# Data-Driven Decentralized Learning Regulation for Networked Interconnected Systems Using Generalized Fuzzy Hyperbolic Models

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***Abstract*—In this article, a decentralized event-triggered (ET) regulation problem is tackled for networked interconnected systems (NISs) with control constraints and unmatched interference. Foremost, the decentralized regulation issue is converted into the optimal control problems for the associated auxiliary subsystem. In confronting the unavailability of system dynamics, the utilization of generalized fuzzy hyperbolic models-assisted identifier provides a novel perspective to devise the efficacious control policy for the constrained NISs. For the sake of mitigating the communication workload, a new dual threshold functions-based adaptive ET scheme (DTAETS) is put forward by incorporating the current data and latest ET signal. Moreover, we present a data-driven decentralized reinforcement learning algorithm to acquire the solution of DTAETS-boosted Hamilton–Jacobi–Isaacs equation. Then, the uniformly ultimately bounded stability of auxiliary subsystem and the weight estimation error is assured. Ultimately, a numeral experiment is conducted to substantiate the validity of the theoretical results.**

***Index Terms*—Data-driven reinforcement learning (RL), event-triggered scheme (ETS), generalized fuzzy hyperbolic models (GFHMs), networked interconnected systems (NISs).**

## I. Introduction

NOWADAYS, several data-driven techniques for complex nonlinear systems have extracted plentiful interests on basis of input–output data. As representative approximation instruments, neural network (NN) [1], [2] and fuzzy logic systems (FLSs) [3], [4] have been frequently adopted to reconstruct the unknown system parameters. Nevertheless, the activation function in NN demands to be appropriately selected according to specific environment, which increases the difficulty of implementing NN. Meanwhile, certain crucial dynamic behaviors are beyond the representation capabilities of FLSs [5]. In response to the aforementioned drawbacks, the generalized fuzzy hyperbolic models (GFHMs) were proposed as a novel type of fuzzy approximator, which can effectively approximate any nonlinear function over compact sets [6]. From the modeling perspective, it is not essential for GFHMs to assign the accurate premise structure and the relevant variable space. Moreover, the quantity of tuning parameters in GFHMs is comparatively less in contrast to Takagi–Sugeno fuzzy model [7], [8], [9], [10], [11]. It indicates that the involved computational pressure can be further lessened on account of GFHMs. According to the aforementioned merits, it is relatively significant to construct the GFHMs-assisted identifier for complex networked systems such that the unknown system parameters can be approximately obtained.

With the advanced deployment of Internet and industry technology, networked interconnected systems (NISs) have gained abundant concentrations. As a consequence of the interconnected characteristic, several decentralized regulation schemes have been put forward to stabilize the NISs [12], [13], [14]. Within the decentralized regulation framework, the focused control problem can be composed into multiple optimal regulation issues for nominal/auxiliary subsystems. To be specific, a series of nominal subsystems can be presented for NISs with matched interconnection [15], while the auxiliary subsystems with supplemental control signal are constructed under the unmatched situation [16]. On the ground of such an approach, abundant research results [17], [18], [19] have been published with regard to different control problems. For instance, the authors in [20] investigated a decentralized tracking control issue for unknown NISs via a local observer. In [21], an FLSs-based event-triggered (ET) control strategy was proposed for the constrained NISs. From the abovementioned content, the unavailable dynamics of NISs is capable to be estimated by applying NN and FLSs. Nonetheless, GFHMs can efficaciously alleviate the computational burden during the modeling and parameter identification process. Therefore, a GFHMs-assisted reconstruction

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model is taken into consideration for NISs with mismatched interconnection.

In addressing the aforementioned optimal regulation problem, reinforcement learning (RL) [22], [23], [24] presents a valid strategy to approximate the solution of Hamilton–Jacobi–Isaacs equation (HJIE). Initially, the time-triggered HJIE [25], [26] was deduced by sampling the system state at each time instant. It is of apparentness that such a mechanism will result in a large amount of dispensable computation. To deal with this deficiency, ET scheme (ETS) [27], [28], [29], [30] was provided to govern the transmitted information according to the signal variation degree. Furthermore, an adaptive ETS (AETS) was proposed with the aim of regulating the triggered threshold in adaptive manner. To mention a few, Sun et al. [31] presented an AETS with dynamic threshold modeled by inverse tangent function. In [32], an error-dependent function was utilized to construct the adjustment rule of dynamic threshold in AETS. However, the aforesaid literature only contain a single threshold function and the previous triggered data packages. In practical implementations, the large errors may emerge in the triggered condition as a result of complicated exogenous disturbance [33]. It not only leads to the release of redundant state information into the communication network, but also affects the regulation performance. Under the incentive of this problem, we make efforts to introduce the dual threshold functions and amalgamate the current sampled data and the latest released signal into them.

In view of the discussion above, a data-driven decentralized ET regulation problem is tackled for the constrained NISs via GFHMs-assisted identifier. The distinguishing features of this article are reflected in the following aspects.

1) The integral RL in [34] and [35] has removed the requirement of drift dynamics. Nevertheless, the devised control strategies in [34] and [35] still depend on the input dynamics. To further relax this restriction, we deploy a GFHMs-assisted identifier to simultaneously estimate the unknown drift and input dynamics of each auxiliary subsystem. By virtue of this mechanism, a data-driven decentralized RL algorithm is proposed for the constrained-input NISs with mismatched disturbance and interconnection.
2) A novel dual threshold functions-based AETS (DTAETS) is established to manage the sampled data transmission. Owing to the applied dual threshold functions, the proposed DTAETS not only makes the threshold adjustment process more flexible, but also can decrease the network occupancy in comparison with [31], [32]. Despite the improved AETS in [33] with similar structure, the adopted dual threshold functions can be promptly determined by the current error data. Meanwhile, the related parameters in DTAETS are simplified such that the computational complexity is efficiently reduced.
3) In the existing literature [16], [19], [21], [36], the decentralized controller design for NISs has been presented and the stability performance can also be guaranteed. However, these control strategies did not comprehensively take the control constraints and external interference into account. In practical reality, inherent physical properties and complex disturbance may have adverse impacts on stabilizing the NISs. Toward this end, the constrained-input NISs with mismatched interference are considered so as to present more general control scheme in contrast to [16], [19], [21], [36].

*Notations:* The standard mathematical symbols are adopted in this article. Specially, $C^{\dagger}$ denotes the Moore–Penrose pseudoinverse of $C$. $\otimes$ stands for the Kronecker product. $I_k$ denotes the $k \times 1$ column vector of ones. $I_{k\times k}$ represents the $k \times k$ identity matrix.

## II. Problem Descriptions

In this section, the constrained-input NISs with mismatched interconnection are presented initially. Thereafter, the decentralized regulation issue is converted into $N$ optimal control problems for auxiliary subsystems within RL framework. Under the communication constraints, the DTAETS is presented to alleviate the transmission burden and the corresponding HJIE for $\alpha$th auxiliary subsystem is deduced.

### A. Constrained NISs With Unmatched Disturbance

The constrained-input NISs are composed of $N$ subsystems with the following dynamics:

$$\begin{aligned}\dot{x}_\alpha(t) &= l_\alpha(x_\alpha(t)) + C_\alpha u_\alpha(t)\\ &\quad + h_\alpha(x_\alpha(t))d_\alpha(t) + \Delta l_\alpha(x(t))\\ x_\alpha(0) &= x_\alpha^0, \;\; \alpha = 1, 2, \ldots, N\end{aligned} \tag{1}$$

in which $x_\alpha = [x_{\alpha 1}^T, x_{\alpha 2}^T, \ldots, x_{\alpha n_\alpha}^T]^T \in \mathbb{R}^{n_\alpha}$ denotes the $\alpha$th subsystem state with the initial value $x_\alpha^0$. $x = [x_1^T, x_2^T, \ldots, x_N^T]^T \in \mathbb{R}^{\sum_{\alpha=1}^N n_\alpha}$ represents the whole state. $u_\alpha \in \aleph_\alpha \subseteq \mathbb{R}^{z_\alpha}$ is the constrained input of $\alpha$th subsystem, $\aleph_\alpha = \{u_\alpha | u_\alpha = [u_{\alpha 1}, u_{\alpha 2}, \ldots, u_{\alpha z_\alpha}]^T, |u_{\alpha\beta}| \leq \zeta_\alpha, \beta = 1, 2, \ldots, z_\alpha\}$, and $\zeta_\alpha > 0$ represents the upper bound. $l_\alpha(x_\alpha) \in \mathbb{R}^{n_\alpha}$, $\Delta l_\alpha(x) \in \mathbb{R}^{n_\alpha}$, $C_\alpha = [C_{\alpha 1}^T, C_{\alpha 2}^T, \ldots, C_{\alpha n_\alpha}^T]^T \in \mathbb{R}^{n_\alpha \times z_\alpha}$, and $h_\alpha(x_\alpha) = [h_{\alpha 1}^T, h_{\alpha 2}^T, \ldots, h_{\alpha n_\alpha}^T]^T \in \mathbb{R}^{n_\alpha \times m_\alpha}$ stand for the unknown system drift dynamics, the nonlinear interconnected term, the unknown control matrix, and the disturbance dynamics, respectively. $d_\alpha \in \mathbb{R}^{m_\alpha}$ serves as the exogenous interference with $d_\alpha \in L_2[0, \infty)$ and $h_\alpha(x_\alpha) \neq C_\alpha$. For the convenience, the notation $t$ is omitted in the subsequent descriptions.

*Assumption 1 (see [16], [37]):* For every $\alpha \in \{1, 2, \ldots, N\}$, the equilibrium point of the $\alpha$th subsystem is assumed to be $x_\alpha = 0$ while $u_\alpha = 0$, $d_\alpha = 0$ and $\Delta l_\alpha(x) = 0$. In addition, the nonlinear interconnection $\Delta l_\alpha(x)$ satisfies the mismatched situation, i.e., $\Delta l_\alpha(x) = \lambda_\alpha(x_\alpha)\phi_\alpha(x)$. $\lambda_\alpha(x_\alpha) = [\lambda_{\alpha 1}(x_\alpha)^T, \lambda_{\alpha 2}(x_\alpha)^T, \ldots, \lambda_{\alpha n_\alpha}(x_\alpha)^T]^T \in \mathbb{R}^{n_\alpha \times q_\alpha}$ is the known function with $\lambda_\alpha(x_\alpha) \neq C_\alpha$. $\phi_\alpha(x) \in \mathbb{R}^{q_\alpha}$ denotes the uncertain function and its bounded condition can be represented by

$$||\phi_\alpha(x)|| \leq \sum_{r=1}^N \pi_{\alpha r} F_{\alpha r}(x_r) \tag{2}$$

in which $\pi_{\alpha r}$ and $F_{\alpha r}(x_r)$ $(r = 1, 2, \dots, N)$ are the nonnegative parameter and the positive-definite function, respectively. Meanwhile, $\phi_\alpha(0) = 0$ and $F_{\alpha r}(0) = 0$.

Define $F_r(x_r) \triangleq \max_{1\le\alpha\le N}\{F_{\alpha r}(x_r)\}$. Afterwards, condition (2) is developed as

$$||\phi_\alpha(x)|| \le \sum_{r=1}^{N} c_{\alpha r}F_r(x_r), \quad c_{\alpha r} \ge \frac{\pi_{\alpha r}F_{\alpha r}(x_r)}{F_r(x_r)} \tag{3}$$

where $c_{\alpha r}$ stands for the nonnegative constant.

*Assumption 2 (see[37]):* For $\forall x_\alpha \in \mathbb{R}^{n_\alpha}$ $(\alpha = 1, 2, \dots, N)$, there exist positive constants $D_{C_\alpha}$, $D_{\lambda_\alpha}$, and $D_{h_\alpha}$ such that $||C_\alpha|| \le D_{C_\alpha}$, $||\lambda_\alpha(x_\alpha)|| \le D_{\lambda_\alpha}$, and $||h_\alpha(x_\alpha)|| \le D_{h_\alpha}$ hold. Besides, the input dynamics $C_\alpha$ satisfies $\text{rank}(C_\alpha) = z_\alpha < n_\alpha$ and $C_\alpha^T\lambda_\alpha(x_\alpha) = 0$.

Focusing on the aforementioned NISs under Assumptions 1 and 2, this article aims to propose a decentralized control scheme such that the entire system state is uniformly ultimately bounded (UUB). Nevertheless, it is relatively tough to present the desired regulation scheme due to the unmatched coupled interconnected term $\Delta l_\alpha(x)$. In order to obviate this intractable conundrum, the decentralized controller design is converted into devising $N$ optimal control strategies for the corresponding auxiliary subsystems.

### B. Decentralized Optimal Control Strategy

The following auxiliary subsystem related to the $\alpha$th subsystem (1) is presented:

$$\begin{aligned}\dot{x}_\alpha &= l_\alpha(x_\alpha) + C_\alpha u_\alpha + h_\alpha(x_\alpha)d_\alpha \\ &\quad + (I_{n_\alpha\times n_\alpha} - C_\alpha C_\alpha^\dagger)\lambda_\alpha(x_\alpha)v_\alpha\end{aligned} \tag{4}$$

in which $v_\alpha \in \mathbb{R}^{q_\alpha}$ represents the auxiliary control signal.

