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Finite-time adaptive event-triggered asynchronous state estimation for Markov jump systems with cyber-attacks

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Abstract
This article considers the issue of finite-time H_∞ asynchronous state estimation for event-triggered nonlinear Markovian jump systems subject to cyber-attacks. An adaptive event-triggered scheme is introduced to cope with the capacity constraint of the networked resources. It is assumed that the transmitted sensor measurements may experience randomly malicious cyber-attacks. Considering the effect of the adaptive event-triggered scheme and the occurrence of cyber-attacks, we establish a new state estimation error system model. Sufficient conditions of the finite-time boundedness and the H_∞ finite-time boundedness are developed for the augmented system. Then, the design methods of the asynchronous estimator gains are derived, which can ensure the H_∞ finite-time boundedness of the estimation error system. Finally, a numerical example is given to show the effectiveness of the theoretical results.

KEYWORDS
asynchronous state estimation, cyber-attacks, event-trigger, finite-time, Markov jump systems

1 | INTRODUCTION

Markovian jump systems (MJSs) have attracted much attention in the past few decades, owing to their well descriptions for the practical systems, such as truck-trailer systems and mass-spring system, which may be subject to unpredictable variations in their structures, possibly caused by sudden environment disturbance, random failures of the components and so on.[1-4] Close attention has been paid to the filter design and stability analysis for MJSs and many important results have been appeared.[5-8] The authors in Reference 5 studied the problem of stabilization and H_∞ performance of event-triggered semi-Markovian systems with actuator saturation. In Reference 6, the authors have investigated the reliable filtering for the nonlinear continuous-time Markov jump system. However, most of the existing publications are concerned with the asymptotic stability of the MJSs in the infinite time interval. In practice, the industrial production requires to achieve the desired performance in a finite time.[9] Therefore, widespread attentions of researchers has been devoted to the finite time performance of the systems, including finite-time control[10] and finite-time state estimation problems.[11,12] For instance, the finite-time event-driven control problem was addressed in Reference 10 for networked switched system with cyber-attacks. The event-triggered filter design was concerned in Reference 11 for networked systems suffering deception attacks. However, most of the existing results focus on the synchronous state estimation or control problem. The finite-time asynchronous state estimation for MJSs is scarce, which motivates this article.

In recent years, networked control systems (NCSs) have been a hot research topic due to the advantages of low cost, flexible architectures, convenient system diagnosis, and maintenance.[13-17] It should be noted that because of the characteristics of sharing communication network and limited network bandwidth, there exist some unavoidable phenomena in NCSs such as network-induced delays, data packet dropout and so on, which motivate the researchers to design specific data transmission schemes for various NCSs to optimize the network resources. As important data transmission strategies, event-triggered schemes have been proved to be effective in decreasing the number of transmission of communication network while maintaining the system performances.[18] Under the event-triggered scheme, sensor measurements transmitted or not are according to the prescribed event-triggered condition.[19-21] Only when the sampled measurements exceeds the event-triggered condition, they can be sent into the network, which increases the utilization ratio of the network recourses significantly. Up to now, various event-triggered schemes have been proposed for multi-agent systems,[4] nonlinear interconnected control systems[22] and neural networks,[23] and so forth. However, the available related event-triggered methods for MJSs are not fully investigated, which deserves further investigation.

In addition, NCSs also face the challenges of malicious attacks which aim to prevent the system components from accessing precise control signals and sensor measurements, resulting in the control systems failing and leading to economic lost and calamitous consequence. Generally speaking, the cyber-attacks can be categorized into denial-of-service (DoS) attacks[24] and deception attacks.[25] DoS attacks attempt to block sensor/control data from accessing the communication channel and preventing them reaching their destinations. Deception attacks destroy the NCSs by tampering the control signals and sensor measurements during transmission,[26,27] which is more concealed. Nowadays, security issues of NCSs have become one of the major concerns and received more and more attention. A series of novel approaches have been proposed to decrease the influence of cyber-attacks.[28,29] Specifically, in Reference 28, the secure distributed optimal frequency regulation was addressed for power grid under cyber-attacks. The output consensus problem was investigated considering the occurrence of the random DoS attacks in Reference 29. However, there are few approaches reported to protect MJSs from cyber-attacks. Due to the fact that the networked systems are vulnerable to malicious cyber-attacks, it is needed to proposed an effective security method to eliminate the impacts of cyber-attacks against MJSs, which motivates the current study.

In this article, the finite-time non-fragile asynchronous state estimator design issue is discussed for MJSs under AETS and cyber-attacks. The main contributions are summarized in the following. 1) The phenomena of network resource constraint and cyber-attacks are taken into account. In order to reduce the amount of the networked data transmissions, an AETS is introduced. The transmitted sensor measurements via network channel are assumed to encounter cyber-attacks. 2) In view of the effect of the AETS and cyber-attacks, an estimation error system model is presented by taking the asynchronous modes between the system and the estimator into account. 3) Sufficient conditions are given, respectively, under which the estimation error system is FTB and H_∞ FTB. In addition, the design approach of the secure asynchronous state estimator gains are developed.

2 PRELIMINARIES

Consider the following continuous-time nonlinear MJS

$$\begin{cases} \dot{x}(t) = A_{r_t}x(t) + A_{\omega r_t}\omega(t) + \alpha(t)H_{r_t}h(x(t)) \\ y(t) = C_{r_t}x(t) \\ z(t) = L_{r_t}x(t) \end{cases} \tag{1}$$

where $x(t) \in R^n$, $y(t) \in R^m$, and $z(t) \in R^p$ are the system state, the measurement output, and the signal to be estimated, respectively. $\omega(t) \in R^q$ is the disturbance input which belongs to $\mathcal{L}_2[0,\infty)$ and subjects to

$$\int_0^{+\infty} \omega^T(t)\omega(t)dt \leq \overline{\omega}^2 \tag{2}$$

$h(x(t))$ is a nonlinear function of the state. The stochastic variable $\alpha(t) \in \{0,1\}$ and satisfies $\Pr\{\alpha(t)=1\} = \overline{\alpha}$, $\Pr\{\alpha(t)=0\} = 1-\overline{\alpha}$. The matrices $A_{r_t}, A_{\omega r_t}, C_{r_t}, H_{r_t}$, and L_{r_t} are the system matrices with appropriate dimensions. $r_t \in \mathbb{S}\,(t \geq 0, \mathbb{S} = \{1,2,\dots,r\})$ is used to denote the continuous-time Markov jump process. r_t satisfies the following transition probability matrix $\Pi = (\pi_{ij})_{r\times r}$ described by

$$\Pr\{r_{t+\Delta t} = j | r_t = i\} = \begin{cases} \pi_{ij}\Delta t + o(\Delta t), & i \neq j \\ 1 + \pi_{ij}\Delta t + o(\Delta t), & i = j \end{cases}$$

where $\Delta t > 0$, $\lim_{h\to 0}\frac{o(\Delta t)}{\Delta t} = 0$, $\pi_{ij} \geq 0$, for $i,j \in \mathbb{S}$, π_{ij} represents the transition rate from mode i to mode j at instant $t+\Delta t$ if $j \neq i$. $\pi_{ii} = -\sum_{j=1,j\neq i}^{r}\pi_{ij}$.

Assumption 1. The nonlinear function $h(x(t))$ is assumed to satisfy the following condition:

$$\|h(x(t)) - h(\hat{x}(t))\| \leq \|G(x(t)-\hat{x}(t))\| \tag{3}$$

where $x(t) \in R^n$, G is a known matrix with compatible dimensions.

The main objective of this article is to design an asynchronous non-fragile state estimator to estimate the state of system (1). In most of the existing state estimation method, the measurement output $y(t)$ is assumed to be transmitted to the state estimator via an ideal channel, which is unreality obviously. In this article, we assume there exist cyber-attacks on the constrained network transmission channel. Thus, the input of the state estimator $\tilde{y}(t)$ is not equal to the measurement output $y(t)$.

