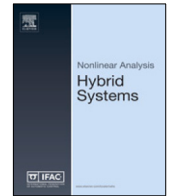




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# Nonlinear Analysis: Hybrid Systems

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## Secure state estimation for complex networks with multi-channel oriented round robin protocol

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### ABSTRACT

This paper focuses on the secure state estimation problem for complex networks (CNs) which are compromised by deception attack and constrained with limited communication resource. Firstly, a multi-channel oriented round robin (RR) protocol is proposed to schedule the data transferred over the communication network consisting of multiple transmission channels. The extended RR protocol can not only avoid the data collision caused by limited communication resource, but also fully utilize the sliced network bandwidth. Then, a state estimation error model is constructed by further considering the influence of deception attack. Following the model, efficient state estimators are designed based on analyzing the sufficient conditions that assuring the stability of the formulated state estimation error system. Finally, numerical results are presented to validate the theoretical outcomes.

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## 1. Introduction

Nowadays, complex networks (CNs) are being viewed as a desirable structure to depict many real-world systems, such as urban traffic networks, social networks, food webs and power grids etc [1–4]. In general, a specific CN is constituted by numerous nodes with dynamic behaviors and multiple links each of which indicates the certain relationship between a pair of nodes. Since the ubiquity of CNs, great interests have been stimulated in studying some important research issues concerning the characteristics of CNs, e.g., synchronization, optimization and state estimation. The latter problem has been gaining special attentions because it is demanding to acquire accurate system states but which can be hardly achieved with available measurement outputs. In the existing studies, lots of strategies on state estimation for diverse CNs have been reported [5–10]. To specifically mention a few, the authors in [5] proposed an information fusion approach to implement state estimation over discrete-time CNs with time-delays; towards discrete-time nonlinear singularly perturbed CNs, a new state estimation scheme was designed to fix the discrepancies caused by the two time scales in [6]; focusing on fractional-order stochastic CNs affected by cyber attacks, an adaptive event-triggered nonfragile state estimation method was reported in [10].

As well known, the signal transmission within each node (i.e., subsystem) of modern CNs is widely conducted through shared communication networks with the prevalence of networked control systems. The introduced wired or wireless

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networks can connect geographically distributed system components like sensors, estimators and actuators, so as to cost-efficiently and flexibly enable remote management and control. However, communication networks also induce some critical challenges, especially the data collision caused by limited network bandwidth [11] and the security issue incurred by cyber attacks [12]. Therefore, the state estimation for CNs should take the two issues into serious consideration.

For avoiding potential data collision, an effective method is to use data scheduling protocol to arrange the transmission order of each node rationally. The commonly available protocols include try-once-discard (TOD) protocol [13,14], stochastic communication (SC) protocol [15,16] and round robin (RR) protocol [17–19]. Among which, RR protocol schedules competitors in a periodic manner, i.e., each node will take turns to access the communication network to transfer signals. Given the easy implementation and the fairness of data transmission, RR protocol has been extensively adopted in many applications. In the literature, lots of scholars have been attracted to investigate RR-based state estimation for CNs [20–22], but the corresponding works are mainly based on single-channel communication and then assume that only one node can transmit data at each time instant. With the development of networking technology, e.g., wavelength division multiplexing (WDM) and orthogonal frequency division multiplexing (OFDM), the network bandwidth is generally divided into multiple transmission channels so as to realize flexible and fine-grained bandwidth sharing, and then multiple nodes can be admitted to access the communication network at each time instant. Under multi-channel communication model, traditional RR protocol is not applicable and then promoted to be extended into multi-channel scenario for achieving efficient channel utilization. Therefore, an unexplored issue of state estimation for CNs with multi-channel oriented RR protocol is raised, which greatly motivates our study.

Focusing on security problem of CNs, many types of cyber attacks have been investigated, such as denial of services (DoS) attack [23–25], replay attack [26,27] and deception attack. From the requirement of data integrity, deception attack is particularly concerned due to that the corresponding attacker always tries to falsify original data transferred over the network. For example, the authors in [28] exploited the partial-nodes-based state estimation problem for distributed-delayed CNs with stochastic disturbances and deception attack; recursive filters were designed for CNs influenced by state saturations and deception attack in [29,30]; a secure synchronization problem for a class of nonlinearly coupled CNs with deception attack was studied in [31]. Although fruitful results on performance guarantee for CNs with deception attack have been presented, but under the above mentioned multi-channel oriented RR protocol, the state estimation over deception attack-influenced CNs needs to be further investigated.

Inspired by the aforementioned discussion, we will dedicate to design secure state estimators for CNs under deception attack and multi-channel oriented RR protocol in this paper. The major contributions of the work can be summarized as follows:

- For CNs with multiple transmission channels, an extended RR protocol is proposed for scheduling competitive nodes so as to efficiently mitigate data collision as well as utilize the finite communication bandwidth.
- By taking account of the deception attack driven by a Bernoulli process and the presented multi-channel oriented RR protocol, a new state estimation model for CNs is constructed to formulate the studied problem.
- The sufficient conditions for assuring the stability of the defined state estimation errors are derived, then an algorithm is designed accordingly to enable secure state estimators.

The rest of the paper is arranged as follows. In Section 2, a discrete-time CN model is established based on the description of deception attack and multi-channel oriented RR protocol, which is followed by the formulation of the studied state estimation problem. The main results on the desired state estimators are given in Section 3. Numerical experiments are conducted in Section 4 to evaluate the efficiency of the work. The conclusion of the paper and some future research issues are presented in Section 5.

## 2. Problem statement

The diagram of the studied state estimation system is depicted in Fig. 1. As shown, a discrete-time CN with  $N$  coupled nodes is considered. Each node  $i$  ( $1 \leq i \leq N$ ) is specifically formulated as:

$$\begin{cases} x_i(t+1) = \ell(x_i(t)) + \sum_{j=1}^N u_{ij} Q x_j(t) + A_i v(t) + D_i \omega(t), \\ y_i(t) = B_i x_i(t) + C_i v(t), \end{cases} \quad (1)$$

where  $x_i(t) \in \mathbb{R}^{n_x}$  and  $y_i(t) \in \mathbb{R}^{n_y}$  are the state vector and measurement output of node  $i$ , respectively;  $v(t)$  is an external disturbance and supposed to be constrained by  $\|v(t)\| \leq \bar{v}$ ;  $\omega(t)$  is a zero-mean Gaussian white noise satisfying  $\mathbb{E}\{\omega^T(t)\omega(t)\} = \bar{\omega}$ ;  $\ell(\cdot)$  is a nonlinear vector-valued function;  $Q = \text{diag}\{q_1, q_2, \dots, q_{n_x}\} > 0$  is an inner coupling matrix;  $u_{ij} \geq 0$  ( $i \neq j$ ) denotes the relationship between node  $i$  and node  $j$ , i.e.,  $u_{ij} > 0$  if the two nodes are connected with each other, otherwise  $u_{ij} = 0$ , furthermore,  $u_{ii}$  is set to be  $-\sum_{j=1, j \neq i}^N u_{ij}$ .

**Assumption 1.** The nonlinear function  $\ell(\cdot)$  in the system (1) is continuous with  $\ell(0) = 0$ , and satisfies:

$$[\ell(x) - \ell(z)]^T [\ell(x) - \ell(z)] \leq (x - z)^T R^T R (x - z), \quad \forall x, z \in \mathbb{R}^n, \quad (2)$$

where  $R$  is a real matrix with compatible dimension.

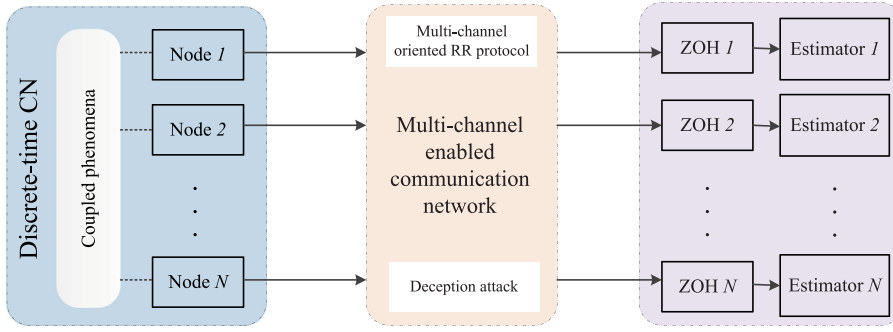


Fig. 1. Diagram of the envisioned state estimation system.