By means of Assumption 2, it can be deduced that $C_\alpha^\dagger\lambda_\alpha(x_\alpha) = (C_\alpha^TC_\alpha)^{-1}C_\alpha^T\lambda_\alpha(x_\alpha) = 0$. Therefore, the $\alpha$th auxiliary subsystem (4) is rewritten as

$$\dot{x}_\alpha = l_\alpha(x_\alpha) + C_\alpha u_\alpha + h_\alpha(x_\alpha)d_\alpha + \lambda_\alpha(x_\alpha)v_\alpha. \tag{5}$$

For the purpose of achieving the problem conversion, the performance function associated with the $\alpha$th auxiliary subsystem (5) is given by

$$J_\alpha(x_\alpha) = \int_t^\infty [\Psi_\alpha(x_\alpha(\iota)) + \Xi_\alpha(x_\alpha(\iota), u_\alpha(\iota), v_\alpha(\iota), d_\alpha(\iota))]d\iota \tag{6}$$

in which $\Psi_\alpha(x_\alpha) = \varsigma_\alpha F_\alpha^2(x_\alpha)$ and $\Xi_\alpha(x_\alpha, u_\alpha, v_\alpha, d_\alpha) = x_\alpha^TQ_\alpha x_\alpha + \vartheta_\alpha(u_\alpha) + \delta_\alpha v_\alpha^Tv_\alpha - \gamma_\alpha^2d_\alpha^Td_\alpha$. $\varsigma_\alpha > 0$, $\delta_\alpha > 0$ and $\gamma_\alpha > 0$ are the given parameters. $Q_\alpha \in \mathbb{R}^{n_\alpha\times n_\alpha}$ denotes the positive definite matrix. Motivated by [37], the function $\vartheta_\alpha(u_\alpha)$ is determined by $2\zeta_\alpha\sum_{\beta=1}^{z_\alpha}\int_0^{u_{\alpha\beta}}\Pi^{-1}(\chi_\alpha/\zeta_\alpha)d\chi_\alpha$, from which $\Pi(\cdot)$ is selected as the hyperbolic tangent function in this article.

In accordance with the performance function $J_\alpha(x_\alpha)$ in (6), its optimal value is stated as

$$J_\alpha^*(x_\alpha) = \min_{u_\alpha, v_\alpha}\max_{d_\alpha} J_\alpha(x_\alpha). \tag{7}$$

To transform the aforesaid optimization problem, the Hamilton function can be obtained as follows:

$$\begin{aligned}&H(\nabla J_\alpha^*(x_\alpha), x_\alpha, u_\alpha, v_\alpha, d_\alpha) \\ &= (\nabla J_\alpha^*(x_\alpha))^T\left(l_\alpha(x_\alpha) + C_\alpha u_\alpha + h_\alpha(x_\alpha)d_\alpha + \lambda_\alpha(x_\alpha)v_\alpha\right) \\ &\quad + \Psi_\alpha(x_\alpha) + x_\alpha^TQ_\alpha x_\alpha + \vartheta_\alpha(u_\alpha) + \delta_\alpha v_\alpha^Tv_\alpha - \gamma_\alpha^2d_\alpha^Td_\alpha\end{aligned} \tag{8}$$

where $\nabla J_\alpha^*(x_\alpha) = \partial J_\alpha^*(x_\alpha)/\partial x_\alpha$.

Attributed to the Bellman's optimality principle, the optimization problem in (7) is converted into acquiring the solution of HJIE described by

$$\min_{u_\alpha, v_\alpha}\max_{d_\alpha} H(\nabla J_\alpha^*(x_\alpha), x_\alpha, u_\alpha, v_\alpha, d_\alpha) = 0. \tag{9}$$

Then, the optimal control strategies and worst-case interference policy are elicited as

$$\begin{cases} u_\alpha^*(x_\alpha) = -\zeta_\alpha\Pi\left(\frac{1}{2\zeta_\alpha}C_\alpha^T\nabla J_\alpha^*(x_\alpha)\right) \\ v_\alpha^*(x_\alpha) = -\frac{1}{2\delta_\alpha}\lambda_\alpha^T(x_\alpha)\nabla J_\alpha^*(x_\alpha) \\ d_\alpha^*(x_\alpha) = \frac{1}{2\gamma_\alpha^2}h_\alpha^T(x_\alpha)\nabla J_\alpha^*(x_\alpha). \end{cases} \tag{10}$$

Taking expressions (8)–(10) into account, the HJIE for the $\alpha$th auxiliary subsystem (5) is calculated as

$$\begin{aligned}&(\nabla J_\alpha^*(x_\alpha))^T\left(l_\alpha(x_\alpha) + C_\alpha u_\alpha^*(x_\alpha) + h_\alpha(x_\alpha)d_\alpha^*(x_\alpha)\right. \\ &\left.+\lambda_\alpha(x_\alpha)v_\alpha^*(x_\alpha)\right) + x_\alpha^TQ_\alpha x_\alpha + \vartheta_\alpha\left(u_\alpha^*(x_\alpha)\right) \\ &+ \Psi_\alpha(x_\alpha) + \delta_\alpha||v_\alpha^*(x_\alpha)||^2 - \gamma_\alpha^2||d_\alpha^*(x_\alpha)||^2 = 0.\end{aligned} \tag{11}$$

*Remark 1:* In the performance function $J_\alpha(x_\alpha)$ in (6), the utilization of specific component will be explained as follows. First of all, the term $x_\alpha^TQ_\alpha x_\alpha$ is employed to reflect the $\alpha$th auxiliary subsystem state. Stimulated by [16], [18], [35], [37], $\Psi_\alpha(x_\alpha)$ and $\vartheta_\alpha(u_\alpha)$ are relevant to the interconnection term in NISs (1) and the constrained control input, respectively. In accordance with the impacts of auxiliary control signal and exogenous disturbance, the terms $\delta_\alpha v_\alpha^Tv_\alpha$ and $-\gamma_\alpha^2d_\alpha^Td_\alpha$ are also considered in the performance function.

### C. DTAETS-Boosted HJIE for $\alpha$th Auxiliary Subsystem

In NISs, the system state information can be transferred via a confined-bandwidth communication network. To prevent the superfluous transmission, a conventional AETS [31] with single threshold is adopted according to the following condition:

$$e_{\alpha,s}^T\Theta_\alpha e_{\alpha,s} > \eta_\alpha(t_s^\alpha + \bar{l}_\alpha h)x_\alpha^T(t_s^\alpha)\Theta_\alpha x_\alpha(t_s^\alpha) \tag{12}$$

where $e_{\alpha,s} = x_\alpha(t_s^\alpha) - x_\alpha(t_s^\alpha + \bar{l}_\alpha h)$ with $s \in \{0, 1, \dots\}$ and $\bar{l}_\alpha \in \{1, 2, \dots\}$ is the difference of the last triggered time $t_s^\alpha$ and the current sampling time $t_s^\alpha + \bar{l}_\alpha h$. $h$ denotes the sampling period. $\Theta_\alpha \in \mathbb{R}^{n_\alpha\times n_\alpha}$ represents the positive-definite weight matrix. $\eta_\alpha(t_s^\alpha + \bar{l}_\alpha h)$ serves as the adjustable threshold function.

It is noteworthy that the redundant state information will not be transmitted if condition (12) is not gratified, which can obviate the communication constraints. Nonetheless, the aforesaid AETS only takes account of the previous triggered data in the right side of (12). To further decrease the network workload and improve the flexibility of threshold adjustment, we endeavor to combine the data packets at the last triggered time and the current sampling time. Toward this end, a novel DTAETS is proposed to regulate the signal transmission with the following triggered condition:

$$e_{\alpha,s}^T\Theta_\alpha e_{\alpha,s} \; > \; \eta_{1,\alpha}(t_s^\alpha+\bar{l}_\alpha h)x_\alpha^T(t_s^\alpha)\Theta_\alpha x_\alpha(t_s^\alpha) + \eta_{2,\alpha}(t_s^\alpha+\bar{l}_\alpha h)x_\alpha^T(t_s^\alpha+\bar{l}_\alpha h)\Theta_\alpha x_\alpha(t_s^\alpha+\bar{l}_\alpha h) \tag{13}$$

where $\eta_{1,\alpha}(t_s^\alpha+\bar{l}_\alpha h)$ and $\eta_{2,\alpha}(t_s^\alpha+\bar{l}_\alpha h)$ represent the dual adaptive threshold functions. Furthermore, their specific value can be obtained by

$$\begin{cases}\eta_{1,\alpha}(t_s^\alpha+\bar{l}_\alpha h)=\underline{\eta}_{1,\alpha}+(\overline{\eta}_{1,\alpha}-\underline{\eta}_{1,\alpha})e^{-\epsilon_{1,\alpha}e_{\alpha,s}^T\Theta_\alpha e_{\alpha,s}}\\ \eta_{2,\alpha}(t_s^\alpha+\bar{l}_\alpha h)=\underline{\eta}_{2,\alpha}+(\overline{\eta}_{2,\alpha}-\underline{\eta}_{2,\alpha})e^{-\epsilon_{2,\alpha}e_{\alpha,s}^T\Theta_\alpha e_{\alpha,s}}\end{cases} \tag{14}$$

in which $\underline{\eta}_{\xi,\alpha} > 0$ and $\overline{\eta}_{\xi,\alpha} > 0$ denote the lower and the upper bounds of $\eta_{\xi,\alpha}(t_s^\alpha+\bar{l}_\alpha h)$ $(\xi=1,2)$ with $\overline{\eta}_{1,\alpha} < \lambda_{\min}(\Theta_\alpha)/(2\lambda_{\max}(\Theta_\alpha))$. The positive scalars $\epsilon_{1,\alpha}$ and $\epsilon_{2,\alpha}$ are applied to govern the sensitivity of $\eta_{1,\alpha}(t_s^\alpha+\bar{l}_\alpha h)$ and $\eta_{2,\alpha}(t_s^\alpha+\bar{l}_\alpha h)$, respectively.

On these grounds, the triggered time sequence is determined according to

$$t_{s+1}^\alpha = t_s^\alpha + \min_{\bar{l}_\alpha\geq 1}\{\bar{l}_\alpha h|\bar{l}_\alpha \text{satisfies condition (13)}\}. \tag{15}$$

Facilitated by zero-order hold approach and the proposed DTAETS, the desired optimal control strategies are acquired for $\forall t\in[t_s^\alpha, t_{s+1}^\alpha)$

$$\begin{cases}u_\alpha^*(x_\alpha(t))=u_\alpha^*(x_\alpha(t_s^\alpha))=-\zeta_\alpha\Pi\left(\frac{1}{2\zeta_\alpha}C_\alpha^T\nabla J_\alpha^*(x_\alpha(t_s^\alpha))\right)\\ v_\alpha^*(x_\alpha(t))=v_\alpha^*(x_\alpha(t_s^\alpha))=-\frac{1}{2\delta_\alpha}\lambda_\alpha^T(x_\alpha)\nabla J_\alpha^*(x_\alpha(t_s^\alpha)).\end{cases} \tag{16}$$

In order to simplify later analysis, the notation $x_\alpha(t_s^\alpha)$ is replaced with $x_{\alpha,s}$ and the time scope $t\in[t_s^\alpha, t_{s+1}^\alpha)$ is omitted unless stated otherwise. Then, the DTAETS-boosted HJIE for the $\alpha$th auxiliary subsystem is deduced as

$$\begin{aligned}&(\nabla J_\alpha^*(x_\alpha))^T\left(l_\alpha(x_\alpha)+C_\alpha u_\alpha^*(x_{\alpha,s})+h_\alpha(x_\alpha)d_\alpha^*(x_\alpha)\right.\\ &\left.+\lambda_\alpha(x_\alpha)v_\alpha^*(x_{\alpha,s})\right)+x_\alpha^TQ_\alpha x_\alpha+\vartheta_\alpha\left(u_\alpha^*(x_{\alpha,s})\right)\\ &+\Psi_\alpha(x_\alpha)+\delta_\alpha||v_\alpha^*(x_{\alpha,s})||^2-\gamma_\alpha^2||d_\alpha^*(x_\alpha)||^2=0.\end{aligned} \tag{17}$$

*Remark 2:* From the perspective of mitigating the computational and communication burden, the optimal ET control strategies $u_\alpha^*(x_{\alpha,s})$ and $v_\alpha^*(x_{\alpha,s})$ in (16) will update at each ET time instant. For the update rule of the worst interference, the ET disturbance policy is utilized in [34] to relieve the computational pressure. Under the more practical consideration, the worst interference strategy $d_\alpha^*(x_\alpha)$ can be determined at each sampled time instant similar to [38]. On this basis, the aforementioned DTAETS-boosted HJIE can be formulated for the $\alpha$th auxiliary subsystem.

### D. Stability Analysis

Prior to proceeding, the following assumption is involved in assisting the stability analysis.

*Assumption 3 (see[34]):* Supposing that $||\nabla J_\alpha^*(x_\alpha)|| \leq \nabla J_{\alpha M}$, where $\nabla J_{\alpha M}$ is a positive scalar. In addition, there exist two positive Lipschitz constants $\mathcal{K}_{u_\alpha^*}$ and $\mathcal{K}_{v_\alpha^*}$ such that $||u_\alpha^*(x_\alpha)-u_\alpha^*(x_{\alpha,s})||\leq\mathcal{K}_{u_\alpha^*}||e_{\alpha,s}||$ and $||v_\alpha^*(x_\alpha)-v_\alpha^*(x_{\alpha,s})||\leq\mathcal{K}_{v_\alpha^*}||e_{\alpha,s}||$ hold.

*Theorem 1:* Take account of $N$ constrained-input auxiliary subsystems (5), the associated performance function (6) and the triggered condition of DTAETS (13). Under Assumptions 1–3, the decentralized ET control strategy composed of $u_\alpha^*(x_{\alpha,s})$ $(\alpha=1,2,\ldots,N)$ in (16) can actuate the NISs (1) with unmatched disturbance to be UUB if there exist $N$ scalars $\varsigma_\alpha^* > 0$ such that $\varsigma_\alpha\geq\varsigma_\alpha^*$ and the following inequality hold:

$$\lambda_{\min}(Q_\alpha)-\frac{\mathcal{K}_{u_\alpha^*}^2(2\overline{\eta}_{1,\alpha}+\overline{\eta}_{2,\alpha})\lambda_{\max}(\Theta_\alpha)}{\lambda_{\min}(\Theta_\alpha)-2\overline{\eta}_{1,\alpha}\lambda_{\max}(\Theta_\alpha)} > 0. \tag{18}$$

*Proof:* See Appendix A. ■

## III. GFHMs-Assisted System Identification

Foremost, we demonstrate the relative definition and properties of GFHMs in this section. Afterwards, a GFHMs-based identifier is constructed to approximate the unknown parameters in the $\alpha$th auxiliary subsystem (5). In addition, the adaptive rules of the estimated weights for the deployed identifier are provided. Meanwhile, the convergence of the state estimation error and the boundedness of the estimated weights are analyzed.