Remark 1. It has been demonstrated that the state estimator can not be implemented exactly.[30,31] Due to the fact that uncertainties are unavoidable because of the unexpected errors or complex environment, it is necessary to design non-fragile state estimator which is insensitive to fluctuations of its gains.

In this article, the following nonfragile H_∞ state estimator will be designed:

$$\begin{cases} \dot{\hat{x}}(t) = A_{r_t}\hat{x}(t) + \overline{\alpha}H_{r_t}h(\hat{x}(t)) + (K_{\sigma_t} + \Delta K_{\sigma_t})[\hat{y}(t) - \tilde{y}(t)] \\ \hat{y}(t) = C_{r_t}\hat{x}(t) \\ \hat{z}(t) = L_{r_t}\hat{x}(t) \end{cases} \tag{4}$$

where $\hat{x}(t) \in R^n$ and $\hat{z}(t) \in R^p$ are the estimates of $x(t)$ and $z(t)$, respectively; K_{σ_t} is the state estimator gain to be estimated. ΔK_{σ_t} is the disturbance of the state estimator gain and $\Delta K_{\sigma_t} = F_{\sigma_t}\Delta_{\sigma_t}R_{\sigma_t}$. $\sigma_t \in \mathbb{L}\ (t \geq 0, \mathbb{L} = \{1,2,\dots,l\})$ satisfies the following probability matrix $\Lambda = (\lambda_{i\phi})_{r\times l}$ described by

$$\Pr\{\sigma_t = \phi | r_t = i\} = \lambda_{i\phi}$$

where $\lambda_{i\phi} \in [0,1]$ and $\sum_{\phi=1}^{l}\lambda_{i\phi} = 1$.

For convenience, in the sequel, r_t and σ_t will be replaced by $i \in \mathbb{S}$ and $\phi \in \mathbb{L}$, respectively.

In this article, we assume the sensor and the state estimator are connected by communication network which is vulnerable to cyber-attacks. An event-triggered device between the sensor and the state estimator is adopted to save the precious network resources and reduce the unnecessary transmission. If the following predefined condition is violated, the newly sampled data will be released into the network

$$e_k^T(t)\Omega_i e_k(t) \leq \rho_i(t) y^T\left(i_k^d h\right)\Omega_i y\left(i_k^d h\right) \tag{5}$$

where h is the sampling period, $e_k(t) = y(t_k h) - y(i_k^d h)$, $\Omega_i > 0$, $i_k^d h = t_k h + dh$, $d = 0,1,2,\dots,t_{k+1}-t_k$. $y(i_k^d h)$ is the current measured output, $y(t_k h)$ is the latest transmitted one. The threshold condition in (5) satisfies

$$\dot{\rho}_i(t) = \frac{1}{\rho_i(t)}\left(\frac{1}{\rho_i(t)} - \mu_i\right) e_k^T(t)\Omega_i e_k(t) \tag{6}$$

where $\mu_i \geq 1$, $\rho_i(0) \in (0, 1]$.

Under the effect of the event-trigger condition (5), the holding interval of the Zero order hold at the state estimator $\Pi = [t_k h, t_{k+1}h)$ can be divided into $\bigcup_{d=0}^{\overline{d}} \Pi_d$, $\overline{d} = t_{k+1} - t_k - 1$, $\Pi_d = [i_k^d h, i_k^d h + h)$.

Remark 2. The event-triggered condition (5) is adopted to reduce the unnecessary transmissions via communication network. Noted that $\rho_i(t)$ can be dynamically adjusted according to the state variation of the system and the desired system performance. When $\dot{\rho}_i(t) = 0$, $\rho_i(t)$ will become a constant, (5) will turn to the event-triggered condition in Reference 19.

Define $\tau(t) = t - i_k^d h$, it yields that $0 \leq \tau(t) < h$, the transmitted measurement output under the AETS (5) can be rewritten as

$$y(t_k h) = y(t - \tau(t)) + e_k(t) \tag{7}$$

It is assumed that the triggered measurement outputs $y(t_k h)$ will undergo malicious cyber-attacks, the input of the state estimator can be described as

$$\tilde{y}(t) = s(t_k h)y(t_k h) + (1 - s(t_k h))f(y(t_k h)) \tag{8}$$

where $f(y(t_k h))$ is the cyber-attacks with unknown but energy-bounded value. $s(t_k h)$ is a random variable taking values in {0,1} and $E\{s(t_k h)\} = \overline{s}$.

Remark 3. When $s(t_k h) = 1$, the triggered measurement output $y(t_k h)$ will be arrive at the filter successfully; when $s(t_k h) = 0$, the cyber-attacks are launched and the transmitted measurement output $y(t_k h)$ will be replaced by $f(y(t_k h))$.

Assumption 2. The cyber-attacks are assumed to satisfy the following condition:

$$y^T(t_k h)N^T N y(t_k h) - f^T(y(t_k h))f(y(t_k h)) \geq 0 \tag{9}$$

where N is a constant matrix.

Define $e(t) = x(t) - \hat{x}(t)$, $\tilde{z}(t) = z(t) - \hat{z}(t)$. Combine(1), (4), (7), and (8), we can obtain state estimation error system as:

$$\begin{aligned}
\dot{e}(t) = {} & (A_i + K_\phi + \Delta K_\phi)e(t) + A_{\omega i}\omega(t) + \overline{\alpha}H_i(h(x(t) - h(\hat{x}(t))) \\
& - (K_\phi + \Delta K_\phi)[C_i x(t) - \overline{s}(C_i x(t - \tau(t)) + e_k(t)) - (1 - \overline{s})f(y(t - \tau(t)))] \\
& + (\alpha(t) - \overline{\alpha})H_i h(x(t)) + (s(t_k h) - \overline{s})(K_\phi + \Delta K_\phi)\left[C_i x(t - \tau(t)) + e_k(t) - f(y(t - \tau(t)))\right]
\end{aligned} \tag{10}$$

$$\tilde{z}(t) = L_i e(t) \tag{11}$$

Setting $\xi(t) = \begin{bmatrix} x^T(t) & e^T(t) \end{bmatrix}^T$, from (1) and (10), we can calculate that

$$\begin{aligned}
\dot{\xi}(t) = {} & \bar{A}_{i\phi}\xi(t) + \overline{s}\bar{A}_{\tau_{i\phi}}\xi(t - \tau(t)) + \overline{s}\bar{A}_{e_\phi}e_k(t) + \bar{A}_{\omega_i}\omega(t) + (1 - \overline{s})\bar{A}_{e_\phi}f(y(t - \tau(t))) + \overline{\alpha}\overline{H}_i g(\xi(t)) \\
& + (\alpha(t) - \overline{\alpha})\hat{H}_i g(\xi(t)) - (\overline{s} - s(t_k h))[\bar{A}_{\tau_{i\phi}}\xi(t - \tau(t)) + \bar{A}_{e_\phi}e_k(t) - \bar{A}_{e_\phi}f(y(t_k h))]
\end{aligned} \tag{12}$$

where

$$\bar{A}_{i\phi} = (\tilde{A}_{i\phi} + \Delta A_\phi), \bar{A}_{\tau_{i\phi}} = \tilde{A}_{\tau_{i\phi}} + \Delta A_{\tau_{i\phi}}, \bar{A}_{e_\phi} = \tilde{A}_{e_\phi} + \Delta A_{e_\phi}$$

$$\tilde{A}_{i\phi} = \begin{bmatrix} A_i & 0 \\ -K_\phi C_i & A_i + K_\phi C_i \end{bmatrix}, \Delta A_{i\phi} = S_1 \Delta K_\phi C_i S_2$$

$$\tilde{A}_{\tau_{i\phi}} = \begin{bmatrix} 0 & 0 \\ K_\phi C_i & 0 \end{bmatrix}, \Delta A_{\tau_{i\phi}} = S_1 \Delta K_\phi C_i S_3, \tilde{A}_{e_\phi} = \begin{bmatrix} 0 \\ K_\phi \end{bmatrix}, \Delta A_{e_\phi} = S_1 \Delta K_\phi,$$

$$\bar{A}_{\omega_i} = \begin{bmatrix} A_{\omega_i} \\ A_{\omega_i} \end{bmatrix}, \overline{H}_i = \begin{bmatrix} H_i & 0 \\ 0 & H_i \end{bmatrix}, \hat{H}_i = \begin{bmatrix} H_i & 0 \\ H_i & 0 \end{bmatrix}, h(e(t)) = h(x(t) - h(\hat{x}(t))$$

$$S_1 = \begin{bmatrix} 0 \\ I \end{bmatrix}, S_2 = \begin{bmatrix} -I & I \end{bmatrix}, S_3 = \begin{bmatrix} I & 0 \end{bmatrix}, g(\xi(t)) = \begin{bmatrix} h(x(t)) \\ h(e(t)) \end{bmatrix}$$

