Dynamic-Memory Event-Triggered Sliding-Mode Secure Control for Nonlinear Semi-Markov Jump Systems With Stochastic Cyber Attacks

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***Abstract*— This paper presents an investigation of the sliding-mode secure control for nonlinear semi-Markov jump systems in the presence of non-periodic denial-of-service attacks and false data injection attacks. First of all, we employ Takagi-Sugeno fuzzy model to describe the nonlinear semi-Markov jump control systems. In order to optimize transmission efficiency and control performance, a novel dynamic-memory event-triggered mechanism is developed by incorporating auxiliary dynamic variable and historical transmitted data. Then, a memory-based fuzzy sliding surface is put forward to attenuate the influences of stochastic cyber attacks with the aid of event-triggered state information. Moreover, by utilizing Lyapunov stability theory, sufficient conditions are derived to guarantee the exponentially mean-square stability of the system with an H_∞ performance index, even in the cases of generally uncertain and unknown transition rates. Furthermore, a memory-based sliding mode secure controller is designed to ensure the reachability of the predefined switching surface and desirable sliding motion within finite time. Finally, the efficacy of the proposed control scheme is demonstrated through a tunnel diode circuit model.**

***Note to Practitioners*—This study focuses on the issue of secure control for nonlinear semi-Markov jump systems, which holds practical significance across various domains, including applications in robotic manipulators, circuit models, and DC motors. We broaden the scope by considering more general jump parameter matrices to align more closely with the real-world system environment. Moreover, networked environments pose two primary challenges: network bandwidth constraints and the external network attacks. To tackle these issues, this paper introduces an innovative dynamic memory event triggering mechanism to enhance network transmission efficiency and optimize communication resource utilization. Meanwhile, to address cyber attacks resulting from the inherent openness of networks, the current study adopts a defensive sliding mode control strategy to provide robust protection against network attacks and disturbances. The effectiveness of the suggested approach is confirmed through a circuit system model. It is worth mentioning that the proposed method has the potential for broader application within real-world engineering scenarios, particularly those involving network-based nonlinear systems with various practical constraints.**

***Index Terms*— Sliding mode control, nonlinear semi-Markovian systems, stochastic cyber-attacks, event-triggered mechanism.**

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I. Introduction

In RECENT decades, a great deal of literature has emerged on fuzzy-based control strategies which aim to approximate complex nonlinear systems by decomposing the input space into numerous subspaces through a fuzzy blend of local linear systems associated with each subspace. Various control methods, such as sliding mode control [1], output feedback control [2] and finite-time control [3], have been utilized in the context of Takagi-Sugeno (T-S) fuzzy systems to effectively address associated control issues. It is noticed that sliding mode control (SMC) has garnered particular interest in the field of systems science and control engineering due to its robustness against external disturbances. SMC has been employed to address diverse issues through the utilization of appropriate controller design conditions [4]. Within a finite-time frame, the attainment of convergence for the sliding-mode surface can be ensured by designing sufficiently large control signals, which effectively counteract the adverse influences of uncertainty and nonlinearity [5], [6]. As the state variable trajectories reach the predetermined sliding surface, they exhibit insensitivity towards uncertainties and nonlinearities. Recently, a few studies on sliding-mode control method for T-S fuzzy systems have emerged [1], [7]. However, there remain some worthwhile explorations to further develop, which serves as the motivation for this study.

Markov jump systems (MJSs) are stochastic systems with multiple modes and possess a powerful modeling ability for practical systems subject to stochastic changes, such as random system faults or communication failures [6], [8], [9], [10]. However, the MJSs have limitations due to the memoryless nature of their transition probability, which complies with an exponential distribution and remains independent of the sojourn-time. To address this limitation, semi-Markov jump systems (S-MJSs) with sojourn-time-dependent transition probabilities have emerged as an extension solution, which has led to the exploration of various topics [11], [12]. For instance, the design of SMC for a specific category of nonlinear S-MJSs is studied in [1], and the issue of model predictive control concerning discrete-time S-MJSs has been addressed in [13]. Additionally, the problem of output feedback control for particular continuous-time hidden S-MJSs incorporating time delays is investigated in [14].

In addition, the constrained communication resources and the transmission of extensive data may significantly impact the performance of S-MJSs. Therefore, the event-triggered mechanism (ETM) has gained significant traction due to its simplistic task model and remarkable ability to reduce signal redundancy [15], [16]. The introduction of event-triggered scheme has been and became prevalent in control fields in recent years [17], [18], [19]. For example, the event-triggered H_∞ control problem for nonlinear systems with data losses was examined in [20]. Additionally, the event-triggered fault detection for networked fuzzy systems was explored in [21]. To further reduce energy consumption and enhance the suitability for dynamic terms and switching modes in practical systems, several more efficient and universal ETMs have been proposed. For instance, the authors [22] designed a dynamic-memory ETM, which was applied to fault detection problems. The authors [23] investigated an adaptive ETM for nonlinear systems with network attacks. The authors [24] addressed the resilient control problem by involving memory-based ETM for a specific category of networked systems to deal with deception attacks. Drawing upon the above literature, we know that it is necessary to use historical trigger data in the event triggering mechanism to improve the performance of the system. However, the inclusion of historical packets results in the release of additional packets, which potentially leads to resource wastage. Thus, the utilization of the dynamic variable becomes essential to regulate data transmission. Prompted by these challenges, this paper introduces a new dynamic memory-event-triggered mechanism (DMETM) for S-MJSs, which effectively optimizes the utilization of communication resources while conserving the limited bandwidth.

The development of networks has increased the efficiency of data transmission. For networked control systems, the communication transmission channels are often subject to attacks from network hackers. Network security is particularly important as attacks can significantly degrade system performance. Therefore, it is necessary to account for the impact of cyber attacks when modeling and analyzing networked systems [16]. Recently, researchers have extensively investigated the security concerns of various networked systems under cyber attacks. For instance, the authors in [25] explored the design of secure filters for switched systems subject to false data injection (FDI) attacks. The authors in [26] addressed the issue of reliable control with memory ETM under the threat of deception attacks. The authors in [27] studied fuzzy-based filter design in nonlinear systems with deception and denial of service (DoS) attacks. Up to now, the majority of published research has focused on single cyber attacks. However, real-world systems may encounter a wide range of cyber attacks. In order to accurately represent network environments, this article incorporates multiple cyber attacks, including FDI attacks and DoS attacks. It is worth noting that the consideration of multiple network attacks in S-MJSs has often been overlooked, which serve as one of the primary motivations of this paper.

In light of the previous discussions, this study examines the novel dynamic memory-event-triggered SMC for fuzzy S-MJSs with multiple cyber attacks. The main and innovative features of this research can be summarized as follows:

(1) Unlike the existing results in [5] and [28], this paper proposes a novel memory-event triggered strategy for networked S-MJS incorporating historical errors and the change of state error, which reduces the burden on the network more effectively and is also more resistant to the effects of DoS attacks. Moreover, this approach utilizes mode-dependent and parameter-dependent Lyapunov-Krasovskii functionals, which can significantly decrease conservatism.

(2) In contrast to [6], [29], and [30], this paper considers a new memory-based sliding mode surface along with its corresponding SMC law to combat network attacks and exogenous disturbances that are prone to occur in some real engineering systems such as robot manipulators and DC motors, while ensuring the finite-time accessibility of the sliding surface.

(3) The current study takes into account a more realistic scenario with multiple network attacks, differing from [31] where a single attack is considered. Furthermore, we propose a set of sufficient conditions to ensure the globally exponential stability and H_∞ performance of the nonlinear S-MJSs with general uncertain transition rates, particularly for completely unknown situations.

The subsequent sections of this paper are structured as follows. In Section II, we provide a comprehensive exposition of the fuzzy S-MJSs model under cyber attacks, as well as constructing a novel switched dynamic fuzzy control system. Section III focuses on the stability analysis and sliding mode controller design for networked fuzzy S-MJSs, where the key findings are presented. In Section IV, an electric circuit model is introduced to illustrate the effectiveness of the control method. Finally, we conclude this paper in Section V.

Notations

$*$	Symmetric terms of a matrix
I	Identity matrix
A^{T}	Transposition of matrix A
A^{-1}	Inversion of matrix A
$sym\{B\}$	$B^T + B$
$\|\cdot\|$	Euclidean norm of the vector

II. Problem Formulation

A. Model of Nonlinear S-MJSs

Consider a category of nonlinear systems characterized by parameter uncertainties as Figure 1, which can be precisely depicted by the subsequent fuzzy model consisting of r fuzzy rules.

Plant Rule i: IF $\gamma_j(t)$ is F_{ij} $(i = 1, 2 \cdots, r;$ $j = 1, 2, \cdots, p)$, THEN

$$\begin{cases} \dot{x}(t) = A_{i\varsigma_t}x(t) + B_{i\varsigma_t}\bar{u}(t) + C_{i\varsigma_t}\omega(t), \\ z(t) = D_{i\varsigma_t}x(t) + E_{i\varsigma_t}\omega(t), \end{cases} \quad (1)$$

where $x(t) \in \mathcal{R}^n$ and $u(t) \in \mathcal{R}^m$ denote the state and control input of the system, respectively. The matrices $A_{i\varsigma_t}$, $B_{i\varsigma_t}$, $C_{i\varsigma_t}$ $D_{i\varsigma_t}$ and $E_{i\varsigma_t}$ are well-defined constant matrices, which possess suitable dimensions. The exogenous disturbance $\omega(t)$ is an element of $L_2[0, \infty)$. The continuous-time semi-Markov process $\{\varsigma_t, t \geq 0\}$ occurs within a space $S = \{1, 2, \cdots s\}$ with transition probabilities described by:

$$\Pr\{\varsigma_{t+\mu} = c|\varsigma_t = v\} = \begin{cases} \psi_{vc}(\mu)\mu + o(\mu), c \neq v, \\ 1 + \psi_{vc}(\mu)\mu + o(\mu), c = v, \end{cases} \quad (2)$$

where $\mu > 0$ represents sojourn time of the current mode independent of t, and we have $\lim_{\mu \to 0} o(\mu)/\mu = 0$. If $c \neq v$, then $\psi_{vc}(\mu) > 0$ and $\psi_{vv}(\mu) = -\sum_{c \neq v}^{s} \psi_{vc}(\mu) < 0$. Supposing that the controller suffers randomly occurring injection attacks, the control input $u(t)$ would be modeled as follows

$$\bar{u}(t) = u(t) + \beta(t)H_{\varsigma_t}\chi(x(t), t),$$

where H_{ς_t} is an unknown weighting matrix with a norm of $\|H_{\varsigma_t}\| < \hbar_{\varsigma_t}$, and $\|\chi(x(t), t)\| \leq \xi_{\varsigma_t}(x(t), t)$. The variable $\beta(t)$ is a Bernoulli variable with $\Pr\{\beta(t) = 1\} = E\{\beta(t)\} = \bar{\rho}$ and $\Pr\{\beta(t) = 0\} = 1 - E\{\beta(t)\} = 1 - \bar{\rho}$, where $\bar{\rho} \in [0, 1]$.