### A. Description of GFHMs

To begin with, the generalized fuzzy hyperbolic rule base (RB) is defined as follows.

*Definition 1 (see[39]):* Considering the multi-input variable $\bar{x}=[\bar{x}_1,\bar{x}_2,\ldots,\bar{x}_n]^T$ and the unique output $\bar{z}$, an RB is known as the generalized fuzzy hyperbolic RB as long as it can satisfy the following requirements.

1) The $\tau$th fuzzy rule $(\tau=1,2,\ldots,2^K;\ K=\sum_{\hbar=1}^n\kappa_\hbar)$ is presented as follows.
   **IF** $(\bar{x}_1-\bar{d}_{11})$ is $W_{11}$,..., $(\bar{x}_1-\bar{d}_{1\kappa_1})$ is $W_{1\kappa_1}$ and $(\bar{x}_2-\bar{d}_{21})$ is $W_{21}$,..., $(\bar{x}_2-\bar{d}_{2\kappa_2})$ is $W_{2\kappa_2}$ and...and $(\bar{x}_n-\bar{d}_{n1})$ is $W_{n1}$,..., $(\bar{x}_n-\bar{d}_{n\kappa_n})$ is $W_{n\kappa_n}$ **THEN** $\bar{z}^\tau=s_{W_{11}}+\cdots+s_{W_{1\kappa_1}}+\cdots+s_{W_{n\kappa_n}}$.
   In the aforementioned rule, $\kappa_\hbar$ $(\hbar=1,2,\ldots,n)$ denotes the number of transformations in regard to $\bar{x}_\hbar$. $\bar{d}_{\hbar j}$ $(j=1,2,\ldots,\kappa_\hbar)$ is the $j$th transformation bias of $\bar{x}_\hbar$. $W_{\hbar j}$ represents the fuzzy set related to $\bar{x}_\hbar-\bar{d}_{\hbar j}$ including a positive subset $\mathcal{P}_{\hbar j}$ and a negative subset $\mathcal{N}_{\hbar j}$. Then, $\vec{x}=[\bar{x}_1-\bar{d}_{11},\ldots,\bar{x}_n-\bar{d}_{n\kappa_n}]^T$ is regarded as the generalized input variable.
2) The existence of $s_{W_{\hbar j}}$ is relevant with $W_{\hbar j}$. Concretely, if $W_{\hbar j}$ exists in **IF**, $s_{W_{\hbar j}}$ also exists in **THEN** and vice versa. In addition, $2^K$ fuzzy rules are contained in the RB, which illustrates that all combinations of $\mathcal{P}_{\hbar j}$ and $\mathcal{N}_{\hbar j}$ are taken into account.

In order to provide the specific definition and the associated characteristic of GFHMs, two lemmas are outlined as follows.

*Lemma 1 (see[40]):* For a system in accordance with Definition 1, we denote the $l$th element of $\vec{x}$ as $\vec{x}_l = \bar{x}_\hbar - \bar{d}_{\hbar j}$ ($l = \kappa_1 + \cdots + \kappa_{\hbar-1} + j$). Meanwhile, the corresponding membership functions can be obtained by

$$\varpi_{\mathcal{P}_l}(\vec{x}_l) = e^{-\frac{(\vec{x}_l-\nu_l)^2}{2}},\ \varpi_{\mathcal{N}_l}(\vec{x}_l) = e^{-\frac{(\vec{x}_l+\nu_l)^2}{2}} \tag{19}$$

in which $\nu_l > 0$ stands for a scalar. Then, the targeted system can be formulated as follows:

$$\bar{z} = \mathcal{W}^T \tanh(\mathcal{V}\vec{x}) + \delta \tag{20}$$

where $\delta \in \mathbb{R}$, $\mathcal{W} = [\mathcal{W}_1, \ldots, \mathcal{W}_K]^T \in \mathbb{R}^K$, and $\mathcal{V} = \text{diag}\{\nu_1, \ldots, \nu_K\} \in \mathbb{R}^{K\times K}$ are the bias, the weight vector and inter weight, respectively. Furthermore, $\tanh(\mathcal{V}\vec{x}) = [\tanh(\nu_1\vec{x}_1), \ldots, \tanh(\nu_K\vec{x}_K)]^T \in \mathbb{R}^K$, and the presented model (20) is called as GFHM.

*Lemma 2 (see[6]):* Given an unknown smooth function $F(x)$ and any constant $\epsilon > 0$, there exists a fuzzy function $l(x)$ belonging to the set of GFHMs such that the following condition satisfies:

$$|F(x) - l(x)| \leq \epsilon. \tag{21}$$

*Remark 3:* In Lemma 1, the detailed definition of GFHMs is introduced and the inter weight $\mathcal{V}$ is chosen as an identity matrix in this article. Moreover, Lemma 2 reveals the universal approximation ability of GFHMs similar to NN structure [41]. Therefore, its weights can be effectively regulated in virtue of several learning techniques. Owing to the abovementioned traits, GFHMs-assisted identifier can be deployed to tackle the unavailability of system dynamics.

### B. *Identifier Design for $\alpha$th Auxiliary Subsystem*

Due to the unknown dynamics $l_\alpha(x_\alpha)$ and $C_\alpha$, the proposed optimal control policies (16) are inapplicable for $\alpha$th auxiliary subsystem (5). To cope with this difficulty, we will present the GFHMs-assisted identifier structure to approximate the unavailable dynamics.

Apparently, the $\alpha$th constrained auxiliary subsystem (5) is equivalent to

$$\begin{aligned}\dot{x}_\alpha = {} & F_{G_\alpha}(x_\alpha) + G_\alpha x_\alpha + C_\alpha u_\alpha \\ & + h_\alpha(x_\alpha)d_\alpha + \lambda_\alpha(x_\alpha)v_\alpha \end{aligned} \tag{22}$$

where $F_{G_\alpha}(x_\alpha) = l_\alpha(x_\alpha) - G_\alpha x_\alpha$. $G_\alpha = [G_{\alpha 1}^T, \ldots, G_{\alpha n_\alpha}^T]^T$ denotes a designable matrix with compatible dimensions.

In order to approximate the $n_\alpha$ elements of $F_{G_\alpha}(x_\alpha)$, we subsequently adopt the $n_\alpha$ GFHMs with the same premise architecture $\{k, \ldots, k\}$. Subsequently, the dynamics (22) is represented by

$$\begin{cases}\dot{x}_{\alpha 1} = b_{\alpha 1}^* + (\mathcal{W}_{\alpha 1}^*)^T \tanh(X_{m\alpha} - \mathcal{D}_{\alpha 1}^*) + G_{\alpha 1}x_\alpha \\ \quad +\varepsilon_{\alpha 1} + C_{\alpha 1}u_\alpha + h_{\alpha 1}(x_\alpha)d_\alpha + \lambda_{\alpha 1}(x_\alpha)v_\alpha \\ \quad\quad \vdots \\ \dot{x}_{\alpha n_\alpha} = b_{\alpha n_\alpha}^* + (\mathcal{W}_{\alpha n_\alpha}^*)^T \tanh(X_{m\alpha} - \mathcal{D}_{\alpha n_\alpha}^*) + G_{\alpha n_\alpha}x_\alpha \\ \quad +\varepsilon_{\alpha n_\alpha} + C_{\alpha n_\alpha}u_\alpha + h_{\alpha n_\alpha}(x_\alpha)d_\alpha + \lambda_{\alpha n_\alpha}(x_\alpha)v_\alpha \end{cases} \tag{23}$$

in which $b_{\alpha 1}^*, \ldots, b_{\alpha n_\alpha}^* \in \mathbb{R}$; $\mathcal{W}_{\alpha 1}^*, \ldots, \mathcal{W}_{\alpha n_\alpha}^* \in \mathbb{R}^{kn_\alpha}$; $\mathcal{D}_{\alpha 1}^*, \ldots, \mathcal{D}_{\alpha n_\alpha}^* \in \mathbb{R}^{kn_\alpha}$; and $X_{m\alpha} = x_\alpha \otimes I_k \in \mathbb{R}^{kn_\alpha}$. $\varepsilon_{\alpha 1}, \ldots, \varepsilon_{\alpha n_\alpha} \in \mathbb{R}$ are the approximation errors.

Define $X_\alpha \triangleq [\underbrace{X_{m_\alpha}^T, \ldots, X_{m\alpha}^T}_{n_\alpha}]^T \in \mathbb{R}^{kn_\alpha^2}$. Then, model (23) is formulated as

$$\begin{aligned}\dot{x}_\alpha = {} & B_\alpha^* + (\mathcal{W}_\alpha^*)^T \tanh(X_\alpha - \mathcal{D}_\alpha^*) + \varepsilon_\alpha \\ & + G_\alpha x_\alpha + C_\alpha u_\alpha + h_\alpha(x_\alpha)d_\alpha + \lambda_\alpha(x_\alpha)v_\alpha \end{aligned} \tag{24}$$

where $\mathcal{W}_\alpha^* = \text{diag}\{\mathcal{W}_{\alpha 1}^*, \ldots, \mathcal{W}_{\alpha n_\alpha}^*\} \in \mathbb{R}^{kn_\alpha^2 \times n_\alpha}$, $B_\alpha^* = [b_{\alpha 1}^*, \ldots, b_{\alpha n_\alpha}^*]^T \in \mathbb{R}^{n_\alpha}$, $\mathcal{D}_\alpha^* = [(\mathcal{D}_{\alpha 1}^*)^T, \ldots, (\mathcal{D}_{\alpha n_\alpha}^*)^T]^T \in \mathbb{R}^{kn_\alpha^2}$, and $\varepsilon_\alpha = [\varepsilon_{\alpha 1}, \ldots, \varepsilon_{\alpha n_\alpha}]^T \in \mathbb{R}^{n_\alpha}$.

By denoting the estimated state as $\hat{x}_\alpha$, the identified dynamics can be obtained as follows:

$$\begin{aligned}\dot{\hat{x}}_\alpha = {} & \hat{B}_\alpha + \hat{\mathcal{W}}_\alpha^T \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha) + G_\alpha \hat{x}_\alpha \\ & + \hat{C}_\alpha u_\alpha + h_\alpha(\hat{x}_\alpha)d_\alpha + \lambda_\alpha(\hat{x}_\alpha)v_\alpha \end{aligned} \tag{25}$$

in which $\hat{B}_\alpha$, $\hat{\mathcal{W}}_\alpha$, $\hat{\mathcal{D}}_\alpha$, and $\hat{C}_\alpha$ are the estimated parameters of $B_\alpha^*, \mathcal{W}_\alpha^*, \mathcal{D}_\alpha^*$, and $C_\alpha$, respectively. $\hat{X}_\alpha = [\hat{X}_{m\alpha}^T, \ldots, \hat{X}_{m\alpha}^T]^T \in \mathbb{R}^{kn_\alpha^2}$ with $\hat{X}_{m\alpha} = \hat{x}_\alpha \otimes I_k$ serves as the augmented estimated state vector.

Subtracting (25) from (24), the state estimation error system is formulated as

$$\begin{aligned}\dot{e}_{G,\alpha} = {} & \tilde{B}_\alpha + \tilde{\mathcal{W}}_\alpha^T \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha) + \tilde{C}_\alpha u_\alpha + G_\alpha e_{G,\alpha} \\ & + (\mathcal{W}_\alpha^*)^T[\tanh(X_\alpha - \mathcal{D}_\alpha^*) - \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha)] \\ & + \bar{h}_\alpha(x_\alpha, \hat{x}_\alpha)d_\alpha + \bar{\lambda}_\alpha(x_\alpha, \hat{x}_\alpha)v_\alpha + \varepsilon_\alpha \end{aligned} \tag{26}$$

where $e_{G,\alpha} = x_\alpha - \hat{x}_\alpha$ is the state estimation error. $\tilde{B}_\alpha = B_\alpha^* - \hat{B}_\alpha$, $\tilde{\mathcal{W}}_\alpha = \mathcal{W}_\alpha^* - \hat{\mathcal{W}}_\alpha$, $\tilde{C}_\alpha = C_\alpha - \hat{C}_\alpha$, and $\tilde{\mathcal{D}}_\alpha = \mathcal{D}_\alpha^* - \hat{\mathcal{D}}_\alpha$ represent the weight estimation errors. $\bar{h}_\alpha(x_\alpha, \hat{x}_\alpha) = h_\alpha(x_\alpha) - h_\alpha(\hat{x}_\alpha)$ and $\bar{\lambda}_\alpha(x_\alpha, \hat{x}_\alpha) = \lambda_\alpha(x_\alpha) - \lambda_\alpha(\hat{x}_\alpha)$. $\tilde{X}_\alpha = X_\alpha - \hat{X}_\alpha$ denotes the augmented state error.

### C. *Convergence Analysis*

To conveniently assist the subsequent analysis, the following assumption can be given and has been utilized in [6] and [39].

*Assumption 4:* There exist constants $\varepsilon_{\alpha M} > 0, \theta_\alpha > 0$ and positive definite matrix $P_\alpha$ such that $||\varepsilon_\alpha|| \leq \varepsilon_{\alpha M}$, $\tilde{\mathcal{D}}_\alpha^T \tilde{\mathcal{D}}_\alpha \leq \theta_\alpha \tilde{X}_\alpha^T \tilde{X}_\alpha$, and $(\mathcal{W}_\alpha^*)^T \mathcal{W}_\alpha^* \leq P_\alpha$.

