

# Decentralized event-triggered synchronization control for complex networks with nonperiodic DoS attacks

Yan Li<sup>1</sup> | Feiyu Song<sup>1</sup>  | Jinliang Liu<sup>1,2</sup>  | Xiangpeng Xie<sup>3,4</sup>  | Engang Tian<sup>5</sup> 

<sup>1</sup>College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, China

<sup>2</sup>College of Automation Electronic Engineering, Qingdao University of Science and Technology, Qingdao, China

<sup>3</sup>Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing, China

<sup>4</sup>School of Information Science and Engineering, Chengdu University, Chengdu, China

<sup>5</sup>School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, Shanghai, China

## Correspondence

Jinliang Liu, College of Information Engineering, Nanjing University of Finance and Economics, Nanjing, Jiangsu 210023, China.

Email: liujinliang@vip.163.com

## Funding information

Natural Science Foundation of Jiangsu Province of China, Grant/Award Numbers: BE2020001-3, BK20202011, BK20211290; National Natural Science Foundation of China, Grant/Award Number: 61973152; Natural Science Foundation of the Jiangsu Higher Education Institutions of China, Grant/Award Number: 19KJA510005; Qing Lan Project

## Abstract

This article focuses on the issue of decentralized event-triggered synchronization control for complex networks (CNs) under nonperiodic denial-of-service (DoS) attacks. First, to alleviate the pressure on network bandwidth, a decentralized event-triggered scheme is employed at each coupled node to decide whether the synchronization signal is transmitted to the communication network. Then, an event-based synchronization error model is established for CNs under DoS attacks, where the communication network is assumed to be composed of multiple transmission channels and DoS attacks will independently compromise each of channels. Based on the constructed model, sufficient conditions that assuring the secure synchronization of the system are analyzed with the assistance of Lyapunov stability theory. Meanwhile, the synchronization controller gains are designed by solving a set of linear matrix inequalities. The efficiency of the study is finally validated by simulations.

## KEYWORDS

complex networks (CNs), decentralized synchronization control, DoS attacks, event-triggered scheme

## 1 | INTRODUCTION

Complex networks (CNs) are now widely used to model many practical systems with large scale, such as food webs, subway, power grid and so forth.<sup>1-3</sup> A typical CN is generally composed by a number of nodes each of which represents a real individual and links denoting the relationships between different individuals. Given the architecture characteristic

of CNs, an important issue how to synchronize all coupled nodes is raised, and has attracting many research attentions.<sup>4-6</sup> For example, on the basis of intermittent control method, the authors in Reference 7 introduced the synchronization of CNs with stochastic disturbance. In Reference 8, the pinning synchronization of delayed CNs under self-triggered control was discussed. A synchronization error model was adopted in References 9,10, by which the synchronization problem for CNs can be transformed into verifying the convergence of the defined dynamic error. However, with the introducing of communication network, the complexity of synchronization in CNs will be inevitably intensified.

In networked systems, the communication network enables remote, flexible, and cost-efficient control, but with the continuous soaring of sampled data, the network resources are becoming limited and thus degrade system performance.<sup>11-13</sup> To mitigate resource-constrained dilemma, event-triggered strategies, by which data is transmitted only while the specific conditions are satisfied, are extensively studied.<sup>14-17</sup> In Reference 18, a representative event-triggered scheme was adopted and it can be seen that this scheme indeed reduces network burden by avoiding the transmission of unnecessary packets. Based on Reference 18, a series works that focus on integrating event-triggered method into CNs were presented. For instance, the authors in Reference 19 designed a kind of event-based state estimators in CNs with time delay and random coupling weights. In Reference 20, a  $H_\infty$  state estimation problem for CNs with event-triggered mechanism, random distributed delays, and state saturation was discussed. In the case of event-triggered communication, a new filter scheme for time-varying nonlinear CNs with the influence of quantization was proposed in Reference 21. Moreover, decentralized event-triggered strategies have also been exploited given the diversity of the coupled nodes in CNs. In Reference 22, the authors discussed a decentralized event-triggered mechanism for synchronization of CNs with different dynamics of nodes. Under decentralized event-triggered scheme, the non-fragile control problem for T-S fuzzy neural networks was studied in Reference 23.

Despite attractive advantages of event-triggered mechanism, the performance of networked systems is still challenged by varies of cyber attacks given the openness of communication network.<sup>24-26</sup> In practice, some typical attacks, for example, replay attacks,<sup>27,28</sup> deception attacks,<sup>29,30</sup> and denial-of-service (DoS) attacks, have gained considerable research attentions. Replay attacks degrade system performance via repeatedly transmitting previously sampled data. Under replay attacks, the secure control problem of cyber-physical systems was studied to ensure the  $H_\infty$  stability in Reference 28. Different from replay attacks, deception attacks maliciously inject false data to tamper with normal data, which increases the difficulty of attack detection. In Reference 29, the problem of filter design to ensure the security of discrete time-delay systems with stochastic deception attacks and sensor saturation was studied.

Comparing with the replay attacks and deception attacks, DoS attacks are especially common since that they can easily cause large transmission delay and serious packet loss by wasting network resources. In Reference 31, a resilient event-triggered controller was designed for networked control systems under DoS attacks. With the influence of DoS attacks and the change of controller gains, a distributed event-triggered security consensus control strategy was proposed in Reference 32 for multi-agent systems. The authors in Reference 33 studied the state estimation issue over cyber-physical systems affected by the problems of resource constraints, sensor saturation and DoS jamming attacks. It is worth noting that the aforementioned researches are concerned about periodic DoS attacks, however, given the burstiness of actual cyber attacks, it is more critical to consider nonperiodic DoS attacks. In Reference 16, the event-triggered quantized  $H_\infty$  control strategy for networked control systems under nonperiodic DoS attacks had been discussed. Furthermore, the event-based control problem of networked switching systems under nonperiodic DoS attacks was studied in Reference 34. With the expanding of networked systems, it is desirable to slice the bandwidth of the communication network into multiple transmission channels so as to achieve flexible bandwidth sharing at fine granularity among system nodes. Multi-channel scenario is likely to incur asynchronous DoS attacks, that is, each of channels tends to be compromised by different DoS attacks, which will complicate the system analysis. In Reference 35, the distributed consensus control issue over multi-agent systems under asynchronous DoS attacks was investigated. Focusing on cyber-physical systems with multiple transmission channels, the input-to-state stabilizing control problem under DoS attacks was studied in Reference 36.

Motivated by the above investigation, this article dedicates to realize decentralized synchronization control for resource-constrained CNs with multiple transmission channels under nonperiodic DoS attacks. The major contributions of the study can be listed as follows:

- Taking both network resource constraints and nonperiodic DoS attacks into consideration, a decentralized event-triggered scheme is adopted for CNs with multiple transmission channels.
- A decentralized event-based synchronization error model is established to formulate the considered synchronization control problem over CNs under nonperiodic asynchronous DoS attacks.

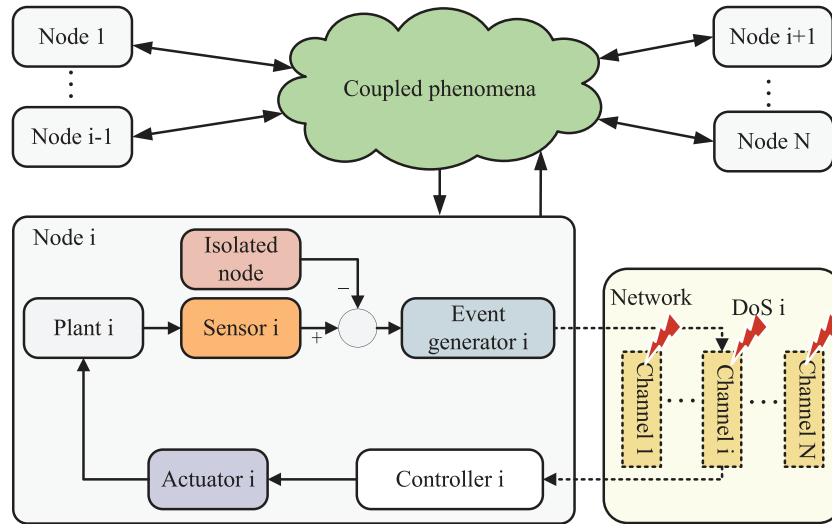


FIGURE 1 The structure of the considered CN system

- The sufficient conditions that guaranteeing the synchronization of the formulated system are obtained based on Lyapunov stability analysis, furthermore, the controller feedback gains, and parameter matrices of event-triggered scheme are simultaneously derived via linear matrix inequality (LMI) technique.

The rest of the article is organized as follows. In Section 2, a decentralized event-based synchronization error model for CNs affected by nonperiodic asynchronous DoS attacks is constructed, which is followed by the introduction of some preliminary knowledge. The main results of the study are presented in Section 3. A numerical example is shown in Section 4 to verify the efficiency of the proposed method. The article is concluded in Section 5.

*Notation 1.*  $\mathbb{R}^{n_x}$  represents the  $n_x$ -dimensional Euclidean space, and  $\mathbb{R}^{n_x \times n_x}$  denotes the matrix of  $n_x \times n_x$  dimensions.  $\mathbb{N}$  stands for the set of positive integers.  $M^T$  is the transpose of matrix  $M$ .  $\|\cdot\|$  represents the Euclidean norm.  $\text{diag}\{\dots\}$  describes the block-diagonal matrix and  $*$  represents the symmetric term in the matrix.  $\lambda_{\min}(A)$  and  $\lambda_{\max}(A)$  stand for the minimum and maximum eigenvalue of matrix  $A$ , respectively.

## 2 | PROBLEM FORMULATION AND PRELIMINARIES

The framework of the considered CN under nonperiodic asynchronous DoS attacks is depicted in Figure 1. As shown, the CN consists of  $N$  coupled nodes (referred to as node), for each node  $i$  ( $1 \leq i \leq N$ ), the sensor, event generator, controller, actuator are included and an isolated node is embedded. The isolated nodes co-located at  $N$  nodes are assumed to be identical, and then the studied synchronization control is how to synchronize each node to its embedded isolated node. We would like to note that the communication network is divided into multiple transmission channels, and each of channels serves one node for transmitting the data released by the event generator to the controller. Then, nonperiodic DoS attacks are assumed to independently occur on each channel. For the convenience of description, the DoS attack that compromising the channel  $i$  is referred to as DoS  $i$ .

We consider the following CN system:<sup>37,38</sup>

$$\dot{x}_i(t) = Ax_i(t) + v(x_i(t)) + \sigma \sum_{j=1}^N c_{ij} \Gamma x_j(t) + Bu_i(t), \quad (1)$$

for  $1 \leq i \leq N$ ,  $x_i(t) \in \mathbb{R}^{n_x}$  represents the state vector of node  $i$  and  $u_i(t)$  represents the corresponding control input vector,  $A$  and  $B$  are the parameter matrices with appropriate dimensions,  $v: \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x}$  is a continuous nonlinear vector function,  $\sigma > 0$  is the strength of the network coupling,  $\Gamma = \text{diag}\{t_1, t_2, \dots, t_{n_x}\}$  is the coupling matrix in the CN,  $C = [c_{ij}]_{N \times N}$  is the

network matrix, where  $c_{ij} > 0$  ( $i \neq j$ ) if there is an undirected link between node  $i$  and node  $j$ , otherwise,  $c_{ij} = 0$ , and for node  $i$ ,  $c_{ii} = -\sum_{j=1, j \neq i}^N c_{ij}$ .

**Remark 1.** The inner-coupling weight  $t_i$  in Equation (1) is uncertain and it satisfies  $\underline{t}_i \leq t_i \leq \bar{t}_i$ , where  $\underline{t}_i$  and  $\bar{t}_i$  are known constants.

**Assumption 1** (39,40). The vector-valued function  $v(\cdot)$  in the system Equation (1) is a continuous nonlinear function satisfying the following Lipschitz constraints:

$$\|v(x) - v(y)\| \leq \|\Lambda(x - y)\|, \quad (2)$$

where the upper bound of  $v(\cdot)$  is represented by a constant matrix  $\Lambda$ .

Let  $s(t) \in \mathbb{R}^{n_x}$  represent the state of each isolated node, which satisfies:

$$\dot{s}(t) = As(t) + v(s(t)), \quad (3)$$

and the synchronization error can be described by  $\varepsilon_i(t) = x_i(t) - s(t)$ , then the error system can be formulated as:

$$\dot{\varepsilon}_i(t) = A\varepsilon_i(t) + v_i(t) + \sigma \sum_{j=1}^N c_{ij} \Gamma \varepsilon_j(t) + Bu_i(t), \quad (4)$$

where  $v_i(t) = v(x_i(t)) - v(s(t))$ . In the rest of the section, we will further update the formulated system synchronization error model by taking nonperiodic asynchronous DoS attacks and decentralized event-triggered scheme into account.