In this paper, we examine the transition rates of the semi-Markov process, some of which are only partially accessible or not exactly accessible, and determine the lower and upper bounds. The values of $\psi_{vc}(\mu)$ lie in the range of $[\underline{\psi}_{vc}, \overline{\psi}_{vc}]$, where $\underline{\psi}_{vc}$ and $\overline{\psi}_{vc}$ are known constants. We denote $\psi_{vc}(\mu) = \psi_{vc} + \Delta\psi_{vc}(\mu)$, where $\psi_{vc} = \frac{1}{2}(\underline{\psi}_{vc} + \overline{\psi}_{vc})$, and $|\Delta\psi_{vc}(\mu)| \leq \lambda_{vc}$, where $\lambda_{vc} = \frac{1}{2}(\overline{\psi}_{vc} - \underline{\psi}_{vc})$, for brevity, we define $\Lambda = \Lambda_{v,k} \cup \Lambda_{v,uk}$

$$\Lambda_{v,k} = \{k : \psi_{vc} \ is \ partially \ accessible, v \in s\},$$
$$\Lambda_{v,uk} = \{k : \psi_{vc} \ is \ completely \ unaccessible, v \in s\}.$$

Suppose $\Lambda_{v,k} \neq \varnothing$ and $\Lambda_{v,uk} \neq \varnothing$. The set $\Lambda_{v,k}$ can be represented as $\{k_{v,1}, k_{v,2}, \cdots, k_{v,q}\}$, where $1 < q < s$. Using fuzzy blending, we obtain the following for mode v:

$$\dot{x}(t) = \sum_{i=1}^{r} q_{iv}(\gamma(t))(A_{iv}x(t) + B_{iv}\bar{u}(t) + C_{iv}\omega(t)), \quad (3)$$

where $\gamma(t) = [\gamma_1(t), \gamma_2(t), \cdots \gamma_p(t)]^T$. The function $q_i(\gamma(t)) = \prod_{j=1}^{p} \varsigma_{ij}(\gamma_j(t)) / \sum_{i=1}^{r} \prod_{j=1}^{p} \varsigma_{ij}(\gamma_j(t))$ is the membership function, where $\varsigma_{ij}(\gamma_j(t))$ denotes the grade membership of $\gamma_j(t)$ in ς_{ij}. It is noteworthy that $\varsigma_{ij}(\gamma_j(t)) \geq 0$, $q_i(\gamma(t)) \in [0, 1]$, and $\sum_{i=1}^{r} q_i(\gamma(t)) = 1$.

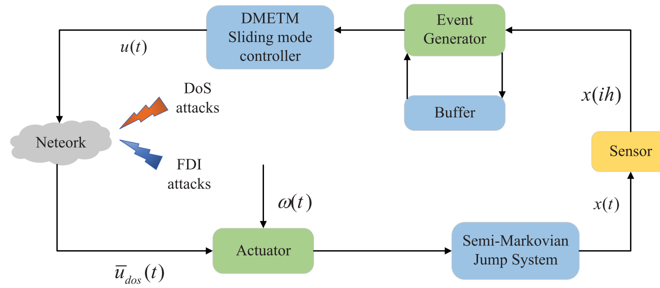

Fig. 1. The structure of networked S-MJSs.

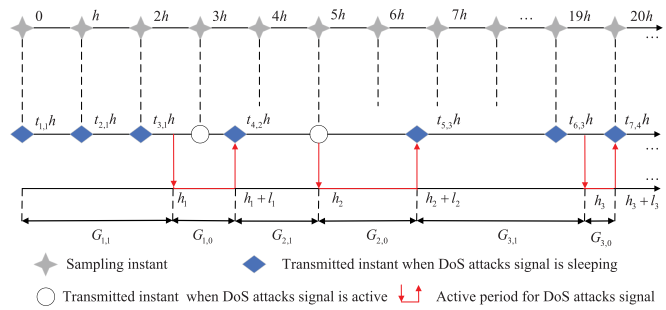

Fig. 2. Example of DMETM under DoS attacks.

B. DoS Attacks

In addition to FDI attacks, this paper explores the impacts of non-periodic energy-limited DoS attacks on the transmission network between the actuator and controller, as illustrated in Figure 2. The energy limitation of the DoS jamming signal renders it inactive during a specific duration, as it conserves energy for subsequent attacks. In order to determine the presence of DoS attacks, a variable $\upsilon(t)$ is introduced.

$$\upsilon(t) = \begin{cases} 1, & t \in [h_l, h_l + l_l), \\ 0, & t \in [h_l + l_l, h_{l+1}), \end{cases} \quad (4)$$

where h_l and $h_l + l_l$ represent the moments of inactivity and activity associated with DoS attacks in transmission network. Therefore, we can determine the starting and ending instants of the DoS sleeping period as $0 = h_1 < h_1 + l_1 < \cdots < h_l < h_l + l_l < \cdots$. To simplify analysis, let $G_{l,1} = [h_l, h_l + l_l)$ and $G_{l,0} = [h_l + l_l, h_{l+1})$.

Assumption 2.1: Assume that the maximum duration of communication interruption periods in DoS attacks, denoted as $q_{\max}$, satisfies $q_{\max} \geq \sup_{l \in N}\{h_{l+1} - h_l - l_l\}$, where the lower and upper bounds of communication recovery periods are meet the constraint $l_{\min} \leq l_l \leq l_{\max}$.

Assumption 2.2: During the occurrence of DoS attacks, the cumulative count of communication interruption and recovery transitions, denoted as $\chi(t)$, satisfies $\chi(t) \leq b + \frac{t}{\Omega_a}$ with $b \geq 0$ and $\Omega_a \in R_{\geq 0}$.

C. Dynamic-Memory Event-Triggered Mechanism

To conserve bandwidth resources in S-MJSs, the DMETM is utilized. Let h and $t_k h$ denote the sampling period and the most recent transmission instant, respectively. The subsequent triggering instant as $t_{k+1}h$ is determined by the following equation:

$$\begin{aligned} t_{k+1}h &= t_k h + \min_{d\geq 0}\{d|F(t,\vartheta(t))>0\},\\ F(t,\vartheta(t)) &= \sum_{n=1}^{m}\mu_n e_n^T(t)\Omega_v e_n(t) - \sigma\vartheta(t)\tilde{x}^T(t)\Omega_v\tilde{x}(t),\\ \vartheta(t) &= d_0 + (d_1-d_0)\varrho(t),\\ \varrho(t) &= \max\{e^{-\lambda_1\|\sum_{n=1}^{m}\mu_n e_n(t)\|}, e^{-\lambda_2\|\sum_{n=1}^{m}\mu_n E_n(t)\|}\}, \end{aligned} \quad (5)$$

where $\sigma \in (0,1)$, $0 \leq d_0 \leq d_1 \leq 1$ and λ_1, λ_2 are positive values. The matrix Ω_v is a symmetric positive definite matrix. Additionally, m denotes the number of packets recently released and stored in the buffer, and $\sum_{n=1}^{m}\mu_n = 1$ where $\mu_n \in [0,1]$. Furthermore, we have $\tilde{x}(t) = (1/m)\sum_{n=1}^{m}x(t_{k-n+1}h)$ and $e_n(t) = x(t_{k-n+1}h) - x(t_k h + dh)$, $E_n(t) = e_n(t) - e_{n-1}(t)$.

Remark 2.1: The essential role of the state error change $\left\|\sum_{n=1}^{m}\mu_n E_n(t)\right\|$ in event-triggered strategy cannot be underestimated. It is noteworthy that the event-triggering parameter $\vartheta(t)$ in (5), bounded between d_0 and d_1, undergoes adaptive adjustments through the function $\varrho(t)$. The term $\left\|\sum_{n=1}^{m}\mu_n e_n(t)\right\|$ cleverly improves the system performance by capturing the historical errors as illustrated in [28]. However, it is inadequate to adjust the dynamic parameter by relying solely on historical errors, especially in the case where the state change errors described above are large and the history errors are small. To overcome this complexity, the trigger proposed in (5) incorporates both the state error and the change of error to modify the dynamic parameter, which leads to a significant reduction in energy consumption and improves the suitability for dynamic terms.

Remark 2.2: The DMETM (5) is proposed not only for its ability to adjust inter-event time flexibly but also for its better adaptation to DoS attacks, which is superior to other DMETM constructions described in [5]. In the simulation example section, we will find that the designed DMETM is more likely to obtain vertex data of the system state curve due to the introduction of historical triggered signals. Additionally, the parameter μ_n represents the weighting assigned to each triggered packet. In general, packets closer to the present are given a more important status, i.e. $\mu_n \geq \mu_{n+1} (n = 1, \ldots, m-1)$. When $\lambda_1 = 0$, $\lambda_2 = 0$ and $d_1 = 1$, the internal dynamic variables $\vartheta(t)$ becomes 1, then DMETM (5) simplifies to the conventional memory-based ETM described in [24]. Furthermore, the DMETM (5) will be degraded to a memoryless ETM if $m = 1$.