Definition 1. (Finite-time bounded (FTB)) For given scalars $\varphi_2 > \varphi_1 > 0$ and $J_i > 0$, the system (12) is said to be FTB with respect to $(\varphi_1, \varphi_2, T, J_i, \overline{\omega})$, if

$$\sup_{-t_M \le t \le 0} \mathcal{E}\left\{\xi^T(t)J_i\xi(t), \dot{\xi}^T(t)J_i\dot{\xi}(t)\right\} \le \varphi_1^2 \Rightarrow \mathcal{E}\left\{\xi^T(t)J_i\xi(t)\right\} \le \varphi_2^2 \tag{13}$$

The objective of this article is to design a non-fragile asynchronous state estimator for system (1), such that system (11) and (12) are FTB and satisfy a H_∞ prescribed performance level. The following two requirements should be ensured:

(1) The system (12) is FTB with respect to $(\varphi_1, \varphi_2, T, J_i, \overline{\omega})$
(2) Under zero initial state, the following condition holds for $\omega(t) \in \mathcal{L}_2[0, \infty)$:

$$\mathcal{E}\left\{\int_0^T \tilde{z}^T(t)\tilde{z}(t)dt\right\} < \gamma^2 \mathcal{E}\left\{\int_0^T \omega^T(t)\omega(t)dt\right\} \tag{14}$$

3 | MAIN RESULTS

In this section, we will develop a non-fragile state estimator design method. First, the FTB and H_∞ performance analysis problem will be conducted. Then, the parameters of the non-fragile state estimator and the event generator will be designed.

Theorem 1. *For given positive scalars $\overline{\alpha}$, β, $\overline{s}$, h, μ_i, $\overline{\omega}$, φ_1, T, and matrices K_ϕ, F_ϕ, R_ϕ, $J_i > 0$, the augmented system (12) is FTB with $(\varphi_1, \varphi_2, T, J_i, \overline{\omega})$ if there exist positive scalars $\varrho_p (p = 1, 2, 3, 4, 5)$, $\lambda > 0$, φ_2 and matrices $Q_1 = diag\{Q_{11}, Q_{12}\} > 0$, $Q_2 = diag\{Q_{21}, Q_{22}\} > 0$, $P_i = diag\{P_{1i}, P_{2i}\} > 0$, $D_i > 0$, $\Omega_i > 0$, and U with appropriate dimensions satisfying $\begin{bmatrix} Q_2 & U \\ U & Q_2 \end{bmatrix} \ge 0$ such that for each $i \in \mathbb{S}$, $\phi \in \mathbb{L}$, the following inequalities hold*

$$\overline{\Xi}^{i\phi} = \begin{bmatrix} \overline{\Phi}_{11}^{i\phi} & * & * & * \\ h\overline{\Phi}_{21}^{i\phi} & -P_iQ_2^{-1}P_i & * & * \\ h\overline{\Phi}_{31}^{i\phi} & 0 & -P_iQ_2^{-1}P_i & * \\ \overline{\Phi}_{41}^{i\phi} & 0 & 0 & -I \end{bmatrix} < 0 \tag{15}$$

$$\varrho_1 J_i < P_i < \varrho_2 J_i, Q_1 < \varrho_3 J_i, Q_2 < \varrho_4 J_i, 0 < D_i < \varrho_5 I \tag{16}$$

$$\varrho_1^{-1}(\Delta\varphi_1^2 + \varrho_5\overline{\omega}^2)e^{\beta T} < \varphi_2^2, \Delta = \varrho_2 + \tau_M\varrho_3 + h^3\varrho_4 \tag{17}$$

where

$$\overline{\Phi}_{11}^{i\phi} = \begin{bmatrix} \Gamma_1^{i\phi} & * & * & * & * & * & * \\ \overline{s}\bar{A}_{\tau_{i\phi}}^T P_i + Q_2 - U^T & \Gamma_2^i & * & * & * & * & * \\ U^T & Q_2 - U^T & -e^{\beta h}Q_1 - Q_2 & * & * & * & * \\ \overline{s}\bar{A}_{e\phi}^T P_i & 0 & 0 & -\mu_i\Omega_i & * & * & * \\ \bar{A}_{\omega_i}^T P_i & 0 & 0 & 0 & -D_i & * & * \\ (1-\overline{s})\bar{A}_{e\phi}^T P_i & 0 & 0 & 0 & 0 & -I & * \\ \overline{\alpha}\overline{H}_i^T P_i & 0 & 0 & 0 & 0 & 0 & \Gamma_3^i \end{bmatrix}$$

$$\Gamma_1^{i\phi} = P_i\bar{A}_{i\phi} + \bar{A}_{i\phi}^T P_i + Q_1 - Q_2 - \beta P_i + \lambda\overline{G}^T\overline{G} + \sum_{j=1}^{r} \pi_{ij}P_j$$
$$\Gamma_2^i = -2Q_2 + U + U^T + S_3^T C_i^T \Omega_i C_i S_3, \Gamma_3^i = \delta_\alpha^2 \tau_M^2 \hat{H}_i^T Q_2 \hat{H}_i - \lambda I$$
$$\overline{\Phi}_{21}^{i\phi} = \begin{bmatrix} P_i\bar{A}_{i\phi} & \bar{s}P_i\bar{A}_{\tau_{i\phi}} & 0 & \bar{s}P_i\bar{A}_{e\phi} & P_i\bar{A}_{\omega_i} & (1-\bar{s})P_i\bar{A}_{e\phi} & \overline{\alpha}P_i\overline{H}_i \end{bmatrix}$$
$$\overline{\Phi}_{31}^{i\phi} = \begin{bmatrix} 0 & P_i\delta_s\bar{A}_{\tau_{i\phi}} & 0 & \delta_s P_i\bar{A}_{e\phi} & 0 & -\delta_s P_i\bar{A}_{e\phi} & 0 \end{bmatrix}$$
$$\overline{\Phi}_{41}^{i\phi} = \begin{bmatrix} 0 & NC_iS_3 & 0 & N & 0 & 0 & 0 \end{bmatrix}, \delta_\alpha^2 = \overline{\alpha}(1-\overline{\alpha}), \delta_s^2 = \bar{s}(1-\bar{s})$$

Proof. Choose the following Lyapunov functional:

$$V(\xi(t), i, t) = V_1(\xi(t), i, t) + V_2(\xi(t), i, t) + V_3(\xi(t), i, t) + V_4(\xi(t), i, t) \tag{18}$$

where

$$V_1(\xi(t), i, t) = \xi^T(t)P_i\xi(t)$$
$$V_2(\xi(t), i, t) = \int_{t-h}^{t} e^{\beta(t-s)}\xi^T(s)Q_1\xi(s)ds$$
$$V_3(\xi(t), i, t) = h\int_{-h}^{0}\int_{t+s}^{t} e^{\beta(t-s)}\dot{\xi}^T(v)Q_2\dot{\xi}(v)dvds$$
$$V_4(\xi(t), i, t) = \frac{1}{2}\rho_i^2(t)$$