The introduced CN is supposed to be threatened by deception attack and constrained by limited communication bandwidth, then the system model will be updated with the depiction of the considered deception attack and the proposed multi-channel oriented RR protocol used for avoiding data collision in the bandwidth-limited communication network with multiple channels.

2.1. Deception attack and multi-channel oriented RR protocol

Given the randomness of cyber attacks, a Bernoulli variable  $\zeta_t \in \{0, 1\}$  with  $Pr\{\zeta_t = 1\} = \bar{\zeta}$  and  $Pr\{\zeta_t = 0\} = 1 - \bar{\zeta}$  ( $0 \leq \bar{\zeta} \leq 1$ ) is introduced to describe the behavior of the considered deception attack. To be specific,  $\zeta_t = 1$  indicates that the communication network operates normally at the time instant  $t$ , otherwise a malicious act is launched by the attacker. Then, the attack-influenced measurement signal of node  $i$  is formulated as:

$$\tilde{y}_i(t) = \zeta_t y_i(t) + (1 - \zeta_t) \tilde{h}(t), \tag{3}$$

where  $\tilde{h}(t)$  is the falsified data with  $\|\tilde{h}(t)\| \leq \tilde{h}$ .

**Remark 1.** By referring to the existed studies [14,32], a Bernoulli variable is used to depict the envisioned deception attack, and the probability of the attack can be evaluated via monitoring the communication network. Moreover, from the perspective of attacker, the energy bounded injected data  $\tilde{h}(t)$  is adopted since that high-energy deception signal is more likely to be detected.

For the multi-channel enabled communication network with limited bandwidth, we suppose that the network bandwidth is divided into  $M$  ( $M < N$ ) channels, so the signals of  $N$  nodes cannot be transmitted simultaneously at each time instant. Given the merits of RR protocol, we thereby dedicate to propose a multi-channel oriented RR protocol to avoid data collision as well as to realize efficient channel utilization under multi-channel communication scenario.

Under the proposed multi-channel oriented RR protocol,  $M$  nodes will be scheduled to deliver measurement signals over the communication network, i.e., each of the arranged nodes will be assigned a dedicated channel, at each time instant. Let  $o(t)$  be the set of the indexes of the selected nodes at the time instant  $t$  ( $t = 1, 2, \dots$ ), and it is defined as:

$$o(t) = \{ \text{mod}(M * (t - 1), N) + 1, \text{mod}(M * (t - 1) + 1, N) + 1, \dots, \text{mod}(M * (t - 1) + (M - 1), N) + 1 \}, \tag{4}$$

where  $\text{mod}(\cdot, \cdot)$  denotes modular operation, and it is apparently that  $o(t) \subseteq \{1, 2, \dots, N\}$  and  $|o(t)| = M$ . Based on the definition, it can be found that  $o(t)$  is a periodic sequence with the cycle of  $K$  time instants, where  $K = [M, N]/M$  and  $[M, N]$  represents the least common multiple of  $M$  and  $N$ .

The following example is presented to further demonstrate the proposed RR protocol. In the example, we assume that there is a CN with three nodes and the network bandwidth is sliced into two channels, i.e.,  $N = 3$  and  $M = 2$ , then it has  $K = 3$ . According to Eq. (4), we have:

$$\begin{aligned} o(1) &= \{1, 2\}, & o(2) &= \{3, 1\}, & o(3) &= \{2, 3\}, \\ o(4) &= \{1, 2\}, & o(5) &= \{3, 1\}, & o(6) &= \{2, 3\}, \\ & \dots & & & & \end{aligned}$$

which validates that  $o(t)$  is a periodic sequence with the cycle of  $K = 3$  time instants.

**Remark 2.** Based on the above illustration, it can be found that the proposed RR protocol gives all nodes fair opportunities in accessing the multi-channel enabled communication network, and thus keeps the advantages of traditional RR protocol. Furthermore, none of channels is idle at each time instant under such a protocol, which means that the limited network bandwidth is always fully utilized.

With the multi-channel oriented RR protocol, the measurement signal used by the state estimator of node  $i$  is:

$$\bar{y}_i(t) = \begin{cases} \tilde{y}_i(t), & \text{if } i \in o(t), \\ \tilde{y}_i(t-1), & \text{otherwise,} \end{cases} \quad (5)$$

where a zero-order holder (ZOH) is deployed with the state estimator as shown in Fig. 1. Then, by defining the following matrix:

$$\Phi_{\epsilon(t)} = \sum_{i \in o(t)} \phi_i, \quad (6)$$

in which  $\phi_i = \text{diag}\{\delta(i-1)I, \delta(i-2)I, \dots, \delta(i-N)I\}$ ,  $\delta(\cdot) \in \{0, 1\}$  is the Kronecker delta function,  $\epsilon(t) \in \{1, 2, \dots, K\}$  and  $\epsilon(t) = \text{mod}(t-1, K) + 1$ , we can get:

$$\bar{y}(t) = \Phi_{\epsilon(t)}\tilde{y}(t) + (I - \Phi_{\epsilon(t)})\tilde{y}(t-1), \quad (7)$$

where

$$\bar{y}(t) = [\bar{y}_1^T(t), \bar{y}_2^T(t), \dots, \bar{y}_N^T(t)]^T, \quad \tilde{y}(t) = [\tilde{y}_1^T(t), \tilde{y}_2^T(t), \dots, \tilde{y}_N^T(t)]^T.$$

Thus, the augmented model of the CN with the depicted deception attack and multi-channel oriented RR protocol can be formulated as:

$$\begin{cases} x(t+1) = \ell(x(t)) + (U \otimes Q)x(t) + Av(t) + D\omega(t), \\ \bar{y}(t) = \zeta_t \Phi_{\epsilon(t)} Bx(t) + \zeta_t \Phi_{\epsilon(t)} Cv(t) + (1 - \zeta_t)\Phi_{\epsilon(t)}\tilde{h}(t) + (I - \Phi_{\epsilon(t)})\tilde{y}(t-1), \end{cases} \quad (8)$$

where

$$\begin{aligned} A &= [A_1^T, A_2^T, \dots, A_N^T]^T, \quad B = \text{diag}\{B_1, B_2, \dots, B_N\}, \quad C = [C_1^T, C_2^T, \dots, C_N^T]^T, \\ D &= [D_1^T, D_2^T, \dots, D_N^T]^T, \quad x(t) = [x_1^T(t), x_2^T(t), \dots, x_N^T(t)]^T, \\ \ell(x(t)) &= [\ell^T(x_1(t)), \ell^T(x_2(t)), \dots, \ell^T(x_N(t))]^T, \quad U = [u_{ij}]_{N \times N}. \end{aligned}$$

## 2.2. Problem formulation

On the basis of the augmented system model (8), the following secure state estimators are then designed:

$$\hat{x}(t+1) = \ell(\hat{x}(t)) + (U \otimes Q)\hat{x}(t) + L_{\epsilon(t)}(\bar{y}(t) - B\hat{x}(t)), \quad (9)$$

where

$$\begin{aligned} \hat{x}(t) &= [\hat{x}_1^T(t), \hat{x}_2^T(t), \dots, \hat{x}_N^T(t)]^T, \quad \ell(\hat{x}(t)) = [\ell^T(\hat{x}_1(t)), \ell^T(\hat{x}_2(t)), \dots, \ell^T(\hat{x}_N(t))]^T, \\ L_{\epsilon(t)} &= \text{diag}\{L_{\epsilon(t),1}, L_{\epsilon(t),2}, \dots, L_{\epsilon(t),N}\}, \end{aligned}$$

$\hat{x}_i(t)$  is the estimation of  $x_i(t)$ , and  $L_{\epsilon(t),i}$  is the parameter of the state estimator for node  $i$  which will be determined shortly.

