Event-Triggered Sliding-Mode Control for Fuzzy Semi-Markovian Jump Systems With Dead Zone Input and Application to Circuit Systems

Yushun Tan, Jiajing Liu, Xiangpeng Xie, *Senior Member, IEEE*, Engang Tian, and Jinliang Liu, *Member, IEEE*

***Abstract*—This brief investigates event-triggered sliding mode control (SMC) for fuzzy semi-Markovian jump systems (S-MJSs) subject to limited communication capacity, in which we consider the network-induced transmission delay, dead-zone input and partially unknown transition rates of jumping mode in a whole framework. First, an event-triggered mechanism (ETM) is employed in the sensor-to-controller channel to save network resources efficiently. Then, a novel integral sliding surface is obtained with the aid of the event-triggered state information, and some new sufficient conditions on the stochastic stability of the overall closed-loop system are given in the sense of partially unknown transition rates via stochastic stability theory and matrix inequality method. Furthermore, a mode-depended sliding mode controller is designed to guarantee the reachability of the sliding surface in finite time by using the Lyapunov stability theory. Finally, a TD circuit model is provided to illustrate the validity of the proposed control scheme.**

***Index Terms*—Sliding mode control, fuzzy semi-Markovian systems, dead-zone input, event-triggered mechanism.**

I. Introduction

In RECENT decades, Markovian jump systems (MJSs) have received significant investigation owing to their powerful capability for the physical systems subject to the abruptly random fluctuations [1]. As a kind of more general MJSs, S-MJSs are attracting extensive attention from a large number of researchers. The time-varying transition rates (TRs) of S-MJSs are related to the sojourn time, in contrast to the fixed TRs in MJSs [2]. So S-MJSs are more appropriate to reflect physical systems with uncertain TRs than MJSs. The investigation and control synthesis of S-MJSs in industrial engineering are quite fruitful [3], [4], [5]. Meanwhile, the intervention of realistic factors and the nonlinear feature of the physical model make the theoretical analysis of S-MJSs more complicated. Thus, the Takagi-Sugeno (T-S) fuzzy method is proposed to efficiently solve nonlinear characteristics. Over the years, many scholars gave a large number of meaningful work of S-MJSs by T-S fuzzy method [6], [7].

Due to the limitation of communication bandwidth, some unexpected interfering factors, such as data packet loss and communication delays, may occur, which will interfere with the controller to obtain the jumping mode information of systems accurately [8]. The above phenomenon will seriously degrade the performance of the system and lead to more complex control design. Thus, the ETM is proposed to lower the congestion of communication network by decreasing the quantity of unessential information during the transmission process. Event-triggered control problems of S-MJSs have attracted widespread concern and plenty of literature have been published [9], [10], [11], [12], [13]. However, the existing event-triggered control results are not concerned with the S-MJSs, which constitutes the first motivation of the current work.

As is known to all, the SMC has been proven as a valid strategy for handling the stabilization problems in many nonlinear complex systems on account of its strong robustness and fast response [14]. Thus, a mass of excellent SMC accomplishments have been published in recent years [3], [15]. According to the exiting works associating with SMC of S-MJSs, the time-varying TRs can be handled by different methods. Moreover, dead-zone input, as a class of significant nonlinear input, has attracted extensive attentions and plentiful results have been given [16], [17]. Despite the fact, the aforementioned works cannot be simply applied to the event-triggered SMC design for fuzzy S-MJSs with dead-zone input owing to its special structure. So how to construct the sliding surface and design the reliable sliding mode controller subject to event-triggered communication and dead-zone input is a challenging work, which constitutes the second motivation of this brief.

Manuscript received 30 November 2022; revised 3 February 2023; accepted 11 March 2023. Date of publication 14 March 2023; date of current version 29 August 2023. This work was supported in part by the National Natural Science Foundation of China under Grant 61973152, Grant 62273174, and Grant 62022044; and in part by the Jiangsu Natural Science Foundation for Distinguished Young Scholars under Grant BK20190039. This brief was recommended by Associate Editor X. Wu. *(Corresponding author: Jinliang Liu.)*

Yushun Tan and Jiajing Liu are with the School of Applied Mathematics, Nanjing University of Finance and Economics, Nanjing 210023, China (e-mail: tyshun994@163.com).

Xiangpeng Xie is with the Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210096, Jiangsu, China (e-mail: xiexp@njupt.edu.cn).

Engang Tian is with the School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China (e-mail: tianengang@163.com).

Jinliang Liu is with the School of Optical-Electrical and Computer Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China, and also with the School of Information Engineering, Nanjing University of Finance and Economics, Nanjing 210023, Jiangsu, China (e-mail: liujinliang@vip.163.com).

Color versions of one or more figures in this article are available at https://doi.org/10.1109/TCSII.2023.3257088.

Digital Object Identifier 10.1109/TCSII.2023.3257088

Inspired by these ideas, we investigate event-triggered SMC for fuzzy S-MJSs with dead-zone input in this brief. Compared with [2], more general S-MJSs have been studied, and the core and novel characteristics of this brief can be generalized as: (1) Propose an event-triggered integral-type fuzzy sliding mode control technique for networked S-MJSs to reduce network bandwidth burden and save network resources. (2) Investigate the stochastic stability of the closed-loop fuzzy S-MJSs and a set of sufficient conditions are proposed subject to general uncertain TRs, especially for completely unknown situations. (3) Under the premise of dead-zone nonlinear input, the designed SMC law guarantees the finite-time accessibility of the sliding surface. What'more, the SMC law makes the realization of the stability of nonlinear systems more convenient in practical applications.

Notations: sym$\{B\}$ is $B^T + B$.

