

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins



Hybrid-triggered-based security controller design for networked control system under multiple cyber attacks



Jie Cao a, Da Ding a, Jinliang Liu a,*, Engang Tian b, Songlin Hu c, Xiangpeng Xie c

- ^a College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, PR China
- ^b Department of Control Science and Engineering, University of Shanghai for Science and Technology, Shanghai 200093, PR China
- ^c Institute of Advanced Technology and the Jiangsu Engineering Laboratory of Big Data Analysis and Control for Active Distribution Network, Nanjing University of Posts and Telecommunications, Nanjing, Jiangsu210023, PR China

ARTICLE INFO

Article history: Received 17 April 2020 Received in revised form 2 September 2020 Accepted 19 September 2020 Available online 9 October 2020

Keywords:
Networked control system
Hybrid-triggered scheme
Multiple cyber attacks
Denial of service attacks
Deception attacks

ABSTRACT

This paper addresses the hybrid-triggered security control design for the networked control system (NCS) under multiple cyber attacks. A hybrid-triggered scheme (HTS) is introduced to mitigate the burden of the network transmission. Besides, a new model of multiple cyber attacks is built by simultaneously considering deception attacks and DoS attacks. In addition, A novel NCS model based on multiple cyber attacks is established with the hybrid-triggered scheme. Then, based on Lyapunov stability theory, criteria for guaranteeing the closed-loop system stability and achieving hybrid-triggered security controller design are derived. Finally, an illustrative example is given to validate the usefulness of the theoretical results.

© 2020 Elsevier Inc. All rights reserved.

1. Introduction

Networked control system (NCS) represents a control system where the feedback loop is closed via some communication networks. A vital feature of an NCS is that various system components, such as sensors, controllers, and actuators, allow being spatially distributed over a communication network medium [35]. Different from the traditional control system, NCS offers significant advantages in terms of reduced cost, convenient installation and maintenance, eased power requirements, high reliability, etc. Naturally, NCS has found extensive applications in a variety of areas including power grids, transportation networks, sensor networks and multi-agent systems. It should be pointed out that the introduction of the networks into the control system inevitably causes several network-induced constraints during system analysis and syntheses, such as data packet dropouts and communication delay. These problems will significantly degrade system performance or even lead to system instability [21]. In order to tackle these network-induced constraints for NCS, multidisciplinary research efforts with both control and communication theories are demanded, which has been witnessed by a large amount of literature over the past few years [2,13,17,31]. For example, in [32] the problem of communication constraint and topology switching are studied for distributed control of a sensor-network-based large-scale NCS. The authors in [25] investigated the problem of delay and data loss in a nonlinear NCS by using an adaptive back stepping control approach. The authors in [17] studied the controller design of NCS considering the random cyber attack and hybrid-triggered scheme.

E-mail address: liujinliang@vip.163.com (J. Liu).

^{*} Corresponding author.

Over the past decade, the problem of how to reduce the transmission of redundant data and thus improve the utilization efficiency of network bandwidth has been regarded as a hot research topic in both control and signal processing areas [1,6,7,9]. In order to reduce the impact of redundant data collected by traditional time-triggered schemes on network bandwidth, event-triggered schemes have been intensively studied [3,8,26,29]. A prominent feature of event-triggered schemes is that data transmissions are only invoked when a certain event triggering is violated, thereby making network resources occupied when "needed". Such an event-triggered scheme is of great significance for NCS when the system approaches its steady-state, and no external disturbance acts on the system. In this case, the system state will have little fluctuation between any two consecutive samplings and transmissions and apparently, the time-triggered scheme will still transmit these little-fluctuated data through the bandwidth-limited communication channels, leading to a waste of scarce network resources. For example, the authors in [29] proposed a novel event-triggered scheme which adopts a threshold in the triggering condition to decide whether to deliver the sample data. In addition, the authors applied the event-triggered scheme in [26] to study the problem of limited communication capabilities. A new mixed sampling scheme is proposed in [19], where self-triggered and event-triggered schemes are studied to improve energy efficiency for a wireless NCS. A novel hybridtriggering mechanism switching between the event-triggering mechanism and the time-triggering mechanism is introduced to reduce the load on the network bandwidth and improve the system performance for NCS in [15]. In [11], a hybrid communication mechanism is proposed to transmit the following measurements when the measured value released by the event-triggered is lost.

The introduction of a communication network into NCS not only brings network-induced constraints but also makes the NCS more open to the cyber world. Specifically, the modern NCSs face serious security threats, as testified by a number of security incidents in the past decade. For example, Black-Energy 3 attacked the Ukrainian power grid in 2015, causing damage to multiple substations and causing massive power outages; the Venezuelan Guri hydropower plant suffered a cyber attack in 2019, the destruction resulted in the interruption of power supply in 18 of the 23 states, and a large-scale power outage paralyzed traffic, interrupted communications, and prevented flights from taking off and landing normally. For NCS, the security estimation and control problems have yet received a great deal of attention, see, e.g., [4,5,14,27,28,34,36]. As acknowledged in the literature, denial of service(DoS) attack [23] and deception attacks [24] are deemed as the two most representative cyber attacks in NCS. In particular, a DoS attack disrupts the stability and operation of NCS by blocking signals and data from arriving at their destinations, while a deception attack injects malicious and falsified data into the sensor or control data transmission channels with an ultimate aim to disrupt system stability. For example, in [12], considering periodic DoS attacks, the authors redesign resilient event-triggered parameters and controllers to achieve the stability required by NCS. Paper [33] investigated the robust output consensus conundrum of heterogeneous linear multi-agent systems considering the impact of stochastic DoS attacks. In [10], resilient attack detection estimators are constructed to deal with problems caused by cyber attacks such as blocking wireless transmission channels and intentionally modify the system's state. By considering stochastic cyber attacks, the problem of finite-time H_{∞} filter with the event-triggered mechanism for networked state-dependent uncertain systems is studied in [16]. Several effective security control methods have been available to deal with DoS attack or deception attack separately. However, we should note that when network communication resources are limited, how to achieve secure event-triggered control of NCS in the simultaneous presence of multiple DoS attack and deception attack remains insufficiently explored, which motivates this study.

In this paper, by proposing a hybrid triggered scheme, the problem of security controller design for NCS under multiple cyber attacks will be investigated. A hybrid-triggered scheme governed by a Bernoulli random variable will be developed to reduce the redundant data transmissions. Multiple cyber attacks will be taken into consideration during the security controller design.

This paper is organized as follows. In Section 2, a mathematical model for the NCS with the hybrid-triggered scheme under multiple cyber-attacks is formulated. Based on the Lyapunov stability theory and LMI techniques, Section 3 shows the main results of the NCS with sufficient stability conditions and controller design method. Moreover, an example is given to show the effectiveness of the designed controller in Section 4. Section 5 presents the conclusion and prospects.

Notation: \mathbb{R}_n represents the n-dimensional Euclidean space, $\mathbb{R}_{n \times m}$ denotes the set of $n \times m$ real matrices; \mathbb{N} represents the natural number; \mathbb{N}^T denotes the transposition of matrix \mathbb{N} ; \mathbb{N} >0, for $\mathbb{N} \in \mathbb{R}$ means that \mathbb{N} is real symmetric positive definite; \mathbb{N} represents the identity matrix, and 0 represents the zero matrices with appropriate dimension; $\mathbb{E}\{A\}$ represents the mathematical expectation of \mathbb{N} ; for a symmetric matrix $\begin{bmatrix} A & * \\ B & C \end{bmatrix}$ with two symmetric matrices \mathbb{N} and \mathbb{N} denotes the entries implied by symmetry.

2. System description

Consider the following networked control system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(t) \in \mathbb{R}_m$ represents the state vector, $u(t) \in \mathbb{R}_n$ represents output vector, matrices A and B are constant matrices of appropriate dimensions.

We aim to deal with the problem of security controller design for NCS with multiple cyber attacks and hybrid-triggered scheme, where the system is controlled through a communication network, as shown in Fig. 1. The controller is described as

$$u(t) = Kx(t) \tag{2}$$

where $K \in \mathbb{R}_{n \times m}$ denotes the controller gain matrix to be designed.

A hybrid-triggering mechanism is introduced into the system to seek a better balance between data transmission and data collection. The switching pattern of the hybrid-triggered scheme is assumed to follow the Bernoulli distribution, More specifically, under the time-triggered scheme, the data to be transmitted over the network can be written as

$$x_n(t) = x(t - \eta(t)) \tag{3}$$

where $\eta(t) \in [0, \eta_m], \eta_m$ is the upper bound of the network induced delay. When the event-triggered scheme is activated, signals transmitted over the network should meet the following condition:

$$e_i(t)^T \Omega e_i(t) > \sigma^2 x^T (t_i h + nh) \Omega x (t_i h + nh) \tag{4}$$

where $\sigma \in [0,1), \Omega > 0, e_i(t) = x(t_ih) - x(t_ih + nh), j, n \in \mathbb{N}, x(t_ih)$ indicates the latest transmitted data, $x(t_ih + nh)$ represents the current sampled data. Then the next triggering instant can be computed as:

$$t_{j+1}h = t_j h + \inf_{n \ge 1} \geqslant \left\{ nh|e_j(t)^T \Omega e_j(t) > \sigma^2 x^T (t_j h + nh) \Omega x (t_j h + nh) \right\}$$

$$\tag{5}$$

Define $\tau(t) = t - t_i h - nh$, the sampled data can be presented as

$$x_e(t) = x(t - \tau(t)) + e_i(t) \tag{6}$$

Remark 1. Due to the limitation of hardware equipment and operating environment, network resources are usually limited. The author in [29] proposed an event-triggered scheme to get better performance and lower network bandwidth requirements under limited network resources. Only when condition (4) is satisfied, the sampled data can be delivered over the network.