Letting  $e_i(t) = x_i(t) - \hat{x}_i(t)$ , then the following error dynamics of the state estimators can be obtained:

$$\begin{aligned} e(t+1) &= F(e(t)) + (U \otimes Q - L_{\epsilon(t)}B)e(t) + (A - \zeta_t L_{\epsilon(t)}\Phi_{\epsilon(t)}C)v(t) \\ &\quad + D\omega(t) + (L_{\epsilon(t)}B - \zeta_t L_{\epsilon(t)}\Phi_{\epsilon(t)}B)x(t) - (1 - \zeta_t) \\ &\quad \times L_{\epsilon(t)}\Phi_{\epsilon(t)}\tilde{h}(t) - L_{\epsilon(t)}(I - \Phi_{\epsilon(t)})\tilde{y}(t-1), \end{aligned} \quad (10)$$

where

$$\begin{aligned} F(e(t)) &= [\ell^T(e_1(t)), \ell^T(e_2(t)), \dots, \ell^T(e_N(t))]^T, \\ \ell(e_i(t)) &= \ell(x_i(t)) - \ell(\hat{x}_i(t)), \quad e(t) = [e_1^T(t), e_2^T(t), \dots, e_N^T(t)]^T. \end{aligned}$$

We further define  $\mathcal{E}(t) = [e^T(t), \bar{y}^T(t-1), x^T(t)]^T$ , and thus the augmented estimation error system below can be derived:

$$\mathcal{E}(t+1) = \bar{B}_{\epsilon(t)}\mathcal{E}(t) + \bar{F}(t) + \bar{A}_{\epsilon(t)}v(t) + \bar{H}_{\epsilon(t)}\tilde{h}(t) + W(t), \quad (11)$$

where

$$\begin{aligned} \bar{B}_{\epsilon(t)} &= \begin{bmatrix} U \otimes Q - L_{\epsilon(t)}B & -L_{\epsilon(t)}(I - \Phi_{\epsilon(t)}) & \mathcal{N}_1 \\ 0 & I - \Phi_{\epsilon(t)} & \zeta_t \Phi_{\epsilon(t)}B \\ 0 & 0 & U \otimes Q \end{bmatrix}, \quad \bar{F}(t) = \begin{bmatrix} F(e(t)) \\ 0 \\ \ell(x(t)) \end{bmatrix}, \\ \bar{A}_{\epsilon(t)} &= \begin{bmatrix} A - \zeta_t L_{\epsilon(t)}\Phi_{\epsilon(t)}C \\ \zeta_t \Phi_{\epsilon(t)}C \\ A \end{bmatrix}, \quad \bar{H}_{\epsilon(t)} = \begin{bmatrix} (\zeta_t - 1)L_{\epsilon(t)}\Phi_{\epsilon(t)} \\ (1 - \zeta_t)\Phi_{\epsilon(t)} \\ 0 \end{bmatrix}, \\ \mathcal{N}_1 &= L_{\epsilon(t)}B - \zeta_t L_{\epsilon(t)}\Phi_{\epsilon(t)}B, \quad W(t) = D[\omega^T(t), \omega^T(t), \omega^T(t)]^T. \end{aligned}$$

Based on the above formulation, the objective of this paper is to design appropriate state estimators to guarantee the stability of the system (11). At the end of this section, the definition and lemma below are given to assist the presentation of the main results.

**Definition 1** ([33]). The system (11) is said to be exponentially ultimately bounded in mean square if there exist constants  $\alpha \in [0, 1)$ ,  $\beta > 0$ , and  $\kappa > 0$ , such that:

$$\mathbb{E}\{\|\mathcal{E}(t)\|^2 \mid \mathcal{E}(0)\} \leq \alpha^t \beta + \kappa. \tag{12}$$

**Lemma 1** ([34]). For any matrices  $Z > 0$  and  $X$ , the following matrix inequality will hold:

$$-XZ^{-1}X^T \leq Z - X - X^T. \tag{13}$$

### 3. Main results

In this section, the sufficient conditions that assuring the ultimate boundedness of the system (11) in mean square sense are derived firstly. Then, the design method for the secure state estimators is introduced.

**Theorem 1.** Given estimator gain matrices  $L_r$  ( $r = 1, 2, \dots, K$ ), positive scalars  $\mu_1, \mu_2, \rho$  and  $\bar{\zeta}$ , the system (11) is exponentially ultimately bounded in mean square if there exist positive definite matrices  $\mathcal{P}_{r,s}$  ( $s = 1, 2, 3$ ) and matrix  $R$  with appropriate dimensions such that:

$$\hat{\mathcal{E}}_r = \begin{bmatrix} \hat{\Sigma}_1 & * & * \\ \hat{\Sigma}_2 & -\mathcal{P}_{r+1}^{-1} & * \\ \hat{\Sigma}_3 & 0 & -\mathcal{P}_{r+1}^{-1} \end{bmatrix} < 0, \tag{14}$$

where

$$\hat{\Sigma}_1 = \begin{bmatrix} -(1-\rho)\mathcal{P}_r + \bar{\mathcal{R}}^T \bar{\mathcal{R}} & * & * & * \\ 0 & -\mathbf{I} & * & * \\ 0 & 0 & -\mu_1 I & * \\ 0 & 0 & 0 & -\mu_2 I \end{bmatrix}, \bar{\mathcal{R}} = \begin{bmatrix} R & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & R \end{bmatrix},$$

$$\mathcal{P}_r = \text{diag}\{\mathcal{P}_{r,1}, \mathcal{P}_{r,2}, \mathcal{P}_{r,3}\}, \mathcal{P}_{K+1} = \mathcal{P}_1, \mathbf{I} = \text{diag}\{I_1, I, I_1\},$$

$$\hat{\Sigma}_2 = [\bar{B}_{r,1} \quad \mathbf{I} \quad \bar{H}_{r,1} \quad \bar{A}_{r,1}], \hat{\Sigma}_3 = [\vartheta \bar{B}_{r,2} \quad 0 \quad \vartheta \bar{H}_{r,2} \quad \vartheta \bar{A}_{r,2}],$$

$$\bar{B}_{r,1} = \begin{bmatrix} U \otimes Q - L_r B & -L_r(I - \Phi_r) & \mathcal{A}_2 \\ 0 & I - \Phi_r & \bar{\zeta} \Phi_r B \\ 0 & 0 & U \otimes Q \end{bmatrix}, \bar{B}_{r,2} = \begin{bmatrix} 0 & 0 & -L_r \Phi_r B \\ 0 & 0 & \Phi_r B \\ 0 & 0 & 0 \end{bmatrix},$$

$$\bar{A}_{r,1} = \begin{bmatrix} A - \bar{\zeta} L_r \Phi_r C \\ \bar{\zeta} \Phi_r C \\ A \end{bmatrix}, \bar{A}_{r,2} = \begin{bmatrix} -L_r \Phi_r C \\ \Phi_r C \\ 0 \end{bmatrix}, \bar{H}_{r,1} = \begin{bmatrix} (\bar{\zeta} - 1)L_r \Phi_r \\ (1 - \bar{\zeta})\Phi_r \\ 0 \end{bmatrix},$$

$$\bar{H}_{r,2} = \begin{bmatrix} L_r \Phi_r \\ -\Phi_r \\ 0 \end{bmatrix}, \vartheta = \sqrt{\bar{\zeta}(1 - \bar{\zeta})}, \mathcal{A}_2 = L_r B - \bar{\zeta} L_r \Phi_r B.$$