■

Along system (12), it can be deduced that:

$$\mathcal{E}\{\dot{V}_1(\xi(t), i, t)\} = \sum_{\phi=1}^{l} \lambda_{i\phi}\left[\beta V_1(\xi(t), i, t) - \beta\xi^T(t)P_i\xi(t) + \sum_{j=1}^{r}\xi^T(t)\pi_{ij}P_j\xi(t) + 2\xi^T(t)P_i\dot{\xi}(t)\right] \tag{19}$$
$$\mathcal{E}\{\dot{V}_2(\xi(t), i, t)\} = \sum_{\phi=1}^{l} \lambda_{i\phi}\left[\beta V_2(\xi(t), i, t) + \xi^T(t)Q_1\xi(t) - e^{\beta h}\xi^T(t-h)Q_1x(t-h)\right] \tag{20}$$
$$\mathcal{E}\{\dot{V}_3(\xi(t), i, t)\} = \sum_{\phi=1}^{l} \lambda_{i\phi}\left[\beta V_3(\xi(t), i, t) + h^2\dot{\xi}^T(t)Q_2\dot{\xi}(t) - h\int_{t-h}^{t} e^{\beta h}\dot{\xi}^T(s)Q_2\dot{\xi}(s)ds\right] \tag{21}$$

Note that[22,32]

$$\dot{V}_4(\xi(t), t) \leq (C_ix(t-\tau(t)))^T\Omega_i(C_ix(t-\tau(t))) - \mu_i e_k^T(t)\Omega_i e_k(t) \tag{22}$$

By lemma 1 in Reference 33, for Q_2 and U satisfy

$$\begin{bmatrix} Q_2 & U \\ * & Q_2 \end{bmatrix} \geq 0$$

it follows that

$$-h\int_{t-h}^{t} e^{\beta h}\xi^{T}(s)Q_2\dot{\xi}(s)ds \leq \zeta^T(t)\Sigma\zeta(t) \tag{23}$$

in which

$$\Sigma = \begin{bmatrix} -Q_2 & Q_2 - U & U \\ * & -2Q_2 + U + U^T & Q_2 - U \\ * & * & -Q_2 \end{bmatrix}, \zeta(t) = \begin{bmatrix} \xi(t) \\ \xi(t-\tau(t)) \\ \xi(t-\tau_M) \end{bmatrix}$$

Note that

$$h^2\dot{\xi}^T(t)Q_2\dot{\xi}(t) = \sum_{\phi=1}^{l} \lambda_{i\phi}h^2 \left[\mathcal{A}_{i\phi}^T Q_2 \mathcal{A}_{i\phi} + h^2\delta_s^2 \mathcal{B}_{i\phi}^T Q_2 \mathcal{B}_{i\phi} + \delta_\alpha^2 h^2 g^T(\xi(t))\hat{H}_i^T Q_2 \hat{H}_i g(\xi(t))\right] \tag{24}$$

where

$$\begin{aligned}\mathcal{A}_{i\phi} &= \bar{A}_{i\phi}\xi(t) + \bar{s}\bar{A}_{\tau_{i\phi}}\xi(t-\tau(t)) + \bar{s}\bar{A}_{e_\phi}e_k(t) + \bar{A}_{\omega_i}\omega(t) \\ &\quad + (1-\bar{s})\bar{A}_{e_\phi}f(y(t-\tau(t))) + \overline{\alpha}\overline{H}_i g(\xi(t)) \\ \mathcal{B}_{i\phi} &= \bar{A}_{\tau_{i\phi}}\xi(t-\tau(t)) + \bar{A}_{e_\phi}e_k(t) - \bar{A}_{e_\phi}f(y(t-\tau(t)))\end{aligned}$$

One can easily get the following inequality from (3):

$$\lambda g^T(\xi(t))g(\xi(t)) - \lambda\xi^T(t)\overline{G}^T\overline{G}\xi(t) \le 0 \tag{25}$$

in which $\overline{G} = diag\{G, G\}$.

According to Assumption 2, we can get the cyber-attacks satisfy

$$[C_iS_3\xi(t-\tau(t)) + e_k(t)]^T N^T N[C_iS_3\xi(t-\tau(t)) + e_k(t)] - f^T(y(t-\tau(t)))f(y(t-\tau(t))) \ge 0 \tag{26}$$

Combing (19)–(26), for $t \in [0, T)$, we have

$$\begin{aligned}\mathcal{E}\dot{V}(\xi(t),t) \le \sum_{\phi=1}^{l} \lambda_{i\phi} \Bigg\{ & \beta V(\xi(t),t) + \sum_{j=1}^{r} \xi^T(t)\pi_{ij}P_j\xi(t) + 2\xi^T(t)P_i\mathcal{A}_{i\phi} + \xi^T(t)Q_1\xi(t) \\ & + h^2\mathcal{A}_{i\phi}^T Q_2\mathcal{A}_{i\phi} + h^2\delta_s^2\mathcal{B}_{i\phi}^T Q_2\mathcal{B}_{i\phi} + \delta_\alpha^2 h^2 g^T(\xi(t))\hat{H}_i^T Q_2\hat{H}_i g(\xi(t)) \\ & - e^{\beta h}\xi^T(t-h)Q_1 x(t-h) + \zeta^T(t)\Sigma\zeta(t) \\ & - \lambda g^T(\xi(t))g(\xi(t)) + \lambda\xi^T(t)\overline{G}^T\overline{G}\xi(t) - \xi(t)^T S_2^T G^T G S_2\xi(t) - \beta\xi^T(t)P_i\xi(t) \\ & + [C_iS_3\xi(t-\tau(t)) + e_k(t)]^T N^T N[C_iS_3\xi(t-\tau(t)) + e_k(t)] - f^T(y(t-\tau(t)))f(y(t-\tau(t))) \\ & + (C_i x(t-\tau(t)))^T\Omega_i(C_i x(t-\tau(t))) - \mu_i e_k^T(t)\Omega_i e_k(t) \Bigg\}\end{aligned} \tag{27}$$

Applying Schur complement lemma, from (15) and $0 < D_i < \varrho_5 I$ in Theorem 1, one can derive the following

$$\begin{aligned}\mathcal{E}\dot{V}(\xi(t),t) &< \beta V(\xi(t),t) + \omega^T(t)D_i\omega(t) \\ &< \beta V(\xi(t),t) + \varrho_5\omega^T(t)\omega(t)\end{aligned} \tag{28}$$

which implies

$$\mathcal{E}\left\{\frac{d}{dt}\left(e^{-\beta t}V(\xi(t),t)\right)\right\} < \varrho_5\omega^T(t)\omega(t) \tag{29}$$

Integrating both sides of (29) from 0 to T, we obtain

$$\int_0^T \mathcal{E}\left\{\frac{d}{dt}\left(e^{-\beta s}V(\xi(s),s)\right)\right\} ds < \varrho_5\int_0^T \omega^T(s)\omega(s)ds \tag{30}$$

Noted that $\beta > 0$, one can easily get from (30) that

$$e^{-\beta t}\mathcal{E}\{V(\xi(t),t)\} \le \mathcal{E}\{V(\xi(0),0)\} + \varrho_5\mathcal{E}\left\{\int_0^T \omega^T(s)\omega(s)ds\right\} \tag{31}$$

Recalling (2), (16), and (17), we have

$$\begin{aligned}\mathcal{E}\{\xi^T(t)P_i\xi(t)\} &< \mathcal{E}\{V(\xi(t),t))\}\\ &< e^{\beta T}\mathcal{E}\{V(\xi(0),0)\} + e^{\beta t}\varrho_5\overline{\omega}^2\\ &< e^{\beta T}(\Delta\varphi_1^2 + \varrho_5\overline{\omega}^2)\end{aligned} \tag{32}$$

In view of $\sigma_1 J_i < P_i$ in (16), we have

$$\begin{aligned}\mathcal{E}\{\xi^T(t)P_i\xi(t)\} &= \mathcal{E}\left\{\xi^T(t)J_i^{\frac{1}{2}}\left(J_i^{-\frac{1}{2}}P_iJ_i^{-\frac{1}{2}}\right)J_i^{\frac{1}{2}}\xi(t)\right\}\\ &> \varrho_1\mathcal{E}\left\{\xi^T(t)J_i\xi(t)\right\}\end{aligned} \tag{33}$$

Then, combine (32) and $e^{\beta T}(\Delta\varphi_1^2 + \varrho_5\overline{\omega}^2) < \varphi_2^2$ in (17), we derive

$$\mathcal{E}\{\xi^T(t)J_i\xi(t)\} < \varrho_1^{-1}e^{\beta T}(\Delta\varphi_1^2 + \varrho_5\overline{\omega}^2) < \varphi_2^2 \tag{34}$$

This completes the proof.

