

Event-Based Secure Leader-Following Consensus Control for Multiagent Systems With Multiple Cyber Attacks

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Abstract—This article concentrates on event-based secure leader-following consensus control for multiagent systems (MASs) with multiple cyber attacks, which contain replay attacks and denial-of-service (DoS) attacks. A new multiple cyber-attacks model is first built by considering replay attacks and DoS attacks simultaneously. Different from the existing researches on MASs with a fixed topological graph, the changes of communication topologies caused by DoS attacks are considered for MASs. Besides, an event-triggered mechanism is adopted for mitigating a load of network bandwidth by scheduling the transmission of sampled data. Then, an event-based consensus control protocol is first developed for MASs subjected to multiple cyber attacks. In view of this, by using the Lyapunov stability theory, sufficient conditions are obtained to ensure the mean-square exponential consensus of MASs. Furthermore, the event-based controller gain is derived by solving a set of linear matrix inequalities. Finally, an example is simulated for confirming the effectiveness of the theoretical results.

Index Terms—Event-triggered mechanism, leader-following consensus control, multiagent systems (MASs), multiple cyber attacks.

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I. INTRODUCTION

AS AN important topic in the study of multiagent systems (MASs), the consensus problem has fascinated considerable attention due to its widespread applications in a long list of domains, such as traffic control, robots, power systems, unmanned aerial vehicles, and military [1]–[8]. In recent years, various consensus control problems have been studied for MASs, especially the leader-following consensus issue. A number of notable achievements have been acquired [9]–[14]. For example, the containment control problem for MASs with multiple leaders was addressed in [9]. Yin *et al.* [12] investigated the adaptive event-triggered consensus issue of the leader-following MASs with input saturation. In [13], the leader-following consensus for MASs was discussed with the event-triggered scheme. In reality, lots of factors can account for the unsatisfactory system performance and security crisis of MASs, such as the resource constraints problem and malicious attack signal, which impels us to study the event-based security leader-following consensus control of MASs.

Due to the limited communication resources and massive transmission data in the networked systems, it has been a vital research topic that how to improve the resource utilization rate to relieve the network load when guaranteeing the system performance [15], [16]. The time-triggered control strategy is usually adopted in the networked systems, where signal sampling and data delivering are performed periodically. However, when the current sampling data are almost the same with the latest transmitted data, large amounts of similar data are conveyed in the network, which results in network congestion. In order to overcome this drawback, multifarious event-triggered control strategies are developed to save limited communication resources, including continuous event-triggered mechanism [7], discrete event-triggered mechanism [17], distributed event-triggered mechanism [18], [19], and hybrid-triggered mechanism [20]. Among these control strategies, the discrete event-triggered mechanism proposed in [17] has received widespread attention since it is based on the discrete supervision of the system states and whether the sampled signal should be sent out or not depends on whenever its threshold is violated. A number of achievements have been procurable [21]–[23]. By applying the event-triggered scheme [17], Gu *et al.* [24] developed the filter design for a class of cyber-physical systems.

The event-triggered mechanism proposed in [17] was adopted in [25] where the problem of event-triggered predictive control for nonlinear systems was addressed. In [26], the leader-following consensus issue of second-order MASs was investigated by employing the event-triggered mechanism in [17].

In addition, system security is also a research hotspot of the networked systems. Cyber attack is one of the major threats to system security since it can bring system instability or even system paralysis [27]–[30]. Consequently, it is extremely important to consider the influences of cyber attacks in the modeling and analysis of networked systems. Recently, the security issue of diverse networked systems subjected to cyber attacks has been widely investigated, see [24], [31], and [32] for deception attacks; [33]–[35] for replay attacks; and [36]–[39] for denial-of-service (DoS) attacks. By taking the replay attacks into account, Zhu and Martinez [34] investigated the secure performance analysis for a class of networked control systems. In [35], the problem of secure fusion estimation was studied for cyber-physical systems under replay attacks. By considering the impacts of DoS attacks, the authors designed the state estimator for cyber-physical systems in [36]. The design of an observer-based event-triggered controller for networked linear systems was presented in [37] with the consideration of DoS attacks. The resilient H_∞ filtering problem of networked systems with DoS attacks was addressed in [38]. Up to now, most published achievements are concerned with single cyber attack, however, the actual system may be subjected to a variety of cyber attacks. In order to represent actual circumstances, the multiple cyber attacks are considered in this article, which contains replay attacks and DoS attacks. To the best of our knowledge, there has been no relevant result on the event-based secure leader-following consensus control for MASs under multiple cyber attacks, which motivates our present work.

Inspired by the aforementioned analysis, this article investigates the event-based secure leader-following consensus control for MASs with multiple cyber attacks. Then, the main contributions of this article are summarized as follows.

- 1) A mathematical model of multiple cyber attacks is established for MASs under the consideration of replay attacks and DoS attacks.
- 2) Different from the MASs with fixed topological graph [12], [13], the changes of communication topologies brought by DoS attacks are considered for the studied MASs.
- 3) A novel control protocol is first proposed for the MASs subject to multiple cyber attacks by the utilization of the event-triggered mechanism.
- 4) Sufficient conditions are acquired to guarantee the leader-following consensus of MASs in exponentially mean square. Moreover, the precise expressions of the desired controller gains are derived by solving a series of linear matrix inequalities.

The remainder of this article is organized as follows. The graph theory and problem formulation are provided in Section II. The main results of the event-based leader-following consensus control issue for the discussed MASs are given in Section III. In Section IV, the usefulness of the

designed method is illustrated through a simulated example. Finally, Section V presents the conclusions.

Notation: R^m and $R^{m \times n}$ represent the m -dimensional Euclidean space and the set of $m \times n$ real matrices, respectively; the superscript T denotes matrix transposition; I_N and I_{2N} indicate N -dimensional and $(2 \times N)$ -dimensional identity matrices, respectively; I denotes the identity matrix with appropriate dimension; \otimes is the Kronecker product of two matrices; for $S \in R^{m \times m}$, the notation $S > 0$ represents that the matrix S is real symmetric positive definite. $\varpi_m(A)$ and $\varpi_n(A)$ stand for the maximum and minimum eigenvalues of matrix A , respectively. $*$ in a symmetric matrix represents the entries implied by symmetry. $\ell_{2,n}^-$ denotes that $\ell_{2,n}$ belongs to time interval $[\ell_{1,n}, \ell_{2,n})$. $\mathbf{E}\{V(t)\}$ denotes the mathematical expectation of $V(t)$. \mathbb{N} stands for the set of non-negative integers.

II. PROBLEM FORMULATION AND MODELING

A. Graph Theory

In a directed graph $\mathbb{G} = (\mathbb{V}, \mathbb{E}, \mathbb{A})$, $\mathbb{V} = \{1, 2, \dots, N\}$, $\mathbb{E} \subseteq \{(i, j), i, j \in \mathbb{V}\}$, and $\mathbb{A} = [a_{ij}]_{N \times N}$ denote a set of nodes, a set of edges, and a weighted adjacency matrix with non-negative elements a_{ij} , $i, j = 1, 2, \dots, N$, respectively. An edge $(j, i) \in \mathbb{E}$ in graph \mathbb{G} denotes that agent i can acquire the information of agent j . $a_{ij} > 0$ only when $(i, j) \in \mathbb{E}$; $a_{ij} = 0$ when $(i, j) \notin \mathbb{E}$. $D = \text{diag}\{\text{indeg}_1, \text{indeg}_2, \dots, \text{indeg}_N\}$ is represented as the in-degree matrix, where $\text{indeg}_i = \sum_{j \in \mathbb{M}_i} a_{ij}$ for agent i . $\mathbb{M}_i = \{j | (i, j) \in \mathbb{E}\}$ is the set of neighbors of agent i . Define a Laplacian matrix $L = [l_{ij}]_{N \times N}$ of \mathbb{G} as $L = D - \mathbb{A}$.

B. Problem Formulation

Consider the following MAS composed of N followers and one leader labeled as node 0. The dynamics of the i th agent and the leader are described as

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Fh(x_i(t), t) + Bu_i(t) \\ \dot{x}_0(t) = Ax_0(t) + Fh(x_0(t), t) \end{cases} \quad (1)$$

where $x_i(t) \in R^a$ and $h(x_i(t), t) \in R^a$ are the state vectors and its nonlinear dynamics of the i th agent, respectively; $u_i(t) \in R^b$ is the control output vector of the i th agent; $i = \{1, 2, \dots, N\} \triangleq S_N$; and A , F , and B are constant matrices with appropriate dimensions.

Assumption 1 [12]: The nonlinear function $h: R^a \rightarrow R^a$ satisfies the Lipschitz condition, that is, for $\forall x, z \in R^a$, there exists a scalar $\epsilon > 0$ such that the following inequality holds:

$$\|h(x) - h(z)\| \leq \epsilon \|x - z\|. \quad (2)$$

Remark 1: In a directed communication graph \mathbb{G} , if there exists a directed spanning tree, then its Laplacian matrix L has N eigenvalues which contain a simple eigenvalue 0 and $N - 1$ eigenvalues with positive real parts. According to this fact, one can draw the conclusion that every agent in the MAS (1) possesses not only one neighboring agent, which is significant to design the following event-triggered communication mechanism in this article.

Remark 2: In the past decades, the consensus control problem of MASs has aroused much attention from various communities due to its extensive applications in practice, such

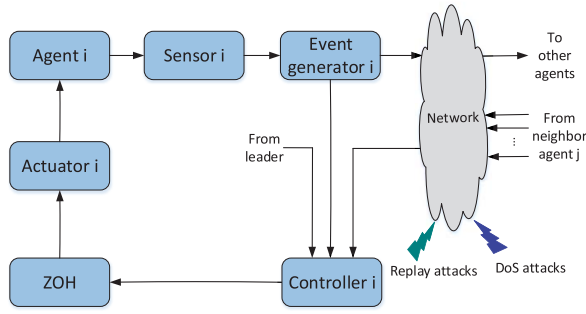


Fig. 1. Structure of event-based control for agent i .

as sensors networks, unmanned aerial, mobile robot networks, etc. In general, the MASs can be categorized as leaderless and leader-follower systems [9], [12], [40]. In this article, the leader-following consensus control for the MAS (1) with one leader is investigated.

