U}_1$, $\hat{U}_2$, $\hat{W}_1$, $\hat{W}_2$, Y_{sj}, M_{sj} with compatible dimensions and scalars e_1, e_2, such that (35) and the following linear matrix inequalities hold with $l_j(\pi(\hat{x}))-\chi_j l_j(\pi(x)) \geq 0$:*

$$\hat{\Phi}_{ij}^q-\hat{\zeta}_i<0(i,j\in\mathcal{R};q=1,2) \tag{73}$$

$$\chi_i\hat{\Phi}_{ii}^q-\chi_i\hat{\zeta}_i+\hat{\zeta}_i<0(i\in\mathcal{R};q=1,2) \tag{74}$$

$$\chi_j\hat{\Phi}_{ij}^q+\chi_i\hat{\Phi}_{ji}^q-\chi_j\hat{\zeta}_i-\chi_i\hat{\zeta}_j+\hat{\zeta}_i+\hat{\zeta}_j<0(i<j;i,j\in\mathcal{R};q=1,2) \tag{75}$$

$$\begin{bmatrix}-\mu_2 P_2 & * \\ P_2 & -P_1\end{bmatrix}\leq 0 \tag{76}$$

$$\begin{bmatrix}-\mu_1 e^{2(\alpha_1+\alpha_2)h}P_2 & * \\ P_1 & -P_2\end{bmatrix}\leq 0 \tag{77}$$

$$\begin{bmatrix} -\mu_{3-q}\hat{Q}_{3-q} & * \\ I & -\hat{Q}_q \end{bmatrix} \leq 0 \tag{78}$$

$$\begin{bmatrix} -\mu_{3-q}\hat{R}_{3-q} & * \\ P_{3-q} & e_q^2\hat{R}_q - 2e_qP_q \end{bmatrix} \leq 0 \tag{79}$$

$$\begin{bmatrix} -\mu_{3-q}\hat{Z}_{3-q} & * \\ P_{3-q} & e_q^2\hat{Z}_q - 2e_qP_q, \end{bmatrix} \leq 0 \tag{80}$$

where

$$\hat{\Phi}_{ij}^1 = \begin{bmatrix} \hat{\lambda}_{11}^1 & * & * & * & * & * \\ \hat{\lambda}_{21}^1 & \hat{\lambda}_{22}^1 & * & * & * & * \\ \hat{\lambda}_{31}^1 & \hat{\lambda}_{32}^1 & \hat{\lambda}_{33}^1 & * & * & * \\ \bar{B}_{ij}^TP_1^T & 0 & 0 & -\gamma^2I & * & * \\ -\tilde{L}_{ij} & 0 & 0 & 0 & -I & * \\ \sqrt{h}P_1\bar{A}_{ij} & 0 & 0 & 0 & 0 & \hat{\lambda}_{77}^1 \end{bmatrix}$$

$$\hat{\Phi}_{ij}^2 = \begin{bmatrix} \hat{\lambda}_{11}^2 & * & * & * & * & * & * & * & * \\ \hat{\lambda}_{21}^2 & \hat{\lambda}_{22}^2 & * & * & * & * & * & * & * \\ \hat{\lambda}_{31}^2 & \hat{\lambda}_{32}^2 & \hat{\lambda}_{33}^2 & * & * & * & * & * & * \\ \hat{\lambda}_{41}^2 & 0 & 0 & -\hat{\Omega} & * & * & * & * & * \\ \bar{B}_{ij}^TP_2^T & 0 & 0 & 0 & -\gamma^2I & * & * & * & * \\ -\tilde{L}_{ij} & 0 & 0 & 0 & 0 & -I & * & * & * \\ \sqrt{h}P_2\bar{A}_{ij} & \hat{\lambda}_{72}^2 & 0 & \hat{\lambda}_{74}^2 & \hat{\lambda}_{75}^2 & 0 & \hat{\lambda}_{77}^2 & * & * \\ 0 & \hat{\lambda}_{82}^2 & 0 & \hat{\lambda}_{84}^2 & 0 & 0 & 0 & -m_1I & * \\ 0 & \hat{\lambda}_{92}^2 & 0 & H_1^T & 0 & 0 & 0 & 0 & -m_1^{-1}I \end{bmatrix}$$

