Stabilization of Networked Control Systems With Hybrid-Driven Mechanism and Probabilistic Cyber Attacks

Jinliang Liu, Zheng-Guang Wu, Dong Yue, *Senior Member, IEEE*, and Ju H. Park

***Abstract*—This paper investigates the controller design problem of networked control systems subject to cyber attacks. A hybrid-triggering communication strategy is employed to save the limited communication resources. State measurements are transmitted over a communication network and may be corrupted by cyber attacks. The aim of this paper is to design a controller for a new closed-loop system model with consideration of randomly occurring cyber attacks and the hybrid-triggering scheme. A stability criterion is obtained for the system stabilization by employing Lyapunov stability theory and stochastic analysis techniques. Moreover, the desired controller gain is derived by resorting to some matrix inequalities. Finally, a numerical example is exploited to demonstrate the usefulness of the proposed scheme.**

***Index Terms*—Cyber attacks, hybrid-triggering scheme, networked control systems (NCSs), time delay.**

I. Introduction

In NETWORKED control systems (NCSs), selecting appropriate transmission strategy has been a hot research topic in recent years. The information among the control components in NCSs is exchanged through a shared bandwidth limited communication network. There are some challenging problems in networked transmission, such as, network-induced delays, packet dropouts, and quantization problems [1]–[8]. These imperfections induced by communication networks inevitably degrade the system performance and destabilize NCSs. In the past few years, a lot of research interest has been devoted to deal with these issues, see [9], [10] and references therein.

It should be noted that communication constraints have to be improved since large control information needs to be transmitted by the band-limited communication network [11]. How to save the scarce network band-width resources while guaranteeing the performance of NCSs is a practical and valuable problem. Recently, event-triggering communication scheme is proposed to overcome this drawback and has stirred considerable interests [12], [13]. Compared with the traditional time-triggering communication [14], [15], event-triggering communication can choose sampled data packets to be released into the network more efficiently. If the prescribed event-triggering condition is satisfied, the corresponding sampled data packets are discarded purposely [16], [17]. The occupancy of a network resource is reduced under event-triggering schemes while system performance is ensured. Recently, various event-triggering communication schemes have been proposed [18]–[21]. For example, Yue *et al.* [18] discussed the controller design for NCSs under the event-triggering scheme. Li *et al.* [19] investigated the fault detection problem for event-triggered nonlinear networked systems. In [21], the event-triggering $\mathcal{L}_2$ control for a sampled-data control system is studied. However, in the aforementioned works, the adopted event-triggering communication scheme is fixed, which ignores the variation of the network utilization. In practical systems, it is necessary to take the variation of the network loads into consideration and construct a more flexible communication scheme. Based on the work of [18], Liu *et al.* [22] addressed the desired controller design for NCSs under a hybrid-triggering scheme. Considering random occurring nonlinear perturbations and actuator faults, a reliable controller is designed in [23] for hybrid-triggering T–S fuzzy systems. In [24], the problem of $\mathcal{H}_\infty$ filter is designed for neural networks with deception attacks based on a hybrid-triggering scheme. Liu *et al.* [25] are concerned with hybrid-driven $\mathcal{H}_\infty$ filtering for a class of Takagi–Sugeno fuzzy systems with quantization. Hybrid-driven-based $\mathcal{H}_\infty$ control is investigated in [26] for networked cascade control systems with actuator saturations and stochastic cyber attacks. However, limited network resources and cyber attacks have not been fully investigated, which are common phenomena in an unreliable communication network. This is the first motivation of this paper.

Manuscript received December 28, 2017; revised July 27, 2018; accepted December 15, 2018. Date of publication January 4, 2019; date of current version January 19, 2021. The work of J. Liu was supported by the National Natural Science Foundation of China under Grant 61403185 and Grant 61773218. The work of J. H. Park was supported by the Basic Science Research Programs through the National Research Foundation of Korea (NRF) funded by the Ministry of Education under Grant NRF-2017R1A2B2004671. This paper was recommended by Associate Editor W.-K. V. Chan. *(Corresponding author: Ju H. Park.)*

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Digital Object Identifier 10.1109/TSMC.2018.2888633

Recently, researchers have paid increasing attention to cyber attacks in NCSs. Due to the strong opening-up property of a shared network, especially the wireless networks, the measurement and control commands are transmitted over an unprotected communication network and are susceptible to be corrupted by adversaries [27], [28]. Malicious attacks can block the communication link and significantly impair the system performance or even lead to loss of control of NCSs. Studying the influence of these especially malicious attacks has been a fundamental research topic [29]–[35]. Generally speaking, the cyber attacks mentioned in the literature can be categorized as denial of service (DoS) attacks [36], [37] and deception attacks [32], [38]. DoS attacks destroy the data availability in control systems, which cause a long delay of the information exchange and packet loss. Deception attacks affect the integrity of data, which can be classified as data replay attacks [33] and false data injection attacks [34]. For instance, in [30], a resilient $\mathcal{H}_\infty$ load frequency control scheme is investigated for event-triggered multiarea power systems with DoS attacks. A variation of the receding-horizon control under the replay attacks is studied in [33]. The design problem of false data injection attacks against the output tracking control of networked systems is addressed in [34]. In [35], the observer-based event-triggering consensus control problem is considered for a class of multiagent systems with lossy sensors and cyber attacks. However, the deception attacks have not yet been fully investigated. How to cope with the deception attacks and reduce the computation burden is still a challenging problem, which motivates our interest in the analysis on effects of deception attacks.

Motivated by the above discussions, a novel control method against deception attacks is proposed for NCSs in this paper. Comparing with the previous work in the literature, the main contributions of this paper is as follows.

1) The proposed model against deception attacks is based on a hybrid-triggering communication strategy, in which random variables describing the switching mode between the time-triggering scheme and the event-triggering scheme obey a Bernoulli distribution.
2) The deception attacks, resulting in the violation of data integrity and/or authenticity, are assumed to satisfy a nonlinear function and have random occurrence.
3) Sufficient conditions are derived which, when satisfied, can guarantee the stability of the developed model. Furthermore, the parameter of the desired controller gain is obtained by the solution to a set of matrix inequalities.

The rest of this paper is organized as follows. The problem under consideration is introduced in Section II. Section III presents the sufficient stabilization conditions and the controller design method. A simulated example is given in Section IV to demonstrate the effectiveness of the design method. The conclusions are provided in Section V.

Notation: $\mathbb{R}^n$ stands for n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ denotes the the set of $n \times m$ real matrices; $\mathcal{E}$ denotes the expectation operator; the superscript T stands for matrix transposition; the notation $X > 0$, for $X \in \mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite; Prob$\{X\}$ is the occurrence probability of event X; for a matrix A_1 and two symmetric matrices A_2 and A_3, $\begin{bmatrix} A_2 & * \\ A_1 & A_3 \end{bmatrix}$ denotes a symmetric matrix, where $*$ denotes the entries implied by symmetry.

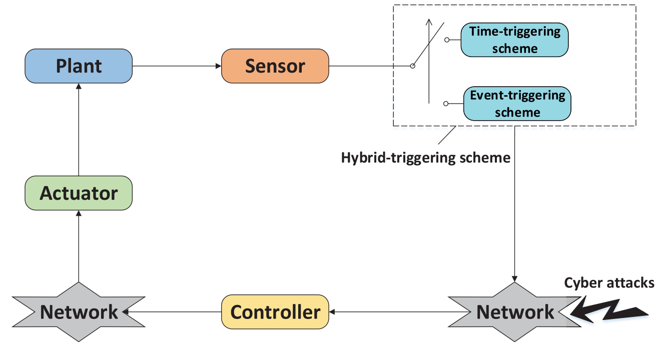

Fig. 1. Structure of hybrid-driven NCSs under cyber attacks.

II. System Description

Consider the following continuous-time system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^m$ is the control input, and A and B are the parameter matrices with appropriate dimensions.

In this paper, the system (1) is assumed to be controlled through constrained networks. The network-induced delay is unavoidable in sensor-controller channel and controller-actuator channel. The following controller is designed to stabilize the system (1):

$$u(t) = Kx(t_k h),\ t \in \left[t_k h + \tau_{t_k}, t_{k+1}h + \tau_{t_{k+1}}\right) \tag{2}$$

where h represents the sampling period, $\{t_1, t_2, t_3, \ldots\} \subset \{1, 2, 3, \ldots\}$, $t_k h$ represents the sampling instant which arrives at the actuator successfully, τ_{t_k} is the corresponding network-induced delay. K is the controller feedback gain to be designed later.

As shown in Fig. 1, a hybrid-triggering scheme [22] between the sensor and the controller is introduced to pursue a better tradeoff between communication and performance. The switching mode between the two communication schemes is designed by the human and assumed to follow a Bernoulli distribution.

When the "time-triggering scheme" is selected, the sequence of the sampling instants will be $t_k h = kh$, $(k = 1, 2, 3, \ldots)$. Define $\tau(t) = t - t_k h$, (2) can be rewritten as

$$u(t) = Kx(t - \tau(t)) \tag{3}$$

where $\tau(t) \in [0, \tau_M]$, τ_M is the upper bound of the network-induced delay. The drawback of this method is that the periodic sampled data is transmitted even when the output does not significantly change the control signal. Thus, many redundant packets are transmitted through the communication network.