**Proof.** We firstly construct the following Lyapunov function:

$$V(t) = \mathcal{E}^T(t) \mathcal{P}_{\epsilon(t)} \mathcal{E}(t), \tag{15}$$

given that  $\mathbb{E}\{\zeta_t - \bar{\zeta}\} = 0$ ,  $\mathbb{E}\{(\zeta_t - \bar{\zeta})^2\} = \bar{\zeta}(1 - \bar{\zeta}) = \vartheta^2$ , then the expectation of the difference of  $V(t)$  can be computed as:

$$\begin{aligned} \mathbb{E}\{\Delta V(t)\} &= \mathbb{E}\{\mathcal{E}^T(t+1) \mathcal{P}_{\epsilon(t+1)} \mathcal{E}(t+1) - \mathcal{E}^T(t) \mathcal{P}_{\epsilon(t)} \mathcal{E}(t)\} \\ &= \mathbb{E}\{[\bar{B}_{\epsilon(t),1} \mathcal{E}(t) + \bar{F}(t) + \bar{H}_{\epsilon(t),1} \mathbf{h}(t) + \bar{A}_{\epsilon(t),1} v(t)]^T \mathcal{P}_{\epsilon(t+1)} \\ &\quad \times [\bar{B}_{\epsilon(t),1} \mathcal{E}(t) + \bar{F}(t) + \bar{H}_{\epsilon(t),1} \mathbf{h}(t) + \bar{A}_{\epsilon(t),1} v(t)] \\ &\quad + \vartheta^2 [\bar{B}_{\epsilon(t),2} \mathcal{E}(t) + \bar{H}_{\epsilon(t),2} \mathbf{h}(t) + \bar{A}_{\epsilon(t),2} v(t)]^T \mathcal{P}_{\epsilon(t+1)} \\ &\quad \times [\bar{B}_{\epsilon(t),2} \mathcal{E}(t) + \bar{H}_{\epsilon(t),2} \mathbf{h}(t) + \bar{A}_{\epsilon(t),2} v(t)] - \mathcal{E}^T(t) \mathcal{P}_{\epsilon(t)} \eta(t)\} \\ &\quad + \mathbb{E}\{W^T(t) \mathcal{P}_{\epsilon(t+1)} W(t)\} \\ &= \mathbb{E}\{\xi^T(t) \mathcal{E}_{\epsilon(t+1)} \xi(t) - \mathcal{E}^T(t) \mathcal{P}_{\epsilon(t)} \mathcal{E}(t)\} + \mathbb{E}\{W^T(t) \mathcal{P}_{\epsilon(t+1)} W(t)\}, \end{aligned} \tag{16}$$

where

$$\xi(t) = [\mathcal{E}(t) \quad \bar{F}(t) \quad \bar{h}(t) \quad v(t)], \quad \mathcal{E}_{\epsilon(t+1)} = \begin{bmatrix} \mathbf{0} & * & * \\ \Sigma_2 & -\mathcal{P}_{\epsilon(t)}^{-1} & * \\ \Sigma_3 & \mathbf{0} & -\mathcal{P}_{\epsilon(t)}^{-1} \end{bmatrix},$$

$$\Sigma_2 = [\bar{B}_{\epsilon(t),1} \quad \mathbf{I} \quad \bar{H}_{\epsilon(t),1} \quad \bar{A}_{\epsilon(t),1}], \quad \Sigma_3 = [\vartheta \bar{B}_{\epsilon(t),2} \quad \mathbf{0} \quad \vartheta \bar{H}_{\epsilon(t),2} \quad \vartheta \bar{A}_{\epsilon(t),2}].$$

On the basis of Eq. (16) and taking  $\|v(t)\| \leq \bar{v}$  and  $\|\bar{h}(t)\| \leq \bar{h}$  into consideration, we can get:

$$\begin{aligned} \mathbb{E}\{\Delta V(t)\} &\leq \mathbb{E}\{\xi^T(t)\bar{\mathcal{E}}_{\epsilon(t)}\xi(t) - \rho V(t)\} + \mu_1 \bar{h}^2 + \mu_2 \bar{v}^2 \\ &+ \mathbb{E}\{W^T(t)\mathcal{P}_{\epsilon(t+1)}W(t)\}, \end{aligned} \tag{17}$$

where

$$\bar{\mathcal{E}}_{\epsilon(t)} = \mathcal{E}_{\epsilon(t+1)} - \text{diag}\{(1 - \rho)\mathcal{P}_{\epsilon(t)}, \mathbf{0}, \mu_1 I, \mu_2 I\} = \begin{bmatrix} \Sigma_1 & * & * \\ \Sigma_2 & -\mathcal{P}_{\epsilon(t)}^{-1} & * \\ \Sigma_3 & \mathbf{0} & -\mathcal{P}_{\epsilon(t)}^{-1} \end{bmatrix},$$

$$\Sigma_1 = \begin{bmatrix} -(1 - \rho)\mathcal{P}_{\epsilon(t)} & * & * & * \\ \mathbf{0} & \mathbf{0} & * & * \\ \mathbf{0} & \mathbf{0} & -\mu_1 I & * \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & -\mu_2 I \end{bmatrix}.$$

According to Assumption 1, it can be obtained that:

$$\bar{F}^T(t)\bar{F}(t) \leq \mathcal{E}^T(t)\bar{\mathcal{R}}^T\bar{\mathcal{R}}\mathcal{E}(t). \tag{18}$$

Moreover, it is obviously that:

$$\begin{aligned} \mathbb{E}\{W^T(t)\mathcal{P}_{\epsilon(t+1)}W(t)\} &\leq \lambda_{\max}(\mathcal{P}_{\epsilon(t+1)})\mathbb{E}\{W^T(t)W(t)\} \\ &= 3\bar{\omega}\lambda_{\max}(D^T\mathcal{P}_{\epsilon(t+1)}D). \end{aligned} \tag{19}$$

By combining Eqs. (17)–(19), it has:

$$\begin{aligned} \mathbb{E}\{\Delta V(t)\} &\leq \mathbb{E}\{\xi^T(t)\bar{\mathcal{E}}_{\epsilon(t)}\xi(t) - \rho V(t) + \mathcal{E}^T(t)\bar{\mathcal{R}}^T\bar{\mathcal{R}}\mathcal{E}(t) - \bar{F}^T(t)\bar{F}(t)\} \\ &+ \mu_1 \bar{h}^2 + \mu_2 \bar{v}^2 + 3\bar{\omega}\lambda_{\max}(D^T\mathcal{P}_{\epsilon(t+1)}D) \\ &= \mathbb{E}\{\xi^T(t)\hat{\mathcal{E}}_{\epsilon(t)}\xi(t) - \rho V(t)\} + \mu_1 \bar{h}^2 + \mu_2 \bar{v}^2 \\ &+ 3\bar{\omega}\lambda_{\max}(D^T\mathcal{P}_{\epsilon(t+1)}D). \end{aligned} \tag{20}$$

Then, with the assistance of the Schur complement method, we can derive:

$$\mathbb{E}\{\Delta V(t)\} \leq -\rho\mathbb{E}\{V(t)\} + J, \tag{21}$$

with the holding of Eq. (14), where  $J = \mu_1 \bar{h}^2 + \mu_2 \bar{v}^2 + 3\bar{\omega}\lambda_{\max}(D^T\mathcal{P}_{\epsilon(t+1)}D)$ . So, for any scalar  $\sigma > 0$ , it can be gotten that:

$$\begin{aligned} \mathbb{E}\{\sigma^{t+1}V(t+1)\} - \mathbb{E}\{\sigma^t V(t)\} &= \sigma^{t+1}(\mathbb{E}\{V(t+1)\} - \mathbb{E}\{V(t)\}) + \sigma^t(\sigma - 1)\mathbb{E}\{V(t)\} \\ &\leq \sigma^t(\sigma - \rho\sigma - 1)\mathbb{E}\{V(t)\} + \sigma^{t+1}J. \end{aligned} \tag{22}$$

Letting  $\sigma = 1/(1 - \rho)$  and summing up both sides of Eq. (22) from 0 to  $\iota - 1$  with respect to  $t$ , we thereby obtain:

$$\mathbb{E}\{\sigma^\iota V(\iota)\} - \mathbb{E}\{V(0)\} \leq \frac{\sigma(1 - \sigma^\iota)}{1 - \sigma}J, \tag{23}$$

which implies that:

$$\begin{aligned} \mathbb{E}\{V(\iota)\} &\leq \sigma^{-\iota}\left(\mathbb{E}\{V(0)\} + \frac{\sigma}{1 - \sigma}J\right) + \frac{\sigma}{\sigma - 1}J \\ &= (1 - \rho)^\iota(\mathbb{E}\{V(0)\} - \bar{\rho}) + \bar{\rho}, \end{aligned} \tag{24}$$

where  $\bar{\rho} = \frac{J}{\rho}$ . We further would like to note that  $\mathbb{E}\{V(t)\} \geq \lambda_{\min}(\mathcal{P}_{\epsilon(t)})\mathbb{E}\{\|\mathcal{E}(t)\|^2\}$ , and thus arrive at:

$$\mathbb{E}\{\|\mathcal{E}(t)\|^2\} \leq \frac{\Omega}{\lambda_{\min}(\mathcal{P}_{\epsilon(t)})} = \alpha^t \beta + \kappa, \tag{25}$$

in which

$$\Omega = (1 - \rho)^\iota(\mathbb{E}\{V(0)\} - \bar{\rho}) + \bar{\rho}, \quad \alpha = 1 - \rho, \quad \beta = \frac{\mathbb{E}\{V(0)\} - \bar{\rho}}{\lambda_{\min}(\mathcal{P}_{\epsilon(t)})}, \quad \kappa = \frac{\bar{\rho}}{\lambda_{\min}(\mathcal{P}_{\epsilon(t)})}.$$

Therefore, the established estimation error system (11) is exponentially ultimately bounded in mean square according to Definition 1. So far, the proof is completed.

