

# Interval Type-2 Fuzzy-Model-Based Filtering for Nonlinear Systems With Event-Triggering Weighted Try-Once-Discard Protocol and Cyberattacks

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**Abstract**—This article addresses the fuzzy filtering problem for nonlinear systems based on the interval type-2 fuzzy model. To make better use of the constrained resources for communication, a novel event-triggering weighted try-once-discard protocol (ET-WTODP) is proposed. Different from the existing diverse protocols, ET-WTODP only allows a portion of components with relatively greater deviation to transmit at each triggering moment. The fuzzy filter is constructed with imperfectly matched membership functions and mode-dependent parameters. Denial-of-service (DoS) attacks frequently occur in communication networks are also considered. The overall fuzzy filtering error system (OFES) model is established taking into account ET-WTODP and DoS attacks. The sufficient conditions for the OFES to be asymptotically stable with a preset  $H_\infty$  performance level are identified by using the Lyapunov stability theory to obtain the desired asynchronous filter. Finally, numerical simulations are presented for verification that the proposed filtering approach is feasible and valid.

**Index Terms**—Denial-of-service (DoS) attacks, event-triggering scheme, interval type-2 (IT2) fuzzy system, weighted try-once-discard protocol (WTODP).

## I. INTRODUCTION

WITH the advancement of networked control technology, the Takagi–Sugeno (T–S) fuzzy model has gained recognition as a potent instrument for handling intricate nonlinear systems [1], [2]. The traditional interval type-1 T–S fuzzy model can

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effectively capture the system nonlinearities through the precise membership function. However, due to the uncertainty of the parameters, it can be difficult to obtain the precise membership function for approximating a nonlinear system in practice [3], [4]. To address the increasing complexity of the modern industry, the interval type-2 (IT2) T–S fuzzy model has been widely used to overcome this challenge [4], [5], [6]. On the other hand, the filtering or state estimation problem has been of interest as an important research theme in system control and signal processing [7], [8], [9], [10]. Among various filtering techniques that have been developed,  $H_\infty$  filtering stands out for its supply of ensured levels of noise attenuation and is robust to model uncertainties, without prior statistical information about the external noises [11]. For instance, in [12], the acknowledgment-based  $H_\infty$  filtering was addressed for IT2 fuzzy system with an attack-resilient event-triggered scheme. In [13], a nonfragile  $H_\infty$  filter was designed for fuzzy-model-based nonlinear switched systems under quantization, while the parallel distribution compensation (PDC) design concept constrains flexibility in the design process and makes it more difficult to devise fuzzy filters. Therefore, the design of the non-PDC IT2 fuzzy filter has to be investigated with desperate need, and this becomes one of the impetuses for the research in this article.

The evolution of wireless communication networks facilitates the information exchange between separate system components whereas greater communication and computing resources are needed for high-frequency data transmission [14], [15]. Aiming to economize on the limited network bandwidth, rapidly growing concerns have been seen in the survey of various event-triggering schemes and data scheduling protocols over recent years. The event-triggering scheme, which releases those sampled data violating a certain event-triggering condition, was first proposed in [16] and [17], and then improved in [18], [19], and [20], dedicated to reducing the transmission of redundant data. In comparison with event-triggering schemes, data scheduling protocols, such as round-robin protocol [21], stochastic communication protocol [22], weighted try-once-discard protocol (WTODP) [23], and FlexRay protocol [24] permit merely one sensor node to transmit at each time instant to avoid data conflicts by properly sequencing the transmission of each participating sensor node. The study focusing on event-triggered and protocol-based nonlinear systems has attracted plenty of

attention from scholars. For example, the event-triggered distributed filtering problem was addressed in [25] and investigated in [26], respectively. In [27], the  $H_\infty$  filtering problem was addressed for a round-robin protocol-based nonlinear system. Considering stochastic communication protocol, the  $H_\infty$  filtering problem for high-rate communication network-based systems was studied in [11]. However, the investigation of IT2 fuzzy-model-based filtering considering both the event-triggering scheme and data scheduling protocol concurrently has not been fully studied, which sheds some light on our research.

Besides limited network bandwidth, another critical problem of networked systems that requires serious consideration is network security [28]. Consequently, continuous interest has been aroused in the modeling and synthetical analysis of cyberattacks [29], [30], [31], [32], [33]. As the most typical type of cyberattacks, DoS attacks are intended to jam the channels of the network at an unpredictable time so that the signal that should be sent cannot be successfully transmitted over the network. Meanwhile, with DoS attacks described by the periodic attack model, the resilient  $H_\infty$  filtering for nonlinear systems was studied in [34]. Moreover, the mathematical model based on the Bernoulli distributed variable has been formulated in [35], which conveniently reflects the probability characteristics of DoS attacks. Accordingly, the problem of  $H_\infty$  filtering for IT2 fuzzy systems becomes more challenging because of the combined effect of ET-WTODP and DoS attacks, which has not been adequately addressed, and this is another incentive for this article.

Summarizing the abovementioned discussions, this article aims to devise an asynchronous fuzzy filter such that the overall fuzzy filtering error system (OFES) satisfies asymptotic stability with a certain  $H_\infty$  performance level. Although the asynchronous filtering for Markov switched systems was studied in [36], considering event-triggered WTODP and deception attacks, this method cannot apply when DoS attacks occur under the ET-WTODP strategy. In addition, different from the event-triggered WTODP where the event-triggering scheme and WTODP occur sequentially in [36], the ET-WTODP in our work combines the event-triggering scheme and WTODP concurrently. The primary contributions of our work are as follows.

- 1) A novel data transmission strategy ET-WTODP, which simultaneously integrates the event-triggering scheme with WTODP is proposed. This strategy can dynamically regulate the number of sensor nodes that transmit data based on the relationship between the thresholds and each component of the measured output.
- 2) The fuzzy filter is developed with imperfectly matched MFs under the non-PDC framework to increase its flexibility, and the mode-dependant gain parameters in the filter mirror the impact of the separate transmission cases under ET-WTODP.
- 3) A novel model of the OFES is formulated and sufficient conditions are deduced to guarantee the asymptotic stability of the OFES with preset disturbance attenuation performance index.

The rest of this article is organized as follows. The IT2 fuzzy-model-based networked system, asynchronous fuzzy filter, and

other introductions to the proposed ET-WTODP strategy are described in Section II. The main results are derived and the relevant filter gains are determined in Section III. The feasibility of the designed filter is verified by a simulation example in Section IV. Finally, Section V concludes this article.

*Notations:* The notations utilized in this article are standard. Specially,  $\mathcal{H}_e\{X\}$  stands for  $X + X^T$ .

## II. PROBLEM FORMULATION

### A. IT2 Fuzzy Networked System

Consider the networked system based on IT2 fuzzy model with  $v$  rules as follows.

*Plant rule  $i$ :* IF  $t_1(x(h))$  is  $H_1^i$ ,  $t_2(x(h))$  is  $H_2^i$ ,  $\dots$ ,  $t_n(x(h))$  is  $H_n^i$ , THEN

$$\begin{cases} x(h+1) = A_i x(h) + B_i \omega(h) \\ y(h) = C_i x(h) + D_i \omega(h) \\ z(h) = L_i x(h) \end{cases} \quad (1)$$

where  $H_a^i$  denotes the fuzzy sets,  $t_a(x(h))$  is the premise variables ( $i = 1, 2, \dots, v$ ,  $a = 1, 2, \dots, n$ ),  $v$  and  $n$  represent the number of IF-THEN rules and premise variables for the system.  $x(h) \in \mathbb{R}^{n_x}$  is the system state,  $y(h) \in \mathbb{R}^{n_y}$  represents the measured output, and  $\omega(h) \in \mathbb{R}^{n_\omega}$  is external disturbance satisfying  $\omega(h) \in \mathbb{L}_2[0, \infty)$ , respectively.  $z(h) \in \mathbb{R}^{n_z}$  is the output to be estimated.  $A_i, B_i, C_i, D_i$ , and  $L_i$  are constant matrices with proper dimensions. The following interval sets define the firing strength of the  $i$ th rule:

$$\mathcal{E}_i(x(h)) = [\underline{\epsilon}_i(x(h)), \bar{\epsilon}_i(x(h))]$$

where  $\epsilon_i(x(h)) = \prod_{a=1}^n \underline{\mu}_{H_a^i}(t_a(x(h))) \geq 0$ ,  $\bar{\epsilon}_i(x(h)) = \prod_{a=1}^n \bar{\mu}_{H_a^i}(t_a(x(h))) \geq 0$ .  $\underline{\epsilon}_i(x(h))$ , and  $\bar{\epsilon}_i(x(h))$  represent lower and upper grades of membership (LUM) with  $\bar{\epsilon}_i(x(h)) \geq \underline{\epsilon}_i(x(h))$ .  $\underline{\mu}_{H_a^i}(t_a(x(h)))$  and  $\bar{\mu}_{H_a^i}(t_a(x(h)))$  are the lower and upper MFs (LUMFs) with  $\bar{\mu}_{H_a^i}(t_a(x(h))) \geq \underline{\mu}_{H_a^i}(t_a(x(h))) \geq 0$ .