When the "event-triggering scheme" is chosen, the signals transmitted through the network are supposed to exceed the following event-triggering condition [18]:

$$e_k(t)^{\mathrm{T}}\Omega e_k(t) \le \sigma x^{\mathrm{T}}(t_kh+lh)\Omega x(t_kh+lh) \tag{4}$$

where $e_k(t) = x(t_kh) - x(t_kh+lh)$, $\Omega > 0$ is a matrix with appropriate dimension, $l = 1, 2, \ldots$ and $\sigma \in [0, 1)$ is a given positive scalar, $x(t_kh)$ is the latest transmitted signal at the latest triggering time t_kh. Whether the latest sampled signals $x(t_kh+lh)$ are delivered or not is dependent on the condition (4). For convenience of analysis, we will partition the interval $[t_kh+\tau_{t_k}, t_{k+1}h+\tau_{t_{k+1}})$ into several subintervals. There exists d satisfying $[t_kh+\tau_{t_k}, t_{k+1}h+\tau_{t_{k+1}}) = \bigcup_{l=0}^{d}\Lambda_l$, where $\Lambda_l = [t_kh+lh+\tau_{t_{k+l}}, t_kh+lh+h+\tau_{t_{k+l+1}}]$, $l = 0, \ldots, d$, $d = t_{k+1} - t_k - 1$. Define $\eta(t) = t - t_kh - lh$, one has $0 \le \tau_{t_{k+l}} \le \eta(t) \le \eta_M$, in which $\eta_M \triangleq h + \tau_{t_{k+l+1}}$. From the definitions of $\eta(t)$ and $e_k(t)$, we can get $x(t_kh) = x(t-\eta(t)) + e_k(t)$. Then, (2) can be rewritten as

$$u(t) = K[x(t-\eta(t)) + e_k(t)]. \tag{5}$$

Under the hybrid-triggering scheme proposed in [22], the controller can be switched between (3) and (5)

$$\begin{aligned} u(t) = {} & \alpha(t)Kx(t-\tau(t)) \\ & + (1-\alpha(t))K[x(t-\eta(t)) + e_k(t)] \end{aligned} \tag{6}$$

where $\alpha(t)$ is a random variable obeying Bernoulli distribution with its mathematical expectation Prob $\{\alpha(t)\} = \bar{\alpha}$.

Remark 1: $\alpha(t)$ is used to adjust the transmission strategy according to the variation of the network utilization. Specifically, $\alpha(t) = 1$ means the time-triggering scheme is active, then, the closed system transmits the sampled data periodically; $\alpha(t) = 0$ means the event-triggering scheme is active, then, the sampled data is delivered conditionally.

The network with limited bandwidth is probably vulnerable to cyber attacks, due to the fact that limited communication/computing capacity is a stumbling block to the use of sophisticated encryption for secure transmission. Aiming to affect the system performance, the attackers could destroy or modify some significant data. In this paper, we address the cyber attacks which resulting in the violation of data integrity and/or authenticity.

Considering the unreliable network channel, the control input may subject to cyber attacks which occur in a probabilistic way. From an adversary scenario, the attacks aiming to destabilize the plant can be launched at any time. Thus, the controller with cyber attacks can be described as

$$u(t) = (1-\theta(t))u_0(t) + \theta(t)Kf(x(t-d(t))) \tag{7}$$

where $u_0(t) = \alpha(t)Kx(t-\tau(t)) + (1-\alpha(t))K(x(t-\eta(t)) + e_k(t))$, $f(x(t-d(t)))$ is the signal sent by attackers, $d(t) \in [0, d_M]$, d_M is a positive number. $\theta(t) \in \{0, 1\}$, $\theta(t)$ is a Bernoulli distributed variable governing the probabilistic cyber attacks with that Prob$\{\theta(t) = 1\} = \bar{\theta}$, Prob $\{\theta(t) = 0\} = 1 - \bar{\theta}$.

It should be pointed out that there exists a minimum probability of random attacks by the attackers to achieve the attacking purpose [3], [39]. The adversaries could launch the deception attacks with a specific probability. The random variable $\theta(t)$ describes the physical property of the cyber attacks such that $\theta(t) = 1$ means the cyber attacks is implemented and $\theta(t) = 0$ means the data transmission is normal.

From (1) and (7), the hybrid-triggering closed-loop system can be formulated as follows:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + (1-\theta(t))Bu_0(t) + \theta(t)BKf(x(t-d(t))) \\ &= Ax(t) + (1-\bar{\theta})B[\bar{\alpha}Kx(t-\tau(t)) \\ &\qquad + (1-\bar{\alpha})(Kx(t-\eta(t)) + Ke_k(t))] \\ &\quad + (1-\bar{\theta})(\alpha(t)-\bar{\alpha})B[Kx(t-\tau(t)) \\ &\qquad - (Kx(t-\eta(t)) - Ke_k(t))] \\ &\quad + \big(\bar{\theta}-\theta(t)\big)[\bar{\alpha}BKx(t-\tau(t)) \\ &\qquad + (1-\bar{\alpha})(BKx(t-\eta(t)) + BKe_k(t))] \\ &\quad + (\bar{\theta}-\theta(t))(\alpha(t)-\bar{\alpha})B[Kx(t-\tau(t)) \\ &\qquad - (Kx(t-\eta(t)) - Ke_k(t))] \\ &\quad + \bar{\theta}BKf(x(t-d(t))) \\ &\quad + \big(\theta(t)-\bar{\theta}\big)BKf(t-d(t)). \end{aligned} \tag{8}$$

Remark 2: In the literature, cyber attacks are intermittent or random and are assumed to satisfy an arbitrary limited magnitude (see [3], [24], [26], [35], [39]–[41]), which is reasonable according to security requirements. Inspired by the above-mentioned work about cyber attacks, the signal sent by attackers in this paper is modeled as a nonlinear function satisfying an inequality. The probability of the signal successfully sent by the attackers obeys Bernoulli distribution.

Definition 1 [14]: For any $\epsilon > 0$, there is a $\psi(\epsilon) > 0$ such that $\mathcal{E}\{|x(t)|^2\} < \epsilon$, $t > 0$ when $\mathcal{E}\{|x(0)|^2\} < \psi(\epsilon)$, then, the closed-loop system in (8) is said to be mean square stable; If $\lim_{t\to\infty}\mathcal{E}\{|x(t)|^2\} = 0$ for any initial conditions, then, the closed-loop system in (8) is said to be mean square asymptotically stable (MSAS).

Assumption 1 [3], [26]: The randomly occurring cyber attacks function $f(x)$ is assumed to satisfy

$$f^{\mathrm{T}}(x)f(x) \le x^{\mathrm{T}}G^{\mathrm{T}}Gx \tag{9}$$

where G is a known constant matrix representing the upper bound of the nonlinearity.

Lemma 1 [42]: For given $x, y \in \mathbb{R}^n$, and positive definite matrix $\Lambda \in \mathbb{R}^{n\times n}$, the following inequality holds:

$$2x^{\mathrm{T}}y \le x^{\mathrm{T}}\Lambda x + y^{\mathrm{T}}\Lambda^{-1}y. \tag{10}$$

Lemma 2 [43]: Assume $\tau_m < \tau(t) < \tau_M$, $0 < d(t) < d_M$, Θ_1, Θ_2, Θ_3, Θ_4, and Ω are matrices with appropriate dimensions, then

$$\begin{aligned} &(\tau(t)-\tau_m)\Theta_1 + (\tau_M-\tau(t))\Theta_2 \\ &\quad + d(t)\Theta_3 + (d_M-d(t))\Theta_4 + \Omega < 0 \end{aligned} \tag{11}$$

if and only if

$$\begin{cases} (\tau_M-\tau_m)\Theta_1 + d_M\Theta_3 + \Omega < 0 \\ (\tau_M-\tau_m)\Theta_2 + d_M\Theta_3 + \Omega < 0 \\ (\tau_M-\tau_m)\Theta_1 + d_M\Theta_4 + \Omega < 0 \\ (\tau_M-\tau_m)\Theta_2 + d_M\Theta_4 + \Omega < 0. \end{cases}$$

III. Main Results

In the following, sufficient stability conditions are developed for closed-loop system (8).

Theorem 1: For given probability scalars $\bar{\alpha}$, $\bar{\theta}$, time delays τ_M, η_M, d_M, trigger parameter σ, and matrix K, the closed-loop system in (8) is MSAS if there exist matrices $P > 0$, $Q_1 > 0$, $Q_2 > 0$, $Q_3 > 0$, $R_1 > 0$, $R_2 > 0$, $R_3 > 0$, $\Omega > 0$, M, N, T, S, U, and V with appropriate dimensions such that the following inequalities hold for $l = 1, \ldots, 8$

$$\Xi(l) = \begin{bmatrix} \Phi_1 & * \\ \Phi_2 & \Phi_3 \end{bmatrix} < 0 \tag{12}$$

where

$$\Phi_1 = \begin{bmatrix} \Xi_{11} + \Upsilon + \Upsilon^{\mathrm{T}} & * & * & * \\ \Xi_{21}(l) & \Xi_{22} & * & * \\ \Xi_{31} & 0 & \Xi_{33} & * \\ \Xi_{41} & 0 & 0 & \Xi_{44} \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} \Xi_{51} & 0 & 0 & 0 \\ \Xi_{61} & 0 & 0 & 0 \\ \Xi_{71} & 0 & 0 & 0 \\ \Xi_{81} & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_3 = \begin{bmatrix} \Xi_{55} & * & * & * \\ 0 & \Xi_{66} & * & * \\ 0 & 0 & \Xi_{77} & * \\ 0 & 0 & 0 & -P \end{bmatrix}$$

$$\Xi_{11} = \begin{bmatrix} \Phi_{11} & * \\ \Phi_{21} & \Phi_{22} \end{bmatrix}, \quad \mathbb{A} = PA + A^{\mathrm{T}}P + Q_1 + Q_2 + Q_3$$

$$\Phi_{11} = \begin{bmatrix} \mathbb{A} & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & -Q_3 & * \\ \hat{\theta}\bar{\alpha}K^{\mathrm{T}}B^{\mathrm{T}}P & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \hat{\theta}\hat{\alpha}K^{\mathrm{T}}B^{\mathrm{T}}P & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hat{\theta}\hat{\alpha}K^{\mathrm{T}}B^{\mathrm{T}}P & 0 & 0 & 0 \\ \bar{\theta}K^{\mathrm{T}}B^{\mathrm{T}}P & 0 & 0 & 0 \end{bmatrix}$$