$$\hat{\lambda}_{11}^1 = P_1\bar{A}_{ij} + \bar{A}_{ij}^TP_1^T + 2\alpha_1P_1 + \hat{Q}_1 + f_1\hat{R}_1 + f_1\hat{Z}_1,\ f_1 = -\frac{1}{h}e^{-2\alpha_1h}$$

$$\hat{\lambda}_{21}^1 = f_1(\hat{R}_1 - \hat{U}_1 + \hat{Z}_1 - \hat{W}_1),\ \hat{\lambda}_{22}^1 = f_1\left(-2\hat{R}_1 + \hat{U}_1 + \hat{U}_1^T - 2\hat{Z}_1 + \hat{W}_1 + \hat{W}_1^T\right)$$

$$\hat{\lambda}_{31}^1 = f_1(\hat{U}_1 + \hat{W}_1),\ \hat{\lambda}_{32}^1 = f_1(\hat{R}_1 - \hat{U}_1 + \hat{Z}_1 - \hat{W}_1),\ \hat{\lambda}_{33}^1 = f_1(\hat{R}_1 + \hat{Z}_1 + h\hat{Q}_1)$$

$$\hat{\lambda}_{77}^1 = -2e_1P_1 + e_1^2\hat{R}_1 - 2e_1P_1 + e_1^2\hat{Z}_1$$

$$\hat{\lambda}_{11}^2 = P_2\bar{A}_{ij} + \bar{A}_{ij}^TP_2^T - 2\alpha_2P_2 + \hat{Q}_2 + f_2\hat{R}_2 + f_2\hat{Z}_2$$

$$\hat{\lambda}_{21}^2 = f_2(\hat{R}_2 - \hat{U}_2 + \hat{Z}_2 - \hat{W}_2) - (I + \Delta_r)\bar{C}_{ij}^TP_2^T,\ f_2 = \frac{1}{h}e^{2\alpha_2h}$$

$$\hat{\lambda}_{22}^2 = f_2\left(-2\hat{R}_2 + \hat{U}_2 + \hat{U}_2^T - 2\hat{Z}_2 + \hat{W}_2 + \hat{W}_2^T\right) + \sigma H_1^T\bar{C}_i^T\Omega\bar{C}_iH_1$$

$$\hat{\lambda}_{31}^2 = f_2(\hat{U}_2 + \hat{W}_2),\ \hat{\lambda}_{32}^2 = f_2(\hat{R}_2 - \hat{U}_2 + \hat{Z}_2 - \hat{W}_2)$$

$$\hat{\lambda}_{33}^2 = f_2(\hat{R}_2 + \hat{Z}_2 + h\hat{Q}_2),\ \hat{\lambda}_{41}^2 = -(I + \Delta_r)Y_j^T,\ \hat{\lambda}_{72}^2 = \sqrt{h}(I + \Delta_r)P_2\bar{C}_{ij}$$

$$\hat{\lambda}_{74}^2 = \sqrt{h}(I + \Delta_r)Y_j,\ \hat{\lambda}_{75}^2 = -\sqrt{h}(I + \Delta_r)\bar{B}_{ij}^TP_2^T,\ \hat{\lambda}_{82}^2 = -\sqrt{m_1}H_2Y_jH_1$$

$$\hat{\lambda}_{77}^2 = -2e_2P_2 + e_2^2\hat{R}_2 - 2e_2P_2 + e_2^2\hat{Z}_2,\ \hat{\lambda}_{84}^2 = \sqrt{m_1h}H_2Y_jH_1,\ \hat{\lambda}_{92}^2 = H_1^T\bar{C}_j^TH_2$$

$$Y_j = diag\{Y_{1j}, Y_{2j}, \ldots, Y_{mj}\}, M_j = diag\{M_{1j}, M_{2j}, \ldots, M_{mj}\}.$$

Moreover, the sth security-guaranteed state estimator gains and coupling gains are obtained

$$K_{sj} = \left(P_{22}^{(s)}\right)^{-1} Y_{sj}, D_{sj} = \left(P_{22}^{(s)}\right)^{-1} M_{sj}, s \in \mathcal{V}. \tag{81}$$