Theorem 2. *For given positive scalars $\overline{\alpha}$, β, $\overline{s}$, h, μ_i, $\overline{\omega}$, φ_1, T and matrices K_ϕ, F_ϕ, R_ϕ, $J_i > 0$, the augmented system (11) and (12) is H_∞ FTB with $(\varphi_1, \varphi_2, T, J_i, \gamma, \overline{\omega})$ if there exist positive scalars $\varrho_p (p = 1, 2, 3, 4, 5)$, λ, φ_2, γ and matrices $Q_1 = diag\{Q_{11}, Q_{12}\} > 0$, $Q_2 = diag\{Q_{21}, Q_{22}\} > 0$, $P_i = diag\{P_{1i}, P_{2i}\} > 0$, $\Omega_i > 0$, and U with appropriate dimensions satisfying $\begin{bmatrix} Q_2 & U \\ U & Q_2\end{bmatrix} \geq 0$ such that (16) and the following constraints hold for each $i \in \mathbb{S}$, $\phi \in \mathbb{L}$*

$$\begin{bmatrix} \Phi_{11}^{i\phi} & * & * & * & * \\ h\overline{\Phi}_{21}^{i\phi} & -P_iQ_2^{-1}P_i & * & * & * \\ h\overline{\Phi}_{31}^{i\phi} & 0 & -P_iQ_2^{-1}P_i & * & * \\ \overline{\Phi}_{41}^{i\phi} & 0 & 0 & -I & * \\ \overline{\Phi}_{51}^{i} & 0 & 0 & 0 & -I \end{bmatrix} < 0 \tag{35}$$

$$\varrho_1^{-1}(\Delta\varphi_1^2 + \gamma^2 e^{-\beta T}\overline{\omega}^2) < e^{-\beta T}\varphi_2^2 \tag{36}$$

where

$$\Phi_{11}^{i\phi} = \begin{bmatrix} \Gamma_1^{i\phi} & * & * & * & * & * & * \\ \overline{s}\bar{A}_{\tau_{i\phi}}^T P_i + Q_2 - U^T & \Gamma_2^{i\phi} & * & * & * & * & * \\ U^T & Q_2 - U^T & -e^{\beta h}Q_1 - Q_2 & * & * & * & * \\ \overline{s}\bar{A}_{e\phi}^T P_i & 0 & 0 & -\mu_i\Omega_i & * & * & * \\ \bar{A}_{\omega_i}^T P_i & 0 & 0 & 0 & -\gamma^2 e^{\beta T} I & * & * \\ (1-\overline{s})\bar{A}_{e\phi}^T P_i & 0 & 0 & 0 & 0 & -I & * \\ \overline{\alpha}\overline{H}_i^T P_i & 0 & 0 & 0 & 0 & 0 & \Gamma_3^i \end{bmatrix}$$

$$\overline{\Phi}_{51}^{i} = \begin{bmatrix} L_i & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Other symbols are defined in Theorem 1.

Proof. Letting $D_i = \gamma^2 e^{-\beta T}\overline{\omega}^2$, (15) can be rewritten as

$$\begin{bmatrix} \Phi_{11}^{i\phi} & * & * & * \\ h\overline{\Phi}_{21}^{i\phi} & -I & * & * \\ h\overline{\Phi}_{31}^{i\phi} & 0 & -P_iQ_2^{-1}P_i & * \\ \overline{\Phi}_{41}^{i\phi} & 0 & 0 & -P_iQ_2^{-1}P_i \end{bmatrix} < 0 \tag{37}$$

∎

The definition of Φ_{11}^{i} is given in Theorem 2. By Theorem 1, the FTB of the system (12) can be guaranteed by (16), (36), and (37) with respect to $(\varphi_1, \varphi_2, T, J_i, \overline{\omega})$.

By using Schur complement, from (28) and (35), we can derive

$$\mathcal{E}\dot{V}(\xi(t), t) < \beta V(\xi(t), t) + \gamma^2 e^{-\beta T}\omega^T(t)\omega(t) - \tilde{z}^T(t)\tilde{z}(t) \tag{38}$$

which can be represented as

$$\mathcal{E}e^{-\beta t}\dot{V}(\xi(t), t) < e^{-\beta t}[\gamma^2 e^{-\beta T}\omega^T(t)\omega(t) - \tilde{z}^T(t)\tilde{z}(t)] \tag{39}$$

Integrating both sides of (39) from 0 to T, under zero initial condition, one can get

$$\int_0^T e^{-\beta t}[\tilde{z}^T(t)\tilde{z}(t) - \gamma^2 e^{-\beta T}\omega^T(t)\omega(t)]dt < -\int_0^T \mathcal{E}e^{-\beta t}\dot{V}(\xi(t), t)dt \leq V(x(0)) = 0 \tag{40}$$

It yields that

$$\int_0^T \tilde{z}^T(t)\tilde{z}(t)dt < \int_0^T \gamma^2\omega^T(t)\omega(t)dt \tag{41}$$

The proof of the theorem is completed.

Remark 4. The main difficulty in deriving the main results is how to deal with the adopted AETS and the asynchronous modes information between the system and the state estimator. Inspired by References 22,35, we overcome these difficulties and derive the sufficient conditions under which the estimation error system is H_∞ FTB. Besides, the desired non-fragile asynchronous state estimator are designed which can be applicable event if the MJSs are subject to limited network resources and cyber-attacks.

Theorem 3. *For given positive scalars $\overline{\alpha}$, β, $\overline{s}$, h, μ_i, $\overline{\omega}$, φ_1, T, θ_i and matrices F_ϕ, R_ϕ, $J_i > 0$, the augmented system (11) and (12) is FTB with regard to $(\varphi_1, \varphi_2, T, J_i, \gamma, \overline{\omega})$, if there exist positive scalars ε_1, ε_2, ε_3, φ_2, and H_∞ disturbance attenuation level γ, and matrices $Q_1 = diag\{Q_{11}, Q_{12}\} > 0$, $Q_2 = diag\{Q_{21}, Q_{22}\} > 0$, $P_i = diag\{P_{1i}, P_{2i}\} > 0$, $\Omega_i > 0$, and U satisfying with appropriate dimensions $\begin{bmatrix} Q_2 & U \\ U & Q_2 \end{bmatrix} \geq 0$ such that (16), (36) and the following inequality hold for each $i \in \mathbb{S}$, $\phi \in \mathbb{L}$:*

$$\begin{bmatrix} \tilde{\Xi}^{i\phi} & * & * & * & * & * & * \\ M_{1i\phi}^T & -\varepsilon_1 I & * & * & * & * & * \\ M_{2i\phi} & 0 & -\varepsilon_1 I & * & * & * & * \\ M_{3i\phi}^T & 0 & 0 & -\varepsilon_2 I & * & * & * \\ M_{4i\phi} & 0 & 0 & 0 & -\varepsilon_2 I & * & * \\ M_{5i\phi}^T & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * \\ M_{6i\phi} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{42}$$