$$\Phi_{22} = \begin{bmatrix} -Q_1 & * & * & * & * \\ 0 & \sigma\Omega & * & * & * \\ 0 & 0 & -Q_2 & * & * \\ 0 & 0 & 0 & -\Omega & * \\ 0 & 0 & 0 & 0 & -\bar{\theta}I \end{bmatrix}$$

$$\Upsilon = \begin{bmatrix} \Upsilon_1 & -N & S-T & -S & 0 & 0 \end{bmatrix}$$

$$\Upsilon_1 = \begin{bmatrix} M+T+U & V-U & -V & N-M \end{bmatrix}$$

$$\Xi_{21}(1) = \begin{bmatrix} \sqrt{\tau_M}M^{\mathrm{T}} \\ \sqrt{\eta_M}T^{\mathrm{T}} \\ \sqrt{d_M}U^{\mathrm{T}} \end{bmatrix}, \Xi_{21}(2) = \begin{bmatrix} \sqrt{\tau_M}M^{\mathrm{T}} \\ \sqrt{\eta_M}T^{\mathrm{T}} \\ \sqrt{d_M}V^{\mathrm{T}} \end{bmatrix}$$

$$\Xi_{21}(3) = \begin{bmatrix} \sqrt{\tau_M}M^{\mathrm{T}} \\ \sqrt{\eta_M}S^{\mathrm{T}} \\ \sqrt{d_M}U^{\mathrm{T}} \end{bmatrix}, \Xi_{21}(4) = \begin{bmatrix} \sqrt{\tau_M}M^{\mathrm{T}} \\ \sqrt{\eta_M}S^{\mathrm{T}} \\ \sqrt{d_M}V^{\mathrm{T}} \end{bmatrix}$$

$$\Xi_{21}(5) = \begin{bmatrix} \sqrt{\tau_M}N^{\mathrm{T}} \\ \sqrt{\eta_M}T^{\mathrm{T}} \\ \sqrt{d_M}U^{\mathrm{T}} \end{bmatrix}, \Xi_{21}(6) = \begin{bmatrix} \sqrt{\tau_M}N^{\mathrm{T}} \\ \sqrt{\eta_M}T^{\mathrm{T}} \\ \sqrt{d_M}V^{\mathrm{T}} \end{bmatrix}$$

$$\Xi_{21}(7) = \begin{bmatrix} \sqrt{\tau_M}N^{\mathrm{T}} \\ \sqrt{\eta_M}S^{\mathrm{T}} \\ \sqrt{d_M}U^{\mathrm{T}} \end{bmatrix}, \Xi_{21}(8) = \begin{bmatrix} \sqrt{\tau_M}N^{\mathrm{T}} \\ \sqrt{\eta_M}S^{\mathrm{T}} \\ \sqrt{d_M}V^{\mathrm{T}} \end{bmatrix}$$

$$\Xi_{31} = \begin{bmatrix} \Xi_{311} & \Xi_{312} \end{bmatrix}$$

$$\Xi_{311} = \begin{bmatrix} \sqrt{\tau_M}PA & 0 & 0 & \sqrt{\tau_M}\hat{\theta}\bar{\alpha}PBK & 0 \\ \sqrt{\eta_M}PA & 0 & 0 & \sqrt{\eta_M}\hat{\theta}\bar{\alpha}PBK & 0 \\ \sqrt{d_M}PA & 0 & 0 & \sqrt{d_M}\hat{\theta}\bar{\alpha}PBK & 0 \end{bmatrix}$$

$$\Xi_{312} = \begin{bmatrix} \sqrt{\tau_M}\hat{\theta}\hat{\alpha}PBK & 0 & \sqrt{\tau_M}\hat{\theta}\hat{\alpha}PBK & \sqrt{\tau_M}\bar{\theta}PBK \\ \sqrt{\eta_M}\hat{\theta}\hat{\alpha}PBK & 0 & \sqrt{\eta_M}\hat{\theta}\hat{\alpha}PBK & \sqrt{\eta_M}\bar{\theta}PBK \\ \sqrt{d_M}\hat{\theta}\hat{\alpha}PBK & 0 & \sqrt{d_M}\hat{\theta}\hat{\alpha}PBK & \sqrt{d_M}\bar{\theta}PBK \end{bmatrix}$$

$$\Xi_{41} = \begin{bmatrix} \Xi_{411} & \Xi_{412} \end{bmatrix}$$

$$\Xi_{411} = \begin{bmatrix} 0 & 0 & 0 & \gamma_1\hat{\theta}\sqrt{\tau_M}PBK & 0 \\ 0 & 0 & 0 & \gamma_1\hat{\theta}\sqrt{\eta_M}PBK & 0 \\ 0 & 0 & 0 & \gamma_1\hat{\theta}\sqrt{d_M}PBK & 0 \end{bmatrix}$$

$$\Xi_{412} = \begin{bmatrix} -\gamma_1\hat{\theta}\sqrt{\tau_M}PBK & 0 & -\gamma_1\hat{\theta}\sqrt{\tau_M}PBK & 0 \\ -\gamma_1\hat{\theta}\sqrt{\eta_M}PBK & 0 & -\gamma_1\hat{\theta}\sqrt{\eta_M}PBK & 0 \\ -\gamma_1\hat{\theta}\sqrt{d_M}PBK & 0 & -\gamma_1\hat{\theta}\sqrt{d_M}PBK & 0 \end{bmatrix}$$

$$\Xi_{51} = \begin{bmatrix} \Xi_{511} & \Xi_{512} \end{bmatrix}$$

$$\Xi_{511} = \begin{bmatrix} 0 & 0 & 0 & \gamma_2\bar{\alpha}\sqrt{\tau_M}PBK & 0 \\ 0 & 0 & 0 & \gamma_2\bar{\alpha}\sqrt{\eta_M}PBK & 0 \\ 0 & 0 & 0 & \gamma_2\bar{\alpha}\sqrt{d_M}PBK & 0 \end{bmatrix}$$

$$\Xi_{512} = \begin{bmatrix} \gamma_2\hat{\alpha}\sqrt{\tau_M}PBK & 0 & \gamma_2\hat{\alpha}\sqrt{\tau_M}PBK & 0 \\ \gamma_2\hat{\alpha}\sqrt{\eta_M}PBK & 0 & \gamma_2\hat{\alpha}\sqrt{\eta_M}PBK & 0 \\ \gamma_2\hat{\alpha}\sqrt{d_M}PBK & 0 & \gamma_2\hat{\alpha}\sqrt{d_M}PBK & 0 \end{bmatrix}$$

$$\Xi_{61} = \begin{bmatrix} \Xi_{611} & \Xi_{612} \end{bmatrix}$$

$$\Xi_{611} = \begin{bmatrix} 0 & 0 & 0 & \gamma_1\gamma_2\sqrt{\tau_M}PBK & 0 \\ 0 & 0 & 0 & \gamma_1\gamma_2\sqrt{\eta_M}PBK & 0 \\ 0 & 0 & 0 & \gamma_1\gamma_2\sqrt{d_M}PBK & 0 \end{bmatrix}$$

$$\Xi_{612} = \begin{bmatrix} -\gamma_1\gamma_2\sqrt{\tau_M}PBK & 0 & -\gamma_1\gamma_2\sqrt{\tau_M}PBK & 0 \\ -\gamma_1\gamma_2\sqrt{\eta_M}PBK & 0 & -\gamma_1\gamma_2\sqrt{\eta_M}PBK & 0 \\ -\gamma_1\gamma_2\sqrt{d_M}PBK & 0 & -\gamma_1\gamma_2\sqrt{d_M}PBK & 0 \end{bmatrix}$$

$$\Xi_{71} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\sqrt{\tau_M}PBK \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\sqrt{\eta_M}PBK \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\sqrt{d_M}PBK \end{bmatrix}$$

$$\Xi_{81} = \begin{bmatrix} 0 & \bar{\theta}G & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\hat{\theta} = 1 - \bar{\theta}, \hat{\alpha} = 1 - \bar{\alpha}, \gamma_1^2 = \bar{\alpha}\hat{\alpha}, \gamma_2^2 = \bar{\theta}\hat{\theta}$$

$$\Xi_{22} = \mathrm{diag}\{-R_1, -R_2, -R_3\}$$

$$\Xi_{33} = \mathrm{diag}\left\{-PR_1^{-1}P, -PR_2^{-1}P, -PR_3^{-1}P\right\}$$

$$\Xi_{44} = \Xi_{55} = \Xi_{66} = \Xi_{77} = \Xi_{33}.$$

Proof: Choose a Lyapunov–Krasovskii functional candidate as

$$\begin{aligned} V(t) = {} & x^{\mathrm{T}}(t)Px(t) + \int_{t-\tau_M}^{t} x^{\mathrm{T}}(s)Q_1x(s)ds \\ & + \int_{t-\eta_M}^{t} x^{\mathrm{T}}(s)Q_2x(s)ds + \int_{t-d_M}^{t} x^{\mathrm{T}}(s)Q_3x(s)ds \\ & + \int_{t-\tau_M}^{t}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)R_1\dot{x}(v)dvds \\ & + \int_{t-\eta_M}^{t}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)R_2\dot{x}(v)dvds \\ & + \int_{t-d_M}^{t}\int_{s}^{t} \dot{x}^{\mathrm{T}}(v)R_3\dot{x}(v)dvds. \end{aligned} \tag{13}$$

Taking derivative on $V(t)$ and taking expectation on them, we have

$$\begin{aligned}\mathcal{E}\{\dot{V}(t)\} &= \mathcal{E}\{2x^{\mathrm{T}}(t)P\dot{x}(t)\} + x^{\mathrm{T}}(t)\mathcal{Q}x(t)\\ &- x^{\mathrm{T}}(t-\tau_M)Q_1x(t-\tau_M)\\ &- x^{\mathrm{T}}(t-\eta_M)Q_2x(t-\eta_M)\\ &- x^{\mathrm{T}}(t-d_M)Q_3x(t-d_M) + \mathcal{E}\{\dot{x}^{\mathrm{T}}(t)\bar{R}\dot{x}(t)\}\\ &- \int_{t-\tau_M}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)ds - \int_{t-\eta_M}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)ds\\ &- \int_{t-d_M}^{t}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)ds \end{aligned}\tag{14}$$

where $\mathcal{Q} = Q_1 + Q_2 + Q_3$ and $\bar{R} = \tau_M R_1 + \eta_M R_2 + d_M R_3$.