Proof. When $q = 2$, by considering the impact of quantization, the matrix Φ_{ij}^2 can be expressed as

$$\Phi_{ij}^2 = \Psi_{ij}^2 + sym\left\{H_E^T \triangle_r H_F\right\}, \tag{82}$$

where

$$\Psi_{ij}^2 = \begin{bmatrix} \lambda_{11}^2 & * & * & * & * & * & * \\ \tilde{\lambda}_{21}^2 & \lambda_{22}^2 & * & * & * & * & * \\ \lambda_{31}^2 & \lambda_{32}^2 & \lambda_{33}^2 & * & * & * & * \\ \tilde{\lambda}_{41}^2 & 0 & 0 & -\Omega & * & * & * \\ \lambda_{51}^2 & 0 & 0 & 0 & -\gamma^2 I & * & * \\ -\tilde{L}_{ij} & 0 & 0 & 0 & 0 & -I & * \\ \sqrt{h}P_2\bar{A}_{ij} & \tilde{\lambda}_{72}^2 & 0 & \tilde{\lambda}_{74}^2 & \tilde{\lambda}_{75}^2 & 0 & \lambda_{77}^2 \end{bmatrix}$$

$$\tilde{\lambda}_{21}^2 = \tilde{\phi}_{21}^2 + \tilde{\psi}_{21}^2, \tilde{\phi}_{21}^2 = f_2(R_2 - U_2 + Z_2 - W_2)$$

$$\tilde{\psi}_{21}^2 = -\bar{C}_{ij}^T P_2^T, \tilde{\lambda}_{41}^2 = -K_j^T H_2^T P_2^T, \tilde{\lambda}_{72}^2 = \sqrt{h}P_2\bar{C}_{ij}$$

$$\tilde{\lambda}_{74}^2 = \sqrt{h}P_2H_2K_j, \tilde{\lambda}_{75}^2 = -\sqrt{h}P_2\bar{B}_{ij}$$

$$H_E = \begin{bmatrix} -H_2K_jH_1 & 0_{1\times 6} \end{bmatrix}$$

$$H_F = \begin{bmatrix} 0 & H_1^T\bar{C}_j^T H_2^T & 0 & H_1^T & 0_{1\times 3}. \end{bmatrix}$$

Based on Lemma 2 and (82), the following inequality can be acquired:

$$\Psi_{ij}^2 + m_1 H_E^T H_E + m_1^{-1} H_F^T H_F < 0. \tag{83}$$

By utilizing Schur complement, the equality (83) can be described as

$$\Phi_{ij}^2 = \begin{bmatrix} \Psi_{ij}^2 & * & * \\ H_E & -m_1 I & * \\ H_F & 0 & -m_1^{-1} I. \end{bmatrix} < 0 \tag{84}$$

Based on $(R_2 - e_2^{-1}P_2)R_2^{-1}(R_2 - e_2^{-1}P_2) \geq 0, (Z_2 - e_2^{-1}P_2)Z_2^{-1}(Z_2 - e_2^{-1}P_2) \geq 0$, one has

$$\begin{cases} -P_2R_2^{-1}P_2 \leq -2e_2P_2 + e_2^2R_2 \\ -P_2Z_2^{-1}P_2 \leq -2e_2P_2 + e_2^2Z_2. \end{cases} \tag{85}$$

Then, replace $-P_2R_2^{-1}P_2$ and $-P_2Z_2^{-1}P_2$ in (84) with $-2e_2P_2 + e_2^2R_2$ and $-2e_2P_2 + e_2^2Z_2$, respectively.

Define $X = I$, $\theta_2 = \text{diag}\{\underbrace{X, \ldots, X}_{7}, P_2^{-1}, X\}$, $P_2 = \text{diag}\{P_{21}, P_{22}\}$, $P_{22} = \text{diag}\{P_{22}^1, \ldots, P_{22}^m\}$, $Y_j = P_{22}K_j$, $M_j = P_{22}D_j$, $\hat{Q}_2 = XQ_2X$, $\hat{R}_2 = XR_2X$, $\hat{U}_2 = XU_2X$, $\hat{Z}_2 = XZ_2X$, $\hat{W}_2 = XW_2X$, and $\hat{\Omega}_2 = X\Omega_2X$. Premultiply and postmultiply Φ_{ij}^2

with θ_2 and θ_2^T. The following inequality can be derived:

$$\dot{V}_2(t) + z^T(t)z(t) - \gamma^2\bar{w}^T(t)\bar{w}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\xi^T(t)\hat{\Phi}^2_{ij}\xi(t). \tag{86}$$