*Theorem 2:* For the $\alpha$th auxiliary subsystem (5), the identifier (25) is deployed via the following adaptive laws of the estimated weights $\hat{B}_\alpha$, $\hat{\mathcal{W}}_\alpha$, $\hat{\mathcal{D}}_\alpha$, and $\hat{C}_\alpha$:

$$\begin{cases} \dot{\hat{B}}_\alpha = \varrho_\alpha^1 e_{G,\alpha} \\ \dot{\hat{\mathcal{W}}}_{\alpha j} = \varrho_{\alpha j}^2 e_{G,\alpha j} \tanh(\hat{X}_{m\alpha} - \hat{\mathcal{D}}_{\alpha j}) \\ \dot{\hat{C}}_\alpha = e_{G,\alpha} u_\alpha^T \varrho_\alpha^3 \\ \dot{\hat{\mathcal{D}}}_{\alpha j} = -\varrho_{\alpha j}^4 (e_{G,\alpha} \otimes I_k) \end{cases} \tag{27}$$

where $\varrho_\alpha^1$, $\varrho_{\alpha j}^2$, $\varrho_{\alpha j}^4 \in \mathbb{R}$ are positive scalars and $\varrho_\alpha^3 \in \mathbb{R}^{z_\alpha \times z_\alpha}$ denotes a positive definite matrix with $j = 1, 2, \ldots, n_\alpha$. $e_{G,\alpha j}$ represents the $j$th component of $e_{G,\alpha}$. Then, $e_{G,\alpha}$ can converge to an adjustable neighborhood of the origin and the weight estimation errors $\tilde{B}_\alpha$, $\tilde{\mathcal{W}}_\alpha$, $\tilde{C}_\alpha$, $\tilde{\mathcal{D}}_\alpha$ are bounded under Assumptions 1–4.

*Proof:* See Appendix B. ■

Through the aforementioned proof, the state estimation error $e_{G,\alpha}$ can be kept within a narrow scope. Hence, it is sensible to reconstruct the $\alpha$th auxiliary subsystem as follows:

$$\begin{aligned} \dot{x}_\alpha &= \bar{B}_\alpha + \bar{\mathcal{W}}_\alpha^T \tanh(X_\alpha - \bar{\mathcal{D}}_\alpha) + G_\alpha x_\alpha \\ &\quad + \bar{C}_\alpha u_\alpha + h_\alpha(x_\alpha) d_\alpha + \lambda_\alpha(x_\alpha) v_\alpha \\ &= \hat{l}_\alpha(x_\alpha) + \bar{C}_\alpha u_\alpha + h_\alpha(x_\alpha) d_\alpha + \lambda_\alpha(x_\alpha) v_\alpha \end{aligned} \tag{28}$$

where $\hat{l}_\alpha(x_\alpha) = \bar{B}_\alpha + \bar{\mathcal{W}}_\alpha^T \tanh(X_\alpha - \bar{\mathcal{D}}_\alpha) + G_\alpha x_\alpha$. $\bar{B}_\alpha$, $\bar{\mathcal{W}}_\alpha$, $\bar{\mathcal{D}}_\alpha$, and $\bar{C}_\alpha$ denote the finally converged weights of $\hat{B}_\alpha$, $\hat{\mathcal{W}}_\alpha$, $\hat{\mathcal{D}}_\alpha$, and $\hat{C}_\alpha$, respectively. For convenience, the mathematical symbols $l_\alpha(x_\alpha)$ and $C_\alpha$ will continue to be utilized without indicating that they are the converged values during the identification process. So far, the unknown parameters of each auxiliary subsystem have been approximately attained. However, the analytical solution of DTAETS-boosted HJIE (17) still cannot be derived owing to the nonlinearity of $\alpha$th auxiliary subsystem (5). Aiming to overcome this obstacle, an effective algorithm will be put forward to obtain the related numerical solution in the succeeding section.

## IV. Data-Driven Decentralized RL Algorithm

In this section, a data-driven decentralized RL algorithm within critic-only NN architecture is devised to acquire the numerical solution of the targeted HJIE (17). Moreover, the validity analysis of the proposed algorithm is also conducted by demonstrating the UUB stability of $\alpha$th auxiliary subsystem (5) and weight estimation error.

### A. Critic-Only NN Structure

Owing to the universal approximation ability, a unique critic NN is adopted to reconstruct $J_\alpha^*(x_\alpha)$ in (7)

$$J_\alpha^*(x_\alpha) = w_\alpha^T \varphi_\alpha(x_\alpha) + \varpi_\alpha(x_\alpha) \tag{29}$$

in which $w_\alpha \in \mathbb{R}^{n_\alpha^c}$ is the critic weight with $n_\alpha^c$ being the quantity of neurons. $\varphi_\alpha(x_\alpha) = [\varphi_{\alpha 1}(x_\alpha), \ldots, \varphi_{\alpha n_\alpha^c}(x_\alpha)]^T \in \mathbb{R}^{n_\alpha^c}$ serves as activation function with linearly independent components. $\varpi_\alpha(x_\alpha)$ represents the residual error.

From (29), the derivative of $J_\alpha^*(x_\alpha)$ can be calculated as

$$\nabla J_\alpha^*(x_\alpha) = \nabla \varphi_\alpha^T(x_\alpha) w_\alpha + \nabla \varpi_\alpha(x_\alpha). \tag{30}$$

Afterwards, the ET control signals $u_\alpha^*(x_\alpha, s)$, $v_\alpha^*(x_\alpha, s)$ in (16) and the worst disturbance $d_\alpha^*(x_\alpha)$ in (10) can be derived as follows:

$$\begin{cases} u_\alpha^*(x_{\alpha,s}) = -\zeta_\alpha \Pi(\mathcal{A}_{\alpha,1}(x_{\alpha,s})) + \varpi_{u_\alpha^*}(x_{\alpha,s}) \\ v_\alpha^*(x_{\alpha,s}) = -\frac{1}{2\delta_\alpha} \lambda_\alpha^T(x_{\alpha,s}) \left[\nabla \varphi_\alpha^T(x_{\alpha,s}) w_\alpha + \nabla \varpi_\alpha(x_{\alpha,s})\right] \\ d_\alpha^*(x_\alpha) = \frac{1}{2\gamma_\alpha^2} h_\alpha^T(x_\alpha) \left[\nabla \varphi_\alpha^T(x_\alpha) w_\alpha + \nabla \varpi_\alpha(x_\alpha)\right] \end{cases} \tag{31}$$

where $\mathcal{A}_{\alpha,1}(x_{\alpha,s}) = \frac{1}{2\zeta_\alpha} C_\alpha^T \nabla \varphi_\alpha^T(x_{\alpha,s}) w_\alpha$, $\varpi_{u_\alpha^*}(x_{\alpha,s}) = -\frac{1}{2}(I_{z_\alpha} - \tanh^2(\xi)) C_\alpha^T \nabla \varpi_\alpha(x_{\alpha,s})$ with $\xi$ determined between $\frac{1}{2\zeta_\alpha} C_\alpha^T \nabla J_\alpha^*(x_{\alpha,s})$ and $\mathcal{A}_{\alpha,1}(x_{\alpha,s})$.

In actual circumstance, the control strategies $u_\alpha^*(x_\alpha, s)$ and $v_\alpha^*(x_\alpha, s)$ in (31) cannot be executed due to the unidentified weight $w_\alpha$. For conquering this obstacle, $w_\alpha$ will be replaced by its approximation $\hat{w}_\alpha$, and the following form is obtained:

$$\hat{J}_\alpha(x_\alpha) = \hat{w}_\alpha^T \varphi_\alpha(x_\alpha), \ \nabla \hat{J}_\alpha(x_\alpha) = \nabla \varphi_\alpha^T(x_\alpha) \hat{w}_\alpha. \tag{32}$$

On account of (32), the approximate values of $u_\alpha^*(x_{\alpha,s})$, $v_\alpha^*(x_{\alpha,s})$ and $d_\alpha^*(x_\alpha)$ are elicited as

$$\begin{cases} \hat{u}_\alpha(x_{\alpha,s}) = -\zeta_\alpha \Pi\left(\frac{1}{2\zeta_\alpha} C_\alpha^T \nabla \varphi_\alpha^T(x_{\alpha,s}) \hat{w}_\alpha\right) \\ \hat{v}_\alpha(x_{\alpha,s}) = -\frac{1}{2\delta_\alpha} \lambda_\alpha^T(x_{\alpha,s}) \nabla \varphi_\alpha^T(x_{\alpha,s}) \hat{w}_\alpha \\ \hat{d}_\alpha(x_\alpha) = \frac{1}{2\gamma_\alpha^2} h_\alpha^T(x_\alpha) \nabla \varphi_\alpha^T(x_\alpha) \hat{w}_\alpha. \end{cases} \tag{33}$$

Utilizing $\nabla \hat{J}_\alpha(x_\alpha)$ in (32) and $\hat{u}_\alpha(x_{\alpha,s})$, $\hat{d}_\alpha(x_\alpha)$, $\hat{v}_\alpha(x_{\alpha,s})$ in (33) to, respectively, substitute for $\nabla J_\alpha^*(x_\alpha)$, $u_\alpha$, $d_\alpha$, $v_\alpha$ in (8), the approximate Hamilton function is inferred as follows:

$$\begin{aligned} &\hat{H}(\nabla \hat{J}_\alpha(x_\alpha), x_\alpha, \hat{u}_\alpha(x_{\alpha,s}), \hat{v}_\alpha(x_{\alpha,s}), \hat{d}_\alpha(x_\alpha)) \\ &= \hat{w}_\alpha^T \nabla \varphi_\alpha(x_\alpha) \Big(l_\alpha(x_\alpha) + C_\alpha \hat{u}_\alpha(x_{\alpha,s}) + h_\alpha(x_\alpha) \hat{d}_\alpha(x_\alpha) \\ &\quad + \lambda_\alpha(x_\alpha) \hat{v}_\alpha(x_{\alpha,s})\Big) + \Psi_\alpha(x_\alpha) + x_\alpha^T Q_\alpha x_\alpha \\ &\quad + \vartheta_\alpha(\hat{u}_\alpha(x_{\alpha,s})) + \delta_\alpha ||\hat{v}_\alpha(x_{\alpha,s})||^2 - \gamma_\alpha^2 ||\hat{d}_\alpha(x_\alpha)||^2. \end{aligned} \tag{34}$$

Define $e_{c_\alpha}$ as the approximation error of Hamilton function. Then, it can be derived that

$$\begin{aligned} e_{c_\alpha} &= \hat{H}(\nabla \hat{J}_\alpha(x_\alpha), x_\alpha, \hat{u}_\alpha(x_{\alpha,s}), \hat{v}_\alpha(x_{\alpha,s}), \hat{d}_\alpha(x_\alpha)) \\ &\quad - H(\nabla J_\alpha^*(x_\alpha), x_\alpha, u_\alpha^*(x_{\alpha,s}), v_\alpha^*(x_{\alpha,s}), d_\alpha^*(x_\alpha)) \\ &= \hat{w}_\alpha^T \psi_\alpha + \Psi_\alpha(x_\alpha) + x_\alpha^T Q_\alpha x_\alpha + \vartheta_\alpha(\hat{u}_\alpha(x_{\alpha,s})) \\ &\quad + \delta_\alpha ||\hat{v}_\alpha(x_{\alpha,s})||^2 - \gamma_\alpha^2 ||\hat{d}_\alpha(x_\alpha)||^2 \end{aligned} \tag{35}$$

with $\psi_\alpha = \nabla \varphi_\alpha(x_\alpha)(l_\alpha(x_\alpha) + C_\alpha \hat{u}_\alpha(x_{\alpha,s}) + h_\alpha(x_\alpha) \hat{d}_\alpha(x_\alpha) + \lambda_\alpha(x_\alpha) \hat{v}_\alpha(x_{\alpha,s}))$.

In order to attain a sufficiently small $e_{c_\alpha}$, the following weight tuning rule is applied to minimize $E_{c_\alpha} = \frac{1}{2} e_{c_\alpha}^T e_{c_\alpha}$:

$$\dot{\hat{w}}_\alpha = -\frac{\rho_\alpha}{(1 + \psi_\alpha^T \psi_\alpha)^2} \frac{\partial E_{c_\alpha}}{\partial \hat{w}_\alpha} = -\frac{\rho_\alpha \psi_\alpha}{(1 + \psi_\alpha^T \psi_\alpha)^2} e_{c_\alpha} \tag{36}$$

in which $\rho_\alpha > 0$ denotes the learning rate.

Denoting $\tilde{w}_\alpha = w_\alpha - \hat{w}_\alpha$ as the weight estimation error, one has

$$\dot{\tilde{w}}_\alpha = -\rho_\alpha \frac{\psi_\alpha \psi_\alpha^T}{(1+\psi_\alpha^T\psi_\alpha)^2}\tilde{w}_\alpha + \frac{\rho_\alpha\psi_\alpha}{(1+\psi_\alpha^T\psi_\alpha)^2}\varpi_{H_\alpha} \quad (37)$$

with $\varpi_{H_\alpha} = -\nabla\varpi_\alpha^T(x_\alpha)(l_\alpha(x_\alpha) + C_\alpha\hat{u}_\alpha(x_{\alpha,s}) + h_\alpha(x_\alpha)\hat{d}_\alpha(x_\alpha) + \lambda_\alpha(x_\alpha)\hat{v}_\alpha(x_{\alpha,s}))$.

## B. Validity Analysis

In what follows, we will prove the UUB stability of the $\alpha$th auxiliary subsystem (5) and the weight estimation error $\tilde{w}_\alpha$ so as to validate the aforementioned algorithm. For this purpose, a vital assumption is imposed to promote this analysis.