This article aims at designing an event-based control strategy such that the system (1) under multiple cyber attacks can reach mean-square exponential consensus. For this purpose, the event-triggered mechanism is adopted for the MAS, the structure of which is exhibited in Fig. 1. In this framework, the sensor is time triggered while the controller, the zero-order hold (ZOH) and the actuator are event triggered. The state of each agent i is sampled in a sampling period h , then the sampled data can be sent out only when the given event-triggering condition is satisfied. Then, one can obtain the following sequence of transmitting instant for agent i :

$$t_{\lambda+1}^i = t_{\lambda}^i + \min_{f^i \geq 1} \{f^i h | (\psi^i(t_{\lambda}^i h))^T \Phi_i \psi^i(t_{\lambda}^i h) > \sigma \xi_i^T(t_{\lambda}^i h + f^i h) \Phi_i \xi_i(t_{\lambda}^i h + f^i h)\} \quad (3)$$

where parameter $\sigma \in [0, 1)$, $\Phi_i > 0$, $f^i = 1, 2, \dots$, and $\psi^i(t_{\lambda}^i h) = x_i(t_{\lambda}^i h) - x_i(t_{\lambda}^i h + f^i h)$, $i \in \mathcal{S}_N$, $h > 0$, $t_{\lambda}^i h$ denotes the latest transmitting instant, $\xi_i(t_{\lambda}^i h + f^i h) = \sum_{j=1}^{M_i} a_{ij} [x_i(t_{\lambda}^i h + f^i h) - x_j(t_{\lambda}^i h + f^i h)]$.

Remark 3: From (3), one can see that not all sampled data but the measurement data satisfying the threshold in (3) will be transmitted via the network. Resultly, the communication resources can be saved and the network load can also be relieved. Note that when $\sigma = 0$, all sampling data are sent out, which means that the event-triggered mechanism turns to be the time-triggered mechanism.

Remark 4: In comparison with periodic sampling (time-triggered scheme), the event-triggered mechanism adopted in this article is based on the discrete supervision of system states, where the data $x_i(t_{\lambda}^i h)$ will be broadcasted depends on whether the predetermined condition is violated or not. It should be mentioned that the event-triggered algorithm (3) first proposed in [17] has been widely applied to the modeling and analysis of networked systems [12], [24], [36], [41]. However, the event-based secure leader-following consensus control of MASs with multiple cyber attacks has not been investigated so far. Therefore, the event-triggered mechanism (3) is employed to mitigate a load of network bandwidth for the addressed MAS (1) in this article.

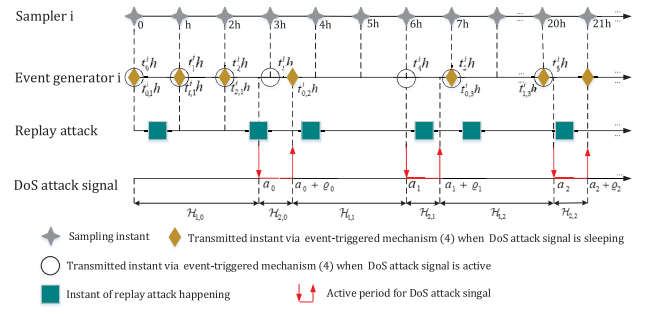


Fig. 2. Example of multiple cyber-attacks model.

Due to the openness of the communication network, the networked systems are susceptible to cyber attacks. As exhibited in Fig. 1, in this article, a class of multiple cyber attacks is taken into consideration, including replay attacks and DoS attacks. Fig. 2 shows an example of transmitted signals via the event-triggered mechanism (3) under multiple attacks model for agent i , which may be helpful to understand the mechanism of launching a multiple cyber attack.

The replay attackers launch an attack at time t by executing the following steps [35].

- 1) Record transmitted signals via the network from initial instant t_0 to current instant t . The set of the recorded data is represented as $\mathcal{T}_i(t) \triangleq \{\tilde{x}_i(t_0), \tilde{x}_i(t_1), \dots, \tilde{x}_i(t_m)\}$, $0 \leq t_0 < t_1 < \dots < t_m < t$.
- 2) Select one arbitrary data $\tilde{x}_i(t_r)$ from $\mathcal{T}_i(t)$ for replay.

In order to describe replay attacks, a Bernoulli random variable $\alpha_i(t)$ is introduced, whose expectation and mathematical variance are denoted as $\bar{\alpha}_i$ and ρ_i^2 , respectively. With the consideration of replay attacks, the i th agent's transmitted signal under replay attacks is presented as

$$\hat{x}_i(t) = \alpha_i(t) \tilde{x}_i(t_r) + (1 - \alpha_i(t)) \tilde{x}_i(t) \quad (4)$$

where $\alpha_i(t) \in \{0, 1\}$. $\alpha_i(t) = 1$ denotes that replay attacks happen and the normal data are replaced by $\tilde{x}_i(t_r)$; $\alpha_i(t) = 0$ indicates the absence of replay attacks. $\tilde{x}_i(t)$ is the normal signal of agent i , which will be broadcasted over the network; t_r is the previous instant and t is the current instant, obviously, one can obtain that $0 < t_r < t$, namely, $t_r = t - d(t)$, where $d(t) \in (0, d_M)$, d_M is the upper bound of $d(t)$. It is worth mentioning that the random variables $\alpha_i(t)$, $i \in \mathcal{S}_N$ are independent.

At the same time, the influences of DoS attacks are also taken into account. It needs to be mentioned that a DoS attack calls for a certain amount of energy. Suppose that the energy of the DoS jamming signal is limited, then the DoS jamming signal is sleeping at a certain period to save energy for the next attack. Therefore, a reasonable assumption is formalized as follows.

Assumption 2: Whether DoS attacks occur or not can be described by a variable $\beta(t)$, which satisfies

$$\beta(t) = \begin{cases} 1, & t \in [a_n, a_n + \varrho_n) \\ 0, & t \in [a_n + \varrho_n, a_{n+1}), n \in \mathbb{N} \end{cases}$$

where $[a_n, a_n + \varrho_n)$ and $[a_n + \varrho_n, a_{n+1})$ denote the sleeping period and active period of the DoS jamming signal,

respectively; and ϱ_n represents the length of the n th sleeping period. Then, we can obtain the starting instants, and ending instants of the DoS sleeping period satisfy $0 \leq a_0 < a_0 + \varrho_0 < a_1 < \dots < a_n < a_n + \varrho_n < a_{n+1} < \dots$.

Based on Assumption 2, denoting D_M and D_m as a uniform upper bound on the lengths of the DoS active periods $a_{n+1} - a_n - \varrho_n$ and a uniform lower bound on the lengths of the DoS sleeping periods ϱ_n , respectively, then one can acquire the following inequalities:

$$\begin{cases} D_M \geq \sup_{n \in \mathbb{N}} \{a_{n+1} - a_n - \varrho_n\} \\ D_m \leq \inf_{n \in \mathbb{N}} \{\varrho_n\}. \end{cases} \quad (5)$$

Besides, with the number of DoS attacks sleep/active transitions in the interval $[0, t)$ being represented as $o(t)$, then there exist $b_0 \geq 0$ and $f_a \geq h$ so as to satisfy the following condition:

$$o(t) \leq b_0 + \frac{t}{f_a}. \quad (6)$$

In order to facilitate analysis, denote $\mathcal{H}_{1,n} \triangleq [a_n, a_n + \varrho_n)$, $\mathcal{H}_{2,n} \triangleq [a_n + \varrho_n, a_{n+1})$, $\mathcal{D}_{\lambda,n}^i \triangleq [t_{\lambda,n}^i h, t_{\lambda+1,n}^i h)$ for $\forall n \in \mathbb{N}$, $t_{0,n} h \triangleq a_n$. The corresponding network-induced delay of transmitted instant $t_{\lambda,n}^i h$ is defined as $\delta_{\lambda,n}^i$. Denote $\bar{\delta} = \max\{\delta_{\lambda,n}^i\}$. Inspired by [20] and [37], to perform easier analysis, $\mathcal{D}_{\lambda,n}^i$ can be divided as $\mathcal{M}_{\lambda,n}^{\chi^i} \cup \mathcal{M}_{\lambda,n}^{p_{\lambda,n}+1}$, where

$$\begin{cases} \mathcal{M}_{\lambda,n}^{\chi^i} = [t_{\lambda,n}^i h + (\chi^i - 1)h + \bar{\delta}, t_{\lambda,n}^i h + \chi^i h + \bar{\delta}) \\ \chi^i \in \{1, 2, \dots, p_{\lambda,n}\} \\ \mathcal{M}_{\lambda,n}^{p_{\lambda,n}+1} = [t_{\lambda,n}^i h + p_{\lambda,n} h + \bar{\delta}, t_{\lambda+1,n}^i h) \\ p_{\lambda,n} \triangleq \inf\{\chi^i \in \mathbb{N} | t_{\lambda,n}^i h + \chi^i h \geq t_{\lambda+1,n}^i h\} \\ \lambda \in \{0, 1, 2, \dots, q(n)\} \triangleq \mathcal{U}(n) \\ q(n) = \sup\{\lambda \in \mathbb{N} | t_{\lambda,n}^i h \leq a_n + \varrho_n\}, n \in \mathbb{N}. \end{cases}$$

Then, the interval $\mathcal{H}_{1,n}$ can be rewritten as $\mathcal{H}_{1,n} = \bigcup_{\lambda=0}^{\mathcal{U}(n)} \bigcup_{\chi^i=1}^{p_{\lambda,n}+1} \{\mathcal{M}_{\lambda,n}^{\chi^i} \cap \mathcal{H}_{1,n}\}$. As a result, define

$$\begin{cases} \delta_{\lambda,n}^i(t) = t - t_{\lambda,n}^i h - (\theta^i - 1)h \\ \psi_{\lambda,n}^i(t) = x_i(t_{\lambda,n}^i h) - x_i(t_{\lambda,n}^i h + (\theta^i - 1)h) \end{cases} \quad (7)$$

where $t \in \mathcal{M}_{\lambda,n}^{\theta^i} \cap \mathcal{H}_{1,n}$, $\theta^i = 1, 2, \dots, p_{\lambda,n} + 1$. From (7), one can acquire that $0 < \delta_{\lambda,n}^i \leq \delta_{\lambda,n}^i(t) \leq h + \bar{\delta} \triangleq \delta_M$, δ_M is the upper bound of $\delta_{\lambda,n}^i(t)$. Further, the event-triggered sampled state $x_i(t_{\lambda,n}^i h)$ can be given as follows:

$$\tilde{x}_i(t) = x_i(t_{\lambda,n}^i h) = \psi_{\lambda,n}^i(t) + x_i(t - \delta_{\lambda,n}^i(t)) \quad (8)$$

with $\psi_{\lambda,n}^i(t)$ violating the following inequality:

$$(\psi_{\lambda,n}^i(t))^T \Phi_i \psi_{\lambda,n}^i(t) - \sigma \xi_i^T (t - \delta_{\lambda,n}^i(t)) \Phi_i \xi_i (t - \delta_{\lambda,n}^i(t)) \leq 0. \quad (9)$$

Then, by considering the impacts of replay attacks and DoS attacks, the actual control input of agent i received from agent j can be expressed as

$$\bar{x}_j(t) = \begin{cases} \alpha_j(t) \tilde{x}_j(t_r) + (1 - \alpha_j(t)) \tilde{x}_j(t) \\ t \in \mathcal{D}_{\lambda,n}^j \cap \mathcal{H}_{1,n} \\ 0, t \in \mathcal{H}_{2,n}. \end{cases} \quad (10)$$

Remark 5: In (10), when $t \in \mathcal{D}_{\lambda,n}^j \cap \mathcal{H}_{1,n}$ and $\alpha_j(t) = 0$, it means that no cyber attack occurs in the network, in this case, $\bar{x}_j(t) = \tilde{x}_j(t)$; when $t \in \mathcal{D}_{\lambda,n}^j \cap \mathcal{H}_{1,n}$ and $\alpha_j(t) = 1$, it means that only replay attacks happen, in such a situation, $\bar{x}_j(t) = \tilde{x}_j(t_r)$; when $t \in \mathcal{H}_{2,n}$, we can obtain $\bar{x}_j(t) = 0$, which means that the communication paralysis appears due to the presence of DoS attacks.