Similar to [37], system (1) can be derived by the following explicit expression:

$$\begin{cases} x(h+1) = \sum_{i=1}^v \epsilon_i(x(h)) [A_i x(h) + B_i \omega(h)] \\ y(h) = \sum_{i=1}^v \epsilon_i(x(h)) [C_i x(h) + D_i \omega(h)] \\ z(h) = \sum_{i=1}^v \epsilon_i(x(h)) L_i x(h) \end{cases} \quad (2)$$

where

$$\epsilon_i(x(h)) = \frac{\varrho_i(x(h))}{\sum_{i=1}^v \varrho_i(x(h))} \geq 0, \quad \sum_{i=1}^v \epsilon_i(x(h)) = 1$$

$$\varrho_i(x(h)) = \underline{\chi}_i(x(h)) \underline{\epsilon}_i(x(h)) + \bar{\chi}_i(x(h)) \bar{\epsilon}_i(x(h)).$$

$\epsilon_i(x(h))$  is the normalized membership.  $\underline{\chi}_i(x(h)) \in [0, 1]$  and  $\bar{\chi}_i(x(h)) \in [0, 1]$  are nonlinear weighting functions (NWFs) with  $\underline{\chi}_i(x(h)) + \bar{\chi}_i(x(h)) = 1$ .

### B. Event-Triggering WTODP

It is illustrated in Fig. 1 that  $y(h)$  is delivered via the communication link and scheduled by a data transmission strategy. For analysis simplify, we suppose there are  $n_y$  sensor

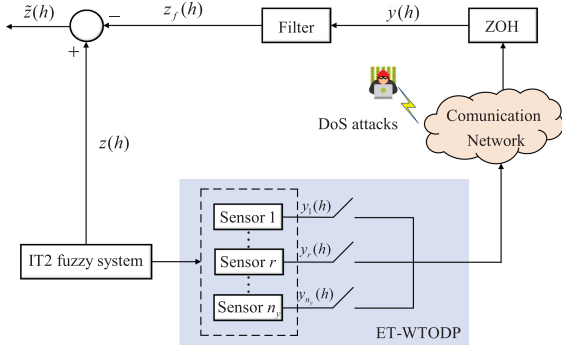


Fig. 1. Diagram of nonlinear system with ET-WTODP strategy and DoS attacks.

nodes, and define  $y(h) = [y_1^T(h), y_2^T(h), \dots, y_{n_y}^T(h)]^T$  with  $y_r(h)$  ( $r = 1, 2, \dots, n_y$ ) denoting the measured output of the  $r$ th sensor.

Define  $\mu_r(h)$  for  $r$ th sensor node

$$\mu_r(h) = (y_r(l_{m-1}^r) - y_r(h))^T \phi_r (y_r(l_{m-1}^r) - y_r(h)) \quad (3)$$

where  $y_r(l_{m-1}^r)$  represents the signal sent by sensor node  $r$  to the filter at the previous time, and  $\phi_r > 0$  is a given weight parameter. Thus,  $\mu_r(h)$  shows the deviation of the present measured output of the  $r$ th sensor node from its previously transmitted one arriving at the filter. According to conventional WTODP [4], the index of the sensor node selected to be sent at time instant  $h$  can be denoted as

$$\psi(h) = \arg \max_{1 \leq r \leq n_y} \mu_r(h) \quad (4)$$

where  $\psi(h) \in \{1, 2, \dots, n_y\}$ , that is, each sampling instant allows only one sensor node to transmit.

Different from the WTODP mentioned above, a novel data transmission strategy that integrates the event-triggering scheme and WTODP simultaneously is proposed in this article. Given two positive thresholds  $L_{\max}$  and  $L_{\min}$  which satisfy  $L_{\max} > L_{\min}$ , as well as a positive scalar  $\lambda_r$ . For  $r \in \{1, 2, \dots, n_y\}$ , we are going to consider the following three scenarios.

- 1) *Case A*: If there exist  $\mu_r(h) \geq L_{\max} y_r^T(h) \lambda_r y_r(h)$ , all those  $y_r(h)$  will be delivered at time instant  $h$ .
- 2) *Case B*: If none of the  $\mu_r(h)$  satisfy *Case A* but there are any  $\mu_r(h) \geq L_{\min} y_r^T(h) \lambda_r y_r(h)$  exist, only the  $r$ th sensor node with the greatest  $\mu_r(h)$  will be delivered.
- 3) *Case C*: If there exist no  $\mu_r(h)$  satisfying *Case A* and *B*, but all  $\mu_r(h) < L_{\min} y_r^T(h) \lambda_r y_r(h)$ , then, no signal will be transmitted at time instant  $h$ .

For the sake of analysis, we set  $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$  to represent those instants satisfying *Case A, B, C*, respectively. Then, we will analyze the transmission of the measured output  $y(h)$  under the abovementioned three cases.

*Case A*: For  $h \in \mathcal{S}_1$ , there exist  $\mu_r(h)$  satisfying

$$\mu_r(h) \geq L_{\max} y_r^T(h) \lambda_r y_r(h). \quad (5)$$

Given the proposed ET-WTODP strategy and the potential DoS attacks arising from the openness of the communication network, the comprehensive updating rule for  $y_r(l_m^r)$  under

zero-order holder scheme is described as follows:

$$y_r(l_m^r) = \begin{cases} (1 - \nu(h))y_r(h), & \text{if } y_r(h) \text{ satisfies (5)} \\ y_r(l_{m-1}^r), & \text{otherwise} \end{cases} \quad (6)$$

where  $y_r(l_m^r)$  denotes the measured output arrived at the filter from the  $r$ th sensor at current instant.  $\nu(h)$  is a Bernoulli-distributed variable satisfying

$$\text{Prob}\{\nu(h) = 0\} = 1 - \bar{\nu}, \text{Prob}\{\nu(h) = 1\} = \bar{\nu} \quad (7)$$

where  $\bar{\nu} \in [0, 1]$  is a given constant. When  $y_r(h)$  satisfies (5),  $\nu(h) = 0$  means sensor node  $r$  is not suffering from DoS attacks, and the obtainable signal for the filter is  $y_r(h)$ ,  $\nu(h) = 1$  represents sensor node  $r$  is under DoS attacks and the corresponding filter input is 0.

Define  $e(h) = [e_1^T(h), \dots, e_{n_y}^T(h)]^T$ ,  $e_r(h) = y_r(l_m^r) - y_r(h)$ , ( $r = 1, 2, \dots, n_y$ ). When  $y_r(h)$  satisfies (5),  $e_r(h) = (1 - \nu(h))y_r(h) - y_r(h)$ , the value of  $e_r^T(h)\phi_r e_r(h)$  is 0 or  $y_r^T(h)\phi_r y_r(h)$ , then we get  $L_{\max} y_r^T(h) \lambda_r y_r(h) \geq e_r^T(h)\phi_r e_r(h)$  with  $L_{\max} \lambda_r \geq \phi_r$ . When  $y_r(h)$  does not satisfy (5), we have  $y_r(l_m^r) = y_r(l_{m-1}^r)$ , then  $e_r^T(h)\phi_r e_r(h) = \mu_r(h)$ , which means  $L_{\max} y_r^T(h) \lambda_r y_r(h) > e_r^T(h)\phi_r e_r(h)$ . Then, it is not tough to get that

$$\sum_{r=1}^{n_y} [L_{\max} y_r^T(h) \lambda_r y_r(h) - e_r^T(h)\phi_r e_r(h)] \geq 0 \quad (8)$$

and the formula for the following holds:

$$L_{\max} y^T(h) \Lambda y(h) - e^T(h) \Phi e(h) \geq 0 \quad (9)$$

where

$$\Lambda = \text{diag}\{\lambda_1, \lambda_2, \dots, \lambda_{n_y}\}, \Phi = \text{diag}\{\phi_1, \phi_2, \dots, \phi_{n_y}\}.$$

*Case B*: For  $h \in \mathcal{S}_2$ , there exist  $\mu_r(h)$  satisfying

$$\mu_r(h) \geq L_{\min} y_r^T(h) \lambda_r y_r(h). \quad (10)$$

In this case, the finally selected sensor is determined based on (4), and the signal arrived at the filter which is denoted as  $\bar{y}_r(h)$  is given as

$$\bar{y}_r(h) = \begin{cases} (1 - \nu(h))y_r(h), & \text{if } r = \psi(h) \\ \bar{y}_r(h-1), & \text{otherwise.} \end{cases} \quad (11)$$

Define  $\bar{y}(h) = [\bar{y}_1^T(h), \bar{y}_2^T(h), \dots, \bar{y}_{n_y}^T(h)]^T$ , and we have

$$\bar{y}(h) = (1 - \nu(h))\Psi_{\psi(h)} y(h) + \tilde{\Psi}_{\psi(h)} \bar{y}(h-1) \quad (12)$$

where  $\Psi_{\psi(h)} = \text{diag}\{\delta(\psi(h) - 1), \dots, \delta(\psi(h) - n_y)\}$ ,  $\tilde{\Psi}_{\psi(h)} = I - \Psi_{\psi(h)}$ , and  $\bar{y}(h) \in \mathbb{R}^{n_y}$  is the filter input.  $\delta(\cdot) \in \{0, 1\}$  is the Kronecker delta function.

*Remark 1*: It is noted that the measured output of sensor node  $r$  available for the filter at the present instant is denoted as  $y_r(l_m^r)$  in *Case A* and  $\bar{y}_r(h)$  in *Case B* separately to distinguish (7) from (12). Similarly, the signal sent by sensor node  $r$  to the filter at the previous time in this article is denoted in different notation (i.e.,  $y_r(l_{m-1}^r)$  in *Case A* and  $\bar{y}_r(h-1)$  in *Case B*) just for the convenience of later derivation.