According to the hybrid-triggered scheme proposed in [18], combining with Eq. (3) and Eq. (6), the data transmission in the network can be written as

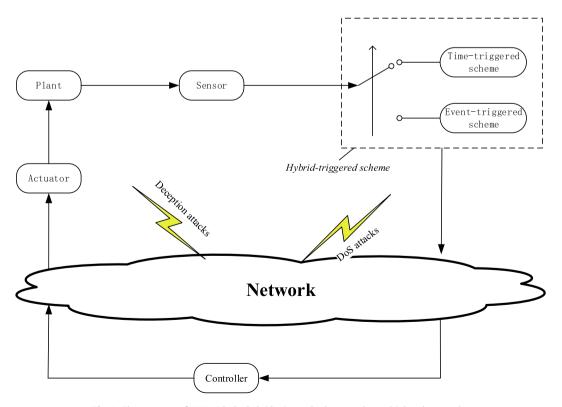


Fig. 1. The structure of NCS with the hybrid-triggered scheme under multiple cyber attacks.

$$x_H(t) = \alpha(t)x(t - \eta(t)) + (1 - \alpha(t))[x(t - \tau(t)) + e_i(t)]$$

$$\tag{7}$$

where $\alpha(t)$ is a random Bernoulli distribution variable, $\mathbb{E}\{\alpha(t)\}=\overline{\alpha}$, and the mathematical variance of $\alpha(t)$ can be described as ζ_1^2 .

Remark 2. Different from the event-triggering mechanism and time-triggering mechanism, the hybrid-triggered scheme described by the random Bernoulli distribution variable $\alpha(t)$. When $\alpha(t) = 1$, the time-triggered scheme is activated, the system periodically sends sampled data, the Eq. (7) can be expressed as $x_H(t) = x(t - \eta(t))$; otherwise, event-triggered scheme is activated, the sampled data can be delivered when the system satisfies the trigger conditions, then the Eq. (7) can be present as $x_H(t) = [x(t - \tau(t)) + e_i(t)]$.

Remark 3. The hybrid-triggered scheme was first proposed in [18], the purpose of the hybrid-triggered scheme is to improve the effectiveness and reduce the burden of network bandwidth for NCS. Similar hybrid-triggered schemes have been studied in [11,15]. A detailed comparison and description of the hybrid-triggered schemes is beyond the scope of this paper.

Due to the openness of the network, data cannot be transmitted securely using encryption algorithms solely. Therefore, attackers can easily cause severe data loss and data tampering. Cyber attacks will seriously affect the normal operation of the system and thus should be carefully modeled and addressed. This paper presents multiple cyber attacks, including randomly occurring deception attacks and DoS attacks.

When deception attack is active, the transmitted data can be replaced. The transmitted data can then be expressed as:

$$\mathbf{x}_{D}(t) = \beta(t)\mathbb{E}(\mathbf{x}(t - \lambda(t))) + (1 - \beta(t))\mathbf{x}_{H}(t) \tag{8}$$

where $x_H(t)$ represents the data normally transmitted by the system during the sleep period of the deception attack, as given in (7). The random Bernoulli distribution variable $\beta(t)$ is used to indicate the possibility of spoofing attacks. $\mathbb{E}\{\beta(t)\} = \overline{\beta}$, and ζ_2^2 is used to denote the mathematical variance of $\beta(t)$.

Assumption 1. [17] For given real constant matrix \mathbb{G} , the deception attack $\mathbb{F}(x(t))$ satisfies

$$\|\mathbb{F}(x(t-\lambda(t)))\|_{2} \le \|\mathbb{G}x(t-\lambda(t))\|_{2} \tag{9}$$

Remark 4. The nonlinear function $\mathbb{F}(x(t-\lambda(t)))$ represents the deception attack signal which replaces the normal data transmission. When $\beta(t)=0$, the network is suffering from the deception attacks, the data normally transmitted by the system can be replaced with $\mathbb{F}(x(t-\lambda(t)))$; otherwise, $\beta(t)=1$, the system has not been subjected to spoofing attacks, which have no impact on the network, and data will be transmitted usually.

Next, we let $\delta(t)$ represent the frequency of DoS attacks, which satisfies the following:

$$\delta(t) = \begin{cases} 1, & t \in [h_i, h_i + l_i) \\ 0, & t \in [h_i + l_i, h_{i+1}) \end{cases}$$
(10)

where h_i denotes the start moment of the i_{th} sleep cycle time, l_i represents the duration of the i_{th} sleep period. In addition, h_i and l_i satisfy the following relationship inequalities: $0 \le h_0 < h_1 < h_1 + l_1 < \cdots < h_i < h_i + l_i < \cdots$. For the purpose of simplicity, $\mathbb{D}_{i,1} \triangleq [h_i, h_i + l_i)$ and $\mathbb{D}_{i,0} \triangleq [h_i, h_{i+1}]$ are given.

Assumption 2. [20] Assume that there exists a uniform upper bound b_{max} indicates the periods when the DoS attack is active, and a unified lower bound l_{min} represents normal communication time, then b_{max} and l_{min} satisfy the following inequalities:

$$\begin{cases}
l_{min} \leqslant & \inf_{i \in \mathbb{N}} \{l_i\} \\
b_{max} \geqslant & \sup_{i \in \mathbb{N}} \{h_i - h_{i-1} - l_{i-1}\}
\end{cases}$$
(11)

Assumption 3. [20] n(t) is defined as the total number of sleep/active transitions when the DoS attack is active, there exist $a \ge 0$ and $\varpi_D \in \mathbb{R}$ which should meet the following:

$$n(t) \leqslant a + \frac{h}{\varpi_{\mathsf{D}}} \tag{12}$$

Remark 5. When $\delta(t) = 1$, the DoS attack is sleeping, in other words, the DoS attack has no effects on the data transmission; while $\delta(t) = 0$, DoS attacks block data transmission and prevent the sampled data from being transmitted to the controller. The actual input of the controller, under DoS attacks and deception attacks, can be presented as follows:

$$\widetilde{\chi}(t) = \delta(t) \{ \beta(t) \mathbb{F}(\chi(t - \lambda(t))) + (1 - \beta(t)) \chi_H(t) \}$$

$$\tag{13}$$

Remark 6. From (13), the following three cases are noted: 1).when $\delta(t) = 0$, DoS attack is active and the controller will not receive any signal from the network; 2). when $\delta(t) = 1$ and $\beta(t) = 1$, the controller receives the transmitted data subject to only the deception attacks; 3). when $\delta(t) = 1$ and $\beta(t) = 0$, the data received by the controller is $x_H(t)$.

Because the DoS attack can block communication, the data transfer mechanism needs to be redesigned.

$$t_{i,j}h = \{t_{j,h} \text{ satisfying } 5 | t_{j,h} \in \mathbb{D}_{i-1,1}\} \cup \{h_i\} \tag{14}$$

where $j_l, t_j, i, l \in \mathbb{N}, j$ denotes the triggering time in i_{th} DoS attack period, $j \in \{1, 2, \dots, j_i\}$.

For $n \in \{1, 2, \dots, v_{i,j}\}$, define that

where $v_{i,j} \triangleq \sup\{n \in \mathbb{N} | t_{i,j}h + nh < t_{i,j+1}h\}$, then the event intervals $\varpi_{i,j}$ can be expressed as follows:

$$\varpi_{i,j} = \bigcup_{n=1}^{\nu_{i,j}} \left[t_{i,j} h + (n-1)h, t_{i,j} h + nh \right] \cup \left(t_{i,j} h + \nu_{i,j} h, t_{i,j+1} h \right)$$
(16)

Note that

$$\mathbb{D}_{i,1} = \cup_{k=0}^{k_i} \left\{ \boldsymbol{\varpi}_{i,j} \cap \mathbb{D}_{i,1} \right\} \subseteq \cup_{i=0}^{j_i} \boldsymbol{\varpi}_{i,j} \tag{17}$$

set

$$\Theta_{i,i}^n = h_{i,i}^n \cap \mathbb{D}_{i,1} \tag{18}$$

then combining (15)–(17), the interval $D_{i,1}$ can be rewritten as:

$$\mathbb{D}_{i,1} = \bigcup_{i=0}^{j_i} \bigcup_{n=1}^{v_{ij}+1} \Theta_{i,i}^n \tag{19}$$