When $q = 1$, by using same analysis method above, we can get

$$\dot{V}_1(t) + z^T(t)z(t) - \gamma^2\bar{w}^T(t)\bar{w}(t) \leq \sum_{i=1}^{r}\sum_{j=1}^{r} l_i(\pi(x))l_j(\pi(\hat{x}))\xi^T(t)\hat{\Phi}^1_{ij}\xi(t). \tag{87}$$

For inequality (32), by using Schur complement theory, then premultiplying and postmultiplying with diag$\{P_2^{-1}, P_2^{-1}\}$ and its transposition, one can derive (76). For inequality (33), following the similar method above, we can obtain (77). Following the similar method, it can be easily derived that three inequalities of (35) ensure (78) to (80) holding, respectively. According to Theorem 2, it yields that the system (28) is exponentially stable. Moreover, owing to $Y_j = P_{22}K_j$ and $M_j = P_{22}D_j$, the state estimator gains and coupling gains can be derived as $K_j = P_{22}^{-1}Y_j$ and $D_j = P_{22}^{-1}M_j$. That completes the proof. □

4 | NUMERICAL EXAMPLES

In this section, an illustrative example is utilized to testify the effectiveness of the proposed distributed state estimators for nonlinear networked systems against DoS attacks.

Consider the following system:

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{2} l_i(\pi(x))\{A_i x(t) + B_i\omega(t)\} \\ y_s(t) = \sum_{i=1}^{2} l_i(\pi(x))C_i x(t) \\ z(t) = \sum_{i=1}^{2} l_i(\pi(x))L_i x(t), \end{cases}$$

where

$$A_1 = \begin{bmatrix} -0.68 & 0.4 \\ 0.25 & -0.59 \end{bmatrix}, A_2 = \begin{bmatrix} -0.3 & 0.2 \\ 0.5 & -0.6 \end{bmatrix}, B_1 = \begin{bmatrix} 0.01 \\ 0.04 \end{bmatrix}, B_2 = \begin{bmatrix} 0.03 \\ 0.08 \end{bmatrix}, C_1 = \begin{bmatrix} -0.2 & 0.1 \\ -0.3 & 0.1 \end{bmatrix}$$
$$C_2 = \begin{bmatrix} -0.4 & 0.2 \\ -0.5 & 0.1 \end{bmatrix}, L_1 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, L_2 = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, l_1(\pi(x)) = \sin^2 t, l_2(\pi(x)) = \cos^2 t.$$

The initial conditions of the system are given by $x(t) = \begin{bmatrix} -1 & 1 \end{bmatrix}^T$, $\hat{x}_1(t) = \begin{bmatrix} -0.3 & 0.3 \end{bmatrix}^T$, $\hat{x}_2(t) = \begin{bmatrix} -0.4 & 0.4 \end{bmatrix}^T$, $\hat{x}_3(t) = \begin{bmatrix} -0.5 & 0.5 \end{bmatrix}^T$, and $\hat{x}_4(t) = \begin{bmatrix} -0.6 & 0.6 \end{bmatrix}^T$. The function of external disturbance is supposed as $\omega(t) = \begin{cases} \sin(\pi t), 0 \leq t \leq 20 \\ 0, \text{else} \end{cases}$.

The DoS attacks related parameters are given in Table 1. As shown in Figure 3, the matrix Λ is given as

$$\Lambda = \begin{bmatrix} 1 & 0 & 0 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & -1 & 2 & 0 \\ 0 & 0 & -1 & 1. \end{bmatrix}$$

TABLE 1 Denial-of-service-related parameters

	$n=0$	$n=1$	$n=2$	$n=3$	$n=4$		$n=5$	$n=6$	$n=7$	$n=8$	$n=9$
ℓ_n	0	2.20	1.92	1.91	2.53	ℓ_n	2.96	3.22	1.96	2.33	3.10
$\iota_{\max}$	0	0.29	0.12	0.21	0.19	$\iota_{\max}$	0.08	0.11	0.26	0.32	0.34
w_n	0	2.49	4.53	6.65	9.37	w_n	12.41	15.74	17.96	20.61	24.05