II. Problem Formulation

A. T-S Fuzzy Model of S-MJSs

The T-S fuzzy model is represented by IF-THEN rules to solve nonlinear S-MJSs on the probability space (Γ, F, P) as follows:

Plant Rule i: IF $\mu_j(t)$ is F_{ij} $(i = 1, 2, \ldots, r; j = 1, 2, \ldots, p)$, THEN

$$\begin{cases} \dot{x}(t) = \mathcal{A}_i(r_t)x(t) + \mathcal{B}_i(r_t)(\varphi(z(t)) + h(x(t), t)), \\ x(0) = \psi(0), \end{cases} \tag{1}$$

where $x(t) \in \mathcal{R}^n$, $z(t) \in \mathcal{R}^m$ represent the system state and control input; $\mathcal{A}_i(r_t)$,$\mathcal{B}_i(r_t)$ are known constant matrices with proper dimensions; $\varphi(z(t))$ denotes the nonlinear function in regard to $z(t)$; $h(x(t), t)$ is the uncertain parameter; $\mu_1(t), \mu_2(t), \ldots, \mu_p(t)$ are available premise variable; F_{ij} are the fuzzy set. $\{r_t, t \geq 0\}$ represents a continuous-time semi-Markov process taking values in a discrete space $S = \{1, 2, \ldots, s\}$ with TPs described by:

$$\Pr\{r_{t+\delta} = v | r_t = g\} = \begin{cases} \xi_{gv}(\delta)\delta + o(\delta), & g \neq v, \\ 1 + \xi_{gv}(\delta)\delta + o(\delta), & g = v, \end{cases} \tag{2}$$

where $\delta > 0$ denotes the sojourn time independent of t and $\lim_{\delta \to 0} o(\delta)/\delta = 0$, $\xi_{gv}(\delta) > 0$ if $g \neq v$ and $\xi_{gg}(\delta) = -\sum_{g \neq v}^{s} \xi_{gv}(\delta) < 0$.

In this brief, consider transition rates about the semi-Markov process are partially inaccessible or not exactly accessible, and the upper and lower bounds exist. $\xi_{gv}(\delta) \in [\underline{\xi}_{gv}, \overline{\xi}_{gv}]$, where $\underline{\xi}_{gv}$ and $\overline{\xi}_{gv}$ are known constants representing the lower and the upper bounds. Then, denote $\xi_{gv}(\delta) = \xi_{gv} + \Delta\xi_{gv}(\delta)$, where $\xi_{gv} = \frac{1}{2}(\underline{\xi}_{gv} + \overline{\xi}_{gv})$, $|\Delta\xi_{gv}(\delta)| \leq \lambda_{gv}$ with $\lambda_{gv} = \frac{1}{2}(\overline{\xi}_{gv} - \underline{\xi}_{gv})$. For brevity, define $\Lambda = \Lambda_{g,k} \cup \Lambda_{g,uk}$ with

$$\Lambda_{g,k} = \big\{k : \xi_{gv} \textit{ is not exacly konwn}, v \in s\big\},$$
$$\Lambda_{g,uk} = \big\{k : \xi_{gv} \textit{ is completely unknown}, v \in s\big\}.$$

Consider $\Lambda_{g,k} \neq \varnothing$ and $\Lambda_{g,uk} \neq \varnothing$. Then, indicate the following set $\Lambda_{g,k} = \{k_{g,1}, k_{g,2}, \ldots, k_{g,c}\}$, $1 < c < s$.

The system (1) can be deduced as:

$$\dot{x}(t) = \sum_{i=1}^{r} q_i(\mu(t))(\mathcal{A}_i(g)x(t) + \mathcal{B}_i(g)(\varphi(z(t)) + h(x(t), t))), \tag{3}$$

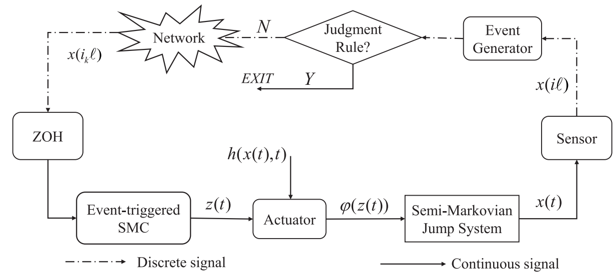

Fig. 1. The diagram of control system with event generator.

where $\mu(t) = [\mu_1(t), \mu_2(t), \ldots, \mu_p(t)]^T$, $q_i(\mu(t)) = \prod_{j=1}^{p} \varsigma_{ij}(\mu_j(t)) / \sum_{i=1}^{r} \prod_{j=1}^{p} \varsigma_{ij}(\mu_j(t))$ is the membership function. $\varsigma_{ij}(\mu_j(t))$ is the grade membership of $\mu_j(t)$ in ς_{ij}. Here we have $\varsigma_{ij}(\mu_j(t)) \geq 0$ and $q_i(\mu(t))$ satisfies $1 \geq q_i(\mu(t)) \geq 0$, $\sum_{i=1}^{r} q_i(\mu(t)) = 1$ for $t > 0$. For the T-S fuzzy S-MJSs (3), the object of this brief is to construct reliable control input $z(t)$ to achieve the stabilization of system under the implementation of nonlinear control inputs $\varphi(z(t))$.

The control framework of the systems is illustrated in Figure 1.

B. Event-Triggered Mechanism

Design an event generator to cope with energy consumption and computational constraints. The triggering threshold is proposed as

$$\begin{aligned} \varepsilon\mathbb{E}\big(x(i_k\ell)^T\Gamma_g x(i_k\ell)\big) < \mathbb{E}([x(i_k + l)\ell - x(i_k\ell)]^T \\ \Gamma_g[x(i_k + l)\ell - x(i_k\ell)]) \end{aligned} \tag{4}$$

where $\varepsilon \in [0, 1)$ represents a known triggering parameter. Γ_g denotes a symmetric positive definite matrix corresponding to mode g. ℓ is the sampling period.

Remark 1: In the judgment rule (4), $i_k\ell(i_k \in N, k = 0, 1 \cdots \infty, i_0 = 0\})$ denotes the release instant. Suppose that the network-induced delay is $b_k \in [b_m, b_M]$. Denote $t_k = i_k\ell + b_k$ to simplify the representation, and assume the initial state $x(0)$ is successfully transmitted.

Similar to [18], the $[t_k, t_{k+1})$ can be separated into $\cup_{l=1}^{\hat{p}_k} D_l = [t_k, t_{k+1})$, where $D_l = [t_k + j\ell, t_k + j\ell + \ell)$ and $\hat{p}_k = \min\{p | t_k + (l-1)\ell \geq t_{k+1}, p = 1, 2, \ldots, \}$. For $t \in D_l$, define $\tau_k(t)$ and $e_k(t)$ as $\tau_k(t) = t - (i_k + l)\ell$ and $e_k(t) = x(i_k\ell) - x((i_k + l)\ell)$. We can easily get $x(i_k\ell) = e_k(t) + x(t - \tau_k(t))$, and $b_m \leq \tau_k(t) < b_k + \ell \leq b_M + \ell = \bar{b}_M$.