Case C: For  $h \in \mathcal{S}_3$ , all  $\mu_r(h)$  satisfy

$$\mu_r(h) < L_{\min} y_r^T(h) \lambda_r y_r(h). \quad (13)$$

In this case, by adopting the hold input strategy, the obtainable data of the filter is

$$\bar{y}_r(h) = \bar{y}_r(h-1). \quad (14)$$

According to (13) and (14), for all  $y_r(h)$ , it can be derived that  $L_{\min} y_r^T(h) \lambda_r y_r(h) > (\bar{y}_r(h-1) - y_r(h))^T \phi_r (\bar{y}_r(h-1) - y_r(h))$ , that is

$$\sum_{r=1}^{n_y} [L_{\min} y_r^T(h) \lambda_r y_r(h) - e_r^T(h) \phi_r e_r(h)] > 0. \quad (15)$$

Then, it is easily derived that

$$L_{\min} y^T(h) \Lambda y(h) - e^T(h) \Phi e(h) \geq 0. \quad (16)$$

*Remark 2:* The ET-WTODP strategy is proposed based on the event-triggering property of WTODP, which simultaneously integrates the event-triggering scheme with the WTODP. According to the selection principle interpreted above, the superiority of ET-WTODP is that it can dynamically adjust the number of the selected sensors based on the relationship between the thresholds  $L_{\max}$ ,  $L_{\min}$  and each measured output component  $y_r(h)$ . With limited transmission resources, this strategy will be more beneficial to save network bandwidth and guarantee the filtering performance, which is also later proved in the simulation results.

*Remark 3:* Unlike traditional WTODP, which delivers only one sensor node per moment, multiple nodes may be sent at each time under ET-WTODP. Moreover, no signals are transmitted when all the measured output components show a small deviation from the last transmitted ones.

### C. Asynchronous Fuzzy-Model-Based Filter

The filter with asynchronous MFs is constructed as follows.

*Filter rule j:* IF  $g_1(x(h))$  is  $G_1^j$ ,  $g_2(x(h))$  is  $G_2^j$ , ...,  $g_u(x(h))$  is  $G_u^j$ , THEN

$$\begin{cases} x_f(h+1) = A_{fj,\psi(h)} x_f(h) + B_{fj,\psi(h)} \bar{y}(h) \\ z_f(h) = C_{fj,\psi(h)} x_f(h) \end{cases} \quad (17)$$

where  $G_b^j$  denotes the fuzzy sets,  $g_b(x(h))$  is the premise variables ( $j = 1, 2, \dots, v, b = 1, 2, \dots, u$ ),  $v$  and  $u$  represent the number of IF-THEN rules of the filter and the premise variables.  $x_f(h) \in \mathbb{R}^{n_x}$  and  $z_f(h) \in \mathbb{R}^{n_z}$  are the filter state and estimation output.  $A_{fj,\psi(h)}$ ,  $B_{fj,\psi(h)}$ , and  $C_{fj,\psi(h)}$  are filter parameter matrices to be devised. The firing strength of the  $j$ th rule is defined as

$$\mathcal{T}_j(x(h)) = [\underline{\tau}_j(x(h)), \bar{\tau}_j(x(h))]$$

where  $\underline{\tau}_j(x(h)) = \prod_{b=1}^u \underline{\mu}_{G_b^j}(g_b(x(h))) \geq 0$ ,  $\bar{\tau}_j(x(h)) = \prod_{b=1}^u \bar{\mu}_{G_b^j}(g_b(x(h))) \geq 0$ .  $\underline{\tau}_j(x(h))$  and  $\bar{\tau}_j(x(h))$  denote LUM with  $\bar{\tau}_j(x(h)) \geq \underline{\tau}_j(x(h))$ .  $\underline{\mu}_{G_b^j}(g_b(x(h)))$  and  $\bar{\mu}_{G_b^j}(g_b(x(h)))$  are the LUMFs with  $\bar{\mu}_{G_b^j}(g_b(x(h))) \geq \underline{\mu}_{G_b^j}(g_b(x(h))) \geq 0$ .

The fuzzy filter is then expressed as

$$\begin{cases} x_f(h+1) = \sum_{j=1}^v \tau_j(x(h)) [A_{fj,\psi(h)} x_f(h) + B_{fj,\psi(h)} \bar{y}(h)] \\ z_f(h) = \sum_{j=1}^v \tau_j(x(h)) C_{fj,\psi(h)} x_f(h) \end{cases} \quad (18)$$

where

$$\tau_j(x(h)) = \frac{\varepsilon_j(x(h))}{\sum_{j=1}^v \varepsilon_j(x(h))} \geq 0, \quad \sum_{j=1}^v \tau_j(x(h)) = 1$$

$$\varepsilon_j(x(h)) = \underline{\iota}_j(x(h)) \underline{\tau}_j(x(h)) + \bar{\iota}_j(x(h)) \bar{\tau}_j(x(h)).$$

$\tau_j(x(h))$  is the normalized membership.  $\underline{\iota}_j(x(h)) \in [0, 1]$  and  $\bar{\iota}_j(x(h)) \in [0, 1]$  are NWFs with  $\underline{\iota}_j(x(h)) + \bar{\iota}_j(x(h)) = 1$ .

*Remark 4:* The dissipative filtering problem without synchronous premise variables requirement was considered in [38] and [39] for Markov jump systems and nonlinear systems, respectively. Since identical MFs between the plant and filter are hardly satisfied again in network environments, the fuzzy filter in our article is designed with imperfect matching MFs, which increases the design flexibility and strengthens robustness in comparison with the PDC methods. Moreover, the filter parameter matrices  $A_{fj,\psi(h)}$ ,  $B_{fj,\psi(h)}$ , and  $C_{fj,\psi(h)}$  are associated with  $\psi(h)$ , which makes them more practical.

*Remark 5:* Despite some filtering problems that have been carried out in [12] and [40], the problem that we are addressing in this article is not the same as the existing ones. For example, in [40], an event-triggered positive polynomial fuzzy filter was designed with the consideration of disturbance and time delay without communication protocols and cyberattacks. By applying the switched system method, an event-triggered attack-resilient  $H_\infty$  filter was devised in [12] for the IT2 fuzzy system with quantization and DoS attacks. Nevertheless, the abovementioned references do not apply to scenarios where DoS attacks occur in the network with the ET-WTODP strategy. Therefore, considering the non-PDC scheme, ET-WTODP strategy, and DoS attacks, we propose an asynchronous fuzzy filter design method for nonlinear systems in our work.

Define  $\eta(h) = [x^T(h) \quad \bar{y}^T(h-1) \quad x_f^T(h)]^T$ ,  $y(l_m) = [y_1^T(l_m^1), \dots, y_{n_y}^T(l_m^{n_y})]^T$ , and let  $\tilde{z}(h) = z(h) - z_f(h)$ ,  $m = \psi(h)$ ,  $n = \psi(h+1)$ . For instant set  $\mathcal{S}_1$ , recalling (2), (6), and (18), we have

$$\begin{cases} \eta(h+1) = \sum_{i=1}^v \sum_{j=1}^v \varepsilon_i(x(h)) \tau_j(x(h)) [\tilde{A}_{ij,m} \eta(h) \\ \quad + \tilde{B}_{ij,m} \omega(h) + \tilde{C}_{ij,m} y(l_m)] \\ \tilde{z}(h) = \sum_{i=1}^v \sum_{j=1}^v \varepsilon_i(x(h)) \tau_j(x(h)) \tilde{L}_{ij,m} \eta(h), h \in \mathcal{S}_1 \end{cases} \quad (19)$$

where

$$\tilde{A}_{ij,m} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{fj,m} \end{bmatrix}, \quad \tilde{B}_{ij,m} = [B_i^T \quad 0 \quad 0]^T$$

$$\tilde{C}_{ij,m} = [0 \quad I \quad B_{fj,m}^T]^T, \quad \tilde{L}_{ij,m} = [L_i \quad 0 \quad -C_{fj,m}].$$

For  $h \in \mathcal{S}_2$ , according to (2), (12), and (18), one obtains

$$\begin{cases} \eta(h+1) = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h)) \tau_j(x(h)) [(\tilde{A}_{1ij,m} \\ + \tilde{A}_{2ij,m}) \eta(h) + (\tilde{B}_{1ij,m} + \tilde{B}_{2ij,m}) \omega(h)] \\ \tilde{z}(h) = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h)) \tau_j(x(h)) \tilde{L}_{ij,m} \eta(h), h \in \mathcal{S}_2 \end{cases} \quad (20)$$

where

$$\tilde{A}_{1ij,m} = \begin{bmatrix} A_i & 0 & 0 \\ \rho \Psi_m C_i & \tilde{\Psi}_m & 0 \\ \rho B_{fj,m} \Psi_m C_i & B_{fj,m} \tilde{\Psi}_m & A_{fj,m} \end{bmatrix}$$

$$\tilde{A}_{2ij,m} = \begin{bmatrix} 0 & 0 & 0 \\ -\tilde{\nu}(h) \Psi_m C_i & 0 & 0 \\ -\tilde{\nu}(h) B_{fj,m} \Psi_m C_i & 0 & 0 \end{bmatrix}$$

$$\tilde{B}_{1ij,m} = \begin{bmatrix} B_i \\ \rho \Psi_m D_i \\ \rho B_{fj,m} \Psi_m D_i \end{bmatrix}, \tilde{B}_{2ij,m} = \begin{bmatrix} B_i \\ -\tilde{\nu}(h) \Psi_m D_i \\ -\tilde{\nu}(h) B_{fj,m} \Psi_m D_i \end{bmatrix}$$

$$\rho = 1 - \bar{\nu}, \tilde{\nu}(h) = \nu(h) - \bar{\nu}.$$

For  $h \in \mathcal{S}_3$ , based on (2), (14), and (18), we get

$$\begin{cases} \eta(h+1) = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h)) \tau_j(x(h)) [\bar{A}_{ij,m} \eta(h) \\ + \bar{B}_{ij,m} \omega(h)] \\ \tilde{z}(h) = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h)) \tau_j(x(h)) \bar{L}_{ij,m} \eta(h), h \in \mathcal{S}_3 \end{cases} \quad (21)$$

where

$$\bar{A}_{ij,m} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & I & 0 \\ 0 & B_{fj,m} & A_{fj,m} \end{bmatrix}, \bar{B}_{ij,m} = [B_i^T \ 0 \ 0]^T.$$

For ease of description, (19), (20), and (21) are collectively referred as the OFES. In light of the foregoing elaboration, our goal is to devise an asynchronous fuzzy filter (18) that can satisfy the following requirements.