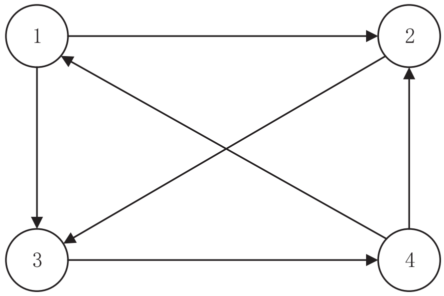

FIGURE 3 The structure of the estimators's networks

In the following, two cases are considered for the proposed method. In Case 1, quantization mechanism and ETS are not considered for nonlinear networked systems with DoS attacks. In Case 2, quantization mechanism and ETS are adopted for the system subjected to DoS attacks.

Case 1. Set the distributed event-generator parameters $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_4 = 0$, which declare that the ETS is not used. Meanwhile, the parameter $b = 0$ means that the quantization mechanism is not adopted. Set the H_∞ disturbance attenuation level $\gamma = 1.2$ and the DoS attacks parameters $\iota_{\max} = 0.35$, $\ell_{\min} = 1.89$, $e_1 = 1$, $e_2 = 1$, $\alpha_1 = 0.05$, $\alpha_2 = 0.5$, and $\mu_1 = \mu_2 = 1.01$ such that inequality (35) holds.

The state estimators gains and coupling gains can be obtained by solving Theorem 3

$$K_{11} = \begin{bmatrix} -0.0327 & -0.0518 \\ 0.0263 & 0.0218 \end{bmatrix}, K_{12} = \begin{bmatrix} -0.1382 & -0.1860 \\ 0.0326 & 0.0130 \end{bmatrix}, K_{21} = \begin{bmatrix} -0.0378 & -0.0544 \\ 0.0369 & 0.0330 \end{bmatrix}$$

$$K_{22} = \begin{bmatrix} -0.1311 & -0.1838 \\ 0.0616 & 0.0118 \end{bmatrix}, K_{31} = \begin{bmatrix} -0.0056 & -0.0105 \\ 0.0094 & 0.0051 \end{bmatrix}, K_{32} = \begin{bmatrix} -0.0761 & -0.1065 \\ 0.0014 & 0.0247 \end{bmatrix}$$

$$K_{41} = \begin{bmatrix} -0.0525 & -0.0785 \\ 0.0444 & 0.0449 \end{bmatrix}, K_{42} = \begin{bmatrix} -0.1690 & -0.2173 \\ 0.0696 & 0.0179 \end{bmatrix}, D_{11} = \begin{bmatrix} 1.5877 & -1.1930 \\ 4.4378 & -3.0940 \end{bmatrix}$$

$$D_{12} = \begin{bmatrix} 0.7710 & -1.0227 \\ 1.0975 & -0.9935 \end{bmatrix}, D_{21} = \begin{bmatrix} 1.0074 & -0.6036 \\ 2.3095 & -1.6707 \end{bmatrix}, D_{22} = \begin{bmatrix} 0.6802 & -0.8544 \\ 0.5956 & -0.5937 \end{bmatrix}$$

$$D_{31} = \begin{bmatrix} 1.0769 & -0.5245 \\ 1.5812 & -1.2932 \end{bmatrix}, D_{32} = \begin{bmatrix} 0.9685 & -1.0593 \\ 0.3368 & -0.3988 \end{bmatrix}, D_{41} = \begin{bmatrix} 0.9998 & -1.0966 \\ 7.6405 & -5.1338 \end{bmatrix}$$

$$D_{42} = \begin{bmatrix} 0.9107 & -1.3719 \\ 1.9426 & -1.5953 \end{bmatrix}.$$

Based on the obtained gains, the simulation results of case 1 are shown in Figures 4 to 6 by using MATLAB. In Figure 4, the red lines denote $x(t)$ and the blue lines represent $\hat{x}_1(t)$. Figure 5 shows the state estimation error systems can rapidly reach stable when the DoS attacks occur. The response of state estimation error $\tilde{z}(t)$ is given in Figure 6.