Remark 6: In reality, the networked systems are vulnerable to various cyber attacks resulting from the openness of the communication network. Note that there is a lot of literature about a single cyber attack while the research on multiple cyber attacks is deficient. In view of this, the effects of replay attacks and DoS attacks are concurrently discussed in this article, then the new mathematical model (10) of multiple cyber attacks is first established for the networked MASs.

Due to the DoS attacks, the communication topologies are no longer always fixed in this article. When the DoS jamming signal is sleeping in $\bigcup_{n \in \mathbb{N}} \mathcal{H}_{1,n}$, the communication topology of MASs is connected, where the data transmission is successful. When the DoS attacks occur in active periods $\bigcup_{n \in \mathbb{N}} \mathcal{H}_{2,n}$, consider the effects of DoS attacks on the communication links \mathbb{E} rather than the nodes \mathbb{V} . Under such circumstances, the original communication topology with a directed spanning tree is destroyed as a disconnected topological graph where the agents are assumed to give up communications. It should be pointed out that the impacts of DoS attacks will be wiped out at time a_{n+1} by some repairing efforts, then the topological graph is recovered to $\tilde{\mathbb{G}}$ with a directed spanning tree before the next DoS attack. The time spent in repatching the topological graph may be short such that it can be negligible. A piecewise constant function $\tau(t) : [0, +\infty) \rightarrow M = \{1, 2, \dots, m\}$ is utilized to depict the switching among different topology diagrams; $\mathbb{G}^{\tau(t)} \in \tilde{\mathbb{G}}$, for $t \in \mathcal{H}_{1,n}$, $n \in \mathbb{N}$, where $\tilde{\mathbb{G}} = \{\mathbb{G}^1, \mathbb{G}^2, \dots, \mathbb{G}^m\}$, $m \geq 1$, represents the set of all possible topological graphs when the MASs recover from DoS attacks.

On the basis of the analysis above, the event-based control protocol for system (1) is designed as follows:

$$u_i(t) = \begin{cases} -\phi K \left\{ \sum_{j=1}^{\mathbb{M}_i} a_{ij}^{\tau(t)} [\tilde{x}_i(t) - \bar{x}_j(t)] \right. \\ \left. + d_i^{\tau(t)} [\tilde{x}_i(t) - x_0(t)] \right\}, t \in \mathcal{D}_{\lambda,n}^i \cap \mathcal{H}_{1,n} \\ 0, t \in \mathcal{H}_{2,n} \end{cases} \quad (11)$$

where $\lambda \in \mathcal{U}(n)$, $n \in \mathbb{N}$, $\phi > 0$ denotes the coupling strength of the MASs, K is the controller gain to be determined later; $a_{ij}^{\tau(t)}$ stands for the adjacency element of $\mathbb{G}^{\tau(t)}$; and $d_i^{\tau(t)} = 1$ means that follower i can receive the information from the leader.

Remark 7: The structure of event-based control for agent i is illustrated in Fig. 1, where the information of agent i is sampled and then the measurements can be delivered to other neighbor agents via the network with multiple cyber attacks. The control input of agent i is the signals received from its neighbor agent j ($j \in \mathbb{M}_i$) and the leader, which means that the controller update frequency of agent i relies on the event instants of its own, the leader, and all neighbor agents. Then, the control output is transmitted to the actuator of agent i . The

event generator i is utilized to relieve the load of network bandwidth. It needs to be pointed out that a ZOH, located between the controller i and the actuator i , is adopted to maintain the control input of agent i immutable when there is no latest transmitted data reaching the controller i .

Remark 8: It should be noticed that this article considers attacks on the communication links \mathbb{E} instead of the nodes \mathbb{V} . When the DoS attacks take place in the network, it leads to disconnected communication topology, then there is no control input for MASs, namely, $u_i(t) = 0$ ($t \in \cup_{n \in \mathbb{N}} \mathcal{H}_{2,n}$). Then, the event-triggered control law (11) is proposed for MAS (1) under multiple cyber attacks to cut down the controller update frequency.

Define error $e_i(t) = x_i(t) - x_0(t)$, by combining (1), (8), (10), and (11), and let

$$\begin{aligned} x(t) &= [x_1^T(t), x_2^T(t), \dots, x_N^T(t)]^T \\ e(t) &= [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T \\ \check{\alpha}(t) &= \text{diag}\{\alpha_1(t), \alpha_2(t), \dots, \alpha_N(t)\} \\ \psi_{\lambda,n}(t) &= \left[\left(\psi_{\lambda,n}^1 \right)^T(t), \left(\psi_{\lambda,n}^2 \right)^T(t), \dots, \left(\psi_{\lambda,n}^N \right)^T(t) \right]^T \\ h(x(t), t) &= [h^T(x_1(t), t), h^T(x_2(t), t), \dots, h^T(x_N(t), t)]^T \\ H(x(t), t) &= [h^T(x_1(t), t) - h^T(x_0(t), t), h^T(x_2(t), t) \\ &\quad - h^T(x_0(t), t), \dots, h^T(x_N(t), t) - h^T(x_0(t), t)]^T \\ x(t - \delta_{\lambda,n}(t)) &= [x_1^T(t - \delta_{\lambda,n}^1(t)), x_2^T(t - \delta_{\lambda,n}^2(t)) \\ &\quad \dots, x_N^T(t - \delta_{\lambda,n}^N(t))]^T. \end{aligned}$$

Then we can obtain the consensus tracking error system

$$\dot{e}(t) = \begin{cases} (I_N \otimes A)e(t) + (I_N \otimes F)H(x(t), t) \\ - \phi(I_N \otimes B) \left\{ [L^{\tau(t)} \otimes K]e(t - \delta_{\lambda,n}(t)) \right. \\ + (D^{\tau(t)} \otimes K)\psi_{\lambda,n}(t) - (A^{\tau(t)} \otimes K)\psi_{\lambda,n}(t) \\ \left. - \check{\alpha}(t)(A^{\tau(t)} \otimes K)[e(t - r_{\lambda,n}(t)) + \psi_{\lambda,n}(t_r) \right. \\ \left. - e(t - \delta_{\lambda,n}(t)) - \psi_{\lambda,n}(t)] \right\} \\ + (D_0^{\tau(t)} \otimes K)\psi_{\lambda,n}(t) \Big\}, t \in \mathcal{D}_{\lambda,n}^i \cap \mathcal{H}_{1,n}, \lambda \in \mathcal{U}(n) \\ (I_N \otimes A)e(t) + (I_N \otimes F)H(x(t), t), t \in \mathcal{H}_{2,n} \end{cases} \quad (12)$$

where $r_{\lambda,n}(t) = d(t) + \delta_{\lambda,n}(t - d(t))$, and $r_{\lambda,n}(t) \in [0, r_M)$, r_M is the upper bound of $r_{\lambda,n}(t)$.

For technical convenience, denote

$$\begin{aligned} \eta(t) &= \begin{bmatrix} x(t) \\ e(t) \end{bmatrix}, \quad \tilde{F}(x(t), t) = \begin{bmatrix} h(x(t), t) \\ H(x(t), t) \end{bmatrix} \\ \bar{A} &= \text{diag}\{I_N \otimes A, I_N \otimes A\}, \quad \bar{F} = \text{diag}\{I_N \otimes F, I_N \otimes F\} \\ D_0^{\tau(t)} &= \text{diag}\{d_1^{\tau(t)}, d_2^{\tau(t)}, \dots, d_N^{\tau(t)}\}, \quad \hat{\alpha}(t) = \text{diag}\{\check{\alpha}(t), \check{\alpha}(t)\} \\ \bar{B}_1^{\tau(t)} &= \begin{bmatrix} 0_{N \times N} & \bar{L}^{\tau(t)} \otimes B \\ 0_{N \times N} & \bar{L}^{\tau(t)} \otimes B \end{bmatrix}, \quad \bar{L}^{\tau(t)} = L^{\tau(t)} + D_0^{\tau(t)} \\ \bar{B}_2^{\tau(t)} &= \begin{bmatrix} A^{\tau(t)} \otimes B - D^{\tau(t)} \otimes B - D_0^{\tau(t)} \otimes B \\ A^{\tau(t)} \otimes B - D^{\tau(t)} \otimes B - D_0^{\tau(t)} \otimes B \end{bmatrix} \\ \bar{B}_3^{\tau(t)} &= \begin{bmatrix} 0_{N \times N} & A^{\tau(t)} \otimes B \\ 0_{N \times N} & A^{\tau(t)} \otimes B \end{bmatrix}, \quad \bar{B}_4^{\tau(t)} = \begin{bmatrix} A^{\tau(t)} \otimes B \\ A^{\tau(t)} \otimes B \end{bmatrix} \\ H_1 &= [I_N \quad 0_{N \times N}], \quad H_2 = [0_{N \times N} \quad I_N] \end{aligned}$$

where $0_{N \times N}$ represents the N -dimensional zero matrix, which can be abbreviated as 0 sometimes. It needs to be pointed out

that in the following text, the element 0 in a matrix denotes the zero matrix with appropriate dimensions.

Then, by combining (1) and (12), the consensus tracking error system can be written as

$$\dot{\eta}(t) = \begin{cases} \bar{A}\eta(t) + \bar{F}\tilde{F}(x(t), t) - \phi\bar{B}_1^{\tau(t)}\bar{K}\eta(t - \delta_{\lambda,n}(t)) \\ + \phi\bar{B}_2^{\tau(t)}\bar{K}_1\psi_{\lambda,n}(t) + \phi\hat{\alpha}(t)\bar{B}_3^{\tau(t)}\bar{K}[\eta(t - r_{\lambda,n}(t)) \\ - \eta(t - \delta_{\lambda,n}(t))] + \phi\hat{\alpha}(t)\bar{B}_4^{\tau(t)}\bar{K}_1[\psi_{\lambda,n}(t_r)] \\ - \psi_{\lambda,n}(t), t \in \mathcal{D}_{\lambda,n}^i \cap \mathcal{H}_{1,n}, \lambda \in \mathcal{U}(n) \\ \bar{A}\eta(t) + \bar{F}\tilde{F}(x(t), t), t \in \mathcal{H}_{2,n} \end{cases} \quad (13)$$

where $\bar{K} = I_{2N} \otimes K$, $\bar{K}_1 = I_N \otimes K$; and $\zeta(t)$ is the supplemented initial condition of the state $\eta(t)$ with $\zeta(0) \triangleq \zeta_0$.