Now, for $i \in \mathbb{N}, j \in \{1, 2, \dots, j_i\}$, two piecewise functions can be defined as:

$$\tau_{i,j}(t) = \begin{cases}
 t - t_{i,j}h, & t \in \Theta_{i,j}^{1} \\
 t - t_{i,j}h - h, & t \in \Theta_{i,j}^{2} \\
 \vdots & \vdots \\
 t - t_{i,j}h - v_{i,j}h, & t \in \Theta_{i,j}^{v_{i,j}+1}
\end{cases}$$
(20)

and

$$e_{i,j}(t) = \begin{cases} 0, & t \in \Theta_{i,j}^{1} \\ x(t_{i,j}h) - x(t_{i,j}h + h), & t \in \Theta_{i,j}^{2} \\ \vdots & \\ x(t_{i,j}h) - x(t_{i,j}h + v_{i,j}h), & t \in \Theta_{i,j}^{v_{i,j}+1} \end{cases}$$
(21)

According to the inequality (4), and combining (20) and (21), the event-triggered condition can be expressed as:

$$e_{i,j}(t)^{\mathsf{T}}\Omega e_{i,j}(t) > \sigma^{2} x^{\mathsf{T}} \Big(t - \eta_{i,j}(t) \Big) \Omega x \Big(t - \eta_{i,j}(t) \Big) \tag{22}$$

Then under the event-triggering mechanism, the transmitted data $\tilde{x}_e(t)$ can be described as:

$$\tilde{x}_e(t) = x(t - \tau_{i,i}(t)) + e_{i,i}(t), t \in \varpi_{i,i} \cap \mathbb{D}_{i,1}$$

$$\tag{23}$$

The output u(t) can be obtained by combining (2), (13) and (23):

$$u(t) = \begin{cases} K\beta(t)\mathbb{F}(x(t)) + K(1 - \beta(t))(1 - \alpha(t))(x(t - \tau(t)) + e_k(t)) \\ + K(1 - \beta(t))\alpha(t)x(t - \eta(t)), & t \in \mathbb{D}_{i-1,1} \\ 0, & t \in \mathbb{D}_{i-1,0} \end{cases}$$
(24)

then the system can be rewritten as:

$$\begin{cases} \dot{x}(t) = \begin{cases} Ax(t) + BK\beta(t)\mathbb{F}(x(t)) + BK(1-\beta(t))(1-\alpha(t))\big(x(t-\tau(t)) + e_j(t)\big) + BK(1-\beta(t))\alpha(t) *x(t-\eta(t)), & t \in \mathbb{D}_{i-1,1} \\ Ax(t), & t \in \mathbb{D}_{i-1,0} \end{cases} \\ x(t) = \varphi(t), \end{cases}$$

$$(25)$$

3. Main results

Sufficient conditions satisfied the exponentially stable is obtained for the system(25) in this section. In addition, the hybrid-triggered-based controller is designed by considering multiple cyber attacks.

Lemma 1 [22]. For any vectors $a, b \in \mathbb{R}_n$, and $Q \in \mathbb{R}_{n \times n}$ is a positive definite matrix, the following inequality holds:

$$2a^{\mathsf{T}}b \leqslant a^{\mathsf{T}}Qa + b^{\mathsf{T}}Q^{-1}b \tag{26}$$

Lemma 2 [29]. Suppose $\tau(t) \in [0, \tau_m], \eta(t) \in [0, \eta_m]$, matrices $\Pi_1, \Pi_2, \Pi_3, \Pi_4$ and Ξ with appropriate dimensions, then

$$\tau(t)\Pi_1 + (\tau_m - \tau(t))\Pi_2 + \eta(t)\Pi_3 + (\eta_m - \eta(t))\Pi_4 + \Xi < 0,$$

if and only if

$$\begin{cases} \tau_{m}\Pi_{1} + \eta_{m}\Pi_{3} + \Xi < 0, \\ \tau_{m}\Pi_{2} + \eta_{m}\Pi_{3} + \Xi < 0, \\ \tau_{m}\Pi_{1} + \eta_{m}\Pi_{4} + \Xi < 0, \\ \tau_{m}\Pi_{2} + \eta_{m}\Pi_{4} + \Xi < 0. \end{cases}$$

$$(27)$$

Lemma 3 [30]. For given positive matrix X, Y and scalar θ , the following inequality holds:

$$-XY^{-1}X \leqslant -2\theta X + \theta^2 Y \tag{28}$$

Theorem 1. For given positive scalars ρ_1 , $\eta_1(t=1,2)$, trigger parameter σ , DoS parameters a, ϖ_D , l_{min} , b_{max} , matrix G and K, the system (25) is asymptotically stable if there exist positive matrix $\Omega > 0$, $P_1 > 0$, $Q_{1s} > 0$, $Z_{1s} > 0$ (s = 1,2,3), matrix L_q , M_q , N_q (q = 1,2,3,4) with appropriate dimensions, the following inequalities hold:

$$\Phi_l < 0 \tag{29}$$

$$P_1 \leqslant \eta_2 P_2 \tag{30}$$

$$P_2 \leq \eta_1 e^{2(\rho_1 + \rho_2)h} P_1$$
 (31)

$$Q_{is} \leqslant \eta_{3,-i}Q_{(3-i)s} \tag{32}$$

$$Z_{\text{K}} \leqslant \eta_{3} Z_{(3-1)} \tag{33}$$

$$\frac{2\rho_{1}l_{min}-2(\rho_{1}+\rho_{2})h-2\rho_{2}b_{max}-ln\eta_{1}\eta_{2}}{\varpi_{D}}>0\tag{34}$$

where the elements of the matrix Φ_i are given in Appendix A.

Proof. Choose the following Lyapunov functional for the system (25)

$$V_{\phi(t)}(t) = V_{1\phi(t)}(t) + V_{2\phi(t)}(t) + V_{3\phi(t)}(t)$$
(35)

where

$$V_{1\phi(t)}(t) = \chi(t)^T P_{\phi(t)} \chi(t)$$

$$V_{2\phi(t)}(t) = \int_{t-\tau_m}^t e(\cdot) x(s)^T Q_{\phi(t)1} x(t) ds + \int_{t-\eta_m}^t e(\cdot) x(s)^T Q_{\phi(t)2} x(s) ds + \int_{t-\lambda_m}^t e(\cdot) x(s)^T Q_{\phi(t)3} x(s) ds$$

$$V_{3\phi(t)}(t) = \int_{t-\tau_m}^t \int_{s}^t e(\cdot)\dot{x}(v)^T Z_{\phi(v)1}\dot{x}(v) dv ds + \int_{t-\tau_m}^t \int_{s}^t e(\cdot)\dot{x}(v)^T Z_{\phi(t)2}\dot{x}(v) dv ds + \int_{t-\lambda_m}^t \int_{s}^t e(\cdot)\dot{x}(v)^T Z_{\phi(t)3}\dot{x}(v) dv ds$$

in which $e(\cdot) = e^{2(-1)^{\phi(t)} \rho_{\phi(t)}(t-s)}, P_{\phi(t)}, Q_{\phi(t)}, Z_{\phi(t)}$ are symmetric positive matrices and $\phi(t)$ can be defined as follows:

$$\phi(t) = \begin{cases} 1, & t \in [-h, 0] \cup \left(\cup_{i \in \mathbb{N}} \mathbb{D}_{i, 1}\right) \\ 2, & t \in \cup_{i \in \mathbb{N}} \mathbb{D}_{i, 0} \end{cases}$$

$$(36)$$

For the case of $\phi(t) = 1$, by taking the time derivative and mathematical expectation of (35), we can obtain:

$$\begin{split} \mathbb{E}\Big\{\dot{V}_{1}(t)\Big\} \leqslant -2\rho_{1}V_{1}(t) + 2\rho_{1}V_{11}(t) + 2\mathbb{E}\big\{x^{T}(t)\rho_{1}\dot{x}(t)\big\} + x^{T}(t)Q_{11}x(t) + x^{T}(t)Q_{12}x(t) + x^{T}(t)Q_{13}x(t) \\ &+ \tau_{m}\mathbb{E}\big\{\dot{x}^{T}(t)Z_{11}\dot{x}(t)\big\} + \eta_{m}\mathbb{E}\big\{\dot{x}^{T}(t)Z_{11}\dot{x}(t)\big\} + \lambda_{m}\mathbb{E}\big\{\dot{x}^{T}(t)Z_{11}\dot{x}(t)\big\} - e^{-2\rho_{1}\tau_{m}}x^{T}(t - \tau_{m})Q_{11}x(t - \tau_{m}) \\ &- e^{-2\rho_{1}\eta_{m}}x^{T}(t - \eta_{m})Q_{12}x(t - \eta_{m}) - e^{-2\rho_{1}\lambda_{m}}x^{T}(t - \lambda_{m})Q_{12}x(t - \lambda_{m}) - \int_{t - \tau_{m}}^{t} e^{-2\rho_{1}\tau_{m}}\dot{x}^{T}(s)Z_{11}\dot{x}(s)ds \\ &- \int_{t - n}^{t} e^{-2\rho_{1}\eta_{m}}\dot{x}^{T}(s)Z_{12}\dot{x}(s)ds - \int_{t - \lambda_{m}}^{t} e^{-2\rho_{1}\lambda_{m}}\dot{x}^{T}(s)Z_{13}\dot{x}(s)ds \end{split} \tag{37}$$