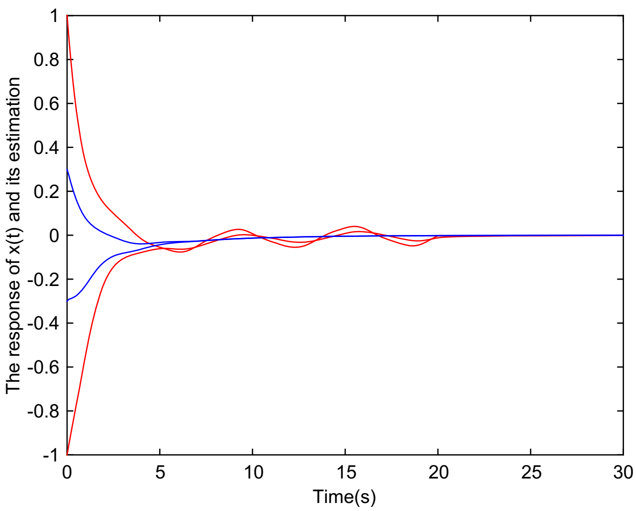

FIGURE 4 Response of state $x(t)$ and its estimation $\hat{x}_1(t)$ in case 1 [Colour figure can be viewed at wileyonlinelibrary.com]

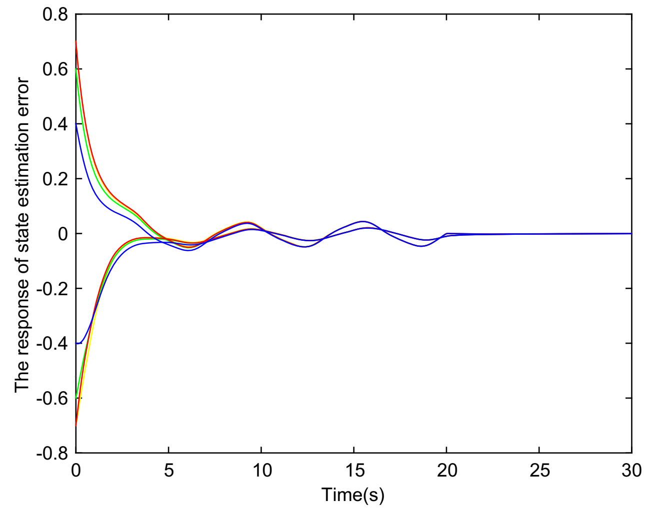

FIGURE 5 Response of estimation error $e(t)$ in case 1 [Colour figure can be viewed at wileyonlinelibrary.com]

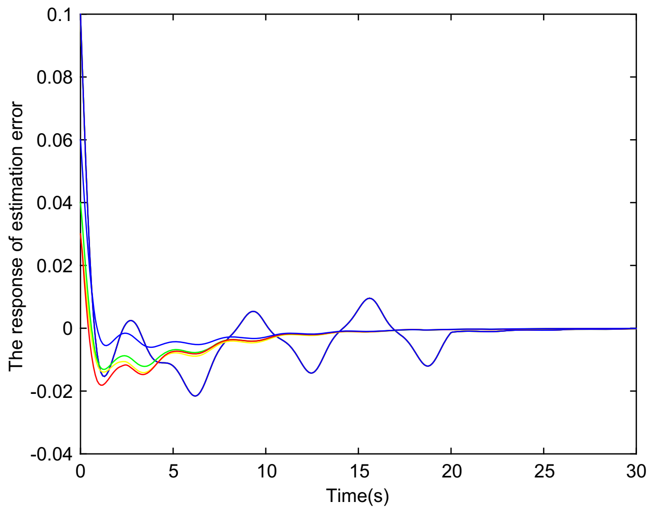

FIGURE 6 Response of estimation error $\tilde{z}(t)$ in case 1 [Colour figure can be viewed at wileyonlinelibrary.com]

Case 2. Set the parameters of event-triggered parameters $\sigma_1 = 0.4$, $\sigma_2 = 0.5$, $\sigma_3 = 0.6$, and $\sigma_4 = 0.3$ and the quantization parameter $b = 0.818$. In this case, an ETS and a quantization mechanism are adopted. Set the H_∞ disturbance attenuation level $\gamma = 1.2$ and the DoS attacks parameters $\iota_{\max} = 0.35$, $\ell_{\min} = 1.89$, $e_1 = 1$, $e_2 = 1$, $\alpha_1 = 0.05$, $\alpha_2 = 0.5$, and $\mu_1 = \mu_2 = 1.01$ such that inequality (35) holds.