Based on the event-triggered rule (4), for $t \in [t_k, t_{k+1})$, it satisfies

$$\begin{aligned} \mathbb{E}\big({e_k}^T(t)\Gamma_g e_k(t)\big) \leq \varepsilon\mathbb{E}([e_k(t) + x(t - \tau_k(t))]^T \\ \Gamma_g[e_k(t) + x(t - \tau_k(t))]). \end{aligned} \tag{5}$$

C. Event-Triggered SMC Dynamics

Consider a fuzzy-logic integral sliding mode to decide the corresponding event-triggered SMC dynamics

$$\begin{aligned} s(t) = \mathcal{G}(g)x(t) - \int_0^t \sum_{i=1}^{r} q_i(\mu(t))(\mathcal{G}(g)(\mathcal{A}_i(g) \\ + \mathcal{B}_i(g)\mathcal{K}(g))x(s))ds, \end{aligned} \tag{6}$$

where $\mathcal{K}(g) \in \mathcal{R}^{n_i}$ is real matrices to be pre-defined to make $\mathcal{A}_i(g) + \mathcal{B}_i(g)\mathcal{K}(g)$ is Hurwitz and $\mathcal{G}(g)^T \in \mathcal{R}^n$ is selected to enable $\mathcal{G}(g)\sum_{i=1}^{r} q_i(\mu(t))\mathcal{B}_i(g)$ is nonsingular. Then, $\dot{s}(t)$ is inferred as:

$$\dot{s}(t) = \sum_{i=1}^{r} q_i(\mu(t))\mathcal{G}(g)[\mathcal{B}_i(g)((\varphi(z(t)) + h(x(t), t)) - \mathcal{B}_i(g)\mathcal{K}(g)x(t)]. \tag{7}$$

When the trajectories of states reach onto the designed sliding surface, there are $\dot{s}(t) = 0$ and $s(t) = 0$. Therefore, the event-triggered equivalent control from (7) is deduced as:

$$\varphi(u_{eq}) = \mathcal{K}(g)x(i_k\ell) - h(x(t), t)). \tag{8}$$

Under the control input $\varphi(u_{eq})$, $x(t - \tau_k(t))$ and e_k, (3) can be deduced by:

$$\dot{x}(t) = \sum_{i=1}^{r} q_i(\mu(t))(\mathcal{A}_i(g)x(t) + \mathcal{B}_i(g)\mathcal{K}(g) \times x(t - \tau_k(t)) + \mathcal{B}_i(g)\mathcal{K}(g)e_k(t)). \tag{9}$$

Definition 1 [3]: Define the dead-zone function $\varphi(z(t))$ as:

$$\varphi(z(t)) = \begin{cases} \varphi_a(z(t))(z(t) - z_a), & z(t) > z_a, \\ 0, & -z_b \le z(t) \le z_a, \\ \varphi_b(z(t))(z(t) + z_b), & z(t) < -z_b, \end{cases} \tag{10}$$

where $\varphi_a(z(t)) \ge h_a > 0$ and $\varphi_b(z(t)) \ge h_b > 0$ are the nonlinear functions of $z(t)$, z_a, z_b, h_a, h_b are positive constants.

Lemma 1 [19]: For any real number ρ, and any square matrix Y, there holds $\rho(Y + Y^T) \le \rho^2\bar{R} + Y\bar{R}^{-1}Y^T$ for any matrix $\bar{R} > 0$.

Assumption 1: $||h(x(t), t)|| \le \sigma$ and σ is a given positive constant.

Assumption 2: The dead-zone function $\varphi(z(t))$ is antisymmetry about the origin, so $z_a = z_b = z_\dagger$, $h_a = h_b = h$. Based on above condition, the (10) can be rewritten as

$$\begin{aligned} (z(t) - z_\dagger)\varphi(z(t)) &\ge h(z(t) - z_\dagger)^2, \quad z(t) > z_\dagger, \\ (z(t) + z_\dagger)\varphi(z(t)) &\ge h(z(t) + z_\dagger)^2, \quad z(t) < -z_\dagger. \end{aligned} \tag{11}$$

III. Main Results

In this section, Theorem 1 investigates the stochastic stability of the closed-loop fuzzy S-MJSs (9), and a series of stochastic stability conditions are proposed. Theorem 2 focuses on analyzing the reachability of the sliding surface.

Theorem 1: For given scalars $b_M > b_m > 0$, $\varepsilon \in [0, 1)$, the resultant closed-loop system (9) running on the sliding mode surface $s(t) = 0$ is stochastically stable, if there exist matrices $P_g > 0$, $T_{g,v} > 0$, $Z_{g,v} > 0$, $H_{g,v} > 0$, $\Gamma_g > 0$ and matrices $M_{ng}(n = 1, 2, 3)$ with compatible dimensions such that the following inequality constraints hold

Case I, if $g \in \Lambda_{g,k}, \forall l \in \Lambda_{g,uk}$, $\Lambda_{g,k} = \{k_{g,1}, k_{g,2}, \ldots k_{g,o1}\}$,

$$\begin{pmatrix} \Theta_{ig}^1 & \Pi & \hat{M}_{1g} & \hat{M}_{2g} & \hat{M}_{3g} & \mathcal{C}_g^{12} \\ \star & -\frac{1}{b_M}R_3 & 0 & 0 & 0 & 0 \\ \star & \star & -\frac{1}{b_M}R_3 & 0 & 0 & 0 \\ \star & \star & \star & -\frac{1}{b}R_3 & 0 & 0 \\ \star & \star & \star & \star & -\frac{1}{b}R_3 & 0 \\ \star & \star & \star & \star & \star & \mathcal{C}_g^{13} \end{pmatrix} < 0, \tag{12}$$