The following can be obtained by using the free weighting matrix method

$$\begin{aligned} & 2\theta(t)L_{1}\left[x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t}\dot{x}(s)ds\right]=0\\ & 2\theta(t)L_{2}\left[x(t-\tau(t))-x(t-\tau_{m})-\int_{t-\tau_{m}}^{t-\tau(t)}\dot{x}(s)ds\right]=0\\ & 2\theta(t)M_{1}\left[x(t)-x(t-\eta(t))-\int_{t-\eta(t)}^{t}\dot{x}(s)ds\right]=0\\ & 2\theta(t)M_{2}\left[x(t-\eta(t))-x(t-\eta_{m})-\int_{t-\eta_{m}}^{t-\eta(t)}\dot{x}(s)ds\right]=0\\ & 2\theta(t)M_{2}\left[x(t-\chi(t))-x(t-\eta_{m})-\int_{t-\chi(t)}^{t}\dot{x}(s)ds\right]=0\\ & 2\theta(t)N_{1}\left[x(t)-x(t-\chi(t))-\int_{t-\chi(t)}^{t}\dot{x}(s)ds\right]=0\\ & 2\theta(t)N_{2}\left[x(t-\chi(t))-x(t-\chi(t))-\int_{t-\chi(t)}^{t-\chi(t)}\dot{x}(s)ds\right]=0 \end{aligned} \tag{38}$$

and by applying Lemma 1, we can obtain

$$-2\theta(t)L_{1}\int_{t-\tau(t)}^{t}\dot{x}(s)ds\leqslant\tau_{m}\theta^{T}(t)L_{1}e^{2\rho_{1}\tau_{m}}Z_{11}^{-1}L_{1}^{T}\theta(t)+\int_{t-\tau(t)}^{t}\dot{x}^{T}(s)e^{-2\rho_{1}\tau_{m}}Z_{11}\dot{x}(s)ds\\ -2\theta(t)L_{2}\int_{t-\tau_{m}}^{t-\tau(t)}\dot{x}(s)ds\leqslant\tau_{m}\theta^{T}(t)L_{2}e^{2\rho_{1}\tau_{m}}Z_{11}^{-1}L_{2}^{T}\theta(t)+\int_{t-\tau_{m}}^{t-\tau(t)}\dot{x}^{T}(s)e^{-2\rho_{1}\tau_{m}}Z_{11}\dot{x}(s)ds\\ -2\theta(t)M_{1}\int_{t-\eta(t)}^{t}\dot{x}(s)ds\leqslant\eta_{m}\theta^{T}(t)M_{1}e^{2\rho_{1}\eta_{m}}Z_{12}^{-1}M_{1}^{T}\theta(t)+\int_{t-\eta(t)}^{t}\dot{x}^{T}(s)e^{-2\rho_{1}\eta_{m}}Z_{11}\dot{x}(s)ds\\ -2\theta(t)M_{2}\int_{t-\eta_{m}}^{t-\eta(t)}\dot{x}(s)ds\leqslant\eta_{m}\theta^{T}(t)M_{2}e^{2\rho_{1}\eta_{m}}Z_{12}^{-1}M_{2}^{T}\theta(t)+\int_{t-\eta(t)}^{t-\eta(t)}\dot{x}^{T}(s)e^{-2\rho_{1}\eta_{m}}Z_{11}\dot{x}(s)ds\\ -2\theta(t)N_{1}\int_{t-\lambda(t)}^{t}\dot{x}(s)ds\leqslant\lambda_{m}\theta^{T}(t)N_{1}e^{2\rho_{1}\lambda_{m}}Z_{13}^{-1}N_{1}^{T}\theta(t)+\int_{t-\lambda(t)}^{t}\dot{x}^{T}(s)e^{-2\rho_{1}\lambda_{m}}Z_{11}\dot{x}(s)ds\\ -2\theta(t)N_{2}\int_{t-\lambda(t)}^{t-\lambda(t)}\dot{x}(s)ds\leqslant\lambda_{m}\theta^{T}(t)N_{2}e^{2\rho_{1}\lambda_{m}}Z_{13}^{-1}N_{1}^{T}\theta(t)+\int_{t-\lambda(t)}^{t}\dot{x}^{T}(s)e^{-2\rho_{1}\lambda_{m}}Z_{11}\dot{x}(s)ds$$

where matrices $L_1, L_2, M_1, M_2, N_1, N_2$ with appropriate dimensions, $\theta(t) = [\theta_1 \ \theta_2 \ \theta_3]$, $\theta_1 = [x^T(t) \ x^T(t - \tau(t)) \ x^T(t - \tau_m)]$, $\theta_2 = [x^T(t - \eta(t)) \ x^T(t - \eta_m) \ x^T(t - \lambda(t))]$, $\theta_3 = [x^T(t - \lambda_m) \ \mathbb{F}^T(x(t - \lambda(t))) \ e_k^T(t)]$.

$$\mathbb{E}\{\dot{x}^{T}(t)Z_{11}\dot{x}(t)\} = A_{o}^{T}Z_{11}A_{o} + \zeta_{1}^{2}A_{\alpha}^{T}Z_{11}A_{\alpha} + \zeta_{2}^{2}A_{\beta}^{T}Z_{11}A_{\beta} + \zeta_{1}^{2}\zeta_{2}^{2}A_{\alpha\beta}^{T}Z_{11}A_{\alpha\beta}
\mathbb{E}\{\dot{x}^{T}(t)Z_{12}\dot{x}(t)\} = A_{o}^{T}Z_{12}A_{o} + \zeta_{1}^{2}A_{\alpha}^{T}Z_{12}A_{\alpha} + \zeta_{2}^{2}A_{\beta}^{T}Z_{12}A_{\beta} + \zeta_{1}^{2}\zeta_{2}^{2}A_{\alpha\beta}^{T}Z_{12}A_{\alpha\beta}
\mathbb{E}\{\dot{x}^{T}(t)Z_{13}\dot{x}(t)\} = A_{o}^{T}Z_{13}A_{o} + \zeta_{1}^{2}A_{\alpha}^{T}Z_{13}A_{\alpha} + \zeta_{2}^{2}A_{\beta}^{T}Z_{13}A_{\beta} + \zeta_{1}^{2}\zeta_{2}^{2}A_{\alpha\beta}^{T}Z_{13}A_{\alpha\beta}$$
(40)

where

$$\begin{split} &A_o = Ax(t) + BK\big[\bar{\beta}\mathbb{F}(x(t-\lambda(t))) + \big(1-\bar{\beta}\big)\bar{\alpha}x(t-\eta(t)) + \big(1-\bar{\alpha}\big)\big(1-\bar{\beta}\big)\big[x(t-\tau(t)) + e_j(t)\big]\big] \\ &A_\alpha = (\alpha(t) - \bar{\alpha})BK\big[\big(x(t-\tau(t)) - x(t-\eta(t)) - e_j(t)\big)\big(1-\bar{\beta}\big)\big] \\ &A_\beta = \big(\beta(t) - \bar{\beta}\big)BK\big[\mathbb{F}(x(t-\lambda(t))) - \bar{\alpha}x(t-\tau(t)) - (1-\bar{\alpha}\big)\big(x(t-\eta(t)) + e_j(t)\big)\big] \\ &A_{\alpha\beta} = (\alpha(t) - \bar{\alpha})\big(\beta(t) - \bar{\beta}\big)BK\big[x(t-\eta(t)) + e_j(t) - x(t-\tau(t))\big] \end{split}$$

From the event-triggered condition (4), it follows

$$\sigma^2 x^T \left(t - \eta_{i,j}(t) \right) \Omega x \left(t - \eta_{i,j}(t) \right) - e_{i,j}^T(t) \Omega e_{i,j}(t) > 0 \tag{41}$$

By recalling the Assumption 1, the following condition can be obtained

$$x^{T}(t-\lambda(t))\mathbb{G}^{T}\mathbb{G}x(t-\lambda(t)) - \mathbb{F}^{T}(x(t-\lambda(t)))\mathbb{F}(x(t-\lambda(t))) \leq 0$$

$$\tag{42}$$

Applying Schur complement and Lemma 2, the following can be obtained by combining (35)-(41):

$$\mathbb{E}\Big\{\dot{V}_{1}(t)\Big\} \leqslant -2\rho_{1}V_{1}(t) + \theta^{T}(t)\Big[\Upsilon_{11}^{1} + \tau_{m}\mathbb{E}\Big\{\dot{x}^{T}(t)Z_{11}\dot{x}(t)\Big\} + \eta_{m}\mathbb{E}\Big\{\dot{x}^{T}(t)Z_{12}\dot{x}(t)\Big\} \\
+ \lambda_{m}\mathbb{E}\Big\{\dot{x}^{T}(t)Z_{13}\dot{x}(t)\Big\}\tau_{m}L_{1}e^{2\rho_{1}\tau_{m}}Z_{11}^{-1}L_{1}^{T} + \tau_{m}L_{2}e^{2\rho_{1}\tau_{m}}Z_{11}^{-1}L_{2}^{T}\eta_{m}M_{1}e^{2\rho_{1}\eta_{m}}Z_{12}^{-1}M_{1}^{T} \\
+ \eta_{m}M_{2}e^{2\rho_{1}\eta_{m}}Z_{12}^{-1}M_{2}^{T}\lambda_{m}N_{1}e^{2\rho_{1}\lambda_{m}}Z_{13}^{-1}N_{1}^{T} + \lambda_{m}N_{2}e^{2\rho_{1}\lambda_{m}}Z_{13}^{-1}N_{2}^{T}\Big]\theta(t) \tag{43}$$