- 1) For  $\omega(h) = 0$ , the OFES within different instant sets  $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$  is asymptotically stable.
- 2) Under the zero-initial condition, for any  $\omega(h) \neq 0$ , the OFES satisfies

$$E \left\{ \sum_{h=0}^{\infty} \|\tilde{z}(h)\|^2 \right\} \leq \gamma^2 \sum_{h=0}^{\infty} \|\omega(h)\|^2 \quad (22)$$

and guarantees a predefined  $H_\infty$  attenuation level  $\gamma$ .

*Remark 6:* Since the measured output arrived at the fuzzy filter is different in  $\mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$ , the OFES is derived in three forms of expression separately for different time sets. To ensure the OFES satisfies asymptotic stability with predefined  $H_\infty$  performance, sufficient conditions for different transmission cases will be analyzed in the sequel.

### III. SECURITY FUZZY FILTER DESIGN

Theorems 1 and 2 are presented for the analysis and synthesis of the OFES with DoS attacks and ET-WTODP strategy.

#### A. Stability Analysis

*Theorem 1:* For given gain matrices  $A_{fj,m}, B_{fj,m}$ , and  $C_{fj,m}$ , positive parameters  $\phi_r, \lambda_r, L_{\max}, L_{\min}, \Lambda, \Phi, \bar{\Phi}, \bar{\nu}, \rho, \kappa, \varphi, \theta, \vartheta, \gamma, \tau_j(x(h)) - \zeta_j \epsilon_j(x(h)) > 0$  ( $\zeta_j > 0$ ), the asymptotic stability of the OFES with  $H_\infty$  performance level  $\gamma$  is obtained if there are matrices  $P_m > 0, M, N, Q$  exist, satisfying the inequalities below ( $i \leq j$ )

$$\Pi_{ijmn} + \Pi_{jimn} - 2M < 0 \quad (23)$$

$$\zeta_j \Pi_{ijmn} + \zeta_i \Pi_{jimn} - \zeta_j M - \zeta_i M + 2M < 0 \quad (24)$$

$$\Theta_{ijmn} + \Theta_{jimn} - 2N < 0 \quad (25)$$

$$\zeta_j \Theta_{ijmn} + \zeta_i \Theta_{jimn} - \zeta_j N - \zeta_i N + 2N < 0 \quad (26)$$

$$\Omega_{ijmn} + \Omega_{jimn} - 2Q < 0 \quad (27)$$

$$\zeta_j \Omega_{ijmn} + \zeta_i \Omega_{jimn} - \zeta_j Q - \zeta_i Q + 2Q < 0 \quad (28)$$

where

$$\Pi_{ijmn} = \begin{bmatrix} F_{1ijmn} & * & * & * \\ \tilde{C}_{ij,m}^T P_n \tilde{A}_{ij,m} & \tilde{C}_{ij,m}^T P_n \tilde{C}_{ij,m} & * & * \\ 0 & 0 & -\bar{\Phi} & * \\ F_{2ijmn} & \tilde{B}_{ij,m}^T P_n \tilde{C}_{ij,m} & 0 & F_{3ijmn} \end{bmatrix}$$

$$\Theta_{ijmn} = \begin{bmatrix} \Gamma_{1ijmn} & * \\ \Gamma_{2ijmn} & \Gamma_{3ijmn} \end{bmatrix}$$

$$\Omega_{ijmn} = \begin{bmatrix} \Upsilon_{1ijmn} & * & * \\ 0 & -\bar{\Phi} & * \\ \Upsilon_{2ijmn} & 0 & \Upsilon_{3ijmn} \end{bmatrix}$$

$$F_{1ijmn} = \tilde{A}_{ij,m}^T P_n \tilde{A}_{ij,m} + \tilde{L}_{ij,m}^T \tilde{L}_{ij,m} + \tilde{C}_i^T \theta \tilde{C}_i - P_m$$

$$F_{2ijmn} = \tilde{B}_{ij,m}^T P_n \tilde{A}_{ij,m} + D_i^T \theta \tilde{C}_i$$

$$F_{3ijmn} = \tilde{B}_{ij,m}^T P_n \tilde{B}_{ij,m} + D_i^T \theta D_i - \gamma^2 I$$

$$\bar{\Phi} = \varphi \Phi, \theta = \varphi L_{\max} \Lambda, \tilde{C}_i = C_i E, E = [I \ 0 \ 0]$$

$$\Gamma_{1ijmn} = \tilde{A}_{1ij,m}^T P_n \tilde{A}_{1ij,m} + \tilde{L}_{ij,m}^T \tilde{L}_{ij,m} \\ + \tilde{A}_{2ij,m}^T P_n \tilde{A}_{2ij,m} - P_m$$

$$\Gamma_{2ijmn} = \tilde{B}_{1ij,m}^T P_n \tilde{A}_{1ij,m} + \tilde{B}_{2ij,m}^T P_n \tilde{A}_{2ij,m}$$

$$\Gamma_{3ijmn} = \tilde{B}_{1ij,m}^T P_n \tilde{B}_{1ij,m} + \tilde{B}_{2ij,m}^T P_n \tilde{B}_{2ij,m} - \gamma^2 I$$

$$\bar{A}_{2ij,m} = \begin{bmatrix} 0 & 0 & 0 \\ -\kappa \Psi_m C_i & 0 & 0 \\ -\kappa B_{fj,m} \Psi_m C_i & 0 & 0 \end{bmatrix}$$

$$\bar{B}_{2ij,m} = \begin{bmatrix} B_i \\ -\kappa \Psi_m D_i \\ -\kappa B_{fj,m} \Psi_m D_i \end{bmatrix}, \kappa = \sqrt{\bar{\nu}(1 - \bar{\nu})}$$

$$\Upsilon_{1ijmn} = \bar{A}_{ij,m}^T P_n \bar{A}_{ij,m} + \bar{L}_{ij,m}^T \bar{L}_{ij,m} + \bar{C}_i^T \vartheta \bar{C}_i - P_m$$

$$\Upsilon_{2ijmn} = \bar{B}_{ij,m}^T P_n \bar{A}_{ij,m} + D_i^T \vartheta \bar{C}_i$$

$$\Upsilon_{3ijmn} = \bar{B}_{ij,m}^T P_n \bar{B}_{ij,m} + D_i^T \vartheta D_i - \gamma^2 I, \vartheta = \varphi L_{\min} \Lambda.$$

*Proof:* Construct the mode-dependant Lyapunov function

$$V(h) = \eta^T(h)P_m\eta(h) \quad (29)$$

and define  $\mathcal{I}_D(h) = \Delta V(h) + \tilde{z}^T(h)\tilde{z}(h) - \gamma^2\omega^T(h)\omega(h)$  for derivation convenience.

*Case A:* Let  $\xi_1(h) = [\eta^T(h) \ y^T(l_m) \ e^T(h) \ \omega^T(h)]^T$ . By analyzing the time difference of  $V(h)$ , we obtain

$$\begin{aligned} & E\{\mathcal{I}_D(h)|h \in \mathcal{S}_1\} \\ & \leq E\{\eta^T(h+1)P_n\eta(h+1) - \eta^T(h)P_m\eta(h) + \tilde{z}^T(h)\tilde{z}(h) \\ & \quad - \gamma^2\omega^T(h)\omega(h) + \varphi[L_{\max}y^T(h)\Lambda y(h) - e^T(h)\Phi e(h)]\} \\ & = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\tau_j(x(h))\xi_1^T(h)\Pi_{ijmn}\xi_1(h). \end{aligned} \quad (30)$$

A slack matrix  $M$  with compatible dimension is employed to produce less conservative results

$$\begin{aligned} & \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))(\epsilon_j(x(h)) - \tau_j(x(h)))M \\ & = \sum_{i=1}^v \epsilon_i(x(h)) \left[ \left( \sum_{j=1}^v \epsilon_j(x(h)) - \sum_{j=1}^v \tau_j(x(h)) \right) M \right] = 0. \end{aligned} \quad (31)$$

Substituting (31) into (30), one obtains

$$\begin{aligned} & E\{\mathcal{I}_D(h)|h \in \mathcal{S}_1\} \\ & \leq \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\tau_j(x(h))\xi_1^T(h)\Pi_{ijmn}\xi_1(h) \\ & = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\xi_1^T(h)[\tau_j(x(h))\Pi_{ijmn} \\ & \quad + (\epsilon_j(x(h)) - \tau_j(x(h)))M]\xi_1(h) \\ & = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\xi_1^T(h)[\epsilon_j(x(h))(\zeta_j\Pi_{ijmn} - \zeta_j M \\ & \quad + M) + (\tau_j(x(h)) - \zeta_j\epsilon_j(x(h)))(\Pi_{ijmn} - M)]\xi_1(h) \\ & = \frac{1}{2} \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\xi_1^T(h)[\epsilon_j(x(h)) \\ & \quad \times (\zeta_j\Pi_{ijmn} + \zeta_i\Pi_{jimn} - \zeta_j M - \zeta_i M + 2M) \\ & \quad + (\tau_j(x(h)) - \zeta_j\epsilon_j(x(h))) \\ & \quad \times (\Pi_{ijmn} + \Pi_{jimn} - 2M)]\xi_1(h). \end{aligned} \quad (32)$$