Case II, if $g \in \Lambda_{g,uk}, \forall l \in \Lambda_{g,uk}$, $\Lambda_{g,k} = \{k_{g,1}, k_{g,2}, \ldots k_{g,o2}\}$,

$$P_g - P_l \ge 0,$$

$$\begin{pmatrix} \Theta_{ig}^2 & \Pi & \hat{M}_{1g} & \hat{M}_{2g} & \hat{M}_{3g} & \mathcal{C}_g^{22} \\ \star & -\frac{1}{b_M}R_3 & 0 & 0 & 0 & 0 \\ \star & \star & -\frac{1}{b_M}R_3 & 0 & 0 & 0 \\ \star & \star & \star & -\frac{1}{b}R_3 & 0 & 0 \\ \star & \star & \star & \star & -\frac{1}{b}R_3 & 0 \\ \star & \star & \star & \star & \star & \mathcal{C}_g^{23} \end{pmatrix} < 0, \tag{13}$$

Case III, if $g \in \Lambda_{g,uk}$, $\Lambda_{g,k} = \varnothing$, $l \ne k$, $l \in D_{l,k}$,

$$\begin{pmatrix} \Theta_{ig}^3 & \Pi & \hat{M}_{1g} & \hat{M}_{2g} & \hat{M}_{3g} & \mathcal{C}_g^{32} \\ \star & -\frac{1}{b_M}R_3 & 0 & 0 & 0 & 0 \\ \star & \star & -\frac{1}{b_M}R_3 & 0 & 0 & 0 \\ \star & \star & \star & -\frac{1}{b}R_3 & 0 & 0 \\ \star & \star & \star & \star & -\frac{1}{b}R_3 & 0 \\ \star & \star & \star & \star & \star & \mathcal{C}_g^{33} \end{pmatrix} < 0, \tag{14}$$

where Θ_{ig}^k is described as

$$\begin{pmatrix} \mathcal{C}_{ig}^{k1} & -M_{1g} & \mathcal{B}_i(g)\mathcal{K}(g) & 0 & \mathcal{B}_i(g)\mathcal{K}(g) \\ \star & \Upsilon_2 & -M_{2g} & 0 & 0 \\ \star & \star & \Upsilon_3 & -M_{3g} & \varepsilon\Gamma_g^T \\ \star & \star & \star & -R_2 & 0 \\ \star & \star & \star & \star & (\varepsilon - 1)\Gamma_g \end{pmatrix}.$$

Moreover, the remaining scalars are provided as

$$\begin{aligned} \Upsilon_2 &= -R_1 + M_{2g} + M_{2g}{}^T, \ \Upsilon_3 = \varepsilon\Gamma_g + M_{3g} + M_{3g}{}^T, \\ \hat{M}_{1g} &= [M_{1g}{}^T, 0, 0, 0, 0]^T, \ \hat{M}_{2g} = [0, M_{2g}{}^T, 0, 0, 0]^T, \\ \hat{M}_{3g} &= [0, 0, M_{3g}{}^T, 0, 0]^T, \ b = \bar{b}_M - b_m, \\ \Pi &= [(\mathcal{A}_i(g)), 0, (\mathcal{B}_i(g)\mathcal{K}(g)), 0, (\mathcal{B}_i(g)\mathcal{K}(g))]^T R_3, \\ \mathcal{C}_{ig}^{11} &= sym\{P_g\mathcal{A}_i(g)\} + R_1 + R_2 + M_{1g} + M_{1g}{}^T \\ &\quad + \sum_{v \in \Lambda_{g,k}} \Big[\frac{(\lambda_{gv})^2}{4}T_{gv} + \xi_{gv}(P_v - P_l)\Big], \\ \mathcal{C}_g^{12} &= [(P_{k_{g,1}} - P_l), \ldots, (P_{k_{g,c1}} - P_l)], \\ \mathcal{C}_g^{13} &= [-T_{gk_{g,1}}, \ldots, -T_{gk_{g,c1}}], \\ \mathcal{C}_{ig}^{21} &= sym\{P_g\mathcal{A}_i(g)\} + R_1 + R_2 + M_{1g} + M_{1g}{}^T \\ &\quad + \sum_{v \in \Lambda_{g,k}} \Big[\frac{(\lambda_{gv})^2}{4}Z_{gv} + \xi_{gv}(P_v - P_l)\Big], \\ \mathcal{C}_g^{22} &= [(P_{k_{g,1}} - P_l), \ldots, (P_{k_{g,c2}} - P_l)], \\ \mathcal{C}_g^{23} &= [-Z_{gk_{g,1}}, \ldots, -Z_{gk_{g,c2}}], \\ \mathcal{C}_{ig}^{31} &= sym\{P_g\mathcal{A}_i(g)\} + R_1 + R_2 + M_{1g} + M_{1g}{}^T \\ &\quad + a_g\frac{(\lambda_{ll})^2}{4}H_{gg} + a_g\xi_{ll}(P_g - P_l), \\ \mathcal{C}_g^{32} &= a_g(P_g - P_l), \ \mathcal{C}_g^{33} = -a_gH_{gg}. \end{aligned}$$

Proof: Choose the semi-Markovian Lyapunov functional

$$\mathcal{V}(x(t), r_t) = \sum_{j=1}^{3} \mathcal{V}_j(x(t), r_t),$$

where $\mathcal{V}_1(x(t), r_t) = x(t)P(r_t)x(t)$, $\mathcal{V}_2(x(t), r_t) = \int_{t-b_m}^{t} x(s)R_1x(s)ds + \int_{t-\bar{b}_M}^{t} x(s)R_2x(s)ds$ and $\mathcal{V}_3(x(t), r_t) = \int_{-\bar{b}_M}^{0}\int_{t+\mu}^{t} \dot{x}(s)R_3\dot{x}(s)dsd\mu$.