In Theorem 1, the sufficient conditions that guarantee the stability of the system (11) are derived based on the given state estimator gain matrices, then the following theorem which provides a design approach for the desirable state estimators is presented consequently.

**Theorem 2.** For given positive scalars  $\mu_1, \mu_2, \rho$  and  $\bar{\zeta}$ , the system (11) is exponentially ultimately bounded in mean square if there exist positive definite matrices  $\mathcal{P}_{r,s}, X_{r,s}$  ( $r = 1, 2, \dots, K; s = 1, 2, 3$ ),  $Y_{r,1}$  and matrix  $R$  with appropriate dimensions such that the following linear matrix inequalities (LMIs):

$$\bar{\mathcal{E}}_r = \begin{bmatrix} \bar{\Sigma}_1 & * \\ \Sigma_l & \bar{\mathcal{P}}_d \end{bmatrix} < 0, \tag{26}$$

are satisfied, where

$$\begin{aligned} \bar{\Sigma}_1 &= \begin{bmatrix} \bar{\Sigma}_{11} & * & * & * \\ 0 & \mathbf{I} & * & * \\ 0 & 0 & -\mu_1 I & * \\ 0 & 0 & 0 & -\mu_2 I \end{bmatrix}, \\ \Sigma_l &= [\bar{\Sigma}_{21}^T \ \bar{\Sigma}_{22}^T \ \bar{\Sigma}_{23}^T \ \bar{\Sigma}_{31}^T \ \bar{\Sigma}_{32}^T \ 0]^T, \\ \bar{\mathcal{P}}_d &= \text{diag}\{\bar{\mathcal{P}}_{r,1}, \bar{\mathcal{P}}_{r,2}, \bar{\mathcal{P}}_{r,3}, \bar{\mathcal{P}}_{r,1}, \bar{\mathcal{P}}_{r,2}, \bar{\mathcal{P}}_{r,3}\}, \\ \bar{\Sigma}_{11} &= \begin{bmatrix} -(1-\rho)\mathcal{P}_{r,1} + R^T R & * & * \\ 0 & -(1-\rho)\mathcal{P}_{r,2} & * \\ 0 & 0 & -(1-\rho)\mathcal{P}_{r,3} + R^T R \end{bmatrix}, \\ \bar{\Sigma}_{21} &= [\mathcal{A}_1 \ X_{r,1} \ 0 \ 0 \ (\bar{\zeta} - 1)Y_{r,1}\Phi_r \ \mathcal{A}_2], \\ \bar{\Sigma}_{22} &= [0 \ \mathcal{B} \ 0 \ X_{r,2} \ 0 \ (1-\bar{\zeta})X_{r,2}\Phi_r \ \bar{\zeta}X_{r,2}\Phi_r C], \\ \bar{\Sigma}_{23} &= [0 \ 0 \ X_{r,3}(U \otimes Q) \ 0 \ 0 \ X_{r,3} \ 0 \ X_{r,3}A], \\ \bar{\Sigma}_{31} &= [0 \ 0 \ -\vartheta Y_{r,1}\Phi_r B \ 0 \ 0 \ 0 \ \vartheta Y_{r,1}\Phi_r \ \mathcal{C}_1], \\ \bar{\Sigma}_{32} &= [0 \ \vartheta X_{r,2}\Phi_r B \ 0 \ 0 \ 0 \ 0 \ -\vartheta \Phi_r \ \vartheta X_{r,2}\Phi_r C], \\ \mathcal{A}_1 &= [X_{r,1}(U \otimes Q) - Y_{r,1}C \ -Y_{r,1} + Y_{r,1}\Phi_r \ \mathcal{A}_{11}], \\ \mathcal{A}_2 &= X_{r,1}A - \bar{\zeta}Y_{r,1}\Phi_r C, \ \mathcal{A}_{11} = Y_{r,1}B - \bar{\zeta}Y_{r,1}\Phi_r B, \\ \mathcal{B} &= [\bar{\zeta}X_{r,2}\Phi_r B \ X_{r,2} - X_{r,2}\Phi_r], \ \mathcal{C}_1 = -\vartheta Y_{r,1}\Phi_r C, \\ \bar{\mathcal{P}}_{r,s} &= \mathcal{P}_{r+1,s} - X_{r,s} - X_{r,s}^T, \end{aligned}$$

and the estimator gain matrices can be calculated as  $L_r = X_{r,1}^{-1}Y_{r,1}$ .

**Proof.** Defining  $Y_{r,1} = X_{r,1}L_r$  and  $J = \text{diag}\{\mathcal{I}, X_r, X_r\}$ , where  $X_r = \text{diag}\{X_{r,1}, X_{r,2}, X_{r,3}\}$ , then pre- and post-multiplying  $\bar{\mathcal{E}}_r$  depicted in Eq. (14) by  $J$  and  $J^T$ , respectively, so that we can obtain:

$$\bar{\mathcal{E}}_r = \begin{bmatrix} \bar{\Sigma}_1 & * & * & * & * & * & * \\ \bar{\Sigma}_{21} & \bar{\mathcal{P}}_{r,1} & * & * & * & * & * \\ \bar{\Sigma}_{22} & 0 & \bar{\mathcal{P}}_{r,2} & * & * & * & * \\ \bar{\Sigma}_{23} & 0 & 0 & \bar{\mathcal{P}}_{r,3} & * & * & * \\ \bar{\Sigma}_{31} & 0 & 0 & 0 & \bar{\mathcal{P}}_{r,1} & * & * \\ \bar{\Sigma}_{32} & 0 & 0 & 0 & 0 & \bar{\mathcal{P}}_{r,2} & * \\ 0 & 0 & 0 & 0 & 0 & 0 & \bar{\mathcal{P}}_{r,3} \end{bmatrix} < 0, \tag{27}$$

where  $\bar{\mathcal{P}}_{r,s} = -X_{r,s}\mathcal{P}_{r+1,s}^{-1}X_{r,s}^T$ . Subsequently,  $\bar{\mathcal{E}}_r$  can be derived by replacing each  $\bar{\mathcal{P}}_{r,s}$  in  $\bar{\mathcal{E}}_r$  with corresponding  $\bar{\mathcal{P}}_{r,s}$ . According to Lemma 1, we have  $\bar{\mathcal{P}}_{r,s} \leq \bar{\mathcal{P}}_{r,s}$ , then Eq. (27) can be satisfied if Eq. (26) holds, which proves the theorem.

**Remark 3.** As we presented in the main results, by selecting appropriate Lyapunov function and conducting elaborate analysis, the considered state estimation problem can be effectively resolved in a decentralized manner, then the scalability of the proposed method can be guaranteed. Certainly, we also noticed that only sufficient conditions for assuring the stability of the established state estimation error system are derived in the theorems. However, it is hardly to find the necessary and sufficient conditions for the stability of the complicated networked control systems, so our designed secure state estimation method is feasible and valuable.