In order to mitigate the negative impact of DoS attacks, we propose an improved DMETM on the basis of (5) as follows

$$t_{k+1,l}h = \{t_{k,l}h + dh \text{ satisfying (5)}| \; t_{k,l}h + dh \in G_{l,1},\} \cup \{h_l\} \quad (6)$$

where d belongs to $\{1, 2, \cdots, d(l)\}$ with $d(l) = \sup\{d \in N|t_{k,l}h + dh < h_l + l_l\}$. Similar to [23], we divide the event-triggered interval $\Im_{k,l} = [t_{k,l}h + h_{k,l}, t_{k+1,l}h + h_{k+1,l})$ into multiple sub-intervals as follows

$$\begin{aligned} \Im_{k,l} = &\bigcup_{s=1}^{\theta_{k,l}}[t_{k,l}h + (s-1)h + h_{k,l}, t_{k,l}h + sh + h_{k,l})\\ &\cup [t_{k,l}h + \theta_{k,l}h + h_{k,l}, t_{k+1,l}h + h_{k+1,l}), \end{aligned} \quad (7)$$

where $\theta_{k,l} = \sup\{d \in N|t_{k,l}h + dh < t_{k+1,l}h\}$, $h_{k,l}$ and $h_{k+1,l}$ are transmission delay of the packets $t_{k,l}h$ and $t_{k+1,l}h$. Denote

$$\begin{aligned} N_{k,l}^{s} &= \bigcup_{s=1}^{\theta_{k,l}}[t_{k,l}h + (s-1)h + h_{k,l}, t_{k,l}h + sh + h_{k,l}),\\ N_{k,l}^{\theta_{k,l}+1} &= [t_{k,l}h + \theta_{k,l}h + h_{k,l}, t_{k+1,l}h + h_{k+1,l}). \end{aligned}$$

Thus, we can express the sleeping interval $G_{l,1} = \bigcup_{k=0}^{k(l)}\bigcup_{s=1}^{\theta_{k,l}+1}\{N_{k,l}^{s} \cap G_{l,1}\} = \bigcup_{k=0}^{k(l)}\bigcup_{s=1}^{\theta_{k,l}+1}\{\hbar_{k,l}^{s}\}$. Subsequently, for $t \in \hbar_{k,l}^{l}$, we introduce the the time delay between the current instant and the previous triggering instant $h(t)$ as $h(t) = t - t_{k,l}h - lh$ with

$$\begin{aligned} h_{k,l} \leq h(t) \leq h + h_{k,l}, &\quad t \in \hbar_{k,l}^{s}, s = 1, \cdots, \theta_{k,l}\\ h_{k,l} \leq h(t) \leq h + h_{k+1,l}, &\quad t \in \hbar_{k,l}^{\theta_{k,l}+1} \end{aligned}$$

It follows that $h(t)$ takes values within the interval $0 \leq h(t) < \bar{h}$ with $\bar{h} = \max_{l\in n}\{\max_{k\in n}\{h_{k,l}\}\} + h$. Then, combining with (5), we get $e_{n,l}(t) = x(t_{k-n+1,l}h) - x(t_{k,l}h + lh)$.

Consequently, the memory-based DETM sampled state, denoted by $x(t_{k-n+1}h)$, can be expressed as $x(t_{k-n+1}h) = x(t - h(t)) + e_{n,l}(t)$ for all $t \in \hbar_{k,l}^{s}$.

Remark 2.3: If the condition in (5) is violated, the subsequent data transmission is initiated. However, since the attacker aims to disrupt communication, it follows that the activated data ought not to be situated within the attack active time interval. In this investigation, focusing on the DoS attack, we put forward an attack-resilient event-triggered condition outlined in (6). The time sequences of the triggered events are located within the attack sleeping interval if no triggered data satisfies the condition in (6). Additionally, it can be deduced that the data at instant h_l will be compelled to undergo successful transfer. Consequently, the proposed triggering mechanism serves to sustain the control performance of the system, even when the attacks are infrequent or minor.

D. Event-Based SMC Dynamics

The novel nonlinear event-based sliding-mode function is designed as

$$\begin{aligned} s(t) = &G_v x(t) - \int_0^t \sum_{i=1}^{r} q_i(\gamma(t))G_v A_{iv}x(s)ds\\ &- \sum_{n=1}^{m}\mu_n\int_0^t\sum_{i=1}^{r}q_i(\gamma(t))G_v B_{iv}\\ &\times K_v[x(t-h(s)) + e_{n,l}(s)]ds, \end{aligned} \quad (8)$$

where G_v is selected to ensure that $\sum_{i=1}^{r} q_i(\gamma(t))G_v B_{iv}$ is nonsingular. Then, solving the derivative of (8), we obtain

$$\dot{s}(t) = \sum_{i=1}^{r} q_i(\gamma(t))[G_v B_{iv}(u(t) + \beta(t)H_{\varsigma_t}\chi(x(t), t)) + G_v C_{iv}\omega(t) - \sum_{n=1}^{m} \mu_n G_v \times B_{iv} K_v x(t_{k+1-n}h)]. \tag{9}$$

Setting $s(t) = 0$ and $\dot{s}(t) = 0$, we obtain the equivalent control input under memory-based DETM as follows:

$$u_{eq} = \sum_{n=1}^{m} \mu_n K_v x(t_{k+1-n}h) - (G_v B_{iv})^{-1} \times G_v C_{iv}\omega(t) - \beta(t)H_{\varsigma_t}\chi(x(t), t). \tag{10}$$

Substituting the equivalent controller (10) into (3), and combining the variables $h(t)$ and $e_{n,l}(t)$, it yields that

$$\dot{x}(t) = \sum_{i=1}^{r} q_i(\gamma(t))\{A_{iv}x(t) + \sum_{n=1}^{m} \mu_n B_{iv} K_v \times (x(t - h(t) + e_{n,l}(t)) + \bar{C}_{iv}\omega(t)\}, \tag{11}$$

where $\bar{C}_{iv} = [I - B_{iv}(G_v B_{iv})^{-1}G_v]C_{iv}$.

Remark 2.4: In conventional event-triggered SMC, only the current state information and the last released information are taken into account [1], [6], leaving it susceptible to inaccuracies and inefficiencies in dynamic environments. These constraints will reduce the system robustness against external interference due to insufficient scope of the sampled signals. Consequently, the memory-based SMC has emerged as a critical improvement that will be effective in reducing conservatism by utilizing the abundant historical signals. For $m = 1$, the memory-based sliding mode function will be degraded to conventional integral-type sliding mode function.

Taking into account the concurrent impact of FDI attacks and DoS attacks, the subsequent control law is formulated and deployed:

$$\bar{u}_{dos}(t) = \begin{cases} \bar{u}(t) & t \in G_{l,1} \\ 0 & t \in G_{l,0} \end{cases}$$

Then, the control systems to counteract DoS attacks under the event-based SMC protocol can be derived by:

$$\begin{cases} \dot{x}(t) = \begin{cases} \sum_{i=1}^{r} q_i(\gamma(t))\{A_{iv}x(t) + \sum_{n=1}^{m} \mu_n B_{iv} K_v \\ \times(x(t - h(t)) + e_{n,l}(t)) + \bar{C}_{iv}\omega(t)\}, & t \in G_{l,1}, \\ \sum_{i=1}^{r} q_i(\gamma(t))\{A_i(m)x(t) + C_{iv}\omega(t)\}, & t \in G_{l,0}, \end{cases} \\ x(t) = \zeta(t),\ t \in [-\bar{h}, 0). \end{cases} \tag{12}$$

Definition 2.1 [26]: The system (12) attains global exponential stability, if there exist constants $\varrho > 0$ and $d > 0$ such that $\|x(t)\| \leq \varrho e^{-dt}\|\zeta(0)\|_{\bar{h}}$ for $t > 0$, where $\|\zeta(0)\|_{\bar{h}} = \sup_{-\bar{h}\leq\varkappa\leq 0}\{\zeta(\varkappa), \dot{\zeta}(\varkappa)\}$.

Definition 2.2 [2]: For given scalar $\beta > 0$, the S-MJSs (12) achieves H_∞ performance level β, if the systems is globally exponentially stable with $\omega(t) = 0$ and $\|z(t)\|_2 \leq \beta\|\omega(t)\|_2$ holds for any nonzero $\omega(t) \in L_2[0, \infty)$.

Lemma 1 ([16]): Given any real number ρ and square matrix Y, the inequality $\rho(Y + Y^T) \leq \rho^2\bar{R} + Y\bar{R}^{-1}Y^T$ holds, where $\bar{R}$ is a positive definite matrix.

Lemma 2 ([29]): For each possible mode $\varsigma_t = \upsilon$, if the linear matrix equalities(LMIs):

$$\begin{pmatrix} -I & 0 \\ \varpi_1 \Xi & -I \end{pmatrix} < 0,\ \begin{pmatrix} E & 0 \\ I & \varpi_2 I \end{pmatrix} > 0,\ E < \varpi_3 I,$$
$$\sqrt{r}(\varpi_2 + \varpi_3) - 2\omega_1\sqrt{\lambda_{\min}(\mathcal{B}_\upsilon{}^T\mathcal{B}_\upsilon)} < 0$$

are solvable for $(E, \varpi_1, \varpi_2, \varpi_3)$, then there exists a nonsingular matrix $\mathcal{G}_\upsilon = (\mathcal{B}_\upsilon{}^T E^{-1}\mathcal{B}_\upsilon)\mathcal{B}_\upsilon{}^T E^{-1}$ such that $\mathcal{G}_\upsilon \sum_{i=1}^{r} q_i(\upsilon(t))\mathcal{B}_{i\upsilon}$ is nonsingular. Here, $\Xi = 0.5[(\mathcal{B}_\upsilon - \mathcal{B}_{1\upsilon}), \ldots, (\mathcal{B}_\upsilon - \mathcal{B}_{r\upsilon})]$ is defined, and $\mathcal{B}_\upsilon = \frac{1}{r}\sum_{i=1}^{r} \mathcal{B}_{i\upsilon}$ is the full column rank.

Lemma 3 ([32]): For $h(t) \in [0, \bar{h}]$, if $\begin{bmatrix} T & * \\ Z & T \end{bmatrix} \geq 0$ for any constant matrices $Z \in \mathbb{R}^{n_1\times n_1}$ and $S \in \mathbb{R}^{n_1\times n_1}$, the following inequality holds

$$-\bar{h}\int_{t-\bar{h}}^{t} \dot{x}^T(s)T\dot{x}(s)ds \leq c^T(t)Qc(t),$$

where

$$c(t) = [x(t), x(t - h(t)), x(t - \bar{h})]^T,$$
$$Q = \begin{bmatrix} -T & * & * \\ T + Z & -2T - sym\{Z\} & * \\ -Z & T + Z & -T \end{bmatrix}.$$

III. Main Results

In this section, we present the stability analysis of system (12) under nonperiodic DoS attacks (4) and memory-based DETM (5). The proof is divided into two situations: DoS sleeping and DoS attacking. We provide a set of sufficient conditions for the global exponential stability of fuzzy S-MJSs with $\omega(t) = 0$ in Theorem 3.1. Meanwhile, the disturbance attenuation conditions for H_∞ performance index are presented in Theorem 3.2. Finally, we analyze the reachability of the sliding surface in Theorem 3.3.