By solving the linear matrix inequalities in Theorem 3 via MATLAB, one can obtain

$$K_{11} = \begin{bmatrix} -0.0067 & -0.0137 \\ 0.0057 & 0.0037 \end{bmatrix}, K_{12} = \begin{bmatrix} -0.0543 & -0.0803 \\ 0.0097 & 0.0095 \end{bmatrix}, K_{21} = \begin{bmatrix} -0.0026 & -0.0042 \\ 0.0055 & 0.0045 \end{bmatrix}$$
$$K_{22} = \begin{bmatrix} -0.0550 & -0.0820 \\ 0.0158 & 0.0023 \end{bmatrix}, K_{31} = \begin{bmatrix} -0.0030 & -0.0041 \\ 0.0010 & 0.0035 \end{bmatrix}, K_{32} = \begin{bmatrix} -0.0297 & -0.0453 \\ 0.0064 & 0.0152 \end{bmatrix}$$
$$K_{41} = \begin{bmatrix} -0.0062 & -0.0121 \\ 0.0069 & 0.0077 \end{bmatrix}, K_{42} = \begin{bmatrix} -0.0544 & -0.0761 \\ 0.0154 & 0.0003 \end{bmatrix}, D_{11} = \begin{bmatrix} 1.5221 & -1.2182 \\ 5.0718 & -3.4955 \end{bmatrix}$$
$$D_{12} = \begin{bmatrix} 0.7973 & -1.1258 \\ 1.0896 & -0.9645 \end{bmatrix}, D_{21} = \begin{bmatrix} 0.8225 & -0.4333 \\ 1.9424 & -1.4362 \end{bmatrix}, D_{22} = \begin{bmatrix} 0.7767 & -0.9895 \\ 0.4505 & -0.4706 \end{bmatrix}$$
$$D_{31} = \begin{bmatrix} 1.3202 & -0.7499 \\ 2.2461 & -1.7354 \end{bmatrix}, D_{32} = \begin{bmatrix} 1.1027 & -1.3143 \\ 0.5286 & -0.5650 \end{bmatrix}, D_{41} = \begin{bmatrix} 0.5413 & -0.6205 \\ 5.9936 & -4.0299 \end{bmatrix}$$
$$D_{42} = \begin{bmatrix} 0.8691 & -1.3564 \\ 1.4830 & -1.2087 \end{bmatrix}.$$

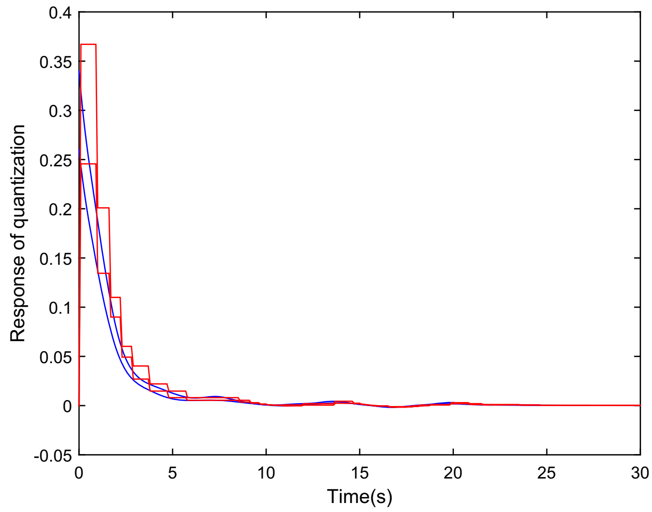

FIGURE 7 Response of quantization in case 2 [Colour figure can be viewed at wileyonlinelibrary.com]

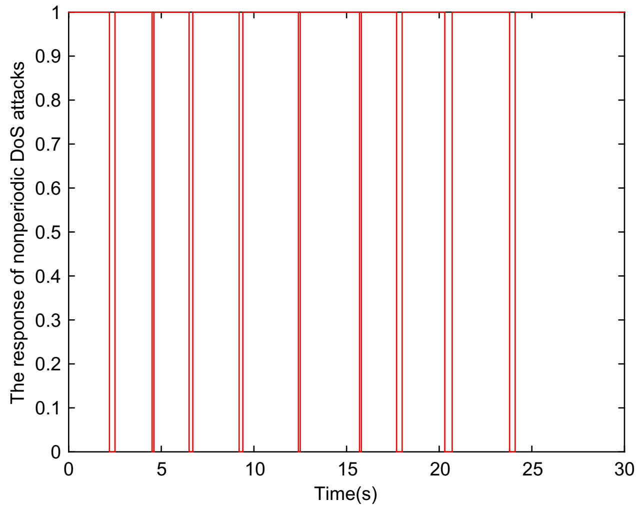

FIGURE 8 Response of denial-of-service (DoS) attacks in case 2 [Colour figure can be viewed at wileyonlinelibrary.com]