## 2.1 | Nonperiodic asynchronous DoS attacks

In the considered CN, the sampling times of the sensors are assumed to be periodic with interval  $h > 0$ , and the transmission of released sampling-data will be influenced by DoS attacks. For DoS  $i$ , we consider a type of energy-limited nonperiodic DoS attacks, in which the attacker will fall to sleep for a period while the energy is exhausted and then become active with the energy restoration, so the behavior of DoS  $i$  can be depicted as:

$$\mathcal{F}^i(t) = \begin{cases} 0, & t \in [f_n^i, f_n^i + g_n^i), \\ 1, & t \in [f_n^i + g_n^i, f_{n+1}^i), \end{cases} \quad (5)$$

where  $\{f_n^i\}_{n \in \mathbb{N}}$  and  $\{g_n^i\}_{n \in \mathbb{N}}$  are the sequences of real numbers, and  $f_{n+1}^i > f_n^i + g_n^i$ . As presented in Equation (5), the  $n$ th action interval of DoS  $i$ , that is,  $D_n^i = [f_n^i, f_{n+1}^i)$ , is divided into a sleeping period, that is,  $\widetilde{D}_n^i = [f_n^i, f_n^i + g_n^i)$ , and an active period, that is,  $\overline{D}_n^i = [f_n^i + g_n^i, f_{n+1}^i)$ . Thus, in the time intervals  $\{\widetilde{D}_n^i\}_{n \in \mathbb{N}}$ , the normal communication is allowed without disturbing of the DoS attack; in the time intervals  $\{\overline{D}_n^i\}_{n \in \mathbb{N}}$ , the normal communication is denied due to the DoS attack.

**Remark 2.** In literatures,<sup>41,42</sup> many various detection methods, such as *activity profiling* and *change-point detection*, have been proposed to effectively identify DoS attacks. Thus, in this article, we suppose that the depicted DoS attacks can always be successfully detected, and then focus on designing secure synchronization controllers.

In practice, it is hard to recover the interrupted communication immediately after the attacker stops launching DoS attacks, we thereby regulate the start point of each action interval of DoS  $i$ , that is,  $f_n^i$  ( $n \in \mathbb{N}$ ), as  $d_n^i = \min_{l \geq 0} \{lh | lh \geq f_n^i\}$ . Then,  $\widetilde{D}_n^i$  and  $\overline{D}_n^i$  are updated as  $\widetilde{D}_n^i = [d_n^i, f_n^i + g_n^i)$  and  $\overline{D}_n^i = [f_n^i + g_n^i, d_{n+1}^i)$  for subsequent design and analysis of controllers.

**Assumption 2** (43). The lower bound for the length of the sleeping periods and the upper bound for the length of the active periods of the attacker are existed, which can be specifically depicted as:

$$\begin{aligned} \inf_{n \in \mathbb{N}} \{f_n^i + g_n^i - d_n^i\} &\geq g_{min}, \\ \sup_{n \in \mathbb{N}} \{d_{n+1}^i - f_n^i - g_n^i\} &\leq d_{max}. \end{aligned} \tag{6}$$

*Remark 3.* For DoS  $i$ , given the limited energy, each of active periods would not last for long time, and each sleeping period should continue for some time to accumulate energy, so it is reasonable to make Assumption 2.

*Remark 4.* A lot of existing researches focus on the influence of periodic DoS attacks on CNs with single transmission channel. However, the attackers are more likely to generate irregular and uncertain DoS signals in practice. Moreover, given the diversity of numerous nodes in CNs, it is more reasonable to consider CNs with multiple transmission channels, and the resulting asynchronous DoS attacks should be seriously concerned. Thus, this article takes non-periodic asynchronous DoS attacks into account, and studies the security issue occurred in synchronization control for CNs.

## 2.2 | Decentralized event-triggered scheme

For the purpose of saving the network bandwidth, a decentralized event-triggered scheme is adopted for the considered CN system. Thus, for each node  $i$  ( $1 \leq i \leq N$ ), an event generator is deployed between the sensor and the corresponding channel as presented in Figure 1. Each event generator follows the similar triggering rule to decide whether the sampled data can be released into the channel, so we then focus on introducing the triggering rule for event generator  $i$ .

Without considering the influence of DoS  $i$ , the following event-triggering condition proposed in Reference 18 can be adopted.

$$(\varepsilon_i(t_k^i h + u^i h) - \varepsilon_i(t_k^i h))^T \Omega_i (\varepsilon_i(t_k^i h + u^i h) - \varepsilon_i(t_k^i h)) > \rho_i \varepsilon_i^T(t_k^i h + u^i h) \Omega_i \varepsilon_i(t_k^i h + u^i h), \tag{7}$$

where,  $\rho_i \in (0, 1)$  is a given parameter;  $\Omega_i$  is a positive symmetric matrix;  $t_k^i h$  denotes the latest triggering instant, in which  $t_k^i$  ( $k \in \{0, 1, 2, \dots\}$ ) are nonnegative integers and the initial state is set to be zero, that is,  $t_0^i = 0$ ;  $t_k^i h + u^i h$  represents the current sampling instant;  $\varepsilon_i(t_k^i h)$  and  $\varepsilon_i(t_k^i h + u^i h)$  indicate the latest transmitted data and the current sampling data, respectively.

Based on the above triggering condition Equation (7), the next triggering instant  $t_{k+1}^i h$  can be written as:

$$\begin{aligned} t_{k+1}^i h &= t_k^i h + \min_{u^i \geq 1} \{u^i h | (\varepsilon_i(t_k^i h + u^i h) - \varepsilon_i(t_k^i h))^T \Omega_i (\varepsilon_i(t_k^i h + u^i h) - \varepsilon_i(t_k^i h)) \\ &> \rho_i \varepsilon_i^T(t_k^i h + u^i h) \Omega_i \varepsilon_i(t_k^i h + u^i h)\}. \end{aligned} \tag{8}$$

*Remark 5.* Apparently, if  $\rho_i = 0$ , the event-triggered scheme is reduced to a time-triggered scheme that every sampling data can be transmitted. Otherwise, if  $\rho_i \in (0, 1)$ , the sampled data can be released only while the condition described in Equation (7) is satisfied.

With the introducing of DoS  $i$ , the triggering condition depicted in Equation (7) can not be directly adopted. Thus, we modify the condition to adapt to the nonperiodic DoS attack. Specifically, the updated event-triggering instant under DoS  $i$  can be written as:

$$t_{k,n}^i h = \{t_{n_j}^i h \text{ satisfying Equation (7)} | t_{n_j}^i h \in \widetilde{D}_n^i\} \cup \{d_n^i\}, \tag{9}$$

where  $n \in \mathbb{N}$  denotes the index of the action interval of DoS  $i$ ;  $k$  represents the index of the updated event-triggering instant in the  $n$ th action interval of DoS  $i$ ,  $k \in \{1, 2, \dots, k^i(n)\} \triangleq \psi^i(n)$  and  $k^i(n) = \sup\{k \in \mathbb{N} | t_{k,n}^i h \leq f_n^i + g_n^i\}$  ( $t_{1,n}^i h \triangleq d_n^i$ );  $t_{n_j}^i$  and  $n_j$  are nonnegative integers, and  $n_j = l$  if  $t_l^i h$  is the  $j$ th triggering instant (i.e.,  $t_l^i h$  satisfying Equation (7)) in  $\widetilde{D}_n^i$ .

*Remark 6.* Considering that the normal communication is blocked in each interval  $\widetilde{D}_n^i$  and the start point of each action interval of DoS attacks, that is,  $d_n^i$ , is a multiple of  $h$ , to assure the time and effective update of system signal, we let the sampled data always be released at the start point of each interval  $\widetilde{D}_n^i$ , that is,  $d_n^i$ , as presented in Equation (9).

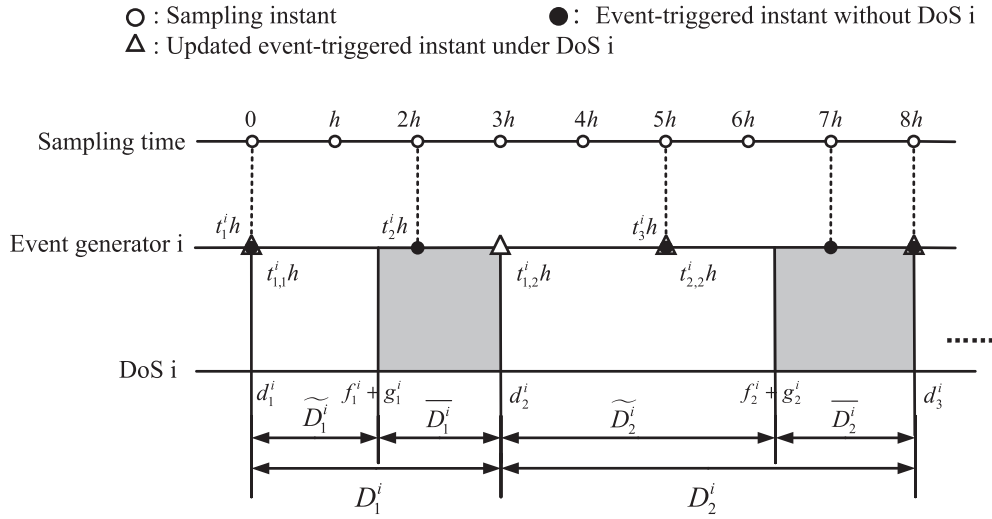


FIGURE 2 Example of the event-triggered scheme under nonperiodic DoS attacks

An example is listed in Figure 2 to further illustrate the event-triggered scheme under nonperiodic DoS attacks. As shown, for node  $i$ , following the event-triggering condition without considering DoS  $i$ , the data sampled at the triggering instants  $t_1^i h, t_2^i h, \dots$  are released. However, due to the influence of DoS  $i$  that the data released in  $\{\overline{D}_n^i\}_{n \in \mathbb{N}}$  will be blocked and only the data released in  $\{\widetilde{D}_n^i\}_{n \in \mathbb{N}}$  can be successfully transmitted, the triggering instants are updated as  $t_{1,1}^i h, \dots, t_{k^{(1)},1}^i h, t_{1,2}^i h, \dots, t_{k^{(2)},2}^i h, \dots$ , according to Equation (9). As presented in the example,  $t_{1,1}^i h = t_1^i h = d_1^i$ ,  $t_{1,2}^i h = d_2^i$ , and  $t_{2,2}^i h = t_3^i h$ .

### 2.3 | Synchronization error system formulation

Based on the above decentralized event-triggered scheme and referring to Reference 44, the control input under the described DoS  $i$  can be expressed as:

$$u_i(t) = \begin{cases} K_i \varepsilon_i(t_{k,n}^i h), & t \in [t_{k,n}^i h, t_{k+1,n}^i h), \\ 0, & t \in [f_n^i + g_n^i, d_{n+1}^i), \end{cases} \quad (10)$$

where,  $t_{k^{(n)}+1,n}^i h = f_n^i + g_n^i$  is defined for uniform description, and  $\varepsilon_i(t_{k,n}^i h)$  denotes the transmitted data between  $t_{k,n}^i h$  and  $t_{k+1,n}^i h$ .

We further define  $V_{k,n}^i \triangleq [t_{k,n}^i h, t_{k+1,n}^i h)$  ( $n \in \mathbb{N}, k \in \psi^i(n)$ ) and substitute Equation (10) into Equation (4), thereby the following event-based synchronization error model under nonperiodic asynchronous DoS attacks can be derived.

$$\dot{\varepsilon}_i(t) = \begin{cases} A \varepsilon_i(t) + v_i(t) + \sigma \sum_{j=1}^N c_{ij} \Gamma \varepsilon_j(t) + B K_i \varepsilon_i(t_{k,n}^i h), & t \in V_{k,n}^i, \\ A \varepsilon_i(t) + v_i(t) + \sigma \sum_{j=1}^N c_{ij} \Gamma \varepsilon_j(t), & t \in D_n^i. \end{cases} \quad (11)$$

For analysis convenience, let  $o_{k,n}^i = \sup\{q \in \mathbb{N} | t_{k,n}^i h + qh < t_{k+1,n}^i h, q = 1, 2, \dots\}$ , so that the time interval  $V_{k,n}^i$  can be written as:

$$V_{k,n}^i = \bigcup_{q=1}^{o_{k,n}^i+1} \varrho_{k,n}^{i,q} \quad (12)$$

where,

$$\begin{cases} \varrho_{k,n}^{i,q} = [t_{k,n}^i h + (q-1)h, t_{k,n}^i h + qh), & q \in \{1, 2, \dots, o_{k,n}^i\}, \\ \varrho_{k,n}^{i,o_{k,n}^i+1} = [t_{k,n}^i h + o_{k,n}^i h, t_{k+1,n}^i h). \end{cases} \quad (13)$$