Theorem 3.1: For given scalars $\bar{h} > 0$ and $\sigma \in [0, 1)$, the nonlinear S-MJSs (12) running on the sliding mode surface $s(t) = 0$ realizes global exponential stability with $\omega(t) = 0$, if there exist symmetric positive definite matrices $P_{f\upsilon}$, $T_{\upsilon,c}$, $Z_{\upsilon,c}$, $H_{\upsilon,c}$, $\Omega_\upsilon > 0$ such that the following inequality constraints hold for given positive scalars $\phi_f, \gamma_f, q_{\max}, l_{\min}$ and $l_{\max}$ $(f = 1, 2, \upsilon \in S)$.

$$P_{1\upsilon} \leq \phi_2 P_{2\upsilon}, \tag{13}$$
$$P_{2\upsilon} \leq e^{2(\gamma_1+\gamma_2)h} P_{1\upsilon}, \tag{14}$$
$$Q_f \leq \phi_{3-f} Q_{3-f}, \tag{15}$$
$$R_f \leq \phi_{3-f} R_{3-f}, \tag{16}$$
$$\frac{2\gamma_1 l_{\min} - 2(\gamma_1 + \gamma_2)h - 2\gamma_2 q_{\max} - \ln(\gamma_1\gamma_2)}{\Omega_a} \geq 0, \tag{17}$$
$$\begin{bmatrix} e^{-2\gamma_f h}H_f & M_f \\ * & e^{-2\gamma_f h}H_f \end{bmatrix} \geq 0, \tag{18}$$

Case I, if $\upsilon \in \Lambda_{\upsilon,k}, \forall l \in \Lambda_{\upsilon,uk}, \Lambda_{\upsilon,k} = \{k_{\upsilon,1}, k_{\upsilon,2}, \cdots k_{\upsilon,q1}\}$,

$$\begin{bmatrix} \Pi_{f\upsilon}^1 & \Sigma_{f\upsilon}^T & C_{f11\upsilon}^T \\ * & -H_f & 0 \\ * & * & C_{12\upsilon} \end{bmatrix} < 0, \tag{19}$$

Case II, if $v \in \Lambda_{v,uk}, \forall l \in \Lambda_{v,uk}, \Lambda_{v,k} = \{k_{v,1}, k_{v,2}, \cdots k_{v,q2}\}$,

$$P_{fv} - P_l \geq 0,\quad \begin{bmatrix} \Pi^2_{fv} & \Sigma^T_{fv} & C^T_{f21v} \\ * & -H_f & 0 \\ * & * & C_{22v} \end{bmatrix} < 0, \tag{20}$$

Case III, if $v \in \Lambda_{v,uk}, \Lambda_{v,k} = \varnothing, l \neq k, l \in \Lambda_{l,k}$,

$$\begin{bmatrix} \Pi^3_{fv} & \Sigma^T_{fv} & C^T_{f31v} \\ * & -H_f & 0 \\ * & * & C_{32v} \end{bmatrix} < 0, \tag{21}$$

where

$$\Pi^j_{1v} = \begin{bmatrix} \Pi^j_{111} & \Pi_{112} & -M_1 & \Pi_{114} \\ * & \Pi_{122} & \Pi_{123} & \Pi_{124} \\ * & * & \Pi_{133} & 0 \\ * & * & * & \Pi_{144} \end{bmatrix},$$

$$\Pi^j_{2v} = \begin{bmatrix} \Pi^j_{211} & \Pi_{212} & -M_1 \\ * & \Pi_{222} & \Pi_{223} \\ * & * & \Pi_{233} \end{bmatrix},$$

$$\Pi^1_{f11} = 2\gamma_f P_{fv} + sym\{A^T_{iv} P_{fv}\} + R_f - e^{-2\gamma_f h} H_f + \sum\nolimits_{c\in\Lambda_{v,k}} [\frac{(\lambda_{vc})^2}{4} T_{vc} + \psi_{vc}(P_{fc} - P_l)],$$

$$\Pi^2_{f11} = 2\gamma_f P_{fv} + sym\{A^T_{iv} P_{fv}\} + R_f - e^{-2\gamma_f h} H_f + \sum\nolimits_{c\in\Lambda_{v,k}} [\frac{(\lambda_{vc})^2}{4} Z_{vc} + \psi_{vc}(P_{fc} - P_l)],$$

$$\Pi^3_{f11} = 2\gamma_f P_{fv} + sym\{A^T_{iv} P_{fv}\} + R_f - e^{-2\gamma_f h} H_f + a_v[\frac{(\lambda_{ll})^2}{4} H_{vv} + \psi_{ll}(P_{fv} - P_l)],$$

$$\Pi_{f12} = P_{fv} B_{iv} K_v + e^{-2\gamma_f h} H_f + M_f,$$

$$\Pi_{f22} = -2e^{-2\gamma_f h} H_f - M^T_f - M_f + \sigma d_1 \Omega_v,$$

$$\Pi_{f23} = e^{-2\gamma_f h} H_f + M_f,$$

$$\Pi_{f33} = -e^{-2\gamma_f h} R_f - e^{-2\gamma_f h} H_f,$$

$$\Pi_{114} = [\mu_1 P_{1v} B_{iv} K_v, \cdots, \mu_m P_{1v} B_{iv} K_v],$$

$$\Pi_{124} = [\underbrace{\frac{\sigma d_1 \Omega_v}{m}, \ldots, \frac{\sigma d_1 \Omega_v}{m}}_{m}],$$

$$\Pi_{144} = [\frac{\sigma d_1 \Omega_v}{m^2} \times I_{m\times m}, + diag\{-\mu_1 \Omega_v, \ldots, -\mu_m \Omega_v\}],$$

$$C_{111v} = [[(P_{fk_{v,1}} - P_l) \cdots (P_{k_{v,q1}} - P_l)]^T, \underbrace{0, \cdots, 0}_{m+2}],$$

$$C_{121v} = [[(P_{fk_{v,1}} - P_l) \cdots (P_{k_{v,q2}} - P_l)]^T, \underbrace{0, \cdots, 0}_{m+2}],$$

$$C_{131v} = [(a_v(P_{fv} - P_l))^T, \underbrace{0, \cdots, 0}_{m+2}],$$

$$C_{211v} = [[(P_{fk_{v,1}} - P_l) \cdots (P_{k_{v,q1}} - P_l)]^T, 0, 0],$$

$$C_{221v} = [[(P_{fk_{v,1}} - P_l) \cdots (P_{k_{v,q2}} - P_l)]^T, 0, 0],$$

$$C_{231v} = [(a_v(P_{fv} - P_l))^T, 0, 0],$$

$$C_{12v} = diag[-T_{vk_{v,1}}, \cdots, -T_{vk_{v,q1}}],$$

$$C_{22v} = diag[-Z_{vk_{v,1}}, \cdots, -Z_{vk_{v,q2}}], C_{32v} = -a_v H_{vv},$$

$$\Sigma_1 = \bar{h}[H_1 A_{iv}, H_1 B_{iv} K_v, 0, \mu_1 H_1 B_{iv} K_v, \cdots, \mu_m H_1 B_{iv} K_v],$$

$$\Sigma_2 = \bar{h}[H_2 A_{iv}, 0, 0].$$

Proof: Choose the semi-Markovian Lyapunov functional

$$V_f(t) = x^T(t) P_{fv} x(t) + \int_{t-\bar{h}}^{t} x^T(s) e(\centerdot) R_f x(s) ds + h \int_{-h}^{0} \int_{t+\sigma}^{t} \dot{x}^T(s) e(\centerdot) H_f \dot{x}(s) ds d\sigma,$$

where $e(\centerdot) = e^{2(-1)^f \gamma_f (t-s)}$, $f \in \{1, 2\}$.
According to the definition of the weak infinitesimal operator as outlined in [33], we obtain

$$\begin{aligned} \dot{V}_1(t) \leq & -2\gamma_1 V_1(t) + 2\gamma_1 x^T(t) P_{1v} x(t) \\ & + sym\{\dot{x}^T(t) P_{1v} x(t)\} + x^T(t)(\sum\nolimits_{v=1}^{s} \psi_{vc} P_{1v}) x(t) \\ & + x^T(t) R_1 x(t) + \bar{h}^2 \dot{x}^T(s) H_1 \dot{x}(s) - x^T(t-\bar{h}) e^{-2\gamma_1 h} \\ & \times R_1 x(t-\bar{h}) - \bar{h} \int_{t-\bar{h}}^{t} \dot{x}^T(s) e^{-2\gamma_1 h} H_1 \dot{x}(s) ds \end{aligned} \tag{22}$$

Based on the definition of $\vartheta(t)$, it follows that $d_0 < \vartheta(t) \leq d_1$ for $t \in [0, \infty)$. From the event-triggered conditions (5), one can derive the inequality for intervals $[t_{k,l}h + h_{k,l}, t_{k+1,l}h + h_{k+1,l})$,

$$0 \leq -(\sum_{n=1}^{m} \mu_n e^T_{n,l}(t) \Omega_v e^T_{n,l}(t) - \sigma d_1 \tilde{x}^T(t) \Omega_v \tilde{x}(t)) \tag{23}$$

Based on Lemma 2.3, it can be derived that $-h \int_{t-\bar{h}}^{t} \dot{x}^T(s) e^{-2\gamma_1 h} H_1 \dot{x}(s) ds \leq \varphi^T(t) Q \varphi(t)$, where $\varphi(t) = [x(t), x(t-h(t)), x(t-\bar{h})]^T$ and Q is defined as follows

$$\begin{bmatrix} -e^{-2\gamma_1 h} H_1 & -e^{-2\gamma_1 h} H_1 + M_1 & -M_1 \\ * & -2e^{-2\gamma_1 h} H_1 - sym\{M_1\} & e^{-2\gamma_1 h} H_1 + M_1 \\ * & * & e^{-2\gamma_1 h} H_1 \end{bmatrix}. \tag{24}$$

Combining (22)-(24), it can be concluded that

$$\dot{V}_1(t) + 2\gamma_1 V_1(t) \leq \sum_{i=1}^{r} q_i(\gamma(t)) {\psi_1}^T(t) \bar{\Psi}_{1iv} \psi_1(t), \tag{25}$$

where $\bar{\Psi}_{1iv}$=$[\Pi_1 + \Sigma^T_1 H^{-1}_1 \Sigma_1 + \Gamma_1]$, $\psi_1(t) = [x(t), x(t-h(t)), x(t-\bar{h}), e_{1,l}(t), \cdots, e_{m,l}(t)]^T$, $\Gamma_1 = diag\{\sum_{c=1}^{s} \psi_{vc} P_{1c}, \underbrace{0, \ldots, 0}_{m+2}\}$, and $\Psi_{1iv} = \Pi_1 + \Sigma^T_1 H^{-1}_1 \Sigma_1$ with

$$\Pi_1 = \begin{bmatrix} \Pi_{11} & \Pi_{112} & -M_1 & \Pi_{114} \\ * & \Pi_{122} & \Pi_{123} & \Pi_{124} \\ * & * & \Pi_{133} & 0 \\ * & * & * & \Pi_{144} \end{bmatrix}$$

and $\Pi_{11} = 2\gamma_1 P_{1v} + sym\{A^T_{iv} P_{1v}\} + R_1 - e^{-2\gamma_1 h} H_1$.