**Table 1**  
Parameters  $L_{r,i}$ , ( $r, i = 1, 2, 3$ ).

	$r = 1$	$r = 2$	$r = 3$
$L_{r,1}$	$\begin{bmatrix} 0.0544 \\ 0.1211 \end{bmatrix}$	$\begin{bmatrix} 0.0635 \\ 0.0239 \end{bmatrix}$	$\begin{bmatrix} -0.0228 \\ -0.0671 \end{bmatrix}$
$L_{r,2}$	$\begin{bmatrix} 0.0873 \\ 0.0161 \end{bmatrix}$	$\begin{bmatrix} -0.0675 \\ -0.1065 \end{bmatrix}$	$\begin{bmatrix} -0.0796 \\ 0.0272 \end{bmatrix}$
$L_{r,3}$	$\begin{bmatrix} -0.0626 \\ -0.0721 \end{bmatrix}$	$\begin{bmatrix} -0.0481 \\ 0.0563 \end{bmatrix}$	$\begin{bmatrix} 0.0563 \\ 0.1054 \end{bmatrix}$

**Remark 4.** State estimation for CNs with deception attack has been exploited in some recent researches, such as [35] and [36], however, the works either did not comprehensively consider the influence of deception attack and using RR protocol to avoid data collision incurred by limited communication resource, or did not share the same design objective of our study, i.e., to devise decentralized state estimators to guarantee the exponential mean square boundedness of the constructed system (11). Moreover, it is also worth noting that many state estimation results for CNs have been obtained based on RR protocol in the existed literature, e.g., [6,37], differing from these studies focused on single-channel communication, our work is concerned with multi-channel communication and then proposes multi-channel oriented RR protocol based state estimation method for CNs.

#### 4. Numerical examples

In this section, two simulation examples are presented to validate the efficiency of our designed state estimation scheme.

**Example 1.** We consider a CN with three nodes (i.e.,  $N = 3$ ) and two transmission channels (i.e.,  $M = 2$ , which implies that  $K = 3$ ), and the system parameters are set as follows [33]:

$$\begin{aligned} A_1 &= [0.08 \ 0.09]^T, & A_2 &= [0.05 \ 0.1]^T, & A_3 &= [0.05 \ 0.1]^T, \\ B_1 &= [0.64 \ -0.15], & B_2 &= [1.1 \ 1.03], & B_3 &= [0.12 \ 0.83], \\ D_1 &= [0.08 \ 0.09]^T, & D_2 &= [0.08 \ 0.09]^T, & D_3 &= [0.05 \ 0.1]^T, \\ C_1 &= 0.3, & C_2 &= 0.3, & C_3 &= 0.2. \end{aligned}$$

We further set  $\ell(x_i(t)) = 0.3x_i(t) - \tanh(0.3x_i(t))$  ( $i = 1, 2, 3$ ) and the upperbound of  $\ell(\cdot)$  is  $R = \text{diag}\{0.3, 0.3\}$ . The coupling configuration matrix is assumed to be  $U = [u_{ij}]_{3 \times 3}$ , where  $u_{ij} = 0.1$  if  $i \neq j$ , otherwise  $u_{ij} = -0.2$ . The inner coupling matrix is given as  $Q = \text{diag}\{0.5, 0.5\}$ . For the considered deception attack, let  $\zeta = 0.5$  and  $\hbar = 0.5$ . Based on the above experiment settings, and letting the initial system and estimation states to be  $x_1(0) = [2 \ -2]^T$ ,  $x_2(0) = [3 \ -3]^T$ ,  $x_3(0) = [2.5 \ -2.5]^T$ ,  $\hat{x}_i(0) = [1 \ -1]^T$  ( $i = 1, 2, 3$ ), we calculate the LMIs in Eq. (26) via MATLAB, and thus get the estimator gain matrices, i.e.,  $L_r = \text{diag}\{L_{r,1}, L_{r,2}, L_{r,3}\}$  ( $r = 1, 2, 3$ ), in Table 1.

Then, the simulation results are presented in Figs. 2–6. As shown in Figs. 2–4, the state curve of each node is very close to its estimation after about  $t = 10$ , which validates the effectiveness of the proposed estimation method. Besides, the occurrence of the deception attack and nodes' estimation errors are given in Fig. 5, it can be found that the estimation errors are minor even under stochastic deception attack with the designed secure state estimators. Certainly, we know that the speed of the convergence of the designed estimators is affected by many factors, e.g., the external disturbance and noise, the non-linearity of the envisioned system, the measurement of the deception attack, and the constructed Lyapunov function. Thus, the decrease of the convergence time can be achieved by improving some adjustable influence factors, such as to effectively estimate the energy upperbound of the considered deception attack, and to construct more accurate Lyapunov function, which will be specifically investigated in our future work. Fig. 6 presents the operation of the multi-channel oriented RR protocol, by which the advantages of the protocol, i.e, high node fairness and bandwidth utilization, are fully demonstrated.

**Example 2.** As we know, the CN formulated by Eq. (1) can be applied to the modeling of the coupled neural networks by viewing  $x_i(t)$  as the neuron state and  $\ell(x_i(t))$  as the activation function [38]. Then, we consider a coupled neural network with five nodes and two communication channels (i.e.,  $N = 5$  and  $M = 2$ , which means that  $K = 5$ ) and set the parameters of the system model of the corresponding CN as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} 0.18 \\ 0.05 \end{bmatrix}, & A_2 &= \begin{bmatrix} 0.15 \\ 0.14 \end{bmatrix}, & A_3 &= \begin{bmatrix} 0.05 \\ 0.12 \end{bmatrix}, & A_4 &= \begin{bmatrix} 0.1 \\ 0.31 \end{bmatrix}, & A_5 &= \begin{bmatrix} 0.2 \\ 0.52 \end{bmatrix}, \\ B_1 &= [0.24 \ -0.15], & B_2 &= [1 \ 0.53], & B_3 &= [0.12 \ 0.8], & B_4 &= [0.4 \ 0.3], \\ B_5 &= [0.12 \ 0.52], & C_1 &= 0.3, & C_2 &= 0.4, & C_3 &= 0.2, & C_4 &= 0.3, & C_5 &= 0.15, \end{aligned}$$



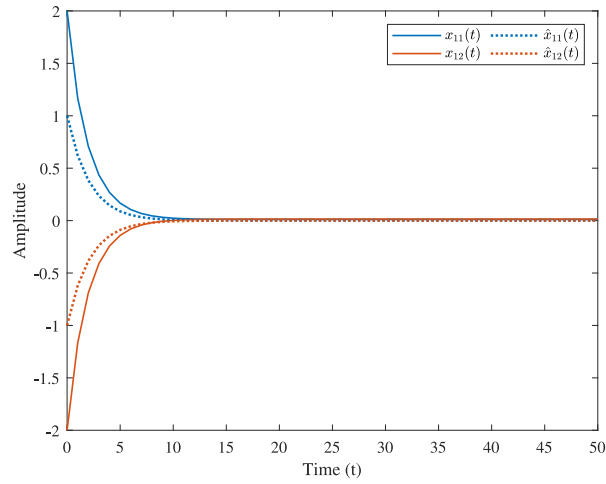


Fig. 2. State Trajectories of node 1 in Example 1.

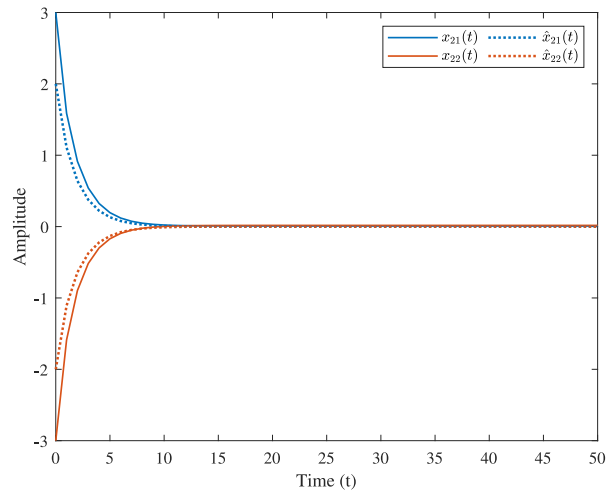


Fig. 3. State Trajectories of node 2 in Example 1.