For each  $k \in \psi^i(n)$  ( $n \in \mathbb{N}$ ), we can define two piecewise functions as follows:

$$\tau_{k,n}^i(t) = \begin{cases} t - t_{k,n}^i h, & t \in \varrho_{k,n}^{i,1}, \\ t - t_{k,n}^i h - h, & t \in \varrho_{k,n}^{i,2}, \\ \vdots \\ t - t_{k,n}^i h - o_{k,n}^i h, & t \in \varrho_{k,n}^{i,o_{k,n}^i+1}, \end{cases} \tag{14}$$

and

$$e_{k,n}^i(t) = \begin{cases} 0, & t \in \varrho_{k,n}^{i,1}, \\ \varepsilon_i(t_{k,n}^i h) - \varepsilon_i(t_{k,n}^i h + h), & t \in \varrho_{k,n}^{i,2}, \\ \vdots \\ \varepsilon_i(t_{k,n}^i h) - \varepsilon_i(t_{k,n}^i h + o_{k,n}^i h), & t \in \varrho_{k,n}^{i,o_{k,n}^i+1}. \end{cases} \tag{15}$$

Obviously,  $\tau_{k,n}^i(t) \in [0, h)$  for  $t \in V_{k,n}^i$ . According to the above two functions,  $\varepsilon_i(t_{k,n}^i h)$  can be described as:

$$\varepsilon_i(t_{k,n}^i h) = \varepsilon_i(t - \tau_{k,n}^i(t)) + e_{k,n}^i(t). \tag{16}$$

Then, the triggering condition of node  $i$  can be represented as:

$$(e_{k,n}^i(t))^T \Omega_i e_{k,n}^i(t) > \rho_i \varepsilon_i^T(t - \tau_{k,n}^i(t)) \Omega_i \varepsilon_i(t - \tau_{k,n}^i(t)). \tag{17}$$

By combining Equations (11) and (16), then the event-based synchronization error model can be formulated as:

$$\begin{cases} \dot{\varepsilon}_i(t) = \begin{cases} A\varepsilon_i(t) + v_i(t) + \sigma \sum_{j=1}^N c_{ij} \Gamma \varepsilon_j(t) + BK_i \varepsilon_i(t - \tau_{k,n}^i(t)) + BK_i e_{k,n}^i(t), & t \in V_{k,n}^i, \\ A\varepsilon_i(t) + v_i(t) + \sigma \sum_{j=1}^N c_{ij} \Gamma \varepsilon_j(t), & t \in D_n^i, \end{cases} \\ \varepsilon_i(t) = \delta_i(t), & t \in [-h, 0], \end{cases} \tag{18}$$

where  $\delta_i(t)$  is the initial function of  $\varepsilon_i(t)$ .

At the end of the section, we would like to give the following definitions and lemma to derive the main results.

**Definition 1** (43). The synchronization error system described by Equation (18) is said to be exponentially stable (ES) if there exist constants  $\epsilon_i > 0$  and  $\iota_i > 0$  such that  $\|\varepsilon_i(t)\| \leq \epsilon_i e^{-\iota_i t} \|\delta_i(0)\|_h$  for all  $t \geq 0$ , where  $\iota_i$  is called the decay rate.

**Definition 2** (44). During the time interval  $[0, t)$ , let  $n_i(t)$  denote the number of off/on transitions in DoS  $i$ , that is,  $n_i(t) = \text{card} \{ \int_n^t + g_n^i \in [0, t) | n \in \mathbb{N} \}$ , where  $\text{card} \{ \cdot \}$  represents the size of the set. Then, for given parameters  $\varsigma_i \geq 0$  and  $\eta_i \geq 0$ ,  $n_i(t)$  satisfies:

$$n_i(t) \leq \eta_i + \frac{t}{\varsigma_i}, \quad t \geq 0. \tag{19}$$

**Lemma 1** (45). For given  $x, y \in \mathbb{R}^{n_x}$ , and positive definite matrix  $R \in \mathbb{R}^{n_x \times n_x}$ , the following inequality holds:

$$2x^T y \leq x^T R x + y^T R^{-1} y. \tag{20}$$

### 3 | MAIN RESULTS

Given that the synchronization means  $\lim_{t \rightarrow \infty} \varepsilon_i(t) = 0$ , then the synchronization control problem can be transformed into proving the stability of the formulated error system. Thus, the sufficient conditions to guarantee the ES of the

system described by Equation (18) are first analyzed based on Lyapunov stability theory, then the controller gains and event-triggering parameters are obtained by using LMI technique in this section.

**Theorem 1.** For given scalars  $\alpha_i^w > 0, \mu_i^w > 0, (1 \leq i \leq N, w = 1, 2), 0 < \rho_i < 1, \text{ sampling period } h > 0, \text{ DoS parameters } \eta_i \geq 0, \zeta_i \geq 0, \text{ matrix } \Lambda \text{ and feedback gain matrices } K_i, \text{ the synchronization error system formulated as Equation (18) is ES with decay rate } \iota_i = \frac{\varphi_i}{2}, \text{ if there exist symmetric positive definite matrices } P_i^w, Q_i^w, R_i^w, \Omega_i \text{ and free weight matrices } M_i^w, N_i^w, \text{ then the following nonlinear matrix inequalities Equation (21) and conditions Equations (22)–(26) can be derived:$

$$\Sigma_i^w = \begin{bmatrix} \Theta_{1i}^w & * & * & * \\ \sqrt{h}(M_i^w)^T & \Theta_{2i}^w & * & * \\ \sqrt{h}(N_i^w)^T & 0 & \Theta_{3i}^w & * \\ \Theta_{4i}^w & 0 & 0 & \Theta_{5i}^w \end{bmatrix} < 0, \tag{21}$$

$$P_i^1 \leq \mu_i^2 P_i^2, \tag{22}$$

$$P_i^2 \leq \mu_i^1 e^{2(\alpha_i^1 + \alpha_i^2)h} P_i^1, \tag{23}$$

$$Q_i^1 \leq \mu_i^2 Q_i^2, Q_i^2 \leq \mu_i^1 Q_i^1, \tag{24}$$

$$R_i^1 \leq \mu_i^2 R_i^2, R_i^2 \leq \mu_i^1 R_i^1, \tag{25}$$

$$0 < \varphi_i = \frac{2\alpha_i^1 g_{\min} - 2(\alpha_i^1 + \alpha_i^2)h - 2\alpha_i^2 d_{\max} - \ln(\mu_i^1 \mu_i^2)}{\zeta_i}, \tag{26}$$

where  $\Theta_{1i}^w = H_i^w + \Delta_i^w + (\Delta_i^w)^T$  with

$$H_i^1 = \begin{bmatrix} \Pi_{1i}^1 & * & * & * & * & * \\ P_i^1 & -I & * & * & * & * \\ K_i^T B^T P_i^1 & 0 & \rho_i \Omega_i & * & * & * \\ K_i^T B^T P_i^1 & 0 & 0 & -\Omega_i & * & * \\ 0 & 0 & 0 & 0 & -e^{-2\alpha_i^1 h} Q_i^1 & * \\ \Pi_{2i}^1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, H_i^2 = \begin{bmatrix} \Pi_{1i}^2 & * & * & * & * \\ P_i^2 & -I & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & -e^{2\alpha_i^2 h} Q_i^2 & * \\ \Pi_{2i}^2 & 0 & 0 & 0 & 0 \end{bmatrix};$$

$$\Pi_{1i}^1 = 2\alpha_i^1 P_i^1 + P_i^1 A + A^T P_i^1 + Q_i^1 + \Lambda^T \Lambda + \sigma c_{ii} \Gamma^T P_i^1, \Pi_{1i}^2 = -2\alpha_i^2 P_i^2 + P_i^2 A + A^T P_i^2 + Q_i^2 + \Lambda^T \Lambda + \sigma c_{ii} \Gamma^T P_i^2,$$

$$\Pi_{2i}^1 = \left[ \sigma c_{i1} P_i^1 \Gamma \quad \dots \quad \sigma c_{i(i-1)} P_i^1 \Gamma \quad \sigma c_{i(i+1)} P_i^1 \Gamma \quad \dots \quad \sigma c_{iN} P_i^1 \Gamma \right]^T,$$

$$\Pi_{2i}^2 = \left[ \sigma c_{i1} P_i^2 \Gamma \quad \dots \quad \sigma c_{i(i-1)} P_i^2 \Gamma \quad \sigma c_{i(i+1)} P_i^2 \Gamma \quad \dots \quad \sigma c_{iN} P_i^2 \Gamma \right]^T,$$

$$\Delta_i^1 = \begin{bmatrix} M_i^1 & 0 & N_i^1 - M_i^1 & 0 & -N_i^1 & 0_{n_x \times (N-1)} \end{bmatrix}, \Delta_i^2 = \begin{bmatrix} M_i^2 & 0 & N_i^2 - M_i^2 & -N_i^2 & 0_{n_x \times (N-1)} \end{bmatrix},$$

$$M_i^1 = \begin{bmatrix} M_{1i}^1 & M_{2i}^1 & M_{3i}^1 & M_{4i}^1 & M_{5i}^1 & M_{6i}^1 & \dots & M_{(N+4)i}^1 \end{bmatrix}, N_i^1 = \begin{bmatrix} N_{1i}^1 & N_{2i}^1 & N_{3i}^1 & N_{4i}^1 & N_{5i}^1 & N_{6i}^1 & \dots & N_{(N+4)i}^1 \end{bmatrix},$$

$$M_i^2 = \begin{bmatrix} M_{1i}^2 & M_{2i}^2 & M_{3i}^2 & M_{4i}^2 & M_{5i}^2 & M_{6i}^2 & \dots & M_{(N+3)i}^2 \end{bmatrix}, N_i^2 = \begin{bmatrix} N_{1i}^2 & N_{2i}^2 & N_{3i}^2 & N_{4i}^2 & N_{5i}^2 & N_{6i}^2 & \dots & N_{(N+3)i}^2 \end{bmatrix},$$

$$\Theta_{2i}^1 = \Theta_{3i}^1 = -e^{-2\alpha_i^1 h} R_i^1, \Theta_{2i}^2 = \Theta_{3i}^2 = -R_i^2, \Theta_{4i}^w = \sqrt{h} \Lambda_i^w R_i^w, \Theta_{5i}^w = -R_i^w,$$

$$\Lambda_i^1 = [A + \sigma c_{ii} \Gamma \quad I \quad BK_i \quad BK_i \quad 0 \quad \sigma c_{i1} \Gamma \quad \dots \quad \sigma c_{i(i-1)} \Gamma \quad \sigma c_{i(i+1)} \Gamma \quad \dots \quad \sigma c_{iN} \Gamma],$$

$$\Lambda_i^2 = [A + \sigma c_{ii} \Gamma \quad I \quad 0 \quad 0 \quad \sigma c_{i1} \Gamma \quad \dots \quad \sigma c_{i(i-1)} \Gamma \quad \sigma c_{i(i+1)} \Gamma \quad \dots \quad \sigma c_{iN} \Gamma].$$

*Proof.* In terms of the system Equation (18), we construct a Lyapunov function as follow:

$$V_i(t) = V_i^{m(t)}(t) = V_{1i}^{m(t)}(t) + V_{2i}^{m(t)}(t) + V_{3i}^{m(t)}(t), \tag{27}$$

where,



$$\begin{aligned} V_{1i}^{m(t)}(t) &= \varepsilon_i^T(t)P_i^{m(t)}\varepsilon_i(t), \\ V_{2i}^{m(t)}(t) &= \int_{t-h}^t \varepsilon_i^T(s)z(\cdot)Q_i^{m(t)}\varepsilon_i(s)ds, \\ V_{3i}^{m(t)}(t) &= \int_{-h}^0 \int_{t+\theta}^t \dot{\varepsilon}_i^T(s)z(\cdot)R_i^{m(t)}\dot{\varepsilon}_i(s)dsd\theta, \end{aligned}$$

and  $m(t) = \begin{cases} 1, & t \in V_{k,n}^i \\ 2, & t \in D_h^i \end{cases}$ ,  $z(\cdot) \triangleq e^{2(-1)^{m(t)}\alpha_i^{m(t)}(t-s)}$ .