*Assumption 5 (see [18]):* $||\nabla\varphi_\alpha(x_\alpha)|| \le \nabla\varphi_{\alpha M}$, $||\nabla\varpi_\alpha(x_\alpha)|| \le \nabla\varpi_{\alpha M}$, $||\varpi_{H_\alpha}|| \le D_{\varpi_{H_\alpha}}$, and $||\varpi_{u_\alpha^*}(x_\alpha)|| \le D_{\varpi_{u_\alpha^*}}$, where $\nabla\varphi_{\alpha M}$, $\nabla\varpi_{\alpha M}$, $D_{\varpi_{H_\alpha}}$, and $D_{\varpi_{u_\alpha^*}}$ represent the positive parameters.

Denote $\Upsilon(\mathcal{A}_{\alpha,\mu}(x_\alpha)) = \zeta_\alpha \tanh(\mathcal{A}_{\alpha,\mu}(x_\alpha))$, $\mu \in \{1,2\}$ with $\mathcal{A}_{\alpha,1}(x_\alpha)$ given by (31) and $\mathcal{A}_{\alpha,2}(x_\alpha) = \frac{1}{2\zeta_\alpha}C_\alpha^T\nabla\varphi_\alpha^T(x_\alpha)\hat{w}_\alpha$. Illuminated by the Taylor's theorem [42], the following result is obtained:

$$\begin{aligned}\Upsilon(\mathcal{A}_{\alpha,1}(x_\alpha)) &= \Upsilon(\mathcal{A}_{\alpha,2}(x_\alpha)) \\ &+ \frac{1}{2}\left(I_{z_\alpha\times z_\alpha} - \epsilon_{\mathcal{A}_{\alpha,2}}(x_\alpha)\right)C_\alpha^T\nabla\varphi_\alpha^T(x_\alpha)\tilde{w}_\alpha \\ &+ O\left((\mathcal{A}_{\alpha,1}(x_\alpha) - \mathcal{A}_{\alpha,2}(x_\alpha))^2\right)\end{aligned} \quad (38)$$

with $\epsilon_{\mathcal{A}_{\alpha,2}}(x_\alpha) = \text{diag}\{\tanh^2(\mathcal{A}_{\alpha,2}(x_\alpha))\}$. $O(\cdot)$ serves as the high-order infinitesimal, which is denoted as $O_{\mathcal{A}_\alpha}$ for brevity. As demonstrated in [37], the bound of $O_{\mathcal{A}_\alpha}$ is described by

$$O_{\mathcal{A}_\alpha} \le 2\sqrt{z_\alpha} + D_{C_\alpha}\nabla\varphi_{\alpha M}||\tilde{w}_\alpha||. \quad (39)$$

*Theorem 3:* Under the consideration of the approximately optimal ET control strategies $\hat{u}_\alpha(x_{\alpha,s})$ and $\hat{v}_\alpha(x_{\alpha,s})$ in (33) with the admissible initial values, the approximately worst interference $\hat{d}_\alpha(x_\alpha)$ in (33), the tuning law of $\hat{w}_\alpha$ in (36), and Assumptions 1–5. If the following conditions are satisfied, the state $x_\alpha$ in the $\alpha$th auxiliary subsystem (5) and the weight estimation error $\tilde{w}_\alpha$ are assured to be UUB:

$$\begin{cases}\lambda_{\min}(Q_\alpha) - 2(1+\delta_\alpha)\frac{\mathcal{K}_{\max}^2\lambda_{\max}(\Theta_\alpha)(2\overline{\eta}_{1,\alpha}+\overline{\eta}_{2,\alpha})}{\lambda_{\min}(\Theta_\alpha)-2\overline{\eta}_{1,\alpha}\lambda_{\max}(\Theta_\alpha)} > 0 \\ \frac{\rho_\alpha}{2}\lambda_{\min}(\Phi_\alpha) - \nabla\varphi_{\alpha M}^2\left(\frac{D_{h_\alpha}^2}{2\gamma_\alpha^2} + \frac{D_{\lambda_\alpha}^2}{\delta_\alpha} + 24D_{C_\alpha}^2\right) > 0\end{cases} \quad (40)$$

in which $\mathcal{K}_{\max} = \max\{\mathcal{K}_{u_\alpha^*}, \mathcal{K}_{v_\alpha^*}\}$ and $\Phi_\alpha = \psi_\alpha\psi_\alpha^T/(1+\psi_\alpha^T\psi_\alpha)^2$.

*Proof:* See Appendix C. ■

## V. Experimental Study

To elaborate the feasibility of the designed decentralized control approach, a numerical experiment is conducted for constrained-input NISs with two subsystems. Borrowed from [37], the relative parameters are provided as follows:

$$\begin{aligned}\dot{x}_1 &= l_1(x_1) + \begin{bmatrix}0\\0.9\end{bmatrix}u_1 + \begin{bmatrix}0.5\\0\end{bmatrix}d_1 + \begin{bmatrix}1\\0\end{bmatrix}\phi_1(x) \\ \dot{x}_2 &= l_2(x_2) + \begin{bmatrix}0\\1.2\end{bmatrix}u_2 + \begin{bmatrix}0.3\\0\end{bmatrix}d_2 + \begin{bmatrix}1\\0\end{bmatrix}\phi_1(x)\end{aligned} \quad (41)$$

in which

$$l_1(x_1) = \begin{bmatrix}x_{12} - x_{11}\\0.5x_{11}^2x_{12} - 0.5(x_{11}+x_{12})\end{bmatrix}$$

$$l_2(x_2) = \begin{bmatrix}0.5x_{22}\\0.5x_{21}\cos^2(x_{22}) - x_{21} - 0.5x_{22}\end{bmatrix}$$

$$\phi_1(x) = (x_{11}+x_{22})\cos(0.5x_{21})\sin^2(\bar{\lambda}_1x_{12})$$

$$\phi_2(x) = 0.5(x_{12}+x_{22})\cos(\bar{\lambda}_2e^{x_{21}^2})$$

and the uncertain values $\bar{\lambda}_1, \bar{\lambda}_2$ are stochastically chosen within $[-1,1]$. The upper bounds of constrained input are determined by $\zeta_1 = \zeta_2 = 0.08$. To ensure Assumption 1, the related parameters can be selected as $c_{11} = c_{12} = 1$, $c_{21} = c_{22} = 0.5$, $F_1(x_1) = ||x_1||$, and $F_2(x_2) = ||x_2||$. From the aforementioned description, it is not difficult to witness that Assumption 2 can also be met. Meanwhile, we consider the initial state of NISs (41) as $x_1(0) = [1.5, -0.8]^T$ and $x_2(0) = [1.5, 0.8]^T$.

In the performance function (6), the specific parameters are presented as $Q_1 = 2I_{2\times2}$, $Q_2 = 3I_{2\times2}$, $\delta_1 = \delta_2 = 0.1$, $\varsigma_1 = 2.5$, $\varsigma_2 = 2.2$, $\gamma_1 = 0.18$, and $\gamma_2 = 0.45$. In addition, the weight matrices in DTAETS (13) are selected as $\Theta_1 = \text{diag}\{0.45, 0.75\}$ and $\Theta_2 = \text{diag}\{0.36, 0.62\}$. The lower and upper bounds of the dual threshold functions (14) become $\eta_{1,1} \in [0.007, 0.025]$, $\eta_{2,1} \in [0.005, 0.015]$, $\eta_{1,2} \in [0.006, 0.022]$, and $\eta_{2,2} \in [0.002, 0.018]$.

Moreover, the activation function $\varphi_\alpha(x_\alpha)$ in critic NN (29) is chosen as $[x_{\alpha1}^3, x_{\alpha1}^2x_{\alpha2}, x_{\alpha1}x_{\alpha2}, x_{\alpha1}x_{\alpha2}^2, x_{\alpha2}^3]^T$ ($\alpha = 1,2$) and its learning rates can be designed as $\rho_1 = 0.38$ and $\rho_2 = 0.12$. The approximated weight $\hat{w}_\alpha$ is set to be $[\hat{w}_{\alpha1}, \hat{w}_{\alpha2}, \hat{w}_{\alpha3}, \hat{w}_{\alpha4}, \hat{w}_{\alpha5}]^T$ with the initialization $\hat{w}_1(0) = 0.034I_5$ and $\hat{w}_2(0) = 0.045I_5$. Similar to [39], an exploratory noise is added into the control input of each auxiliary subsystem for 20 s. In the identified dynamics (25), $G_1 = G_2 = -3I_{2\times2}$. By adopting the proposed algorithm, the dynamics $l_\alpha(x_\alpha)$ and $C_\alpha$ are obtained via GFHMs-assisted identifier, which is concurrently regulated with the evolution of constrained auxiliary subsystem states. To this end, the relevant parameters in (27) are determined by

$$\varrho_1^1 = 1.02,\ \varrho_2^1 = 0.95,\ \varrho_{11}^2 = 1.21,\ \varrho_{12}^2 = 1.48$$

$$\varrho_{21}^2 = 1.16,\ \varrho_{22}^2 = 1.52,\ \varrho_1^3 = 1.07,\ \varrho_2^3 = 1.05$$

$$\varrho_{11}^4 = 1.98,\ \varrho_{12}^4 = 1.53,\ \varrho_{21}^4 = 2.03,\ \varrho_{22}^4 = 1.45.$$

Under such a circumstance, the relative simulation results are depicted in Figs. 1–7. For auxiliary subsystems 1 and 2, the state estimation errors $e_{G,1}$ and $e_{G,2}$ of GFHMs-assisted identifier are plotted in Figs. 1 and 2, respectively. It is witnessed that $e_{G,1}$

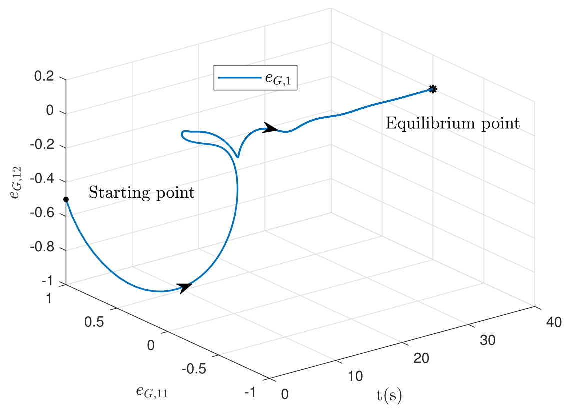


Fig. 1. State estimation error $e_{G,1}$ for auxiliary subsystem 1.

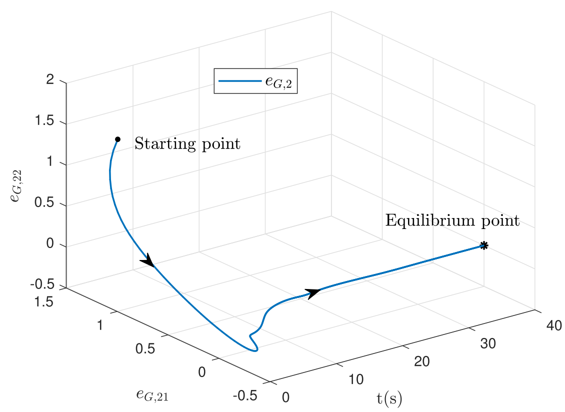


Fig. 2. State estimation error $e_{G,2}$ for auxiliary subsystem 2.

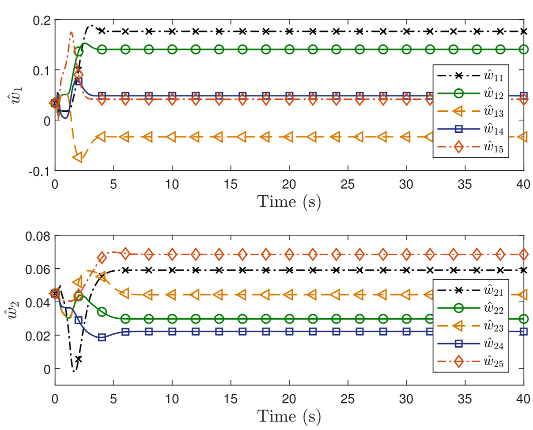


Fig. 3. Evolutions of the critic weights $\hat{w}_1$ and $\hat{w}_2$.

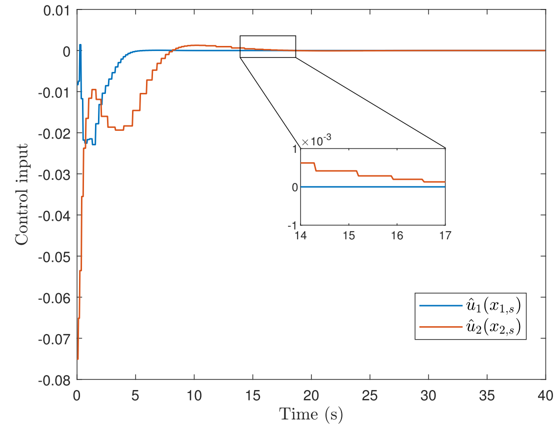


Fig. 4. Responses of the constrained inputs $\hat{u}_1(x_{1,s})$ and $\hat{u}_2(x_{2,s})$.

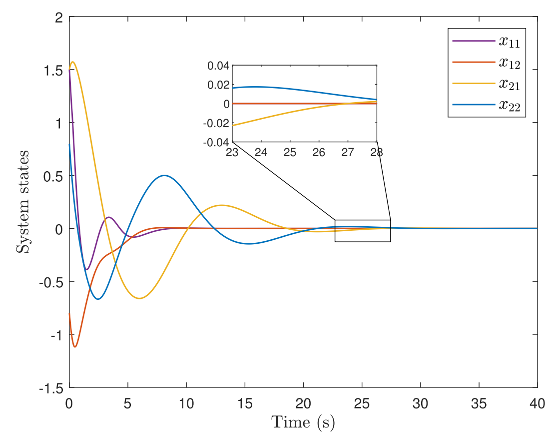


Fig. 5. Trajectories of the whole system states.

and $e_{G,2}$ can ultimately tend to the zero equilibrium point. Thus, the unknown system dynamics can be approximated by implementing the GFHMs-based identifier. Moreover, Fig. 3 displays the weight evolutions of critic-only NN based on the abovementioned approximate parameters. Before the added probing noise is detached, the critic weights $\hat{w}_1$ and $\hat{w}_2$ can converge to the final values $\hat{w}_1 = [0.1763, 0.1405, -0.0332, 0.0484, 0.0415]^T$ and $\hat{w}_2 = [0.0590, 0.0298, 0.0443, 0.0222, 0.0685]^T$. On these grounds, the proposed data-driven decentralized RL algorithm has been executed successfully.