Notice that $\mathcal{E}\{\alpha(t)\} = \bar{\alpha}$, $\mathcal{E}\{\alpha(t)-\bar{\alpha}\} = 0$, $\mathcal{E}\{(\alpha(t)-\bar{\alpha})^2\} = \gamma_1^2$, $\mathcal{E}\{\theta(t)\} = \bar{\theta}$, $\mathcal{E}\{\theta(t)-\bar{\theta}\} = 0$, and $\mathcal{E}\{(\theta(t)-\bar{\theta})^2\} = \gamma_2^2$, then, we can obtain

$$\mathcal{E}\{2x^{\mathrm{T}}(t)P\dot{x}(t)\} = 2x^{\mathrm{T}}(t)P\mathcal{A}, \mathcal{E}\{\dot{x}^{\mathrm{T}}(t)\bar{R}\dot{x}(t)\}\tag{15}$$

$$\begin{aligned}&= \mathcal{A}^{\mathrm{T}}\bar{R}\mathcal{A} + \hat{\theta}^2\gamma_1^2\mathcal{B}_1^{\mathrm{T}}\bar{R}\mathcal{B}_1\\ &+ \gamma_2^2\mathcal{B}_2^{\mathrm{T}}\bar{R}\mathcal{B}_2 + \gamma_2^2\gamma_1^2\mathcal{B}_1^{\mathrm{T}}\bar{R}\mathcal{B}_1 + \gamma_1^2F^{\mathrm{T}}\bar{R}F\end{aligned}\tag{16}$$

where $\mathcal{A} = Ax(t) + B(1-\bar{\theta})[\bar{\alpha}Kx(t-\tau(t)) + (1-\bar{\alpha})Kx(t-\eta(t)) + (1-\bar{\alpha})Ke_k(t)] + \bar{\theta}F$, $\mathcal{B}_1 = BKx(t-\tau(t)) - BKx(t-\eta(t)) - BKe_k(t)$, $\mathcal{B}_2 = \bar{\alpha}BKx(t-\tau(t)) + \hat{\alpha}BKx(t-\eta(t)) + \hat{\alpha}BKe_k(t)$, and $F = BKf(x(t-d(t)))$, in which $\hat{\theta}$, $\hat{\alpha}$, γ_1, and γ_2 are defined in (12).

By employing the free-weighting matrices method [44], [45], for any appropriately dimensioned matrices M, N, S, T, U, and V, the following equations hold:

$$2\xi^{\mathrm{T}}(t)M\left[x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t}\dot{x}(s)ds\right] = 0\tag{17}$$

$$2\xi^{\mathrm{T}}(t)N\left[x(t-\tau(t)) - x(t-\tau_M) - \int_{t-\tau_M}^{t-\tau(t)}\dot{x}(s)ds\right] = 0\tag{18}$$

$$2\xi^{\mathrm{T}}(t)T\left[x(t) - x(t-\eta(t)) - \int_{t-\eta(t)}^{t}\dot{x}(s)ds\right] = 0\tag{19}$$

$$2\xi^{\mathrm{T}}(t)S\left[x(t-\eta(t)) - x(t-\eta_M) - \int_{t-\eta_M}^{t-\eta(t)}\dot{x}(s)ds\right] = 0\tag{20}$$

$$2\xi^{\mathrm{T}}(t)U\left[x(t) - x(t-d(t)) - \int_{t-d(t)}^{t}\dot{x}(s)ds\right] = 0\tag{21}$$

$$2\xi^{\mathrm{T}}(t)V\left[x(t-d(t)) - x(t-d_M) - \int_{t-d_M}^{t-d(t)}\dot{x}(s)ds\right] = 0\tag{22}$$

where

$$\begin{aligned}\xi^{\mathrm{T}}(t) &= \begin{bmatrix}\xi_1^{\mathrm{T}}(t) & \xi_2^{\mathrm{T}}(t) & \xi_3^{\mathrm{T}}(t)\end{bmatrix}\\ \xi_1^{\mathrm{T}}(t) &= \begin{bmatrix}x^{\mathrm{T}}(t) & x^{\mathrm{T}}(t-d(t)) & x^{\mathrm{T}}(t-d_M)\end{bmatrix}\\ \xi_2^{\mathrm{T}}(t) &= \begin{bmatrix}x^{\mathrm{T}}(t-\tau(t)) & x^{\mathrm{T}}(t-\tau_M)\end{bmatrix}\end{aligned}$$

and

$$\xi_3^{\mathrm{T}}(t) = \begin{bmatrix}x^{\mathrm{T}}(t-\eta(t)) & x^{\mathrm{T}}(t-\eta_M) & e_k^{\mathrm{T}}(t) & f^{\mathrm{T}}(x)\end{bmatrix}.$$

By applying Lemma 1, for given real matrices R_1 and R_2, the following inequalities can be obtained:

$$\begin{aligned}-2\xi^{\mathrm{T}}(t)M\int_{t-\tau(t)}^{t}\dot{x}(s)ds &\le \int_{t-\tau(t)}^{t}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)ds\\ &+ \tau(t)\xi^{\mathrm{T}}(t)MR_1^{-1}M^{\mathrm{T}}\xi(t)\end{aligned}\tag{23}$$

$$\begin{aligned}-2\xi^{\mathrm{T}}(t)N\int_{t-\tau_M}^{t-\tau(t)}\dot{x}(s)ds &\le \int_{t-\tau_M}^{t-\tau(t)}\dot{x}^{\mathrm{T}}(s)R_1\dot{x}(s)ds\\ &+ (\tau_M-\tau(t))\xi^{\mathrm{T}}(t)NR_1^{-1}N^{\mathrm{T}}\xi(t)\end{aligned}\tag{24}$$

$$\begin{aligned}-2\xi^{\mathrm{T}}(t)T\int_{t-\eta(t)}^{t}\dot{x}(s)ds &\le \int_{t-\eta(t)}^{t}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)ds\\ &+ \eta(t)\xi^{\mathrm{T}}(t)TR_2^{-1}T^{\mathrm{T}}\xi(t)\end{aligned}\tag{25}$$

$$\begin{aligned}-2\xi^{\mathrm{T}}(t)S\int_{t-\eta_M}^{t-\eta(t)}\dot{x}(s)ds &\le \int_{t-\eta_M}^{t-\eta(t)}\dot{x}^{\mathrm{T}}(s)R_2\dot{x}(s)ds\\ &+ (\eta_M-\eta(t))\xi^{\mathrm{T}}(t)SR_2^{-1}S^{\mathrm{T}}\xi(t)\end{aligned}\tag{26}$$

$$\begin{aligned}-2\xi^{\mathrm{T}}(t)U\int_{t-d(t)}^{t}\dot{x}(s)ds &\le \int_{t-d(t)}^{t}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)ds\\ &+ d(t)\xi^{\mathrm{T}}(t)UR_3^{-1}U^{\mathrm{T}}\xi(t)\end{aligned}\tag{27}$$

$$\begin{aligned}-2\xi^{\mathrm{T}}(t)V\int_{t-d_M}^{t-d(t)}\dot{x}(s)ds &\le \int_{t-d_M}^{t-d(t)}\dot{x}^{\mathrm{T}}(s)R_3\dot{x}(s)ds\\ &+ (d_M-d(t))\xi^{\mathrm{T}}(t)VR_3^{-1}V^{\mathrm{T}}\xi(t).\end{aligned}\tag{28}$$

From Assumption 1, we have

$$\begin{aligned}&\bar{\theta}x^{\mathrm{T}}(t-d(t))G^{\mathrm{T}}Gx(t-d(t))\\ &\quad - \bar{\theta}f^{\mathrm{T}}(x(t-d(t)))f(x(t-d(t))) \ge 0.\end{aligned}\tag{29}$$

From the inequality (4), we have

$$\sigma x^{\mathrm{T}}(t-\eta(t))\Omega x(t-\eta(t)) - e_k^{\mathrm{T}}(t)\Omega e_k(t) \ge 0.\tag{30}$$