Then, taking the derivative of  $V_{1i}^{m(t)}$ ,  $V_{2i}^{m(t)}$ , and  $V_{3i}^{m(t)}$ , the following equations can be obtained:

$$\dot{V}_{1i}^{m(t)} = 2\varepsilon_i^T(t)P_i^{m(t)}\dot{\varepsilon}_i(t), \tag{28}$$

$$\dot{V}_{2i}^{m(t)} = 2(-1)^{m(t)}\alpha_i^{m(t)}V_{2i}^{m(t)}(t) + \varepsilon_i^T(t)Q_i^{m(t)}\varepsilon_i(t) - \varepsilon_i^T(t-h)e^{2(-1)^{m(t)}\alpha_i^{m(t)}h}Q_i^{m(t)}\varepsilon_i(t-h), \tag{29}$$

$$\dot{V}_{3i}^{m(t)} = 2(-1)^{m(t)}\alpha_i^{m(t)}V_{3i}^{m(t)}(t) + h\varepsilon_i^T(t)R_i^{m(t)}\dot{\varepsilon}_i(t) - \int_{t-h}^t \dot{\varepsilon}_i^T(s)e^{2(-1)^{m(t)}\alpha_i^{m(t)}(t-s)}R_i^{m(t)}\dot{\varepsilon}_i(s)ds. \tag{30}$$

By employing the free-weighting matrices method, for appropriately dimensioned matrices  $M_i^{m(t)}$  and  $N_i^{m(t)}$ , we can get:

$$\begin{aligned} 2(\xi_i^{m(t)}(t))^T M_i^{m(t)} \left[ \varepsilon_i(t) - \varepsilon_i(t - \tau_{k,n}^i(t)) - \int_{t-\tau_{k,n}^i(t)}^t \dot{\varepsilon}_i(s)ds \right] &= 0, \\ 2(\xi_i^{m(t)}(t))^T N_i^{m(t)} \left[ \varepsilon_i(t - \tau_{k,n}^i(t)) - \varepsilon_i(t - h) - \int_{t-h}^{t-\tau_{k,n}^i(t)} \dot{\varepsilon}_i(s)ds \right] &= 0, \end{aligned}$$

where  $\xi_i^1(t) = [\varepsilon_i^T(t) \quad v_i^T(t) \quad \varepsilon_i^T(t - \tau_{k,n}^i(t)) \quad (e_{k,n}^i(t))^T \quad \varepsilon_i^T(t - h) \quad \varepsilon_1^T(t) \dots \varepsilon_{i-1}^T(t) \quad \varepsilon_{i+1}^T(t) \dots \varepsilon_N^T(t)]^T$ ,  
 $\xi_i^2(t) = [\varepsilon_i^T(t) \quad v_i^T(t) \quad \varepsilon_i^T(t - \tau_{k,n}^i(t)) \quad \varepsilon_i^T(t - h) \quad \varepsilon_1^T(t) \dots \varepsilon_{i-1}^T(t) \quad \varepsilon_{i+1}^T(t) \dots \varepsilon_N^T(t)]^T$ .

By applying Lemma 1, for given real matrices  $R_i^{m(t)}$ , the following inequalities can be obtained:

$$\begin{aligned} -2(\xi_i^{m(t)}(t))^T M_i^{m(t)} \int_{t-\tau_{k,n}^i(t)}^t \dot{\varepsilon}_i(s)ds &\leq \tau_{k,n}^i(t)(\xi_i^{m(t)}(t))^T M_i^{m(t)} e^{2(-1)^{m(t)+1}\alpha_i^{m(t)}\lambda^{m(t)}(t-s)}(R_i^{m(t)})^{-1}(M_i^{m(t)})^T \xi_i^{m(t)}(t) \\ &\quad + \int_{t-\tau_{k,n}^i(t)}^t \dot{\varepsilon}_i^T(s)e^{2(-1)^{m(t)}\alpha_i^{m(t)}\lambda^{m(t)}(t-s)}R_i^{m(t)}\dot{\varepsilon}_i(s)ds, \end{aligned} \tag{31}$$

$$\begin{aligned} -2(\xi_i^{m(t)}(t))^T N_i^{m(t)} \int_{t-h}^{t-\tau_{k,n}^i(t)} \dot{\varepsilon}_i(s)ds &\leq (h - \tau_{k,n}^i(t))(\xi_i^{m(t)}(t))^T N_i^{m(t)} e^{2(-1)^{m(t)+1}\alpha_i^{m(t)}\lambda^{m(t)}(t-s)}(R_i^{m(t)})^{-1}(N_i^{m(t)})^T \xi_i^{m(t)}(t) \\ &\quad + \int_{t-h}^{t-\tau_{k,n}^i(t)} \dot{\varepsilon}_i^T(s)e^{2(-1)^{m(t)}\alpha_i^{m(t)}\lambda^{m(t)}(t-s)}R_i^{m(t)}\dot{\varepsilon}_i(s)ds, \end{aligned} \tag{32}$$

where  $\lambda^1 = 1, \lambda^2 = 0$ .

Recalling Assumption 1, it is equivalent to:

$$v_i^T(t)v_i(t) \leq \varepsilon_i^T(t)\Lambda^T\Lambda\varepsilon_i(t). \tag{33}$$

Combining the triggering condition Equations (17) and (28)–(33), we can obtain:

$$\begin{aligned} \dot{V}_i(t) &\leq 2(-1)^{m(t)}\alpha_i^{m(t)}V_i^{m(t)}(t) + 2(-1)^{m(t)+1}\alpha_i^{m(t)}\varepsilon_i^T(t)P_i^{m(t)}\varepsilon_i(t) + 2\varepsilon_i^T(t)P_i^{m(t)}\left(A\varepsilon_i(t) + v_i(t) + \sigma \sum_{j=1}^N c_{ij}\Gamma\varepsilon_j(t) \right. \\ &\quad \left. + \lambda^{m(t)}(BK_i\varepsilon_i(t - \tau_{k,n}^i(t)) + BK_i e_{k,n}^i(t))\right) + \varepsilon_i^T(t)Q_i^{m(t)}\varepsilon_i(t) - \varepsilon_i^T(t-h)e^{2(-1)^{m(t)}\alpha_i^{m(t)}h}Q_i^{m(t)}\varepsilon_i(t-h) \end{aligned}$$

$$\begin{aligned}
 &+ h(A\varepsilon_i(t) + v_i(t) + \sigma \sum_{j=1}^N c_{ij} \Gamma \varepsilon_j(t) + \lambda^{m(t)} (BK_i \varepsilon_i(t - \tau_{k,n}^i(t)) + BK_i e_{k,n}^i(t)))^T R_i^{m(t)} (A\varepsilon_i(t) + v_i(t)) \\
 &+ \sigma \sum_{j=1}^N c_{ij} \Gamma \varepsilon_j(t) + \lambda^{m(t)} (BK_i \varepsilon_i(t - \tau_{k,n}^i(t)) + BK_i e_{k,n}^i(t)) + 2(\xi_i^{m(t)}(t))^T M_i^{m(t)} [\varepsilon_i(t) - \varepsilon_i(t - \tau_{k,n}^i(t))] \\
 &+ 2(\xi_i^{m(t)}(t))^T N_i^{m(t)} [\varepsilon_i(t - \tau_{k,n}^i(t)) - \varepsilon_i(t - h)] + h(\xi_i^{m(t)}(t))^T M_i^{m(t)} e^{2(-1)^{m(t)+1} \alpha_i^{m(t)} \lambda^{m(t)} (t-s)} (R_i^{m(t)})^{-1} (M_i^{m(t)})^T \xi_i^{m(t)}(t) \\
 &+ h(\xi_i^{m(t)}(t))^T N_i^{m(t)} e^{2(-1)^{m(t)+1} \alpha_i^{m(t)} \lambda^{m(t)} (t-s)} (R_i^{m(t)})^{-1} (N_i^{m(t)})^T \xi_i^{m(t)}(t) + \varepsilon_i^T(t) \Lambda^T \Lambda \varepsilon_i(t) - v_i^T(t) v_i(t) \\
 &+ \lambda^{m(t)} (\rho_i \varepsilon_i^T(t - \tau_{k,n}^i(t)) \Omega_i \varepsilon_i(t - \tau_{k,n}^i(t)) - (e_{k,n}^i(t))^T \Omega_i e_{k,n}^i(t)). \tag{34}
 \end{aligned}$$

Then, we have:

$$\begin{aligned}
 \dot{V}_i(t) &\leq 2(-1)^{m(t)} \alpha_i^{m(t)} V_i^{m(t)}(t) + (\xi_i^{m(t)}(t))^T [\Theta_{1i}^{m(t)} + hM_i^{m(t)} e^{2(-1)^{m(t)+1} \alpha_i^{m(t)} \lambda^{m(t)} (t-s)} (R_i^{m(t)})^{-1} (M_i^{m(t)})^T \\
 &\quad + hN_i^{m(t)} e^{2(-1)^{m(t)+1} \alpha_i^{m(t)} \lambda^{m(t)} (t-s)} (R_i^{m(t)})^{-1} (N_i^{m(t)})^T + h(\mathbb{A}_i^{m(t)})^T R_i^{m(t)} \mathbb{A}_i^{m(t)}] \xi_i^{m(t)}(t). \tag{35}
 \end{aligned}$$

Given  $\Sigma_i^{m(t)} < 0$  and by referring to Schur complement, it can be found that:

$$\begin{aligned}
 &(\xi_i^{m(t)}(t))^T [\Theta_{1i}^{m(t)} + hM_i^{m(t)} e^{2(-1)^{m(t)+1} \alpha_i^{m(t)} \lambda^{m(t)} (t-s)} (R_i^{m(t)})^{-1} (M_i^{m(t)})^T + hN_i^{m(t)} e^{2(-1)^{m(t)+1} \alpha_i^{m(t)} \lambda^{m(t)} (t-s)} (R_i^{m(t)})^{-1} (N_i^{m(t)})^T \\
 &\quad + h(\mathbb{A}_i^{m(t)})^T R_i^{m(t)} \mathbb{A}_i^{m(t)}] \xi_i^{m(t)}(t) < 0. \tag{36}
 \end{aligned}$$

Hence,

$$\dot{V}_i(t) \leq 2(-1)^{m(t)} \alpha_i^{m(t)} V_i^{m(t)}(t),$$

which means that:

$$V_i(t) \leq \begin{cases} e^{-2\alpha_i^1(t-d_n^i)} V_i^1(d_n^i), & t \in \widetilde{D}_n^i, \\ e^{2\alpha_i^2(t-f_n^i-g_n^i)} V_i^2(f_n^i + g_n^i), & t \in \overline{D}_n^i. \end{cases} \tag{37}$$

For given conditions  $P_i^1 \leq \mu_i^2 P_i^2$ ,  $Q_i^1 \leq \mu_i^2 Q_i^2$  and  $R_i^1 \leq \mu_i^2 R_i^2$  in Equations (22)–(25), it can be easily derived that:

$$V_i^1(d_n^i) \leq \mu_i^2 V_i^2(\widetilde{d}_n^i), \tag{38}$$

where  $\widetilde{d}_n^i = \lim_{t \rightarrow d_n^i-} t$ . Similarly, with other conditions in Equations (22)–(25), we can get:

$$V_i^2(f_n^i + g_n^i) \leq \mu_i^1 e^{2(\alpha_i^1 + \alpha_i^2)h} V_i^1(\widetilde{f}_n^i + \widetilde{g}_n^i), \tag{39}$$

where  $\widetilde{f}_n^i + \widetilde{g}_n^i = \lim_{t \rightarrow (f_n^i + g_n^i)-} t$ .

Then, the following two cases are specifically analyzed:

Case 1 ( $t \in \widetilde{D}_n^i$ ): we can get the following inequalities by employing Equations (37)–(39) and Assumption 2:

$$\begin{aligned}
 V_i^1(t) &\leq e^{-2\alpha_i^1(t-d_n^i)} V_i^1(d_n^i) \leq \mu_i^2 e^{-2\alpha_i^1(t-d_n^i)} V_i^2(\widetilde{d}_n^i) \leq \mu_i^2 V_i^2(\widetilde{d}_n^i) \\
 &\leq \mu_i^2 e^{2\alpha_i^2(d_n^i - f_{n-1}^i - g_{n-1}^i)} V_i^2(f_{n-1}^i + g_{n-1}^i) \leq \mu_i^2 e^{2\alpha_i^2 d_{max}} V_i^2(f_{n-1}^i + g_{n-1}^i) \\
 &\leq \mu_i^1 \mu_i^2 e^{2\alpha_i^2 d_{max} + 2(\alpha_i^1 + \alpha_i^2)h} V_i^1(\widetilde{f}_{n-1}^i + \widetilde{g}_{n-1}^i) \\
 &\leq \mu_i^1 \mu_i^2 e^{2\alpha_i^2 d_{max} + 2(\alpha_i^1 + \alpha_i^2)h - 2\alpha_i^1(f_{n-1}^i + g_{n-1}^i - d_{n-1}^i)} V_i^1(d_{n-1}^i) \leq \mu_i^1 \mu_i^2 e^{2\alpha_i^2 d_{max} + 2(\alpha_i^1 + \alpha_i^2)h - 2\alpha_i^1 g_{min}} V_i^1(d_{n-1}^i) \\
 &\leq \dots \\
 &\leq e^{n_i(t) \times 2(\alpha_i^1 + \alpha_i^2)h + n_i(t) \ln(\mu_i^1 \mu_i^2) + n_i(t) \times 2\alpha_i^2 d_{max} - n_i(t) \times 2\alpha_i^1 g_{min}} V_i^1(0) \\
 &\leq e^{p_i^1(t)} V_i^1(0),
 \end{aligned}$$

where  $p_i^1(t) = (\eta_i + \frac{t}{\varsigma_i}) \times 2(\alpha_i^1 + \alpha_i^2)h + 2\alpha_i^2 d_{max}(\eta_i + \frac{t}{\varsigma_i}) - 2\alpha_i^1 g_{min}(\eta_i + \frac{t}{\varsigma_i}) + (\eta_i + \frac{t}{\varsigma_i}) \ln(\mu_i^1 \mu_i^2)$ . Then, according to Equation (26), we have:

$$V_i^1(t) \leq e^{b_i^1} e^{-\varphi_i t} V_i^1(0), \tag{40}$$

where  $b_i^1 = 2\eta_i(\alpha_i^1 + \alpha_i^2)h + \eta_i \ln(\mu_i^1 \mu_i^2) + 2\alpha_i^2 d_{max} \eta_i - 2\alpha_i^1 g_{min} \eta_i$ .