In Fig. 4, the DTAETS-assisted control strategies $\hat{u}_1(x_{1,s})$ and $\hat{u}_2(x_{2,s})$ are presented under the constraints $|\hat{u}_1(x_{1,s})| \leq 0.08$ and $|\hat{u}_2(x_{2,s})| \leq 0.08$. Evidently, these control strategies are accordance with the restricted condition. Under the decentralized control policy consisting of $\hat{u}_1(x_{1,s})$ and $\hat{u}_2(x_{2,s})$, the whole states of NISs are convergent according to Fig. 5. It indicates that the adopted control scheme is practicable from the perspective of assuring the system stability.

Meanwhile, the DTAETS-based ET interval and dual threshold functions for two auxiliary systems are depicted in Figs. 6 and 7. Since the dual threshold functions have been stable about 10 s, the relevant trajectories are omitted after 10 s. It is not complex to deduce that the dual threshold functions can adaptively regulate and eventually converge to their upper bounds. To further validate the merit of the proposed DTAETS, the comparative results are presented in Table I by utilizing time-triggered scheme (TTS) in [43] and AETS in [31]. Obviously, the released packages and the transmission ratio under DTAETS are both the minimum. Hence, the designed DTAETS can alleviate the network occupancy and computational pressure.

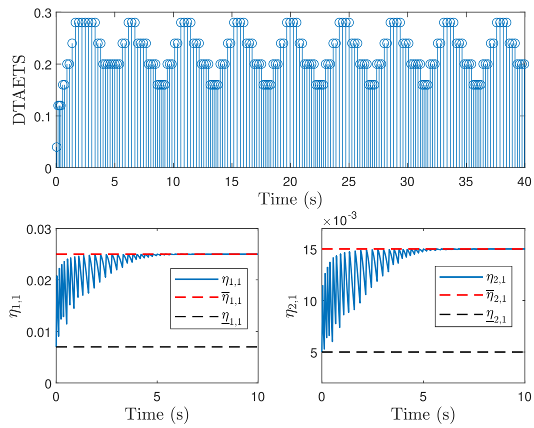


Fig. 6. ET interval and dual threshold functions for auxiliary subsystem 1.

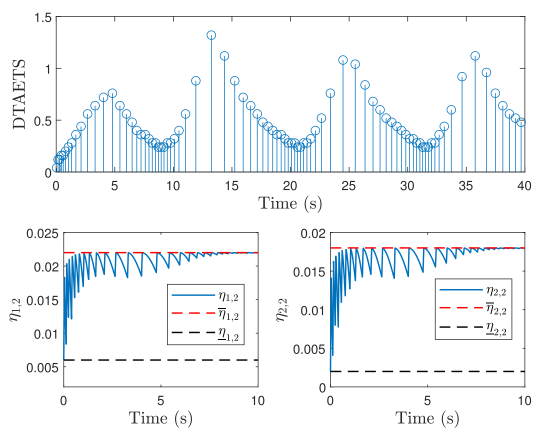


Fig. 7. ET interval and dual threshold functions for auxiliary subsystem 2.

TABLE I
COMPARISON OF DIFFERENT TRIGGERED SCHEMES

| Terms \ schemes | | TTS | AETS | DTAETS |
|---|---|---|---|---|
| Released package | Subsystem 1 | 1000 | 215 | 185 |
| | Subsystem 2 | 1000 | 106 | 85 |
| Transmission ratio | Subsystem 1 | 100% | 21.5% | 18.5% |
| | Subsystem 2 | 100% | 10.6% | 8.5% |

## VI. Conclusion

This article concentrates on the data-driven decentralized controller design for the constrained-input NISs with unknown dynamics. By devising an appropriate performance function, the investigated regulation problem is converted into solving the HJIE for $N$ auxiliary subsystems. On the ground of dual threshold functions, a novel DTAETS is proposed to relieve the computational and communication pressure. Afterwards, the DTAETS-boosted HJIE is constructed for the constrained auxiliary subsystems. With the aim of handling the unavailable system parameters, we present a GFHMs-assisted identifier to approximate the drift and input dynamics. On this basis, a data-driven decentralized RL algorithm is proposed within the critic-sole NN structure. Then, the UUB stability of auxiliary subsystems and the weight estimation error is guaranteed. Eventually, the effectiveness of the presented algorithm can be validated via a simulation experiment. It should be noted that the cyber attacks exist in the open communication network. Thus, the data-driven security control scheme will be explored for the constrained NISs in the future research.

## Appendix A
## Proof of Theorem 1

We take the following Lyapunov function into consideration:

$$L(x) = \sum_{\alpha=1}^{N} J_\alpha^*(x_\alpha). \tag{42}$$

By differentiating (42) along the $N$ trajectories $l_\alpha(x_\alpha) + C_\alpha u_\alpha^*(x_{\alpha,s}) + h_\alpha(x_\alpha) d_\alpha^*(x_\alpha) + \Delta l_\alpha(x)$, one has

$$\begin{aligned}\dot{L}(x) = &\sum_{\alpha=1}^{N} \Big\{ (\nabla J_\alpha^*(x_\alpha))^T \left( l_\alpha(x_\alpha) + C_\alpha u_\alpha^*(x_\alpha) \right.\\ &+ h_\alpha(x_\alpha) d_\alpha^*(x_\alpha) + \lambda_\alpha(x_\alpha)\phi_\alpha(x)) \\ &+ (\nabla J_\alpha^*(x_\alpha))^T C_\alpha \left( u_\alpha^*(x_{\alpha,s}) - u_\alpha^*(x_\alpha) \right) \Big\}. \end{aligned} \tag{43}$$

According to expressions (10) and (11), it yields

$$(\nabla J_\alpha^*(x_\alpha))^T C_\alpha = -2\zeta_\alpha \Pi^{-T} \left( u_\alpha^*(x_\alpha)/\zeta_\alpha \right) \tag{44}$$

$$(\nabla J_\alpha^*(x_\alpha))^T \lambda_\alpha(x_\alpha) = -2\delta_\alpha \left( v_\alpha^*(x_\alpha) \right)^T \tag{45}$$

$$\begin{aligned}&(\nabla J_\alpha^*(x_\alpha))^T \left( l_\alpha(x_\alpha) + C_\alpha u_\alpha^*(x_\alpha) + h_\alpha(x_\alpha) d_\alpha^*(x_\alpha) \right) \\ &\quad = -\Psi_\alpha(x_\alpha) - x_\alpha^T Q_\alpha x_\alpha - \vartheta_\alpha(u_\alpha^*(x_\alpha)) \\ &\qquad + \delta_\alpha ||v_\alpha^*(x_\alpha)||^2 + \gamma_\alpha^2 ||d_\alpha^*(x_\alpha)||^2. \end{aligned} \tag{46}$$

Inserting conditions (44)–(46) into (43), it is not hard to derive that

$$\begin{aligned}\dot{L}(x) = &\sum_{\alpha=1}^{N} \Big\{ -\Psi_\alpha(x_\alpha) - x_\alpha^T Q_\alpha x_\alpha - 2\delta_\alpha \left( v_\alpha^*(x_\alpha) \right)^T \phi_\alpha(x) \\ &- \vartheta_\alpha(u_\alpha^*(x_\alpha)) + \delta_\alpha ||v_\alpha^*(x_\alpha)||^2 + \gamma_\alpha^2 ||d_\alpha^*(x_\alpha)||^2 \\ &+ 2\zeta_\alpha \Pi^{-T} \left( u_\alpha^*(x_\alpha)/\zeta_\alpha \right) \left( u_\alpha^*(x_\alpha) - u_\alpha^*(x_{\alpha,s}) \right) \Big\}. \end{aligned} \tag{47}$$

On the basis of Assumption 1, we have

$$\begin{aligned}&-2\delta_\alpha \left( v_\alpha^*(x_\alpha) \right)^T \phi_\alpha(x) \le 2\delta_\alpha \big|\big| v_\alpha^*(x_\alpha) \big|\big| \big|\big| \phi_\alpha(x) \big|\big| \\ &\qquad \le 2\delta_\alpha \big|\big| v_\alpha^*(x_\alpha) \big|\big| \sum_{r=1}^{N} c_{\alpha r} F_r(x_r). \end{aligned} \tag{48}$$

By virtue of Young's inequality $2\tilde{\alpha}^T \tilde{\beta} \le ||\tilde{\alpha}||^2 + ||\tilde{\beta}||^2$ in [44], Assumptions 2 and 3, the following results are obtained:

$$\begin{aligned}&2\zeta_\alpha \Pi^{-T} \left( u_\alpha^*(x_\alpha)/\zeta_\alpha \right) \left( u_\alpha^*(x_\alpha) - u_\alpha^*(x_{\alpha,s}) \right) \\ &\le ||\zeta_\alpha \Pi^{-1} \left( u_\alpha^*(x_\alpha)/\zeta_\alpha \right)||^2 + ||u_\alpha^*(x_\alpha) - u_\alpha^*(x_{\alpha,s})||^2 \\ &\le \frac{1}{4} D_{C_\alpha}^2 \nabla J_{\alpha M}^2 + \mathcal{K}_{u_\alpha^*}^2 ||e_{\alpha,s}||^2. \end{aligned} \tag{49}$$

On the basis of Assumptions 2 and 3, it can be readily derived from (10) that

$$||v_\alpha^*(x_\alpha)||^2 \le \frac{1}{4\delta_\alpha^2} D_{\lambda_\alpha}^2 \nabla J_{\alpha M}^2 \tag{50}$$

$$||d_\alpha^*(x_\alpha)||^2 \le \frac{1}{4\gamma_\alpha^4} D_{h_\alpha}^2 \nabla J_{\alpha M}^2. \tag{51}$$

Combining formulas (48)–(51), condition (47) is developed as

$$\begin{aligned}\dot{L}(x) \le &- \sum_{\alpha=1}^{N} \left\{ \Psi_\alpha(x_\alpha) + \delta_\alpha^2 ||v_\alpha^*(x_\alpha)||^2 \right.\\ &\left. - 2\delta_\alpha ||v_\alpha^*(x_\alpha)|| \sum_{r=1}^{N} c_{\alpha r} F_r(x_r) \right\}\\ &+ \sum_{\alpha=1}^{N} \left\{ \mathcal{K}_{u_\alpha^*}^2 ||e_{\alpha,s}||^2 - \lambda_{\min}(Q_\alpha)||x_\alpha||^2 \right.\\ &\left. + \left( \frac{1}{4} D_{C_\alpha}^2 + \frac{1+\delta_\alpha}{4\delta_\alpha} D_{\lambda_\alpha}^2 + \frac{D_{h_\alpha}^2}{4\gamma_\alpha^2} \right) \nabla J_{\alpha M}^2 \right\}. \end{aligned} \tag{52}$$

For $t \in [t_s^\alpha, t_{s+1}^\alpha)$, it is apparent that the triggered condition (13) is not satisfied, which means

$$\begin{aligned}&\lambda_{\min}(\Theta_\alpha)||e_{\alpha,s}||^2 \le \lambda_{\max}(\Theta_\alpha)(\overline{\eta}_{1,\alpha}||x_{\alpha,s}||^2 + \overline{\eta}_{2,\alpha}||x_\alpha||^2)\\ &= \lambda_{\max}(\Theta_\alpha)(\overline{\eta}_{1,\alpha}||e_{\alpha,s} + x_\alpha||^2 + \overline{\eta}_{2,\alpha}||x_\alpha||^2)\\ &\le \lambda_{\max}(\Theta_\alpha)(2\overline{\eta}_{1,\alpha}||e_{\alpha,s}||^2 + (2\overline{\eta}_{1,\alpha} + \overline{\eta}_{2,\alpha})||x_\alpha||^2). \end{aligned} \tag{53}$$

Recalling the expression $\overline{\eta}_{1,\alpha} < \lambda_{\min}(\Theta_\alpha)/(2\lambda_{\max}(\Theta_\alpha))$ presented in (14), the following inequality can be acquired:

$$||e_{\alpha,s}||^2 \le \frac{(2\overline{\eta}_{1,\alpha} + \overline{\eta}_{2,\alpha})\lambda_{\max}(\Theta_\alpha)}{\lambda_{\min}(\Theta_\alpha) - 2\overline{\eta}_{1,\alpha}\lambda_{\max}(\Theta_\alpha)} ||x_\alpha||^2. \tag{54}$$

Denote the auxiliary vector $\varpi = [-F_1(x_1), \ldots, -F_N(x_N), \delta_1||v_1^*(x_1)||, \ldots, \delta_N||v_N^*(x_N)||]^T$ and the auxiliary matrix $\tilde{\varsigma} = \text{diag}\{\varsigma_1, \ldots, \varsigma_N\}$. Based on formula (54), expression (52) is rewritten as

$$\dot{L}(x) \le \sum_{\alpha=1}^{N} \left\{ \Lambda_{1,\alpha} - \Lambda_{2,\alpha}||x_\alpha||^2 \right\} - \varpi^T \mathcal{P} \varpi \tag{55}$$

in which

$$\Lambda_{1,\alpha} = \frac{1}{4}\nabla J_{\alpha M}^2 \left( D_{C_\alpha}^2 + \frac{1+\delta_\alpha}{\delta_\alpha} D_{\lambda_\alpha}^2 + \frac{D_{h_\alpha}^2}{\gamma_\alpha^2} \right)$$

$$\Lambda_{2,\alpha} = \lambda_{\min}(Q_\alpha) - \frac{\mathcal{K}_{u_\alpha^*}^2 (2\overline{\eta}_{1,\alpha} + \overline{\eta}_{2,\alpha})\lambda_{\max}(\Theta_\alpha)}{\lambda_{\min}(\Theta_\alpha) - 2\overline{\eta}_{1,\alpha}\lambda_{\max}(\Theta_\alpha)}$$

$$\mathcal{P} = \begin{bmatrix} \tilde{\varsigma} & \tilde{C}^T \\ \tilde{C} & I_{N\times N} \end{bmatrix}, \ \tilde{C} = \begin{bmatrix} c_{11} & c_{12} & \cdots & c_{1\,N} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N1} & c_{N2} & \cdots & c_{NN} \end{bmatrix}.$$

Obviously, $\mathcal{P}$ can be assured as a positive definite matrix by selecting sufficiently large $\varsigma_\alpha$ $(\alpha = 1, 2, \ldots, N)$. Therefore, for $\forall \varsigma_\alpha \ge \varsigma_\alpha^*, -\varpi^T \mathcal{P} \varpi < 0$ holds and condition (55) is expressed as follows:

$$\dot{L}(x) \le \sum_{\alpha=1}^{N} \left\{ \Lambda_{1,\alpha} - \Lambda_{2,\alpha}||x_\alpha||^2 \right\}. \tag{56}$$

It is concluded that when inequality (18) satisfies, $\dot{L}(x) < 0$ if $x_\alpha$ is out of the set $\Im_\alpha$ described by

$$\Im_\alpha = \left\{ x_\alpha : \ \ ||x_\alpha|| \le \sqrt{\frac{\Lambda_{1,\alpha}}{\Lambda_{2,\alpha}}} \right\}.$$

Thus, the UUB stability of the constrained NISs (1) can be guaranteed in light of Lyapunov theorem extension [45], which completes the proof.