*Case B:* According to (20), we define  $\xi_2(h) = [\eta^T(h) \ \omega^T(h)]^T$ . Then, it is derived that

$$\begin{aligned} & E\{\mathcal{I}_D(h)|h \in \mathcal{S}_2\} \\ & = E\{\eta^T(h+1)P_n\eta(h+1) - \eta^T(h)(P_m - \tilde{L}_{ij,m}^T\tilde{L}_{ij,m}) \\ & \quad \times \eta(h) - \gamma^2\omega^T(h)\omega(h)\} \end{aligned}$$

$$= \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\tau_j(x(h))\xi_2^T(h)\Theta_{ijmn}\xi_2(h). \quad (33)$$

*Case C:* Define  $\xi_3(h) = [\eta^T(h) \ e^T(h) \ \omega^T(h)]^T$ . Then, the difference of  $V(h)$  is taken to obtain that

$$\begin{aligned} & E\{\mathcal{I}_D(h)|h \in \mathcal{S}_3\} \\ & \leq E\{\eta^T(h+1)P_n\eta(h+1) - \eta^T(h)P_m\eta(h) + \tilde{z}^T(h)\tilde{z}(h) \\ & \quad - \gamma^2\omega^T(h)\omega(h) + \varphi[L_{\min}y^T(h)\Lambda y(h) - e^T(h)\Phi e(h)]\} \\ & = \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\tau_j(x(h))\xi_3^T(h)\Omega_{ijmn}\xi_3(h). \end{aligned} \quad (34)$$

Subsequently, the slack matrices  $N$  and  $Q$  are introduced for (33) and (34). Then, we have

$$\begin{aligned} & E\{\mathcal{I}_D(h)|h \in \mathcal{S}_2\} \\ & = \frac{1}{2} \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\xi_2^T(h)[\epsilon_j(x(h)) \\ & \quad \times (\zeta_j\Theta_{ijmn} + \zeta_i\Theta_{jimn} - \zeta_j N - \zeta_i N + 2N) \\ & \quad + (\tau_j(x(h)) - \zeta_j\epsilon_j(x(h))) \\ & \quad \times (\Theta_{ijmn} + \Theta_{jimn} - 2N)]\xi_2(h) \end{aligned} \quad (35)$$

$$\begin{aligned} & E\{\mathcal{I}_D(h)|h \in \mathcal{S}_3\} \\ & \leq \frac{1}{2} \sum_{i=1}^v \sum_{j=1}^v \epsilon_i(x(h))\xi_3^T(h)[\epsilon_j(x(h)) \\ & \quad \times (\zeta_j\Omega_{ijmn} + \zeta_i\Omega_{jimn} - \zeta_j Q - \zeta_i Q + 2Q) \\ & \quad + (\tau_j(x(h)) - \zeta_j\epsilon_j(x(h))) \\ & \quad \times (\Omega_{ijmn} + \Omega_{jimn} - 2Q)]\xi_3(h). \end{aligned} \quad (36)$$

In light of the abovementioned three cases, it can be obtained from (23) to (28) that for all  $h \in \mathcal{S}_1, \mathcal{S}_2, \mathcal{S}_3$

$$E\{\mathcal{I}_D(h)\} \leq 0. \quad (37)$$

Subsequently, when  $\omega(h) = 0$ , by utilizing the Schur complement lemma, one gets that  $E\{\Delta V(h)\} \leq 0$ , thus, the OFES satisfies asymptotic stability. Meanwhile, by summing both sides of (37) for time instant  $h$  from 0 to  $\infty$ , the  $H_\infty$  performance is obtained in (22).

*Remark 7:* For obtaining less conservative results, the two approaches listed below have been applied to the aforementioned analyzes.

- 1) Different from the conventional Lyapunov function in [41], the mode-dependant one (29) in our article is identified with  $m$ , which is induced by ET-WTODP. Accordingly, separate Lyapunov functions are provided for different transmission scenarios under ET-WTODP.
- 2) Similar to [37], the slack matrix  $M, N, Q$  with compatible dimensions are introduced to obtain (1), (35), and (36) in order to deal with the imperfectly matched MFs problem.

### B. Asynchronous Fuzzy Filter Design

The filter parameter matrices  $A_{fj,m}$ ,  $B_{fj,m}$ , and  $C_{fj,m}$  reflect the influence of ET-WTODP. In *Case B*, according to WTODP, it can be easily derive that  $m \in \{1, 2, \dots, n_y\}$ . Besides, we define  $m = 0$  for *Case C* since there is no signal transmitted, and define  $m = n_y + 1$  for *Case A*, respectively. That is, under the proposed ET-WTODP,  $m$  will take value in the set  $\{0, 1, \dots, n_y + 1\}$ . The following theorem and algorithm that provide the design method for the desired filter are presented consequently.

**Theorem 2:** For given positive scalars  $\phi_r, \lambda_r, L_{\max}, L_{\min}, \Lambda, \Phi, \bar{\Phi}, \bar{\nu}, \rho, \kappa, \varphi, \theta, \vartheta, \gamma, \tau_j(x(h)) - \zeta_j \epsilon_j(x(h)) > 0$  ( $\zeta_j > 0$ ), the OFES is asymptotically stable with  $H_\infty$  performance level  $\gamma$  if there are matrices  $P_m > 0, X_m > 0, Y_m > 0, Z_m > 0, M, N, Q$  exist such that the consequent linear matrix inequalities (LMIs) hold ( $i \leq j$ )

$$\begin{bmatrix} 2\bar{P}_{mn}^1 & * \\ \bar{\Delta}_{ij,m} + \bar{\Delta}_{ji,m} & 2\mathcal{M} \end{bmatrix} < 0 \quad (38)$$

$$\begin{bmatrix} 2\bar{P}_{mn}^1 & * \\ \sqrt{\zeta_j} \bar{\Delta}_{ij,m} + \sqrt{\zeta_i} \bar{\Delta}_{ji,m} & \sqrt{\zeta_j} \mathcal{M} + \sqrt{\zeta_i} \mathcal{M} \end{bmatrix} < 0 \quad (39)$$

$$\begin{bmatrix} 2\bar{P}_{mn}^2 & * \\ \bar{\Sigma}_{ij,m} + \bar{\Sigma}_{ji,m} & 2\mathcal{N} \end{bmatrix} < 0 \quad (40)$$

$$\begin{bmatrix} 2\bar{P}_{mn}^2 & * \\ \sqrt{\zeta_j} \bar{\Sigma}_{ij,m} + \sqrt{\zeta_i} \bar{\Sigma}_{ji,m} & \sqrt{\zeta_j} \mathcal{N} + \sqrt{\zeta_i} \mathcal{N} \end{bmatrix} < 0 \quad (41)$$

$$\begin{bmatrix} 2\bar{P}_{mn}^3 & * \\ \bar{\Xi}_{ij,m} + \bar{\Xi}_{ji,m} & 2\mathcal{Q} \end{bmatrix} < 0 \quad (42)$$

$$\begin{bmatrix} 2\bar{P}_{mn}^3 & * \\ \sqrt{\zeta_j} \bar{\Xi}_{ij,m} + \sqrt{\zeta_i} \bar{\Xi}_{ji,m} & \sqrt{\zeta_j} \mathcal{Q} + \sqrt{\zeta_i} \mathcal{Q} \end{bmatrix} < 0 \quad (43)$$

with

$$A_{fj,m} = X_m^{-1} \bar{A}_{fj,m}, B_{fj,m} = X_m^{-1} \bar{B}_{fj,m}, (m = n_y + 1)$$

$$A_{fj,m} = Y_m^{-1} \bar{A}_{fj,m}, B_{fj,m} = Y_m^{-1} \bar{B}_{fj,m}, (m \in \{1, \dots, n_y\})$$

$$A_{fj,m} = Z_m^{-1} \bar{A}_{fj,m}, B_{fj,m} = Z_m^{-1} \bar{B}_{fj,m}, (m = 0)$$

$$C_{fj,m} = C_{fj,m}, (m \in \{0, 1, \dots, n_y + 1\})$$

and in which

$$\bar{P}_{mn}^1 = \text{diag}\{P_n - \mathcal{H}_e\{X_m\}, P_n - \mathcal{H}_e\{X_m\}, -I, -\theta\}$$

$$\bar{P}_{mn}^2 = \text{diag}\{P_n - \mathcal{H}_e\{Y_m\}, P_n - \mathcal{H}_e\{Y_m\}, -I\}$$

$$\bar{P}_{mn}^3 = \text{diag}\{P_n - \mathcal{H}_e\{Z_m\}, P_n - \mathcal{H}_e\{Z_m\}, -I, -\vartheta\}$$

$$P_n = \begin{bmatrix} P_{11,n} & * & * \\ P_{21,n} & P_{22,n} & * \\ P_{31,n} & P_{31,n} & P_{33,n} \end{bmatrix}, \bar{\Xi}_{ij,m}$$

$$= \begin{bmatrix} \bar{A}_{ij,m}^T & 0 & \bar{L}_{ij,m}^T & \bar{C}_i^T \\ 0 & 0 & 0 & 0 \\ \bar{B}_{ij,m}^T & 0 & 0 & D_i^T \end{bmatrix}$$