Case 2 ( $t \in D_n^i$ ): with the similar procedure as in Case 1, we can obtain:

$$V_i^2(t) \leq \frac{1}{\mu_i^2} e^{b_i^2} e^{-\varphi_i t} V_i^1(0), \tag{41}$$

where  $b_i^2 = (\eta_i + 1)(2(\alpha_i^1 + \alpha_i^2)h + \ln(\mu_i^1 \mu_i^2) + 2\alpha_i^2 d_{max} \eta_i - 2\alpha_i^1 g_{min})$ .

Base on the above analysis and defining  $W_i = \max\{e^{b_i^1}, \frac{1}{\mu_i^2} e^{b_i^2}\}$ ,  $\varpi_i^1 = \min\{\lambda_{min}(P_i^1, P_i^2)\}$ ,  $\varpi_i^2 = \lambda_{max}(P_i^1) + h\lambda_{max}(Q_i^1) + \frac{h^2}{2} \lambda_{max}(R_i^1)$ ,  $t_i = \frac{\varphi_i}{2}$ , it is available that:

$$V_i(t) \leq W_i e^{-\varphi_i t} V_i^1(0), \tag{42}$$

according to Equations (40) and (41).

Thus, we have:

$$V_i(t) \geq \varpi_i^1 \|\varepsilon_i(t)\|^2, V_i^1(0) \leq \varpi_i^2 \|\delta_i(0)\|_h^2. \tag{43}$$

Combining Equations (42) and (43), it yields that:

$$\|\varepsilon_i(t)\|^2 \leq \sqrt{\frac{W_i \varpi_i^2}{\varpi_i^1}} e^{-t_i t} \|\delta_i(0)\|_h,$$

which validates that the system Equation (18) is ES with decay rate  $t_i$ . ■

It is noteworthy that Theorem 1 only analyzes the ES of the synchronization error system by giving the sufficient conditions, but did not succeed in solving the problem of controllers design. In Theorem 2, we then determine the control parameters  $K_i$  and triggering matrices  $\Omega_i$ .

**Theorem 2.** For given scalars  $\alpha_i^w > 0$ ,  $\mu_i^w > 0$ ,  $\kappa_i^w > 0$ ,  $r_i^w > 0$ ,  $s_i^w > 0$  ( $1 \leq i \leq N$ ,  $w = 1, 2$ ),  $0 < \rho_i < 1$ , sampling period  $h > 0$ , DoS parameters  $\eta_i \geq 0$ ,  $\varsigma_i \geq 0$  and matrix  $\Lambda$ , the system Equation (18) is ES with decay rate  $t_i = \frac{\varphi_i}{2}$ , if there exist symmetric positive definite matrices  $P_i^w$ ,  $X_i^w$ ,  $Y_i$  and suitable dimension matrices  $\overline{Q}_i^w$ ,  $\overline{R}_i^w$ ,  $\overline{M}_i^w$ ,  $\overline{N}_i^w$ ,  $\overline{\Omega}_i$ , so the condition Equation (26) and the following LMIs hold:

$$\overline{\Xi}_i^w = \begin{bmatrix} \overline{\Theta}_{1i}^w & * & * & * & * \\ \sqrt{h}(\overline{M}_i^w)^T & \overline{\Theta}_{2i}^w & * & * & * \\ \sqrt{h}(\overline{N}_i^w)^T & 0 & \overline{\Theta}_{3i}^w & * & * \\ \overline{\Theta}_{4i}^w & 0 & 0 & \overline{\Theta}_{5i}^w & * \\ \overline{\Theta}_{6i}^w & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{44}$$

$$\begin{bmatrix} -\mu_i^2 X_i^2 & * \\ X_i^2 & -X_i^1 \end{bmatrix} \leq 0, \tag{45}$$

$$\begin{bmatrix} -\mu_i^1 e^{2(\alpha_i^1 + \alpha_i^2)h} X_i^1 & * \\ X_i^1 & -X_i^2 \end{bmatrix} \leq 0, \tag{46}$$

$$\begin{bmatrix} -\mu_i^1 \bar{R}_i^1 & * \\ X_i^1 & r_i^2 r_i^2 \bar{R}_i^2 - 2r_i^2 X_i^2 \end{bmatrix} \leq 0, \begin{bmatrix} -\mu_i^2 \bar{R}_i^2 & * \\ X_i^2 & r_i^1 r_i^1 \bar{R}_i^1 - 2r_i^1 X_i^1 \end{bmatrix} \leq 0, \tag{47}$$

$$\begin{bmatrix} -\mu_i^1 \bar{Q}_i^1 & * \\ X_i^1 & s_i^2 s_i^2 \bar{Q}_i^2 - 2s_i^2 X_i^2 \end{bmatrix} \leq 0, \begin{bmatrix} -\mu_i^2 \bar{Q}_i^2 & * \\ X_i^2 & s_i^1 s_i^1 \bar{Q}_i^1 - 2s_i^1 X_i^1 \end{bmatrix} \leq 0, \tag{48}$$

moreover, the control parameters and triggering matrices are  $K_i = Y_i X_i^{-1}$  and  $\Omega_i = X_i^{-1} \bar{\Omega}_i X_i^{-1}$ , respectively, where,

$$\bar{\Theta}_{1i}^1 = \begin{bmatrix} E_{1i}^1 & * & * & * & * & * \\ I & -I & * & * & * & * \\ Y_i^T B^T & 0 & \rho_i \bar{\Omega}_i & * & * & * \\ Y_i^T B^T & 0 & 0 & -\bar{\Omega}_i & * & * \\ 0 & 0 & 0 & 0 & -e^{-2\alpha_i^1 h} \bar{Q}_i^1 & * \\ E_{2i}^1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{\Theta}_{1i}^2 = \begin{bmatrix} E_{1i}^2 & * & * & * & * \\ I & -I & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & -e^{2\alpha_i^2 h} \bar{Q}_i^2 & * \\ E_{2i}^2 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$E_{1i}^1 = 2\alpha_i^1 X_i^1 + AX_i^1 + X_i^1 A^T + \bar{Q}_i^1 + \sigma c_{ii} X_i^1 \Gamma^T, E_{2i}^1 = -2\alpha_i^2 X_i^2 + AX_i^2 + X_i^2 A^T + \bar{Q}_i^2 + \sigma c_{ii} X_i^2 \Gamma^T,$$

$$E_{2i}^1 = [\sigma c_{i1} \Gamma X_i^1 \dots \sigma c_{i(i-1)} \Gamma X_i^1 \quad \sigma c_{i(i+1)} \Gamma X_i^1 \dots \sigma c_{iN} \Gamma X_i^1]^T, E_{2i}^2 = [\sigma c_{i1} \Gamma X_i^2 \dots \sigma c_{i(i-1)} \Gamma X_i^2 \quad \sigma c_{i(i+1)} \Gamma X_i^2 \dots \sigma c_{iN} \Gamma X_i^2]^T,$$

$$\bar{\Delta}_i^1 = [\bar{M}_i^1 \quad 0 \quad \bar{N}_i^1 - \bar{M}_i^1 \quad 0 \quad -\bar{N}_i^1 \quad 0_{n_x \times (N-1)}], \bar{\Delta}_i^2 = [\bar{M}_i^2 \quad 0 \quad \bar{N}_i^2 - \bar{M}_i^2 \quad -\bar{N}_i^2 \quad 0_{n_x \times (N-1)}],$$

$$\bar{\Theta}_{2i}^1 = \bar{\Theta}_{3i}^1 = -e^{-2\alpha_i^1 h} \bar{R}_i^1, \bar{\Theta}_{2i}^2 = \bar{\Theta}_{3i}^2 = -\bar{R}_i^2, \bar{\Theta}_{5i}^w = -2\kappa_i^w X_i^w + \kappa_i^w \kappa_i^w \bar{R}_i^w, \bar{\Theta}_{6i}^1 = [\Lambda X_i^1 \quad 0_{n_x \times (N+4)}], \bar{\Theta}_{6i}^2 = [\Lambda X_i^2 \quad 0_{n_x \times (N+3)}]$$

$$\bar{\Theta}_{4i}^1 = [\sqrt{h}(A + \sigma c_{ii} \Gamma) X_i^1 \quad \sqrt{h}I \quad \sqrt{h}B Y_i \quad \sqrt{h}B Y_i \quad 0 \quad \sqrt{h} \sigma c_{i1} \Gamma X_i^1 \dots \sqrt{h} \sigma c_{i(i-1)} \Gamma X_i^1 \quad \sqrt{h} \sigma c_{i(i+1)} \Gamma X_i^1 \dots \sqrt{h} \sigma c_{iN} \Gamma X_i^1],$$

$$\bar{\Theta}_{4i}^2 = [\sqrt{h}(A + \sigma c_{ii} \Gamma) X_i^2 \quad \sqrt{h}I \quad 0 \quad 0 \quad \sqrt{h} \sigma c_{i1} \Gamma X_i^2 \dots \sqrt{h} \sigma c_{i(i-1)} \Gamma X_i^2 \quad \sqrt{h} \sigma c_{i(i+1)} \Gamma X_i^2 \dots \sqrt{h} \sigma c_{iN} \Gamma X_i^2].$$

*Proof.* According to Schur complement method, the matrix inequalities Equation (21) hold only if the following matrix inequalities hold:

$$\Xi_i^w = \begin{bmatrix} \bar{\Theta}_{1i}^w & * & * & * & * \\ \sqrt{h}(M_i^w)^T & \Theta_{2i}^w & * & * & * \\ \sqrt{h}(N_i^w)^T & 0 & \Theta_{3i}^w & * & * \\ \bar{\Theta}_{4i}^w & 0 & 0 & \bar{\Theta}_{5i}^w & * \\ \bar{\Theta}_{6i}^w & 0 & 0 & 0 & -I \end{bmatrix} < 0, \tag{49}$$

where  $\Theta_{2i}^w$  and  $\Theta_{3i}^w$  are defined in Theorem 1,  $\bar{\Theta}_{1i}^w = \bar{H}_i^w + \Delta_i^w + (\Delta_i^w)^T$ ,

$$\bar{H}_i^1 = \begin{bmatrix} \bar{\Pi}_{1i}^1 & * & * & * & * & * \\ P_i^1 & -I & * & * & * & * \\ K_i^T B^T P_i^1 & 0 & \rho_i \bar{\Omega}_i & * & * & * \\ K_i^T B^T P_i^1 & 0 & 0 & -\bar{\Omega}_i & * & * \\ 0 & 0 & 0 & 0 & -e^{-2\alpha_i^1 h} Q_i^1 & * \\ \bar{\Pi}_{2i}^1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \bar{H}_i^2 = \begin{bmatrix} \bar{\Pi}_{1i}^2 & * & * & * & * \\ P_i^2 & -I & * & * & * \\ 0 & 0 & 0 & * & * \\ 0 & 0 & 0 & -e^{2\alpha_i^2 h} Q_i^2 & * \\ \bar{\Pi}_{2i}^2 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\bar{\Pi}_{1i}^1 = 2\alpha_i^1 P_i^1 + P_i^1 A + A^T P_i^1 + Q_i^1 + \sigma c_{ii} \Gamma^T P_i^1, \bar{\Pi}_{1i}^2 = -2\alpha_i^2 P_i^2 + P_i^2 A + A^T P_i^2 + Q_i^2 + \sigma c_{ii} \Gamma^T P_i^2,$$

$$\bar{\Theta}_{4i}^w = \sqrt{h} P_i^w \Lambda_i^w, \bar{\Theta}_{5i}^w = -P_i^w (R_i^w)^{-1} P_i^w, \bar{\Theta}_{6i}^1 = [\Lambda \quad 0_{n_x \times (N+4)}], \bar{\Theta}_{6i}^2 = [\Lambda \quad 0_{n_x \times (N+3)}].$$

For any positive definite matrices  $R_i^w, P_i^w$  and scalar  $\kappa_i^w > 0$ , it can be obtained that:<sup>46</sup>

$$(R_i^w - (\kappa_i^w)^{-1} P_i^w)(R_i^w)^{-1} (R_i^w - (\kappa_i^w)^{-1} P_i^w) \geq 0. \tag{50}$$

According to Equation (50), we can derive:

$$-P_i^w(R_i^w)^{-1}P_i^w \leq -2\kappa_i^w P_i^w + \kappa_i^w \kappa_i^w R_i^w. \tag{51}$$

Thus, by substituting  $-P_i^w(R_i^w)^{-1}P_i^w$  with  $-2\kappa_i^w P_i^w + \kappa_i^w \kappa_i^w R_i^w$  in Equation (49), it can be observed that  $\Xi_i^w < 0$  hold when the following matrix inequalities set up:

$$\widetilde{\Xi}_i^w = \begin{bmatrix} \widetilde{\Theta}_{1i}^w & * & * & * & * \\ \sqrt{h}(M_i^w)^T & \Theta_{2i}^w & * & * & * \\ \sqrt{h}(N_i^w)^T & 0 & \Theta_{3i}^w & * & * \\ \widetilde{\Theta}_{4i}^w & 0 & 0 & -2\kappa_i^w P_i^w + \kappa_i^w \kappa_i^w R_i^w & * \\ \widetilde{\Theta}_{6i}^w & 0 & 0 & 0 & -I \end{bmatrix} < 0. \tag{52}$$

For  $w = 1$ , define  $X_i^1 = (P_i^1)^{-1}$ ,  $Y_i = K_i X_i^1$ ,  $L_i = \text{diag}\{X_i^1, I, X_i^1, X_i^1, X_i^1, X_i^1, X_i^1, X_i^1, X_i^1, I\}$ ,  $\overline{Q}_i^1 = X_i^1 Q_i^1 X_i^1$ ,  $\overline{R}_i^1 = X_i^1 R_i^1 X_i^1$ ,  $\overline{M}_i^1 = X_i^1 M_i^1 X_i^1$ ,  $\overline{N}_i^1 = X_i^1 N_i^1 X_i^1$ ,  $\overline{\Omega}_i^1 = X_i^1 \Omega_i^1 X_i^1$ , pre- and post-multiplying Equation (52) with  $L_i$ , then Equation (44) with  $w = 1$  can be obtained based on Schur complement, the inequality Equation (44) with  $w = 2$  can also be derived by following the same process, which imply that Equation (44) is equivalent to Equation (21) given in Theorem 1.