where

$$\tilde{\Xi}^{i\phi} = \begin{bmatrix} \tilde{\Phi}_{11}^{i\phi} & * & * & * & * \\ h\tilde{\Phi}_{21}^{i\phi} & \Theta_{4i} & * & * & * \\ h\tilde{\Phi}_{31}^{i\phi} & 0 & \Theta_{4i} & * & * \\ \tilde{\Phi}_{41}^{i\phi} & 0 & 0 & -I & * \\ \overline{\Phi}_{51}^{i\phi} & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\tilde{\Phi}_{11}^{i\phi} = \begin{bmatrix} \tilde{\Gamma}_1^{i\phi} & * & * & * & * & * & * \\ \bar{s}\Theta_{2i\phi} + Q_2 - U^T & \Gamma_2^i & * & * & * & * & * \\ U^T & Q_2 - U^T & -e^{\beta h}Q_1 - Q_2 & * & * & * & * \\ \bar{s}\Theta_{3i\phi} & 0 & 0 & -\mu_i\Omega_i & * & * & * \\ \bar{A}_{\omega_i}^T P_i & 0 & 0 & 0 & -\gamma^2 e^{\beta T} I & * & * \\ (1-\bar{s})\Theta_{3i\phi} & 0 & 0 & 0 & 0 & -I & * \\ \overline{\alpha}\overline{H}_i^T P_i & 0 & 0 & 0 & 0 & 0 & \Gamma_3^i \end{bmatrix}$$

$$\tilde{\Gamma}_1^{i\phi} = \Theta_{1i\phi} + \Theta_{1i\phi}^T + Q_1 - Q_2 - \beta P_i + \lambda \overline{G}^T \overline{G} + \sum_{j=1}^{r} \pi_{ij} P_j$$

$$\tilde{\Phi}_{21}^{i\phi} = \begin{bmatrix} \Theta_{1i\phi} & \bar{s}\Theta_{2i\phi} & 0 & \bar{s}\Theta_{3i\phi} & P_i\bar{A}_{\omega i} & (1-\bar{s})\Theta_{3i} & \overline{\alpha}P_i\overline{H}_i \end{bmatrix}$$

$$\tilde{\Phi}_{31}^{i\phi} = \begin{bmatrix} 0 & \delta_s\Theta_{2i\phi} & 0 & \delta_s\Theta_{3i\phi} & 0 & -\delta_s\Theta_{3i\phi} & 0 \end{bmatrix}$$

$$\Theta_{1i\phi} = \begin{bmatrix} P_{1i}A_i & 0 \\ -Y_{i\phi}C_i & P_{2i}A_i + Y_{i\phi}C_i \end{bmatrix}, \Theta_{2i\phi} = \begin{bmatrix} 0 & 0 \\ Y_{i\phi}C_i & 0 \end{bmatrix}$$

$$\Theta_{3i\phi} = \begin{bmatrix} 0 \\ Y_{i\phi} \end{bmatrix}, \Theta_{4i} = -2\theta_i P_i + \theta_i^2 Q_2$$

$$M_{1i\phi}^T = \begin{bmatrix} \epsilon_1(P_iS_1F_\phi)^T & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{2i\phi} = \begin{bmatrix} R_\phi C_i S_2 & \bar{s}R_\phi C_i S_3 & 0 & \bar{s}R_\phi & 0 & (1-\bar{s})R_\phi & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$M_{3i\phi}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \epsilon_2\tau_M(P_iS_1F_\phi)^T & 0 & 0 \end{bmatrix}$$

$$M_{5i\phi}^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \epsilon_3\tau_M\delta_s(P_iS_1F_\phi)^T \end{bmatrix}$$

$$M_{6i\phi} = \begin{bmatrix} 0 & R_\phi C_i S_3 & 0 & R_\phi & 0 & -R_\phi & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Other symbols are defined in Theorem 2. Moreover, the desired parameter of the state estimator is given by $K_\phi = P_{2i}^{-1}Y_{i\phi}$.

Proof. It can be verified that (35) can be rewritten as

$$\hat{\Xi}^{i\phi} + sym\{M_{1i\phi}\Delta(t)_i M_{2i\phi}\} + sym\{M_{3i\phi}\Delta_i(t)M_{2i\phi}\} + sym\{M_{5i\phi}\Delta_i(t)M_{6i\phi}\} < 0 \tag{43}$$

where

$$\hat{\Xi}^{i\phi} = \begin{bmatrix} \hat{\Phi}_{11}^{i} & * & * & * & * \\ h\hat{\Phi}_{21}^{i} & -P_iQ_2^{-1}P_i & * & * & * \\ h\hat{\Phi}_{31}^{i} & 0 & -P_iQ_2^{-1}P_i & * & * \\ \hat{\Phi}_{41}^{i} & 0 & 0 & -I & * \\ \overline{\Phi}_{51}^{i} & 0 & 0 & 0 & -I \end{bmatrix}$$

$$\hat{\Phi}_{11}^{i\phi} = \begin{bmatrix} \Gamma_1^{i\phi} & * & * & * & * & * & * \\ \bar{s}\tilde{A}_{\tau_{i\phi}}^T P_i + Q_2 - U^T & \Gamma_2^i & * & * & * & * & * \\ U^T & Q_2 - U^T & -e^{\beta h}Q_1 - Q_2 & * & * & * & * \\ \bar{s}\tilde{A}_{e\phi}^T P_i & 0 & 0 & -\mu_i\Omega_i & * & * & * \\ \bar{A}_{\omega_i}^T P_i & 0 & 0 & 0 & -\gamma^2 e^{\beta T} I & * & * \\ (1-\bar{s})\tilde{A}_{e\phi}^T P_i & 0 & 0 & 0 & 0 & -I & * \\ \overline{\alpha}\overline{H}_{i\phi}^T P_i & 0 & 0 & 0 & 0 & 0 & \Gamma_3^i \end{bmatrix}$$

$$\hat{\Gamma}_1^{i\phi} = P_i\tilde{A}_{i\phi} + \tilde{A}_{i\phi}^T P_i + Q_1 - Q_2 - \beta P_i + \lambda\overline{G}^T\overline{G} + \sum_{j=1}^{r} \pi_{ij}P_j$$

$$\hat{\Phi}_{21}^{i\phi} = \begin{bmatrix} P_i\tilde{A}_{i\phi} & \bar{s}P_i\tilde{A}_{\tau_{i\phi}} & 0 & \bar{s}P_i\tilde{A}_{e\phi} & P_i\bar{A}_{\omega_i} & (1-\bar{s})P_i\tilde{A}_{e\phi} & \overline{\alpha}P_i\overline{H}_i \end{bmatrix}$$

$$\hat{\Phi}_{31}^{i\phi} = \begin{bmatrix} 0 & \delta_s P_i\tilde{A}_{\tau_{i\phi}} & 0 & \delta_s P_i\tilde{A}_{e\phi} & 0 & -\delta_s P_i\tilde{A}_{e\phi} & 0 \end{bmatrix}$$

■

Applying lemma 2 in Reference 33, the following (44) can ensure (43) holds if there exist positive scalars $\varepsilon_i (i = 1, 2, 3)$ such that

$$\hat{\Xi}^{i\phi} + \varepsilon_1 M_{1i\phi}M_{1i\phi}^T + (\varepsilon_1^{-1} + \varepsilon_2^{-1})M_{2i\phi}^T M_{2i\phi} + \varepsilon_2 M_{3i\phi}M_{3i\phi}^T + \varepsilon_3 M_{5i\phi}M_{5i\phi}^T + \varepsilon_3^{-1}M_{6i\phi}^T M_{6i\phi} < 0 \tag{44}$$

By using Schur complement lemma, (44) is equivalent to

$$\begin{bmatrix} \hat{\Xi}^{i\phi} & * & * & * & * & * & * \\ M_{1i\phi}^T & -\varepsilon_1 I & * & * & * & * & * \\ M_{2i\phi} & 0 & -\varepsilon_1 I & * & * & * & * \\ M_{3i\phi}^T & 0 & 0 & -\varepsilon_2 I & * & * & * \\ M_{2i\phi} & 0 & 0 & 0 & -\varepsilon_2 I & * & * \\ M_{5i\phi}^T & 0 & 0 & 0 & 0 & -\varepsilon_3 I & * \\ M_{6i\phi} & 0 & 0 & 0 & 0 & 0 & -\varepsilon_3 I \end{bmatrix} < 0 \tag{45}$$

Since

$$(Q_2 - \theta_i P_i)Q_2^{-1}(Q_2 - \theta_i P_i) \geq 0 \tag{46}$$

we can easily get

$$-P_i Q_2^{-1} P_i \geq -2\theta_i P_i + \theta_i^2 Q_2 \tag{47}$$

Define $Y_{i\phi} = P_{2i}K_\phi$ and replace $-P_i Q_2^{-1} P_i$ with $-2\theta_i P_i + \theta_i^2 Q_2$, it is easy to derive that (45) can be guaranteed by (42). This completes the proof.