When $\bar{\Psi}_{1iv} < 0$, it is evident that $\dot{V}_1(t) < -2\gamma_1 V_1(t)$. Next, we divide the corresponding proof into three distinct cases as follows:

Case I: $v \in \Lambda_{v,k}$. Define $\lambda_{v,k}$ as $\lambda_{v,k} = \sum_{v\in\Lambda_{v,k}} \psi_{vc}(\mu)$. Given that $\Lambda_{v,uk} \neq \varnothing$, it follows that $\lambda_{v,k} < 0$. Furthermore, $\sum_{c=1}^{s} \psi_{vc}(\mu) P_{1c}$ is equivalent to the following,

$$\begin{aligned}\sum_{c=1}^{s} \psi_{vc}(\mu) P_{1c} &= (\sum_{c\in\Lambda_{v,k}} + \sum_{c\in\Lambda_{v,uk}})\psi_{vc}(\mu) P_{1c} \\ &= \sum_{c\in\Lambda_{v,k}} \psi_{vc}(\mu) P_{1c} - \lambda_{v,k} \sum_{c\in\Lambda_{v,uk}} \frac{\psi_{vc}(\mu)}{-\lambda_{v,k}} P_{1c}.\end{aligned} \tag{26}$$

Noticing that $0 \leq \psi_{vc}(\mu)/-\lambda_{v,k} \leq 1$ $(c \in \Lambda_{v,k})$ and $\sum_{c\in\Lambda_{v,uk}} \frac{\psi_{vc}(\mu)}{-\lambda_{v,k}} = 1$, for $l \in \Lambda_{v,uk}$, the inequality $\bar{\Psi}_{1iv} < 0$ can be guaranteed by

$$\Psi_{1iv} + diag\{\sum_{c\in\Lambda_{v,k}} \psi_{vc}(\mu)(P_{1c} - P_l), \underbrace{0, \ldots, 0}_{m+2}\} < 0. \tag{27}$$

Referring to (27) and Lemma 2.1, for any $T_{vc} > 0$, we obtain

$$\begin{aligned}&\sum_{c\in\Lambda_{v,k}} \Delta\psi_{vc}(\mu)(P_{1c} - P_l) \\ &\leq \sum_{c\in\Lambda_{v,k}} [\frac{(\lambda_{vc})^2}{4} T_{vc} + (P_{1c} - P_l)(T_{vc})^{-1}(P_{1c} - P_l)^T].\end{aligned} \tag{28}$$

Through the integration of equations (26)-(28) and the utilization of the Schur complement, the inequality $\bar{\Psi}_{1iv} < 0$ can be obtained from the condition (19).

Case II: For $v \in \Lambda_{v,uk}$ and $\Lambda_{v,k} \neq \varnothing$. We can define $\lambda_{v,k} = \sum_{v\in\Lambda_{v,k}} \psi_{vc}(\mu)$, which implies that $\lambda_{v,k} > 0$. Furthermore, we can express $\sum_{c=1}^{s} \psi_{vc}(\mu) P_{1c}$ as follows:

$$\begin{aligned}\sum_{c=1}^{s} \psi_{vc}(\mu) P_{1c} &= \sum_{c\in\Lambda_{v,k}} \psi_{vc}(\mu) P_{1c} + \psi_{vv}(\mu) P_{1v} \\ &\quad - (\psi_{vv}(\mu) + \lambda_{v,k}) \sum_{\substack{c\in\Lambda_{v,uk}\\ c\neq v}} \frac{\psi_{vc}(\mu) P_{1c}}{-(\psi_{vv}(\mu) + \lambda_{v,k})}.\end{aligned} \tag{29}$$

As with Case I, $\forall l \in \Lambda_{v,uk} (l \neq v)$, the guarantee of $\bar{\Psi}_{1iv} < 0$ can be achieved through the following set of inequalities

$$\begin{aligned}&P_{1v} - P_l \geq 0, \\ &\Psi_{1iv} + diag\{\sum_{c\in\Lambda_{v,k}} \psi_{vc}(\mu)(P_{1c} - P_l), \underbrace{0, \ldots, 0}_{m+2}\} < 0.\end{aligned} \tag{30}$$

Therefore, it is established that when $Z_{vc} > 0$,

$$\begin{aligned}\sum_{c\in\Lambda_{v,k}} \psi_{vc}(\mu)(P_{1c} - P_l) &\leq \sum_{c\in\Lambda_{v,k}} \psi_{vc}(P_{1c} - P_l) \\ &+ \sum_{c\in\Lambda_{v,k}} [\frac{(\lambda_{vc})^2}{4} Z_{vc} + (P_{1c} - P_l)(Z_{vc})^{-1}(P_{1c} - P_l)^T].\end{aligned} \tag{31}$$

Combining (29)-(31) and employing the Schur complement, we can deduce that inequality (20) guarantees $\bar{\Psi}_{1iv} < 0$.

Case III: $v \in \Lambda_{v,uk}, \Lambda_{v,k} = \varnothing$. Assume that there exists a value $l \neq v$ with $l \in \Lambda_{l,k}$. Following the method described in literature [30], the parameter a_v can be determined to estimate $\psi_{vv}(\mu)$ as $a_v \psi_{ll}(\mu)$. We denote $\lambda_{v,k}$ as $\psi_{vv}(\mu)$. The expression $\sum_{l=1}^{s} \psi_{vl}(\mu) P_l$ can be represented as:

$$\sum_{l=1}^{s} \psi_{vl}(\mu) P_l = \psi_{vv}(\mu) P_{1v} - \lambda_{v,k} \sum_{l\in\Lambda_{v,uk}} \frac{\psi_{vc}(\mu)}{-\lambda_{v,k}} P_l. \tag{32}$$

Notice that $\sum_{l\in\Lambda_{v,uk}} \psi_{vc}(\mu) = -\psi_{vv}(\mu) = -\lambda_{v,k} > 0$, $\forall l \in \Lambda_{v,uk}$. Therefore, we can conclude that:

$$\bar{\Psi}_{1iv} = \Psi_{1iv} + diag\{a_v \psi_{ll}(\mu)(P_{1v} - P_l), \underbrace{0, \ldots, 0}_{m+2}\}, \tag{33}$$

for any $H_{vv} > 0$,

$$\begin{aligned}\Delta\psi_{ll}(\mu)(P_{1v} - P_l) \leq [&\frac{(\lambda_{ll})^2}{4} H_{vv} + (P_{1v} - P_l) \\ &\times (H_{vv})^{-1}(P_{1v} - P_l)^T].\end{aligned} \tag{34}$$

Through a combination of equations (32)-(34) and the use of the Schur complement, we can establish that inequality $\bar{\Psi}_{1iv} < 0$ if (21) holds. In summary, $\dot{V}_1(t) < -2\gamma_1 V_1(t)$ can be guaranteed.

In the same way, we get

$$\dot{V}_2(t) - 2\gamma_2 V_2(t) \leq \sum_{i=1}^{r} q_i(\gamma(t)) \psi_2{}^T(t) \bar{\Psi}_{2iv} \psi_2(t), \tag{35}$$

where $\bar{\Psi}_{2iv} = [\Pi_2 + \Sigma_2^T H_2^{-1} \Sigma_2 + \Gamma_2]$, $\psi_2(t) = [x(t), x(t - h_l(t)), x(t - \bar{h}),]^T$, $\Gamma_2 = diag\{\sum_{c=1}^{s} \psi_{vc} P_{2c}, 0, 0\}$. It is obvious that

$$\dot{V}_2(t) - 2\gamma_2 V_2(t) < 0, \dot{V}_1(t) < -2\gamma_1 V_1(t). \tag{36}$$

Integrating both sides of equation (36), we obtain

$$\begin{aligned}&V_1(t) \leq e^{-2\gamma_1(t-h_l)} V_1(h_l), \;\; t \in [h_l, h_l + l_l), \\ &V_2(t) \leq e^{2\gamma_1(t-h_l-l_l)} V_1(h_l + l_l), \;\; \in [h_l + l_l, h_{l+1}).\end{aligned} \tag{37}$$

Then

$$\begin{aligned}&V_1(h_l{}^+) \leq \phi_2 V_2(h_l{}^-), \\ &V_1((h_l + l_l)^+) \leq \phi_1 e^{2(\gamma_1+\gamma_2)h} V_2((h_l + l_l)^-).\end{aligned} \tag{38}$$

For $t \in [h_l, h_l + l_l)$, we have

$$\begin{aligned}V_1(t) &\leq e^{-2\gamma_1(t-h_l)} \phi_2 V_2(h_l{}^-) \\ &\leq e^{-2\gamma_1(t-h_l)+2\gamma_2(h_l-h_{l-1}-l_l)} \phi_2 V_2(h_{l-1} + l_{l-1}) \\ &\leq e^{c_1(t)} V_1(0),\end{aligned} \tag{39}$$

where $c_1(t) = (b + \frac{t}{\Omega_a})[2((\gamma_1 + \gamma_2)h) + \gamma_2 q_{\max} - \gamma_1 l_{\min} + \ln(\gamma_1\gamma_2)]$. Then, denote $d = [\gamma_1 l_{\min} - ((\gamma_1 + \gamma_2)h) - \gamma_2 q_{\max} - 0.5\ln(\gamma_1\gamma_2)]/\Omega_a$, we get

$$V_1(t) \leq e^{m_1} e^{-dt} V_1(0),$$

where $m_1 = b[2((\gamma_1 + \gamma_2)h) + \gamma_2 q_{\max} - \gamma_1 l_{\min} + \ln(\gamma_1\gamma_2)]$.

In the same way, we have

$$V_2(t) \leq e^{m_2} e^{-dt} V_1(0),$$

where $m_2 = (b+1)[2((\gamma_1+\gamma_2)h)+\gamma_2 q_{\max}-\gamma_1 l_{\min}+\ln(\gamma_1\gamma_2)]$.