$$\bar{\Delta}_{ij,m} = \begin{bmatrix} \bar{A}_{ij,m}^T & 0 & \bar{L}_{ij,m}^T & \bar{C}_i^T \\ \bar{C}_{ij,m}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \bar{B}_{ij,m}^T & 0 & 0 & D_i^T \end{bmatrix}, \bar{A}_{ij,m}^T$$

$$= \begin{bmatrix} A_i^T X_m^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \bar{A}_{fj,m}^T \end{bmatrix}$$

$$\bar{\Sigma}_{ij,m} = \begin{bmatrix} \bar{A}_{1ij,m}^T & \bar{A}_{2ij,m}^T & \bar{L}_{ij,m}^T \\ \bar{B}_{1ij,m}^T & \bar{B}_{2ij,m}^T & 0 \end{bmatrix}$$

$$\bar{B}_{ij,m}^T = \begin{bmatrix} B_i^T X_m^T & 0 & 0 \end{bmatrix}, \bar{C}_{ij,m}^T = \begin{bmatrix} 0 & X_m^T & \bar{B}_{fj,m}^T \end{bmatrix}$$

$$\bar{A}_{1ij,m}^T = \begin{bmatrix} A_i^T Y_m^T & \rho C_i^T \Psi_m^T Y_m^T & \rho C_i^T \Psi_m^T \bar{B}_{fj,m}^T \\ 0 & \bar{\Psi}_m^T Y_m^T & \bar{\Psi}_m^T \bar{B}_{fj,m}^T \\ 0 & 0 & \bar{A}_{fj,m}^T \end{bmatrix}$$

$$\bar{A}_{2ij,m}^T = \begin{bmatrix} 0 & -\kappa C_i^T \Psi_m^T Y_m^T & -\kappa C_i^T \Psi_m^T \bar{B}_{fj,m}^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\bar{B}_{1ij,m}^T = \begin{bmatrix} B_i^T Y_m^T & \rho D_i^T \Psi_m^T Y_m^T & \rho D_i^T \Psi_m^T \bar{B}_{fj,m}^T \end{bmatrix}$$

$$\bar{B}_{2ij,m}^T = \begin{bmatrix} 0 & -\kappa D_i^T \Psi_m^T Y_m^T & -\kappa D_i^T \Psi_m^T \bar{B}_{fj,m}^T \end{bmatrix}$$

$$\bar{A}_{ij,m}^T = \begin{bmatrix} A_i^T Z_m^T & 0 & 0 \\ 0 & Z_m^T & \bar{B}_{fj,m}^T \\ 0 & 0 & \bar{A}_{fj,m}^T \end{bmatrix}, \bar{B}_{ij,m}^T = \begin{bmatrix} B_i^T Z_m^T & 0 & 0 \end{bmatrix}$$

$$\mathcal{M} = \text{diag}\{-P_m, 0, -\bar{\Phi}, -\gamma^2 I\} - M$$

$$\mathcal{N} = \text{diag}\{-P_m, -\gamma^2 I\} - N$$

$$\mathcal{Q} = \text{diag}\{-P_m, -\bar{\Phi}, -\gamma^2 I\} - Q.$$

*Proof:* Applying the Schur complement lemma to (23)–(28), one can get the following inequalities ( $i \leq j$ ):

$$\begin{bmatrix} 2\tilde{P}_n^1 & * \\ \Delta_{ij,m} + \Delta_{ji,m} & 2\mathcal{M} \end{bmatrix} < 0 \quad (44)$$

$$\begin{bmatrix} 2\tilde{P}_n^1 & * \\ \sqrt{\zeta_j} \Delta_{ij,m} + \sqrt{\zeta_i} \Delta_{ji,m} & \sqrt{\zeta_j} \mathcal{M} + \sqrt{\zeta_i} \mathcal{M} \end{bmatrix} < 0 \quad (45)$$

$$\begin{bmatrix} 2\tilde{P}_n^2 & * \\ \Sigma_{ij,m} + \Sigma_{ji,m} & 2\mathcal{N} \end{bmatrix} < 0 \quad (46)$$

$$\begin{bmatrix} 2\tilde{P}_n^2 & * \\ \sqrt{\zeta_j} \Sigma_{ij,m} + \sqrt{\zeta_i} \Sigma_{ji,m} & \sqrt{\zeta_j} \mathcal{N} + \sqrt{\zeta_i} \mathcal{N} \end{bmatrix} < 0 \quad (47)$$

$$\begin{bmatrix} 2\tilde{P}_n^3 & * \\ \Xi_{ij,m} + \Xi_{ji,m} & 2\mathcal{Q} \end{bmatrix} < 0 \quad (48)$$

$$\begin{bmatrix} 2\tilde{P}_n^3 & * \\ \sqrt{\zeta_j} \Xi_{ij,m} + \sqrt{\zeta_i} \Xi_{ji,m} & \sqrt{\zeta_j} \mathcal{Q} + \sqrt{\zeta_i} \mathcal{Q} \end{bmatrix} < 0 \quad (49)$$