Through the similar derivation based on Schur complement, it is not difficult to confirm that Equations (45)–(48) are equivalent to Equations (22)–(25), respectively. We thus prove the theorem. ■

*Remark 7.* It is worth noting that the system stability performance can be directly reflected by decay rates  $t_i \in (0, 1)$ .<sup>16</sup> That is to say, the larger the  $t_{\min} = \min\{t_i | 1 \leq i \leq N\}$ , the better the stability performance. Thus, to achieve better stability performance, the related parameters should be set to get larger  $t_i$  (consequently with larger  $t_{\min}$ ). From Equation (26), we can derive that:

$$t_i = \frac{\varphi_i}{2} = \frac{2\alpha_i^1 g_{\min} - 2(\alpha_i^1 + \alpha_i^2)h - 2\alpha_i^2 d_{\max} - \ln(\mu_i^1 \mu_i^2)}{2\zeta_i},$$

it is obviously that  $t_i$  is influenced by parameters  $h$ ,  $g_{\min}$ ,  $d_{\max}$ ,  $\alpha_i^1$ ,  $\alpha_i^2$ , and  $\mu_i^w$  ( $w = 1, 2$ ). By using the method of change one parameter, fixed the remaining parameters, we can find that the smaller the  $h/\mu_i^w/d_{\max}/\alpha_i^2$ , the larger the  $t_i$ ; and the greater the  $g_{\min}/\alpha_i^1$ , the larger the  $t_i$ . To be emphasized, for the sampling period  $h$ , it has been demonstrated that a smaller  $h$  leads to better stability performance, but it also increases the burden of sampling and trigger judgment. Thus, too small  $h$  should be to avoid being set.

*Remark 8.* Comparing with References 24,35,36, which studied event-triggered control scheme for networked control systems (NCSs), distributed consensus control issue for multi-agent systems (MASSs) and input-to-state stabilizing control problem for cyber-physical systems (CPSs) by taking DoS attacks into consideration, respectively, the focus of this article is to study decentralized synchronization control issue for event-based CNs under DoS attacks. Moreover, differing from<sup>24</sup> which considered single-channel DoS attacks scenario, this article studies multi-channel scenario with asynchronous DoS attacks given the feature of CNs; although the similar asynchronous DoS attacks scenarios were also considered in References 35,36, but this article further takes network resource constraints into account, and then constructs a decentralized event-triggered scheme under asynchronous DoS attack, which is untouched in References 35,36.

*Remark 9.* Event-triggered scheme that can avoid unnecessary data transmission has gained wide attention. In Reference 15, periodic event-triggered control for linear systems was studied, while a delay-based event-triggered controller was designed in Reference 18. Given the effectiveness of the proposed event-triggered schemes, extensive researches have been conducted based on them. Thus, in this study, we adopt the similar event-triggered condition, interval-partition and time-delay methods as<sup>18</sup> to model a decentralized event-triggered scheme. However, since that asynchronous non-periodic DoS attacks are considered in this article, the designed decentralized event-triggered scheme differs a lot from the mechanisms proposed in References 15,18, furthermore, the ultimate aim of this article is to realize synchronization control for CNs under limited network resources and DoS attacks, which also differs this article from References 15,18.

*Remark 10.* Lyapunov functional approach is a classic method which has been widely used to analyze the stability of control systems. Therefore, we use the approach and combining with LMI technique to analyze the stability of the formulated synchronization error system. Although Lyapunov functional approach was also adopted in References 15,18,24,35,36, but we propose different Lyapunov functions according to our analysis target, and thus different analyzing results are obtained.

#### 4 | NUMERICAL SIMULATION RESULTS

In this section, we conduct simulations to validate the efficiency of our study. Consider the following continuous CN system composed of three nodes (i.e.,  $N = 3$ ).

$$\dot{x}_i(t) = Ax_i(t) + v(x_i(t)) + \sigma \sum_{j=1}^N c_{ij} \Gamma x_j(t) + Bu_i(t), \quad 1 \leq i \leq N, \quad (53)$$

where,

$$x_i(t) = \begin{bmatrix} x_{i1}(t) \\ x_{i2}(t) \end{bmatrix}, A = \begin{bmatrix} -1.6 & 0.8 \\ 1.6 & -0.8 \end{bmatrix}, B = \begin{bmatrix} 0.8 \\ 5 \end{bmatrix}.$$

The coupled network matrix of three nodes in the system is assumed to be:

$$C = \begin{bmatrix} -0.2 & 0.1 & 0.1 \\ 0.1 & -0.2 & 0.1 \\ 0.1 & 0.1 & -0.2 \end{bmatrix}.$$

The nonlinear function  $v(x_i(t))$  is set as follows:

$$v(x_i(t)) = \begin{bmatrix} 0.1x_{i1}(t) - \tanh(0.1x_{i2}(t)) \\ 0.1x_{i2}(t) - \tanh(0.1x_{i1}(t)) \end{bmatrix},$$

and the upper bound matrix of  $v(\cdot)$  is:

$$\Lambda = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.$$

The coupling matrix  $\Gamma$  is set to be  $\text{diag}\{0.5, 0.5\}$ . By referring to References 37,44, we further let the sampling period  $h = 0.05$  s, the coupling strength  $\sigma = 0.8$ ,  $\alpha_1^1 = 0.15$ ,  $\alpha_2^1 = 0.1$ ,  $\alpha_1^2 = 0.55$ ,  $\alpha_2^2 = 0.15$ ,  $\alpha_1^3 = 0.25$ ,  $\alpha_2^3 = 0.05$ ,  $\mu_1^1 = 1.01$ ,  $\mu_2^1 = 1.01$ ,  $\mu_1^2 = 1.35$ ,  $\mu_2^2 = 1.21$ ,  $\mu_1^3 = 1.05$ ,  $\mu_2^3 = 1.01$ ,  $d_{\max} = 2$ ,  $g_{\min} = 2$ ,  $\rho_1 = 0.2$ ,  $\rho_2 = 0.2$ ,  $\rho_3 = 0.3$ ,  $r_i^w = 10$ ,  $s_i^w = 10$  ( $i = 1, 2, 3; w = 1, 2$ ). The state initial conditions are supposed to be:  $x_1^T(0) = [2.5 \quad -1.5]$ ,  $x_2^T(0) = [2 \quad -0.5]$ ,  $x_3^T(0) = [2.6 \quad -1.2]$ ,  $s^T(0) = [-0.5 \quad 0.5]$ .

Based on the above simulation settings, we solve LMIs presented in Theorem 2 by employing MATLAB, and then obtain the following feedback gains and triggering parameter matrices.

$$K_1 = [-0.1691 \quad -0.1597], \quad K_2 = [-0.1428 \quad -0.1571],$$

$$K_3 = [-0.1378 \quad -0.1692], \quad \Omega_1 = \begin{bmatrix} 0.1773 & 0.0370 \\ 0.0370 & 0.0801 \end{bmatrix},$$

$$\Omega_2 = \begin{bmatrix} 0.0976 & 0.0162 \\ 0.0162 & 0.0722 \end{bmatrix}, \quad \Omega_3 = \begin{bmatrix} 0.1165 & 0.0116 \\ 0.0116 & 0.0868 \end{bmatrix}.$$

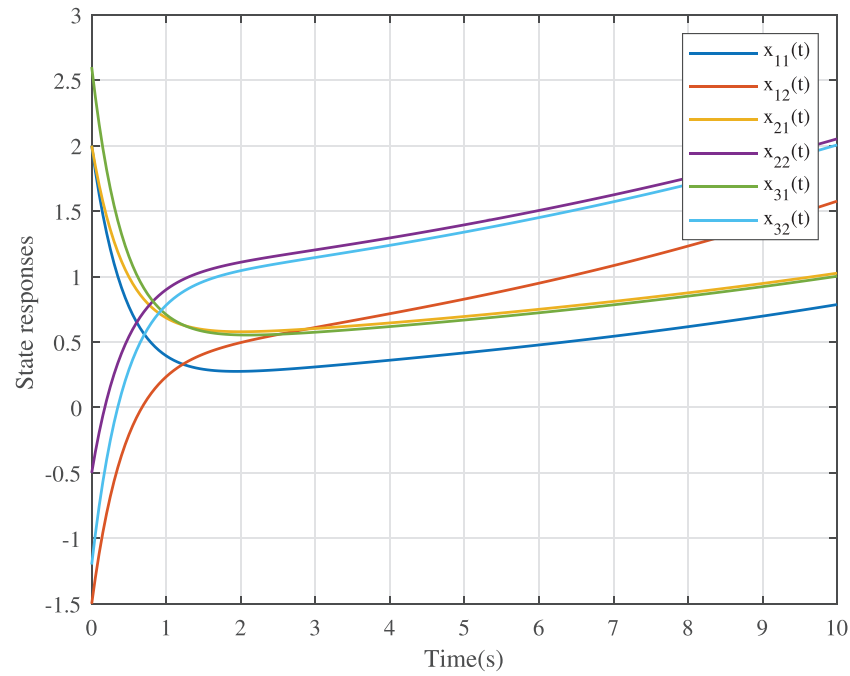


FIGURE 3 The state responses of  $x_i$  ( $i = 1, 2, 3$ ) without control input

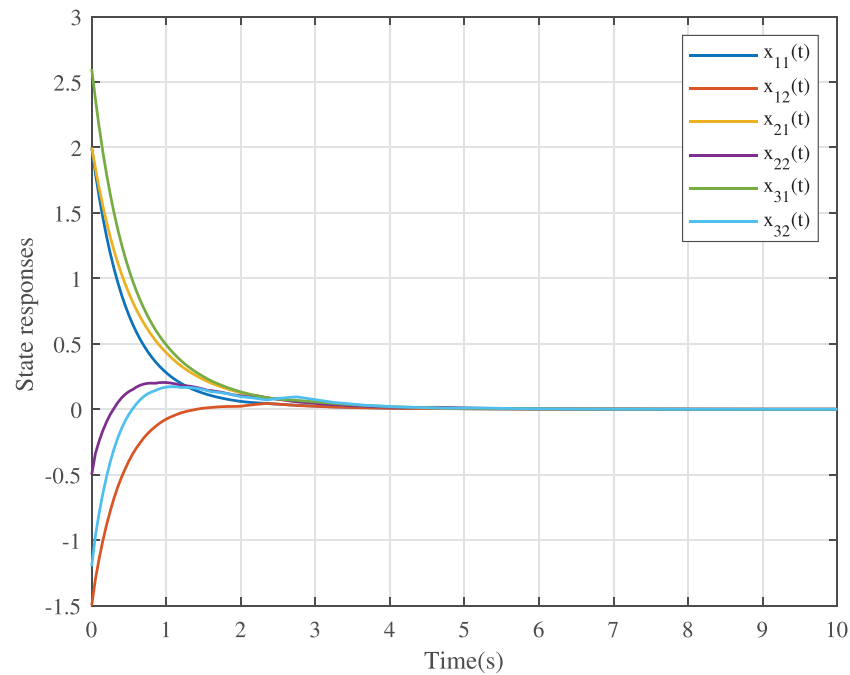


FIGURE 4 The state responses of  $x_i$  ( $i = 1, 2, 3$ ) under control input

According to the calculated results and given parameters, the simulation outcomes are specifically shown in Figures 3–9.

Figures 3 and 4 depict the state trajectories of the three simulated nodes in the CN with and without control input, respectively. From Figure 4, it can be found that the system state can become stable under the designed control strategy, which can not be achieved in Figure 3.