Considering the condition (29), we can get $\mathbb{E}\left\{\dot{V}_1(t)\right\} \leqslant -2\rho_1V_1(t)$, one can get the following by processing $\mathbb{E}\left\{\dot{V}_2(t)\right\}$ in the same way,

$$\mathbb{E}\Big\{\dot{V}_{2}(t)\Big\} \leqslant 2\rho_{2}V_{2}(t) + \theta^{T}(t)\Big[\Upsilon_{11}^{2} + \tau_{m}\mathbb{E}\big\{\dot{x}^{T}(t)Z_{21}\dot{x}(t)\big\} + \eta_{m}\mathbb{E}\big\{\dot{x}^{T}(t)Z_{22}\dot{x}(t)\big\} + \lambda_{m}\mathbb{E}\big\{\dot{x}^{T}(t)Z_{23}\dot{x}(t)\big\}\tau_{m}L_{3}e^{2\rho_{1}\tau_{m}}Z_{21}^{-1}L_{3}^{T} \\ + \tau_{m}L_{4}e^{2\rho_{1}\tau_{m}}Z_{21}^{-1}L_{4}^{T}\eta_{m}M_{3}e^{2\rho_{1}\eta_{m}}Z_{22}^{-1}M_{3}^{T} + \eta_{m}M_{4}e^{2\rho_{1}\eta_{m}}Z_{22}^{-1}M_{4}^{T}\lambda_{m}N_{3}e^{2\rho_{1}\lambda_{m}}Z_{23}^{-1}N_{3}^{T} + \lambda_{m}N_{4}e^{2\rho_{1}\lambda_{m}}Z_{23}^{-1}N_{4}^{T}\Big]\theta(t)$$

$$(44)$$

one can get $\mathbb{E}\left\{\dot{V}_{2}(t)\right\}\leqslant2
ho_{2}V_{2}(t).$

From the inequalities (43) and (44), then we have

$$\begin{cases}
\mathbb{E}\{V_1(t)\} \leqslant e^{-2\rho_1(t-h_i)}V_1(h_i), t \in [h_i, h_i + l_i) \\
\mathbb{E}\{V_2(t)\} \leqslant e^{2\rho_2(t-h_i)}V_2(h_i + l_i), t \in [h_i + l_i, h_{i+1})
\end{cases}$$
(45)

According to (30)-(33), one can be get that

$$\left\{ \mathbb{E}\left\{ V_{1}(h_{i}) \leqslant \eta_{2} \mathbb{E}\left\{ V_{2}(h_{i}^{-}) \right\} \mathbb{E}\left\{ V_{2}(h_{i} + l_{i}) \right\} \leqslant \eta_{1} e^{2(\rho_{1} + \rho_{2})} \mathbb{E}\left\{ V_{1}(h_{i}^{-} + l_{i}^{-}) \right\} \right\}$$

$$\tag{46}$$

For $t \in [h_i, h_i + l_i)$, combining(45) and (46), it can be obtained

$$\mathbb{E}\{V_{1}(t)\} \leqslant e^{-2\rho_{1}(t-h_{i})}\eta_{2}\mathbb{E}\{V_{2}(h_{i}^{-})\}
\leqslant e^{-2\rho_{1}(t-h_{i})+2\rho_{2}(h_{i}-h_{i-1}-l_{i-1})} * \eta_{2}\mathbb{E}\{V_{2}(h_{i-1}+l_{i-1})\}
\leqslant \cdots
\leqslant e^{\rho(t)}(\eta_{1}\eta_{2})^{n(t)}\mathbb{E}\{V_{1}(h_{0})\}$$
(47)

where $\rho(t) = 2[(\rho_1 + \rho_2)hn(t) + \rho_1(-l_0 - l_1 - l_2 - \cdots - l_{i-2} - l_{i-1}) + \rho_2(h_i - h_{i-1} - l_0 - l_1 - l_2 - \cdots - l_{i-2} - l_{i-1})]$, then similar to the (47), for $t \in [h_i + i, l_{i+1})$, one can get the following:

$$\mathbb{E}\{V_{2}(t)\} \leqslant \eta_{1}e^{2\rho_{2}(t-h_{i}-l_{i})+2(\rho_{1}+\rho_{2})}\mathbb{E}\{V_{1}(h_{i}^{-}+l_{i}^{-})\}
\leqslant e^{2\rho_{2}(t-h_{i}-l_{i})+2(\rho_{1}+\rho_{2})-2\rho_{1}(t-h_{i})} * \eta_{1}\mathbb{E}\{V_{1}(h_{i-1})\} \leqslant \cdots \leqslant \frac{e^{\varrho(t)}}{\eta_{2}}(\eta_{1}\eta_{2})^{(n(t)+1)}\mathbb{E}\{V_{1}(\tilde{h}_{0})\}$$
(48)

with $\varrho(t) = 2(n(t) + 1)[(\rho_1 + \rho_2)h - \rho_{1min} + \rho_2 b_{max}].$

Define $M = max\left\{e^{a_1}, \frac{e^{a_2}}{\eta_2}\right\}$, according to (47)-(48), combining with the frequency of DoS attack, then one can get the inequality:

$$\mathbb{E}\{V_{\phi(t)}(t)\} \leqslant Me^{-dt}\mathbb{E}\{V_1(h_0)\}\tag{49}$$

where $a_1 = 2b_1 \Big[h(\rho_1 + \rho_2) - \rho_1 l_{min} + \rho_2 b_{max} + \frac{ln(\rho_1\rho_2)}{2}\Big], a_2 = 2(b_1 + 1) \Big[h(\rho_1 + \rho_2) - \rho_1 l_{min} + \rho_2 b_{max} + \frac{ln(\rho_1\rho_2)}{2}\Big], d_1 = \frac{2\rho_1 \Delta_{min} - 2h(\rho_1 + \rho_2) - 2\rho_2 b_{max} - ln\rho_1\rho_2}{m\rho_1}.$

From the definition of $V_{\phi(t)}(t)$,

$$\mathbb{E}\{V_1(0)\} \leqslant c_1 \|\phi\|_h^2, \mathbb{E}\{V_{\phi(t)}(t)\} \leqslant c_2 \|x(t)\|_h^2 \tag{50}$$

where $c_1 = max\Big\{\zeta_{max}(P_i) + h\zeta_{max}(Q_{11} + Q_{12} + Q_{13}) + \frac{h^2}{2}\zeta_{max}(Z_{11} + Z_{12} + Z_{13})\Big\}, c_2 = min\{\zeta_{min}(P_i)\}.$

Then combine (49) and (50), then we have the following:

$$\mathbb{E}\{V_{\phi(t)}(t)\} \leqslant \sqrt{\frac{c_2 M}{c_1}} e^{-\frac{d}{2}t} \|\varphi\|_h^2 \tag{51}$$

That means that we can get the system (25) is exponential stable when the decay rate is $\frac{d}{2}$.

One can get the sufficient conditions from Theorem 1, which can ensure mean-square exponential consensus of the system (25). Based on Theorem 1, the hybrid-triggered controller gain of NCS under multiple cyber attacks will be derived in the following.

This completes the proof.

Theorem 2. For given positive scalars ρ_1 , $\eta_1(i=1,2;s=1,2,3)$, trigger parameter σ , DoS parameters a, ϖ_D , l_{min} , b_{max} and matrix \mathbb{G} , the system (25) is asymptotically stable if there exist positive matrix $\hat{\Omega} > 0$, $P_{\phi(t)} > 0$, $\hat{Q}_{is} > 0$, $\hat{Z}_{is} > 0$, $X_i > 0$ (i=1,2;s=1,2,3), matrix Y, \hat{L}_q , \hat{M}_q , \hat{N}_q (q=1,2,3,4) with appropriate dimensions, the following linear matrix inequalities hold:

$$\hat{\Phi}_{l} < 0 \tag{52}$$

$$\begin{bmatrix} -\eta_2 X_2 & * \\ X_2 & -X_1 \end{bmatrix} \leqslant 0 \tag{53}$$

$$\begin{bmatrix} -\eta_1 e^{2(\rho_1 + \rho_2)h} X_1 & * \\ X_1 & X_2 \end{bmatrix} \leqslant 0 \tag{54}$$

$$\begin{bmatrix} -\eta_{(3-i)s} \widehat{Q}_{(3-i)s} & * \\ X_{3-i} & -2\nu_{is} X_{is} + \nu_{is}^2 Q_{is} \end{bmatrix} \leqslant 0$$
 (55)

$$\begin{bmatrix} -\eta_{(3-i)s} \hat{Q}_{(3-i)s} & * \\ X_{3-i} & -2v_{is}X_{is} + v_{x}^{2}O_{is} \end{bmatrix} \le 0$$
(56)

the elements of the matrix $\hat{\Phi}_{l}$ can be obtained in Appendix B.