Remark 5. Although some asynchronous state estimation and control problem have been conducted in References 34-36, but the addressed issue in this article is different from the existing ones. In Reference 34, the asynchronous state estimation problem was investigated for Markovian jump neural networks with randomly occurring nonlinearities, parameter uncertainties, and sensor saturations. In Reference 35, the authors considered the stochastically passive asynchronous control for MJSs. The authors in Reference 36 proposed a novel asynchronous output feedback controller design method. However, the above mentioned references are based on the assumption that the network-based communication resources are not limited and the addressed systems work in safe environments, which is actually unrealistic. To be more realistic, in this article, with consideration of the rare network resources and the effect of the cyber-attacks, we present a finite-time adaptive event-triggered asynchronous state estimator design approach for MJSs.

Remark 6. It should be noted that advanced analytical technology is crucial in decreasing conservation degree of the state estimation. Superior methods to deal with the Integral term in (20) lead to less conservative. In this article, lemma 1 in Reference 27 is applied to deal with the Integral term in (20). Other methods, such as delay departioning method and Wirtinger-based integral inequality, also can be used to reduce the conservatism, But the complexity will be increased in the analysis and the derived conditions.

4 | NUMERICAL EXAMPLES

In this section, we present a simulation example to illustrate the effectiveness of the proposed state estimation method for the MJSs subject to cyber-attacks.

Consider the system (1) with the following parameters

$$A_1 = \begin{bmatrix} -3 & 1 \\ -0.9 & -1 \end{bmatrix}, A_2 = \begin{bmatrix} -2 & 1 \\ -1 & -2 \end{bmatrix}, A_3 = \begin{bmatrix} -1 & 0 \\ -1 & -2 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, C_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.8 \end{bmatrix}, C_3 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.8 \end{bmatrix}$$
$$L_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}, L_2 = \begin{bmatrix} 0.3 & 0.1 \\ -0.25 & 0.4 \end{bmatrix}, L_3 = \begin{bmatrix} 0.3 & 0.1 \\ -0.25 & 0.4 \end{bmatrix}$$
$$A_{w1} = \begin{bmatrix} 1 \\ -0.4 \end{bmatrix}, A_{w2} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix} A_{w3} = \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix}$$
$$H_1 = \begin{bmatrix} 0.3 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}, H_2 = \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.2 \end{bmatrix}, H_3 = \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.05 \end{bmatrix}$$

Assume that the uncertain matrix parameters and uncertainties are as follows

$$F_3 = F_2 = F_1 = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}, \quad R_3 = R_2 = R_1 = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}, \quad \Delta_1(t) = \Delta_2(t) = \sin(t)I$$

The nonlinear function and the cyber-attacks are

$$h_3(x(t)) = h_2(x(t)) = h_1(x(t)) = \begin{bmatrix} 0.3tanh(x_1(t)) \\ 0.2tanh(x_2(t)) \end{bmatrix}, \quad f(y(t_k h)) = \begin{bmatrix} 0.01tanh(x_1(t)) \\ 0.03tanh(x_2(t)) \end{bmatrix}$$

which satisfy the constraint (3) and (9) with

$$G = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad N = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.03 \end{bmatrix}$$

The initial conditions of system (1) and the state estimator (4) are chosen by $\hat{x}^T(t) = \begin{bmatrix} -0.2 & 0.6 \end{bmatrix}, x^T(t) = \begin{bmatrix} -0.2 & 0.8 \end{bmatrix}$. The disturbance input is assumed to be $\omega(t) = 0.01e^{-2t}$. The transition rate matrix $\Pi = \begin{bmatrix} -0.5 & 0.2 & 0.3 \\ 0.1 & -0.3 & 0.2 \\ 0.1 & 0.3 & -0.4 \end{bmatrix}$.

Set $J_1 = J_2 = J_3 = I$, $T = 3$, $\varphi_1 = 1$, $\overline{\omega} = 1$, $\beta = 1.2$, $\overline{\alpha} = 0.3$, $\overline{s} = 0.4$, $h = 0.05$, $\mu_1 = 2$, $\mu_2 = 4$, $\mu_3 = 8$, $\theta_1 = 0.1$, $\theta_2 = 0.2$, $\theta_3 = 0.3$. Applying the Matlab toolbox, by Theorem 3, we can derive $\varphi_2 = 5.9161$, $\gamma = 12.2378$, the desired estimator gains and the triggering matrices are

$$K_1 = \begin{bmatrix} 0.3329 & 0.0166 \\ 0.0267 & 0.1078 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.2371 & -0.0199 \\ -0.0134 & 0.2460 \end{bmatrix}, \quad K_3 = \begin{bmatrix} 0.0373 & 0.0146 \\ 0.0067 & 0.2425 \end{bmatrix}$$

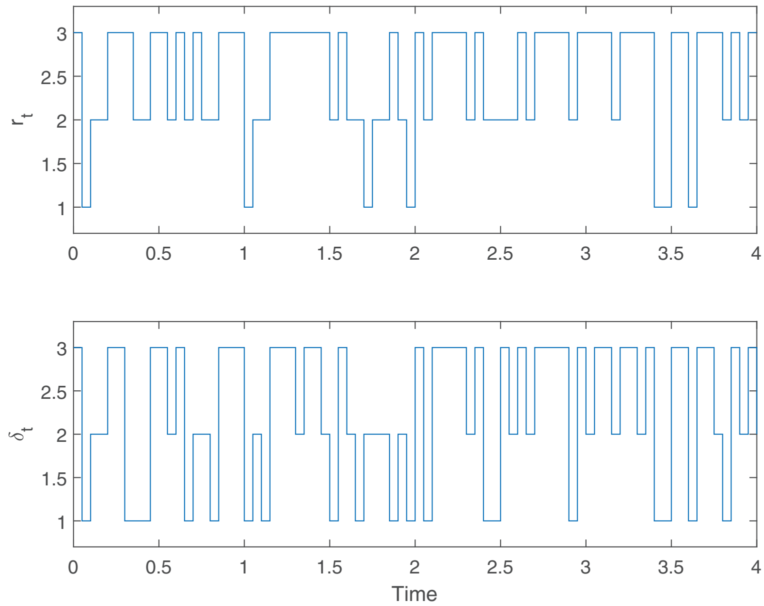

FIGURE 1 Modes of system and estimator

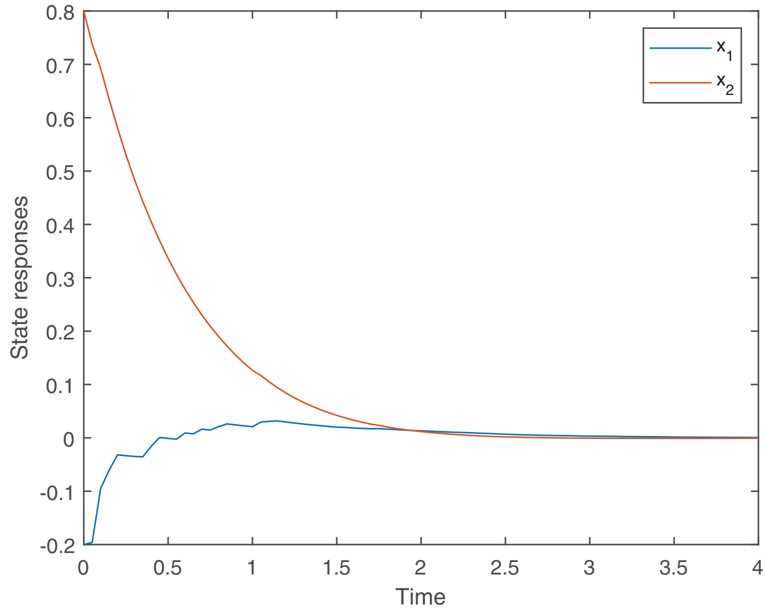

FIGURE 2 State response

$$\Omega_1 = \begin{bmatrix} 1.0793 & 0.0846 \\ 0.0846 & 1.4767 \end{bmatrix}, \quad \Omega_2 = \begin{bmatrix} 2.3207 & 0.0482 \\ 0.0482 & 2.7909 \end{bmatrix}, \quad \Omega_3 = \begin{bmatrix} 4.0975 & 0.0328 \\ 0.0328 & 2.3763 \end{bmatrix} \tag{48}$$