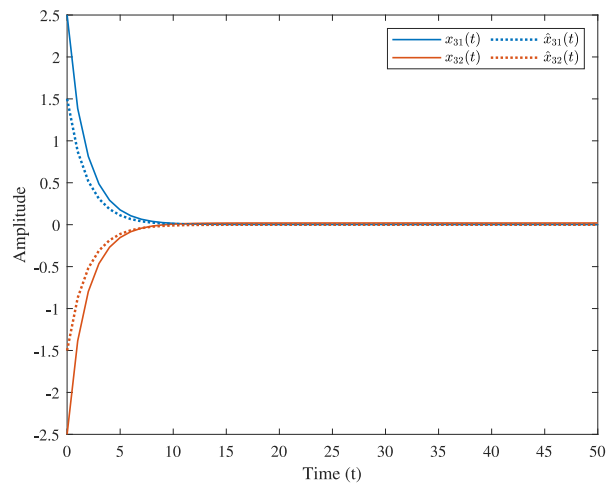


Fig. 4. State Trajectories of node 3 in Example 1.

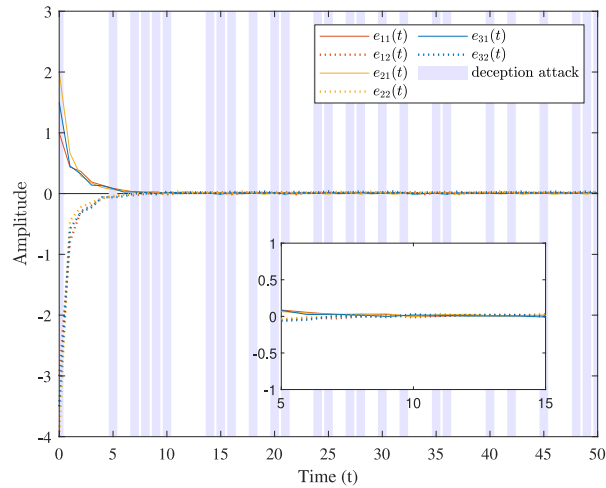


Fig. 5. The deception attack and estimation errors in Example 1.

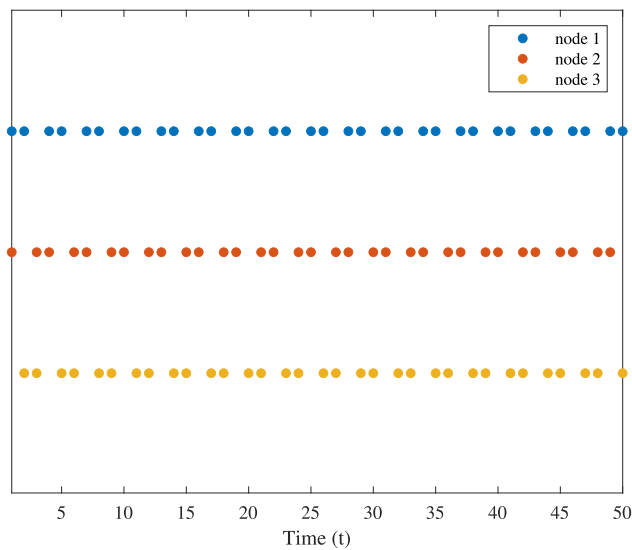


Fig. 6. The selected nodes under multi-channel oriented RR protocol in Example 1.

$$D_1 = \begin{bmatrix} 0.04 \\ 0.06 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 0.06 \\ 0.07 \end{bmatrix}, \quad D_3 = \begin{bmatrix} 0.06 \\ 0.07 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 0.15 \\ 0.05 \end{bmatrix}, \quad D_5 = \begin{bmatrix} 0.04 \\ 0.1 \end{bmatrix},$$

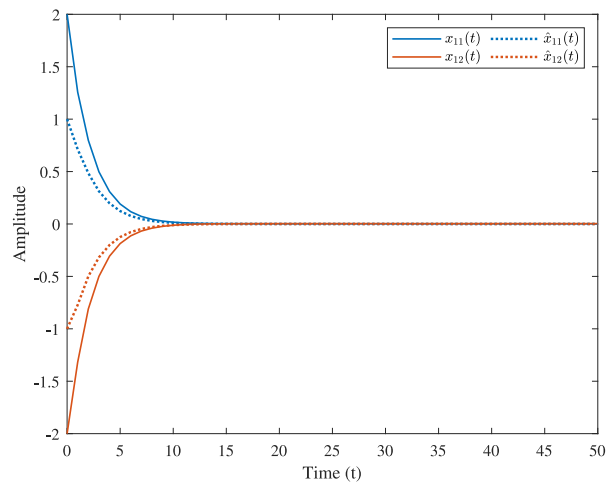
$$Q = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.6 \end{bmatrix}, \quad U = [u_{ij}]_{5 \times 5} = \begin{bmatrix} -0.8 & 0.4 & 0.3 & 0 & 0.1 \\ 0.4 & -0.8 & 0.2 & 0.2 & 0 \\ 0.3 & 0.2 & -1.5 & 0.7 & 0.3 \\ 0 & 0.2 & 0.7 & -1 & 0.1 \\ 0.1 & 0 & 0.3 & 0.1 & -0.5 \end{bmatrix}.$$

The settings of the nonlinear function  $\ell(x_i(t))$  ( $i = 1, 2, \dots, 5$ ) and the deception attack are the same as that in Example 1. Then, based on the given initial conditions:

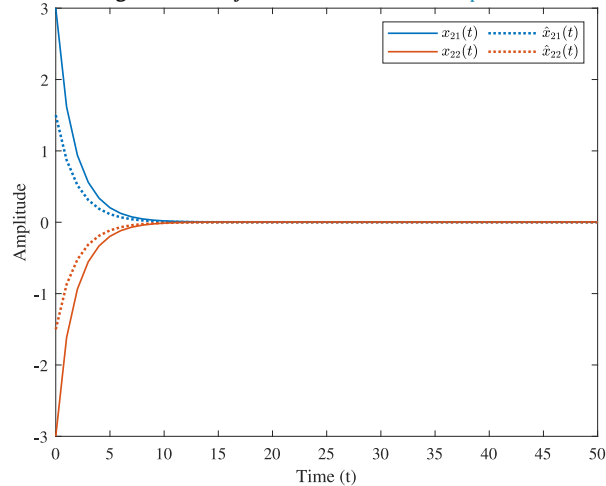
$$\begin{cases} x_1(0) = [2 & -2]^T, & \hat{x}_1(0) = [1 & -1]^T, \\ x_2(0) = [3 & -3]^T, & \hat{x}_2(0) = [1.5 & -1.5]^T, \\ x_3(0) = [4 & -4]^T, & \hat{x}_3(0) = [2 & -2]^T, \\ x_4(0) = [6 & -6]^T, & \hat{x}_4(0) = [4 & -4]^T, \\ x_5(0) = [5 & -5]^T, & \hat{x}_5(0) = [3 & -3]^T, \end{cases}$$

**Table 2**  
Parameters  $L_{r,i}, (r, i = 1, 2, \dots, 5)$ .

	$r = 1$	$r = 2$	$r = 3$	$r = 4$	$r = 5$
$L_{r,1}$	$\begin{bmatrix} 0.0233 \\ 0.0498 \end{bmatrix}$	$\begin{bmatrix} 0.0031 \\ 0.0009 \end{bmatrix}$	$\begin{bmatrix} 0.0532 \\ 0.0366 \end{bmatrix}$	$\begin{bmatrix} -0.0782 \\ 0.0354 \end{bmatrix}$	$\begin{bmatrix} -0.0847 \\ 0.0377 \end{bmatrix}$
$L_{r,2}$	$\begin{bmatrix} -0.0894 \\ -0.0072 \end{bmatrix}$	$\begin{bmatrix} 0.0009 \\ 0.0008 \end{bmatrix}$	$\begin{bmatrix} -0.1184 \\ -0.0362 \end{bmatrix}$	$\begin{bmatrix} -0.0313 \\ 0.0168 \end{bmatrix}$	$\begin{bmatrix} -0.1147 \\ -0.0354 \end{bmatrix}$
$L_{r,3}$	$\begin{bmatrix} -0.0607 \\ -0.2812 \end{bmatrix}$	$\begin{bmatrix} 0.0049 \\ 0.0227 \end{bmatrix}$	$\begin{bmatrix} -0.0376 \\ -0.1757 \end{bmatrix}$	$\begin{bmatrix} -0.0180 \\ -0.0792 \end{bmatrix}$	$\begin{bmatrix} -0.0376 \\ -0.1759 \end{bmatrix}$
$L_{r,4}$	$\begin{bmatrix} -0.2392 \\ -0.0960 \end{bmatrix}$	$\begin{bmatrix} 0.0386 \\ 0.1173 \end{bmatrix}$	$\begin{bmatrix} -0.0843 \\ -0.0377 \end{bmatrix}$	$\begin{bmatrix} -0.1426 \\ -0.0552 \end{bmatrix}$	$\begin{bmatrix} -0.0368 \\ -0.1267 \end{bmatrix}$
$L_{r,5}$	$\begin{bmatrix} -0.0322 \\ -0.0850 \end{bmatrix}$	$\begin{bmatrix} 0.0086 \\ 0.0178 \end{bmatrix}$	$\begin{bmatrix} -0.0191 \\ -0.0486 \end{bmatrix}$	$\begin{bmatrix} -0.0168 \\ -0.0447 \end{bmatrix}$	$\begin{bmatrix} -0.0301 \\ -0.0776 \end{bmatrix}$