Combining (14)–(30), we can obtain that

$$\begin{aligned}\mathcal{E}\{\dot{V}(t)\} &\le 2x^{\mathrm{T}}(t)P\mathcal{A} + x^{\mathrm{T}}(t)(Q_1+Q_2+Q_3)x(t)\\ &- x^{\mathrm{T}}(t-\tau_M)Q_1x(t-\tau_M) - x^{\mathrm{T}}(t-\eta_M)Q_2x(t-\eta_M)\\ &- x^{\mathrm{T}}(t-d_M)Q_3x(t-d_M) + \mathcal{A}^{\mathrm{T}}\bar{R}\mathcal{A} + \hat{\theta}\gamma_1^2\mathcal{B}_1^{\mathrm{T}}\bar{R}\mathcal{B}_1\\ &+ \gamma_2^2\mathcal{B}_2^{\mathrm{T}}\bar{R}\mathcal{B}_2 + \gamma_2^2\gamma_1^2\mathcal{B}_3^{\mathrm{T}}\bar{R}\mathcal{B}_3\\ &+ \gamma_1^2\big(BKf(x(t-d(t)))^{\mathrm{T}}\bar{R}BKf(x(t-d(t))\big)\\ &+ 2\xi^{\mathrm{T}}(t)M[x(t)-x(t-\tau(t))]\\ &+ 2\xi^{\mathrm{T}}(t)T[x(t)-x(t-\eta(t))]\\ &+ 2\xi^{\mathrm{T}}(t)N[x(t-\tau(t))-x(t-\tau_M)]\\ &+ 2\xi^{\mathrm{T}}(t)S[x(t-\eta(t))-x(t-\eta_M)]\\ &+ \tau(t)\xi^{\mathrm{T}}(t)MR_1^{-1}M^{\mathrm{T}}\xi(t)\\ &+ 2\xi^{\mathrm{T}}(t)V[x(t-d(t))-x(t-d_M)]\\ &+ (\tau_M-\tau(t))\xi^{\mathrm{T}}(t)NR_1^{-1}N^{\mathrm{T}}\xi(t)\\ &+ \eta(t)\xi^{\mathrm{T}}(t)TR_2^{-1}T^{\mathrm{T}}\xi(t)\\ &+ (\eta_M-\eta(t))\xi^{\mathrm{T}}(t)SR_2^{-1}S^{\mathrm{T}}\xi(t)\\ &+ 2\xi^{\mathrm{T}}(t)U[x(t)-x(t-d(t))]\\ &+ d(t)\xi^{\mathrm{T}}(t)UR_3^{-1}U^{\mathrm{T}}\xi(t)\\ &+ (d_M-d(t))\xi^{\mathrm{T}}(t)VR_3^{-1}V^{\mathrm{T}}\xi(t)\\ &+ \bar{\theta}x^{\mathrm{T}}(t-d(t))G^{\mathrm{T}}Gx(t-d(t)) - e_k^{\mathrm{T}}(t)\Omega e_k(t)\\ &- \bar{\theta}f^{\mathrm{T}}(x(t-d(t)))f(x(t-d(t)))\end{aligned}$$

$$\begin{aligned}&+\sigma x^{\mathrm{T}}(t-\eta(t))\Omega x(t-\eta(t))\\&\le \xi^{\mathrm{T}}(t)\Big(\Xi_{11}+\Upsilon+\Upsilon^{\mathrm{T}}+\Xi_{21}^{\mathrm{T}}\Xi_{22}^{-1}\Xi_{21}+\Xi_{31}^{\mathrm{T}}\Xi_{33}^{-1}\Xi_{31}\\&\quad+\Xi_{41}^{\mathrm{T}}\Xi_{44}^{-1}\Xi_{41}+\Xi_{51}^{\mathrm{T}}\Xi_{55}^{-1}\Xi_{51}\Xi_{61}^{\mathrm{T}}\Xi_{66}^{-1}\Xi_{61}\\&\quad+\Xi_{71}^{\mathrm{T}}\Xi_{77}^{-1}\Xi_{71}-\Xi_{81}^{\mathrm{T}}\Xi_{81}\Big)\xi(t).\end{aligned}\tag{31}$$

By using Schur complement equivalence and Lemma 2, it can be concluded that $\mathcal{E}\{\dot{V}(t)\}<0$ which can be ensured by (12). Since $V(t)>0$, $\mathcal{E}\{\dot{V}(t)\}<0$, it is easy to derive that $\mathcal{E}\{|x(t)|^2\}\to 0$ as $t\to\infty$. This completes the proof. ■

Theorem 1 presents a sufficient condition which guarantees the mean square asymptotical stability of the augmented system (8). Based on Theorem 1, the following theorem is devoted to designing the controller in the form of (7).

Theorem 2: For given probability scalars $\bar{\alpha}$, $\bar{\theta}$, time delays τ_M, η_M, d_M, trigger parameter σ, scalars ε_1, ε_2, ε_3, and ε_4, the closed-loop system in (8) with controller feedback gain $K=YX^{-1}$ is MSAS, if there exist matrices $X>0$, $\bar{Q}_1>0$, $\bar{Q}_2>0$, $\bar{Q}_3>0$, $\bar{R}_1>0$, $\bar{R}_2>0$, $\bar{R}_3>0$, $\bar{\Omega}>0$, $\bar{M}$, $\bar{N}$, $\bar{T}$, $\bar{S}$, $\bar{U}$, and $\bar{V}$ with appropriate dimensions such that the following linear matrix inequalities hold for $l=1,\dots,8$

$$\bar{\Xi}(l)=\begin{bmatrix}\bar{\Phi}(1) & *\\ \bar{\Phi}(2) & \bar{\Phi}(3)\end{bmatrix}<0\tag{32}$$

where the elements of the matrix $\bar{\Xi}(l)$ are given in the Appendix. Other symbols have been defined in Theorem 1.

Proof: Due to

$$(R_k-\varepsilon_k^{-1}P)R_k^{-1}(R_k-\varepsilon_k^{-1}P)\ge 0,\ (k=1,2,3)\tag{33}$$

we can obtain from (33) that

$$-PR_k^{-1}P\le -2\varepsilon_k P+\varepsilon_k^2 R_k,\ (k=1,2,3).\tag{34}$$

Replace the terms $\mathrm{diag}\{-PR_1^{-1}P,-PR_2^{-1}P,-PR_3^{-1}P\}$ by $\mathrm{diag}\{-2\varepsilon_1X+\varepsilon_1^2\bar{R}_1,-2\varepsilon_2X+\varepsilon_2^2\bar{R}_2,-2\varepsilon_3X+\varepsilon_3^2\bar{R}_3\}$ in $\Xi(l)$ which is defined in (12), then $\breve{\Xi}(l)$ is obtained.

Define $X=P^{-1}$, $Y=KX$, $XQ_1X=\bar{Q}_1$, $XQ_2X=\tilde{Q}_2$, $XQ_3X=\bar{Q}_3$, $XR_1X=\bar{R}_1$, $XR_2X=\bar{R}_2$, $XR_3X=\bar{R}_3$, $X\Omega X=\bar{\Omega}$, $XMX=\bar{M}$, $XNX=\bar{N}$, $XTX=\bar{T}$, $XSX=\bar{S}$, $XUX=\bar{U}$, $XVX=\bar{V}$, pre- and post-multiply $\breve{\Xi}(l)$ with $J=\mathrm{diag}\{X,X,\dots,I\}$ and its transpose. Then recalling $-X^{\mathrm{T}}X\le -2\varepsilon_4X+\varepsilon_4^2I$, replace $-X^{\mathrm{T}}X$ by $-2\varepsilon_4X+\varepsilon_4^2I$, then (32) can be derived. This completes the proof. ■

Assume that $\alpha(t)\equiv 1$, the state measurements are transmitted according to time-triggering scheme, the closed-loop system (8) can be written as

$$\begin{aligned}\dot{x}(t)&=Ax(t)+(1-\bar{\theta})BKx(t-\tau(t))\\&\quad+\bar{\theta}BKf(x(t-d(t)))\\&\quad+\big(\bar{\theta}-\theta(t)\big)\big[BKx(t-\tau(t))-BKf(x(t-d(t)))\big].\end{aligned}\tag{35}$$

The following result may be expected by the same derivation method in Theorem 2.

Corollary 1: Let the scalars $\bar{\theta}$, τ_M, d_M, ε_1, ε_3, ε_4, and controller feedback gain $K=YX^{-1}$ be given. Then, the closed-loop system in (35) is MSAS, if there exist matrices $X>0$, $\bar{Q}_1>0$, $\bar{Q}_3>0$, $\bar{R}_1>0$, $\bar{R}_3>0$, $\hat{M}$, $\hat{N}$, $\hat{U}$, and $\hat{V}$ such that the following linear matrix inequalities hold for $l=1,2,3,4$

$$\hat{\Xi}(l)<0\tag{36}$$

where

$$\hat{\Xi}(l)=\begin{bmatrix}\hat{\Xi}_{11}+\hat{\Upsilon}+\hat{\Upsilon}^{\mathrm{T}} & * & * & * & *\\ \hat{\Xi}_{21}(l) & \hat{\Xi}_{22} & * & * & *\\ \hat{\Xi}_{31} & 0 & \hat{\Xi}_{33} & * & *\\ \hat{\Xi}_{41} & 0 & 0 & \hat{\Xi}_{44} & *\\ \hat{\Xi}_{51} & 0 & 0 & 0 & -I\end{bmatrix}$$

$$\hat{\Xi}_{11}=\begin{bmatrix}\hat{A} & * & * & * & * & *\\ 0 & 0 & * & * & * & *\\ 0 & 0 & -\bar{Q}_3 & * & * & *\\ \hat{\theta}Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & 0 & * & *\\ 0 & 0 & 0 & 0 & -\bar{Q}_1 & *\\ \bar{\theta}Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & 0 & 0 & \hat{X}\end{bmatrix}$$

$$\hat{A}=AX+XA^{\mathrm{T}}+\bar{Q}_1+\bar{Q}_3$$

$$\hat{X}=-2\bar{\theta}\varepsilon_4X+\bar{\theta}\varepsilon_4I$$

$$\hat{\Upsilon}=\begin{bmatrix}\hat{M}+\hat{U} & -\hat{U}+\hat{V} & -\hat{V} & -\hat{M}+\hat{N} & -\hat{N} & 0\end{bmatrix}$$

$$\hat{\Xi}_{21}(1)=\begin{bmatrix}\sqrt{\tau_M}\hat{M}^{\mathrm{T}}\\ \sqrt{d_M}\hat{U}^{\mathrm{T}}\end{bmatrix},\quad \hat{\Xi}_{21}(2)=\begin{bmatrix}\sqrt{\tau_M}\hat{N}^{\mathrm{T}}\\ \sqrt{d_M}\hat{U}^{\mathrm{T}}\end{bmatrix}$$

$$\hat{\Xi}_{21}(3)=\begin{bmatrix}\sqrt{\tau_M}\hat{M}^{\mathrm{T}}\\ \sqrt{d_M}\hat{V}^{\mathrm{T}}\end{bmatrix},\ \hat{\Xi}_{21}(4)=\begin{bmatrix}\sqrt{\tau_M}\hat{N}^{\mathrm{T}}\\ \sqrt{d_M}\hat{U}^{\mathrm{T}}\end{bmatrix}$$

$$\hat{\Xi}_{31}=\begin{bmatrix}\sqrt{\tau_M}AX & 0 & 0 & \hat{\theta}\sqrt{\tau_M}BY & 0 & \bar{\theta}\sqrt{\tau_M}BY\\ \sqrt{d_M}AX & 0 & 0 & \hat{\theta}\sqrt{d_M}BY & 0 & \bar{\theta}\sqrt{d_M}BY\end{bmatrix}$$

$$\hat{\Xi}_{41}=\begin{bmatrix}0 & 0 & 0 & \gamma_2\sqrt{\tau_M}BY & 0 & -\gamma_2\sqrt{\tau_M}BY\\ 0 & 0 & 0 & \gamma_2\sqrt{d_M}BY & 0 & -\gamma_2\sqrt{d_M}BY\end{bmatrix}$$

$$\hat{\Xi}_{22}=\mathrm{diag}\{-\bar{R}_1,-\bar{R}_3\}$$

$$\hat{\Xi}_{33}=\mathrm{diag}\{-2\varepsilon_1X+\varepsilon_1^2\bar{R}_1,-2\varepsilon_3X+\varepsilon_3^2\bar{R}_3\}$$

$$\hat{\Xi}_{44}=\hat{\Xi}_{33},\ \hat{\Xi}_{51}=\begin{bmatrix}0 & \sqrt{\theta}GX & 0 & 0 & 0 & 0\end{bmatrix}.$$