Denoting

$$M = \max\{e^{m_1}, \frac{e^{m_2}}{\phi_2}\},$$

we have

$$V(t) \le Me^{-dt}V_1(0).$$

Give the definition of $V(t)$ as

$$V(t) \ge c_1\|x(t)\|^2,\ \ V_1(0) \le c_2\|\zeta(t)\|^2_{\bar{h}},$$

where $c_1 = \min\{\lambda_{\min}(P_{fv})\}$, $c_2 = \max\{\lambda_{\max}(P_{fv}) + h\lambda_{\max}(R_1) + \frac{h^2}{2}\lambda_{\max}(H_1)\}$.

Thus, one gets

$$\|x(t)\| \le \sqrt{\frac{Mc_2}{c_1}}e^{-dt}\|\zeta(0)\|_{\bar{h}}.$$

To summarize, it can be concluded that the system described by (12) exhibits global exponential stability with a convergence rate of d under the condition that $\omega(t)=0$ based on Definition 2.1.

Theorem 3.2: For given scalars $\bar{h} > 0$ and $\sigma \in [0, 1)$, the resultant nonlinear S-MJSs (12) running on the sliding mode surface $s(t) = 0$ realizes global exponential stability with an H_∞ disturbance attenuation level $\bar{\iota} = \sqrt{(\rho_2/\rho_2)}\beta$, where $\rho_1 = \min\{(1/\phi_2), 1\}, \rho_2 = \max\{(1/\phi_2)e^{2\gamma_2 l_{\max}},\ e^{2\gamma_2 q_{\max}}\}$, if there exist symmetric positive definite matrices P_{fv}, $T_{v,c}$, $Z_{v,c}$, $H_{v,c}$, $\Omega_v > 0$ such that the following inequality constraints hold for the provided positive scalars $\phi_f, \gamma_f, q_{\max}, l_{\min}, l_{\max}$ with (13)-(18) ($f = 1, 2, v \in S$).

Case I, if $v \in \Lambda_{v,k}, \forall l \in \Lambda_{v,uk}, \Lambda_{v,k} = \{k_{v,1}, k_{v,2}, \cdots k_{v,q1}\}$,

$$\begin{bmatrix} \tilde{\Pi}_{fv}{}^1 & \tilde{\Sigma}^T_{fv} & \tilde{C}^T_{f11v} \\ * & -H_f & 0 \\ * & * & C_{12v} \end{bmatrix} < 0, \tag{40}$$

Case II, if $v \in \Lambda_{v,uk}, \forall l \in \Lambda_{v,uk}, \Lambda_{v,k} = \{k_{v,1}, k_{v,2}, \cdots k_{v,q2}\}$,

$$P_{fv} - P_l \ge 0,$$

$$\begin{bmatrix} \tilde{\Pi}_{fv}{}^2 & \tilde{\Sigma}^T_{fv} & \tilde{C}^T_{f21v} \\ * & -H_f & 0 \\ * & * & C_{22v} \end{bmatrix} < 0, \tag{41}$$

Case III, if $v \in \Lambda_{v,uk}, \Lambda_{v,k} = \varnothing, l \ne k, l \in \Lambda_{l,k}$,

$$\begin{bmatrix} \tilde{\Pi}_{fv}{}^3 & \tilde{\Sigma}^T_{fv} & \tilde{C}^T_{f31v} \\ * & -H_f & 0 \\ * & * & C_{32v} \end{bmatrix} < 0, \tag{42}$$

where

$$\tilde{\Pi}_2{}^j = \begin{bmatrix} \tilde{\Pi}_{211}{}^j & \Pi_{212} & -M_1 & \tilde{\Pi}_{214} \\ * & \Pi_{222} & \Pi_{223} & 0 \\ * & * & \Pi_{233} & 0 \\ * & * & * & \tilde{\Pi}_{244} \end{bmatrix},$$

$$\tilde{\Pi}_1{}^j = \begin{bmatrix} \tilde{\Pi}_{111}{}^j & \Pi_{112} & -M_1 & \tilde{\Pi}_{114} & \tilde{\Pi}_{115} \\ * & \Pi_{122} & \Pi_{123} & 0 & \tilde{\Pi}_{125} \\ * & * & \Pi_{133} & 0 & 0 \\ * & * & * & \tilde{\Pi}_{144} & 0 \\ * & * & * & * & \tilde{\Pi}_{155} \end{bmatrix},$$

$$\tilde{\Pi}^1_{f11v} = 2\gamma_f P_{fv} + sym\{A^T_{iv}P_{fv}\} + R_f - e^{-2\gamma_f h}H_f + D^T_{iv}D_{iv} + \sum\nolimits_{c\in\Lambda_{v,k}}[\frac{(\lambda_{vc})^2}{4}T_{vc} + \psi_{vc}(P_{fc} - P_l)],$$

$$\tilde{\Pi}^2_{f11v} = 2\gamma_f P_{fv} + sym\{A^T_{iv}P_{fv}\} + R_f - e^{-2\gamma_f h}H_f + D^T_{iv}D_{iv} + \sum\nolimits_{c\in\Lambda_{v,k}}[\frac{(\lambda_{vc})^2}{4}Z_{vc} + \psi_{vc}(P_{fc} - P_l)],$$

$$\tilde{\Pi}^3_{f11v} = 2\gamma_f P_{fv} + sym\{A^T_{iv}P_{fv}\} + R_f - e^{-2\gamma_f h}H_f + D^T_{iv}D_{iv} + a_v[\frac{(\lambda_{ll})^2}{4}H_{vv} + \psi_{ll}(P_{fv} - P_l)],$$

$$\tilde{\Pi}_{114} = D^T_{iv}E_{iv} + P_{fv}\bar{C}_i(v),$$

$$\tilde{\Pi}_{214} = D^T_{iv}E_{iv} + P_{fv}C_i(v),$$

$$\tilde{\Pi}_{f44} = -\beta^2 I + E^T_{iv}E_{iv},$$

$$\tilde{\Pi}_{115} = [\mu_1 P_{fv}B_{iv}K_v, \cdots, \mu_m P_{1v}B_{iv}K_v],$$

$$\tilde{\Pi}_{125} = [\underbrace{\frac{\sigma d_1\Omega_v}{m}, \ldots, \frac{\sigma d_1\Omega_v}{m}}_{m}],$$

$$\tilde{\Pi}_{155} = [\frac{\sigma d_1\Omega_v}{m^2} \times I_{m\times m} + diag\{-\mu_1\Omega_v, \ldots, -\mu_m\Omega_v\}],$$

$$\tilde{C}_{111v} = [[(P_{fk_{v,1}} - P_l)\cdots(P_{fk_{v,q1}} - P_l)]^T, \underbrace{0, \cdots, 0}_{m+3}],$$

$$\tilde{C}_{121v} = [[(P_{fk_{v,1}} - P_l)\cdots(P_{fk_{v,q2}} - P_l)]^T, \underbrace{0, \cdots, 0}_{m+3}],$$

$$\tilde{C}_{131v} = [(a_v(P_{fv} - P_l))^T, \underbrace{0, \cdots, 0}_{m+3}],$$

$$\tilde{C}_{211v} = [[(P_{fk_{v,1}} - P_l)\cdots(P_{fk_{v,q1}} - P_l)]^T, 0, 0, 0],$$

$$\tilde{C}_{221v} = [[(P_{fk_{v,1}} - P_l)\cdots(P_{fk_{v,q2}} - P_l)]^T, 0, 0, 0],$$

$$\tilde{C}_{231v} = [(a_v(P_{fv} - P_l))^T, 0, 0, 0]$$

$$\tilde{\Sigma}_1 = \bar{h}[H_1A_{iv}, H_1B_{iv}K_v, 0, H_1\bar{C}_i(v), \mu_1 H_1 B_{iv}K_v, \cdots, \mu_m H_1 B_{iv}K_v],$$

$$\tilde{\Sigma}_2 = \bar{h}[H_2A_{iv}, 0, 0, H_2C_{iv}].$$

Proof: Similar to the proof presented in Theorem 3.1, we can deduce the following inequalities if $\omega(t) \ne 0$.

$$\dot{V}_1(t) + 2\gamma_1 V_1(t) + z^T(t)z(t) - \beta^2\omega^T(t)\omega(t) \le \sum_{i=1}^{r} q_i(\gamma(t))\tilde{\psi}_1{}^T(t)[\tilde{\Pi}_1 + \tilde{\Sigma}^T_1 H^{-1}_1\tilde{\Sigma}_1 + \tilde{\Gamma}_1]\tilde{\psi}_1(t),$$

$$\dot{V}_2(t) - 2\gamma_2 V_2(t) + z^T(t)z(t) - \beta^2\omega^T(t)\omega(t) \le \sum_{i=1}^{r} q_i(\gamma(t))\tilde{\psi}_2{}^T(t)[\tilde{\Pi}_2 + \tilde{\Sigma}^T_2 H^{-1}_2\tilde{\Sigma}_2 + \tilde{\Gamma}_2]\tilde{\psi}_2(t),$$

where $\tilde{\Gamma}_1 = diag\{\sum_{c=1}^{s}\psi_{vc}P_{1c}, 0, \ldots, 0\}$, $\tilde{\Gamma}_2 = diag\{\sum_{c=1}^{s}\psi_{vc}P_{2c}, 0, 0, 0\}$, $\tilde{\psi}_1(t) = [x(t), x(t-h(t)), x(t-\bar{h}), \omega(t), e_{1,l}(t), \cdots, e_{m,l}(t)]^T$, $\tilde{\psi}_2(t) = [x(t), x(t-h_l(t)), x(t-\bar{h}), \omega(t)]$ and $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ have been designed in a manner that is similar to Theorem 3.1.