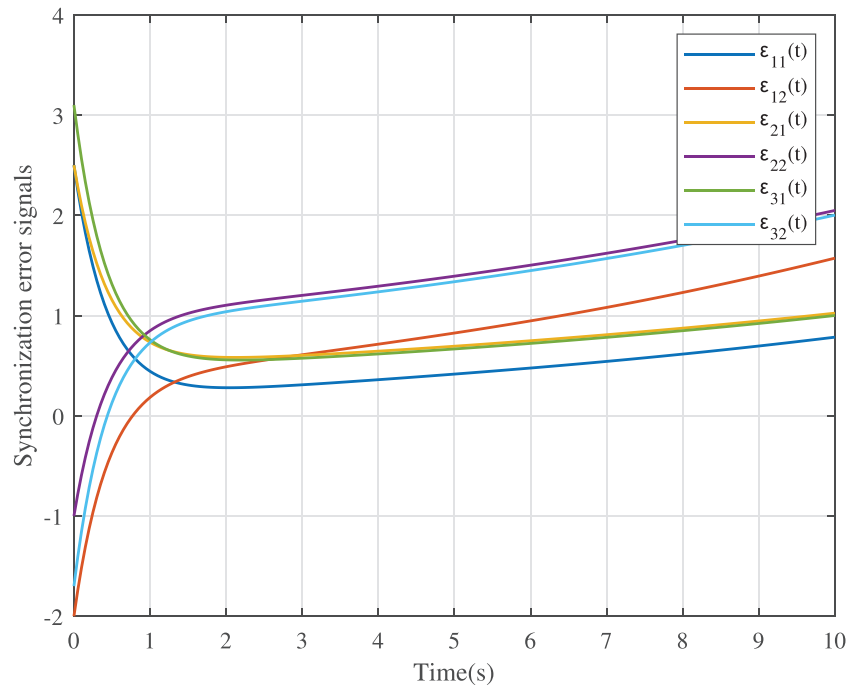


FIGURE 5 Synchronization errors  $\varepsilon_i$  ( $i = 1, 2, 3$ ) without control input

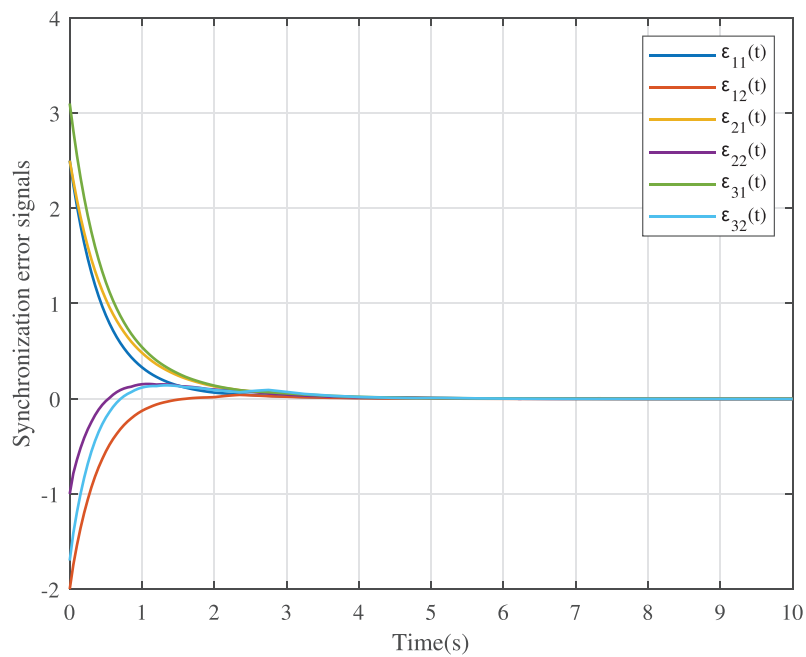


FIGURE 6 Synchronization errors  $\varepsilon_i$  ( $i = 1, 2, 3$ ) under control input  $u_i(t)$

By considering the absence and presence of control input, the responses of the synchronization errors of the three nodes are presented by Figures 5 and 6, respectively. As shown in Figure 6, the synchronization errors tend to zero under the designed control strategy, which can not be realized in Figure 5.

Figure 7 describes the changes of jamming signals when DoS attacks nonperiodically occurred. Figure 8 depicts the trigger intervals and release moments of three event generators. Figure 9 shows the trajectories of corresponding control inputs against DoS attacks.



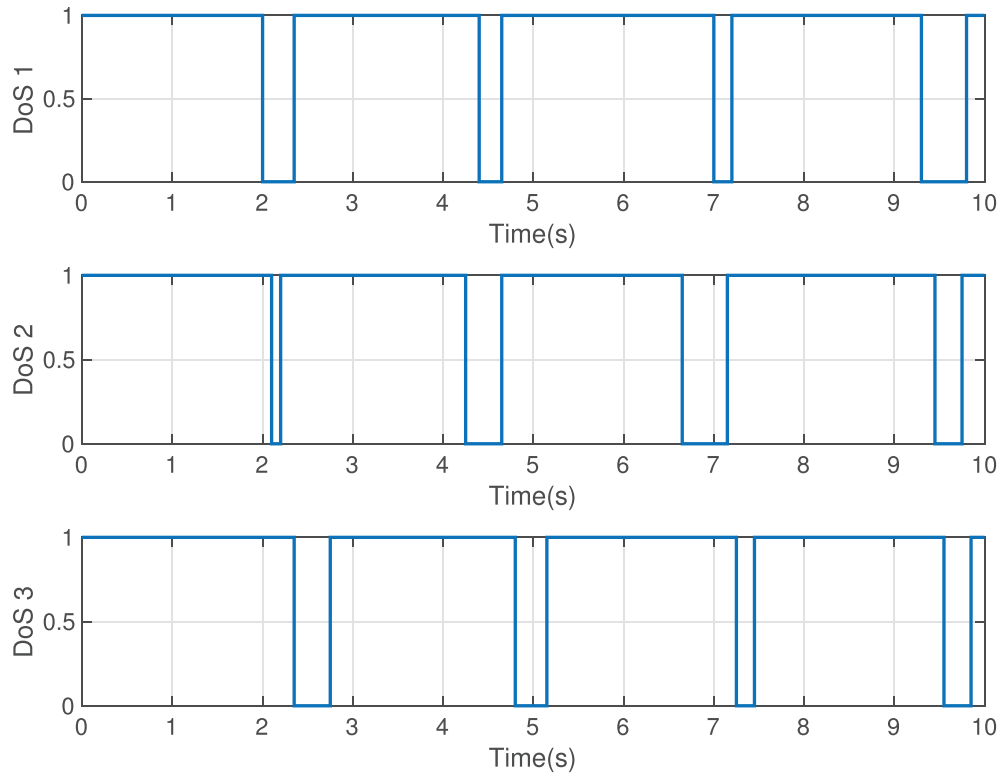


FIGURE 7 The DoS signals

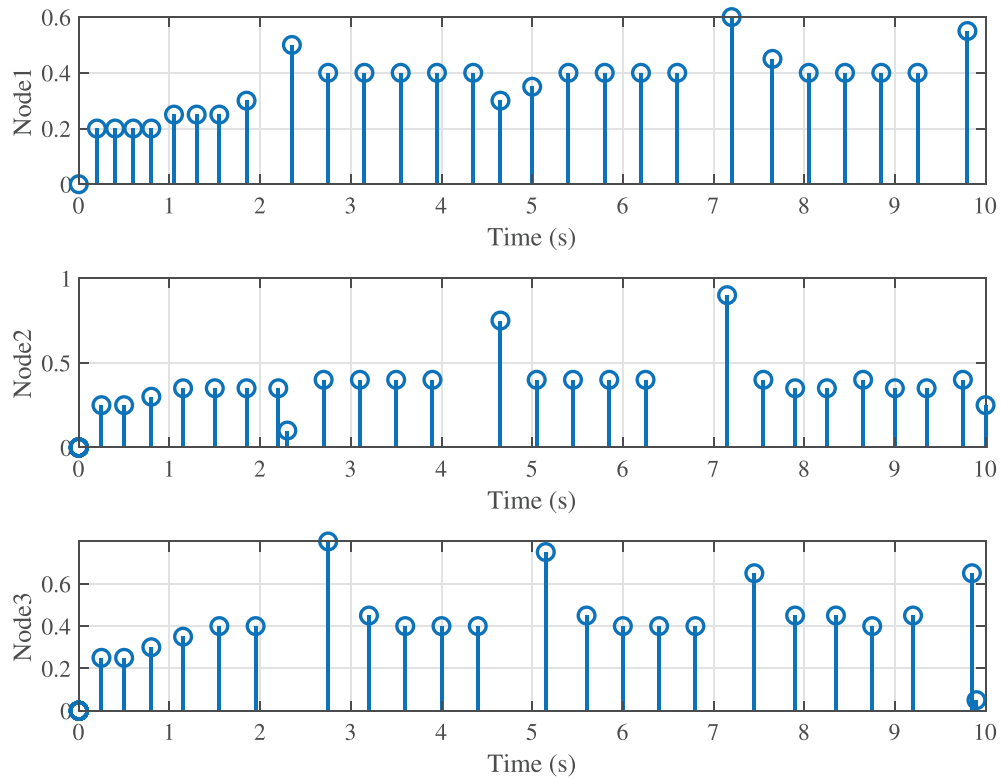


FIGURE 8 The event-triggered instants and intervals

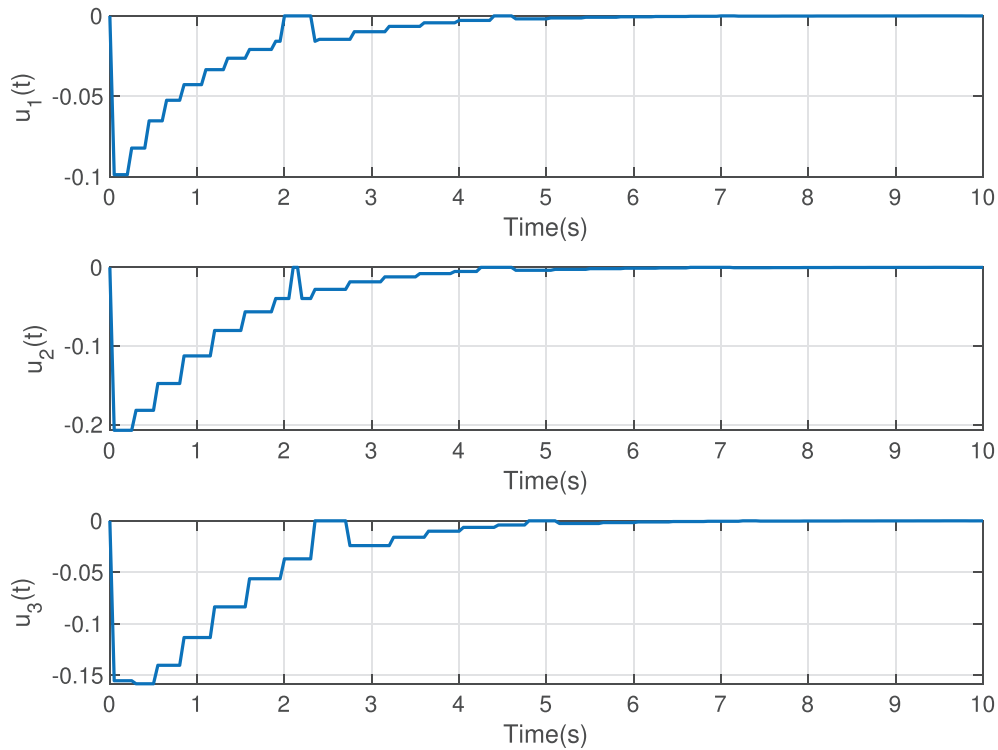


FIGURE 9 Control input  $u_i$  ( $i = 1, 2, 3$ ) under the DoS attacks

TABLE 1  $t_{min}$  for different values of  $h$

$h$	0.01	0.02	0.05	0.10	0.20
$t_{min}$	0.3915	0.3880	0.3775	0.3600	0.3250

TABLE 2  $t_{min}$  for different values of  $\mu_i^w$

$\mu_i^w$	0.85	0.90	1.01	1.25	1.50
$t_{min}$	0.4638	0.4352	0.3775	0.2709	0.1798

TABLE 3  $t_{min}$  for different values of  $d_{max}$

$d_{max}$	0.20	1	2	3	5
$t_{min}$	0.5125	0.4525	0.3775	0.3025	0.1525

To validate the results presented in Remark 7, we then show the variations of  $t_{min}$  (i.e., the system stability performance) with the changes of the related parameters, that is,  $h$ ,  $\mu_i^w$ ,  $d_{max}$ ,  $\alpha_i^2$ ,  $g_{min}$ , and  $\alpha_i^1$ , in Tables 1–6.