## Appendix B
## Proof of Theorem 2

Construct the Lyapunov function as follows:

$$\mathcal{L}_\alpha = \mathcal{L}_{\alpha 1} + \mathcal{L}_{\alpha 2} \tag{57}$$

in which

$$\mathcal{L}_{\alpha 1} = \frac{1}{2\,kn_\alpha} \tilde{X}_\alpha^T \tilde{X}_\alpha = \frac{1}{2} e_{G,\alpha}^T e_{G,\alpha}$$

$$\begin{aligned}\mathcal{L}_{\alpha 2} = &\frac{1}{2}\text{tr}\left\{ \tilde{B}_\alpha (\varrho_\alpha^1)^{-1} \tilde{B}_\alpha^T \right\} + \frac{1}{2}\sum_{j=1}^{n_\alpha} \text{tr}\left\{ \tilde{\mathcal{W}}_{\alpha j} (\varrho_{\alpha j}^2)^{-1} \tilde{\mathcal{W}}_{\alpha j}^T \right\}\\ &+ \frac{1}{2}\text{tr}\left\{ \tilde{C}_\alpha (\varrho_\alpha^3)^{-1} \tilde{C}_\alpha^T \right\} + \frac{1}{2}\sum_{j=1}^{n_\alpha} \text{tr}\left\{ \tilde{\mathcal{D}}_{\alpha j} (\varrho_{\alpha j}^4)^{-1} \tilde{\mathcal{D}}_{\alpha j}^T \right\}. \end{aligned}$$

According to the state estimation error dynamics (26), the derivative of $\mathcal{L}_{\alpha 1}$ can be inferred as

$$\begin{aligned}\dot{\mathcal{L}}_{\alpha 1} = &e_{G,\alpha}^T \tilde{B}_\alpha + e_{G,\alpha}^T \tilde{C}_\alpha u_\alpha + e_{G,\alpha}^T G_\alpha e_{G,\alpha}\\ &+ e_{G,\alpha}^T (\mathcal{W}_\alpha^*)^T [\tanh(X_\alpha - \mathcal{D}_\alpha^*) - \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha)]\\ &+ e_{G,\alpha}^T \tilde{\mathcal{W}}_\alpha^T \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha) + e_{G,\alpha}^T \varepsilon_\alpha\\ &+ e_{G,\alpha}^T \bar{h}_\alpha(x_\alpha, \hat{x}_\alpha) d_\alpha + e_{G,\alpha}^T \bar{\lambda}_\alpha(x_\alpha, \hat{x}_\alpha) v_\alpha. \end{aligned} \tag{58}$$

Based on the inequality $2\tilde{\alpha}^T \tilde{\beta} \le \tilde{\alpha}^T \tilde{\alpha} + \tilde{\beta}^T \tilde{\beta}$ and Assumption 4, we have

$$\begin{aligned}&e_{G,\alpha}^T (\mathcal{W}_\alpha^*)^T [\tanh(X_\alpha - \mathcal{D}_\alpha^*) - \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha)]\\ &\le \frac{1}{2}[\tanh(X_\alpha - \mathcal{D}_\alpha^*) - \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha)]^T\\ &\quad \times [\tanh(X_\alpha - \mathcal{D}_\alpha^*) - \tanh(\hat{X}_\alpha - \hat{\mathcal{D}}_\alpha)]\\ &\quad + \frac{1}{2} e_{G,\alpha}^T (\mathcal{W}_\alpha^*)^T \mathcal{W}_\alpha^* e_{G,\alpha}\\ &\le \frac{1}{2} e_{G,\alpha}^T P_\alpha e_{G,\alpha} + \frac{1}{2}(\tilde{X}_\alpha - \tilde{\mathcal{D}}_\alpha)^T (\tilde{X}_\alpha - \tilde{\mathcal{D}}_\alpha). \end{aligned} \tag{59}$$

Similarly

$$e_{G,\alpha}^T \varepsilon_\alpha \le \frac{1}{2} e_{G,\alpha}^T e_{G,\alpha} + \frac{1}{2}\varepsilon_{\alpha M}^2. \tag{60}$$

Utilizing $||\tilde{\alpha}+\tilde{\beta}||^2 \le 2||\tilde{\alpha}||^2 + 2||\tilde{\beta}||^2$, Assumption 2 and condition (51), it is simple to derive that

$$\begin{aligned} e_{G,\alpha}^T \bar{h}_\alpha(x_\alpha,\hat{x}_\alpha)d_\alpha &\le \frac{1}{2}e_{G,\alpha}^T e_{G,\alpha} + \frac{1}{2}||\bar{h}_\alpha(x_\alpha,\hat{x}_\alpha)d_\alpha||^2 \\ &\le \frac{1}{2}e_{G,\alpha}^T e_{G,\alpha} + \frac{1}{2}||h_\alpha(x_\alpha)-h_\alpha(\hat{x}_\alpha)||^2||d_\alpha||^2 \\ &\le \frac{1}{2}e_{G,\alpha}^T e_{G,\alpha} + \frac{1}{2\gamma_\alpha^4}D_{h_\alpha}^4\nabla J_{\alpha M}^2. \end{aligned} \tag{61}$$

Evidently, it can be obtained from (50) that

$$e_{G,\alpha}^T \bar{\lambda}_\alpha(x_\alpha,\hat{x}_\alpha)v_\alpha \le \frac{1}{2}e_{G,\alpha}^T e_{G,\alpha} + \frac{1}{2\delta_\alpha^2}D_{\lambda_\alpha}^4\nabla J_{\alpha M}^2. \tag{62}$$

Under the combination of (58)–(62), it yields

$$\begin{aligned} \dot{\mathcal{L}}_{\alpha 1} \le\ & e_{G,\alpha}^T\Delta_{1,\alpha}e_{G,\alpha} - \sum_{j=1}^{n_\alpha}(e_{G,\alpha}\otimes I_k)^T\tilde{\mathcal{D}}_{\alpha j} \\ & + \sum_{j=1}^{n_\alpha} e_{G,\alpha j}\tilde{\mathcal{W}}_{\alpha j}^T\tanh(\hat{X}_{m\alpha}-\hat{\mathcal{D}}_{\alpha j}) \\ & + e_{G,\alpha}^T\tilde{B}_\alpha + e_{G,\alpha}^T\tilde{C}_\alpha u_\alpha + \Delta_{2,\alpha} \end{aligned} \tag{63}$$

where

$$\begin{aligned} \Delta_{1,\alpha} &= \frac{P_\alpha}{2} + \frac{kn_\alpha(1+\theta_\alpha)+3}{2}I_{n_\alpha\times n_\alpha} + G_\alpha \\ \Delta_{2,\alpha} &= \frac{1}{2}\varepsilon_{\alpha M}^2 + \frac{1}{2\gamma_\alpha^4}D_{h_\alpha}^4\nabla J_{\alpha M}^2 + \frac{1}{2\delta_\alpha^2}D_{\lambda_\alpha}^4\nabla J_{\alpha M}^2. \end{aligned}$$

By adopting the weight tuning rules (27), the derivative of $\mathcal{L}_{\alpha 2}$ is presented as follows:

$$\begin{aligned} \dot{\mathcal{L}}_{\alpha 2} =\ & \sum_{j=1}^{n_\alpha}(e_{G,\alpha}\otimes I_k)^T\tilde{\mathcal{D}}_{\alpha j} - e_{G,\alpha}^T\tilde{B}_\alpha - e_{G,\alpha}^T\tilde{C}_\alpha u_\alpha \\ & - \sum_{j=1}^{n_\alpha} e_{G,\alpha j}\tilde{\mathcal{W}}_{\alpha j}^T\tanh(\hat{X}_{m\alpha}-\hat{\mathcal{D}}_{\alpha j}). \end{aligned} \tag{64}$$

Furthermore, the following condition can be acquired:

$$\dot{\mathcal{L}}_\alpha \le e_{G,\alpha}^T\Delta_{1,\alpha}e_{G,\alpha} + \Delta_{2,\alpha}. \tag{65}$$

It is apparently concluded that $\Delta_{1,\alpha} < 0$ can be guaranteed by selecting appropriate $G_\alpha$. Thus, $\dot{\mathcal{L}}_\alpha < 0$ holds when $||e_{G,\alpha}|| > \sqrt{\Delta_{2,\alpha}/\lambda_{\min}(-\Delta_{1,\alpha})}$. In view of Lyapunov extension theorem, the boundedness of $\tilde{B}_\alpha$, $\tilde{\mathcal{W}}_\alpha$, $\tilde{C}_\alpha$, and $\tilde{\mathcal{D}}_\alpha$ is ensured.

## Appendix C
## Proof of Theorem 3

Since $x_\alpha$, $x_{\alpha,s}$, and $\tilde{w}_\alpha$ are contained in each auxiliary subsystem, the Lyapunov function is selected as follows:

$$\mathcal{V}_\alpha(t) = \underbrace{J_\alpha^*(x_\alpha)}_{\mathcal{V}_{\alpha,1}} + \underbrace{J_\alpha^*(x_{\alpha,s})}_{\mathcal{V}_{\alpha,2}} + \underbrace{\frac{1}{2}\tilde{w}_\alpha^T w_\alpha}_{\mathcal{V}_{\alpha,3}}. \tag{66}$$

Due to the deployment of DTAETS, the whole analysis process will be completed by the subsequent two scenarios.

*Scenario I:* In this situation, the time scope is expressed as $t \in [t_s^\alpha, t_{s+1}^\alpha)$. Apparently, $\dot{\mathcal{V}}_{\alpha,2} = 0$ and the derivative of $\mathcal{V}_{\alpha,1}$ is inferred as

$$\begin{aligned} \dot{\mathcal{V}}_{\alpha,1} = (\nabla J_\alpha^*(x_\alpha))^T\big(& l_\alpha(x_\alpha) + C_\alpha\hat{u}_\alpha(x_{\alpha,s}) \\ & + h_\alpha(x_\alpha)\hat{d}_\alpha(x_\alpha) + \lambda_\alpha(x_\alpha)\hat{v}_\alpha(x_{\alpha,s})\big). \end{aligned} \tag{67}$$

In virtue of (44)–(46), condition (67) is transformed into

$$\begin{aligned} \dot{\mathcal{V}}_{\alpha,1} =\ & -\Psi_\alpha(x_\alpha) - x_\alpha^TQ_\alpha x_\alpha - \vartheta_\alpha(u_\alpha^*(x_\alpha)) + \gamma_\alpha^2||d_\alpha^*(x_\alpha)||^2 \\ & - \delta_\alpha||v_\alpha^*(x_\alpha)||^2 + 2\gamma_\alpha^2(d_\alpha^*(x_\alpha))^T[\hat{d}_\alpha(x_\alpha) - d_\alpha^*(x_\alpha)] \\ & + 2\zeta_\alpha\Pi^{-T}\left(\frac{u_\alpha^*(x_\alpha)}{\zeta_\alpha}\right)[u_\alpha^*(x_\alpha) - \hat{u}_\alpha(x_{\alpha,s})] \\ & + 2\delta_\alpha(v_\alpha^*(x_\alpha))^T[v_\alpha^*(x_\alpha) - \hat{v}_\alpha(x_{\alpha,s})]. \end{aligned} \tag{68}$$

By adopting Young's inequality and Assumption 2, the following formula can be derived:

$$\begin{aligned} & 2\zeta_\alpha\Pi^{-T}\left(\frac{u_\alpha^*(x_\alpha)}{\zeta_\alpha}\right)[u_\alpha^*(x_\alpha) - \hat{u}_\alpha(x_{\alpha,s})] \\ & \le ||\zeta_\alpha\Pi^{-1}\left(\frac{u_\alpha^*(x_\alpha)}{\zeta_\alpha}\right)||^2 + ||u_\alpha^*(x_\alpha) - \hat{u}_\alpha(x_{\alpha,s})||^2 \\ & \le \frac{1}{4}D_{C_\alpha}^2\nabla J_{\alpha M}^2 + ||u_\alpha^*(x_\alpha) - \hat{u}_\alpha(x_{\alpha,s})||^2. \end{aligned} \tag{69}$$