On basis of the conditions (40)-(42) outlined in Theorem 3.2, it follows that

$$\begin{aligned}&\dot{V}_1(t)+2\gamma_1V_1(t)+z^T(t)z(t)-\beta^2\omega^T(t)\omega(t)<0, t\in G_{l,1},\\&\dot{V}_2(t)-2\gamma_2V_2(t)+z^T(t)z(t)-\beta^2\omega^T(t)\omega(t)<0, t\in G_{l,0}.\end{aligned}\tag{43}$$

For $t\in[0,h_{l+1})$, define $\upsilon_{1l}(t)=(1/\phi_2)e^{-2\gamma_1(h_l-t)}$, $\upsilon_{2l}(t)=e^{2\gamma_2(h_{l+1}-t)}$, $\vartheta_k=2[(\gamma_1+\gamma_2)h+\gamma_2(h_{k+1}-h_k-l_k)]$.

$$\begin{aligned}&\sum_{k=0}^{l}\int_{h_k}^{h_k+l_k}[\upsilon_{1l}(t)(\dot{V}_1(t)+2\gamma_1V_1(t))dt\\&\quad+\int_{h_k+l_k}^{h_{k+1}}\upsilon_{2l}(t)(\dot{V}_2(t)-2\gamma_2V_2(t))dt]\\&\geq\sum_{k=0}^{l}[\frac{1}{\phi_2}e^{2\gamma_1l_k}V_1(h_k+l_k)-\frac{1}{\phi_2}V_1(h_k)\\&\quad+\frac{1}{\phi_2}V_1(h_{k+1})-\phi_1e^{\vartheta_k}V_1(h_k+l_k)]\\&=\frac{1}{\phi_2}[V_1(l_{k+1})-V_1(0)]+\sum_{k=0}^{l}[\frac{1}{\phi_2}e^{2\gamma_1l_k}-\phi_1e^{\vartheta_k}]V_1(h_k+l_k).\end{aligned}$$

We can obtain $\frac{1}{\phi_2}e^{2\gamma_1l_k}-\phi_1e^{\vartheta_k}\geq0$ from (17) and $V_1(0)=0$, $V_1(t)\geq0$. It can be concluded that

$$\begin{aligned}&\sum_{k=0}^{l}\int_{h_k}^{h_k+l_k}[\upsilon_{1l}(t)(V_1(t)+2\gamma_1V_1(t))dt\\&\qquad+\int_{h_k+l_k}^{h_{k+1}}\upsilon_{2l}(t)(V_2(t)-2\gamma_2V_2(t))dt]>0.\end{aligned}\tag{44}$$

One has

$$\begin{aligned}&\sum_{k=0}^{l}\int_{h_k}^{h_k+l_k}[\upsilon_{1l}(t)z^T(t)z(t)dt+\int_{h_k+l_k}^{h_{k+1}}\upsilon_{2l}(t)z^T(t)z(t)dt]\\&\quad\leq\sum_{k=0}^{l}\int_{h_k}^{h_k+l_k}[\upsilon_{1l}(t)\beta^2\omega^T(t)\omega(t)dt\\&\qquad+\int_{h_k+l_k}^{h_{k+1}}\upsilon_{2l}(t)\beta^2\omega^T(t)\omega(t)dt],\end{aligned}\tag{45}$$

which means that

$$\begin{aligned}&\sum_{k=0}^{l}\int_{h_k}^{h_{k+1}}\rho_1z^T(t)z(t)dt\\&\leq\sum_{k=0}^{l}\int_{h_k}^{h_k+l_k}[\upsilon_{1l}(t)z^T(t)z(t)dt+\int_{h_k+l_k}^{h_{k+1}}\upsilon_{2l}(t)z^T(t)z(t)dt]\\&\leq\sum_{k=0}^{l}\int_{h_k}^{h_{k+1}}\rho_2\beta^2\omega^T(t)\omega(t)dt,\end{aligned}\tag{46}$$

where $\rho_1=\min\{(1/\phi_2),1\}$, $\rho_2=\max\{(1/\phi_2)e^{2\gamma_2l_{\max}}, e^{2\gamma_2q_{\max}}\}$. We have

$$\int_0^{h_{l+1}}z^T(t)z(t)dt\leq\frac{\rho_2}{\rho_1}\beta^2\int_0^{h_{l+1}}\omega^T(t)\omega(t)dt.$$

Then, letting $l\to\infty(h_{l+1}\to\infty)$, we get

$$\int_0^{h_{l+1}}z^T(t)z(t)dt\leq\bar{\beta}^2\int_0^{h_{l+1}}\omega^T(t)\omega(t)dt.\tag{47}$$

Based on Definition 2.2, it follows that $\|z(t)\|_2\leq\bar{\beta}\|\omega(t)\|_2$ with $\bar{\beta}=\beta\sqrt{\frac{\rho_2}{\rho_1}}$ for $\omega(t)\in l_2[0,\infty)$. This completes the proof.

Remark 3.1: Regarding the above the stability analysis of systems with time-varying transfer probabilities, the technical approach introduced in this paper is superior to those proposed in literature [33]. We focus on completely unknown situations, specifically Case III of Theorem 3.1, and adopt the procedure explained in [30] to estimate transition rates by utilizing other diagonal components of the transition rate matrix. Subsequently, we suggest an optimal algorithm that can be executed using the MATLAB toolbox, outlined as follows:

$$\begin{aligned}&\max\ a_v>0, s.t, \text{LMIs } (13)-(18), (40)-(42)\\&\text{with feasible } P_{fv},\ T_{v,c},\ Z_{v,c} \text{ and } H_{v,c}.\end{aligned}\tag{48}$$

Theorem 3.3: The SMC law is formulated to ascertain the attainability of the designed integral sliding-mode surface, which ensures the state trajectories can converge to the sliding mode surface in finite time. As a consequence, the achievement of finite-time attractiveness can be attained by

$$\begin{aligned}u(t)&=\sum\nolimits_{n=1}^{m}\mu_nK_vx(t_{k+1-n}h)-\bar{\rho}\hbar_v\xi(v)(x(t),t)\\&\quad-[\varpi\left\|s^T(t)\right\|\|G_vC_{iv}\|+\rho^\star][G_v\bar{B}(v)]^{-1}\frac{s(t)}{\|s(t)\|},\end{aligned}\tag{49}$$

Proof: For Lyapunov function

$$V(t)=\frac{1}{2}s^T(t)s(t),$$

we have

$$\begin{aligned}\dot{V}(t)&=s^T(t)\dot{s}(t)=s^T(t)\sum_{i=1}^{r}q_i(\gamma(t))[G_vB_{iv}(u(t)\\&\quad+\beta(t)H_{\varsigma_t}\chi(x(t),t))+G_vC_{iv}\omega(t)\\&\quad-\sum\nolimits_{n=1}^{m}G_vB_{iv}\mu_nK_vx(t_{k+1-n}h)]\\&\leq\varpi\left\|s^T(t)\right\|\|G_vC_{iv}\|-s^T(t)G_v\bar{B}(v)\\&\quad\times\sum\nolimits_{n=1}^{m}\mu_nK_vx(t_{k+1-n}h)+s^T(t)G_v\bar{B}(v)(u(t)\\&\quad+\bar{\rho}\hbar_v\xi(v)(x(t),t)),\end{aligned}\tag{50}$$

where $\sum_{i=1}^{r}q_i(\gamma(t))G_vB_{iv}=G_v\bar{B}_v$. Based on Lemma 2.2, there exists G_v such that the nonsingularity of $G_v\bar{\mathcal{B}}_v$ can be attained. Substituting (49) into (50), we have

$$\dot{V}(t)\leq-\rho^\star\|s(t)\|=-\sqrt{2}\rho^\star V^{\frac{1}{2}}(t).\tag{51}$$

Apply Dynkin's formula

$$\sqrt{V(t^\star)}-\sqrt{V(0)}\leq-\frac{\sqrt{2}\rho^\star}{2}t^\star,\tag{52}$$

Furthermore, there exists

$$t^\star\leq\frac{\sqrt{2V(0)}}{\rho^\star}.\tag{53}$$

By analyzing (53), it can be deduced that $||(s(t))||=0$ holds for $t\geq t^\star$, which implies the guaranteed satisfaction of $s(t)=0$. Thus, the proof is completed.

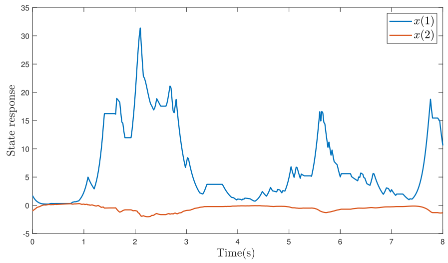

Fig. 3. State response of the system without control.

Remark 3.2: Chattering effect is a widely recognized phenomenon which arises from the utilization of the symbolic function $sgn(s(t))$, as noted in [34]. This phenomenon has the potential to undermine system performance and impose additional strain on the actuator. To effectively reduce the chattering phenomenon, we propose substituting $sgn(s(t))$ with $s(t)/(|s(t)| + \iota)$, where ι is a small constant. In the following simulation example, we demonstrate the efficacy of our proposed design method.

IV. Simulation Example

To establish the validity and practicality of the proposed control scheme, we will present a simulation example of the tunnel diode circuit in this section. Following the approach taken in literature [35], the dynamic equation governing the system is expressed as follows:

$$\begin{cases} \dot{x}_1(t) = -0.1x_1(t) + \dfrac{\alpha_i}{C_0}x_1^3(t) + 10x_2(t) \\ L_0\dot{x}_2(t) = -x_1(t) - R_0x_2(t) + 0.1\omega(t) \\ z(t) = J_0x(t) + 0.1\omega(t) \end{cases}$$

The values of the parameters C_0, L_0, R_0 and J_0 are provided in [35]. We assume that the parameters α_i are selected as $\alpha_1 = 0.01$, $\alpha_2 = 0.02$, and $\alpha_3 = 0.03$. The transitions between modes are following a semi-Markov process. The corresponding transition probability matrix is presented as

$$\begin{pmatrix} -1.5 + \Delta\psi_{11}(\mu) & ? & ? \\ ? & ? & 2.5 + \Delta\psi_{23}(\mu) \\ ? & ? & ? \end{pmatrix}.$$

where ? represents the completely unknown transition probability.

Assume that $|x_1(t)| \leq 3$, $x(t) = [x_1^T(t)\ x_2^T(t)]^T$. Consequence, the fuzzy membership functions are as follows:

$$q_1(x_1(t)) = \begin{cases} (x_1(t) + 3)/3 & -3 \leq x_1(t) \leq 0 \\ -(x_1(t) - 3)/3 & 0 \leq x_1(t) \leq 3 \\ 0 & others \end{cases}$$

$$q_2(x_1(t)) = 1 - q_1(x_1(t))$$

Then, the parameters of the tunnel diode circuit system can be formulated as:

$$\mathcal{A}_1(1) = \begin{bmatrix} -0.1 & 10 \\ -1 & -10 \end{bmatrix}, \mathcal{A}_1(2) = \begin{bmatrix} -4.6 & 10 \\ -1 & -10 \end{bmatrix},$$

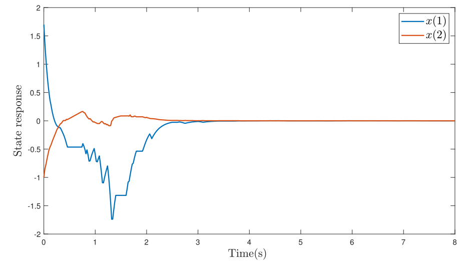

Fig. 4. State response of the system under SMC.