The modes of the system and estimator are given in Figure 1. Under the obtained triggering matrices and the estimator gains in (48), the state response is shown in Figure 2, from which one can see that the augmented system (11) and (12) is H_∞ FTB. Under the AETS, the transmitting instants of modes 1–3 are illustrated in Figure 3, 13% sampled instants are released by the event generator. The adaptive threshold law of modes 1–3 are described in Figure 5. It can be seen that $\rho_1(t)$, $\rho_2(t)$, and $\rho_3(t)$ are dynamically adjusted and finally converge to 0.8836, 0.5553, and 0.1496, respectively. Under the AETS, $\rho_1(t)$, $\rho_2(t)$, and $\rho_3(t)$ can be adjusted with the variation of the state.

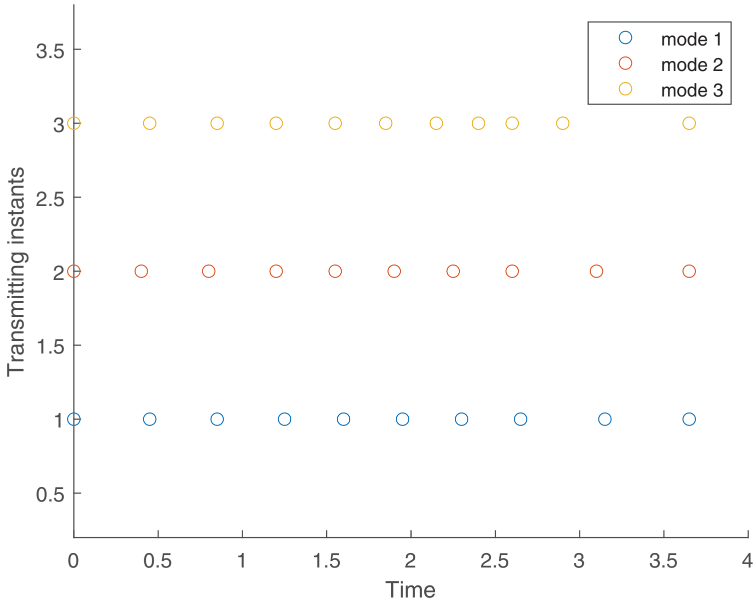

FIGURE 3 Transmitting instants under the AETS

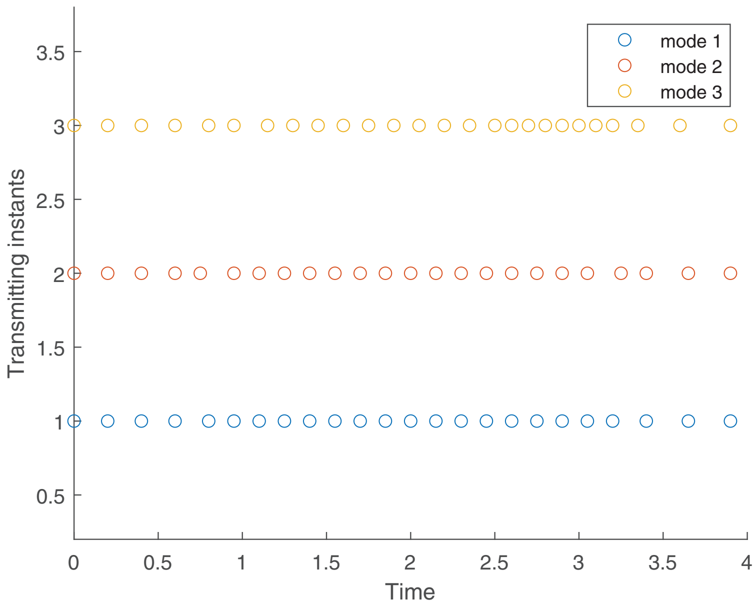

FIGURE 4 Transmitting instants under the periodic event-triggered scheme

Let $\rho_1(t) = \rho_2(t) = 0.01$, the AETS in this article will be reduced to be the periodic event-triggered scheme in Reference 19, the transmitting instants are plotted in Figure 4, 30% sampled instants are released by the event generator. Compared Figure 3 with Figure 4, one can see that the AETS is superiors than the periodic event-triggered scheme.

Based on the simulation results above, it can be verified that the event-triggered state estimator design method can not only save the communication resources, but also can ensure the H_∞ FTB of the augmented systems (11) and (12) subject to cyber-attacks.

Remark 7. It is evident that when the state response becomes close to the steady-state, $\rho_i(t)$ will become constants. The lager of $\rho_i(t)$, the less amount of the transmitted sampled packets, which also have been shown by the simulated results Figures 3–5.

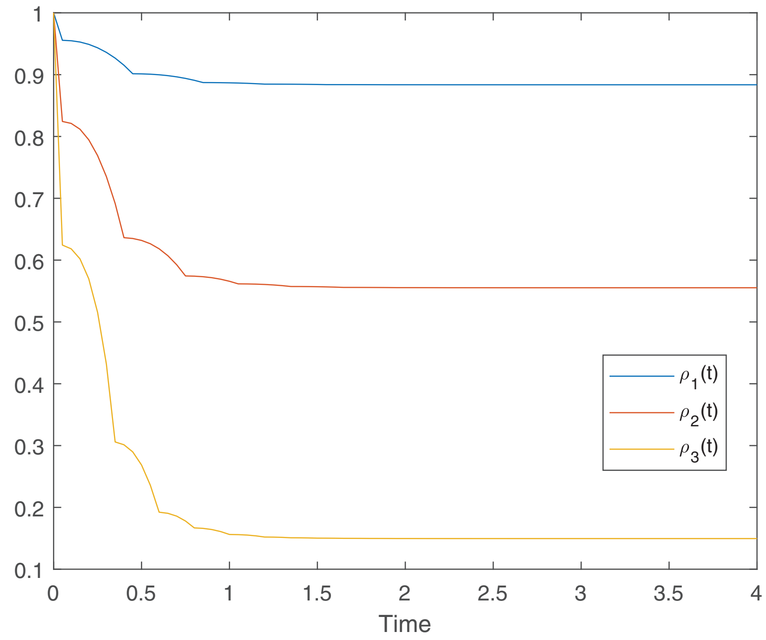

FIGURE 5 Variations of $\rho_1(t)$, $\rho_2(t)$ and $\rho_3(t)$ in modes 1–3

5 | CONCLUSIONS

In this article, the finite-time state estimator is designed for event-trigged asynchronous MJSs with stochastic cyber-attacks. First, an AETS is adopted to improve the efficiency of network resource utilization. Considering the random occurring cyber-attacks, the augmented estimation error system model is constructed. Sufficient conditions are derived, which can, respectively, ensue the estimation error system FTB and H_∞ FTB. The design method of the estimator parameters are gained by solving a set of linear matrix inequalities. Finally, a numerical example is provided to illustrate the effectiveness of the proposed method. In the future, we will investigate the event-triggered control strategy and state estimation problem for asynchronous MJSs, considering the effects of the multiple-attacks and sensor saturation.

CONFLICT OF INTEREST

The authors declare that there is no conflict of interests for this article.

DATA AVAILABILITY STATEMENT

Data sharing not applicable to this article as no datasets were generated or analysed during the current study

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