Remark 9: In (13), an event-based model of MASs under multiple cyber attacks has been established. To the best of our knowledge, the event-based secure leader-following consensus control issue of MASs with multiple cyber attacks is first addressed in this article.

Before we present the main results, the following assumption and definition are given.

Assumption 3: For $t \in \cup_{n \in \mathbb{N}} \mathcal{H}_{1,n}$, the topological graph $\mathbb{G}^{\tau(t)}$ has a directed spanning tree where the leader is the root node.

Definition 1 (Mean-Square Exponential Consensus Tracking [14]): The system (1) with the event-triggered control strategy $u_i(t)$ can reach mean-square exponential stability under multiple cyber attacks when studying the problem of leader-following consensus control for MASs, if there exist such a scalar $w > 0$ and a decay rate ν that the following inequality holds for any $t \geq 0$:

$$\begin{aligned} \mathbf{E} \left\{ \|x_i(t) - x_0(t)\|^2 \right\} \\ \leq w e^{-\nu t} \mathbf{E} \left\{ \|x_i(0) - x_0(0)\|^2 \right\}, i \in \mathcal{S}_N. \end{aligned} \quad (14)$$

III. MAIN RESULTS

Theorem 1: The DoS parameters D_M , D_m , b_0 , and f_a are known. For a given matrix K , and scalars $\sigma \in [0, 1)$, $\bar{\alpha}_i \in (0, 1)$, $\delta_M > 0$, $r_M > 0$, $h > 0$, system (13) is exponentially stable in mean square, if for some prescribed positive scalars $\mu_1, \mu_2, \varsigma = \mu_1 + \mu_2$, ω_1 and ω_2 , there exist symmetric positive-definite matrices Φ_i ($i \in \mathcal{S}_N$), $P_1, P_2, Q_1, Q_2, R_1, R_2, Z_1, Z_2, S_1$, and S_2 , positive scalars ϵ, ϕ , and matrices M_1, M_2, U_1 , and U_2 with appropriate dimensions such that the following inequalities hold:

$$\Pi_1 = \begin{bmatrix} \Gamma_{11}^1 & * & * & * \\ \Gamma_{21}^1 & \Gamma_{22}^1 & * & * \\ \Gamma_{31}^1 & 0 & \Gamma_{33}^1 & * \\ \Gamma_{41}^1 & \Gamma_{42}^1 & \Gamma_{43}^1 & \Gamma_{44}^1 \end{bmatrix} < 0 \quad (15)$$

$$\Pi_2 = \begin{bmatrix} \Gamma_{11}^2 & * & * \\ \Gamma_{21}^2 & \Gamma_{22}^2 & * \\ \Gamma_{31}^2 & \Gamma_{32}^2 & \Gamma_{33}^2 \end{bmatrix} < 0 \quad (16)$$

$$\begin{bmatrix} R_l & * \\ M_l & R_l \end{bmatrix} > 0, \quad \begin{bmatrix} S_l & * \\ U_l & S_l \end{bmatrix} > 0, l = 1, 2 \quad (17)$$

$$\begin{cases} (I_{2N} \otimes P_1) \leq \omega_2 (I_{2N} \otimes P_2) \\ (I_{2N} \otimes P_2) \leq \omega_1 e^{2\varsigma h} (I_{2N} \otimes P_1) \end{cases} \quad (18)$$

$$Q_l \leq \omega_{3-l} Q_{3-l}, \quad R_l \leq \omega_{3-l} R_{3-l}, \quad l = 1, 2 \quad (19)$$

$$\vartheta = \frac{2}{f_a}(\mu_1 D_m - \mu_2 D_M - \zeta h) - \frac{1}{f_a} \ln(\omega_1 \omega_2) > 0 \quad (20)$$

where the elements of Π_1 and Π_2 are given in Appendix A.

Proof: See Appendix B. ■

In Theorem 1, the sufficient conditions are achieved which can guarantee mean-square exponential consensus of system (13). In the following text, the controller gain of MASs with event-triggered mechanism and multiple cyber attacks is derived on the basis of Theorem 1.

Theorem 2: For given DoS parameters D_M , D_m , b_0 , and f_a , and scalars $\sigma \in [0, 1)$, $\bar{\alpha}_i \in (0, 1)$, $\delta_M > 0$, $r_M > 0$, $h > 0$, system (13) is exponentially stable in mean square, if for some prescribed positive scalars μ_1 , μ_2 , $\zeta = \mu_1 + \mu_2$, ω_1 and ω_2 , there exist symmetric positive-definite matrices X_1 , X_2 , $\hat{\Phi}_i$ ($i \in \mathcal{S}_N$), \hat{Q}_1 , \hat{Q}_2 , \hat{R}_1 , \hat{R}_2 , \hat{Z}_1 , \hat{Z}_2 , \hat{S}_1 , and \hat{S}_2 ; positive scalars ϵ , ϕ , v_1 , v_2 , v_3 , v_4 , v_{12} , and v_{22} ; and matrices Y , \hat{M}_1 , \hat{M}_2 , \hat{U}_1 , and \hat{U}_2 with appropriate dimensions such that the inequality (20) and the following linear matrix inequalities hold:

$$\hat{\Pi}_1 = \begin{bmatrix} \hat{\Gamma}_{11}^1 & * & * & * \\ \hat{\Gamma}_{21}^1 & \hat{\Gamma}_{22}^1 & * & * \\ \hat{\Gamma}_{31}^1 & 0 & \hat{\Gamma}_{33}^1 & * \\ \hat{\Gamma}_{41}^1 & \hat{\Gamma}_{42}^1 & \hat{\Gamma}_{43}^1 & \hat{\Gamma}_{44}^1 \end{bmatrix} < 0 \quad (21)$$

$$\hat{\Pi}_2 = \begin{bmatrix} \hat{\Gamma}_{11}^2 & * & * \\ \hat{\Gamma}_{21}^2 & \hat{\Gamma}_{22}^2 & * \\ \hat{\Gamma}_{31}^2 & \hat{\Gamma}_{32}^2 & \hat{\Gamma}_{33}^2 \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} \hat{R}_l & * \\ \hat{M}_l & \hat{R}_l \end{bmatrix} > 0, \begin{bmatrix} \hat{S}_l & * \\ \hat{U}_l & \hat{S}_l \end{bmatrix} > 0, l = 1, 2 \quad (23)$$

$$\begin{bmatrix} -\omega_2(I_{2N} \otimes X_2) & * \\ I_{2N} \otimes X_2 & -(I_{2N} \otimes X_1) \end{bmatrix} \leq 0 \quad (24)$$

$$\begin{bmatrix} -\omega_1 e^{2\zeta h} (I_{2N} \otimes X_1) & * \\ I_{2N} \otimes X_1 & -(I_{2N} \otimes X_2) \end{bmatrix} \leq 0 \quad (25)$$

$$\begin{bmatrix} -\omega_2 \hat{Q}_2 & * \\ I_{2N} \otimes X_2 & -2v_1 (I_{2N} \otimes X_1) + v_1^2 \hat{Q}_1 \end{bmatrix} \leq 0 \quad (26)$$

$$\begin{bmatrix} -\omega_1 \hat{Q}_1 & * \\ I_{2N} \otimes X_1 & -2v_3 (I_{2N} \otimes X_2) + v_3^2 \hat{Q}_2 \end{bmatrix} \leq 0 \quad (27)$$

$$\begin{bmatrix} -\omega_2 \hat{R}_2 & * \\ I_{2N} \otimes X_2 & -2v_{12} (I_{2N} \otimes X_1) + v_{12}^2 \hat{R}_1 \end{bmatrix} \leq 0 \quad (28)$$

$$\begin{bmatrix} -\omega_1 \hat{R}_1 & * \\ I_{2N} \otimes X_1 & -2v_{22} (I_{2N} \otimes X_2) + v_{22}^2 \hat{R}_2 \end{bmatrix} \leq 0 \quad (29)$$

where the elements of $\hat{\Pi}_1$ and $\hat{\Pi}_2$ are given in Appendix C.

Furthermore, the controller gain can be obtained as

$$K = YX_1^{-1}. \quad (30)$$

Proof: See Appendix D. ■

Remark 10: In Theorem 2, according to these known parameters and matrices, unknown quantities Y and X_1 are derived through solving linear matrix inequalities (21)–(29) in MATLAB. Moreover, the controller gain K is acquired by applying equality (30) when the system (13) is exponentially stable in mean square.

Remark 11: With the increasing complexity of the actual system, more agent nodes are needed to describe it, which results in certain computational complexity. In addition,

TABLE I
MEANINGS OF THE PARAMETERS IN SECTION IV

Parameter	Meaning
$x_{i1}(t)$	angle of attack
$x_{i2}(t)$	pitch rate
$\dot{x}_{i1}(t)$	angular velocity of attack
$\dot{x}_{i2}(t)$	pitch acceleration
$u_{i1}(t)$	symmetric elevator position
$u_{i2}(t)$	symmetric pitch thrust velocity nozzle position
A_{Long}	longitudinal state matrix
B_{Long}	longitudinal control input matrix

the application of the processing technology (for instance, Kronecker product) also brings computational complexity. Fortunately, the results of this article are obtained through offline calculation. Besides, with the improvement of hardware equipment, the computing capability is improved, which reduces the influence of computational complexity to some extent.

IV. SIMULATION EXAMPLES

In this section, a simulation example is provided for validating the feasibility of the event-based leader-following control strategy for MASs, where the dynamic of each agent can be described by an F-18 aircraft model [13], [42]. Then, the decoupling linearized longitudinal state dynamical equations of agent i are expressed as

$$\begin{bmatrix} \dot{x}_{i1}(t) \\ \dot{x}_{i2}(t) \end{bmatrix} = A_{Long} \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix} + B_{Long} \begin{bmatrix} u_{i1}(t) \\ u_{i2}(t) \end{bmatrix}$$

where the parameters are given in Table I.