In addition, the controller gain K is also got

$$K = YX_1^{-1} \tag{57}$$

Proof. For any positive e_1 , $(Z_{11} - e_1^{-1}P_1)Z_{11}^{-1}(Z_{11} - e_1^{-1}P_1) \ge 0$, by applying the Lemma 3, we can easily get the following inequalities:

$$-P_1Z_{11}^{-1}P_1 \leqslant -2e_1P_1 + e_1^2Z_{11}(58)$$

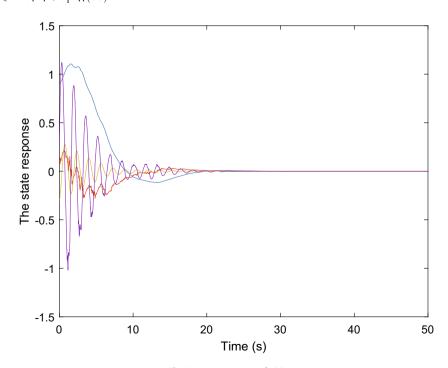


Fig. 2. State response of x(t).

Similarly, one can have

$$\begin{cases}
-P_{1}Z_{12}^{-1}P_{1} \leqslant -2e_{2}P_{1} + e_{2}^{2}Z_{12} \\
-P_{1}Z_{13}^{-1}P_{1} \leqslant -2e_{3}P_{1} + e_{3}^{2}Z_{13} \\
-P_{2}Z_{21}^{-1}P_{2} \leqslant -2e_{4}P_{2} + e_{4}^{2}Z_{21} \\
-P_{2}Z_{22}^{-1}P_{2} \leqslant -2e_{5}P_{2} + e_{5}^{2}Z_{22} \\
-P_{2}Z_{23}^{-1}P_{2} \leqslant -2e_{6}P_{2} + e_{6}^{2}Z_{23}
\end{cases}$$
(59)

Based on (58) and (59), by replacing the $-P_1Z_{11}^{-1}P_1, -P_1Z_{12}^{-1}P_1, -P_1Z_{13}^{-1}P_1, -P_2Z_{21}^{-1}P_2, -P_2Z_{22}^{-1}P_2, -P_2Z_{23}^{-1}P_2$ in with $-2e_1P_1 + e_1^2Z_{11}, -2e_2P_1 + e_2^2Z_{12}, -2e_3P_1 + e_3^2Z_{13}, -2e_4P_2 + e_4^2Z_{21}, -2e_5P_2 + e_5^2Z_{22}, -2e_6P_2 + e_6^2Z_{23}$ respectively. Then we obtain $\phi_{55}^1 = diag\{-2e_1P_1 + e_1^2Z_{11}, -2e_1P_1 + e_1^2Z_{11}, -2e_1P_1 + e_1^2Z_{11}, -2e_2P_1 + e_2^2Z_{12}, -2e_2P_1 + e_2^2Z_{12}, -2e_2P_1 + e_2^2Z_{12}, -2e_2P_1 + e_2^2Z_{12}, -2e_3P_1 + e_3^2Z_{13}, -2e_3P_1 + e_3^2Z_{13}, -2e_3P_1 + e_3^2Z_{13}\}$ and $\phi_{44}^2 = diag\{-2e_4P_2 + e_4^2Z_{21}, -2e_5P_2 + e_5^2Z_{22}, -2e_6P_2 + e_6^2Z_{23}\}$.

When $\phi(t) = 1$, define $X_1 = P_1^{-1}$, $Y = KX_1$, $\widehat{L}_{ij} = X_1L_{ij}X_1$, $\widehat{M}_{ij} = X_1M_{ij}X_1$, $\widehat{N}_{ij} = X_1N_{ij}X_1$ (i = 1,2; j = 1,2,3,···,9), $\widehat{\Omega} = X_1\Omega X_1$, $\widehat{Q}_{1k} = X_1Q_{1k}X_1$, $\widehat{Z}_{1k} = X_1Z_{1k}X_1$ (k = 1,2,3). Multiplying diag $\{X_1, \dots, X_1, I\}$ and its transpose on both side of Φ_1 , then

 $\hat{\Phi}_1 < 0$ can be obtained. Moreover, we can acquire that $\mathbb{E}\{V_1(t)\} < 0$.

When $\phi(t) = 2$, define $X_2 = P_2^{-1}$, $\widehat{L}_{ij} = X_2 L_{ij} X_2$, $\widehat{M}_{ij} = X_2 M_{ij} X_2$, $\widehat{N}_{ij} = X_2 N_{ij} X_2$ (i = 3,4; j = 1,2,3,···,7), $\widehat{\Omega} = X_2 \Omega X_2$, $\widehat{Q}_{2k} = X_2 Q_{2k} X_2$, $\widehat{Z}_{2k} = X_2 Z_{2k} X_2$ (k = 1,2,3). Multiplying diag $\{X_2, \dots, X_2\}$ and its transpose on both side of Φ_2 , then $\widehat{\Phi}_2$ can be

obtained. Further, $\mathbb{E}\{V_2(t)\}$ < 0 can also be obtained.

Applying the similar method, inequalities (54)-(56) hold which follow from the inequalities in (31)-(33). we can get that the system (25) is asymptotically stable in mean square from the results of Theorem 1. At the same time, the required controller state feedback gain can be obtained in (57).

This completes the proof.

4. Simulation example

A frequently used example [11,29], the inverted pendulum on a mobile shopping cart, is considered in this section. The linearized model of this example is constructed as follows:

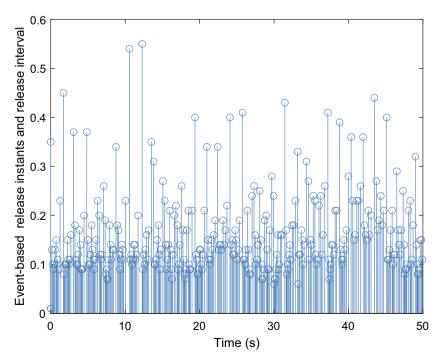


Fig. 3. Release instants and release interval.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{M_2 * g}{M_1} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{g}{L} & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \frac{1}{M_1} \\ 0 \\ \frac{-1}{M_1 * L} \end{bmatrix}$$

In this example, $M_1 = 10kg$, $M_2 = 1kg$, L = 0.75m, $g = 9.8076m/s^2$, where M_1 and M_2 respectively represent the cart mass and the mass of the pendulum bob, the length of the pendulum arm is L and g indicates the gravitational acceleration. Suppose the function of deception attacks is selected as follows:

$$\mathbb{F}(x(t)) = \begin{bmatrix} -tanh(0.25x(t)) \\ -tanh(0.15x(t)) \\ -tanh(0.20x(t)) \\ -tanh(0.10x(t)) \end{bmatrix}$$

According to the Assumption 1, we can obtain $\mathbb{G}=diag\{0.25,0.15,0.20,0.10\}$. Define the sampling period $h=0.01s, l_{min}=1.78, \eta_1=\eta_2=1.01, \rho_1=0.05, \rho_2=0.3, \sigma=0.4, e_1=e_2=e_3=3, e_4=e_5=e_6=10$. Set $\bar{\alpha}=0.35, \bar{\beta}=0.25, b_{max}=0.2$, which means that the network control system is under multiple cyber attacks with the hybrid-triggered scheme. Ω, X_1 and Y can be obtained by solving the above formulas.

$$\Omega = \begin{bmatrix} 0.0011 & 0.0019 & -0.0055 & -0.0008 \\ 0.0019 & 0.0038 & -0.0107 & -0.0018 \\ -0.0055 & -0.0107 & 0.0391 & 0.0050 \\ -0.0008 & -0.0018 & 0.0050 & 0.0015 \end{bmatrix}$$

$$X_1 = 1.0e + 03 * \begin{bmatrix} 7.1336 & -3.0915 & 0.1562 & -0.6325 \\ -3.0915 & 5.2444 & 0.6789 & 0.0763 \\ 0.1562 & 0.6789 & 0.3743 & -0.3393 \\ -0.6325 & 0.0763 & -0.3393 & 4.5615 \end{bmatrix}$$

$$Y = 1.0e + 05 * [-0.2218 & -0.9086 & -0.1101 & 1.2471]$$

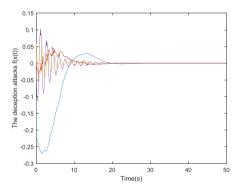


Fig. 4. The response of deception attacks.

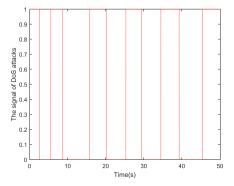


Fig. 5. The response of DoS attacks.