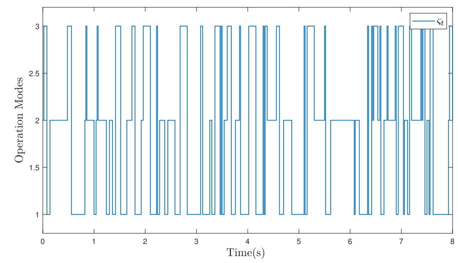

Fig. 5. The mode of semi-Markov jumping systems.

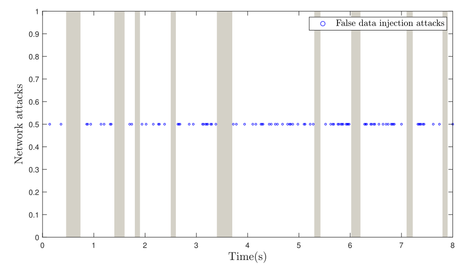

Fig. 6. The instants of network attack.

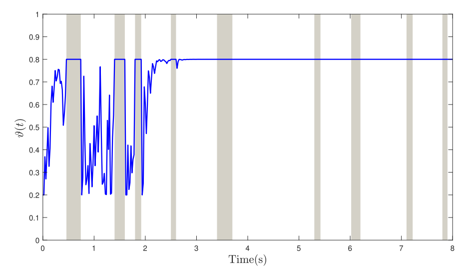

Fig. 7. The auxiliary function trajectory of DMETM.

$$\mathcal{A}_1(3) = \begin{bmatrix} -9.1 & 10 \\ -1 & -10 \end{bmatrix}, \mathcal{A}_2(1) = \begin{bmatrix} -0.1 & 10 \\ -1 & -10 \end{bmatrix},$$
$$\mathcal{A}_2(2) = \begin{bmatrix} -4.6 & 10 \\ -1 & -10 \end{bmatrix}, \mathcal{A}_2(3) = \begin{bmatrix} -13.6 & 10 \\ -1 & -10 \end{bmatrix},$$
$$\mathcal{B}_i(1) = \begin{bmatrix} 0 \\ 1 \end{bmatrix},\ \mathcal{B}_i(2) = \begin{bmatrix} 0 \\ 0.75 \end{bmatrix},\ \mathcal{B}_i(3) = \begin{bmatrix} 0 \\ 0.8 \end{bmatrix},$$
$$\mathcal{K}(v) = \begin{bmatrix} 1.2 & -0.9 \end{bmatrix}, D_{iv} = [1\ 0], E_{iv} = 0.1,$$
$$(i = 1, 2,\ v = 1, 2, 3).$$

The sample time is $h = 0.02$, and disturbance is supposed to be:

$$\omega(t) = \begin{cases} 2 & 0 \leq t < 2 \\ 1 & 2 \leq t < 4 \\ 0 & \text{other} \end{cases}$$

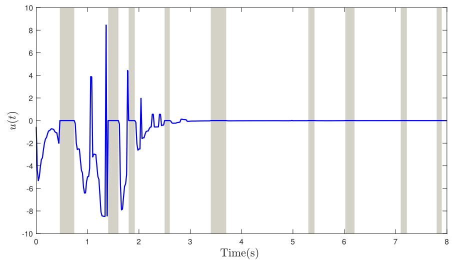

Fig. 8. Control input $u(t)$.

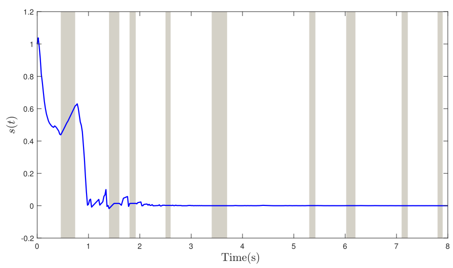

Fig. 9. Sliding surface function $s(t)$.

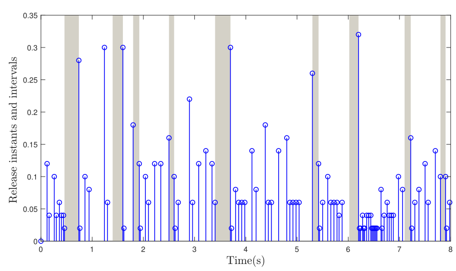

Fig. 10. Release instants and intervals for DETM.

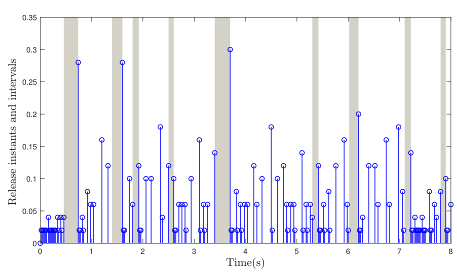

Fig. 11. Release instants and intervals for METM.

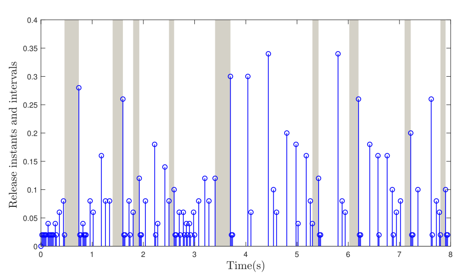

Fig. 12. Release instants and intervals for DMETM.

The parameters about FDI attacks are given as: $\bar{\rho} = 0.35$, $\hbar(1) = e^{-0.5t}\cos(t)$, $\hbar(2) = e^{-0.5t}\sin(t)$, $\hbar(3) = e^{-0.5t}$, $\xi(1)(x(t),t) = 2\sqrt{x_1^2(t)+x_2^2(t)+0.1}\cos(t)$, $\xi(2)(x(t),t) = 2\sqrt{x_1^2(t)+x_2^2(t)+0.1}\sin(t)$, $\xi(3)(x(t),t) = 2\sqrt{x_1^2(t)+x_2^2(t)+0.1}$.

Furthermore, the remaining parameters of the examined system are as follows: $h = 0.02$, $\gamma_1 = 0.02$, $\gamma_2 = 0.5$, $\mu_1 = 0.3$, $\mu_2 = 0.7$, $\phi_1 = \phi_2 = 1.04$, $\sigma = 0.5$, $b = 0.5$, $\mathcal{G}(1) = [0\ 1]$, $d_0 = 0.2$, $d_1 = 0.8$, $h(x(t),t) = 0.4\cos x(t)$, $\Delta\psi_{kl}(\mu) \le |0.2 * \psi_{kl}|$, $\bar{h} = 0.03$, $\lambda_1 = 0.9$, $\lambda_2 = 0.8$, $G(1) = [0\ 1]$, $G(2) = [0\ 1.33]$, $G(3) = [0\ 1.25]$, $\rho^\star = 0.01$. ι in $s(t)/(|s(t)|+\iota)$ is 0.001. Notice that $\psi_{3l}(\mu)(l = 1,2,3)$ are completely inaccessible. To obtain the feasible solutions, we solve the optimization algorithm (48) using the Matlab tool-box to get the following solutions:

$$a_3 = 11.1078,$$
$$\Omega_1 = \begin{pmatrix} 22.2903 & -1.5190 \\ -1.5190 & 22.5978 \end{pmatrix}, \Omega_2 = \begin{pmatrix} 21.9990 & -0.3615 \\ -0.3615 & 22.8333 \end{pmatrix},$$
$$\Omega_3 = \begin{pmatrix} 21.3431 & -0.7911 \\ -0.7911 & 46.6073 \end{pmatrix}, T_{11} = \begin{pmatrix} 9.5158 & 0.0721 \\ 0.0721 & 9.4712 \end{pmatrix},$$
$$Z_{23} = \begin{pmatrix} 9.4478 & 0.0030 \\ 0.0030 & 9.3909 \end{pmatrix}, H_{33} = \begin{pmatrix} 9.5303 & 0.0930 \\ 0.0930 & 9.3715 \end{pmatrix}.$$

We initialize the system with the state vector $x_0 = [\pi/2\ \text{-}1]^T$. Additionally, as portrayed in Fig. 6, the nonperiodic DoS attacks (4) intervals and the FDI attacks are randomly initiated, whereby $G_{l,0}$= {[0.48, 0.76), [1.42, 1.62), [1.82,1.94), [2.52,2.62), [3.42, 3.72), [5.32,5.44),[6.04,6.22),[7.12,7.24),[7.82,7.91)}.

Furthermore, we present the simulation results in Figs. 3-12. Fig. 3 and Fig. 4 depict the state response of S-MJSs with free of control and SMC, respectively. In Fig. 5, the semi-Markovian switching states are illustrated. Additionally, Figs. 7-9 demonstrate the curve of the auxiliary variable function in dynamic METM, the control input, and the sliding surface function $s(t)$, respectively. Figs. 3-9 reveal that the performance of the system is significantly influenced by both DoS attacks and FDI attacks. It can be obtained the control strategy presented in this paper demonstrates remarkable robustness, which enables it to effectively address the impact of cyber attacks and ensure the stability of the system. Moreover, the trajectories of system states and control input $u(t)$ illustrate the effectiveness of the control strategy. Figs. 10-12 display the data-releasing instants and intervals for DETM, METM and DMETM, respectively. Simple statistics shows that the triggering times of DMETM are inferior to METM and DETM, which is consistent with the original idea of design, and also shows the effectiveness of the design method.

V. Conclusion

The paper has explored the event-based secure sliding-mode control for S-MJSs subject to multiple cyber attacks and uncertain TRs. To enhance transmission efficiency and minimize network bandwidth consumption, a new DMETM is implemented. Additionally, the influence of DMETM and multiple cyber attacks are discussed. Next, a memory-based sliding mode surface is designed to handle restrictive conditions, and a new switched fuzzy dynamical system is modeled. Furthermore, sufficient conditions are derived to ensure the attainment of mean-square exponential stability for S-MJSs with an H_∞ performance index in the presence of generally uncertain and unknown TRs. In addition, a based-memory SMC law is developed to achieve finite-time reachability of the designed sliding surface. Finally, simulation results are presented to validate the theoretical findings by employing a tunnel diode circuit model. The proposed SMC strategy can be used in other practical systems such as robotic manipulators [6]. Moreover, the observer-based SMC for S-MJSs with a deterministic switching signal subject to hybrid cyber-attacks is significant in future works.

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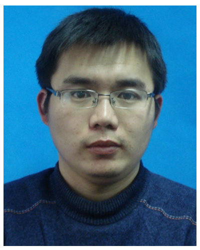

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