In the following, the velocity of 3 Mach and height of 26 kft are chosen as the flight condition of F-18 aircraft, then one can obtain the following system matrices A_{Long} and B_{Long} [43]:

$$A_{Long} = \begin{bmatrix} -0.2296 & 0.993 \\ -0.02436 & -0.2406 \end{bmatrix}$$

$$B_{Long} = \begin{bmatrix} -0.01434 & -0.01145 \\ -1.73 & -0.517 \end{bmatrix}.$$

$F = I_2$, the state vector of agent i is $x_i(t) = [x_{i1}^T(t) \ x_{i2}^T(t)]^T$, control output of agent i is $u_i(t) = [u_{i1}^T(t) \ u_{i2}^T(t)]^T$, then $h(x_i(t), t) = [0.3 \sin(0.4x_{i1}(t)), -0.2 \tan(0.1x_{i2}(t))]^T$. Suppose that there are one leader and eight followers in the agent system, whose communication topology is described as Fig. 3(a). Then, the Laplacian L has the following form:

$$L = \begin{bmatrix} 2 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 3 & 0 & -1 & 0 & -1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 2 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Remark 12: Fig. 3(a) satisfying Assumption 3 shows one kind of communication topology for MASs without DoS attacks. When DoS attacks occur, one possible topology graph of MASs is shown in Fig. 3(b), where there exists no directed

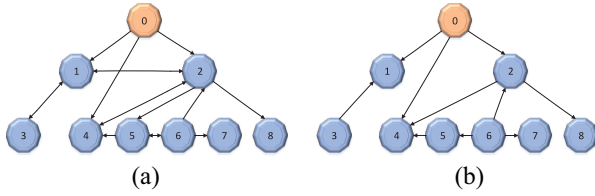


Fig. 3. Communication topology of MASs (a) without DoS attacks and (b) with DoS attacks.

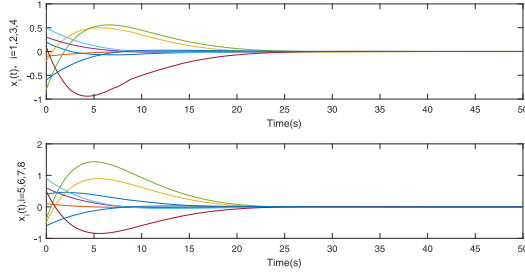


Fig. 4. Responses of $x_i(t)$.

spanning tree and the agents abandon communication. It should be pointed out that due to page limitations, in this section, suppose that the communication topology of MASs without DoS attacks is exhibited in Fig. 3(a). In this case, the controller gain is derived by solving the linear matrix inequalities of Theorem 2 in MATLAB. The other cases of communication topologies for MASs can be discussed and simulated the same way.

Set $\sigma = 0.2$, $\epsilon = 0.2$, $\phi = 0.45$, $\delta_M = 0.2$, $r_M = 0.5$, $\bar{\alpha}_1 = \bar{\alpha}_5 = 0.035$, $\bar{\alpha}_2 = \bar{\alpha}_6 = \bar{\alpha}_8 = 0.04$, $\bar{\alpha}_3 = \bar{\alpha}_4 = 0.03$, $\bar{\alpha}_7 = 0.02$, and $v_1 = v_2 = v_3 = v_4 = 0.1$, $v_{12} = v_{22} = 0.1$. Let $h = 0.01$ s, $D_m = 1.02$ s, $D_M = 1.23$ s, $\omega_1 = \omega_2 = 1.02$, and $\mu_1 = 0.08$, $\mu_2 = 1.06$. Through solving linear matrix inequalities in MATLAB, it can be obtained that

$$Y = \begin{bmatrix} -19.0258 & 63.8747 \\ 63.8747 & -214.0046 \end{bmatrix}, X = \begin{bmatrix} 49.9773 & 1.2792 \\ 1.2792 & 23.3038 \end{bmatrix}$$

$$\Phi_1 = \begin{bmatrix} 85.8297 & -5.2213 \\ -5.2213 & 65.6375 \end{bmatrix}, \Phi_2 = \begin{bmatrix} 195.4735 & 0.6162 \\ 0.6162 & 78.1812 \end{bmatrix}$$

$$\Phi_3 = \begin{bmatrix} 70.5948 & -12.7171 \\ -12.7171 & 196.1262 \end{bmatrix}, \Phi_4 = \begin{bmatrix} 151.4484 & -17.1506 \\ -17.1506 & 122.6408 \end{bmatrix}$$

$$\Phi_5 = \begin{bmatrix} 391.5966 & -0.0590 \\ -0.0590 & 391.7730 \end{bmatrix}, \Phi_6 = \begin{bmatrix} 391.5907 & -0.0400 \\ -0.0400 & 391.7090 \end{bmatrix}$$

$$\Phi_7 = \begin{bmatrix} 391.5809 & -0.0098 \\ -0.0098 & 391.6106 \end{bmatrix}, \Phi_8 = \begin{bmatrix} 391.5809 & -0.0098 \\ -0.0098 & 391.6105 \end{bmatrix}.$$

Then, by utilizing equality (30) in Theorem 2, we can obtain the controller gain

$$K = \begin{bmatrix} -0.4515 & 2.7657 \\ 1.5153 & -9.2664 \end{bmatrix}.$$

The initial conditions are chosen as

$$x_0 = [0.4 \quad -0.5]^T, x_1 = [0.2 \quad -0.1]^T, x_2 = [-0.2 \quad 0.3]^T$$

$$x_3 = [-0.8 \quad 0.5]^T, x_4 = [0.1 \quad -0.6]^T, x_5 = [0.4 \quad 0.1]^T$$

$$x_6 = [-0.5 \quad 0.6]^T, x_7 = [-0.4 \quad 0.9]^T, x_8 = [0.5 \quad -0.6]^T.$$

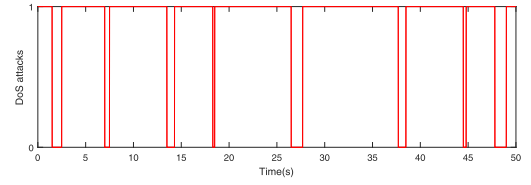


Fig. 5. Signal of DoS attacks.

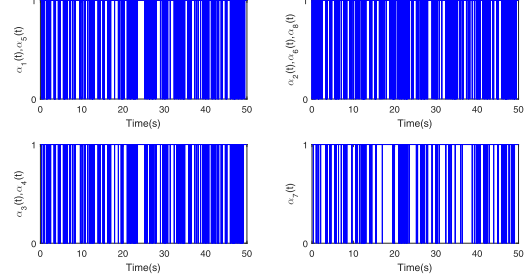


Fig. 6. Bernoulli variables for replay attacks.

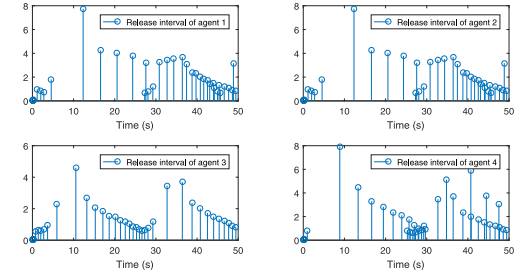


Fig. 7. Release instants and release intervals of Agents 1-4.

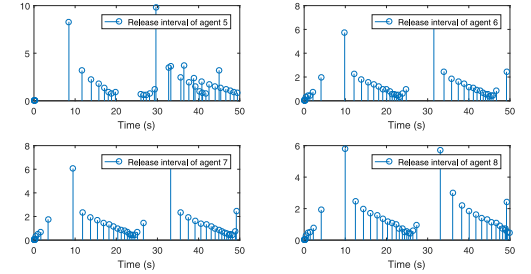


Fig. 8. Release instants and release intervals of Agents 5-8.

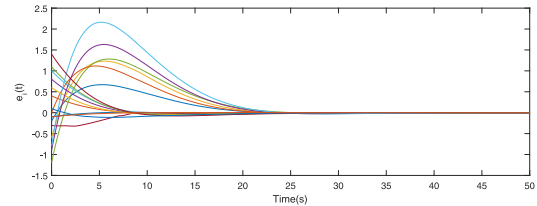


Fig. 9. Responses of errors $e_i(t)$.

Then, by using MATLAB, the simulation results are shown in Figs. 4-10. The state responses of $x_i(t)$ are presented in Fig. 4. The DoS jamming signal with $D_M = 1.23$ and $D_m = 1.02$ is described in Fig. 5. Fig. 6 presents the Bernoulli variables for replay attacks. Figs. 7 and 8 exhibit the release

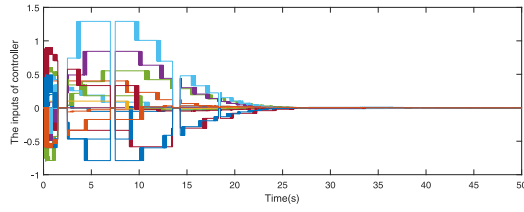


Fig. 10. Responses of controller inputs.

time intervals of eight agents, respectively. Besides, the controller inputs are shown in Fig. 10. The consensus tracking errors $e_i(t)$ are exhibited in Fig. 9, which achieve stability even though the MASs are subjected to multiple cyber attacks. Based on the above analysis, it can be concluded that the event-triggered mechanism adopted in this article can relieve the communication load. The discussed MASs with event-triggered mechanism and multiple cyber attacks can reach consensus. In other words, the designed controller for MASs under multiple cyber attacks performs well.

V. CONCLUSION

In this article, the event-based secure leader-following consensus control for MASs with multiple cyber attacks has been investigated, including replay and DoS attacks. The addressed DoS attacks contribute to the changes of communication topology in MASs. To reduce network bandwidth consumption, the event-triggered mechanism is implemented. Then, the event-based consensus control protocol is first proposed for MASs under multiple cyber attacks. Moreover, sufficient conditions are derived to guarantee the attainment of mean-square exponential consensus for MASs and the event-based controller gain is acquired by applying the Lyapunov stability theory and the linear matrix inequality technique. Finally, the example of the F-18 aircraft model is presented to testify the validity of theoretical results. Future discussion is concerned with the containment control problem about multiple leaders in MASs, along with the attack detection and defense to improve the MASs performance against cyber attacks.