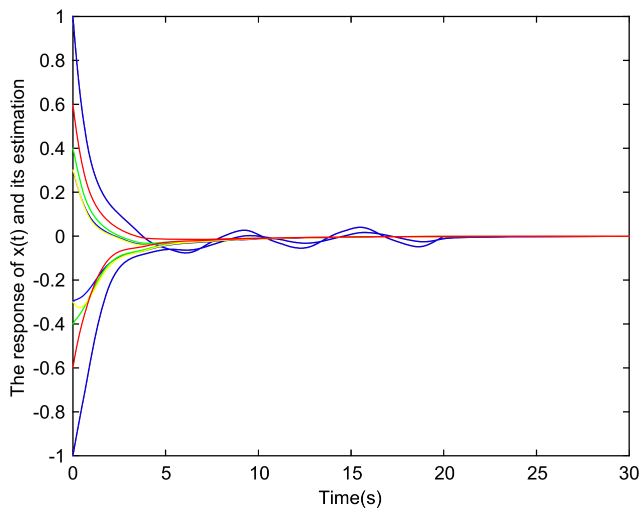

FIGURE 9 Response of state $\check{x}(t)$ and its estimation [Colour figure can be viewed at wileyonlinelibrary.com]

According to the derived gains, the simulation results exhibited in Figures 7 to 10 can be obtained through the MATLAB. In Figure 7, the blue lines represent the normal transmitted data without quantization, and the red lines denote quantized signals. Figure 8 represents the sequence of nonperiodic DoS attacks occurrence. The response of $x(t)$ and its estimation are shown in Figure 9. The graph of event-triggered instants and intervals is shown in Figure 10. Based

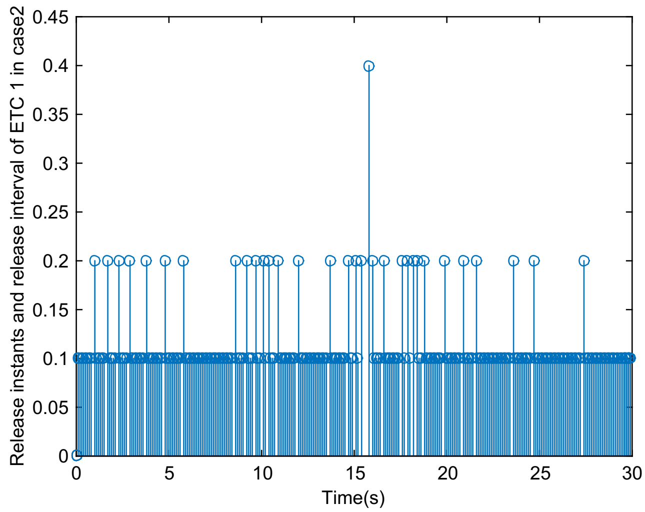

FIGURE 10 Release instants and intervals of event-triggered scheme 1 [Colour figure can be viewed at wileyonlinelibrary.com]

on the graphs above, we can demonstrate the feasibility of the designed distributed state estimation for nonlinear networked systems against DoS attacks.

5 | CONCLUSIONS

This paper has focused on security distributed state estimation for nonlinear networked systems against DoS attacks. First, an ETS and a quantization mechanism are adopted to alleviate the effect of resource constraints. In addition, a T-S fuzzy model is constructed for nonlinear networked systems against DoS attacks. By using Lyapunov stability theory, sufficient conditions ensuring the stability of state estimating error system are obtained. Moreover, the state estimator gains and coupling gains are acquired in terms of linear matrix inequalities. Finally, an example is utilized to verify the usefulness of proposed method. The future research will be connected with H_∞ fusion estimation and quantized filtering for nonlinear networked systems under the consideration of multiple cyber-attacks.

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