**Algorithm 1:** The Proposed Fuzzy Filtering Algorithm.

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1 Given positive scalar  $\phi_r, \lambda_r, L_{max}, L_{min}, \Lambda, \Phi, \bar{\Phi}, \bar{\nu}, \rho,$ 
   $\kappa, \varphi, \theta, \vartheta, \gamma$  and initialize number of sensor nodes  $n_y$ 
  and system initial conditions;
2 for  $h = 0; h \leq T; h = h + 1$  do
3   Calculating  $\epsilon_i(x(h))$  and  $y(h)$  by (2);
4   Selecting one sensor group for transmission based
   on (3);
5   for  $r = 1; r \leq n_y; r = r + 1$  do
6     if  $\mu_r(h) \geq L_{max}y_r^T(h)\lambda_r y_r(h)$  then
7       | Sensor  $r$  is selected;
8     else if  $\mu_r(h) \geq L_{min}y_r^T(h)\lambda_r y_r(h)$  then
9       | Sensor  $r$  is chosen for further selection
       based on (4);
10    else
11      | Sensor  $r$  is not selected;
12    end
13  end
14  for  $r = 1; r \leq n_y; r = r + 1$  do
15    if there exist any selected sensor node then
16      |  $\bar{y}_r(h) = (1 - \nu(h))y_r(h)$ ;
17    else
18      |  $\bar{y}_r(h) = \bar{y}_r(h - 1)$ ;
19    end
20  end
21  Computing  $\tau_j(x(h)), x_f(h)$  and  $z_f(h)$  by (18);
22  Calculating  $z(h), \tilde{z}(h)$  and  $x(h + 1)$  by (2);
23  Updating  $\nu(h)$ ;
24 end

```

---

where

$$\begin{aligned} \tilde{P}_n^1 &= \text{diag}\{-P_n^{-1}, -P_n^{-1}, -I, -\theta\} \\ \tilde{P}_n^2 &= \text{diag}\{-P_n^{-1}, -P_n^{-1}, -I\} \\ \tilde{P}_n^3 &= \text{diag}\{-P_n^{-1}, -P_n^{-1}, -I, -\vartheta\} \\ \Delta_{ij,m} &= \begin{bmatrix} \tilde{A}_{ij,m}^T & 0 & \tilde{L}_{ij,m}^T & \tilde{C}_i^T \\ \tilde{C}_{ij,m}^T & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \tilde{B}_{ij,m}^T & 0 & 0 & D_i^T \end{bmatrix}, \\ \Xi_{ij,m} &= \begin{bmatrix} \tilde{A}_{ij,m}^T & 0 & \tilde{L}_{ij,m}^T & \tilde{C}_i^T \\ 0 & 0 & 0 & 0 \\ \tilde{B}_{ij,m}^T & 0 & 0 & D_i^T \end{bmatrix} \\ \Sigma_{ij,m} &= \begin{bmatrix} \tilde{A}_{1ij,m}^T & \tilde{A}_{2ij,m}^T & \tilde{L}_{ij,m}^T \\ \tilde{B}_{1ij,m}^T & \tilde{B}_{2ij,m}^T & 0 \end{bmatrix}. \end{aligned}$$

Premultiply and postmultiply (44) and (45), (46) and (47), (48) and (49) by  $\text{diag}\{X_m, X_m, I, I, I, I, I, I\}$  and its transpose,  $\text{diag}\{Y_m, Y_m, I, I, I\}$  and its transpose,  $\text{diag}\{Z_m, Z_m, I, I, I, I, I\}$  and its transpose, separately. For  $m = n_y + 1$ , define  $\tilde{A}_{fj,m}^T = A_{fj,m}^T X_m^T$ ,  $\tilde{B}_{fj,m}^T = B_{fj,m}^T X_m^T$ . For  $m \in \{1, 2, \dots, n_y\}$ , define  $\tilde{A}_{fj,m}^T = A_{fj,m}^T Y_m^T$ ,

$\tilde{B}_{fj,m}^T = B_{fj,m}^T Y_m^T$ . For  $m = 0$ , define  $\tilde{A}_{fj,m}^T = A_{fj,m}^T Z_m^T$ ,  $\tilde{B}_{fj,m}^T = B_{fj,m}^T Z_m^T$ . Then, it can be obtained that ( $i \leq j$ )

$$\begin{bmatrix} 2\tilde{P}_{mn}^1 & * \\ \tilde{\Delta}_{ij,m} + \tilde{\Delta}_{ji,m} & 2\mathcal{M} \end{bmatrix} < 0 \quad (50)$$

$$\begin{bmatrix} 2\tilde{P}_{mn}^1 & * \\ \sqrt{\tilde{\zeta}_j} \tilde{\Delta}_{ij,m} + \sqrt{\tilde{\zeta}_i} \tilde{\Delta}_{ji,m} & \sqrt{\tilde{\zeta}_j} \mathcal{M} + \sqrt{\tilde{\zeta}_i} \mathcal{M} \end{bmatrix} < 0 \quad (51)$$

$$\begin{bmatrix} 2\tilde{P}_{mn}^2 & * \\ \tilde{\Sigma}_{ij,m} + \tilde{\Sigma}_{ji,m} & 2\mathcal{N} \end{bmatrix} < 0 \quad (52)$$

$$\begin{bmatrix} 2\tilde{P}_{mn}^2 & * \\ \sqrt{\tilde{\zeta}_j} \tilde{\Sigma}_{ij,m} + \sqrt{\tilde{\zeta}_i} \tilde{\Sigma}_{ji,m} & \sqrt{\tilde{\zeta}_j} \mathcal{N} + \sqrt{\tilde{\zeta}_i} \mathcal{N} \end{bmatrix} < 0 \quad (53)$$

$$\begin{bmatrix} 2\tilde{P}_{mn}^3 & * \\ \tilde{\Xi}_{ij,m} + \tilde{\Xi}_{ji,m} & 2\mathcal{Q} \end{bmatrix} < 0 \quad (54)$$

$$\begin{bmatrix} 2\tilde{P}_{mn}^3 & * \\ \sqrt{\tilde{\zeta}_j} \tilde{\Xi}_{ij,m} + \sqrt{\tilde{\zeta}_i} \tilde{\Xi}_{ji,m} & \sqrt{\tilde{\zeta}_j} \mathcal{Q} + \sqrt{\tilde{\zeta}_i} \mathcal{Q} \end{bmatrix} < 0 \quad (55)$$

where

$$\tilde{P}_{mn}^1 = \text{diag}\{-X_m P_n^{-1} X_m^T, -X_m P_n^{-1} X_m^T, -I, -\theta\}$$

$$\tilde{P}_{mn}^2 = \text{diag}\{-Y_m P_n^{-1} Y_m^T, -Y_m P_n^{-1} Y_m^T, -I\}$$

$$\tilde{P}_{mn}^3 = \text{diag}\{-Z_m P_n^{-1} Z_m^T, -Z_m P_n^{-1} Z_m^T, -I, -\vartheta\}.$$

In addition, for positive-definite matrices  $P_n$ , by adopting the same method as [4], replace  $-X_m P_n^{-1} X_m^T$ ,  $-Y_m P_n^{-1} Y_m^T$ , and  $-Z_m P_n^{-1} Z_m^T$  with  $P_n - \mathcal{H}_e\{X_m\}$ ,  $P_n - \mathcal{H}_e\{Y_m\}$ , and  $P_n - \mathcal{H}_e\{Z_m\}$ , respectively. Subsequently, (38)–(43) are yielded. To make the solution be more concise and clear, the corresponding pseudocode of the filtering algorithm with time window  $T$  is given in Algorithm 2 without loss of generality. At this point, the proof of Theorem 2 has been completed.

#### IV. NUMERICAL EXAMPLE

The following parameters are presented for system (2) in this part to illustrate the effectiveness of the filter against DoS attacks under ET-WTODP

$$A_1 = \begin{bmatrix} -0.53 & -0.44 & 0.12 \\ 0.90 & 0.34 & 0.67 \\ 1.20 & -0.64 & 0.38 \end{bmatrix}, B_1 = \begin{bmatrix} 0.10 & 0.41 & 0.27 \end{bmatrix}^T$$

$$A_2 = \begin{bmatrix} -0.65 & -0.33 & 0.23 \\ 1.2 & 0.55 & 0.77 \\ 1.09 & -0.27 & 0.78 \end{bmatrix}, B_2 = \begin{bmatrix} 0.32 & 0.16 & 0.36 \end{bmatrix}^T$$

$$C_1 = \begin{bmatrix} 1.10 & -0.23 & 0.73 \\ 0.82 & 0.19 & -0.13 \\ 1.04 & 0.65 & -0.24 \end{bmatrix}, L_1 = \begin{bmatrix} -0.13 & 0.54 & 0.38 \end{bmatrix}$$



$$C_2 = \begin{bmatrix} -0.16 & 0 & 0.14 \\ 0 & 1.02 & 0.11 \\ -0.14 & 1.04 & 0.16 \end{bmatrix}, L_2 = \begin{bmatrix} -0.22 & 0.37 & 0.64 \end{bmatrix}$$

$$D_1 = \begin{bmatrix} -0.01 & 0.03 & -0.01 \end{bmatrix}^T, D_2 = \begin{bmatrix} 0.01 & -0.02 & -0.01 \end{bmatrix}^T.$$

The MFs of the plant and filter are the same as those of the plant and controller in [4], except for the following NWFs:

$$\begin{aligned} \underline{\chi}_i(x(h)) &= \bar{l}_j(x(h)) = \sin^2(x_1(h)) \\ \bar{\chi}_i(x(h)) &= \underline{l}_j(x(h)) = \cos^2(x_1(h)). \end{aligned}$$

Assume there are three sensor nodes that are spatially distributed. Consequently, under the proposed ET-WTODP,  $m$  will take value in the set  $\{0, 1, 2, 3, 4\}$ . To be specific,  $m \in \{1, 2, 3\}$  in *Case B*,  $m = 0$  in *Case C* and  $m = 4$  in *Case A*. Choose  $\varphi = 0.5$ ,  $L_{\max} = 1.860$ ,  $L_{\min} = 1.285$ ,  $\gamma = 1.6739$ ,  $\omega(h) = 2e^{-2k}$ . Give the initial conditions  $x(0) = [-1.5 \ 0.5 \ 1]^T$ ,  $x_f(0) = [0.5 \ -0.5 \ 0]^T$ . By Theorem 2, we can obtain the filter gains

$$A_{f10} = \begin{bmatrix} -0.0056 & 0.0078 & -0.0137 \\ 0.0374 & 0.0111 & 0.0261 \\ -0.0003 & 0.0277 & 0.0305 \end{bmatrix},$$

$$A_{f11} = \begin{bmatrix} 0.0071 & 0.0010 & -0.0021 \\ 0.0083 & 0.0007 & 0.0071 \\ -0.0016 & 0.0076 & 0.0098 \end{bmatrix}$$

$$A_{f12} = \begin{bmatrix} -0.0105 & 0.0025 & -0.0018 \\ 0.0273 & -0.0041 & 0.0049 \\ 0.0002 & 0.0094 & 0.0117 \end{bmatrix},$$

$$A_{f13} = \begin{bmatrix} -0.0077 & -0.0007 & -0.0041 \\ 0.0176 & 0.0028 & 0.0039 \\ 0.0090 & 0.0052 & 0.0090 \end{bmatrix}$$