APPENDIX A

ELEMENTS OF Π_1 AND Π_2 IN THEOREM 1

$$\Gamma_{11}^1 = \begin{bmatrix} b_1 & * & * \\ b_2 & b_3 & * \\ -g_1 M_1 & g_1(R_1 + M_1) & -g_1 Q_1 - g_1 R_1 \end{bmatrix}$$

$$b_1 = 2\mu_1(I_{2N} \otimes P_1) + Q_1 + Z_1 - g_1 R_1 - g_2 S_1 + (I_{2N} \otimes P_1)\bar{A} + \bar{A}^T(I_{2N} \otimes P_1)$$

$$b_2 = g_1(R_1 + M_1) - \phi \bar{K}^T (\bar{B}_1^{\tau(t)})^T (I_{2N} \otimes P_1) - \phi \bar{K}^T (\bar{B}_3^{\tau(t)})^T (I_{2N} \otimes P_1)$$

$$b_3 = \sigma H_1^T (\bar{A}^{\tau(t)})^T \Phi \bar{A}^{\tau(t)} H_1 + g_1(-2R_1 - M_1 - M_1^T)$$

$$\Gamma_{21}^1 = \begin{bmatrix} b_4 & 0 & 0 \\ -g_2 U_1 & 0 & 0 \\ \phi \bar{K}_1^T (\bar{B}_4^{\tau(t)})^T \bar{\alpha} (I_{2N} \otimes P_1) & 0 & 0 \end{bmatrix}$$

$$b_4 = \phi \bar{K}^T (\bar{B}_3^{\tau(t)})^T \bar{\alpha} (I_{2N} \otimes P_1) + g_2(S_1 + U_1)$$

$$\Phi = \text{diag}\{\Phi_1, \Phi_2, \dots, \Phi_N\}$$

$$\Gamma_{22}^1 = \begin{bmatrix} b_5 & * & * \\ g_2(S_1 + U_1) & -g_2 Z_1 - g_2 S_1 & * \\ 0 & 0 & -\Phi \end{bmatrix}$$

$$b_5 = \sigma H_1^T (\bar{A}^{\tau(t)})^T \Phi \bar{A}^{\tau(t)} H_1 + g_1(-2S_1 - U_1 - U_1^T)$$

$$\Gamma_{31}^1 = \begin{bmatrix} b_6 & 0 & 0 \\ H_1 \bar{F}^T (I_{2N} \otimes P_1) & 0 & 0 \\ \epsilon H_2 & 0 & 0 \end{bmatrix}$$

$$b_6 = \phi \bar{K}_1^T (\bar{B}_4^{\tau(t)})^T (I_{2N} \otimes P_1) - \phi \bar{K}_1^T (\bar{B}_2^{\tau(t)})^T \bar{\alpha} (I_{2N} \otimes P_1)$$

$$\Gamma_{33} = \text{diag}\{-\Phi, -I, -I\}, \quad g_1 = e^{-2\mu_1 \delta_M}, \quad g_2 = e^{-2\mu_1 r_M}$$

$$\Gamma_{41}^1 = \begin{bmatrix} \delta_M (I_{2N} \otimes P_1) \bar{A} & b_7 & 0 \\ r_M (I_{2N} \otimes P_1) \bar{A} & b_8 & 0 \\ 0 & b_9 & 0 \\ 0 & b_{10} & 0 \end{bmatrix}$$

$$b_7 = -\delta_M \phi (I_{2N} \otimes P_1) \bar{B}_1^{\tau(t)} \bar{K} - \delta_M \phi (I_{2N} \otimes P_1) \bar{\alpha} \bar{B}_3^{\tau(t)} \bar{K}$$

$$b_8 = -r_M \phi (I_{2N} \otimes P_1) \bar{B}_1^{\tau(t)} \bar{K} - r_M \phi (I_{2N} \otimes P_1) \bar{\alpha} \bar{B}_3^{\tau(t)} \bar{K}$$

$$b_9 = \delta_M \phi (I_{2N} \otimes P_1) \rho \bar{B}_3^{\tau(t)} \bar{K}$$

$$b_{10} = r_M \phi (I_{2N} \otimes P_1) \rho \bar{B}_3^{\tau(t)} \bar{K}$$

$$\Gamma_{42}^1 = \begin{bmatrix} \delta_M \phi (I_{2N} \otimes P_1) \bar{\alpha} \bar{B}_3^{\tau(t)} \bar{K} & 0 & \delta_M \phi P_{140} \\ r_M \phi (I_{2N} \otimes P_1) \bar{\alpha} \bar{B}_3^{\tau(t)} \bar{K} & 0 & r_M \phi P_{140} \\ -\delta_M \phi (I_{2N} \otimes P_1) \rho \bar{B}_3^{\tau(t)} \bar{K} & 0 & -\delta_M \phi P_{141} \\ -r_M \phi (I_{2N} \otimes P_1) \rho \bar{B}_3^{\tau(t)} \bar{K} & 0 & -r_M \phi P_{141} \end{bmatrix}$$

$$P_{140} = (I_{2N} \otimes P_1) \bar{\alpha} \bar{B}_4^{\tau(t)} \bar{K}_1, \quad P_{141} = (I_{2N} \otimes P_1) \bar{\alpha} \bar{B}_4^{\tau(t)} \bar{K}_1$$

$$\Gamma_{43}^1 = \begin{bmatrix} \delta_M \phi b_{11} & \delta_M (I_{2N} \otimes P_1) \bar{F} H_3 & 0 \\ r_M \phi b_{12} & r_M (I_{2N} \otimes P_1) \bar{F} H_3 & 0 \\ \delta_M \phi P_{141} & 0 & 0 \\ r_M \phi P_{141} & 0 & 0 \end{bmatrix}$$

$$b_{11} = (I_{2N} \otimes P_1) \bar{B}_2^{\tau(t)} \bar{K}_1 - (I_{2N} \otimes P_1) \bar{\alpha} \bar{B}_4^{\tau(t)} \bar{K}_1$$

$$\bar{\alpha} = \text{diag}\{\bar{\alpha}_1, \dots, \bar{\alpha}_N, \bar{\alpha}_1, \dots, \bar{\alpha}_N\}$$

$$\rho = \text{diag}\{\rho_1, \dots, \rho_N, \rho_1, \dots, \rho_N\}, \quad \rho_i = \sqrt{\bar{\alpha}_i(1 - \bar{\alpha}_i)}$$

$$\Gamma_{44}^1 = \text{diag}\left\{ (I_{2N} \otimes P_1)^{-1} R_1 (I_{2N} \otimes P_1) \right. \\ \left. (I_{2N} \otimes P_1)^{-1} S_1 (I_{2N} \otimes P_1) \right. \\ \left. (I_{2N} \otimes P_1)^{-1} R_1 (I_{2N} \otimes P_1) \right. \\ \left. (I_{2N} \otimes P_1)^{-1} S_1 (I_{2N} \otimes P_1) \right\}$$

$$\bar{K} = I_{2N} \otimes K, \quad \bar{K}_1 = I_N \otimes K$$

$$\Gamma_{11}^2 = \begin{bmatrix} \mathbf{b}_1 & * & * \\ R_2 + M_2 & -2R_2 - M_2 - M_2^T & * \\ -M_2 & R_2 + M_2 & -g_3 Q_2 - R_2 \end{bmatrix}$$

$$\mathbf{b}_1 = -2\mu_2(I_{2N} \otimes P_2) + Q_2 + Z_2 - R_2 - S_2$$

$$+ (I_{2N} \otimes P_2) \bar{A} + \bar{A}^T (I_{2N} \otimes P_2)$$

$$\begin{aligned}\Gamma_{21}^2 &= \begin{bmatrix} S_2 + U_2 & 0 & 0 \\ -U_2 & 0 & 0 \\ H_1 \bar{F}^T (I_{2N} \otimes P_2) & 0 & 0 \\ \epsilon H_2 & 0 & 0 \end{bmatrix} \\ \Gamma_{22}^2 &= \begin{bmatrix} -2S_2 - U_2 - U_2^T & * & * & * \\ S_2 + U_2 & \mathbf{b}_2 & * & * \\ 0 & 0 & -I & * \\ 0 & 0 & 0 & -I \end{bmatrix} \\ \mathbf{b}_2 &= -g_4 Z_2 - S_2, \quad g_3 = e^{2\mu_2 \delta_M}, \quad g_4 = e^{2\mu_2 r_M} \\ \Gamma_{31}^2 &= \begin{bmatrix} \delta_M (I_{2N} \otimes P_2) \bar{A} & 0 & 0 \\ r_M (I_{2N} \otimes P_2) \bar{A} & 0 & 0 \end{bmatrix} \\ \Gamma_{32}^2 &= \begin{bmatrix} 0 & 0 & \delta_M (I_{2N} \otimes P_2) \bar{F} H_3 & 0 \\ 0 & 0 & r_M (I_{2N} \otimes P_2) \bar{F} H_3 & 0 \end{bmatrix} \\ \Gamma_{33}^2 &= \text{diag} \left\{ (I_{2N} \otimes P_2)^{-1} R_2 (I_{2N} \otimes P_2), \right. \\ &\quad \left. (I_{2N} \otimes P_2)^{-1} S_2 (I_{2N} \otimes P_2) \right\}.\end{aligned}$$

APPENDIX B THE PROOF OF THEOREM 1

The following time-varying Lyapunov functional is constructed for system (13):

$$\begin{aligned}V_{\gamma(t)}(t) &= \eta^T(t) (I_{2N} \otimes P_{\gamma(t)}) \eta(t) \\ &\quad + \int_{t-\delta_M}^t \theta(\cdot) \eta^T(v) Q_{\gamma(t)} \eta(v) dv \\ &\quad + \int_{t-r_M}^t \theta(\cdot) \eta^T(v) Z_{\gamma(t)} \eta(v) dv \\ &\quad + \delta_M \int_{-\delta_M}^0 \int_{t+s}^t \theta(\cdot) \dot{\eta}^T(v) R_{\gamma(t)} \dot{\eta}(v) dv ds \\ &\quad + r_M \int_{-r_M}^0 \int_{t+s}^t \theta(\cdot) \dot{\eta}^T(v) S_{\gamma(t)} \dot{\eta}(v) dv ds \quad (31)\end{aligned}$$

where $P_{\gamma(t)} > 0$, $Q_{\gamma(t)} > 0$, $Z_{\gamma(t)} > 0$, $R_{\gamma(t)} > 0$, $S_{\gamma(t)} > 0$, and $\theta(\cdot) \triangleq e^{(-1)^{\gamma(t)} 2\mu_{\gamma(t)}(t-v)}$; $\gamma(t) = 1$ when $t \in \mathcal{H}_{1,n}$ and $\gamma(t) = 2$ when $t \in \mathcal{H}_{2,n}$, $n \in \mathbb{N}$.

The following discussion on two cases of $\gamma(t) = 1$ and $\gamma(t) = 2$ will be given, respectively.

When $\gamma(t) = 1$, for $t \in \mathcal{H}_{1,n}$, $n \in \mathbb{N}$, calculate the derivation and mathematical expectation of $V_1(t)$. In addition, it follows from the inequality (9) that:

$$\begin{aligned}\sigma \eta^T(t - \delta_{\lambda,n}(t)) H_1^T (\mathbb{A}^{\tau(t)})^T \Phi \mathbb{A}^{\tau(t)} H_1 \eta(t - \delta_{\lambda,n}(t)) \\ - \psi_{\lambda,n}^T(t) \Phi \psi_{\lambda,n}(t) \geq 0 \quad (32)\end{aligned}$$

where $\Phi = \text{diag}\{\Phi_1, \Phi_2, \dots, \Phi_N\}$.