Assume that $\alpha(t)\equiv 0$, the state measurements are transmitted through the network based on the event-triggering scheme, the closed-loop system (8) can be written in the following form:

$$\begin{aligned}\dot{x}(t)&=Ax(t)+(1-\bar{\theta})BKx(t-\eta(t))\\&\quad+(1-\bar{\theta})BKe_k(t)+\bar{\theta}BKf(x(t-d(t)))\\&\quad+(\bar{\theta}-\theta(t))\big[BKx(t-\tau(t))+BKe_k(t)\\&\qquad-BKf(x(t-d(t)))\big].\end{aligned}\tag{37}$$

We can obtain the following Corollary 2 by the same deviation method in Theorem 2 and Corollary 1.

Corollary 2: Let $\bar{\theta}$, η_M, d_M, σ, ε_2, ε_3, ε_4, and controller feedback gain $K=YX^{-1}$ be given. Then, the closed-loop system in (37) is MSAS, if there exist matrices $X>0$, $\bar{Q}_2>0$, $\bar{Q}_3>0$, $\bar{R}_2>0$, $\bar{R}_3>0$, $\bar{\Omega}>0$, $\tilde{T}$, $\tilde{S}$, $\tilde{U}$, and $\tilde{V}$ with appropriate dimensions such that the following linear matrix inequalities hold for $l = 1, 2, 3, 4$

$$\tilde{\Xi}(l) = \begin{bmatrix} \tilde{\Xi}_{11} + \tilde{\Upsilon} + \tilde{\Upsilon}^{\mathrm{T}} & * & * & * & * \\ \tilde{\Xi}_{21}(l) & \tilde{\Xi}_{22} & * & * & * \\ \tilde{\Xi}_{31} & 0 & \tilde{\Xi}_{33} & * & * \\ \tilde{\Xi}_{41} & 0 & 0 & \tilde{\Xi}_{44} & * \\ \tilde{\Xi}_{51} & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{38}$$

where

$$\tilde{\Xi}_{11} = \begin{bmatrix} \tilde{A} & * & * & * & * & * & * \\ 0 & 0 & * & * & * & * & * \\ 0 & 0 & -Q_3 & * & * & * & * \\ \hat{\theta} Y^{\mathrm{T}} B^{\mathrm{T}} & 0 & 0 & \sigma \bar{\Omega} & * & * & * \\ 0 & 0 & 0 & 0 & -\bar{Q}_2 & * & * \\ \hat{\theta} Y^{\mathrm{T}} B^{\mathrm{T}} & 0 & 0 & 0 & 0 & -\bar{\Omega} & * \\ \bar{\theta} Y^{\mathrm{T}} B^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & \hat{X} \end{bmatrix}$$

$$\tilde{A} = AX + XA^{\mathrm{T}} + \bar{Q}_1 + \bar{Q}_3$$

$$\tilde{\Upsilon} = \begin{bmatrix} \tilde{T} + \tilde{U} & -\tilde{U} + \tilde{V} & -\tilde{V} & -\tilde{T} + \tilde{S} & -\tilde{S} & 0 & 0 \end{bmatrix}$$

$$\tilde{\Xi}_{21}(1) = \begin{bmatrix} \sqrt{\eta_M} \tilde{T}^{\mathrm{T}} \\ \sqrt{d_M} \tilde{U}^{\mathrm{T}} \end{bmatrix}, \quad \tilde{\Xi}_{21}(2) = \begin{bmatrix} \sqrt{\eta_M} \tilde{S}^{\mathrm{T}} \\ \sqrt{d_M} \tilde{U}^{\mathrm{T}} \end{bmatrix}$$

$$\tilde{\Xi}_{21}(3) = \begin{bmatrix} \sqrt{\eta_M} \tilde{T}^{\mathrm{T}} \\ \sqrt{d_M} \tilde{V}^{\mathrm{T}} \end{bmatrix}, \tilde{\Xi}_{21}(4) = \begin{bmatrix} \sqrt{\eta_M} \tilde{S}^{\mathrm{T}} \\ \sqrt{d_M} \tilde{U}^{\mathrm{T}} \end{bmatrix}$$

$$\tilde{\Xi}_{31} = \begin{bmatrix} \sqrt{\eta_M} \Psi_1 \\ \sqrt{d_M} \Psi_1 \end{bmatrix}, \tilde{\Xi}_{41} = \begin{bmatrix} \sqrt{\eta_M} \Psi_2 \\ \Psi_2 \end{bmatrix}$$

$$\Psi_1 = \begin{bmatrix} AX & 0 & 0 & \hat{\theta} BY & 0 & \hat{\theta} BY & \bar{\theta} BY \end{bmatrix}$$

$$\Psi_2 = \begin{bmatrix} 0 & 0 & 0 & \gamma_2 BY & 0 & \gamma_2 BY & -\gamma_2 BY \end{bmatrix}$$

$$\tilde{\Xi}_{51} = \begin{bmatrix} 0 & \sqrt{\theta} GX & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\tilde{\Xi}_{22} = \mathrm{diag}\{-\bar{R}_2, -\bar{R}_3\}$$

$$\hat{\Xi}_{33} = \mathrm{diag}\{-2\varepsilon_2 X + \varepsilon_2^2 \bar{R}_2, -2\varepsilon_3 X + \varepsilon_3^2 \bar{R}_3\}$$

$$\hat{\Xi}_{44} = \hat{\Xi}_{33}.$$

IV. Simulation Examples

A simulation example is given to illustrate the usefulness of the proposed method in this section.

Consider a special model with parameters given as follows [22]:

$$A = \begin{bmatrix} -2 & -0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad B = \begin{bmatrix} 0.05 \\ 0.02 \end{bmatrix}.$$

The initial state is given as $x_0 = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}^{\mathrm{T}}$, and set the sampling period $h = 1$. The nonlinear attack signals are chosen as

$$f(x(t - d(t))) = \begin{bmatrix} \tanh(-0.3 x_2(t - d(t))) \\ \tanh(0.6 x_1(t - d(t))) \end{bmatrix}$$

which satisfies (9) in Assumption 1, and the nonlinearity bound is $G = \mathrm{diag}\{0.6, 0.3\}$.

In the following, we will present four possible cases to illustrate the usefulness of the proposed results.

Case 1: When there is no cyber attacks, that is, $\theta(t) = 0$, the system reduces to the case in [22]. For given $\bar{\alpha} = 0.2$, $d_M = 0.3$, $\tau_M = 0.1$, $\eta_M = 0.2$, and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 1$, the triggering parameter $\sigma = 0.1$, from Theorem 2, we obtain the parameters of the controller gain and the triggering matrix as

$$K = \begin{bmatrix} -1.2616 & -15.0532 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 7.3326 & 0.0358 \\ 0.0358 & 6.2552 \end{bmatrix}. \tag{39}$$

The state response is shown in Fig. 2.

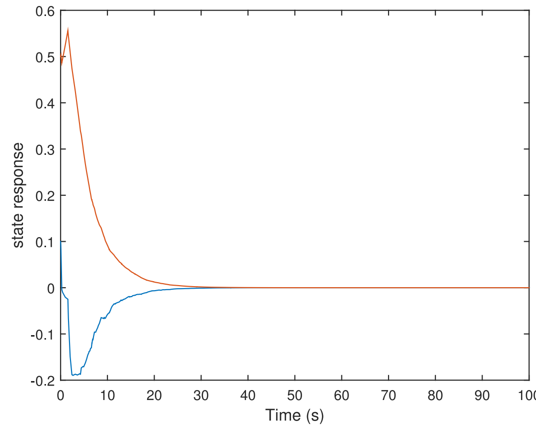

Fig. 2. State response in case 1.

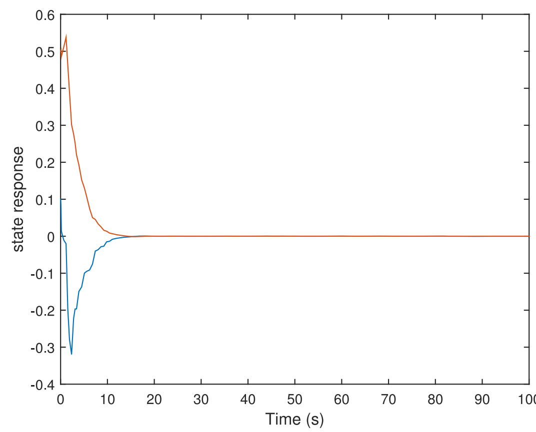

Fig. 3. State response in case 2.