On the basis of the definition of weak infinitesimal operator [5], one gets

$$\begin{aligned}\mathscr{L}\mathcal{V}(x(t), g) = {}& 2x^T(t)P_g\sum_{i=1}^{r} q_i(\mu)[\mathcal{A}_i(g)x(t)\\ &+ \mathcal{B}_i(g)\mathcal{K}(g)x(t-\tau_k(t)) + \mathcal{B}_i(g)\mathcal{K}(g)e_k(t)]\\ &+ x^T(t)(R_1+R_2)x(t) + x^T(t)\sum_{v=1}^{r}\xi_{gv}(\delta)P_vx(t)\\ &+ \bar{b}_M\dot{x}^T(t)R_3\dot{x}(t) - x^T(t-b_m)R_1x(t-b_m)\\ &- x^T(t-\bar{b}_M)R_2x(t-\bar{b}_M) - \int_{t-\bar{b}_M}^{t}\dot{x}^T(s)R_3\dot{x}(s)ds\\ &+ \vartheta_1+\vartheta_2+\vartheta_3, \end{aligned} \tag{15}$$

where $\vartheta_1 = 2x^T(t)M_{1g}[x(t) - x(t-b_m) - \int_{t-b_m}^{t}\dot{x}(s)ds] = 0$, $\vartheta_2 = 2x^T(t-b_m)M_{2g}[x(t-b_m) - x(t-\tau_k(t)) - \int_{t-\tau_k(t)}^{t-b_m}\dot{x}(s)ds] = 0$, $\vartheta_3 = 2x^T(t-\tau_k(t))M_{3g}[x(t-\tau_k(t)) - x(t-\bar{b}_M) - \int_{t-\bar{b}_M}^{t-\tau_k(t)}\dot{x}(s)ds] = 0$ for $M_{ng}(n = 1, 2, 3)$ with compatible dimensions. ■

Considering the event-triggering scheme (5) at $[t_k, t_{k+1})$, it yields

$$\mathbb{E}(\mathscr{L}\mathcal{V}(x(t), g)) \le \mathbb{E}(\sum_{i=1}^{r} q_i(\mu)\{\zeta^T(t)\bar{\Psi}_{ig}\zeta(t)\}), \tag{16}$$

where $\bar{\Psi}_{ig} = \Psi_{ig} + diag\{\sum_{v=1}^{r}\xi_{gv}(\delta)P_v, 0, 0, 0, 0\}$, $\zeta(t) = [x^T(t), x^T(t-b_m), x^T(t-\tau(t)), x^T(t-\bar{b}_M), e_k{}^T(t)]^T$ $\Upsilon_1 = sym\{P_g\mathcal{A}_i(g)\} + R_1 + R_2 + M_{1g} + M_{1g}{}^T + b_mM_{1g}R_3{}^{-1}M_{1g}{}^T$, $\bar{\Upsilon}_2 = -R_1 + M_{2g} + M_{2g}{}^T + b_mM_{2g}R_3{}^{-1}M_{2g}{}^T$, $\bar{\Upsilon}_3 = \varepsilon\Gamma_g^T + M_{3g} + M_{3g}{}^T + b_mM_{3g}R_3{}^{-1}M_{3g}{}^T$, and

$$\Psi_{ig} = \begin{bmatrix} \Upsilon_1 & -M_{1g} & \mathcal{B}_i(g)\mathcal{K}(g) & 0 & \mathcal{B}_i(g)\mathcal{K}(g)\\ \star & \bar{\Upsilon}_2 & -M_{2g} & 0 & 0\\ \star & \star & \bar{\Upsilon}_3 & -M_{3g} & \varepsilon\Gamma_g^T\\ \star & \star & \star & -R_2 & 0\\ \star & \star & \star & \star & (\varepsilon-1)\Gamma_g \end{bmatrix} + \bar{b}_M\begin{bmatrix} (\mathcal{A}_i(g))^T\\ 0\\ (\mathcal{B}_i(g)\mathcal{K}(g))^T\\ 0\\ (\mathcal{B}_i(g)\mathcal{K}(g))^T \end{bmatrix} R_3 \begin{bmatrix} (\mathcal{A}_i(g))^T\\ 0\\ (\mathcal{B}_i(g)\mathcal{K}(g))^T\\ 0\\ (\mathcal{B}_i(g)\mathcal{K}(g))^T \end{bmatrix}^T.$$

Based on the definition of stochastic stability [20], the system (9) is stochastically stable if $\bar{\Psi}_{ig} < 0$. Therefore, the corresponding proof is separated into three cases below.

Case I: $g \in \Lambda_{g,k}$. Define $\lambda_{g,v} = \sum_{v\in\Lambda_{g,k}}\xi_{gv}(\delta)$. Since $\Lambda_{g,uk} \neq \varnothing$, it obviously holds that $\lambda_{g,v} < 0$. $\sum_{v=1}^{s}\xi_{gv}(\delta)P_v$ is equivalent to

$$\sum_{v=1}^{s}\xi_{gv}(\delta)P_v = \sum_{v\in\Lambda_{g,k}}\xi_{gv}(\delta)P_v - \lambda_{g,v}\sum_{v\in\Lambda_{g,uk}}\frac{\xi_{gv}(\delta)}{-\lambda_{g,v}}P_v. \tag{17}$$

Notice $0 \le \xi_{gv}(\delta)/-\lambda_{g,v} \le 1 (v \in \Lambda_{g,k})$, $\sum_{v\in\Lambda_{g,uk}}\frac{\xi_{gv}(\delta)}{-\lambda_{g,v}} = 1$. Therefore, for $\forall l \in \Lambda_{g,uk}$. Thus, for $0 \le \xi_{gv}(\delta) \le -\lambda_{g,v}$, $\bar{\Psi}_{ig} < 0$ can be guaranteed by the inequality

$$\Psi_{ig} + diag\{\sum_{v\in\Lambda_{g,k}}\xi_{gv}(\delta)(P_v - P_l), 0, 0, 0, 0\} < 0. \tag{18}$$

Based on (18) and Lemma 1, for any $T_{gv} > 0$, one gets

$$\sum_{v\in\Lambda_{g,k}}\Delta\xi_{gv}(\delta)(P_v - P_l) \le \sum_{v\in\Lambda_{g,k}}[\frac{(\lambda_{gv})^2}{4}T_{gv} + (P_v - P_l)(T_{gv})^{-1}(P_v - P_l)^T]. \tag{19}$$

By combining (17)-(19) and utilizing the Schur complement, $\bar{\Psi}_{ig} < 0$ give rise to inequality (12).