From Assumption 1, it is easy to achieve that

$$\epsilon^2 \eta^T(t) H_2^T H_2 \eta(t) - \bar{F}^T(x(t), t) H_2^T H_2 \bar{F}(x(t), t) \geq 0. \quad (33)$$

Combining (32) and (33), then by applying Jensen's inequality [28] and Schur complement, we have

$$\mathbf{E}\{\dot{V}_1(t)\} \leq -2\mu_1 \mathbf{E}\{V_1(t)\} + \varphi_1^T(t) \Pi_1 \varphi_1(t) \quad (34)$$

where $\varphi_1(t) = [\eta^T(t) \quad \eta^T(t - \delta_{\lambda,n}(t)) \quad \eta^T(t - \delta_M) \quad \eta^T(t - r_{\lambda,n}(t)) \quad \eta^T(t - r_M) \quad \psi_{\lambda,n}^T(t_r) \quad \psi_{\lambda,n}^T(t) \quad (H_1^T \bar{F}(x(t), t))^T \quad I \quad I \quad I \quad I \quad I]^T$. Due to $\Pi_1 < 0$, it can be acquired that for $t \in \mathcal{H}_{1,n}$, $n \in \mathbb{N}$, $\mathbf{E}\{\dot{V}_1(t)\} \leq -2\mu_1 \mathbf{E}\{V_1(t)\}$ holds.

When $\gamma(t) = 2$, $t \in \mathcal{H}_{2,n}$, $n \in \mathbb{N}$, take the derivation and expectation of $V_2(t)$, and similarly employ the aforementioned method, then we can obtain

$$\mathbf{E}\{\dot{V}_2(t)\} \leq 2\mu_2 \mathbf{E}\{V_2(t)\} + \varphi_2^T(t) \Pi_2 \varphi_2(t) \quad (35)$$

where $\varphi_2(t) = [\eta^T(t) \quad \eta^T(t - \delta_{\lambda,n}(t)) \quad \eta^T(t - \delta_M) \quad \eta^T(t - r_{\lambda,n}(t)) \quad \eta^T(t - r_M) \quad (H_1^T \bar{F}(x(t), t))^T \quad I \quad I \quad I]^T$. By applying $\Pi_2 < 0$, it follows that for $t \in \mathcal{H}_{2,n}$, $n \in \mathbb{N}$, $\mathbf{E}\{\dot{V}_2(t)\} \leq 2\mu_2 \mathbf{E}\{V_2(t)\}$ holds.

For convenience, define $\ell_{J,n} = \begin{cases} a_n, & J = 1 \\ a_n + \varrho_n, & J = 2 \end{cases}$, then

one can obtain $\mathcal{H}_{J,n} = [\ell_{J,n}, \ell_{3-J, J-1+n})$, then it yields that $\mathbf{E}\{V_J(t)\} \leq e^{2(-1)^J \mu_J(t-\ell_{J,n})} \mathbf{E}\{V_J(\ell_{J,n})\}$ holds for $t \in [\ell_{J,n}, \ell_{3-J, J+n-1})$, $J \in \{1, 2\}$. Besides, due to the inequalities in (18) holding, then by some simple calculations, we can achieve the following inequalities:

$$\begin{cases} \mathbf{E}\{V_1(\ell_{1,n})\} \leq \omega_2 \mathbf{E}\{V_2(\ell_{1,n}^-)\} \\ \mathbf{E}\{V_2(\ell_{2,n})\} \leq \omega_1 e^{2\zeta h} \mathbf{E}\{V_1(\ell_{2,n}^-)\}, \quad \zeta = \mu_1 + \mu_2. \end{cases} \quad (36)$$

If $t \in [\ell_{1,n}, \ell_{2,n})$, according to (36), it yields that $\mathbf{E}\{V(t)\} \leq \omega_2 e^{-2\mu_1(t-\ell_{1,n})} \mathbf{E}\{V_2(\ell_{1,n}^-)\}$, then $\mathbf{E}\{V_2(\ell_{1,n}^-)\} \leq e^{2\mu_2(\ell_{1,n}-\ell_{2,n-1})} \mathbf{E}\{V_2(\ell_{2,n-1})\}$ can be derived due to $\ell_{1,n}^- \in [\ell_{2,n-1}, \ell_{1,n})$. By reiterating this process, then we can obtain

$$\mathbf{E}\{V(t)\} \leq e^{o(t)} \mathbf{E}\{V_1(0)\} \quad (37)$$

where $o(t) = (b_0 + [t/f_a])[2(\zeta h + \mu_2 D_M - 2\mu_1 D_m) + \ln(\omega_1 \omega_2)]$. Due to the inequality (20) holding, it is easy to acquire that

$$\mathbf{E}\{V(t)\} \leq e^{\xi_1} e^{-vt} \mathbf{E}\{V_1(0)\} \quad (38)$$

where $\xi_1 = b_0[2(\zeta h + \mu_2 D_M - \mu_1 D_m) + \ln(\omega_1 \omega_2)]$, $v = (1/f_a)[(-\zeta h - \mu_2 D_M + \mu_1 D_m) + (1/2) \ln(\omega_1 \omega_2)]$. Then, by adopting the similar method and combining inequality (20), the following inequality can be obtained:

$$\mathbf{E}\{V(t)\} \leq \frac{e^{\xi_2}}{\omega_2} e^{-vt} \mathbf{E}\{V_1(0)\} \quad (39)$$

where $\xi_2 = (b_0 + 1)[2(\zeta h + \mu_2 D_M - \mu_1 D_m) + \ln(\omega_1 \omega_2)]$.

Moreover, it yields that

$$\mathbf{E}\{V(t)\} \leq \max \left\{ e^{\xi_1}, \frac{e^{\xi_2}}{\omega_2} \right\} e^{-vt} \mathbf{E}\{V_1(0)\}. \quad (40)$$

It follows from the definition of $V(t)$ that:

$$\begin{cases} \mathbf{E}\{V(t)\} \geq \varphi_{\min} \mathbf{E}\{\|\eta(t)\|^2\} \\ \mathbf{E}\{V_1(0)\} \leq \varphi_{\max} \mathbf{E}\{\|\zeta_0\|^2\} \end{cases} \quad (41)$$

where $\varphi_{\min} = \min\{\varpi_n(I_{2N} \otimes P_i)\}$ and $\varphi_{\max} = \max\{\varpi_m(I_{2N} \otimes P_i)\} + h\varpi_m(Q_1) + (h^2/2)\varpi_m(R_1 + S_1)$.

Combining (40) and (41), it is easy to derive that

$$\forall t \geq 0, \mathbf{E}\{\|\eta(t)\|^2\} \leq \varepsilon e^{-\frac{\sigma}{2}t} \mathbf{E}\{\|\zeta_0\|^2\} \quad (42)$$

where $\varepsilon = (\varphi_{\max}/\varphi_{\min}) \cdot \max\{e^{\xi_1}, (e^{\xi_2}/\omega_2)\}$. According to inequality (42) and Definition 1, it can be concluded that if the inequalities (15)–(20) hold, system (13) reaches mean-square exponential stability with decay rate $(\sigma/2)$. That completes the proof.

APPENDIX C
ELEMENTS OF $\hat{\Pi}_1$ AND $\hat{\Pi}_2$ IN THEOREM 2

$$\hat{\Gamma}_{11}^1 = \begin{bmatrix} c_1 & * & * \\ c_2 & c_3 & * \\ -g_\varepsilon \hat{M}_1 & g_1(\hat{R}_1 + \hat{M}_1) & -g_1 \hat{Q}_1 - g_1 \hat{R}_1 \end{bmatrix}$$

$$c_1 = 2\mu_1 \hat{X}_1 + \hat{Q}_1 + \hat{Z}_1 - g_1 \hat{R}_1 - g_2 \hat{S}_1 + \bar{A} \hat{X}_1 + \hat{X}_1 \bar{A}^T$$

$$c_2 = g_1(\hat{R}_1 + \hat{M}_1) - \phi \bar{Y}^T (\bar{B}_1^{\tau(t)})^T - \phi \bar{Y}^T (\bar{B}_3^{\tau(t)})^T$$

$$c_3 = \sigma H_1^T (\bar{A}^{\tau(t)})^T \Phi \bar{A}^{\tau(t)} H_1 + g_1(-2\hat{R}_1 - \hat{M}_1 - \hat{M}_1^T)$$

$$\hat{\Phi} = \text{diag}\{\hat{\Phi}_1, \hat{\Phi}_2, \dots, \hat{\Phi}_N\}$$

$$\hat{\Gamma}_{21}^1 = \begin{bmatrix} \phi \bar{Y}^T (\bar{B}_3^{\tau(t)})^T \bar{\alpha} + g_2(\hat{S}_1 + \hat{U}_1) & 0 & 0 \\ -g_2 \hat{U}_1 & 0 & 0 \\ \phi \bar{Y}_1^T (\bar{B}_4^{\tau(t)})^T \bar{\alpha} & 0 & 0 \end{bmatrix}$$

$$\hat{\Gamma}_{22}^1 = \begin{bmatrix} c_4 & * & * \\ g_2(\hat{S}_1 + \hat{U}_1) & -g_2 \hat{Z}_1 - g_2 \hat{S}_1 & * \\ 0 & 0 & -\hat{\Phi} \end{bmatrix}$$

$$c_4 = \sigma H_1^T (\bar{A}^{\tau(t)})^T \Phi \bar{A}^{\tau(t)} H_1 + g_1(-2\hat{S}_1 - \hat{U}_1 - \hat{U}_1^T)$$

$$\hat{\Gamma}_{31}^1 = \begin{bmatrix} \phi \bar{Y}_1^T (\bar{B}_4^{\tau(t)})^T - \phi \bar{Y}_1^T (\bar{B}_2^{\tau(t)})^T \bar{\alpha} & 0 & 0 \\ H_1 \bar{F}^T & 0 & 0 \\ \varepsilon H_2 \hat{X}_1 & 0 & 0 \end{bmatrix}$$

$$\hat{\Gamma}_{33}^1 = \text{diag}\{-\hat{\Phi}, -I, -I\}, \quad g_1 = e^{-2\mu_1 \delta_M}, \quad g_2 = e^{-2\mu_1 r_M}$$

$$\hat{\Gamma}_{41}^1 = \begin{bmatrix} \delta_M \bar{A} \hat{X}_1 & c_5 & 0 \\ r_M \bar{A} \hat{X}_1 & c_6 & 0 \\ 0 & \delta_M \phi \rho \bar{B}_3^{\tau(t)} \bar{Y} & 0 \\ 0 & r_M \phi \rho \bar{B}_3^{\tau(t)} \bar{Y} & 0 \end{bmatrix}$$

$$c_5 = -\delta_M \phi \bar{B}_1^{\tau(t)} \bar{Y} - \delta_M \phi \bar{B}_3^{\tau(t)} \bar{\alpha} \bar{Y}$$