Case 2: When the system with cyber attacks is under a time-triggering scheme, that is $\alpha(t) = 1$. Set $\bar{\theta} = 0.1$, $\tau_M = 0.1$, $d_M = 0.2$, $\sigma = 0$, and $\varepsilon_1 = \varepsilon_3 = \varepsilon_4 = 1$, from Corollary 1, the controller feedback gain is

$$K = \begin{bmatrix} -4.3025 & -26.3208 \end{bmatrix}. \tag{40}$$

Fig. 3 depicts the state response. The occurrence of cyber attacks is given in Fig. 4.

Case 3: When the closed-loop system with cyber attacks is under an event-triggering scheme, that is, $\alpha(t) = 0$, let $\bar{\theta} = 0.1$, $d_M = 0.2$, $\eta_M = 0.2$, $\sigma = 0.1$, and $\varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$, by applying Corollary 2, the controller feedback gain and event-triggering matrix are as follows:

$$K = \begin{bmatrix} -1.2812 & -17.0583 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 3.2555 & 0.0814 \\ 0.0814 & 3.0223 \end{bmatrix}. \tag{41}$$

The state response is shown in Fig. 5. The release instants and intervals are illustrated in Fig. 6.

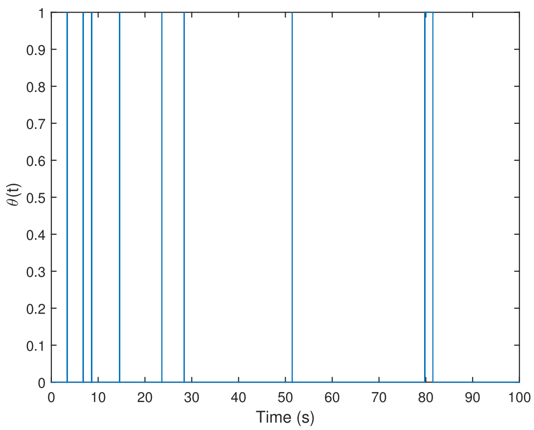

Fig. 4. Occurrence of cyber attacks.

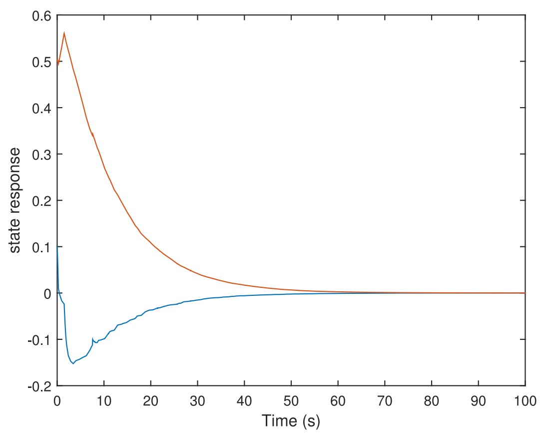

Fig. 5. State response in case 3.

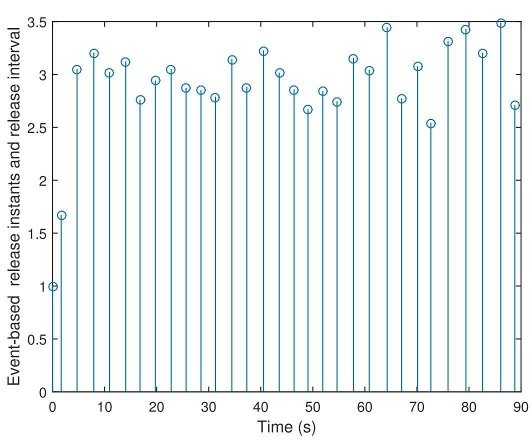

Fig. 6. Release instants and intervals in case 3.

Case 4: When the closed-loop system with cyber attacks is under a hybrid-triggering scheme, set $\bar{\alpha} = 0.2$, $\bar{\theta} = 0.1$, $\tau_M = 0.1$, $d_M = 0.3$, $\eta_M = 0.2$, $\sigma = 0.1$, and $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = \varepsilon_4 = 1$, applying the method in Theorem 2, we can obtain the controller feedback gain and event-triggering matrix

$$K = \begin{bmatrix} -1.8086 & -18.0568 \end{bmatrix}, \quad \Omega = \begin{bmatrix} 23.88 & 0.572 \\ 0.572 & 19.1016 \end{bmatrix}. \tag{42}$$

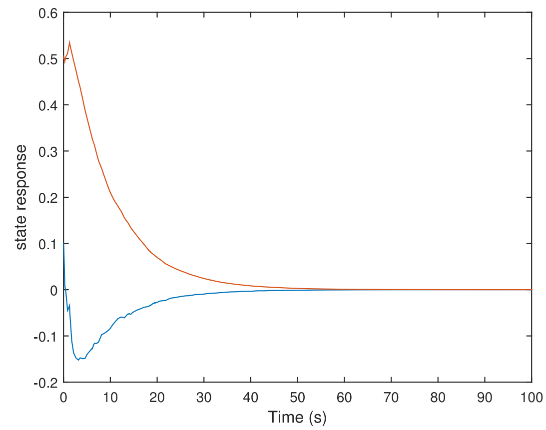

Fig. 7. State response in case 4.

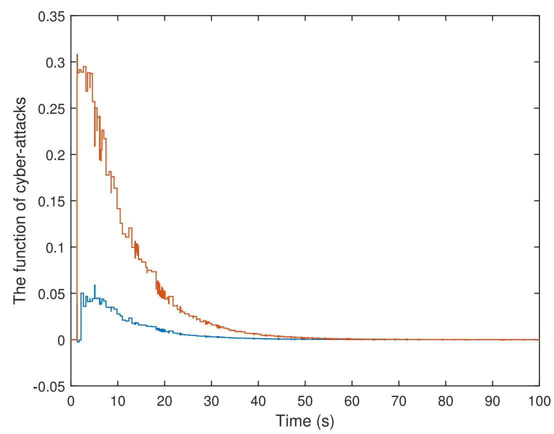

Fig. 8. Attack function $f(x(t - d(t)))$.

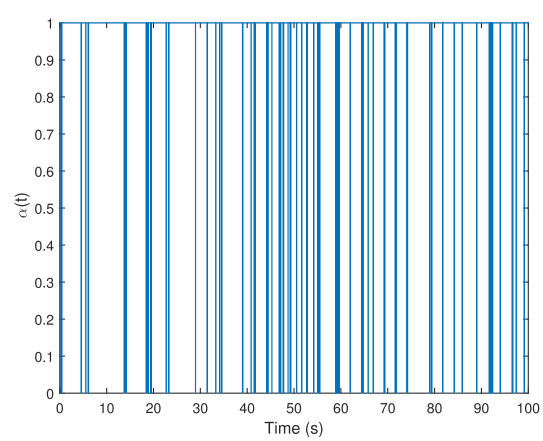

Fig. 9. Bernoulli stochastic variable $\alpha(t)$ with $\bar{\alpha} = 0.2$ in case 4.

The state response is depicted in Fig. 7. Fig. 8 gives the curves of the information $f(x(t - d(t)))$ sent by attackers. The variations of $\alpha(t)$ in this simulation is shown as Fig. 9.

In the simulation, the parameters of the controller and event generator are calculated for the four cases. Figs. 2, 3, 5, and 7 show the state responses under different situations, which illustrate that the system can be stabilized by our method. By comparing Fig. 2 with Fig. 7, we can easily see that the closed-loop system converges to zero more quickly in case 1 which illustrates that the system performance is degraded by cyber attacks. Fig. 6 shows that the transmission frequency is reduced under event-triggering scheme. Moreover, the hybrid-triggering scheme is adopted in this paper to adjust the amount of transmission, which includes the characteristics of the time-triggering scheme and event-triggering scheme.

V. Conclusion

This paper addressed the controller design for a class of hybrid-triggering NCSs. The communication channel between the sensor and the controller is an open network environment which is subject to cyber attacks. The occurrence of cyber attacks is governed by a set of independent random variables satisfying Bernoulli distribution. In order to make full use of the scarce network resources, a hybrid-triggering scheme is adopted, in which the switching from time-triggering scheme to the event-triggering scheme is operated according to a scheduling strategy. Based on the established model, a sufficient condition is obtained to ensure the stability of the closed-loop system. Moreover, the parameters of the hybrid-triggering scheme and the controller gain are co-designed by solving a set of linear matrix inequalities. The simulation example has verified the usefulness of the proposed approach. Follow the investigation initiated in this paper, further research includes stability of nonlinear NCSs with hybrid attacks, observer-based cyber attack detection in complex networks, and controller design for cyber-physical system under hybrid DoS attacks.