For this, we first set  $h = 0.05$ ,  $\mu_i^w = 1.01$  ( $i = 1, 2, 3; w = 1, 2$ ),  $d_{max} = 2$ ,  $\alpha_i^2 = 0.15$ ,  $g_{min} = 2$ , and  $\alpha_i^1 = 0.55$ . Then, we vary the value of one parameter while fix the values of the other parameters at a time. Based on the method, the impacts of  $h$ ,  $\mu_i^w$ ,  $d_{max}$ , and  $\alpha_i^2$  on  $t_{min}$  are presented in Tables 1–4, respectively, and we can see that the larger the  $h$  ( $\mu_i^w/d_{max}/\alpha_i^2$ ), the smaller the  $t_{min}$ . Meanwhile, the relationships between  $g_{min}$ ,  $\alpha_i^1$  and  $t_{min}$  are shown in Tables 5 and 6, respectively, and it can be observed that the larger the  $g_{min}$  ( $\alpha_i^1$ ), the greater the  $t_{min}$ .

TABLE 4  $t_{min}$  for different values of  $\alpha_i^2$ 

$\alpha_i^2$	0.01	0.10	0.15	0.30	0.50
$t_{min}$	0.5210	0.4288	0.3775	0.2238	0.0188

TABLE 5  $t_{min}$  for different values of  $g_{min}$ 

$g_{min}$	0.75	1	2	3	4
$t_{min}$	0.0338	0.1025	0.3775	0.6525	0.9275

TABLE 6  $t_{min}$  for different values of  $\alpha_i^1$ 

$\alpha_i^1$	0.20	0.30	0.55	0.80	1
$t_{min}$	0.0363	0.1338	0.3775	0.6213	0.8163

## 5 | CONCLUSION

In this article, taking both limited network resources and nonperiodic asynchronous DoS attacks into account, the decentralized secure synchronization control issue for CNs is studied. In order to effectively reduce network traffic load under nonperiodical asynchronous DoS attacks, an attack-tolerant decentralized event-triggered strategy is first proposed. A synchronization error model is then constructed to formulate the secure synchronization problem over CNs. Moreover, the sufficient conditions for stabilizing the system, the feedback gain matrices for controlling, and the parameters for triggering are derived by using Lyapunov stability theory and LMI technique. The numerical example finally demonstrates the availability of the proposed strategy. In future work, given the attractiveness of dynamic event-triggered schemes, we will investigate the secure synchronization control of CNs with a dynamic event-triggered mechanism. Apparently, this work lays a good foundation for the future study.

## ACKNOWLEDGMENTS

This work is supported in part by the National Natural Science Foundation of China under Grant 61973152; in part by the Natural Science Foundation of the Jiangsu Higher Education Institutions of China under Grant 19KJA510005; in part by the Natural Science Foundation of Jiangsu Province of China under Grants BK20211290, BK20202011, and BE2020001-3; and in part by the Qing Lan Project.

## CONFLICT OF INTEREST

The authors declared that there is no conflict of interests for this article.

## DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

## ORCID

Feiyu Song  <https://orcid.org/0000-0003-3050-7868>

Jinliang Liu  <https://orcid.org/0000-0001-5489-0246>

Xiangpeng Xie  <https://orcid.org/0000-0003-4822-3134>

Engang Tian  <https://orcid.org/0000-0002-8169-5347>

## REFERENCES

- Zhang J, Lyu M, Karimi HR, Guo P, Bo Y. Robust  $H_\infty$  filtering for a class of complex networks with stochastic packet dropouts and time delays. *Sci World J*. 2014;2014:1-11.
- Ren H, Karimi HR, Lu R, Wu Y. Synchronization of network systems via aperiodic sampled-data control with constant delay and application to unmanned ground vehicles. *IEEE Trans Ind Electron*. 2019;67(6):4980-4990.

3. Hongsri A, Botmart T, Weera W, Junsawang P. New delay-dependent synchronization criteria of complex dynamical networks with time-varying coupling delay based on sampled-data control via new integral inequality. *IEEE Access*. 2021;9:64958-64971.
4. Cheng J, Park JH, Karimi HR, Shen H. A flexible terminal approach to sampled-data exponentially synchronization of Markovian neural networks with time-varying delayed signals. *IEEE Trans Cybern*. 2017;48(8):2232-2244.
5. Wang Y, Karimi HR, Yan H. An adaptive event-triggered synchronization approach for chaotic Lur' systems subject to aperiodic sampled data. *IEEE Trans Circuits Syst II Express Briefs*. 2018;66(3):442-446.
6. Cheng P, He S, Luan X, Liu F. Finite-region asynchronous  $H_\infty$  control for 2D Markov jump systems. *Automatica*. 2021;129:109590.
7. Jiang W, Li L, Tu Z, Feng Y. Semiglobal finite-time synchronization of complex networks with stochastic disturbance via intermittent control. *Int J Robust Nonlinear Control*. 2019;29(8):2351-2363.
8. Zhou X, Li L, Zhao X. Pinning synchronization of delayed complex networks under self-triggered control. *J Franklin Inst*. 2020;358(2):1599-1618.
9. Xing M, Deng F, Zhao X. Synchronization of stochastic complex dynamical networks under self-triggered control. *Int J Robust Nonlinear Control*. 2017;27(16):2861-2878.
10. Liu M, Jiang H, Hu C, Yu Z, Li Z. Pinning synchronization of complex delayed dynamical networks via generalized intermittent adaptive control strategy. *Int J Robust Nonlinear Control*. 2020;30(1):421-442.
11. Wang M, Wang Z, Chen Y, Sheng W. Event-based adaptive neural tracking control for discrete-time stochastic nonlinear systems: a triggering threshold compensation strategy. *IEEE Trans Neural Netw Learn Syst*. 2020;31(6):1968-1981.
12. Liu J, Yin T, Yue D, Karimi HR, Cao J. Event-based secure leader-following consensus control for multiagent systems with multiple cyber attacks. *IEEE Trans Cybern*. 2021;51(1):162-173.
13. Tian E, Wang Z, Zou L, Yue D. Chance-constrained  $H_\infty$  control for a class of time-varying systems with stochastic nonlinearities: the finite-horizon case. *Automatica*. 2019;107:296-305.
14. Peng C, Han Q, Yue D. Communication-delay-distribution-dependent decentralized control for large-scale systems with IP-based communication networks. *IEEE Trans Control Syst Technol*. 2013;21(3):820-830.
15. Heemels W, Donkers MCFT, Teel AR. Periodic event-triggered control for linear systems. *IEEE Trans Automat Contr*. 2012;58(4):847-861.
16. Chen X, Wang Y, Hu S. Event-triggered quantized  $H_\infty$  control for networked control systems in the presence of denial-of-service jamming attacks. *Nonlinear Anal Hybrid Syst*. 2019;33:265-281.
17. Shi Y, Tian E, Shen S, Zhao X. Adaptive memory-event-triggered  $H_\infty$  control for network-based T-S fuzzy systems with asynchronous premise constraints. *IET Control Theory Appl*. 2021;15:534-544.
18. Yue D, Tian E, Han Q. A delay system method for designing event-triggered controllers of networked control systems. *IEEE Trans Automat Contr*. 2013;58(2):475-481.
19. Liu L, Zhang Y, Zhou W, Ren Y, Li X. Event-triggered approach for finite-time state estimation of delayed complex dynamical networks with random parameters. *Int J Robust Nonlinear Control*. 2020;30(14):5693-5711.
20. Li Q, Shen B, Wang Z, Alsaadi FE. Event-triggered  $H_\infty$  state estimation for state-saturated complex networks subject to quantization effects and distributed delays. *J Franklin Inst*. 2018;355(5):2874-2891.
21. Rahimi F, Rezaei H. An event-triggered recursive state estimation approach for time-varying nonlinear complex networks with quantization effects. *Neurocomputing*. 2020;426:104-113.
22. Wang Y, Fan Y, Wang Q, Zhang Y. Stabilization and synchronization of complex dynamical networks with different dynamics of nodes via decentralized controllers. *IEEE Trans Circuits Syst I Regul Pap*. 2012;59(8):1786-1795.
23. Kanakalakshmi S, Sakthivel R, Karthick S, Leelamani A, Parivallal A. Finite-time decentralized event-triggering non-fragile control for fuzzy neural networks with cyber-attack and energy constraints. *Eur J Control*. 2021;57:135-146.
24. Dolk VS, Tesi P, De Persis C, Heemels WPMH. Event-triggered control systems under denial-of-service attacks. *IEEE Trans Control Netw Syst*. 2016;4(1):93-105.
25. Feng J, Xie J, Wang J, Zhao Y. Secure synchronization of stochastic complex networks subject to deception attack with nonidentical nodes and internal disturbance. *Inf Sci*. 2021;547:514-525.
26. Liu J, Wang Y, Zha L, Xie X, Tian E. An event-triggered approach to security control for networked systems using hybrid attack model. *Int J Robust Nonlinear Control*. 2021;31(12):5796-5812.
27. Chen B, Ho DW, Hu G, Yu L. Secure fusion estimation for bandwidth constrained cyber-physical systems under replay attacks. *IEEE Trans Cybern*. 2017;48(6):1862-1876.
28. Su L, Ye D, Zhao X. Static output feedback secure control for cyber-physical systems based on multisensor scheme against replay attacks. *Int J Robust Nonlinear Control*. 2020;30(18):8313-8326.
29. Wang D, Wang Z, Shen B, Alsaadi FE. Security-guaranteed filtering for discrete-time stochastic delayed systems with randomly occurring sensor saturations and deception attacks. *Int J Robust Nonlinear Control*. 2017;27(7):1194-1208.
30. Wang K, Tian E, Liu J, Wei L, Yue D. Resilient control of networked control systems under deception attacks: a memory-event-triggered communication scheme. *Int J Robust Nonlinear Control*. 2020;30(4):1534-1548.
31. Hu S, Yue D, Xie X, Chen X, Yin X. Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks. *IEEE Trans Cybern*. 2018;49(12):4271-4281.
32. Zha L, Liu J, Cao J. Resilient event-triggered consensus control for nonlinear multi-agent systems with DoS attacks. *J Franklin Inst*. 2019;356(13):7071-7090.
33. Liu J, Yin T, Shen M, Xie X, Cao J. State estimation for cyber-physical systems with limited communication resources, sensor saturation and denial-of-service attacks. *ISA Trans*. 2018;104:101-114.

34. Yang F, Gu Z, Tian E, Yan S. Event-based switching control for networked switched systems under nonperiodic DoS jamming attacks. *IET Control Theory Appl.* 2020;14(19):3097-3106.
35. Lu A, Yang G. Distributed consensus control for multi-agent systems under denial-of-service. *Inf Sci.* 2018;439:95-107.
36. Lu A, Yang G. Input-to-state stabilizing control for cyber-physical systems with multiple transmission channels under denial of service. *IEEE Trans Automat Contr.* 2017;63(6):1813-1820.
37. Pan R, Tan Y, Du D, Fei S. Adaptive event-triggered synchronization control for complex networks with quantization and cyber-attacks. *Neurocomputing.* 2020;382:249-258.
38. Zhan T, Ma S, Liu X. Synchronization of singular switched complex networks via impulsive control with all nonsynchronized subnetworks. *Int J Robust Nonlinear Control.* 2019;29(14):4872-4887.
39. He S, Lyu W, Liu F. Robust  $H_\infty$  sliding mode controller design of a class of time-delayed discrete conic-type nonlinear systems. *IEEE Trans Syst Man Cybern Syst.* 2021;51(2):885-892.
40. Nie R, He S, Liu F, Luan X. Sliding mode controller design for conic-type nonlinear semi-Markovian jumping systems of time-delayed Chua's circuit. *IEEE Trans Syst Man Cybern Syst.* 2021;51(4):2467-2475.
41. Carl G, Kesidis G, Brooks R, Rai S. Denial-of-service attack-detection techniques. *IEEE Internet Comput.* 2006;10(1):82-89.
42. Peng C, Sun H. Switching-like event-triggered control for networked control systems under malicious denial of service attacks. *IEEE Trans Automat Contr.* 2020;65(9):3943-3949.
43. Hu S, Yue D, Chen X, Cheng Z, Xie X. Resilient  $H_\infty$  filtering for event-triggered networked systems under nonperiodic DoS jamming attacks. *IEEE Trans Syst Man Cybern Syst.* 2021;51(3):1392-1403.
44. Hu S, Yue D, Han Q, Xie X, Chen X, Dou C. Observer-based event-triggered control for networked linear systems subject to denial-of-service attacks. *IEEE Trans Cybern.* 2019;50(5):1952-1964.
45. Liu J, Wang Y, Cao J, Yue D, Xie X. Secure adaptive-event-triggered filter design with input constraint and hybrid cyber-attack. *IEEE Trans Cybern.* 2021;51(8):4000-4010.
46. Liu J, Suo W, Zha L, Tian E, Xie X. Security distributed state estimation for nonlinear networked systems against DoS attacks. *Int J Robust Nonlinear Control.* 2020;30(3):1156-1180.

**How to cite this article:** Li Y, Song F, Liu J, Xie X, Tian E. Decentralized event-triggered synchronization control for complex networks with nonperiodic DoS attacks. *Int J Robust Nonlinear Control.* 2022;32(3):1633-1653. doi: 10.1002/rnc.5899