By applying the equality (57) in Theorem 2, the controller gain K can also be obtained as

$$K = [-18.9173 \quad -38.9378 \quad 77.3136 \quad 31.1188]$$

Define the initial condition $x_0 = [0.9 \ 0.1 \ -0.3 \ 0.3]$, Fig. 2,3,4,5,5 indicate the simulation results by applying MATLAB. Fig. 2 shows that the system applying the hybrid triggering scheme is exponentially stable suffering multiple network attacks. As can be seen from the from Fig. 3, the amount and frequency of transmitted data is significantly reduced, which reflects the release instants. Fig. 4 shows the f(x(t)) curve describing the deception attacks. From Fig. 5, the sequence of DoS attacks occurrence is presented. According to the above analysis, one can get that the method proposed in this article can reduce the burden of network bandwidth.

5. Conclusions

This paper investigates the security control for the NCS with the hybrid-triggered scheme under multiple cyber attacks. A hybrid-triggered scheme is exploited into the system to decrease network bandwidth consumption. Then when considering the impact of various network attacks, a new mathematical model of NCS with a mixed trigger network is first constructed. Moreover, one can derive sufficient conditions to ensure the stability of the system and the security controller gain is obtained by applying LMI techniques and Lyapunov function. Finally, a simulated example confirms the efficiency of the proposed method. In the future, to improve the capability of the NCS against cyber attacks, attack detection in NCS will be the focus of the following research.

CRediT authorship contribution statement

Jie Cao: Supervision, Methodology, Funding acquisition. **Ding Da:** Data curation, Writing - original draft. **Jinliang Liu:** Conceptualization, Writing - review & editing. **Engang Tian:** Supervision, Formal analysis. **Songlin Hu:** Writing - review & editing. **Xiangpeng Xie:** Software, Validation.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgement

This work is partially supported by National Natural Science Foundation of China under Grant 91646204, 61973152, National Key R&D Program of China under Grant 2017YFD0401001, Natural Science Foundation of the Jiangsu Higher Education Institutions of China under Grant 19KJA510005, Postgraduate Research&Practice Innovation Program of Jiangsu Province under Grant KYCX19-1382.

Appendix A

$$\begin{split} \Phi_1 &= \begin{bmatrix} \Phi_{11}^1 & * & * & * & * & * & * \\ \Phi_{21}^1 & \Phi_{12}^1 & * & * & * & * & * \\ \Phi_{31}^1 & \Phi_{32}^1 & \Phi_{33}^1 & * & * & * & * \\ \Phi_{41}^1 & \Phi_{42}^1 & \Phi_{43}^1 & \Phi_{44}^1 & * & * & * \\ \Phi_{51}^1 & 0 & \Phi_{53}^1 & 0 & \Phi_{55}^1 & * \\ \Phi_{61}^1 & 0 & 0 & 0 & 0 & -I \end{bmatrix}, \Phi_2 &= \begin{bmatrix} \Phi_{11}^2 & * & * & * & * \\ \Phi_{21}^2 & \Phi_{22}^2 & * & * & * \\ \Phi_{21}^2 & \Phi_{22}^2 & * & * & * \\ \Phi_{31}^2 & \Phi_{32}^2 & \Phi_{33}^2 & * & * \\ \Phi_{41}^2 & 0 & 0 & \Phi_{44}^2 \end{bmatrix} \\ \Phi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^1 & * & * & * & * & * \\ \Psi_{11}^2 & * & * & * & * & * \\ \Psi_{11}^2 & * & * & * & * & * \\ \Psi_{11}^2 & * & * & * & * & * \\ \Psi_{11}^2 & * & * & * & * & * \\ \Psi_{21}^1 & * & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & * & * \\ \Psi_{21}^2 & * & * & *$$

$$\begin{split} & \Psi_{13}^{0} = -t_{2}^{-1} - t_{3}^{-1} - t_{3}^{0} - t_{3}^{0$$

Appendix B

$$\begin{split} & \Phi_1 = \begin{bmatrix} \hat{\Phi}_{11}^1 & * & * & * & * & * \\ \hat{\Phi}_{21}^1 & \hat{\Phi}_{12}^1 & * & * & * & * \\ \hat{\Phi}_{31}^1 & \hat{\Phi}_{12}^1 & * & * & * & * \\ \hat{\Phi}_{31}^1 & \hat{\Phi}_{12}^1 & * & * & * & * \\ \hat{\Phi}_{31}^1 & \hat{\Phi}_{12}^1 & \hat{\Phi}_{13}^1 & * & * & * \\ \hat{\Phi}_{31}^1 & 0 & 0 & \hat{\Phi}_{33}^1 & 0 & \hat{\Phi}_{55}^1 & * \\ \hat{\Phi}_{51}^1 & 0 & 0 & 0 & 0 & 0 & -I \end{bmatrix}, \\ & \hat{\Phi}_{11}^1 = \begin{bmatrix} \hat{\Psi}_{11}^1 & * & * & * & * \\ \hat{\Psi}_{21}^1 & \hat{\Psi}_{12}^1 & * & * & * \\ \hat{\Psi}_{21}^1 & \hat{\Psi}_{12}^1 & * & * & * \\ \hat{\Psi}_{21}^1 & \hat{\Psi}_{12}^1 & * & * & * \\ \hat{\Psi}_{31}^1 & \hat{\Psi}_{32}^1 & \hat{\Psi}_{33}^1 & * & * \\ \hat{\Psi}_{11}^1 & * & * & * & * \\ \hat{\Psi}_{21}^1 & \hat{\Psi}_{12}^2 & * & * & * \\ \hat{\Psi}_{31}^1 & \hat{\Psi}_{12}^1 & \hat{\Psi}_{13}^1 & * & * & * \\ \hat{\Psi}_{11}^1 & 2\rho_1 & \hat{\Psi}_{13}^1 & \hat{\Psi}_{14}^1 & * & * & * \\ \hat{\Psi}_{21}^1 & \hat{\Psi}_{22}^1 & \hat{\Psi}_{22}^2 & * & * & * \\ \hat{\Psi}_{21}^1 & \hat{\Psi}_{22}^2 & * & * & * \\ \hat{\Psi}_{31}^1 & \hat{\Psi}_{12}^2 & \hat{\Psi}_{13}^1 & * & * & * \\ \hat{\Psi}_{11}^1 & 2\rho_1 X_1 + 2X_1 + X_1 A^T + \hat{Q}_{11} + \hat{Q}_{12} + \hat{Q}_{13} + \hat{L}_{11} + \hat{L}_{11}^T + \hat{M}_{11} + \hat{M}_{11}^T + \hat{M}_{11}$$

$$\begin{split} & \Psi_{31}^{l} = \begin{bmatrix} \hat{M}_{11}^{l} & \hat{M}_{12}^{l} & \hat{M}_{13}^{l} & \hat{M}_{14}^{l} & \hat{M}_{15}^{l} \\ \hat{M}_{21}^{l} & \hat{M}_{22}^{l} & \hat{M}_{23}^{l} & \hat{M}_{24}^{l} & \hat{M}_{25}^{l} \end{bmatrix}, \Psi_{32}^{l} = \begin{bmatrix} \hat{M}_{16}^{l} & \hat{M}_{17}^{l} \\ \hat{M}_{26}^{l} & \hat{M}_{27}^{l} \end{bmatrix}, \Psi_{33}^{l} = \begin{bmatrix} \hat{M}_{18}^{l} & \hat{M}_{19}^{l} \\ \hat{M}_{28}^{l} & \hat{M}_{29}^{l} \end{bmatrix} \\ & \Psi_{61}^{l} = \begin{bmatrix} \hat{N}_{11}^{l} & \hat{N}_{12}^{l} & \hat{N}_{13}^{l} & \hat{N}_{14}^{l} & \hat{N}_{15}^{l} \\ \hat{N}_{21}^{l} & \hat{N}_{22}^{l} & \hat{N}_{23}^{l} & \hat{N}_{24}^{l} & \hat{N}_{15}^{l} \\ \hat{N}_{21}^{l} & \hat{N}_{25}^{l} & \hat{N}_{24}^{l} & \hat{N}_{15}^{l} \\ \hat{N}_{26}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{63}^{l} = \begin{bmatrix} \hat{N}_{16}^{l} & \hat{N}_{17}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{93}^{l} = \begin{bmatrix} \hat{N}_{16}^{l} & \hat{N}_{17}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{93}^{l} = \begin{bmatrix} \hat{N}_{18}^{l} & \hat{N}_{19}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{63}^{l} = \begin{bmatrix} \hat{N}_{16}^{l} & \hat{N}_{17}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{183}^{l} = \begin{bmatrix} \hat{N}_{18}^{l} & \hat{N}_{19}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{183}^{l} = \begin{bmatrix} \hat{N}_{18}^{l} & \hat{N}_{19}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{183}^{l} = \begin{bmatrix} \hat{N}_{18}^{l} & \hat{N}_{19}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{27}^{l} \end{bmatrix}, \Psi_{183}^{l} = \begin{bmatrix} \hat{N}_{18}^{l} & \hat{N}_{19}^{l} \\ \hat{N}_{28}^{l} & \hat{N}_{29}^{l} \end{bmatrix} \end{bmatrix}$$