Appendix

$$\bar{\Phi}(1) = \begin{bmatrix} \bar{\Xi}_{11} + \bar{\Upsilon} + \bar{\Upsilon}^{\mathrm{T}} & * & * & * \\ \bar{\Xi}_{21}(l) & \bar{\Xi}_{22} & * & * \\ \bar{\Xi}_{31} & 0 & \bar{\Xi}_{33} & * \\ \bar{\Xi}_{41} & 0 & 0 & \bar{\Xi}_{44} \end{bmatrix}$$

$$\bar{\Phi}(2) = \begin{bmatrix} \bar{\Xi}_{51} & 0 & 0 & 0 \\ \bar{\Xi}_{61} & 0 & 0 & 0 \\ \bar{\Xi}_{71} & 0 & 0 & 0 \\ \bar{\Xi}_{81} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Phi}(3) = \begin{bmatrix} \bar{\Xi}_{55} & * & * & * \\ 0 & \bar{\Xi}_{66} & * & * \\ 0 & 0 & \bar{\Xi}_{77} & * \\ 0 & 0 & 0 & -X \end{bmatrix}$$

$$\bar{\Xi}_{11} = \begin{bmatrix} \bar{\Phi}_{11} & * \\ \bar{\Phi}_{21} & \bar{\Phi}_{22} \end{bmatrix}$$

$$\bar{\mathbb{A}} = AX + XA^{\mathrm{T}} + \bar{Q}_1 + \bar{Q}_2 + \bar{Q}_3$$

$$\bar{\Phi}_{11} = \begin{bmatrix} \bar{\mathbb{A}} & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & -\bar{Q}_3 & * \\ \hat{\theta}\bar{\alpha}Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Phi}_{21} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \hat{\theta}\hat{\alpha}Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \hat{\theta}\hat{\alpha}Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & 0 \\ \bar{\theta}Y^{\mathrm{T}}B^{\mathrm{T}} & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Phi}_{22} = \begin{bmatrix} -\bar{Q}_1 & * & * & * & * \\ 0 & \sigma\bar{\Omega} & * & * & * \\ 0 & 0 & -\bar{Q}_2 & * & * \\ 0 & 0 & 0 & -\bar{\Omega} & * \\ 0 & 0 & 0 & 0 & -2\bar{\theta}\varepsilon_4 X + \bar{\theta}\varepsilon_4^2 \end{bmatrix}$$

$$\bar{\Upsilon} = \begin{bmatrix} \bar{\Upsilon}_1 & -\bar{N} & \bar{S} - \bar{T} & -\bar{S} & 0 & 0 \end{bmatrix}$$

$$\bar{\Upsilon}_1 = \begin{bmatrix} \bar{M} + \bar{T} + \bar{U} & \bar{V} - \bar{U} & -\bar{V} & \bar{N} - \bar{M} \end{bmatrix}$$

$$\bar{\Xi}_{21}(1) = \begin{bmatrix} \sqrt{\tau_M}\bar{M}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{T}^{\mathrm{T}} \\ \sqrt{d_M}\bar{U}^{\mathrm{T}} \end{bmatrix}, \bar{\Xi}_{21}(2) = \begin{bmatrix} \sqrt{\tau_M}\bar{M}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{T}^{\mathrm{T}} \\ \sqrt{d_M}\bar{V}^{\mathrm{T}} \end{bmatrix}$$

$$\bar{\Xi}_{21}(3) = \begin{bmatrix} \sqrt{\tau_M}\bar{M}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{S}^{\mathrm{T}} \\ \sqrt{d_M}\bar{U}^{\mathrm{T}} \end{bmatrix}, \bar{\Xi}_{21}(4) = \begin{bmatrix} \sqrt{\tau_M}\bar{M}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{S}^{\mathrm{T}} \\ \sqrt{d_M}\bar{V}^{\mathrm{T}} \end{bmatrix}$$

$$\bar{\Xi}_{21}(5) = \begin{bmatrix} \sqrt{\tau_M}\bar{N}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{T}^{\mathrm{T}} \\ \sqrt{d_M}\bar{U}^{\mathrm{T}} \end{bmatrix}, \bar{\Xi}_{21}(6) = \begin{bmatrix} \sqrt{\tau_M}\bar{N}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{T}^{\mathrm{T}} \\ \sqrt{d_M}\bar{V}^{\mathrm{T}} \end{bmatrix}$$

$$\bar{\Xi}_{21}(7) = \begin{bmatrix} \sqrt{\tau_M}\bar{N}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{S}^{\mathrm{T}} \\ \sqrt{d_M}\bar{U}^{\mathrm{T}} \end{bmatrix}, \bar{\Xi}_{21}(8) = \begin{bmatrix} \sqrt{\tau_M}\bar{N}^{\mathrm{T}} \\ \sqrt{\eta_M}\bar{S}^{\mathrm{T}} \\ \sqrt{d_M}\bar{V}^{\mathrm{T}} \end{bmatrix}$$

$$\bar{\Xi}_{31} = \begin{bmatrix} \bar{\Xi}_{311} & \bar{\Xi}_{312} \end{bmatrix}$$

$$\bar{\Xi}_{311} = \begin{bmatrix} \sqrt{\tau_M}AX & 0 & 0 & \sqrt{\tau_M}\hat{\theta}\bar{\alpha}\ BY & 0 \\ \sqrt{\eta_M}AX & 0 & 0 & \sqrt{\eta_M}\hat{\theta}\bar{\alpha}\ BY & 0 \\ \sqrt{d_M}AX & 0 & 0 & \sqrt{d_M}\hat{\theta}\bar{\alpha}\ BY & 0 \end{bmatrix}$$

$$\bar{\Xi}_{312} = \begin{bmatrix} \sqrt{\tau_M}\hat{\theta}\hat{\alpha}\ BY & 0 & \sqrt{\tau_M}\hat{\theta}\hat{\alpha}\ BY & \sqrt{\tau_M}\bar{\theta}\ BY \\ \sqrt{\eta_M}\hat{\theta}\hat{\alpha}\ BY & 0 & \sqrt{\eta_M}\hat{\theta}\hat{\alpha}\ BY & \sqrt{\eta_M}\bar{\theta}\ BY \\ \sqrt{d_M}\hat{\theta}\hat{\alpha}\ BY & 0 & \sqrt{d_M}\hat{\theta}\hat{\alpha}\ BY & \sqrt{d_M}\bar{\theta}\ BY \end{bmatrix}$$

$$\bar{\Xi}_{41} = \begin{bmatrix} \bar{\Xi}_{411} & \bar{\Xi}_{412} \end{bmatrix}$$

$$\bar{\Xi}_{411} = \begin{bmatrix} 0 & 0 & 0 & \gamma_1\hat{\theta}\sqrt{\tau_M}BY & 0 \\ 0 & 0 & 0 & \gamma_1\hat{\theta}\sqrt{\eta_M}\ BY & 0 \\ 0 & 0 & 0 & \gamma_1\hat{\theta}\sqrt{d_M}\ BY & 0 \end{bmatrix}$$

$$\bar{\Xi}_{412} = \begin{bmatrix} -\gamma_1\hat{\theta}\sqrt{\tau_M}\ BY & 0 & -\gamma_1\hat{\theta}\sqrt{\tau_M}\ BY & 0 \\ -\gamma_1\hat{\theta}\sqrt{\eta_M}\ BY & 0 & -\gamma_1\hat{\theta}\sqrt{\eta_M}\ BY & 0 \\ -\gamma_1\hat{\theta}\sqrt{d_M}\ BY & 0 & -\gamma_1\hat{\theta}\sqrt{d_M}\ BY & 0 \end{bmatrix}$$

$$\bar{\Xi}_{51} = \begin{bmatrix} \bar{\Xi}_{511} & \bar{\Xi}_{512} \end{bmatrix}$$

$$\bar{\Xi}_{511} = \begin{bmatrix} 0 & 0 & 0 & \gamma_2\bar{\alpha}\sqrt{\tau_M}BY & 0 \\ 0 & 0 & 0 & \gamma_2\bar{\alpha}\sqrt{\eta_M}\ BY & 0 \\ 0 & 0 & 0 & \gamma_2\bar{\alpha}\sqrt{d_M}\ BY & 0 \end{bmatrix}$$

$$\bar{\Xi}_{512} = \begin{bmatrix} \gamma_2\hat{\alpha}\sqrt{\tau_M}\ BY & 0 & \gamma_2\hat{\alpha}\sqrt{\tau_M}\ BY & 0 \\ \gamma_2\hat{\alpha}\sqrt{\eta_M}\ BY & 0 & \gamma_2\hat{\alpha}\sqrt{\eta_M}\ BY & 0 \\ \gamma_2\hat{\alpha}\sqrt{d_M}\ BY & 0 & \gamma_2\hat{\alpha}\sqrt{d_M}\ BY & 0 \end{bmatrix}$$

$$\bar{\Xi}_{61} = \begin{bmatrix} \bar{\Xi}_{611} & \bar{\Xi}_{612} \end{bmatrix}$$

$$\bar{\Xi}_{611} = \begin{bmatrix} 0 & 0 & 0 & \gamma_1\gamma_2\sqrt{\tau_M}BY & 0 \\ 0 & 0 & 0 & \gamma_1\gamma_2\sqrt{\eta_M}\ BY & 0 \\ 0 & 0 & 0 & \gamma_1\gamma_2\sqrt{d_M}\ BY & 0 \end{bmatrix}$$

$$\bar{\Xi}_{612} = \begin{bmatrix} -\gamma_1\gamma_2\sqrt{\tau_M}\ BY & 0 & -\gamma_1\gamma_2\sqrt{\tau_M}\ BY & 0 \\ -\gamma_1\gamma_2\sqrt{\eta_M}\ BY & 0 & -\gamma_1\gamma_2\sqrt{\eta_M}\ BY & 0 \\ -\gamma_1\gamma_2\sqrt{d_M}\ BY & 0 & -\gamma_1\gamma_2\sqrt{d_M}\ BY & 0 \end{bmatrix}$$

$$\bar{\Xi}_{71} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\sqrt{\tau_M}\ BY \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\sqrt{\eta_M}\ BY \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \gamma_2\sqrt{d_M}\ BY \end{bmatrix}$$

$$\bar{\Xi}_{81} = \begin{bmatrix} 0 & \bar{\theta}GX & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\Xi}_{22} = \mathrm{diag}\{-\bar{R}_1, -\bar{R}_2, -\bar{R}_3\}$$
$$\bar{\Xi}_{44} = \bar{\Xi}_{55} = \bar{\Xi}_{66} = \bar{\Xi}_{77} = \bar{\Xi}_{33}$$
$$\bar{\Xi}_{33} = \mathrm{diag}\Big\{-2\varepsilon_1 X + \varepsilon_1^2\bar{R}_1, -2\varepsilon_2 X + \varepsilon_2^2\bar{R}_2 - 2\varepsilon_3 X + \varepsilon_3^2\bar{R}_3\Big\}.$$

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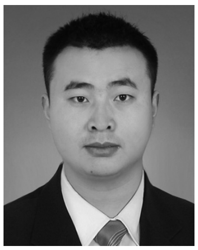

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