Likewise

$$\begin{aligned} & 2\gamma_\alpha^2(d_\alpha^*(x_\alpha))^T[\hat{d}_\alpha(x_\alpha) - d_\alpha^*(x_\alpha)] \\ & \quad \le \gamma_\alpha^2||d_\alpha^*(x_\alpha)||^2 + \gamma_\alpha^2||\hat{d}_\alpha(x_\alpha) - d_\alpha^*(x_\alpha)||^2 \end{aligned} \tag{70}$$

and

$$\begin{aligned} & 2\delta_\alpha(v_\alpha^*(x_\alpha))^T[v_\alpha^*(x_\alpha) - \hat{v}_\alpha(x_{\alpha,s})] \\ & \quad \le \delta_\alpha||v_\alpha^*(x_\alpha)||^2 + \delta_\alpha||v_\alpha^*(x_\alpha) - \hat{v}_\alpha(x_{\alpha,s})||^2. \end{aligned} \tag{71}$$

Recalling $\Psi_\alpha(x_\alpha) > 0$ and $\vartheta_\alpha(u_\alpha^*(x_\alpha)) > 0$, it is simply elicited from (51) and (68)–(71) that

$$\begin{aligned} \dot{\mathcal{V}}_{\alpha,1} \le\ & -x_\alpha^TQ_\alpha x_\alpha + \delta_\alpha||v_\alpha^*(x_\alpha) - \hat{v}_\alpha(x_{\alpha,s})||^2 \\ & + \frac{1}{2\gamma_\alpha^2}D_{h_\alpha}^2\nabla J_{\alpha M}^2 + \gamma_\alpha^2||\hat{d}_\alpha(x_\alpha) - d_\alpha^*(x_\alpha)||^2 \\ & + \frac{1}{4}D_{C_\alpha}^2\nabla J_{\alpha M}^2 + ||u_\alpha^*(x_\alpha) - \hat{u}_\alpha(x_{\alpha,s})||^2. \end{aligned} \tag{72}$$

Applying Assumptions 2, 5 and the inequity $||\tilde{\alpha}+\tilde{\beta}||^2 \le 2||\tilde{\alpha}||^2 + 2||\tilde{\beta}||^2$, we can obtain the following condition on basis of (31) and (33):

$$\begin{aligned} & \gamma_\alpha^2||\hat{d}_\alpha(x_\alpha) - d_\alpha^*(x_\alpha)||^2 \\ & = \gamma_\alpha^2||\frac{1}{2\gamma_\alpha^2}h_\alpha^T(x_\alpha)\left(\nabla\varphi_\alpha^T(x_\alpha)\tilde{w}_\alpha + \nabla\varpi_\alpha(x_\alpha)\right)||^2 \\ & \le \frac{1}{2\gamma_\alpha^2}D_{h_\alpha}^2\left(\nabla\varphi_{\alpha M}^2||\tilde{w}_\alpha||^2 + \nabla\varpi_{\alpha M}^2\right). \end{aligned} \tag{73}$$

According to Assumption 3, one has

$$\delta_\alpha||v_\alpha^*(x_\alpha) - \hat{v}_\alpha(x_{\alpha,s})||^2$$

$$\begin{aligned}&= \delta_\alpha ||v_\alpha^*(x_\alpha) - v_\alpha^*(x_{\alpha,s}) + v_\alpha^*(x_{\alpha,s}) - \hat{v}_\alpha(x_{\alpha,s})||^2\\ &\leq 2\delta_\alpha \mathcal{K}_{v_\alpha^*}^2 ||e_{\alpha,s}||^2 + 2\delta_\alpha ||v_\alpha^*(x_{\alpha,s}) - \hat{v}_\alpha(x_{\alpha,s})||^2\\ &\leq 2\delta_\alpha \mathcal{K}_{v_\alpha^*}^2 ||e_{\alpha,s}||^2 + \frac{D_{\lambda_\alpha}^2}{\delta_\alpha}\left(\nabla\varphi_{\alpha M}^2 ||\tilde{w}_\alpha||^2 + \nabla\varpi_{\alpha M}^2\right).\end{aligned} \tag{74}$$

In view of Assumptions 2, 3, 5 and formulas (38), (39), it can be acquired that

$$\begin{aligned}&||u_\alpha^*(x_\alpha) - \hat{u}_\alpha(x_{\alpha,s})||^2\\ &= ||u_\alpha^*(x_\alpha) - u_\alpha^*(x_{\alpha,s}) + u_\alpha^*(x_{\alpha,s}) - \hat{u}_\alpha(x_{\alpha,s})||^2\\ &\leq 2\mathcal{K}_{u_\alpha^*}^2 ||e_{\alpha,s}||^2 + 2||u_\alpha^*(x_{\alpha,s}) - \hat{u}_\alpha(x_{\alpha,s})||^2\\ &= 2\mathcal{K}_{u_\alpha^*}^2 ||e_{\alpha,s}||^2 + 2||\varpi_{u_\alpha^*}(x_{\alpha,s}) - O_{\mathcal{A}_\alpha}\\ &\quad - \frac{1}{2}\left(I_{z_\alpha\times z_\alpha} - \epsilon_{\mathcal{A}_{\alpha,2}}(x_\alpha)\right) C_\alpha^T \nabla\varphi_\alpha^T(x_\alpha)\tilde{w}_\alpha||^2\\ &\leq 2\mathcal{K}_{u_\alpha^*}^2 ||e_{\alpha,s}||^2 + 4||\varpi_{u_\alpha^*}(x_{\alpha,s})||^2 + 4||O_{\mathcal{A}_\alpha}\\ &\quad + \frac{1}{2}\left(I_{z_\alpha\times z_\alpha} - \epsilon_{\mathcal{A}_{\alpha,2}}(x_\alpha)\right) C_\alpha^T \nabla\varphi_\alpha^T(x_\alpha)\tilde{w}_\alpha||^2\\ &\leq 2\mathcal{K}_{u_\alpha^*}^2 ||e_{\alpha,s}||^2 + 24 D_{C_\alpha}^2 \nabla\varphi_{\alpha M}^2 ||\tilde{w}_\alpha||^2\\ &\quad + 64 z_\alpha + 4 D_{\varpi_{u_\alpha^*}}^2.\end{aligned} \tag{75}$$

Combining expressions (54) and (73)–(75), we restate condition (72) as

$$\begin{aligned}\dot{\mathcal{V}}_{\alpha,1} &\leq -\varSigma_{\alpha,1}||x_\alpha||^2 + \bar{D}_\alpha\\ &\quad + \nabla\varphi_{\alpha M}^2\left(\frac{D_{h_\alpha}^2}{2\gamma_\alpha^2} + \frac{D_{\lambda_\alpha}^2}{\delta_\alpha} + 24 D_{C_\alpha}^2\right)||\tilde{w}_\alpha||^2\end{aligned} \tag{76}$$

where

$$\varSigma_{\alpha,1} = \lambda_{\min}(Q_\alpha) - 2(1+\delta_\alpha)\frac{\mathcal{K}_{\max}^2 \lambda_{\max}(\Theta_\alpha)(2\overline{\eta}_{1,\alpha} + \overline{\eta}_{2,\alpha})}{\lambda_{\min}(\Theta_\alpha) - 2\overline{\eta}_{1,\alpha}\lambda_{\max}(\Theta_\alpha)}$$

$$\begin{aligned}\bar{D}_\alpha &= \frac{D_{h_\alpha}^2}{2\gamma_\alpha^2}(\nabla J_{\alpha M}^2 + \nabla\varpi_{\alpha M}^2) + \frac{1}{4}D_{C_\alpha}^2 \nabla J_{\alpha M}^2\\ &\quad + \frac{1}{\delta_\alpha}D_{\lambda_\alpha}^2 \nabla\varpi_{\alpha M}^2 + 64 z_\alpha + 4 D_{\varpi_{u_\alpha^*}}^2.\end{aligned}$$

Under the consideration of (37) and Assumption 5, the derivative of $\mathcal{V}_{\alpha,3}$ is deduced as follows:

$$\begin{aligned}\dot{\mathcal{V}}_{\alpha,3} &= -\rho_\alpha \tilde{w}_\alpha^T \frac{\psi_\alpha\psi_\alpha^T}{(1+\psi_\alpha^T\psi_\alpha)^2}\tilde{w}_\alpha + \tilde{w}_\alpha^T \frac{\rho_\alpha\psi_\alpha}{(1+\psi_\alpha^T\psi_\alpha)^2}\varpi_{H_\alpha}\\ &\leq -\frac{\rho_\alpha}{2}\lambda_{\min}(\Phi_\alpha)||\tilde{w}_\alpha||^2 + \frac{\rho_\alpha}{2}D_{\varpi_{H_\alpha}}^2.\end{aligned} \tag{77}$$

According to the definition of $\mathcal{V}_\alpha(t)$ in (66), inequities (76) and (77), it is obviously obtained that

$$\dot{\mathcal{V}}_\alpha(t) \leq -\varSigma_{\alpha,1}||x_\alpha||^2 - \varSigma_{\alpha,2}||\tilde{w}_\alpha||^2 + \bar{D}_\alpha + \frac{\rho_\alpha}{2}D_{\varpi_{H_\alpha}}^2 \tag{78}$$

where

$$\varSigma_{\alpha,2} = \frac{\rho_\alpha}{2}\lambda_{\min}(\Phi_\alpha) - \nabla\varphi_{\alpha M}^2\left(\frac{D_{h_\alpha}^2}{2\gamma_\alpha^2} + \frac{D_{\lambda_\alpha}^2}{\delta_\alpha} + 24 D_{C_\alpha}^2\right).$$

Under formula (40), $\dot{\mathcal{V}}_\alpha(t) < 0$ holds when one of the following conditions is gratified:

$$\begin{cases}||x_\alpha|| > \sqrt{(\bar{D}_\alpha + \frac{\rho_\alpha}{2}D_{\varpi_{H_\alpha}}^2)/\varSigma_{\alpha,1}} \triangleq \varOmega_{x_\alpha}\\ ||\tilde{w}_\alpha|| > \sqrt{(\bar{D}_\alpha + \frac{\rho_\alpha}{2}D_{\varpi_{H_\alpha}}^2)/\varSigma_{\alpha,2}} \triangleq \varOmega_{\tilde{w}_\alpha}.\end{cases} \tag{79}$$

Hence, the system state $x_\alpha$ and the weight estimation error $\tilde{w}_\alpha$ can converge to the ultimate bound $\varOmega_{x_\alpha}$ and $\varOmega_{\tilde{w}_\alpha}$, respectively. Subsequently, the relevant analysis at the triggered instant will be conducted.

*Scenario II:* For $t = t_{s+1}^\alpha$, we calculate the difference of $\mathcal{V}_\alpha(t)$ as follows:

$$\begin{aligned}\Delta\mathcal{V}_\alpha(t_{s+1}^\alpha) &= J_\alpha^*(x_\alpha(t_{s+1}^\alpha)) - J_\alpha^*(x_\alpha(t_{s+1}^{\alpha-})) + J_\alpha^*(x_{\alpha,s+1})\\ &\quad - J_\alpha^*(x_{\alpha,s}) + \frac{1}{2}\tilde{w}_\alpha^T(t_{s+1}^\alpha)\tilde{w}_\alpha(t_{s+1}^\alpha)\\ &\quad - \frac{1}{2}\tilde{w}_\alpha^T(t_{s+1}^{\alpha-})\tilde{w}_\alpha(t_{s+1}^{\alpha-})\end{aligned} \tag{80}$$

where $x_\alpha(t_{s+1}^{\alpha-}) = \lim_{\chi\to 0^-} x_\alpha(t_{s+1}^\alpha + \chi)$, $\tilde{w}_\alpha(t_{s+1}^{\alpha-}) = \lim_{\chi\to 0^-} \tilde{w}_\alpha(t_{s+1}^\alpha + \chi)$ with $\chi \in (t_s^\alpha - t_{s+1}^\alpha, 0)$.

As illustrated in Scenario I, $\dot{\mathcal{V}}_\alpha(t) < 0$ holds for $\forall t \in [t_s^\alpha, t_{s+1}^\alpha)$. Therefore, $\mathcal{V}_\alpha(t)$ is monotonically decreasing on the interval $[t_s^\alpha, t_{s+1}^\alpha)$ and it implies

$$\mathcal{V}_\alpha(t_{s+1}^\alpha) < \mathcal{V}_\alpha(t_{s+1}^\alpha + \chi),\ \chi \in (t_s^\alpha - t_{s+1}^\alpha, 0). \tag{81}$$

In light of the properties of limits, it is readily derived from expression (81) that

$$\begin{aligned}&J_\alpha^*(x_\alpha(t_{s+1}^\alpha)) + \frac{1}{2}\tilde{w}_\alpha^T(t_{s+1}^\alpha)\tilde{w}_\alpha(t_{s+1}^\alpha)\\ &< J_\alpha^*(x_\alpha(t_{s+1}^{\alpha-})) + \frac{1}{2}\tilde{w}_\alpha^T(t_{s+1}^{\alpha-})\tilde{w}_\alpha(t_{s+1}^{\alpha-}).\end{aligned} \tag{82}$$

In Scenario I, the boundedness of $x_\alpha$ has been demonstrated, thus, we can obtain $J_\alpha^*(x_{\alpha,s+1}) \leq J_\alpha^*(x_{\alpha,s})$. Based on the aforesaid discussion, $\Delta\mathcal{V}_\alpha(t_{s+1}^\alpha) < 0$ satisfies. It indicates that the UUB stability of $\alpha$th auxiliary subsystem (5) and the weight estimation error $\tilde{w}_\alpha$ is assured.

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