References

- [1] L. Cao, H.-Y. Li, G.-W. Dong, R.-Q. Lu, Event-triggered control for multiagent systems with sensor faults and input saturation, IEEE Trans. Syst., Man, Cybern.: Syst. (2019), https://doi.org/10.1109/TSMC.2019.2938216.
- [2] X.-H. Chang, C. Yang, J. Xiong, Quantized fuzzy output feedback H∞ control for nonlinear systems with adjustment of dynamic parameters, IEEE Trans. Syst., Man, Cybern.: Syst. 49 (10) (2019) 2005–2015.
- [3] J. Cheng, J.H. Park, L.-X. Zhang, Y.-Z. Zhu, An asynchronous operation approach to event-triggered control for fuzzy markovian jump systems with general switching policies, IEEE Trans. Fuzzy Syst. 26 (1) (2018) 6–18.
- [4] L. Ding, Q.-L. Han, B.-D. Ning, D. Yue, Distributed resilient finite-time secondary control for heterogeneous battery energy storage systems under denial-of-service attacks, IEEE Trans. Industr. Inf. 16 (7) (2020) 4909–4919.
- [5] X.-H. Ge, Q.-L. Han, M.-Y. Zhong, X.-M. Zhang, Distributed Krein space-based attack detection over sensor networks under deception attacks, Automatica 109 (2019) 108557.
- [6] X.-H. Ge, Q.-L. Han, Z.-D. Wang, A threshold-parameter-dependent approach to designing distributed event-triggered *H*∞ consensus filters over sensor networks, IEEE Trans. Cybern. 49 (4) (2019) 1148–1159.
- [7] X.-H. Ge, Q.-L. Han, X.-M. Zhang, L. Ding, F.-W. Yang, Distributed event-triggered estimation over sensor networks: A survey, IEEE Trans. Cybern. 50 (3) (2020) 1306–1320.
- [8] Z. Gu, Ju.H. Park, D. Yue, Z.-G. Wu, X.-P. Xie, Event triggered security output feedback control for networked interconnected systems subject to cyberattacks, IEEE Trans. Systems Cybern. Syst. (2020), https://doi.org/10.1109/TSMC.2019.2960115.
- [9] Z. Gu, X.-H. Zhou, T. Zhang, F. Yang, M.-Q. Shen, Event-triggered filter design for nonlinear cyber-physical systems subject to deception attacks, ISA Trans. (2019), https://doi.org/10.1016/j.isatra.2019.02.036.
- [10] Y.-P. Guan, X.-H. Ge, Distributed attack detection and secure estimation of networked cyber-physical systems against false data injection attacks and jamming attacks. IEEE Trans. Signal Inform. Process. Over Networks 4 (1) (2018) 48–59.
- [11] Y.-P. Guan, C. Peng, A hybrid transmission scheme for networked control systems, ISA Trans. 96 (2020) 155-162.
- [12] S.-L. Hu, D. Yue, X.-P. Xie, X.-L. Chen, X.-X. Yin, Resilient event-triggered controller synthesis of networked control systems under periodic dos jamming attacks, IEEE Trans. Syst., Man, Cybern. 49 (12) (2019) 4271–4281.
- [13] O.M. Kwon, M.J. Park, S.M. Lee, J.H. Park, E.J. Cha, Stability for neural networks with time-varying delays via some new approaches, IEEE Trans. Neural Networks 24 (2) (2012) 181–193.
- [14] Y.-Z. Li, D.-W. Shi, T.-W. Chen, False data injection attacks on networked control systems: A stackelberg game analysis, IEEE Trans. Autom. Control 63 (10) (2018) 3503–3509.
- [15] J.-L. Liu, Y.-Y. Gu, X.-P. Xie, D. Yue, J.H. Park, Hybrid-driven-based H∞ control for networked cascade control systems with actuator saturations and stochastic cyber attacks, IEEE Trans. Syst., Man, Cybern.: Syst. 49 (12) (2019) 2452–2463.
- [16] J.-L. Liu, M. Yang, X.-P. Xie, C. Peng, H.-Y. Yan, Finite-time H-infinity filtering for state-dependent uncertain systems with event-triggered mechanism and multiple attacks, IEEE Trans. Circuits Syst. I Regul. Pap. 67 (3) (2020) 1021–2034.
- [17] J.-L. Liu, Z.-G. Wu, D. Yue, J.H. Park, Stabilization of networked control systems with hybrid-driven mechanism and probabilistic cyber attacks, IEEE Trans, Syst., Man, Cybern.: Syst. (2019), https://doi.org/10.1109/TSMC.2018.2888633.
- [18] J.-L. Liu, L.-J. Zha, J. Cao, S.-M. Fei, Hybrid-driven-based stabilisation for networked control systems, IET Control Theory Appl. 10 (17) (2016) 2279–2285
- [19] C. Peng, D. Yue, M.-R. Fei, A higher energy-efficient sampling scheme for networked control systems over ieee 802.15.4 wireless networks, IEEE Trans. Industr. Inf. 12 (5) (2016) 1766–1774.
- [20] C. De Persis, P. Tesi, Input-to-state stabilizing control under denial-of-service, IEEE Trans. Autom. Control 60 (11) (2015) 2930-2944.
- [21] J.-B. Qiu, H.-J. Gao, S.X. Ding, Recent advances on fuzzy-model-based nonlinear networked control systems: A survey, IEEE Trans. Industr. Electron. 63 (2) (2016) 1207–1217.
- [22] Y.-Y. Wang, L.-H. Xie, C.E. De Souza, Robust control of a class of uncertain nonlinear systems, Systems Control Letters 19 (2) (1992) 139-149.
- [23] E.-G. Tian, C. Peng, Memory-Based Event-Triggering H∞ Load Frequency Control for Power Systems Under Deception Attacks, IEEE Trans. Cybern. (2020), https://doi.org/10.1109/TCYB.2020.2972384.
- [24] E.-G. Tian, X.M. Wang, C. Peng, Probabilistic-constrained distributed filtering for a class of nonlinear stochastic systems subject to periodic DOS attacks, IEEE Trans. Circuits Systems-I: Regular Papers (2020), https://doi.org/10.1109/TCSI.2020.3007953.
- [25] C.-W. Wu, J.-X. Liu, X.-J. Jing, H.-Y. Li, L.-G. Wu, Adaptive fuzzy control for nonlinear networked control systems, IEEE Trans. Syst., Man, Cybern.: Syst. 47 (8) (2017) 2420–2430.
- [26] L.-G. Wu, Y.-B. Gao, J.-X. Liu, H.-Y. Li, Event-triggered sliding mode control of stochastic systems via output feedback, Automatica 82 (2017) 79–92.
- [27] F. Yang, Z. Gu, J. Cheng, J.-L. Liu, Event-driven finite-time control for continuous-time networked switched systems under cyber attacks, J. Franklin Inst.-Eng. Appl. Math. (2019), https://doi.org/10.1016/j.jfranklin.2019.09.037.
- [28] Y. Yuan, H.-H. Yuan, L. Guo, H.-J. Yang, S.-L. Sun, Resilient control of networked control system under DoS attacks: a unified game approach, IEEE Trans. Industr. Inf. 12 (5) (2016) 1786–1794.
- [29] D. Yue, E.-G. Tian, Q.-L. Han, A delay system method for designing event-triggered controllers of networked control systems, IEEE Trans. Autom. Control 58 (2) (2013) 475–481.
- [30] D. Yue, E.-G. Tian, Y.-J. Zhang, C. Peng, Delay-distribution-dependent stability and stabilization of t-s fuzzy systems with probabilistic interval delay, IEEE Trans. Syst., Man, Cybern., Part B (Cybernetics) 39 (2) (2009) 503–516.
- [31] D.-W. Zhang, Q.-L. Han, X.-M. Zhang, Network-based modeling and proportional-integral control for direct-drive-wheels systems in wireless network environments, IEEE Trans. Cybern. 50 (6) (2020) 2462–2474.
- [32] D. Zhang, S.K. Nguang, L. Yu, Distributed control of large-scale networked control systems with communication constraints and topology switching, IEEE Trans. Syst., Man, Cybern.: Syst. 47 (7) (2017) 1746–1757.
- [33] D. Zhang, L. Liu, G. Feng, Consensus of heterogeneous linear multiagent systems subject to aperiodic sampled-data and dos attack, IEEE Trans. Cybern. 49 (4) (2019) 1501–1511.
- [34] H. Zhang, P. Cheng, L. Shi, J.-M. Chen, Optimal dos attack scheduling in wireless networked control system, IEEE Trans. Control Syst. Technol. 24 (3) (2016) 843–852.
- [35] X.-M. Zhang, Q.-L. Han, X.-H. Ge, D.-R. Ding, L. Ding, D. Yue, C. Peng, Networked control systems: a survey of trends and techniques, IEEE/CAA J. Automatica Sinica 7 (1) (2020) 1–17.
- [36] X.-M. Zhang, Q.-L. Han, X.-H. Ge, L. Ding, Resilient control design based on a sampled-data model for a class of networked control systems under denial-of-service attacks, IEEE Trans. Cybern. 50 (8) (2020) 3616–3626.