$$c_6 = -r_M \phi \bar{B}_1^{\tau(t)} \bar{Y} - r_M \phi \bar{\alpha} \bar{B}_3^{\tau(t)} \bar{Y}$$

$$\hat{\Gamma}_{42}^1 = \begin{bmatrix} \delta_M \phi \bar{\alpha} \bar{B}_3^{\tau(t)} \bar{Y} & 0 & \delta_M \phi \bar{\alpha} \bar{B}_4^{\tau(t)} \bar{Y}_1 \\ r_M \phi \bar{\alpha} \bar{B}_3^{\tau(t)} \bar{Y} & 0 & r_M \phi \bar{\alpha} \bar{B}_4^{\tau(t)} \bar{Y}_1 \\ -\delta_M \phi \rho \bar{B}_3^{\tau(t)} \bar{Y} & 0 & -\delta_M \phi \rho \bar{B}_4^{\tau(t)} \bar{Y}_1 \\ -r_M \phi \rho \bar{B}_3^{\tau(t)} \bar{Y} & 0 & -r_M \phi \rho \bar{B}_4^{\tau(t)} \bar{Y}_1 \end{bmatrix}$$

$$\hat{\Gamma}_{43}^1 = \begin{bmatrix} \delta_M \phi \bar{B}_2^{\tau(t)} \bar{Y}_1 - \delta_M \phi \bar{\alpha} \bar{B}_4^{\tau(t)} \bar{Y}_1 & \delta_M \bar{F} H_3 & 0 \\ r_M \phi \bar{B}_2^{\tau(t)} \bar{Y}_1 - r_M \phi \bar{\alpha} \bar{B}_4^{\tau(t)} \bar{Y}_1 & r_M \bar{F} H_3 & 0 \\ \delta_M \phi \rho \bar{B}_4^{\tau(t)} \bar{Y}_1 & 0 & 0 \\ r_M \phi \rho \bar{B}_4^{\tau(t)} \bar{Y}_1 & 0 & 0 \end{bmatrix}$$

$$\bar{\alpha} = \text{diag}\{\bar{\alpha}_1, \dots, \bar{\alpha}_N, \bar{\alpha}_1, \dots, \bar{\alpha}_N\}$$

$$\rho = \text{diag}\{\rho_1, \dots, \rho_N, \rho_1, \dots, \rho_N\}, \quad \rho_i = \sqrt{\bar{\alpha}_i(1 - \bar{\alpha}_i)}$$

$$\hat{\Gamma}_{44}^1 = \text{diag}\left\{-2\nu_1 \hat{X}_1 + \nu_1^2 \hat{R}_1, -2\nu_2 \hat{X}_1 + \nu_2^2 \hat{R}_1, \right. \\ \left. -2\nu_1 \hat{X}_1 + \nu_1^2 \hat{R}_1, -2\nu_2 \hat{X}_1 + \nu_2^2 \hat{R}_1\right\}$$

$$\bar{Y} = I_{2N} \otimes Y, \quad \bar{Y}_1 = I_N \otimes Y$$

$$\hat{X}_1 = I_{2N} \otimes X_1, \quad \hat{X}_2 = I_{2N} \otimes X_2$$

$$\hat{\Gamma}_{11}^2 = \begin{bmatrix} \mathbf{c}_1 & * & * \\ \hat{R}_2 + \hat{M}_2 & -2\hat{R}_2 - \hat{M}_2 - \hat{M}_2^T & * \\ -\hat{M}_2 & \hat{R}_2 + \hat{M}_2 & -g_3 \hat{Q}_2 - \hat{R}_2 \end{bmatrix}$$

$$\mathbf{c}_1 = -2\mu_2 \hat{X}_2 + \hat{Q}_2 + \hat{Z}_2 - \hat{R}_2 - \hat{S}_2 + \bar{A} \hat{X}_2 + \hat{X}_2 \bar{A}^T$$

$$\mathbf{c}_2 = -g_4 \hat{Z}_2 - \hat{S}_2, \quad g_3 = e^{2\mu_2 \delta_M}, \quad g_4 = e^{2\mu_2 r_M}$$

$$\hat{\Gamma}_{33}^2 = \text{diag}\left\{-2\nu_3 \hat{X}_2 + \nu_3^2 \hat{R}_2, -2\nu_4 \hat{X}_2 + \nu_4^2 \hat{R}_2\right\}$$

$$\hat{\Gamma}_{21}^2 = \begin{bmatrix} \hat{S}_2 + \hat{U}_2 & 0 & 0 \\ -\hat{U}_2 & 0 & 0 \\ H_1 \bar{F}^T & 0 & 0 \\ \varepsilon H_2 \hat{X}_2 & 0 & 0 \end{bmatrix}$$

$$\hat{\Gamma}_{22}^2 = \begin{bmatrix} -2\hat{S}_2 - \hat{U}_2 - \hat{U}_2^T & * & * & * \\ \hat{S}_2 + \hat{U}_2 & \mathbf{c}_2 & * & * \\ 0 & 0 & -I & * \\ 0 & 0 & 0 & -I \end{bmatrix}$$

$$\hat{\Gamma}_{31}^2 = \begin{bmatrix} \delta_M \bar{A} \hat{X}_2 & 0 & 0 \\ r_M \bar{A} \hat{X}_2 & 0 & 0 \end{bmatrix}, \quad \hat{\Gamma}_{32}^2 = \begin{bmatrix} 0 & 0 & \delta_M \bar{F} H_3 & 0 \\ 0 & 0 & r_M \bar{F} H_3 & 0 \end{bmatrix}$$

APPENDIX D

PROOF OF THEOREM 2

Due to $(R_1 - \nu_1^{-1}(I_{2N} \otimes P_1))R_1^{-1}(R_1 - \nu_1^{-1}(I_{2N} \otimes P_1)) \geq 0$, it is easy to obtain that

$$-(I_{2N} \otimes P_1)R_1^{-1}(I_{2N} \otimes P_1) \leq -2\nu_1(I_{2N} \otimes P_1) + \nu_1^2 R_1. \quad (43)$$

Similarly, we can obtain the following inequalities:

$$\begin{cases} -(I_{2N} \otimes P_1)S_1^{-1}(I_{2N} \otimes P_1) \leq -2\nu_2(I_{2N} \otimes P_1) + \nu_2^2 S_1 \\ -(I_{2N} \otimes P_2)R_2^{-1}(I_{2N} \otimes P_2) \leq -2\nu_3(I_{2N} \otimes P_2) + \nu_3^2 R_2 \\ -(I_{2N} \otimes P_2)S_2^{-1}(I_{2N} \otimes P_2) \leq -2\nu_4(I_{2N} \otimes P_2) + \nu_4^2 S_2. \end{cases}$$

Then, replacing the terms in Γ_{44}^1 and Γ_{33}^2 with $-2\nu_1(I_{2N} \otimes P_1) + \nu_1^2 R_1$, $-2\nu_2(I_{2N} \otimes P_1) + \nu_2^2 S_1$ and $-2\nu_3(I_{2N} \otimes P_2) + \nu_3^2 R_2$, $-2\nu_4(I_{2N} \otimes P_2) + \nu_4^2 S_2$, it follows that:

$$\begin{cases} \Gamma_{44}^1 = \text{diag}\{-2\nu_1(I_{2N} \otimes P_1) + \nu_1^2 R_1 \\ \quad -2\nu_2(I_{2N} \otimes P_1) + \nu_2^2 S_1, -2\nu_1(I_{2N} \otimes P_1) + \nu_1^2 R_1 \\ \quad -2\nu_2(I_{2N} \otimes P_1) + \nu_2^2 S_1\} \\ \Gamma_{33}^2 = \text{diag}\{-2\nu_3(I_{2N} \otimes P_2) + \nu_3^2 R_2 \\ \quad -2\nu_4(I_{2N} \otimes P_2) + \nu_4^2 S_2\}. \end{cases}$$

Let $X_1 = P_1^{-1}$, $\hat{X}_1 = I_{2N} \otimes X_1$, $\bar{X}_1 = I_N \otimes X_1$, $\Delta_1 = \text{diag}\{\hat{X}_1, \hat{X}_1, \hat{X}_1, \hat{X}_1, \bar{X}_1, I, I, \hat{X}_1, \hat{X}_1, \hat{X}_1, \hat{X}_1\}$, then premultiply and postmultiply Π_1 with Δ_1 and Δ_1^T . Define $Y = KX_1$, $\bar{Y} = I_{2N} \otimes Y$, $\bar{Y}_1 = I_N \otimes Y$, $\hat{Q}_1 = \hat{X}_1 Q_1 \hat{X}_1$, $\hat{R}_1 = \hat{X}_1 R_1 \hat{X}_1$, $\hat{Z}_1 = \hat{X}_1 Z_1 \hat{X}_1$, $\hat{S}_1 = \hat{X}_1 S_1 \hat{X}_1$, $\hat{M}_1 = \hat{X}_1 M_1 \hat{X}_1$, $\hat{U}_1 = \hat{X}_1 U_1 \hat{X}_1$, $\hat{\Phi} = \bar{X}_1 \Phi \bar{X}_1$, then we have $\hat{\Pi}_1$. Further, it can be acquired that $\mathbf{E}\{\hat{V}_1(t)\} < 0$.

In addition, denote $X_2 = P_2^{-1}$, $\hat{X}_2 = I_{2N} \otimes X_2$, $\Delta_2 = \text{diag}\{\hat{X}_2, \hat{X}_2, \hat{X}_2, \hat{X}_2, \bar{X}_2, I, I, \hat{X}_2, \hat{X}_2\}$, then premultiply and postmultiply Π_2 with Δ_2 and Δ_2^T . Defining $\hat{Q}_2 = \hat{X}_2 Q_2 \hat{X}_2$, $\hat{R}_2 = \hat{X}_2 R_2 \hat{X}_2$, $\hat{Z}_2 = \hat{X}_2 Z_2 \hat{X}_2$, $\hat{S}_2 = \hat{X}_2 S_2 \hat{X}_2$, $\hat{M}_2 = \hat{X}_2 M_2 \hat{X}_2$, $\hat{U}_2 = \hat{X}_2 U_2 \hat{X}_2$, then we can obtain $\hat{\Pi}_2$; moreover, one can easily obtain that $\mathbf{E}\{\hat{V}_2(t)\} < 0$.

For the first inequality in (18), employing the Schur complement, then by multiplying Ω and Ω^T ($\Omega = \text{diag}\{\hat{X}_2, \hat{X}_2\}$) on both of its sides, we can obtain the inequality (24). Applying the similar method, it follows from the inequalities in (18) and (19) that inequalities (25)–(29) hold. Based on the results of Theorem 1, one can obtain that the system (13) achieves exponentially stability in mean square. Due to $Y = KX_1$, it is easy to derive that the controller gain is $K = YX_1^{-1}$. This completes the proof.

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