**Fig. 7.** State trajectories of node 1 in Example 2.



**Fig. 8.** State trajectories of node 2 in Example 2.

the estimator gain matrices, i.e.,  $L_r = \text{diag}\{L_{r,1}, L_{r,2}, \dots, L_{r,5}\}$  shown in Table 2, can be obtained by solving the LMIs presented in Eq. (26) via MATLAB.

The final simulation results are presented by Figs. 7–13. It can be confirmed that the designed state estimation method can still achieve a satisfactory estimation performance based on Figs. 7–12. The efficiency of the multi-channel oriented RR protocol under the simulated circumstance is validated by Fig. 13.

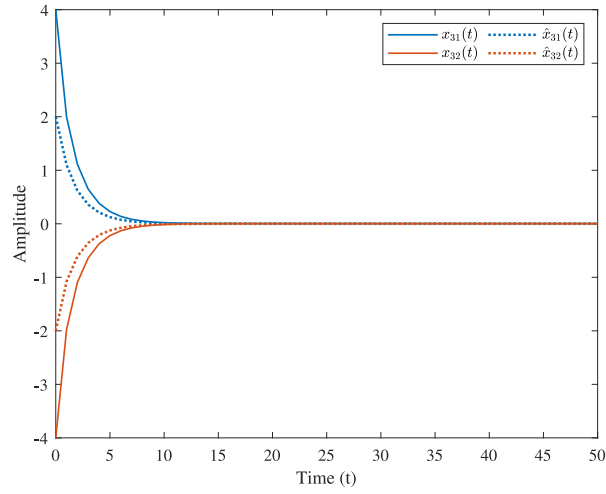


Fig. 9. State trajectories of node 3 in Example 2.

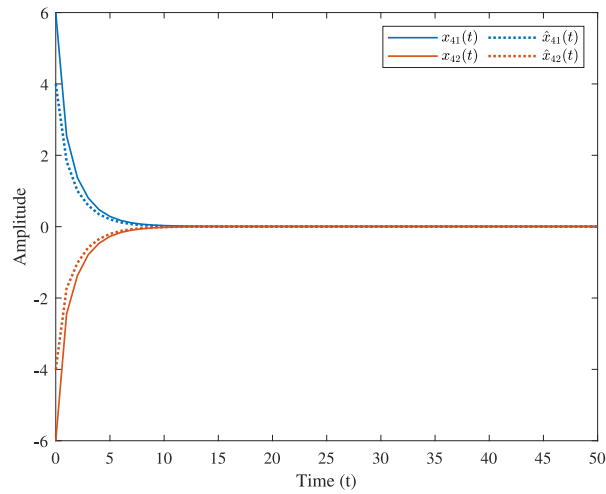


Fig. 10. State trajectories of node 4 in Example 2.

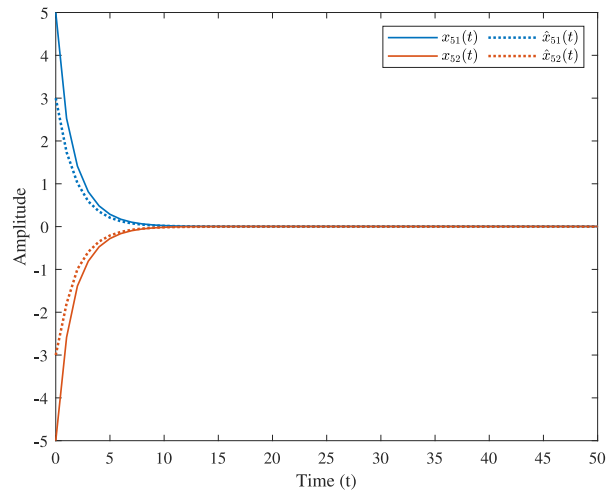


Fig. 11. State trajectories of node 5 in Example 2.

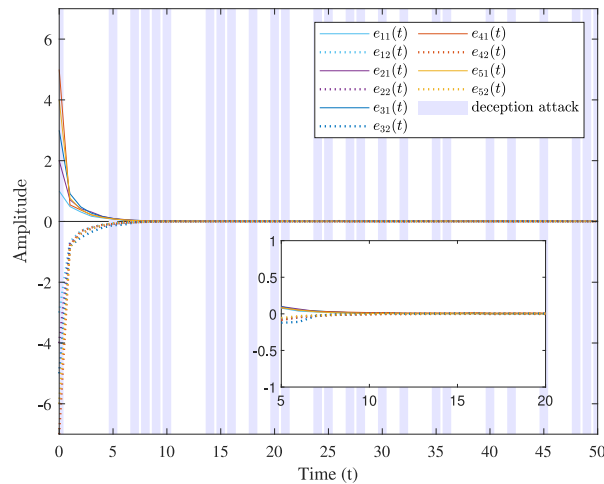


Fig. 12. The deception attack and estimation errors in Example 2.

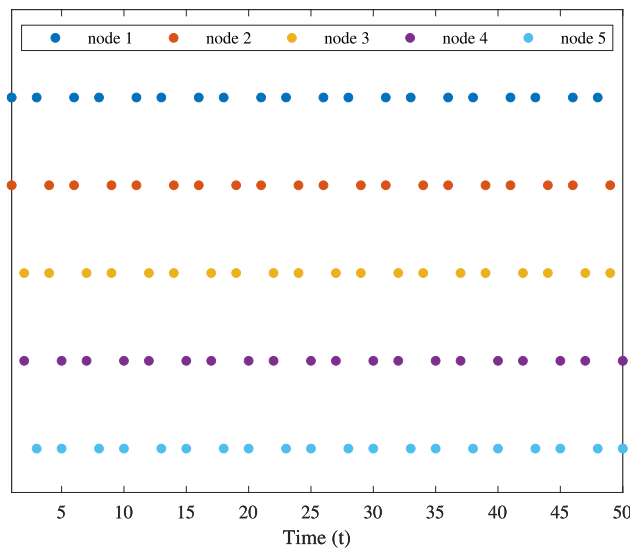


Fig. 13. The selected nodes under multi-channel oriented RR protocol in Example 2.

### 5. Conclusion

In this paper, we study the secure state estimation issue over multi-channel enabled CNs subject to deception attack and limited network resource. To realize conflict-free and fair data transmission, and efficient bandwidth utilization, a multi-channel oriented RR protocol is firstly designed to arrange that a group of nodes can access the network to transmit signals at each time instant. Then, by taking the deception attack depicted by a Bernoulli process into account, a novel state estimation error system is constructed to model the considered secure state estimation problem. Subsequently, the sufficient conditions that assure the established system is exponentially ultimately bounded in mean square are derived, which is followed by the algorithm design for the secure state estimators. The effectiveness of the work is finally illustrated by conducting two numerical simulations. Future research issues may include the design of new data scheduling protocols focused on different performance requirements under multi-channel communication scenario, the control of CNs under influence of various factors such as time-varying network topology, signal saturations and sensor failures.

### CRedit authorship contribution statement

**Yan Li:** Conceptualization, Methodology, Writing – original draft. **Lishuang Wei:** Methodology, Software, Formal analysis. **Jinliang Liu:** Methodology, Formal analysis, Writing – review & editing. **Xiangpeng Xie:** Validation, Software. **Engang Tian:** Validation, Writing – review & editing.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

No data was used for the research described in the article.

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