Case II: $g \in \Lambda_{g,uk}$ and $\Lambda_{g,k} \neq \varnothing$. Letting $\lambda_{g,v} = \sum_{v\in\Lambda_{g,k}}\xi_{gv}(\delta)$, it holds $\lambda_{g,v} > 0$. $\sum_{v=1}^{s}\xi_{gv}(\delta)P_v$ can be written as:

$$\sum_{v=1}^{s}\xi_{gv}(\delta)P_v = \sum_{v\in\Lambda_{g,k}}\xi_{gv}(\delta)P_v + \xi_{gg}(\delta)P_g - (\xi_{gg}(\delta) + \lambda_{g,v})\sum_{\substack{v\in\Lambda_{g,uk}\\ v\neq g}}\frac{\xi_{gv}(\delta)P_v}{-(\xi_{gg}(\delta) + \lambda_{g,v})}. \tag{20}$$

Similar with Case I, $\xi_{gg}(\delta) < 0$, $\bar{\Psi}_{ig} < 0$ can be guaranteed by the following inequalities

$$\begin{aligned}&P_g - P_l \ge 0,\\ &\Psi_{ig} + diag\{\sum_{v\in\Lambda_{g,k}}\xi_{gv}(\delta)(P_v - P_l), 0, 0, 0, 0\} < 0. \end{aligned}\tag{21}$$

And, it holds that for $Z_{gv} > 0$,

$$\sum_{v\in\Lambda_{g,k}}\xi_{gv}(\delta)(P_v - P_l) \le \sum_{v\in\Lambda_{g,k}}\xi_{gv}(P_v - P_l) + \sum_{v\in\Lambda_{g,k}}[\frac{(\lambda_{gv})^2}{4}Z_{gv} + (P_v - P_l)(Z_{gv})^{-1}(P_v - P_l)^T]. \tag{22}$$

Combining (20)-(22) and the Schur complement, it can be deduced that (13) guarantees $\bar{\Psi}_{ig} < 0$ to be held.

Case III: $g \in \Lambda_{g,uk}$, $\Lambda_{g,k} = \varnothing$. Suppose that there exists $l \neq g$, $l \in \Lambda_{l,k}$. Similar to the method in literature [3], $\xi_{gg}(\delta)$ can be estimated by $a_g\xi_{ll}(\delta)$ in which a_g is a parameter to be solved. Denoting $\lambda_{g,v} = \xi_{gg}(\delta)$, $\sum_{l=1}^{s}\xi_{gl}(\delta)P_l$ can be presented as

$$\sum_{v=1}^{s}\xi_{gv}(\delta)P_l = \xi_{gg}(\delta)P_g - \lambda_{g,l}\sum_{l\in\Lambda_{g,uk}}\frac{\xi_{gv}(\delta)}{-\lambda_{g,l}}P_v. \tag{23}$$

Notice that $\sum_{l\in\Lambda_{g,uk}}\xi_{gv}(\delta) = -\xi_{gg}(\delta) = -\lambda_{g,v} > 0$, $\forall l \in \Lambda_{g,uk}$. One gets

$$\bar{\Psi}_{ig} = \Psi_{ig} + diag\{a_g\xi_{ll}(\delta)(P_g - P_l), 0, 0, 0, 0\}. \tag{24}$$

Similarly, for any $H_{gg} > 0$, we have

$$\Delta\xi_{ll}(\delta)(P_g - P_l) \le \Big[\frac{(\lambda_{ll})^2}{4}H_{gg} + (P_g - P_l) \times (H_{gg})^{-1}(P_g - P_l)^T\Big]. \tag{25}$$

By combining (23)-(25) and adopting the Schur complement, $\bar{\Psi}_{ig} < 0$ holds if there exists inequality (14). In summary, the system (9) meets stochastic stability.

Remark 2: The problem of control design on S-MJSs was early proposed in [4] by setting the upper and lower bounds of the TRs. To handle the more general TRs especially for completely unknown situations, we employ the method in [3] to estimate the completely unknown TRs by other diagonal components of TR matrix. For case III in Theorem 1, the TRs from one mode to others are totally inaccessible. Here, an optimal algorithm is proposed as follows:

$$\max\ a_g > 0,\ s.t,\ \text{LMIs (12)} - (14) \quad \text{with feasible } P_g, T_{g,v}, Z_{g,v} \text{ and } H_{g,v}. \tag{26}$$

Theorem 2: The function of sliding mode surface has been designed in (6). The sliding mode controller will be constructed to force the state trajectories of S-MJSs to the sliding surface in finite time. The following SMC law for $t \in [t_k, t_{k+1})$ is executed as:

$$z(t) = -\frac{s(t)}{||s(t)||}(\kappa(t) + z_\dagger), \tag{27}$$

where $\kappa(t) = \frac{1}{h}[\sigma + \|\mathcal{K}(g)\|(1 + \sqrt{\varepsilon\frac{\bar{\lambda}}{\underline{\lambda}}})\|x(t_k\ell)\| + \kappa]$ with κ as a small positive scalar, $\underline{\lambda} = \min_{g\in S}\{\lambda_{\min}(\Gamma_g)\}$ and $\bar{\lambda} = \max_{g\in S}\{\lambda_{\max}(\Gamma_g)\}$.

Proof: Assume Lyapunov functional as $\mathcal{V}(t) = [s^T(t)s(t)]^{\frac{1}{2}}$. One derives that

$$\mathscr{L}\mathcal{V}(t) = \frac{s(t)}{||s(t)||}\sum_{i=1}^{r} q_i(\mu)\mathcal{G}(g)\mathcal{B}_i(g)((\varphi(z(t)) + h(x(t), t) - \mathcal{K}(g)x(t)). \tag{28}$$

Based on the method in [9], select $\mathcal{G}(g)$ to make $\mathcal{G}(g)\sum_{i=1}^{r} q_i(\mu(t))\mathcal{B}_i(g) = 1$. And combining (5), the inequality can be inferred from (28).

$$\begin{aligned}\mathscr{L}\mathcal{V}(t) &= \frac{s(t)}{||s(t)||}(\varphi(z(t)) + h(x(t), t) - \mathcal{K}(g)(e_k(t) + x(t_k\ell)))\\ &\le \frac{s(t)}{||s(t)||}(\varphi(z(t)) + \sigma + \|\mathcal{K}(g)\|(1 + \sqrt{\varepsilon\frac{\bar{\lambda}}{\underline{\lambda}}})\|x(t_k\ell)\|),\end{aligned} \tag{29}$$