$$A_{f14} = \begin{bmatrix} 0.0078 & 0.0146 & -0.0148 \\ 0.0168 & 0.0011 & 0.0123 \\ -0.0142 & 0.0324 & 0.0157 \end{bmatrix},$$

$$A_{f20} = \begin{bmatrix} 0.1578 & 0.0922 & -0.0875 \\ -0.2242 & -0.1357 & -0.1522 \\ -0.1987 & 0.0840 & -0.1528 \end{bmatrix}$$

$$A_{f21} = \begin{bmatrix} 0.0300 & 0.0407 & -0.0418 \\ -0.0407 & -0.0473 & -0.0477 \\ -0.0329 & 0.0205 & -0.0357 \end{bmatrix},$$

$$A_{f22} = \begin{bmatrix} 0.0852 & 0.0026 & -0.0238 \\ -0.1117 & -0.0091 & -0.0594 \\ -0.0582 & 0.0081 & -0.0390 \end{bmatrix}$$

$$A_{f23} = \begin{bmatrix} 0.0872 & 0.0164 & -0.0129 \\ -0.0906 & -0.0129 & -0.0344 \\ -0.0937 & 0.0134 & -0.0335 \end{bmatrix},$$

$$A_{f24} = \begin{bmatrix} 0.0658 & 0.0417 & -0.0484 \\ -0.1050 & -0.0700 & -0.0809 \\ -0.0939 & 0.0444 & -0.0770 \end{bmatrix}$$

$$B_{f10} = \begin{bmatrix} 0.1781 & -0.0247 & 0.0218 \\ 0.0832 & 0.2076 & 0.0304 \\ 0.0648 & -0.0390 & 0.2402 \end{bmatrix},$$

$$B_{f11} = \begin{bmatrix} 0.3009 & 0.0057 & 0.0255 \\ 0.0604 & 0.2411 & -0.0203 \\ 0.0971 & -0.0382 & 0.2379 \end{bmatrix}$$

$$B_{f12} = \begin{bmatrix} 0.2669 & -0.1059 & 0.0124 \\ 0.0364 & 0.3681 & -0.0258 \\ 0.0257 & -0.0253 & 0.2401 \end{bmatrix},$$

$$B_{f13} = \begin{bmatrix} 0.2677 & 0.0104 & -0.0813 \\ 0.0420 & 0.2092 & 0.0366 \\ 0.0291 & -0.0115 & 0.2431 \end{bmatrix}$$

$$B_{f14} = \begin{bmatrix} 0.2116 & 0.0102 & 0.0173 \\ 0.0154 & 0.1757 & -0.0270 \\ 0.0126 & -0.0281 & 0.1867 \end{bmatrix},$$

$$B_{f20} = \begin{bmatrix} 0.3720 & 0.1067 & -0.0837 \\ -0.2147 & 0.0275 & -0.1810 \\ -0.1954 & 0.0577 & 0.0242 \end{bmatrix}$$

$$B_{f21} = \begin{bmatrix} 0.4139 & 0.0652 & -0.0247 \\ 0.0307 & 0.1744 & -0.0863 \\ 0.0831 & -0.0197 & 0.1877 \end{bmatrix},$$

$$B_{f22} = \begin{bmatrix} 0.3659 & -0.0606 & -0.0166 \\ -0.1043 & 0.3787 & -0.1080 \\ -0.0490 & -0.0728 & 0.1918 \end{bmatrix}$$

$$B_{f23} = \begin{bmatrix} 0.3668 & 0.0339 & -0.0271 \\ -0.0739 & 0.1859 & 0.0364 \\ -0.0926 & 0.0087 & 0.2201 \end{bmatrix},$$

$$B_{f24} = \begin{bmatrix} 0.1957 & 0.0184 & 0.0098 \\ 0.0172 & 0.1617 & -0.0302 \\ 0.0120 & -0.0327 & 0.1791 \end{bmatrix}$$

$$C_{f10} = \begin{bmatrix} 0.0598 & -0.0597 & -0.0129 \end{bmatrix},$$

$$C_{f11} = \begin{bmatrix} 0.0574 & -0.0703 & -0.0152 \end{bmatrix}$$

$$C_{f12} = \begin{bmatrix} 0.0602 & -0.0039 & -0.0351 \end{bmatrix},$$

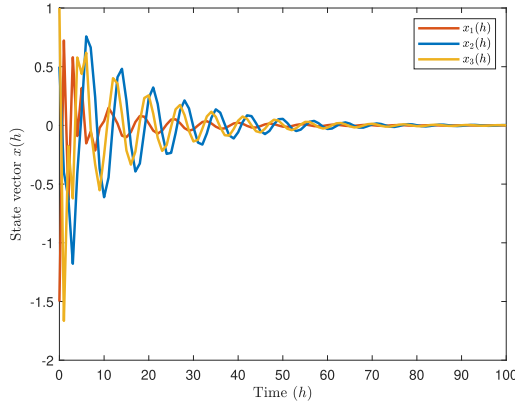


Fig. 2. State trajectories of the IT2 T-S fuzzy system.

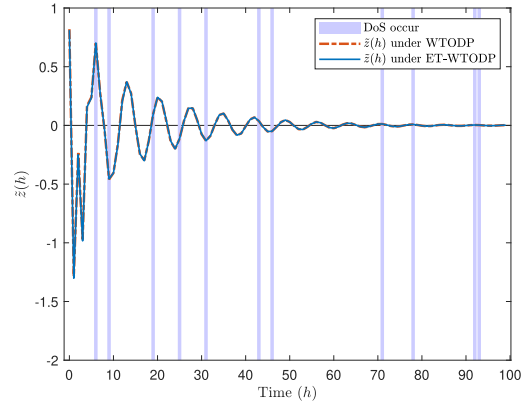
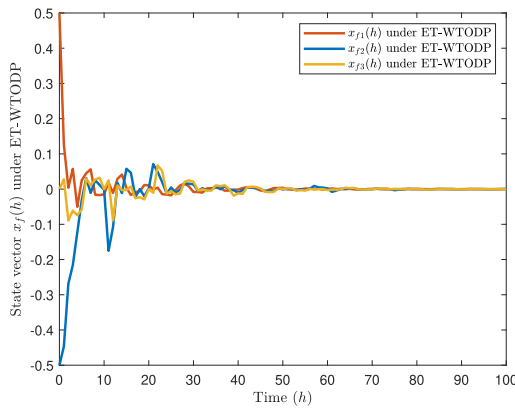
Fig. 4. Trajectories of  $\tilde{z}(h)$ .

Fig. 3. Fuzzy filter state trajectories under ET-WTODP.

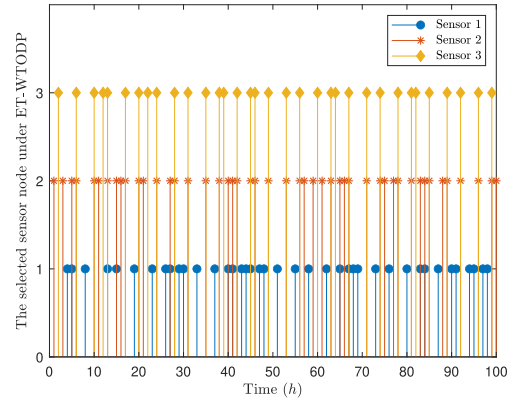


Fig. 5. Selection of sensor nodes under ET-WTODP.

$$\begin{aligned}
 C_{f13} &= \begin{bmatrix} 0.0357 & -0.0199 & 0.0100 \end{bmatrix} \\
 C_{f14} &= \begin{bmatrix} 0.0523 & -0.0489 & -0.0045 \end{bmatrix}, \\
 C_{f20} &= \begin{bmatrix} 0.0553 & -0.1206 & -0.2320 \end{bmatrix} \\
 C_{f21} &= \begin{bmatrix} 0.0491 & -0.0676 & -0.2364 \end{bmatrix}, \\
 C_{f22} &= \begin{bmatrix} 0.0447 & -0.2270 & -0.1885 \end{bmatrix} \\
 C_{f23} &= \begin{bmatrix} 0.0809 & -0.1668 & -0.2862 \end{bmatrix}, \\
 C_{f24} &= \begin{bmatrix} 0.0534 & -0.1253 & -0.2310 \end{bmatrix}.
 \end{aligned}$$

Besides, in order to study the distribution of selected sensor nodes under the designed ET-WTODP, we set the initial value of the filter for comparison with conventional WTODP to be  $x_f^c(0) = x_f(0)$ . The different filter gains  $A_{f11}^c, \dots, A_{f23}^c$ ,  $B_{f11}^c, \dots, B_{f23}^c$ ,  $C_{f11}^c, \dots, C_{f23}^c$  under WTODP have been obtained, which are omitted here owing to space limitation.

Fig. 2 gives the system state trajectories, which indicates the stabilization of the system. Fig. 3 shows the state of the filter under ET-WTODP. When the probability of DoS attacks is

$\bar{\nu} = 0.2$ , the filtering error and occurred situation of DoS attacks are depicted in Fig. 4. With the filter we design, the filtering error under ET-WTODP gradually stabilizes and is prone to zero over the simulation time, which means that the filter can accurately estimate the system output. This is a strong indication of the reliability of the filter and its security against DoS attacks. It is also evident from the trajectories of  $\tilde{z}(h)$  under WTODP that similar filtering capability may be guaranteed under both WTODP and ET-WTODP.

In addition, the distribution of selected sensor nodes under ET-WTODP is shown in Fig. 5, where one sensor node is chosen at instants  $h \in \{1, 2, 3, 4, 8, 9, \dots\}$  and multiple sensor nodes are granted transmission privileges concurrently at instants  $h \in \{6, 10, 13, 15, 17, 20, \dots\}$ . For instance, at  $h = 6$  instant, Sensors 2 and 3 are transmitted, and three sensor nodes are all selected at  $h = 13$ . Moreover, it can be observed that at some instants  $h \in \{7, 9, 14, 18, 25, 32, \dots\}$ , no signal is transmitted. For further comparison, the signal transmission distribution under WTODP is presented in Fig. 6, where merely one sensor node is transported at each given time instant. Statistically speaking, the channel occupancy rate of the ET-WTODP is 84%, which is 16% lower than that of WTODP.

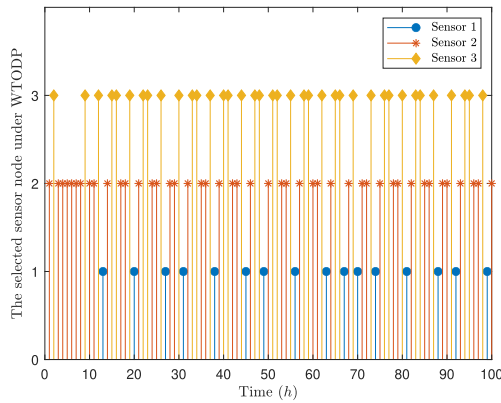


Fig. 6. Selection of sensor nodes under WTODP.

From Figs. 4–6, it is concluded that the proposed ET-WTODP with lower channel occupancy can reduce communication resource consumption than the conventional WTODP, which precisely reflects the advantages of flexibly and dynamically selecting the sensor nodes of the ET-WTODP strategy. Therefore, the ET-WTODP strategy is more conducive to promoting robust filtering performance under limited transmission resources.

## V. CONCLUSION

The problem of  $H_\infty$  filtering for a class of nonlinear systems with ET-WTODP strategy and DoS attacks has been investigated in this article. The proposed ET-WTODP allows flexibility in determining the number of sensor nodes to be selected, improving the occupation rate of the network based on the relationship between the thresholds and each measured output component. To enhance filter design flexibility, an asynchronous filter with imperfect matching MFs, which reflects the effect of ET-WTODP is developed, and then an OFES model that considers both the ET-WTODP and DoS attacks has been constructed. On the basis of the OFES, sufficient conditions that guarantee the asymptotic stability of which with preset  $H_\infty$  performance have been derived, and the filter parameters are then solved by exploiting the LMIs technique. Numerical examples are utilized to confirm the feasibility of the demonstrated filter design approach. The concerned results will be beneficial to the analysis of other fuzzy-model-based hot topics, and the approach proposed in this article is promising to be extended to the filter and control design for switched systems, which will be left to our future work.

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