It can be deduced from (27)

$$z(t) + \frac{s(t)}{||s(t)||}z_\dagger = -\frac{s(t)}{||s(t)||}\kappa(t). \tag{30}$$

According to (11), it holds that

$$(z(t) + \frac{s(t)}{\|s(t)\|}z_\dagger)\varphi(z(t)) \ge h(z(t) + \frac{s(t)}{\|s(t)\|}z_\dagger)^2. \tag{31}$$

Combining (29), (30) and (31), it yields that

$$\mathscr{L}\mathcal{V}(t) \le -h\kappa(t) + (\sigma + \|\mathcal{K}(g)\|(1 + \sqrt{\varepsilon\frac{\bar{\lambda}}{\underline{\lambda}}})\|x(t_k\ell)\|) \tag{32}$$

Substituting $\kappa_i(t)$ into (32) and taking expectation after integrating from 0 to t, it yields

$$\mathbb{E}(\mathcal{V}(t)) \le \mathbb{E}(\mathcal{V}(0)) - \kappa t. \tag{33}$$

From above formula, it gets $\mathbb{E}||(s(t))|| = 0$ if $t \ge \tilde{t} = \frac{\mathbb{E}(\mathcal{V}(0))}{\kappa}$, which means $s(t) = 0$ can be guaranteed by $t \ge \tilde{t}$. The proof is completed. ■

Remark 3: It is well known that chattering effect will be caused by symbolic function $sgn(s(t))$ in [14], which degrades system performance and puts more burden on the actuator. $s(t)/(\|s(t)\| + \gamma)$ can be substituted for $sgn(s(t))$ to reduce the chattering phenomenon effectively, where γ is a small constant. Next, a simulation example will suffice to illustrate the effectiveness of the proposed design method.

IV. Simulation Example

Consider a class of electric circuit model in [6], which includes parasitic capacitor and nonlinear resistor. According to Kirchoff voltage and current law, the relevant equation is described as:

$$\begin{cases} L\dot{I}_L(t) = -RI_L(t) - v_c(t) + U\\ C\dot{v}_c(t) = I_L(t) - \frac{1}{5}(v_c^3(t) - v_c(t)) + bU\end{cases}$$

where the physical meanings of $I_L(t)$ and $v_c(t)$ are inductor current and capacitor voltage. b and R are chosen as $b = 0.5$, $R = 2$. L and C have three different modes as [6].

The matching TR matrix is given by

$$\begin{pmatrix} -2.0 + \Delta\xi_{11}(\delta) & ? & ?\\ ? & ? & 1.0 + \Delta\xi_{23}(\delta)\\ ? & ? & ? \end{pmatrix}.$$

Assume voltage U as the control input and $x(t) = [x_1(t)x_2(t)]^T = [I_L(t)v_c(t)]^T$. Consequence, the membership functions $q_1(x_2(t))$ and $q_2(x_2(t))$ are chosen, as $q_2(x_2(t)) = 1 - q_1(x_2(t))$ with $q_1(x_2(t)) = 1 - \frac{x_2^2(t)}{9}$. It is known that $q_1(x_2(t)) = 1$ and $q_2(x_2(t)) = 0$ when $x_2(t)$ is about 0 rad, and $q_1(x_2(t)) = 0$ and $q_2(x_2(t)) = 1$ when $x_2(t)$ is about 3 rad or -3 rad. Then, two-rule fuzzy model parameters of electric circuit system can be described as:

$$\begin{aligned}
\mathcal{A}_1(1) &= \begin{bmatrix} -20 & -10\\ 10 & 2\end{bmatrix}, \mathcal{A}_2(1) = \begin{bmatrix} -20 & -10\\ 10 & -16\end{bmatrix},\\
\mathcal{A}_1(2) &= \begin{bmatrix} -10 & -5\\ 2 & 0.4\end{bmatrix}, \mathcal{A}_2(2) = \begin{bmatrix} -10 & -5\\ 2 & -3.2\end{bmatrix},\\
\mathcal{A}_1(3) &= \begin{bmatrix} -4 & -2\\ 1 & 0.2\end{bmatrix}, \mathcal{A}_2(3) = \begin{bmatrix} -4 & -2\\ 1 & -1.6\end{bmatrix},\\
\mathcal{B}_i(1) &= \begin{bmatrix} 0\\ 1\end{bmatrix}, \mathcal{B}_i(2) = \begin{bmatrix} 0\\ 0.5\end{bmatrix}, \mathcal{B}_i(3) = \begin{bmatrix} 0\\ 0.8\end{bmatrix},\\
\mathcal{K}(g) &= \begin{bmatrix} 1 & -1\end{bmatrix}, (i = 1, 2, m = 1, 2, 3).
\end{aligned}$$

The dead-zone nonlinear input function is proposed by:

$$\varphi(z(t)) = \begin{cases} (1.5 - 0.6e^{0.4|\sin z(t)|})(z(t) - z_\dagger) & z(t) > z_\dagger\\ 0 & |z(t)| \le z_\dagger\\ (1.5 - 0.6e^{0.4|\sin z(t)|})(z(t) + z_\dagger) & z(t) < -z_\dagger\end{cases}$$

Subsequently, the parameters of the system are given by: $z_\dagger = 2$, $\sigma = 0.5$, $\mathcal{G}(1) = [01]$, $\mathcal{G}(2) = [02]$, $\mathcal{G}(3) = [01.25]$, $h(x(t), t) = 0.4\cos x(t)$, $\Delta\xi_{kl}(\delta) \le |0.15 * \xi_{kl}|$, $b_m = 0.1$ and $b_M = 0.9$, $\ell = 0.03$, $b = 0.5, \kappa = 0.01$. γ in $s(t)/(|s(t)| + \gamma)$ is 0.0001. Notice that $\xi_{3l}(\delta)(l = 1, 2, 3)$ are all unknown, thus, we solve optimization algorithm (26)

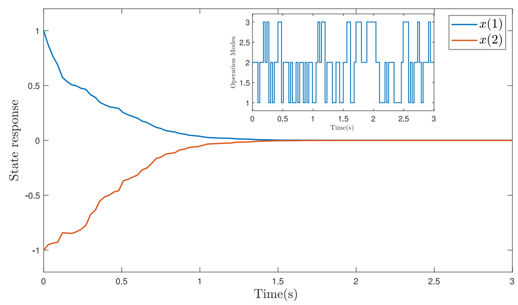

Fig. 2. State response of the system under SMC.

via MATLAB tool-box to obtain the following feasible solutions:

$$a_3 = 12.3019, \Gamma_1 = 92.0687, \Gamma_2 = 87.9311, \Gamma_3 = 87.7904.$$

Choosing $x_0 = [1 \ \ -1]^T$ as the initial state, the simulation results are displayed in Figure 2, which depicts the state response of the system under SMC. Obviously, the state can converge to 0 in 1.5s.

V. Conclusion

In this brief, we have studied the event-triggered SMC for fuzzy S-MJSs under the constraint of dead-zone input and partially known TRs. The sufficient conditions have been proposed to guarantee the stochastic stability of the fuzzy S-MJSs. Then, the SMC law is proposed to realize finite-time reachability of the designed sliding surface. In the end, an electric circuit system is exhibited to validate the merits of theoretical results. The proposed strategy can be developed in other real systems such as DC motors [8]. Moreover, the dead zone input is assumed to be antisymmetric, so the event-triggered control problem of S-MJSs can be extended to the case of asymmetric dead zone, which will be an interesting topic for our future work.

References

[1] D. Zhang, Z. Xu, D. Srinivasan, and L. Yu, "Leader–follower consensus of multiagent systems with energy constraints: A Markovian system approach," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 47, no. 7, pp. 1727–1736, Jul. 2017.

[2] L. Zhang, T. Yang, P. Shi, and M. Liu, "Stability and stabilization of a class of discrete-time fuzzy systems with semi-Markov stochastic uncertainties," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 46, no. 12, pp. 1642–1653, Dec. 2016.

[3] B. Jiang and C. Gao, "Decentralized adaptive sliding mode control of large-scale semi-Markovian jump interconnected systems with dead-zone input," *IEEE Trans. Autom. Control*, vol. 67, no. 3, pp. 1521–1528, Mar. 2022.

[4] J. Huang and Y. Shi, "Stochastic stability of semi-Markov jump linear systems: An LMI approach," in *Proc. 50th IEEE Conf. Decis. Control Eur. Control Conf.*, 2011, pp. 4668–4673.

[5] Y. Tan, Q. Liu, J. Liu, X. Xie, and S. Fei, "Observer-based security control for interconnected semi-Markovian jump systems with unknown transition probabilities," *IEEE Trans. Cybern.*, vol. 52, no. 9, pp. 9013–9025, Sep. 2022.

[6] W. Qi, M. Gao, C. K. Ahn, J. Cao, J. Cheng, and L. Zhang, "Quantized fuzzy finite-time control for nonlinear semi-Markov switching systems," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 67, no. 11, pp. 2622–2626, Nov. 2020.

[7] G. Zhuang, S. Su, J. Xia, and W. Sun, "HMM-based asynchronous H_∞ filtering for fuzzy singular Markovian switching systems with retarded time-varying delays," *IEEE Trans. Cybern.*, vol. 51, no. 3, pp. 1189–1203, Mar. 2021.

[8] Z. Gu, S. Yan, C. K. Ahn, D. Yue, and X. Xie, "Event-triggered dissipative tracking control of networked control systems with distributed communication delay," *IEEE Syst. J.*, vol. 16, no. 2, pp. 3320–3330, Jun. 2022.

[9] X. Fan, Z. Wang, and Z. Shi, "Event-triggered integral sliding mode control for uncertain fuzzy systems," *Fuzzy Sets Syst.*, vol. 416, pp. 47–63, Jul. 2021.

[10] W. Qi, Y. Hou, G. Zong, and C. K. Ahn, "Finite-time event-triggered control for semi-Markovian switching cyber-physical systems with FDI attacks and applications," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 68, no. 6, pp. 2665–2674, Jun. 2021.

[11] Z. Gu, J. H. Park, D. Yue, Z. Wu, and X. Xie, "Event-triggered security output feedback control for networked interconnected systems subject to cyber-attacks," *IEEE Trans. Syst., Man, Cybern., Syst.*, vol. 51, no. 10, pp. 6197–6206, Oct. 2021.

[12] W. Qi, G. Zong, and W. X. Zheng, "Adaptive event-triggered SMC for stochastic switching systems with semi-Markov process and application to boost converter circuit model," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 68, no. 2, pp. 786–796, Feb. 2021.

[13] M. Shen, Y. Gu, J. H. Park, Y. Yi, and W.-W. Che, "Composite control of linear systems with event-triggered inputs and outputs," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol. 69, no. 3, pp. 1154–1158, Mar. 2022.

[14] Y. Wang, Y. Xia, H. Shen, and P. Zhou, "SMC design for robust stabilization of nonlinear Markovian jump singular systems," *IEEE Trans. Autom. Control*, vol. 63, no. 1, pp. 219–224, Jan. 2018.

[15] H. Razmi and S. Afshinfar, "Neural network-based adaptive sliding mode control design for position and attitude control of a quadrotor UAV," *Aerosp. Sci. Technol.*, vol. 91, pp. 12–27, Aug. 2019.

[16] Q. Zhou, S. Zhao, H. Li, R. Lu, and C. Wu, "Adaptive neural network tracking control for robotic manipulators with dead zone," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 30, no. 12, pp. 3611–3620, Dec. 2019.

[17] G. Zong, Y. Wang, H. R. Karimi, and K. Shi, "Observer-based adaptive neural tracking control for a class of nonlinear systems with prescribed performance and input dead-zone constraints," *Neural Netw.*, vol. 147, pp. 126–135, Mar. 2022.

[18] Y. Li and G. Yang, "Model-based adaptive event-triggered control of strict-feedback nonlinear systems," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 29, no. 4, pp. 1033–1045, Apr. 2018.

[19] J. Liu and D. Yue, "Event-triggering in networked systems with probabilistic sensor and actuator faults," *Inf. Sci.*, vol. 240, pp. 145–160, Aug. 2013.

[20] J. Huang and Y. Shi, "Stochastic stability and robust stabilization of semi-Markov jump linear systems," *Int. J. Robust Nonlinear Control*, vol. 23, no. 18, pp. 2028–2043, 2013.